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Knowledge of Individual Histories and Optimal Payment Arrangements

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Abstract

This article reviews recent work that generalizes a random matching model of money to permit there to be a mix of transactions: some accomplished through the use of tangible media of exchange and the rest through some form of credit. The generalizations are accomplished by specifying assumptions about common knowledge of individual histories that are intermediate between no common knowledge and complete common knowledge. One of the specifications permits a simple representation of the sense in which more common knowledge is beneficial. The other permits a comparison between using outside money and using inside money as a medium of exchange.

Not so many years ago, when I used a credit card at some retail outlets, the clerk would consult a printed document that contained the numbers of cards which were not to be accepted. From the well-worn look of the document, it seemed that an updated version was issued, perhaps, once a week. Now, of course, credit cards are checked almost instantaneously as cards are swiped through an electronic reader. A plausible surmise is that a consequence of the change is that credit cards are used more frequently—primarily because more rapid checking of credit histories makes it profitable to offer credit cards to more people. Therefore, the economy comes closer to being cashless. What is perhaps less evident is whether the enhanced ability to maintain, access, and update records is a crucial determinant of how transactions are made. This article reviews recent theoretical work that suggests that such ability is crucial. The work applies *mechanism design*[1] to model environments to illustrate how optimal payment arrangements and the implied level of welfare depend on the ability to maintain, access, and quickly update records of individual histories.

The first section of the article defends my focus on knowledge of individual histories and my use of mechanism design. In particular, I argue, largely on the basis of other people's work, that assumptions about the extent to which knowledge of individual histories is public knowledge ought to play a crucial role in good theories of money. Then I turn to recent applications of that conclusion which take place against the background of random matching models of money. First I describe the consequences for welfare and the use of outside money of a lag in updating the public record of individual histories. (See Kocherlakota and Wallace 1998.) Then I describe the consequences for the roles of inside and outside money of having some people with known histories and others with unknown histories. (See Cavalcanti and Wallace 1999a, b.) (*Inside money* is a form of private credit; it is someone's liability. It could also be called private money. *Outside money,* in contrast, is a net asset for the economy as a whole.)

A Theory of Money

I start from what ought to be common ground: the classical dichotomy and the quantity theory of money. To be precise, I start from a model in two parts: a general competitive equilibrium model (of allocations and relative prices) and a single quantity theory equation, which can be interpreted as a supply-equals-demand-for-money equation. This model is recursive in that the variables in the general equilibrium part are determined without reference to the quantity theory part, which, given those variables, determines the price level. This model has two serious defects: it is incoherent and it fails to address the benefits of monetary exchange.

To see the incoherence, notice that the general equilibrium part of the model is a complete description of a nonmonetary economy with a specification of people, preferences, endowments, resources, and technologies. When I add the single quantity theory equation, I am led to ask who owns the money and why it has value, among other questions. Patinkin (1965) pointed out a symptom of this incoherence: the model fails to satisfy Walras' law.

As regards the second defect, a long tradition asserts that monetary exchange is helpful in overcoming difficulties of exchange, difficulties that economists now call *absence-of-double-coincidence difficulties*. Because the general equilibrium part of the model has complete competitive markets—and, therefore, does not depict absence-of-double-coincidence difficulties—the model cannot display any sense in which monetary exchange is helpful.

The history of monetary theory in the last half century is a history of attempts to overcome these two defects. The incoherence has been relatively easy to overcome. In particular, economists have become adept at formulating coherent intertemporal models in which money is given a role through one of the following devices: real balances as an argument of utility or production functions, cash-in-advance constraints, or transaction costs. These approaches, however, are widely viewed as shortcuts and not as serious attempts to overcome the second defect. Moreover, as I will explain, these approaches are almost certainly not valid shortcuts or summaries of models which do depict the sense in which monetary exchange is helpful.

To proceed to a more detailed discussion of the second defect, I will define *monetary exchange* to be the use of a tangible but intrinsically useless object (for example, shells, stones, or pieces of paper) as a medium of exchange. Then a minimal condition for overcoming the second defect is to have a setting or environment in which the use of such an object, from now on called *money,* is essential in the sense that its presence makes possible outcomes that could not be achieved in its absence.

To make such a claim precise, I need an institution-free way to describe what outcomes can be achieved with and without the use of money in a given environment. That, in turn, calls for the application of mechanism design to the environment. The application of mechanism design leads immediately to a very general result that justifies a focus on knowledge of individual histories. The result is that imperfect knowledge of individual histories is necessary for the essentiality of money. This necessity claim goes back at least to a 1973 paper by Ostroy (1973). (See also Townsend 1989 and Kocherlakota 1998.)

A simple way to prove the necessity claim is to proceed by contradiction. Take any model with complete public knowledge of individual histories and with money and show that the outcome of any game that makes use of money can be duplicated by a game which does not make use of money. By *making use of money,* I mean that holdings of it at some point in time influence what is produced and consumed. By *not making use of money,* I mean that holdings of it are ignored. That is, start with a game that makes use of money. Then create another game that is identical except that the role of money in the first game is replaced by an intangible state variable that exactly mimics the money in terms of individual endowments and individual transitions from one amount of money to another. In addition, let money be ignored in the new game. (Mathematically, let the intangible state variable be in the same set as money. Thus, if money holdings are a nonnegative real number, then let the intangible state variable be a nonnegative real number.) It follows that the two games have the same set of equilibria.

To elaborate a bit, notice that the assumption that individual histories are known implies that whatever is known about holdings of money can also be known about the intangible state variable. Also, because the money is an intrinsically useless object, the intangible state variable can fully substitute for it under whatever constraints applied to the initial allocation—participation constraints or truth-telling constraints. That would not be true if money were a commodity like oil, which has nonmonetary uses. And, again, because the money is an intrinsically useless object, it can be completely ignored—something which would not be true if money were an ordinary commodity. Finally, the assumption that individual histories are known is crucial. If not, the intangible state variable can be misrepresented to an extent that money holdings cannot, because money holdings are tangible.

The necessity result can be used as a basis for a criticism of the shortcut models mentioned earlier and of some other monetary models. In particular, imperfect knowledge of individual histories and its consequences—for example, for credit—do not appear in those models. Therefore, those models are not consistent with the essentiality of money. Put differently, imperfect knowledge of individual histories should appear in those models if they represent valid summaries (sometimes called reduced forms) of models in which money is essential.

For what follows, I want to make a different appeal to the necessity result. Perfect knowledge of individual histories implies no role for money. No knowledge of individual histories, while giving the greatest scope for a role for money, leaves no role for credit in any form. An obvious route to getting a mix of transactions, which is what exists in actual economies, is to specify some degree of imperfect knowledge of individual histories.

The Background Environment

I use a single background environment throughout, one that is adapted from the work of Shi (1995) and Trejos (1995), which, in turn, are adaptations of earlier work of Kiyotaki and Wright (1989). As originally formulated, these were simple settings which made explicit longstanding ideas about the connection between absence-of-double-coincidence difficulties and the need for tangible media of exchange.

In my setting, time is discrete. There are $N > 2$ perishable types of goods in each period of time, N specialization types of people, and a [0,1] continuum of each type. Although I will limit attention to symmetric outcomes, I assume that each person is identified by a (specialization) type—an integer in the set $\{1, 2, ..., N\}$ and a real number in the interval [0,1]—an identification which I will assume throughout to be common knowledge. A type n person consumes only good n and produces only good $n + 1$, modulo N. Each person maximizes expected discounted utility with a discount parameter $\beta \in (0,1)$. The period utility function is $u(x) - y$, where $x \in R_+$ is the amount of the relevant good consumed and $y \in R_+$ the amount of the relevant good produced. The function u is differentiable, is strictly increasing, is strictly concave, and is such that $u(0) = 0$, $u'(0) = \infty$, and there exists $y' > 0$ satisfying $u(y') = y'$. In each period, each person meets one other person at random. That is, the probability of meeting persons of a particular type is equal to that type's weight in the population. For example, the probability that a given person meets someone who produces what the given person consumes is $1/N$.

Why assume random meetings in pairs? Implicit in all descriptions of absence-of-double-coincidence difficulties is that not everyone is together. A general model would posit costs of people getting together. The model studied here is an extreme version in which meeting one person in a period is free and meeting any other person in that period is infinitely costly. When put together with the assumed specialization in consumption and production, such meetings in pairs give rise to a complete absence of double-coincidence meetings. The one free meeting could conceivably be made exogenous or endogenous. (See Corbae, Temzelides, and Wright 2000 for a model in which it is endogenous.) Here it is made exogenous. When these models were first formulated, the randomness was adopted because it is the simplest form that such exogeneity can take. For the analysis that follows, the randomness and implied uncertainty are crucial. The randomness amounts to assuming that a person may or may not encounter a consumption opportunity and may or may not encounter an earnings opportunity. This is a complete-economy version of the kind of uncertainty regarding expenditures and receipts that has long been a part of well-known partial equilibrium models of money demand. (See, for example, Miller and Orr 1966 and Goldman 1974.) More generally, some such uncertainty has almost always been assumed in inventory theory. Therefore, it should not be regarded as a strange ingredient in a model of trade.

The absence of double-coincidence meetings and the perishable nature of the produced goods imply that no trade takes place without some tangible asset or some form of credit. When these models were initially formulated, the goal was to ensure the essentiality of tangible assets. This was done by ruling out credit of any kind via the assumption that people are anonymous. A different way to rule out credit, which is convenient for what I want to discuss, is to assume that people have known identities as described above, but that they cannot commit to future actions and that each person's history is private information. Throughout, I will maintain the no-commitment assumption. However, I will adopt less extreme variants of the private-history assumption.

The potentially helpful role of such knowledge is related to the uncertainty implied by the random meetings. The criterion for the mechanism design problem I study is an ex ante represengive-agent criterion, ex ante in being before the assignment of types and initial money holdings. If there were no incentive constraints, then the best outcome according to that criterion would be consumption and production equal to y^* in every single-coincidence meeting, where y^* is the solution to the maximization of $z(y) \equiv u(y) - y$ by the choice of y. This outcome is unattainable under a fixed stock of outside money if no commitment and privacy of histories are assumed and if money is valuable in the sense that expected discounted utility is weakly increasing in money holdings.

The proof is by contradiction. If that outcome were attained, then expected discounted utility at the start of each period, before meetings, would be constant and, in particular, would not depend on money holdings. But then people would be unwilling to produce to acquire money. Nor would they be willing to produce without acquiring money: with no subsequent knowledge on the part of anyone else about whether people produced or not, there could not be a penalty for failing to produce. More generally, sizable output would not be produced in a meeting between a producer who has experienced a long run of being a producer and a consumer who has experienced a long run of being a consumer. The producer, as a result of previous trades, would have a lot of money and, therefore, would require a lot of money in order to produce much now. But the consumer would have little money because of previous expenditures. The existence of such meetings suggests that there is a beneficial role for other devices which can help free people from dependence on their recent trades. As I will show, some knowledge of individual histories makes that possible.

Although it is not crucial for many of the results to follow, I will use the simplifying assumption throughout that money is indivisible and that each person can hold at most one unit of it. I also assume that money is perfectly durable.

An Updating Lag

As part of a research initiative on payments sponsored by the Research Department of the Federal Reserve Bank of Minneapolis, Narayana Kocherlakota and I took up the question of how to represent the role of technological advances in payment arrangements. We began with the necessity result and the background environment just described. Therefore, we knew about the best mechanism in two extreme cases regarding knowledge of individual histories: with no public knowledge, all trade has to involve money, while with complete public knowledge, money is superfluous (the necessity result). We decided to formulate intermediate situations. Our first thought was to follow some of the literature on bounded rationality and assume that a limited chunk of most recent history is public knowledge. At least as we conceived of this approach, it would not work in the setting just described. Even knowing what people did last period seems to be equivalent to knowing everything in that setting. In particular, it would seem sufficient to know whether potential producers in single-coincidence meetings in the last period produced the "right" amount. Therefore, we looked for an alternative way of specifying intermediate situations.

The alternative we pursued is a lag in updating the public record of individual histories. (See Kocherlakota and Wallace 1998.) Suppose that in each period t, there is a complete record of individual histories, but only up to $t - K$ for some positive integer K. Now consider the possibility that each producer in a single-coincidence meeting produces a positive amount y, as a sort of gift, and that anyone who is discovered to have not produced y never receives production from anyone else. If a producer considers defecting in some period by not producing, then the producer looks forward to K periods during which he or she will be an undiscovered defector. During that time the defector will not produce, but will consume. Obviously, then, the sacrifice in terms of a future payoff from defecting in some period is decreasing in K and approaches zero as $K \to \infty$. In that sense, such a lag in updating histories works. It also seems attractive in terms of our original goal, which was to relate technological advances to the way transactions are made. Such advances have made it possible to quickly update the public record of individual histories.

The formulation we adopted was not a deterministic lag, but a probabilistic lag. We assumed that each period there is a probability, denoted ρ, that histories are updated fully. This specification produces an average lag in updating, which is 1/ρ periods. Thus, a defector looks forward to, on average, 1/ρ periods during which he or she is undiscovered. This is a bit simpler than a deterministic lag.

The only tangible asset in the model is a fixed stock of outside money denoted by m, where $m \in [0,1]$ is the amount per specialization type. (If holdings are symmetric across specialization types, then m is the fraction who have a unit of money and $1 - m$ is the fraction who do not.)

The timing is as follows. At the start of a period, a drawing determines whether the public record of individual histories is updated. Then meetings occur. Then the next period starts. Consideration is limited to a simple class of deterministic allocations that are symmetric over specialization types and are stationary. Given the symmetry, only single-coincidence meetings are relevant. In each such meeting, there is a (potential) producer and a (potential) consumer. Let $y_{ij} \in R_+$ denote production when the producer has i units of money and the consumer has j units, and let $a_{ij} \in \{0,1\}$ denote whether there is an exchange of money holdings, where $a_{ij} = 0$ means no exchange and $a_{ij} = 1$ means an exchange. Let (y,a) denote the collection of pairs (y_{ij},a_{ij}) for $i, j \in \{0,1\} \times \{0,1\}$.

Here I will say that (y,a) is (weakly) implementable if there exists some game which has a subgame perfect Nash equilibrium with an outcome of (y,a). To formulate the claim about the set of (y,a) that is implementable, introducing notation for expected discounted utilities is helpful. Thus, let v_i and v_i' be the expected discounted utilities of a nondefector and an undiscovered defector, respectively, with i units of money at the start of a period just before the new drawing that determines whether histories are updated. Also, for a single-coincidence meeting in which the producer has i units of money and the consumer has j units, let P_{ij} and C_{ij} be producer and consumer payoffs, respectively, from following (y,a) when everyone else follows (y,a). Then

$$(1) \qquad P_{ij} \equiv -y_{ij} + \beta[a_{ij}v_j + (1-a_{ij})v_i]$$

and

$$(2) \qquad C_{ij} \equiv u(y_{ij}) + \beta[a_{ij}v_i + (1-a_{ij})v_j].$$

Then I can express v_i as

$$(3) \qquad v_i = \sum_{j=0}^{1}(m_j/N)(P_{ij}+C_{ji}) + \Big[1 - \sum_{j=0}^{1}(m_j/N)\Big]\beta v_i$$

where $m_0 \equiv 1 - m$ and $m_1 \equiv m$. [For a given (y,a), equation (3) is a pair of linear simultaneous equations in v_0 and v_1 which have a unique solution in terms of (y,a).]

I write the expression for v' under the assumptions that defection once discovered gives a payoff of zero, that everyone else follows (y,a), and that the options in any meeting are to behave according to (y,a) or to have no trade in that meeting. Then

$$(4) \qquad v'_i/(1-\rho) = \sum_{j=0}^{1}(m_j/N)[\max(P'_{ij},0) + \max(C'_{ji},0)] + \left[1 - \sum_{j=0}^{1}(m_j/N)\right]\beta v'_i$$

where P'_{ij} and C'_{ji} are given by equations (1) and (2), respectively, except that v'_k appears in place of v_k, and where the maximization functions appear because it is costless for a defector to defect again. [That is, v'_i equals the product of ρ and zero plus the product of (1−ρ) and the right side of equation (4).] For a given (y,a), equation (4) is a pair of simultaneous equations in v'_0 and v'_1. Although equation (4) is nonlinear because of the maximization terms, it, too, has a unique solution in terms of (y,a).

The claim about implementability is that (y,a) is (weakly) implementable if and only if there exist v and v' such that (3) and (4) hold, $v_1 \geq v_0$, $v'_1 \geq v'_0$ (the free disposal conditions),

$$(5) \qquad P_{ij} \geq \beta v'_i$$

and

$$(6) \qquad C_{ij} \geq \beta v'_j.$$

The complete proof is given in Kocherlakota and Wallace 1998. Here I want to outline the sufficiency part, which shows that if (y,a) satisfies inequalities (5) and (6), then (y,a) is implementable. I can associate with any (y,a) the following game. In each meeting, the two people move simultaneously and choose from the set {*yes*, *no*}. If either plays *no*, then the meeting is autarkic: each leaves the meeting with what was brought into the meeting, and the person who plays *no* becomes an undiscovered defector. If both play *yes*, then the action called for by (y,a) is carried out. The following are proposed equilibrium strategies:

- After a defection has become public knowledge, producers play *no*.
- If a defection has not become public knowledge or has not been witnessed, then everyone plays *yes*.
- If a defection has not become public knowledge but has been witnessed, then the strategy corresponding to the maximization terms in (4) is used.[2]

Given the constraints, it follows that this is a subgame perfect Nash equilibrium and that the associated outcome is (y,a) because no one defects.

Kocherlakota and Wallace 1998 shows that for sufficiently small values of ρ, those close enough to zero, almost all trade involves the use of outside money and that for values of ρ close enough to one, outside money is not needed. Aside from that description, the main result obtained there is that ex ante welfare, measured before initial assignments of money, is increasing in ρ. This follows because v' is decreasing in ρ. Although hardly a surprise, I know of no other model that displays a sense in which technological advances improve welfare through their effect on the way transactions are made. Notice also that the monotonicity of welfare seems not to depend on the assumption that money is indivisible and that there is a unit upper bound on individual holdings.

Known and Unknown Histories

The Kocherlakota-Wallace specification implies a mix of transactions made using outside money and transactions made using gifts. I now turn to work in which transactions are made using inside money and gifts.

In Cavalcanti and Wallace 1999a, b, we pursued an idea first broached in a conversation with the late Rao Aiyagari. The idea is to have some people whose histories are known and others whose histories are unknown and to have the former be issuers of inside money and the latter be users of inside money. Behind this idea is the notion that issuers of inside money are making promises of some sort—perhaps to redeem inside money—and that people with known histories can be made to keep promises.

To pursue that idea, we made two amendments to the background environment described earlier. We assumed that a given fraction of each specialization type, denoted B, have known individual histories and that the rest, the fraction $1 - B$, have unknown individual histories, where B is a parameter. The parameter B can be interpreted as the society's capacity for keeping track of individual histories. We also assumed that each person is equipped with a printing press that can turn out indivisible and perfectly durable objects called *notes*. Each press turns out uniform notes, but the notes turned out by any two presses are distinguishable. The last proviso is a way to rule out counterfeiting. In Cavalcanti and Wallace 1999a, b, we called those with known histories *bankers* and everyone else *nonbankers*. While I will stick with those labels here, notice that the only distinction between bankers and nonbankers is what is commonly known about their histories. This specification is, of course, another way to describe situations that are intermediate between complete privacy of individual histories, $B = 0$, and complete public knowledge of individual histories, $B = 1$.

If $B = 0$ (everyone is a nonbanker), then, not surprisingly, the existence of the printing presses does not matter. To see this, suppose that $B = 0$ and that there is a fraction of nonbankers whose notes are treated uniformly and accepted by other nonbankers. Then the note issuers never produce—in particular, they do not produce to acquire notes because they can always issue new notes. And, of course, the nonissuers never destroy notes. Therefore, the stock of notes is growing without bound. That, in turn, precludes the existence of an equilibrium in which such notes are valuable. When $B > 0$, the same argument does not apply. Those with known histories can be induced by the threat of punishment to produce in exchange for a note and to destroy the note. Despite that possibility, I will ignore note issue by nonbankers because I am looking for optima, and I suspect that note issue by nonbankers would not be optimal because the nonbanker issuers would never produce. From now on, then, *notes* refers to notes issued by bankers.

In Cavalcanti and Wallace 1999a, we studied mechanisms which are simple in that and other respects. We looked at mechanisms which are symmetric across specialization types and in which all notes, all those issued by bankers, are treated symmetrically in equilibrium. We also imposed stationarity, which includes the requirement that the stock of notes held by nonbankers be constant and that actions of bankers depend on only one feature of their histories: whether or not they have defected. We also assumed that note holdings are observed. The crucial feature that permits existence of steady states with valuable notes is the possibility of bankers being punished if they defect. That threat induces bankers to produce to acquire a note even though such production is a gift because a note is useless to them.

In Cavalcanti and Wallace 1999a, we ignored outside money and attempted to say things about the solution to the following optimum problem. Subject to participation constraints and to the steady-state conditions, choose what happens in meetings to maximize nonbankers' expected discounted utility subject to the choice leaving bankers no worse off than nonbankers. We chose that objective because history has no shortage of people and groups proposing to governments that they be allowed to issue objects that resemble the notes in our model.[3] They generally say that their scheme is intended to improve the welfare of others. Our objective took that professed goal seriously. At the same time, we do not expect the issuers to end up worse off if their scheme is accepted.

Despite all the simplifying assumptions, this was not a simple optimum problem. In addition to variables that describe when notes get transferred and issued and destroyed, the choice variables include five distinct output amounts: the amount produced in exchange for a note in a single-coincidence meeting between nonbankers, the amount produced by a nonbanker in exchange for a note from a banker, the amount produced in single-coincidence meetings between bankers, and two amounts produced by bankers in meetings with nonbanker consumers: one amount when the nonbanker has a note and one when the nonbanker does not have a note. Thus, for example, we did not impose that the amount a banker gets from a nonbanker when a note is issued is the same as that produced by a banker when redeeming a note or that either is the same as the amount that a note trades for among nonbankers. In particular, then, notes can be redeemed for more than they trade for among nonbankers which, in turn, exceeds what is given up to acquire a note from a banker. Such a mechanism has notes bearing interest in an expected value sense.

Unfortunately, we were able to say very little about the solution to the optimum problem we posed. We showed that an optimum has notes being issued, being used by nonbankers, and being redeemed. But in other respects we could say very little. We were not able to demonstrate that the objective is increasing in B, that the constraint that bankers be no worse off than nonbankers is binding, or that an optimum has notes bearing interest in the sense described above. Subsequently, we realized that our model could be used to compare inside and outside money as alternative ways of supporting exchange. That comparison appears in Cavalcanti and Wallace 1999b, where we produced the following strong result: the set of implementable outcomes using outside money is a strict subset of outcomes using inside money. This is the result I will discuss in detail.

What exactly am I comparing? The outside-money world has a constant stock of outside money and no note issue. Having no note issue is implementable because if bankers are not threatened with future punishment for not producing in exchange for a note, then they are willing not to produce to get one. Given this behavior of bankers, then, notes, being intrinsically useless, can be ignored for the usual reason: if each person thinks that others in the future will not produce to acquire notes, then no one currently produces to acquire them. I have also imposed considerable symmetry and stationarity. Thus, I limit what happens in a single-coincidence meeting to depend at most on the identity (banker or nonbanker) and state (having zero or one unit of outside money) of the producer and the consumer. It follows that there are 16 potential output levels. Although some are obviously constrained to be zero by participation constraints—in particular, nonbankers never give gifts—it is important to notice that the outside-money arrangement allows for gifts from bankers to each other and to nonbankers.

The inside-money world either has no outside money or outside money exists and is ignored. Given that outside money is an intrinsically useless object, ignoring it is implementable. Then to facilitate a comparison with the outside-money world, the same kind of symmetry and stationarity is imposed. However, one distinction is important. As in the outside-money world, in the inside-money world, the state for a nonbanker is note holdings. However, the state for a banker is not note holdings, because bankers can always issue notes.

I assume that each banker is in one of two states, labeled 0 or 1. I need at least two states for bankers if I am to accomplish the subset claim. Although these states do not correspond to something tangible that bankers hold, bankers can be made to carry around these states because their histories are known.[4] Thus, I can propose something like the following: half the bankers of each specialization type start in state 0, and half start in state 1. In each period bankers switch to the other state, and only those in state 1 issue a note in a meeting with a nonbanker without a note. As part of stationarity, I only consider steady states in which the fractions who are nonbankers in each state and who are bankers in each state are constant.

A unified notation can describe outcomes under either the inside-money or the outside-money arrangement. Although I will not present all the details, some are needed. Most of the notation is, again, for single-coincidence meetings. I need three variables for such meetings: one to describe production, one to describe the state transition for producers, and one to describe the state transition for consumers. Thus, I let production (and consumption) in a single-coincidence meeting be denoted by y_{ij}^{kl}. The superscripts denote identity: k, $l \in \{b$ (banker), n (nonbanker)$\}$, with k denoting the identity of the producer and l the identity of the consumer. The subscripts denote states, with i denoting the state of the producer and j

the state of the consumer. I let $p_{ij}^{kl} \in \{0,1\}$ denote the state transition of the producer and let $q_{ij}^{kl} \in \{0,1\}$ denote the state transition of the consumer, where the superscripts and subscripts have the same meanings as they do for production. Here 0 in the range means keep the current state and 1 means switch to the other state. Notice that were I describing only the use of outside money, I could get by with a single money transfer variable that describes whether or not the trading partners exchange money holdings as in the Kocherlakota-Wallace model. Here, because bankers can issue notes in the inside-money world, a nonbanker can be given a note no matter the state of the banker. Hence, I need separate state-transition variables. I also need some notation to describe the possibility that a banker gives a gift of money, either outside money or a note, in a no-coincidence meeting with a nonbanker. (Although nonbankers never give gifts, a general notation for such gifts is helpful.) I let $r_{ij}^{kl} \in \{0,1\}$ denote whether a person with identity k in state i switches states in a no-coincidence meeting with a person with identity l in state j. (Again, 0 in the range means keep the current state and 1 means switch states.) Finally, I need notation for the distribution of bankers and nonbankers across states. I let x_i^k with $k \in \{b,n\}$ and $i \in \{0,1\}$ denote the fraction of each production-consumption specialization type who have identity k (b for banker, n for nonbanker) and who are in state i. Because each person must be in one of the states, these fractions satisfy

$$(7) \qquad \textstyle\sum_i x_i^b = B$$

and

$$(8) \qquad \textstyle\sum_i x_i^n = 1 - B.$$

Denote an allocation by (y,p,q,r,x), where each symbol is the relevant collection of production, state transition, and distribution variables. The steady-state conditions are easily expressed in terms of (y,p,q,r,x); I will not repeat them here. In addition, restrictions are implied by the preservation of outside-money holdings in all meetings and by the preservation of note holdings in meetings between nonbankers. Listed without qualifications, they are

$$(9) \qquad p_{ii}^{kl} = q_{ii}^{kl} = r_{ii}^{kl} = 0$$

$$(10) \qquad p_{ij}^{kl} = q_{ij}^{kl}$$

and

$$(11) \qquad r_{ij}^{kl} = r_{ji}^{lk}.$$

Equation (9) says that if both people in a meeting have the same state, then neither can switch to a different state. Equations (10) and (11) say that one person in a meeting switches to a different state if and only if the other does. The crucial way in which the inside- and outside-money worlds differ is that the state transitions are more constrained in the outside-money world. When outside money is used, (9), (10), and (11) must hold in all meetings. When inside money is used, they must hold only in meetings between nonbankers, when $k = l = n$.

It is again convenient to express participation constraints in terms of expected discounted utilities. Let v_i^k denote the expected discounted utility for a person with identity k who starts a period in state i. The stationarity implies that v_i^k can be expressed implicitly in terms of an allocation by

$$(12) \qquad N(1-\beta)v_i^k = \textstyle\sum_{l,j} x_j^l \{u(y_{ji}^{lk}) - y_{ij}^{kl} + \beta[q_{ji}^{lk} + p_{ij}^{kl} + (N-2)r_{ij}^{kl}](v_{i'}^k - v_i^k)\}$$

where $i' \neq i$. Notice that for a given allocation (y,p,q,r,x), equation (12) consists of two pairs of equations, each pair being two simultaneous linear equations in v_0^k and v_1^k. Those equations have a unique solution.

Aside from the free disposal conditions, I consider three constraints. The first concerns production by bankers,

$$(13) \qquad -y_{ij}^{bl} + \beta[p_{ij}^{bl} v_{i'}^b + (1-p_{ij}^{bl})v_i^b] \geq 0.$$

Here $i' \neq i$. The others require that nonbankers have nonnegative gains from trade when they consume and when they produce. They are, respectively,

$$(14) \qquad u(y_{ji}^{ln}) + \beta[q_{ji}^{ln} v_{i'}^n + (1-q_{ji}^{ln})v_i^n] \geq \beta v_i^n$$

and

$$(15) \qquad -y_{ij}^{nl} + \beta[p_{ij}^{nl} v_{i'}^n + (1-p_{ij}^{nl})v_i^n] \geq \beta v_i^n$$

where, again, $i' \neq i$.

The *implementability claim* is that a steady-state allocation (y,p,q,r,x), either an inside-money one or an outside-money one, is implementable if and only if there exist v_i^k such that constraints (12)–(15) and the free disposal conditions hold. The same kind of game as described for the Kocherlakota-Wallace model can be used here to show that any such allocation (y,p,q,r,x) is weakly implementable. (See Cavalcanti and Wallace 1999b for a proof.)

In inequalities (13)–(15), the left sides represent payoffs from not defecting and the right sides represent payoffs from defecting. For bankers, the payoff from defecting is permanent autarky; for nonbankers, the payoff is autarky in the meeting only. One consequence of (15) is that a necessary condition for positive nonbanker production is that the nonbanker switches states. In other words, nonbankers do not engage in gift-giving. That is not true of bankers. Inequality (13) does not imply that a banker must switch states in order to produce. Notice also that the only banker activity that is constrained is banker production. That is because banker defection leads to autarky in the meeting and then to permanent autarky. (See Cavalcanti and Wallace 1999b.) It follows that a banker may be tempted to defect only when the banker is asked to experience current disutility, namely, when asked to produce. In particular, a banker is never tempted to defect by issuing a note when the allocation says that a note should not be issued, as is always the case in what I am calling *outside-money allocations* and as may be the case for *inside-money allocations*. Finally, notice that inside- and outside-money allocations are not distinguished by different participation constraints. Therefore, as asserted above, the two kinds of allocations are distinguished by the different restrictions on state transitions that come from applying (9), (10), and (11) to all meetings in the case of outside money and only to meetings between nonbankers in the case of inside money.

With a more constrained law of motion for individual states in the case of outside money than in the case of inside money and with identical participation constraints, the strict set inclusion result is an obvious consequence. The subset result is immediate. The strictness is achieved by giving an example of an outcome that is implementable in the case of inside money, but not in the case of outside money. That is easy to do. The example I give involves having a banker consumer issue a note in any meeting with a nonbanker producer who does not have a note. That cannot happen in the case of outside money because the steady-state conditions would be violated. All bankers would have to have a unit of outside money and sometimes surrender it. If bankers always have outside money, then they can never acquire a unit. By the steady-state conditions, this implies that bankers never surrender a unit, a contradiction.

In Cavalcanti and Wallace 1999b, we made substantial use of the unit upper bound on money holdings only when giving an example of an inside-money outcome which cannot be achieved using outside money. Therefore, I suspect that the strict set inclusion applies quite generally. In particular, it should survive both more general money holdings and richer dependence on banker histories. In the case of outside money, a banker's ability to acquire production from a nonbanker is tied to the banker's holdings of outside money, which depend on the banker's previous trades. Using inside money, the banker can always issue a note and, therefore, is not constrained by recent trades. Nor does it seem essential that the inside-money world have no outside money. I suspect that I can reinterpret any inside-money steady state as one in which the stock of notes held by nonbankers is a mix of outside and inside money.

As regards interpretation, to some extent the inside-money world looks like a world of private bankers issuing notes under an arrangement in which they agree to redeem each other's notes. A modern analog of our notes is stored value cards. However, because our bankers are identical to everyone else and because they receive goods when issuing notes and give goods when redeeming notes, it is at least as apt to view the inside-money world as a world of trade credit in which the issuers of trade credit redeem each other's liabilities and in which these liabilities trade hands among nonissuers.

Whatever I call the notes and their issuers, the inside-money world just described is a unified system in the sense that all notes are treated symmetrically. The same model can be used to describe systems that are not unified. Here is one such system. Suppose that the set of bankers is itself divided into equal-measure subsets. Then the following is certainly implementable in the inside-money world. A banker producer is not punished for treating a nonbanker with a "wrong" note, a note issued by a banker not in its subset, exactly as the banker treats a nonbanker without a note. This is consistent with nonbankers treating all notes symmetrically because the probability of a note being right or wrong does not depend on who issued it. The lack of punishment for not redeeming wrong notes implies that banker production for nonbankers with wrong notes is the same as production for nonbankers without notes. Since this restriction is not present in the case of outside money, I conjecture that the subset result fails when such a nonunified system is imposed. In other words, if the mechanism design problem is constrained to have a nonunified inside-money system, then there is a role for both inside and outside money in the sense that the sets of outcomes using exclusively one or the other are not subsets of each other. Another conceivable way to get roles for both outside and inside money is to make the knowledge of individual histories of bankers imperfect.[5]

Concluding Remarks

Has anything worthwhile come out of this modeling endeavor? Are there new insights? I have emphasized two results. In the Kocherlakota-Wallace (1998) representative-agent setting, shortening the lag with which individual histories are made public knowledge enhances welfare. In the Cavalcanti-Wallace (1999a, b) setting in which some people have known histories and others unknown histories, inside money can achieve strictly more allocations than outside money. Although the first result is not surprising, it does require a background setting in which transactions are difficult. The comparison of inside and outside money seems to be new. Moreover, once elaborated, that result seems quite plausible: the use of outside money is more restrictive because it ties trading opportunities more closely to past transactions than does the use of inside money.

Finally, it is worth noting that the mechanism design approach bypasses the usual industrial organization categories of competition, oligopoly, and monopoly. While people in the second setting, including those with known histories, are selfish and individually small, they are not price-taking competitors in the usual sense. They behave in ways that would not fit into a standard competitive analysis. Given the obvious role for joint behavior in the financial system—for example, as is required for the operation of clearinghouses—it seems desirable to use frameworks which allow for such arrangements.

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[1]For an exposition of mechanism design, see Kreps 1990, chap. 18.

[2]According to this game, a zero payoff for discovered defectors is achieved through global autarky, and v' is the discounted utility of an undiscovered defector and of someone who has witnessed an undiscovered defection. There are also games that punish only the defector. In such a game, the crucial meeting is between a discovered defector with money who is a potential consumer and a nondefector producer without money. While autarky in such a meeting can be Nash, it has the defector playing a weakly dominated strategy (not offering his or her money) and is not robust to cooperative defection by the pair in the meeting. Suppose, instead, that the discovered defector gives up money for ε amount of the good, where ε is small. Then no matter how small is ε, playing *yes* is not a weakly dominated strategy for the defector, and there is no cooperative defection by the pair from that trade. Because this is also true if $\varepsilon = 0$, a zero payoff for a discovered defector can be achieved in this way, with punishment only of the defector.

[3]A famous instance is John Law's banking proposal, which was rejected by the English, but, at least for a time, accepted by the French in the early 18th century.

[4]These states for bankers are an example of the kind of intangible state variable used in the earlier argument for the claim that imperfect knowledge of individual histories is necessary for the essentiality of money.

[5]This idea is pursued in Mills 2000.

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