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Narrow Banking Meets the Diamond-Dybvig Model

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Abstract

A version of the Diamond-Dybvig model of banking is used to evaluate the *narrow banking proposal,* the idea that banks should be required to back demand deposits entirely by safe short-term assets. It is shown that the mere existence of an amount of safe short-term assets outside the banking system that exceeds banking system liabilities does not make the proposal either innocuous or desirable. In fact, despite such existence, using narrow banking to cope with banking system illiquidity eliminates the role of the banking system.

The current version of the 100 percent–reserve banking proposal, called the *narrow banking proposal,* begins with an observation: The magnitude of safe short-term assets outside the banking system exceeds the magnitude of banks' demand deposit liabilities. Therefore, say the proponents of narrow banking, why not avoid the problems of an illiquid banking system portfolio—such as the threat of bank runs and the accompanying need for deposit insurance, regulation, and bailouts—by forcing a rearrangement of asset holdings in the economy? Why not require that demand deposits be backed entirely by safe short-term assets? This is the narrow banking proposal.[1] However, this proposal both begins and ends with the same observation. That is, there is no theory or model of banking from which the proposal emerges. In particular, no model is offered which is consistent with the pervasiveness of illiquid banking systems and which also implies that the narrow banking proposal is desirable. This is a serious omission, for two reasons: the supposed problem that the narrow banking proposal is intended to solve would not exist if the banking system were not illiquid, and an explanation in the form of a theory or model of illiquid banking systems is likely to suggest that benefits accompany such systems.

Models do exist, built on Diamond and Dybvig 1983, that are consistent with illiquid banking. These models offer a plausible explanation of the role played by an illiquid banking system; the explanation suggests that some benefits accompany banking system illiquidity. Although the original version of the model seems ill-suited to address the narrow banking proposal because, in that version, the banking system holds all the assets in the economy, simple extensions of the model may be made that are consistent with assets being held outside the banking system. It is presumably such extensions that Diamond and Dybvig have in mind in their critical discussions of the narrow banking proposal. (See Diamond and Dybvig 1986 and Dybvig 1993.) Diamond and Dybvig say that the proposal makes sense only if the safe short-term assets outside the banking system in the actual economy represent excess liquidity, a fact which they doubt. Therefore, Diamond and Dybvig suggest, implementation of the narrow banking proposal would have undesirable consequences.

My purpose here is simply to make Diamond and Dybvig's argument explicit. I set out a version of the Diamond-Dybvig model and point out what the model implies about the narrow banking proposal. My version of the model supports their position: there can be large amounts of safe short-term assets outside the banking system, but narrow banking is undesirable. It is undesirable relative to something that bears some resemblance to our current banking system and undesirable relative to something resembling, if anything, a banking system with a large amount of liabilities subordinate to its demand deposit liabilities.

Of course, for a variety of reasons, advocates of the narrow banking proposal may be skeptical of this version of the Diamond-Dybvig model and, therefore, skeptical of its implications for their proposal. As with any banking model, this one does not capture some features of actual banking systems. Such skepticism, though, is hardly a persuasive argument for the narrow banking proposal. Any proposal for a major change in policy should be supported by a coherent view of the phenomenon under consideration. In this case, the phenomenon is illiquid banking. Such a view should imply that the policy is desirable, and such a proposal should argue that its view of the phenomenon should be accepted. The advocates of narrow banking have not even begun this process of argumentation. Not only have advocates of narrow banking not made a case, but some of them seem unaware that a plausible model of illiquid banking systems does not lend support to their proposal.[2]

Preliminaries

Before examining the model, I will review a few basic concepts. The term *illiquid banking system* refers to a property of a consolidated balance sheet. The consolidation is over banks and may also include the central bank, the government, and even those in debt to banks. The *illiquidity* property of this balance sheet means that not all banking system obligations can be met if all holders of those obligations simultaneously claim what they have been promised. More generally, if the obligations are deposits that give the owners of the deposits the right to decide when to withdraw, then the banking system is illiquid if there is some possible pattern of withdrawals that cannot be accommodated. The most typical example of such a system is a fractional reserve banking system under a commodity standard such as the gold standard. Such a banking system has demand liabilities that exceed its reserves (in the form of the commodity standard).[3]

Banking system illiquidity seems to open the door to potential difficulties, and history is rife with instances of what are variously called *bank panics* or *bank runs,* which are generally viewed as realizations of the potential difficulties (Friedman and Schwartz 1963). Are these realizations inevitable? Can they be avoided, and at what cost? The narrow banking proposal says, Let's eliminate the potential difficulties by eliminating illiquid banks. That may be a good idea, or it may be silly—as silly as a proposal to reduce automobile accidents by limiting automobile speeds to zero. Here I will appraise the narrow banking proposal using a model.

Why use a model? The alternatives are to look at history or to try an experiment using the narrow banking proposal. As far as history is concerned, even if narrow banking had been in effect in the past, without a model we would not even know what to look for to judge narrow banking's success or failure. The same difficulty arises when we consider an experiment. Moreover, experimenting on the actual economy may be very costly, particularly if narrow banking is not, in fact, a good idea. Using a model amounts to experimenting on an analog of the actual economy. Such experimentation is much cheaper, mainly because it avoids the risks of experimenting on the actual economy.

A Version of the Diamond-Dybvig Model

We want a plausible model that explains illiquid banking and lets us judge how people will be affected by various rules imposed on the banking system. Such a model is set out here.[4] The model has three main ingredients, each of which, as will be explained below, is plausible:

- *Individual uncertainty* about desired time profiles of consumption, including the assumption (referred to as *private information uncertainty*) that realizations of this uncertainty are known to the person, but not to others.

- *Investment technologies* that offer a trade-off between those with good short-term returns and those with good long-term returns.
- *Isolation of people* from each other in a way that, among other things, forces the banking system to deal with depositors on a first-come, first-served basis.

The first ingredient, individual uncertainty, gives rise to a role for assets that can be cashed in at the request of the holder—something like actual demand deposit and savings deposit accounts. This ingredient is plausible in that such uncertainty has long been used to explain why people do not plan the pattern of their expenditures so as to avoid holding low-return assets such as checking accounts and traveler's checks. The second ingredient, investment technologies that offer a trade-off between short-term and long-term technologies, makes it easy to have both banking system illiquidity and safe short-term assets outside the banking system. This ingredient seems eminently plausible as a feature of technologies in actual economies. The third ingredient, isolation that forces first-come, first-served treatment of depositors, is also plausible—if only because something like it is necessary to account for the dominant role of the first-come, first-served principle in almost all retail trade.

When the above ingredients are combined in a way that produces a model of a complete economy, I am able to describe what is feasible in that economy as well as which feasible things are desirable. *Desirability* will be judged by the extent to which people's preferences are satisfied—preferences that take into account the uncertainty people face about desired time profiles of consumption. According to the model, a *best feasible outcome* has features that resemble these:

- Demand deposits.
- An illiquid banking system where the consolidation is best viewed as over banks and over those indebted to banks.
- Safe assets outside the banking system.

This is the sense in which the model explains an illiquid banking system that has safe assets outside the banking system.

The model is essentially the same as the one in Diamond-Dybvig 1983. There are three dates indexed by t, where t = 0, 1, 2, and there is one good per date. The economy is endowed with only date 0 good, an amount that is normalized, per person, to unity. There are two linear constant returns-to-scale technologies. There is a short-term technology with gross return R_1; output at $t + 1$ per unit input into this technology at t is R_1. There is a long-term technology with gross two-period return R_2; output at $t + 2$ per unit input into this technology at t is R_2. There is also a return for liquidation of investment in the long-term technology after one period; this gross return is denoted r_1. I assume $R_2 > (R_1)^2 > (r_1)^2 > 0$, so that, according to the technology, it is best to provide for date 2 consumption by investment in the long-term technology and to provide for date 1 consumption by investment in the short-term technology.[5]

At date 0 there are a large number of identical people, and each person is uncertain about what his or her preferences over consumption at dates 1 and 2 will be. Those preferences may be of an *impatient* type, labeled *type 1*, or of a *patient* type, labeled *type 2.* At the beginning of date 1, each person learns his or her type. This is private information; a person's type is known to that person, but not to anyone else. The preferences of each type 1 (impatient) person after learning his or her type are given by the utility function $u^1(x,y)$, while the preferences of each type 2 (patient) person are given by the utility function $u^2(x,y)$, where in both functions x is date 1 consumption and y is date 2 consumption. The accompanying chart depicts the relationship between the utility functions of the two types using indifference curves. The chart indicates that, at any consumption pair, each impatient person is willing to sacrifice less of date 1 consumption, per unit of additional date 2 consumption, than is a patient person; that is, at any consumption pair, the impatient indifference curve through that pair is steeper than that of the patient indifference curve through that pair.[6]

We let p be both the fraction of people who will turn out to be impatient and the subjective probability at date 0 that each person will be impatient. Welfare is judged at date 0 by the magnitude of expected utility, $pu^1(x,y) + (1–p)u^2(x,y)$. Finally, I assume that people are isolated from each other at date 1, so that at date 1 they cannot get together and coordinate what they do, and so that, if a banking system exists, then it must accommodate withdrawal demands at date 1 sequentially, one person at a time.[7]

Implications of the Model

Before I use the model to appraise narrow banking, I describe some of its other relevant implications.

The Best Feasible Symmetric Outcome

I now describe the *best feasible symmetric outcome* in the model. Let (c_1^i,c_2^i) for i = 1, 2 be a symmetric allocation—symmetric in the sense that everyone at date 0 is given the same type-contingent consumption pairs, where the subscript represents the date and the superscript i represents the type. In other words, a symmetric allocation is a consumption pair for each type. The solution to the following problem is the best feasible symmetric outcome.

Upper Bound Problem. Choose (c_1^i,c_2^i) for i = 1, 2, and choose x_s, x_l, and x_{l1} (all nonnegative) to maximize $pu^1(c_1^1,c_2^1) + (1–p)u^2(c_1^2,c_2^2)$, subject to

(1) $x_s + x_l \leq 1$

(2) $pc_1^1 + (1–p)c_1^2 \leq R_1x_s + r_1x_{l1}$

(3) $pc_2^1 + (1–p)c_2^2$
$\leq R_2(x_l–x_{l1}) + R_1[R_1x_s + r_1x_{l1} – pc_1^1 – (1–p)c_1^2]$

(4) $x_{l1} \leq x_l$

(5) $u^i(c_1^i,c_2^i) \geq u^i(c_1^j,c_2^j)$

for i = 1, 2 and j = 1, 2, where x_s is investment per person at date 0 in the short-term technology, x_l is investment per person at date 0 in the long-term technology, and x_{l1} is the amount per person of long-term investment liquidated at date 1.

Conditions (1)–(4) are *resource constraints.* Constraint (1) says that total investment cannot exceed the initial amount of date 0 good in the economy. Constraint (2) says that total date 1 consumption cannot exceed the returns on investment, which consist of the return on the short-term investment plus the return on the amount of long-term investment liquidated at date 1. Constraint (3) says that total date 2 consumption cannot exceed the date 2 returns on investment, which consist of the return on long-term investment not liquidated at date 1 plus the return on short-term investment made at date 1. Constraint (4) says that the total liquidated long-term investment is bounded above by long-term investment. Constraint (5) is called an *individual incentive compatibility constraint.* This constraint says that for each type, the consumption pair designated for that type should give at least as much utility as the consumption pair designated for the other type. Constraint (5) is required because of the assumption that knowledge of type is private information. Under that assumption, it is well known that any achievable allocation must satisfy such an incentive constraint.[8]

The preceding problem is called an *upper bound problem* because it does not include a constraint dealing with sequential service. Despite this, I will show that the solution to the problem is achievable. Thus I begin by describing some properties of the solution.

PROPOSITION 1. *Any solution to the upper bound problem is such that all date* 1 *consumption is supported by investment in the short-term technology and all date* 2 *consumption is supported by investment in the long-term technology* (*so that there is no liquidation of long-term investment*), *constraints* (1)–(3) *hold at equality, and type* 1 (*impatient*) *persons receive relatively more date* 1 *consumption and relatively less date* 2 *consumption than type* 2 (*patient*) *persons.* (*That is,* $c_1^{1*} > c_1^{2*}$ *and* $c_2^{1*} < c_2^{2*}$*, where an asterisk denotes a solution.*)

The proof of this proposition appears in the Appendix. The investment claims are obvious, given that there is no uncertainty about the total amount of consumption at each date. The claims about the consumption pairs follow from the difference in preferences and the incentive constraints.

Next I argue that the solution to the upper bound problem can be achieved. This is done in two steps. First, I show that if all resources have been invested at date 0 according to the solution to the upper bound problem, then each person is willing at date 1 to receive the solution pair intended for his or her true type, no matter what declarations of type that person thinks others may make. Second, I show that individual defection at date 0 from the upper bound solution is not in any person's interest.

Let N denote the total number of people. It is known that Np are truly impatient and $N(1–p)$ are truly patient. Suppose people who show up at date 1 to declare their type are given a choice between the consumption pair intended for type 1 and the pair intended for type 2, until the number claiming to be a particular type is equal to what is known to be the true number of that type; after that, each person is given the consumption pair for the other type.[9] This scheme, which is a kind of suspension scheme, assures that Np people will end up with the consumption pair intended for type 1 people and $N(1–p)$ people will end up with the pair intended for type 2 people. Therefore, this scheme assures that each person who shows up to declare his or her type at date 1 early enough to have a choice will be faced with two and only two options: the pair intended for type 1 and the pair intended for type 2—no matter what those who showed up earlier have declared and no matter what those who show up later will declare. It follows, then, from the individual incentive compatibility constraint—constraint (5)—that each such person who has a choice will truthfully declare his or her type. Therefore, each such person will receive the consumption pair intended for his or her true type.

To consider possible defection at date 0, suppose that each person begins with one unit of the date 0 good. I now show that no one would like to depart from a mutual arrangement in which each contributes (deposits) one unit at date 0 and receives in exchange a promise of the two consumption pairs that solve the upper bound problem. To show this I must describe what is received by a person who defects. Under my assumptions about isolation at date 1, each person who defects is entirely on his or her own, a situation described by the term *autarky.* The best such a person can do in this situation is receive the consumption pairs that solve the following problem.

Autarky Problem. Choose (c_1^i, c_2^i) for i = 1, 2 and choose x_s, x_l, and x_{l1}^i for i = 1, 2 (all nonnegative) to maximize $pu^1(c_1^1,c_2^1) + (1–p)u^2(c_1^2,c_2^2)$, subject to (1) and to

(6) $c_1^i \le R_1 x_s + r_1 x_{l1}^i$

(7) $c_2^i \le R_2(x_l – x_{l1}^i) + R_1(R_1 x_s + r_1 x_{l1}^i – c_1^i)$

(8) $x_{l1}^i \le x_l$

where x_{l1}^i is the amount of long-term investment liquidated at date 1 in the event that a person turns out to be type i and where each of conditions (6)–(8) holds for i = 1, 2.

I now prove Proposition 2, which assures that there will be no defection.

PROPOSITION 2. *Any solution to the upper bound problem is better than the solution to the autarky problem.*

The proof of this proposition appears in the Appendix. There are two steps to the argument: (a) The consumption 4-tuple that solves the autarky problem is feasible for the upper bound problem, and (b) any consumption 4-tuple that solves the upper bound problem is not feasible under autarky. Step (a) is demonstrated by showing that satisfaction of (6)–(8) for each i implies satisfaction of the constraints of the upper bound problem. Step (b) is demonstrated by showing that the autarky constraints are inconsistent with different consumption pairs for the two types, with all date 1 consumption financed by investment in the short-term technology, and with all date 2 consumption financed by investment in the long-term technology.

Proposition 2 says that in this model there are gains to be made through trade. One source of gain comes from financing all date 1 consumption by the short-term technology and all date 2 consumption by the long-term technology; this can happen under autarky only if the consumption pair does not vary with a person's type—a consumption outcome which, as shown above, is never best for the upper bound problem. In addition, for given investments, trade permits more possibilities for the consumption pairs for the two types than are possible under autarky. This is the only source of gain in the original version of the Diamond-Dybvig model; that version has a single investment technology.

Assets Outside the Banking System

I have described a way to achieve the upper bound solution by means of a banking system which holds all the wealth in the economy and which has liabilities consisting entirely of deposits. I now explain how, when assets are held outside the banking system, this optimum can be achieved in other ways.

As Jacklin (1987) notes, bank deposits such as those discussed in the previous section are somewhat unusual because they are not permitted to be completely withdrawn at either date. Jacklin shows that there is a simple way to change the portfolios to permit complete withdrawal at each date. The amended portfolios are of special interest here because they imply that there are assets outside the banking system.

According to Proposition 1, each person is consuming at least $c_1^{2}*$ at date 1 and at least $c_2^{1}*$ at date 2. Thus the optimal consumption pairs can be supported by the following date 0 arrangement. Each person individually invests $c_1^{2}*/R_1$ in the short-term technology, invests $c_2^{1}*/R_2$ in the long-term technology, and invests the remainder with the bank. The bank invests $p(c_1^{1}*–c_1^{2}*)/R_1$ per person in the short-term technology and $(1–p)(c_2^{2}*–c_2^{1}*)/R_2$ per person in the long-term technology. It offers each depositor the right to withdraw $(c_1^{1}*–c_1^{2}*)$ at date 1 or $(c_2^{2}*–c_2^{1}*)$ at date 2. When combined with the suspension procedure described above, this scheme achieves the optimum in the same sense as does the scheme in which the bank holds all the assets in the economy.

According to the model, the maximum magnitude of assets outside the banking system relative to assets inside the banking system is determined by the degree of difference between the two types of persons. If the types are not very different (in the sense that the difference between the slopes of their indifference curves at any consumption pair are small), then the consumption pairs that solve the upper bound problem are not very different and most assets can be held outside the banking system. An interpretation of a small difference between types is that the magnitude of uncertainty for each person about the desired timing of consumption is small relative to average consumption.

The sense in which this model accounts for assets outside the banking system is precisely the possibility that the solution to the upper bound problem is achieved as just described, with total assets outside the banking system as of date 0 equal to $N(c_1^{2}*/R_1+ c_2^{1}*/R_2)$—or with almost that amount outside the banking system. As I noted, this amount, relative to assets in the banking system, can be small or large depending on how different are the preferences. The amount invested in the short-term technology, or an amount almost that large, is the model's analog of the observation that inspires the current version of the narrow banking proposal, namely, the existence of safe short-term assets outside the banking system. However, before I use the model to discuss the narrow banking proposal, I must first consider a potential special role for the investment in long-term technology—investment that provides the minimum amount of date 2 consumption that each person receives according to the upper bound solution. This minimum amount is equal to $Nc_2^{1}*/R_2$.

Such investment could be held by the banking system and be matched by a liability that is distinct from deposits—a liability that does not give the holder the option to withdraw the liability at date 1. Moreover, such a liability could be made subordinate to the deposit liabilities, in the sense that such claims are paid at date 2 only after all deposit claims against the bank, both at date 1 and at date 2, have been satisfied. It seems clear that a sufficient amount of such assets can substitute for suspension as a way of reassuring the patient (type 2) persons. (See Drees 1989, De Nicolo 1995.) Suspension reassures those who turn out to be patient that their promised date 2 payouts will, in fact, be made. A different way to reassure them is for the bank to hold sufficient assets, the claims on which are subordinate to deposit claims. The investment equal to $Nc_2^{1}*/R_2$ can play the role of such assets. If the consumption pairs that solve the upper bound problem are not too different from one another, then such a scheme can fully reassure depositors because potential liquidations will permit the banking system to meet any pattern of deposit withdrawals. When depositors are reassured in this way, each person will withdraw in accord with his or her true type and there will be no actual liquidations. Notice that the investment supporting the minimum amount of date 1 consumption, $Nc_1^{2}*/R_1$, cannot play such a role because it is needed at date 1. However, $Nc_1^{2}*/R_1$ is the model's analog of the investment outside the banking system (the safe short-term assets) on which the narrow banking proposal focuses.

The Narrow Banking Proposal

I now use the model to interpret and appraise the narrow banking proposal. In my model each person as of date 0 has the same deposit, a deposit which gives that person certain withdrawal options. Given those options, and consistent with my earlier definition of illiquidity, I will say that the banking system is *liquid* if it can accommodate any pattern of withdrawals and is *illiquid* otherwise. That is, the quality of liquidity or illiquidity is a property of the banking system at date 0. The system is liquid if its asset holdings at date 0 and its promises to depositors are such that it can meet any pattern of withdrawals, not just the pattern that is consistent with truthful declaration of types. I will interpret the narrow banking proposal as one requiring the banking system to be liquid without any reliance on liabilities subordinate to deposits.

Proposition 3 demonstrates that any feasible allocation consistent with such a liquid banking system is achievable under autarky. In that sense the narrow banking proposal eliminates the banking system. Proposition 3 also demonstrates that any banking system without liabilities subordinate to deposits, which accomplishes anything relative to autarky, and which, in particular, achieves the upper bound solution, is an illiquid banking system.

PROPOSITION 3. *If the banking system has no liabilities subordinate to its deposit liabilities and is liquid, then the implied consumption pairs are achievable under autarky.*

The proof of this proposition appears in the Appendix. The idea behind it is that if the banking system has to be able to meet any pattern of withdrawals, including everyone claiming to be impatient and everyone claiming to be patient, and has no liabilities subordinate to deposits, then it is in the situation of an individual acting alone.

Proposition 3 is the model's criticism of narrow banking. It shows that in this model narrow banking eliminates the role of banking. The proposition implies that using narrow banking to cope with the potential problems of banking illiquidity is analogous to reducing automobile accidents by limiting automobile speeds to zero.

Concluding Remarks

The results given above are only as convincing as the model from which they are derived. It is relevant, therefore, to compare this model to other ways of explaining banking system illiquidity. It is also relevant to consider discrepancies between the model and the actual economy.

As far as I know, there are only two other explanations of illiquid banking system portfolios. Calomiris and Kahn (1991) explain a bank liability payable on demand as a device to keep bankers from embezzling funds. The other explanation is that banking system illiquidity is the result of risk-taking incentives generated by public policies—policies like improperly priced deposit insurance and central bank last resort lending. In contrast, the Diamond-Dybvig explanation rests on individual uncertainty about desired time profiles for consumption and a trade-off between technologies that give good short-term and long-term rates of return. The embezzlement explanation seems better suited to explaining terms of loans to small firms than to explaining the form of deposits at banks, while the public policy explanation seems inconsistent with the pervasiveness of banking system illiquidity. In any case, these different explanations are not mutually exclusive. As long as a model assigns some role to the very plausible ingredients of the Diamond-Dybvig explanation, the criticism of narrow banking given here still applies.

There are at least three notable discrepancies between the model set out here and the actual economy. First, although the deposits in the model are similar to savings deposits and demand deposits in the actual economy (in that withdrawal is at the request of the depositor), deposits in the model do not serve as vehicles for making third-party payments, as do demand deposits in the actual economy. The significance of this omission is hard to judge because there are no plausible models in which transacting is difficult and where assets that resemble deposits play a role in overcoming the difficulties.

Second, although I show that one asset arrangement which is consistent with the model's best symmetric allocation has assets outside the banking system and an illiquid banking system portfolio, that arrangement is not the only one which is consistent with the model's best feasible symmetric allocation. The model determines features of total portfolios, but within a certain range it does not determine how these portfolios are divided between individual investments and those investments made through the banking system. To explain why banking (at least in the United States) resembles the outcome that has a large amount of assets outside the banking system and that has banking system illiquidity, it is tempting to appeal to two aspects of bank regulation that are not in the model: the United States taxes deposits through reserve requirements and prevents banks from holding some kinds of assets. A tax on bank deposits would make the equilibrium amount of deposits even less than the minimum amount described earlier in connection with the best feasible symmetric outcome. Restrictions on bank assets could limit the amount of liabilities that are willingly held and that are subordinate to deposits.[10] In my model the holding of subordinate liabilities is costless as long as such liabilities do not exceed the amount required to support the minimum amount of date 2 consumption. However, that result obtains only if the banking system is allowed to hold the right assets. In my version of the Diamond-Dybvig model these assets are long-term investment; in other versions of the model they are risky assets.

The third discrepancy is that there is a sense in which my model does not display the kind of banking difficulties that inspired the narrow banking proposal: bank panics and runs. In my model a threatened suspension eliminates any banking difficulties. However, versions of the Diamond-Dybvig model exist which come closer to displaying banking difficulties. One version includes uncertainty about the proportions of people who are patient and impatient. In some versions the best arrangements have actual suspensions (as opposed to just threatened suspensions) in which those people who arrive early do better than those who arrive late. This situation resembles what is observed in bank runs. (See, for example, Wallace 1988 and 1990.) Such versions of the model are no kinder to the narrow banking proposal than is the version set out here.

More generally, any model that relies on the three plausible Diamond-Dybvig ingredients to explain banking system illiquidity will ascribe benefits to having deposits backed by other than safe short-term assets. For this reason I do not think that the above discrepancies between the model and the actual economy make a case for narrow banking. Until advocates of narrow banking provide a plausible explanation of banking system illiquidity that is consistent with their proposal, I must conclude that a persuasive case for narrow banking has not been made. My model shows that sufficient justification for the proposal is not given merely by observing that the magnitude of safe short-term assets outside the banking system exceeds the magnitude of banks' demand deposit liabilities. It is somewhat ironic that, according to my model, assets that may play a useful role in helping overcome potential banking difficulties are not safe, short-term assets but rather long-term assets, those that are not being used to support short-term spending.

[1] As might be expected, there are many versions of the narrow banking proposal. For a discussion of some of them, see Greenbaum and Thakor 1995 (chaps. 10 and 11 and, in particular, pp. 572–73) and Phillips 1995. There are also long-standing precursors of the current proposal. In the *Wealth of Nations,* Adam Smith (1789, bk. 2, chap. 2) urged bankers to match the maturity structures of their assets and liabilities. For a discussion of the 100 percent–reserve requirement proposal, see Friedman 1959.

[2] For example, neither Litan 1987 nor Phillips 1995 cites Diamond and Dybvig 1983 or, for that matter, any model of illiquid banking.

[3] Measuring illiquidity simply by examining reserves and liabilities of the banking system may, however, be misleading. If a central bank is committed to aid banks, then illiquidity should be judged by consolidating over both banks and the central bank. Also, if bank assets include loans they can call in, then it makes sense to consolidate over both the banks and those in debt to them. If the debtors can pay their obligations in a form that satisfies the holders of bank liabilities, then the system is less illiquid than would be indicated by not consolidating over them.

[4] As will be made clear as I proceed, nothing about the model set out here is new.

[5] The original version of the model had a single technology; in effect, it assumed that $r_1 = R_1$. The generalization to $r_1 < R_1$ was first studied by Bhattacharya and Gale (1987) and by Okuda (1989).

[6] Formally, I assume the following: $u^i(x,y)$ is strictly increasing and strictly concave, and type 1 is impatient and type 2 is patient in the sense that $u^1_1(x,y)/u^1_2(x,y) > u^2_1(x,y) \div$

$u_2^2(x,y)$ for each (x,y), where $u_j^i(x,y)$ denotes the partial derivative of u^i with respect to its jth argument. In the original version of the Diamond-Dybvig model, u^1 and u^2 each display perfect substitution between consumption at different dates with different marginal rates of substitution. The more plausible, smooth preferences used here were first introduced into the model by Jacklin (1987).

[7]The need for some such assumption was first pointed out by Jacklin (1987). For a detailed discussion of the assumption, see Wallace 1988.

[8]The role of constraint (5) can be illustrated by the following example. Suppose $u^i(c_1^i,c_2^i) = v(c_1^i) + \beta_i v(c_2^i)$, with $\beta_2 > \beta_1$. Then it is easy to show that if constraint (5) were not imposed, the solution to the upper bound problem would be $c_1^1 = c_1^2$ and $c_2^2 > c_2^1$. (Both types receive the same date 1 consumption, while type 2 receives greater date 2 consumption.) However, this would lead both types to claim to be type 2. Thus, for such preferences—which satisfy my assumptions—the solution with constraint (5) is different from the one without it.

Notice that the incentive constraints are stated as weak inequalities. The accompanying assumption is that if a person is indifferent between telling the truth and not telling the truth, then that person tells the truth.

[9]An important feature of these deposits is that people are offered a choice of two discrete patterns of payouts; they are not offered interest rates and allowed to withdraw any amount. This somewhat unrealistic feature can be avoided by changing the model from one with two types to a more plausible model with a continuum of types. Such a version of the model is studied by Lin (forthcoming). The implications I point out here hold also for that version of the model.

[10]Improperly priced deposit insurance is another explanation. For an analysis of this explanation in the context of a version of the Diamond-Dybvig model, see Hazlett 1992.

Appendix
Proposition Proofs

Here I develop the proofs for the three propositions discussed in the preceding paper.

PROPOSITION 1. *Any solution to the upper bound problem is such that all date* 1 *consumption is supported by investment in the short-term technology and all date* 2 *consumption is supported by investment in the long-term technology* (*so that there is no liquidation of long-term investment*), *constraints* (1)–(3) *hold at equality, and type* 1 (*impatient*) *persons receive relatively more date* 1 *consumption and relatively less date* 2 *consumption than type* 2 (*patient*) *persons.* (*That is,* $c_1^1* > c_1^2*$ *and* $c_2^1* < c_2^2*$, *where an asterisk denotes a solution.*)

Proof. The investment claims are obvious. Although the result about the consumption pairs is well known, a review of the argument may be instructive. Suppose a solution to the upper bound problem assigns the consumption pair *A* in the accompanying chart to type 2, the patient type. I want to show that the pair assigned to type 1, the impatient type, is within the shaded region, the region (strictly) southeast of *A.* I do this by eliminating every other possibility. The chart shows the indifference curves for the two types that go through point *A.* In accord with my assumption, the patient indifference curve is flatter than the impatient indifference curve. The pair assigned to the impatient type cannot be southwest of *A* because then the impatient type would prefer *A;* also, the pair cannot be northeast of *A* because then the patient type would prefer that pair to *A.* In either case, constraint (5) would be violated. Nor can the pair be northwest of *A.* If the pair were, then for the impatient type to be willing to receive that pair rather than *A*, the pair would have to be on or above the impatient indifference curve through *A.* If so, however, then the pair would have to be above the patient indifference curve through *A*, which violates constraint (5). Finally, I show that the pair assigned to the impatient type is not *A.*

Although such an assignment would satisfy constraint (5), that assignment cannot solve the maximum problem because there are other pairs—indicated by (B_1^1,B_2^1) for the impatient and (B_1^2,B_2^2) for the patient—that satisfy (5), that do not use more resources than *A* for both, and that make both types better off. This can be seen as follows. Since the slopes of the indifference curves through *A* differ, there exists a line through *A* with slope denoted $\alpha < 0$, which, as shown in the accompanying chart, is between the two indifference curves near *A.* The condition that the *B* pairs be on this line is

(A1) $$(B_2^2–B_2^1)/(B_1^2–B_1^1) = \alpha$$

while the condition that the pairs use the same amount of resources as pair *A* for both types is

(A2) $$pB_j^1 + (1–p)B_j^2 = A_j$$

for j = 1, 2. Thus I have three linear equations in four unknowns, a fact that leaves me free to choose the *B* pairs sufficiently close to *A* so that they are between the two indifference curves. This choice will assure that each type is better off than it would be at *A* and that constraint (5) holds. Q.E.D.

PROPOSITION 2. *Any solution to the upper bound problem is better than the solution to the autarky problem.*

Proof. There are two steps to the argument: (a) The consumption 4-tuple that solves the autarky problem is feasible for the upper bound problem, and (b) any consumption 4-tuple that solves the upper bound problem is not feasible under autarky.

(a) I denote with two asterisks the solution to the autarky problem, and I let

(A3) $$x_{l1}** = px_{l1}^1** + (1–p)x_{l1}^2**$$

which is the weighted average of the amounts of liquidation of long-term investment under autarky. Since autarky consumption pairs by definition satisfy constraint (5), I need only find a triplet (x_s,x_l,x_{l1}) so that these and the autarky solution consumption pairs satisfy (1)–(4). I propose that

(A4) $$(x_s,x_l,x_{l1}) = (x_s**,x_l**,x_{l1}**).$$

Since (1) is a constraint common to both problems and (4) is implied by condition (8) and my definition of $x_{l1}**$, it remains only to show that (2) and (3) hold. The weighted average of the two equations of (6), with weight p on i = 1 and weight (1–p) on i = 2, implies (2), while the same weighted average of the two equations of (7) implies (3).

(b) Let $c*$'s and $x*$'s be a solution to the upper bound problem. I know that these satisfy the claims in Proposition 1. I do a proof by contradiction and suppose that there are $x**$'s such that these and the $c*$'s satisfy the autarky constraints. Again I let (A3) hold. Since the $c*$'s are such that the two types receive different consumption pairs, it follows from (6) that either $x_{l1}** > 0$ or $x_s** > x_s*$. That is, either there is liquidation under autarky or there is more investment in the short-term technology under autarky.

Since the $x**$'s and $c*$'s satisfy the autarky constraints, it follows from (6) that

(A5) $$pc_1^1* + (1–p)c_1^2* \leq R_1x_s** + r_1x_{l1}**.$$

And since a solution to the upper bound problem satisfies (2) with equality and has no liquidation of long-term investment, it follows that

(A6) $$R_1x_s* \leq R_1x_s** + r_1x_{l1}**.$$

Since $R_1 > r_1$, it follows that

(A7) $$x_s** + x_{l1}** – x_s* \geq 0.$$

From (7) it follows that

$$\text{(A8)} \quad pc_2^1{*} + (1-p)c_2^2{*} \le R_2(x_l{**}-x_{l1}{**}) + R_1\{R_1x_s{**} + r_1x_{l1}{**} - [pc_1^1{*} + (1-p)c_1^2{*}]\}.$$

And since a solution to the upper bound problem satisfies (3) with equality, has all date 2 consumption financed by long-term investment, and has no liquidation of long-term investment, it follows that

$$\text{(A9)} \quad R_2x_l{*} \le R_2(x_l{**}-x_{l1}{**}) + R_1(R_1x_s{**}+r_1x_{l1}{**}-R_1x_s{*}).$$

This, in turn, can be written as

$$\text{(A10)} \quad R_2x_l{*} \le R_2(x_l{**}+x_s{**}-x_s{*}) - [R_2-(R_1)^2](x_s{**}+x_{l1}{**}-x_s{*}) - R_1(R_1-r_1)x_{l1}{**}.$$

Since

$$\text{(A11)} \quad x_s{**} + x_{l1}{**} - x_s{*} \ge 0$$

and since either $x_{l1}{**} > 0$ or $x_s{**} > x_s{*}$, it follows that

$$\text{(A12)} \quad R_2x_l{*} < R_2(x_l{**}+x_s{**}-x_s{*}).$$

Since the upper bound investments satisfy (1) at equality, it follows that the autarky investments violate (1), which is a contradiction. Q.E.D.

PROPOSITION 3. *If the banking system has no liabilities subordinate to its deposit liabilities and is liquid, then the implied consumption pairs are achievable under autarky.*

Proof. For this proof I use an extra subscript to distinguish between investments made individually and those made through the bank. I also let d_j^i denote the deposit payout to type i at date j. In order to be feasible, and given no bank assets subordinate to deposits, the consumptions are related to these amounts by

$$\text{(A13)} \quad c_1^i \le R_1x_{sa} + r_1x_{la1}^i + d_1^i$$

and

$$\text{(A14)} \quad c_2^i \le R_2(x_{la}-x_{la1}^i) + R_1(R_1x_{sa}+r_1x_{la1}^i+d_1^i-c_1^i) + d_2^i$$

for i = 1 and 2, where the subscript a (for *autarkic*) denotes investment made individually. If the banking system is liquid, then it can accommodate any pattern of withdrawals by appropriate asset liquidations. In particular, it can accommodate a pattern in which everyone claims to be type 1, with asset liquidation denoted x_{lb1}^1, and a pattern in which everyone claims to be type 2, with asset liquidation denoted x_{lb1}^2, where the subscript b (for *bank*) denotes investment made through the bank. That is, for i = 1 and 2 I have that

$$\text{(A15)} \quad d_1^i \le R_1x_{sb} + r_1x_{lb1}^i$$

and

$$\text{(A16)} \quad d_2^i \le R_2(x_{lb}-x_{lb1}^i) + R_1(R_1x_{sb}+r_1x_{lb1}^i-d_1^i).$$

If I add $R_1x_{sa} + r_1x_{la1}^i$ to each side of (A15) and use (A13), then I get (6); while if I add

$$\text{(A17)} \quad R_2(x_{la}-x_{la1}^i) + R_1(R_1x_{sa}+r_1x_{la1}^i+d_1^i-c_1^i)$$

to each side of (A16) and use (A14), then I get (7). Therefore, the consumption pairs are feasible under autarky. Q.E.D.

References

Bhattacharya, Sudipto, and Gale, Douglas. 1987. Preference shocks, liquidity, and central bank policy. In *New approaches to monetary economics*, ed. William A. Barnett and Kenneth J. Singleton, pp. 69–88. New York: Cambridge University Press.

Calomiris, Charles W., and Kahn, Charles M. 1991. The role of demandable debt in structuring optimal banking arrangements. *American Economic Review* 81 (June): 497–513.

De Nicolo, Gianni. 1995. Run-proof banking without suspension and deposit insurance. Manuscript. Brandeis University.

Diamond, Douglas W., and Dybvig, Philip H. 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91 (June): 401–19.

___________. 1986. Banking theory, deposit insurance, and bank regulation. *Journal of Business* 59 (January): 55–68.

Drees, Burkhard. 1989. Financial intermediation in generalized Diamond-Dybvig models. Ph.D. dissertation. University of Minnesota.

Dybvig, Philip H. 1993. Remarks on banking and deposit insurance. *Federal Reserve Bank of St. Louis Review* (January/February): 21–24.

Friedman, Milton. 1959. *A program for monetary stability*. New York: Fordham University Press.

Friedman, Milton, and Schwartz, Anna Jacobson. 1963. *A monetary history of the United States, 1867–1960.* Princeton, N.J.: Princeton University Press (for National Bureau of Economic Research).

Greenbaum, Stuart I., and Thakor, Anjan V. 1995. *Contemporary financial intermediation.* Orlando, Fla.: Dryden Press.

Hazlett, Denise. 1992. Deposit insurance and regulation in a Diamond-Dybvig banking model with a risky technology. Ph.D. dissertation. University of Minnesota.

Jacklin, Charles J. 1987. Demand deposits, trading restrictions, and risk sharing. In *Contractual arrangements for intertemporal trade*, ed. Edward C. Prescott and Neil Wallace, pp. 26–47. Minnesota Studies in Macroeconomics, vol. 1. Minneapolis, Minn.: University of Minnesota Press.

Lin, Ping. Forthcoming. Banking, incentive constraints, and demand deposit contracts with nonlinear returns. *Economic Theory*.

Litan, Robert E. 1987. *What should banks do?* Washington, D.C.: Brookings Institution.

Okuda, Hidenobu. 1989. Models of taxation on the financial system in less developed countries. Ph.D. dissertation. University of Minnesota.

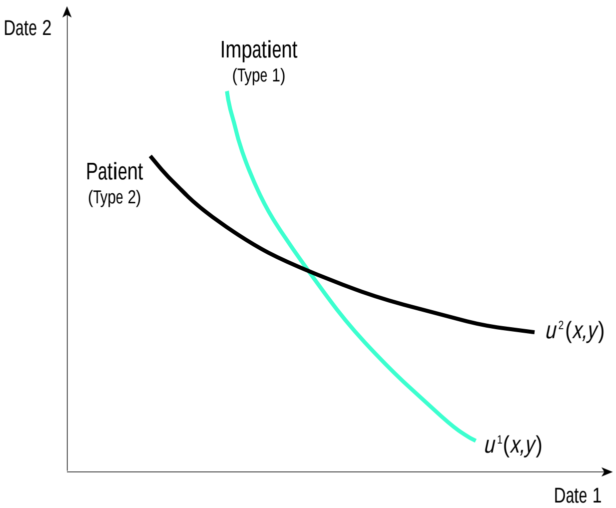
Phillips, Ronnie J. 1995. Narrow banking reconsidered: The functional approach to financial reform. Public Policy Brief 17. Jerome Levy Economics Institute.

Smith, Adam. [1789] 1937. *An inquiry into the nature and causes of the wealth of nations.* Reprint, ed. Edwin Cannan. New York: Random House.

Wallace, Neil. 1988. Another attempt to explain an illiquid banking system: The Diamond and Dybvig model with sequential service taken seriously. *Federal Reserve Bank of Minneapolis Quarterly Review* 12 (Fall): 3–16.

___________. 1990. A banking model in which partial suspension is best. *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (Fall): 11–23.

Consumption Preferences
of Impatient and Patient People



Why Optimal Consumption Is Different for the Two Types of People