http://clevelandfed.org/research/review/
Economic Review 1998 Q3

An Introduction to the Search Theory of Unemployment

by Terry J. Fitzgerald

Terry J. Fitzgerald is an economist at the Federal Reserve Bank of Cleveland. He thanks Paul Gomme and Randall Wright for their comments, and Jennifer Ransom and Jeff Schwarz for their research assistance.

Introduction

On any given day, during economic busts and economic booms alike, millions of Americans are unable to find desirable employment despite their best efforts. Understanding the reasons for this fact is a chief concern for economists and policymakers, since it is necessary for designing good labor market policies. Unemployment not only creates hardships for those it encompasses, but it also *seems* to represent a vast pool of idle economic resources.

Classical labor theory is not well suited to thinking about unemployment, for within this framework the amount of labor that workers supply is exactly equal to the amount of labor demanded by firms at the equilibrium wage—therefore, there is no unemployment. This feature of classical theory has contributed to the historical interpretation of unemployment, or at least a portion of unemployment, as a disequilibrium or an involuntary phenomena. While such terminology has permeated discussions of unemployment, it has done little to enhance our understanding of the underlying determinants of unemployment or its behavior through time and across countries.[1]

A different approach to the study of unemployment, which sought to directly explain the frequency and duration of unemployment spells, took root during the 1970s. The building block of this approach is the simple observation that finding a good job (or a good worker, in the case of a firm) is an uncertain process which requires both time and financial resources. This assumption stands in contrast to the classical model, in which workers and firms are assumed to have full information at no cost about job opportunities and workers. The alternative approach, referred to as the *search theory of unemployment,* seeks to understand unemployment in the context of a model in which the optimizing behavior of workers and firms gives rise to an *equilibrium* rate of unemployment. Furthermore, it has the potential to explain the striking fact that while millions of workers are unemployed, firms are simultaneously looking to fill millions of jobs.

1 See Rogerson (1997) for an excellent discussion of the language used to discuss unemployment.

The search theory approach to understanding unemployment flourished during the 1980s and 1990s. Incorporating the simple observation that searching is costly into a theory of labor markets has resulted in a rich set of models which have helped us not only to understand how unemployment responds to various policies and regulations, but also to gain a better understanding of other labor market issues including job creation and destruction, business cycle characteristics, and the effects of labor market policies on the aggregate economy more generally.

Unfortunately, while economists have found modern search theory an invaluable tool for understanding unemployment (as well as numerous other issues), the insights provided by this approach remain largely unfamiliar to noneconomists. This is partly a reflection of the old language of unemployment—terminology such as "full employment" and the "natural rate of unemployment"—continuing to dominate discussions of unemployment in the media and politics. This review is an attempt to reach out to those readers who are interested in acquiring a modern perspective on unemployment by providing an introduction to the search theory of unemployment.

In this article I present a model of job search and analyze how an unemployed worker's decision environment affects not only her employment decisions, but also the overall level of unemployment.[2] The model focuses on an unemployed worker's decision to accept an offered job or to continue searching for a better job. This is one of the earliest search models used in labor market analysis; its virtue is that it provides a simple framework capturing many of the central ideas upon which labor search theory is based, as well as interesting economic insights. Far richer models which capture many additional characteristics of labor markets have been developed, but these models are much more complex and will not be discussed here.

I. A Model of Job Search

Consider an unemployed worker who is searching for a job by visiting area firms, looking through help wanted ads, etc. Although the worker likely has many job opportunities, she has incomplete information as to the location of her best opportunities. Hence, she must spend time and resources searching, and she must hope she has luck finding one of her better opportunities quickly. In any given week the worker may receive a job offer at some wage w. The decision she faces is whether to accept that offer and forego the possibility of finding a better job, or to continue searching and hope that she is fortunate enough to get a better offer in the near future.

This scenario is captured in a model of job search using the following assumptions. First, each week the worker receives one wage offer. In order to capture the uncertainty of job offers, I assume that this offer is drawn at random from an urn containing wage offers between $\underline{w}$ and $\bar{w}$. Draws from this urn are independent from week to week, so the size of next week's offer is not influenced by the size of this week's offer. While I will interpret draws as weekly wage rates, they can be thought of more generally as capturing the total desirability of a job, which could depend on hours, location, prestige, and so on. For simplicity, assume that all jobs require the same number of hours and are of the same overall quality, so that jobs differ only in terms of the wage.

Each week the unemployed worker must decide whether to accept the wage offer w, or to reject the offer and wait for a better one. If she rejects the offer, the worker receives unemployment income of w^u dollars and draws a new wage offer the following week. For simplicity, wage offers from previous weeks cannot be recalled and accepted, an assumption which has no impact on the worker's decision to accept or reject this week's offer. While I will interpret unemployment income w^u as being unemployment compensation, it may also include factors such as the pecuniary value of leisure and home production activities less the cost of searching.

If the worker accepts the wage offer, she continues to work at that wage until she is fired (assume that the worker cannot search for a better job during this time). An employed worker faces a constant probability α of being fired at the end of each week. When an employed worker is fired, she becomes unemployed and begins searching for a new job the following week. Because an employed worker would never choose to quit her job in this model, I have omitted that possibility. Workers in the model are either employed or unemployed and actively searching for employment. No worker is out of the labor force (that is, not seeking employment).

2 The presentation of the model in this paper was largely drawn from chapter 2 of Sargent (1987), which provides a more advanced overview of search theory. Insights into the model were also drawn from lecture notes provided by Randy Wright.

Workers seek to maximize the expected present value of their lifetime wage income, which is written as

$$(1)\quad E\sum_{t=0}^{\infty}\beta^t y_t,$$

where β is a discount factor between 0 and 1, and y_t denotes the worker's income in period t.[3] Note that $(y_t = w^u)$ if the worker is unemployed, and $(y_t = w)$ if the worker is employed at wage w. The factor β determines the rate at which workers discount their future earnings and can also be written as $1/(1 + r)$, where r is a real rate of interest. While workers in the model have the good fortune of living forever, this assumption can be thought of as an approximation of the case where workers have many periods left to live.

Now consider the unemployed worker's decision problem in more detail. In evaluating a wage offer w, her decision will depend on how the current offer compares to other offers which she may receive. If the chances of receiving a substantially better offer next period are good, then the worker may choose to reject the current offer with the expectation of receiving a better one in the near future. A worker who rejects an offer foregoes income this week in the amount of the offer wage w, less the amount of unemployment compensation w^u. That loss must be balanced against the potential gain from receiving a higher wage offer next week, which the worker would receive in all future weeks until she is fired. In other words, the worker must compare the expected present value of her income if she rejects the offer with the expected present value of her income if she accepts the wage offer. As we will see, just how high the wage offer must be for the worker to accept depends on the exact shape of the wage offer distribution, the probability of being fired, the level of unemployment compensation, and the rate at which the worker discounts future earnings.

The specific mathematical structure of an unemployed worker's decision problem is laid out in the appendix, along with the description of a solution strategy. While the formulation of this problem makes use of mathematical techniques that are likely to be unfamiliar to non-economists, the underlying intuition of the problem is relatively straightforward and will be highlighted here. Recall that in making her decision, the unemployed worker must compare the expected lifetime incomes of accepting or rejecting a particular offer. I describe the unemployed worker's decision problem using the following notation. Let $v^{wait}(w)$ be the expected present value of lifetime income if she rejects a wage offer w and waits for a better offer; let $v^{accept}(w)$ be the expected present value of lifetime income if she accepts w; and let $v^{offer}(w)$ be the expected present value of lifetime income upon drawing a wage offer w. Each of these three functions assumes that the unemployed worker will behave optimally (that is, makes the best decisions) in future periods so as to maximize expected lifetime income as given by (1).

First consider the value of rejecting an offer and waiting for a better offer:

$$(2)\quad v^{wait}(w) = w^u + \beta E v^{offer},$$

where Ev^{offer} is the expected value of $v^{offer}(w)$. The value of waiting includes the unemployment compensation which the worker receives this week, plus the discounted expected value of drawing a new wage offer next week. Notice that $v^{wait}(w)$ is a constant, which I will write as v^{wait}, since Ev^{offer} does not vary with w. This reflects the fact that next week's wage offer is independent of this week's offer, so the value of rejecting an offer and waiting for a new offer is the same regardless of this week's offer.

Next consider the value of accepting a wage offer w:

$$(3)\quad v^{accept}(w) = w + \beta\alpha E v^{offer} + \beta(1-\alpha)v^{accept}(w).$$

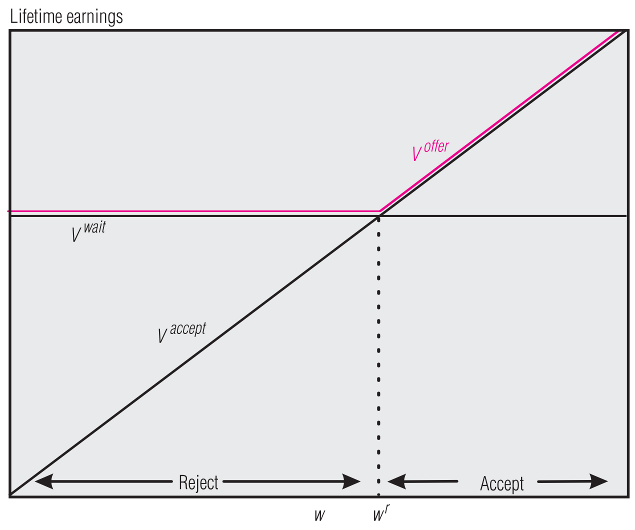
If the worker accepts a wage offer w, she receives income w this week. At the end of the week she is fired with probability α, in which case she receives the discounted expected value of receiving a new offer next week, $\beta E v^{offer}$, or she continues on the job with probability $(1-\alpha)$, in which case she receives the discounted value of accepting the same wage offer next week, $\beta v^{accept}(w)$. This equation can be rewritten

$$(4)\quad v^{accept}(w) = \frac{w + \beta\,\alpha\,Ev^{offer}}{1-\beta(1-\alpha)}.$$

3 The assumption that workers maximize expected lifetime income can be interpreted in several ways: 1) workers are risk-neutral, so they do not care about smoothing consumption; 2) workers are able to perfectly insure themselves against any idiosyncratic income risk, so the worker first maximizes expected income and then arranges her consumption stream so as to maximize utility; or 3) c and w can be reinterpreted as being the utility value of being unemployed and of working at a job with wage w respectively, in which case equation (1) can be reinterpreted as expected, discounted utility.

FIGURE 1

Expected Lifetime Earnings



Notice that $v^{accept}(w)$ increases linearly with w.

The problem for a worker with an offer w in hand is deciding whether to accept the offer, which has value v^{accept}, or reject the offer, which has value v^{wait}. The value of having an offer w in hand is given by

$$(5) \quad v^{offer}(w) = \max \{v^{accept}(w), v^{wait}\},$$

which takes into account that offers will be accepted only when accepting is more beneficial than waiting.

A solution to this problem is characterized by functions $v^{offer}(w)$ and $v^{accept}(w)$, and a constant v^{wait}, that satisfy equations (2), (4), and (5). Associated with the function $v^{offer}(w)$ is a decision rule which indicates whether the worker accepts or rejects each wage offer w between $\underline{w}$ and $\bar{w}$. Unfortunately, computing these functions is not as straightforward as it might first appear. The function $v^{accept}(w)$ and the constant v^{wait} which define $v^{offer}(w)$ depend themselves on $v^{offer}(w)$ through the term Ev^{offer}. None of these elements can be solved for independently.

To gain insight into the nature of the solution to the unemployed worker's job decision, it is helpful to graph $v^{accept}(w)$ and v^{wait} against the value of the wage offer w (figure 1). The decision to accept or reject each wage offer w depends on whether v^{wait} is greater than or less than $v^{accept}(w)$. The figure shows that this decision takes a particularly simple form. For values of w less than w^r, v^{wait} is greater than $v^{accept}(w)$, so the worker is better off rejecting the offer. For w greater than w^r, v^{wait} is less than $v^{accept}(w)$, so the worker is better off accepting the offer. The function $v^{offer}(w)$ is defined by the maximum of these two functions, and is illustrated in blue. Notice that w^r will depend on the specific value of v^{wait} and the function $v^{accept}(w)$, which themselves depend on $v^{offer}(w)$ through the term Ev^{offer}. Furthermore, Ev^{offer} depends on the value of w^r, and it will be helpful to make this dependence explicit by writing $Ev^{offer}(w^r)$.

The wage w^r is called the *reservation wage* and represents the lowest wage offer that an unemployed worker will accept. As I will show in the next section, the exact value of the reservation wage depends on the wage offer distribution, the firing rate α, unemployment compensation w^u, and the discount factor β.

Solving for the Reservation Wage

Next I briefly describe how to solve for the value of the reservation wage. As shown in figure 1, the reservation wage w^r is the value of w which satisfies

$$(6) \quad v^{accept}(w^r) = v^{wait}$$

or, using equations (2) and (4),

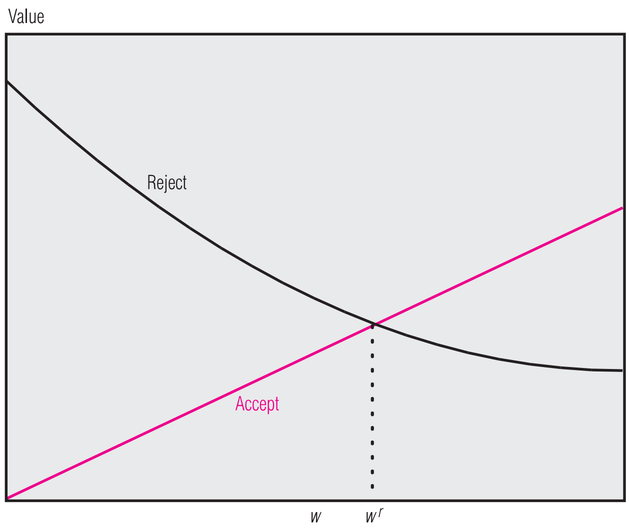
$$(7) \quad \frac{w^r + \beta\alpha Ev^{offer}(w^r)}{1-\beta(1-\alpha)} = w^u + \beta\alpha Ev^{offer}(w^r)$$

This expression says that the reservation wage is the wage at which the value of accepting the wage offer (the left side) is equal to the value of rejecting the offer (the right side). That is, the reservation wage is the wage at which the worker is just indifferent between accepting or rejecting the offer. Before we can solve this equation for w^r, we must first provide an explicit expression for $Ev^{offer}(w^r)$.

Obtaining an expression for $Ev^{offer}(w^r)$ requires that I be explicit about the distribution of wage offers that are contained in the wage offer urn. Assume that these offers are uniformly distributed between $\underline{w}$ and $\bar{w}$. This implies that all wages between $\underline{w}$ and $\bar{w}$ are equally likely to be drawn and makes the computation of $Ev^{offer}(w^r)$ straightforward. It is proportional to the area under the v^{offer} curve between $\underline{w}$ and $\bar{w}$. After doing some algebra, one finds that

FIGURE 2

Determination of the Reservation Wage



(8) $$Ev^{offer}(w^r) = \left(\frac{1}{1-\beta}\right)\left[\frac{w^u + s(\bar{w} - w^r)^2}{2(\bar{w} - \underline{w})}\right]$$

where $s = 1/(1 - \beta(1 - \alpha))$.

Using equation (8) to substitute Ev^{offer} out of equation (7), one obtains a single equation containing w^r:

(9) $$w^r = w^u + \left[\frac{\beta(1-\alpha)}{1-\beta(1-\alpha)}\right]\frac{(\bar{w} - w^r)^2}{2(\bar{w} - \underline{w})}.$$

However, w^r appears on both sides of this equation, so a little more work is needed.

To simplify notation in what follows, define a new function,

(10) $$\varphi(w^r) \equiv \left[\frac{\beta(1-\alpha)}{1-\beta(1-\alpha)}\right]\frac{(\bar{w} - w^r)^2}{2(\bar{w} - \underline{w})},$$

which is the second term on the right side of equation (9). This function can be interpreted as the expected benefit of drawing a new wage when the unemployed worker has an offer w^r in hand. Notice that this function is decreasing in w^r, which indicates that the expected gains from drawing a new wage diminish as w^r increases. If w^r is set to $\bar{w}$, this function is 0, reflecting the fact there can be no gain from drawing a new offer since $\bar{w}$ is the highest possible wage. Equation (9) can be rewritten

(11) $$w^r = w^u + \varphi(w^r).$$

I have arrived at a single equation, (11), which determines the value of the reservation wage w^r given values for all the parameters in the model. The left side can be regarded as the benefit of accepting a wage offer at the reservation wage. The more selective a person is (e.g., the higher her reservation wage), the higher the value of accepting a job offer at the reservation wage. Hence the left side of the equation is increasing in w^r. The right side can be regarded as the value of rejecting the offer and waiting. It includes the value of unemployment compensation w^u plus the expected gain from drawing a new age. The expected gain from receiving additional wage offers again depends on how selective the person is. The pickier she is, the lower the chances of getting such an offer and the lower the value of waiting. Thus the right side is decreasing in w^r. The equilibrium reservation wage is the wage at which the benefit of accepting is equal to the benefit of rejecting.

The next question to consider is whether a unique value of w^r exists which satisfies equation (11). Figure 2 graphs both sides of this equation. Denote the left side, w, the "accept curve," and the right side, $w^u + \varphi(w)$, the "reject curve." Since the accept curve is increasing in w and the reject curve is decreasing in w, the intersection of the two curves, if one exists, will be unique. However, there may not exist such a value. In this case the solution to the problem will correspond to a reservation wage of $\underline{w}$ (or lower) or to a reservation wage of w (or higher). In any case, the functions $v^{offer}(\cdot)$ and $v^{accept}(\cdot)$ and the constant v^{wait} which solve the problem are unique, as is the decision rule for accepting and rejecting wage offers within the set of possible wage offers. Figure 2 will be useful later when we discuss how changes in various parameter values impact the reservation wage.[4]

It is interesting to note that the reservation wage behavior of the unemployed worker in this model is observable in "real world" behavior. Each week many unemployed workers choose to continue their job searches even though they could accept low-paying jobs at, for instance, a local fast food restaurant. They obviously do so with the expectation that they will find a better job in the near future.

4 Equation (9) is quadratic in w^r, and the quadratic formula can be used to directly solve for w^r.

Unemployment Duration and Unemployment Rates

Although this model abstracts from the behavior of firms and the process by which the wage distribution is determined, unemployment durations and unemployment rates can be constructed nonetheless. First, assume that there are many identical workers in the model who act independently and have independent wage offer draws when unemployed. Also assume that the firing of employed workers occurs at the end of each week. At the beginning of the next week, each unemployed worker arrives at a firm, receives a wage offer w, and decides whether to accept or to reject that offer.

By setting the reservation wage w^r relatively high, the worker is less likely to receive an acceptable wage offer and will, on average, spend more time waiting for an acceptable offer than if she set w^r lower. The probability of accepting a wage offer, called the *job-acceptance rate* or the *hazard rate,* is simply equal to the fraction of offers greater than or equal to w^r. Let ψ denote the job-acceptance rate. Because the wage offer distribution is uniform, ψ is computed as

$$(12)\quad \psi = \frac{\bar{w} - w^r}{\bar{w} - \underline{w}} .$$

The average number of weeks it takes to receive an acceptable offer, referred to as the average waiting time, is given by $(1/\psi)$. Notice that if w^r is equal to $\bar{w}$, the job-acceptance rate is 0 and the average waiting time is infinity since there is zero chance of drawing $\bar{w}$ from the uniform distribution. If w^r is equal to $\underline{w}$, the job acceptance rate is 1 and the average waiting time is one week. That is, a job is always accepted in the first week upon becoming unemployed.

Given the assumptions on the transition between employment and unemployment, the average duration of unemployment is the average waiting time less one week, $[(1/\psi) - 1]$. So, for example, if the job-acceptance rate is 1, then the average duration of unemployment is 0 weeks since all unemployed workers except a job offer at the beginning of the week. If the job-acceptance rate is 0.10, or one out of 10, then the average waiting time is 10 weeks and the average duration of unemployment is nine weeks.

The path of the unemployment rate through time can be computed for any given initial unemployment rate u_1 as follows. Let u_t be the fraction of workers who are unemployed during week t (the unemployment rate), and let L denote the total population. Total unemployment is thus (Lu_t), while total employment is $(L - Lu_t)$. Given u_t, we can compute u_{t+1} by keeping track of how many workers enter and exit unemployment each week. This is expressed as

$$(13)\quad Lu_{t+1} = Lu_t(1 - \psi) + [(L - Lu_t)\alpha(1 - \psi)].$$

This equation says that total unemployment next week (Lu_{t+1}) is equal to the number of unemployed workers this week who do not accept a job at the start of next week $[Lu_t(1 - \psi)]$, plus the total number of employed workers this week who are fired and do not accept a job at the start of next week $[(L - Lu_t)\alpha(1 - \psi)]$. This equation can be rewritten by dividing through by L and rearranging terms to get a simpler expression for u_{t+1}, referred to as the *law of motion* for u_t:

$$(14)\quad u_{t+1} = \alpha(1 - \psi) + [(1 - \psi)(1 - \alpha)]u_t.$$

Given any unemployment rate u_1, equation (14) can be used to compute the path of the unemployment rate through time. One property of this law of motion for u_t is that the unemployment rate converges to the same level for any given initial unemployment rate u_1. The unemployment rate to which these paths converge can be computed from equation (14) by setting $u_{t+1} = u_t = u_s$. Solving for u_s produces

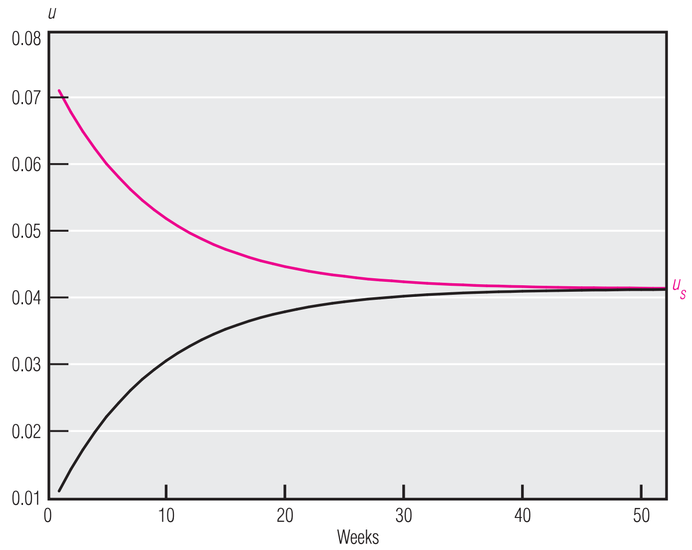
$$(15)\quad u_s = \frac{\alpha(1 - \psi)}{\alpha(1 - \psi) + \psi}.$$

The value u_s is the *steady state unemployment rate.* It is the point at which the flow of people into unemployment equals the flow of workers out of unemployment, so that the unemployment rate remains constant through time.

Notice that steady state unemployment depends only upon the firing rate α and the job-acceptance rate ψ. It is easy to show that higher firing rates and lower job-acceptance rates each imply higher unemployment rates, results which match one's intuition. Remember that while the firing rate α was exogenously given (that is, given as a parameter and not part of the solution), the job-acceptance rate ψ is endogenously determined (that is, not given as a parameter but part of the solution) and depends on all the parameters in the model. Thus, through ψ the steady state unemployment depends on all the parameters in the model.

FIGURE 3

Unemployment Paths



At this point let me briefly return to two points raised in the introduction. First, notice that unemployment in this model arises solely from incomplete information about wages and jobs that is costly to acquire; unemployment here is not a disequilibrium phenomena. Unemployment occurs even though all workers behave optimally and results from the costly but socially beneficial activity of achieving good matches between workers and jobs. Second, the model illustrates that distinctions between voluntary and involuntary unemployment are unclear and not useful. Here unemployment is voluntary in the sense that workers choose to reject wage offers. But unemployment is involuntary in the sense that any unemployed worker (whom we know has only received wage offers less than w^r) would prefer to switch places with any employed worker (who is receiving a wage of w^r or larger).

II. A Numerical Example

In order to make the insights provided by this model concrete, it is helpful to work with a numerical example. Consider a distribution of wage offers that is uniform between 200 and 800. This means that an unemployed worker is equally likely to receive any wage offer between \$200 and \$800 per week, and implies an average wage offer of \$500. Let the firing rate α be 0.005, or 1/2 percent per week, and the discount rate β equal 0.999, which corresponds to a 5 percent annual real interest rate. Lastly, set unemployment compensation w^u to \$200. After solving the model for these parameter values, I will discuss how the reservation wage, the average duration of unemployment, and the unemployment rate respond to changes in these values.

Take a guess at what the reservation wage is for this example. Will the worker hold out for a wage greater than \$500, the average wage offer? The answer is yes. In fact, the reservation wage w^r is \$737.62. If you think this number is surprisingly large, consider the fact that with a firing rate α of 0.005, the average length of employment, which is given by $1/\alpha$, is 200 weeks or almost four years. This means that once a wage offer is accepted, the worker expects to receive that wage for the next four years—thus providing an incentive to hold out for a relatively high wage. Of course, every week an offer is rejected is a week with foregone wage income, so the worker doesn't hold out for \$800.

The job-acceptance rate for this example is equal to 0.104. This says that each week there is a 10.4 percent chance of receiving an acceptable wage offer—which is any wage greater than or equal to \$737.62. This job-acceptance rate implies an average duration of unemployment of 8.6 weeks.

Finally, the steady state unemployment rate is 0.041, or 4.1 percent. Figure 3 illustrates the time paths for two different initial unemployment rates, 7.1 percent and 1.1 percent. Each of these paths converges to the steady state rate of 0.041. As discussed in the previous section, this convergence to steady state occurs for any initial unemployment rate.

What causes the unemployment rate to be above or below the steady rate in the first period? Loosely speaking, one could imagine that a one-time unexpected shock hits the economy which changes the unemployment rate. For example, this could reflect a temporary increase (decrease) in the wage offer distribution, perhaps due to a productivity shock, which implies that more (fewer) wage offers are above (below) the reservation wage and thereby lowering (increasing) unemployment. After this temporary shift, the wage offer distribution returns to its original form, and the unemployment rate steadily returns to its steady state value. Throughout the remainder of this paper, I will focus on the determinants of the steady state unemployment rate.

FIGURE 4A

Effect of Higher Real Interest Rate

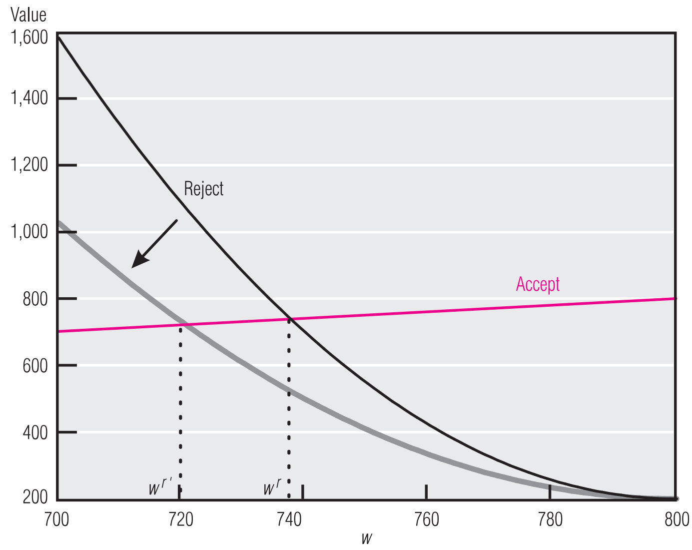
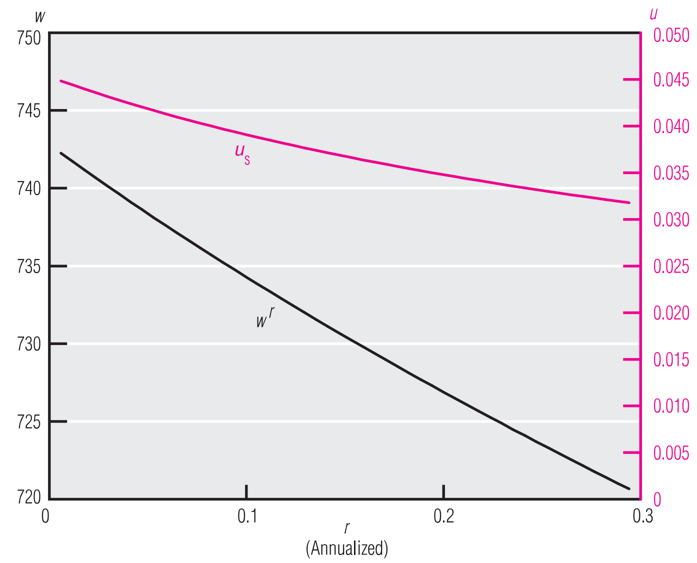


FIGURE 4B

Effect of Changes in the Real Interest Rate



The Effects of Changes in the Environment

While the model presented here is relatively stark and simple, it nonetheless provides interesting insights on how elements of the economic environment influence the unemployment rate. In the following subsections I explore how changes in the discount rate, firing rate, wage offer distribution, and unemployment compensation each influence the solution to the numerical example. For each of these elements I first examine the effect on the reservation wage, then trace the effects on the average duration of unemployment and the steady state unemployment rate.

Changes in the Discount Rate

First consider the effect of an increase in the real interest rate, which implies a lower value for the discount factor β (recall that $\beta = 1/(1+r)$). It is not immediately obvious how this change will effect the reservation wage or the unemployment rate. However, intuition suggests that because a higher interest rate implies discounting future earnings more rapidly, an increase in the real interest rate lowers the benefits of waiting for a higher wage. This suggests that the reservation wage will decrease. Indeed, figure 4a shows that a higher interest rate causes the reject curve to shift inward, resulting in a lower reservation wage.

The lower reservation wage implies a higher job-acceptance rate, lower unemployment duration, and lower steady state unemployment. That is, higher real interest rates lead to lower steady state unemployment. It is informative to consider extreme cases to gain insight into the underlying logic of the model. For example, consider setting the real interest rate infinitely large, which corresponds to setting β to 0. In this case the worker completely discounts future earnings. Thus, she sets her reservation wage to \$200, accepts any job offer, and the unemployment rate is 0. Unemployment in the model is due, in part, to workers' willingness to wait for a high wage offer.

Figure 4b shows how the reservation wage and unemployment rate vary with the real interest rate. Both are steadily decreasing in the interest rate. As the interest rate approaches 0 (the case in which workers do not discount the future at all), the reservation wage and the unemployment rate increase to \$743 and 4.5 percent, respectively.

FIGURE 5A

Effect of Higher Firing Rate

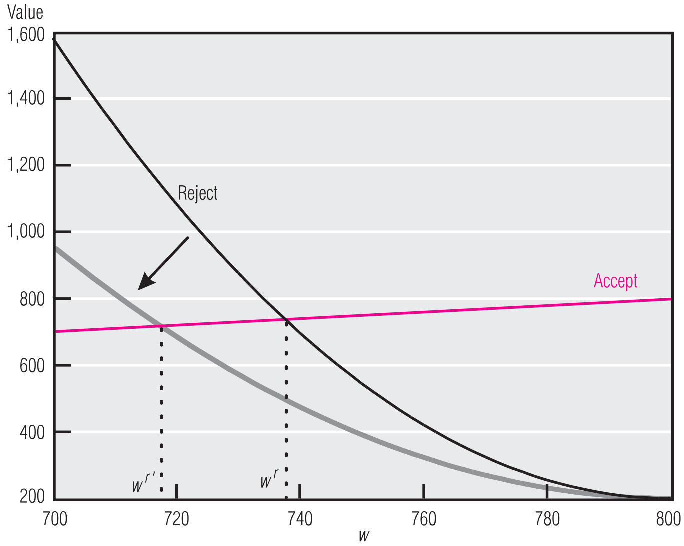
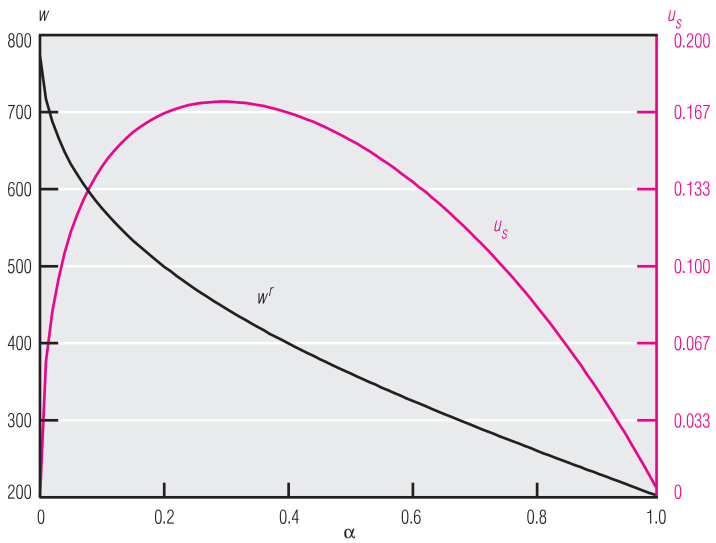


FIGURE 5B

Effect of Changes in the Firing Rate



Changes in the Firing Rate

Next suppose there is an exogenous increase in the firing rate α, perhaps resulting from a change in government regulations. While the effect on the reservation wage may not be apparent in this case, it seems obvious that an increase in the firing rate must result in an increase in the unemployment rate. Figure 5a illustrates that an increase in α causes the reject curve to shift inward toward 0. This results in a decrease in the reservation wage and a corresponding increase in the job-acceptance rate. This finding is not particularly surprising since, all things being equal, an increase in the firing rate reduces the expected length of time at a given job, and thus reduces the benefit of waiting for a relatively high wage offer. For example, if you are likely to hold the same job for only a few months, then it is not worth spending a long time searching for a high wage job.

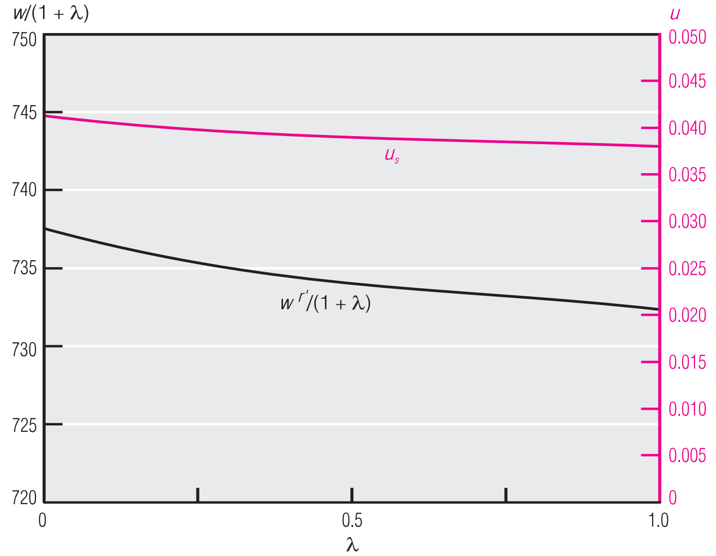
Surprisingly, the effect of an increase in the firing rate on the unemployment rate is ambiguous and depends on the size of the increase. Figure 5b shows that the unemployment rate increases steadily as the firing rate rises to roughly 0.30, but then declines for higher firing rates. To understand why this occurs, note that the reservation wage falls as the firing rate rises. This implies that the job-acceptance rate is *increasing* with α and the average duration of unemployment is *falling.*

Thus there are two competing effects on the steady state unemployment rate.The increase in the firing rate raises unemployment, while the increase in the job-acceptance rate lowers unemployment. Which effect dominates depends upon the specific numerical values used in the example and the magnitude of the increase in the firing rate. In the extreme case where all workers are fired every period ($\alpha = 1$), unemployed workers accept all wage offers ($w^r = 200$) and the unemployment rate is 0. The average weekly wage that workers receive falls from \$737.62 when the unemployment rate is 4.1 percent, to \$500 when the unemployment rate is 0; meanwhile, the expected lifetime earnings, Ev^{offer}, of an unemployed worker fall from \$740,086 to \$500,000. This example makes clear that policies which reduce unemployment do not necessarily benefit workers.

The potential for such surprising effects is one reason that it is important to rigorously model economic behavior. While intuition is certainly useful as a guide, relying on intuition alone often provides an incomplete picture, and is sometimes just plain wrong.

FIGURE 6

Effect of Changes in the Mean of Wage Distribution



Changes in the Wage Offer Distribution

Next I address the impact of changes in the wage offer distribution. More specifically, I examine the effect of a permanent upward shift in the entire wage offer distribution, perhaps resulting from a permanent increase in productivity, and the effect of an increase in the "riskiness" of the wage offer distribution.

An Upward Shift in the Distribution

Suppose that the distribution of wages were to increase by the fraction λ, or λ times 100 percent, as the result of a permanent, across-the-board increase in productivity. This implies that the uniform distribution of wage offers shifts from [200,800] to $[200(1+\lambda), 800(1+\lambda)]$.

Consider two cases. First, suppose that unemployment compensation, w^u, increases by the same percentage as all the wage offers. Equation (9), which determines the reservation wage, could then be rewritten

$$(16)\quad w^{r\prime} = (1+\lambda)w^u + \left[\frac{\beta(1-\alpha)}{1-\beta(1-\alpha)}\right] \times \frac{((1+\lambda)\overline{w} - w^{r\prime})^2}{2((1+\lambda)\overline{w} - (1+\lambda)\underline{w})},$$

where $w^{r\prime}$ is the reservation wage given the new wage offer distribution. With a little bit of algebra, it is straightforward to show that

$$w^{r\prime} = (1+\lambda)w^r.$$

That is, the reservation wage increases by the same percentage as the wage offers.

Next consider what happens to the job-acceptance rate, ψ', which is determined by

$$(17)\quad \psi' = \frac{(1+\lambda)\overline{w} - (1+\lambda)w^r}{(1+\lambda)\overline{w} - (1+\lambda)\underline{w}} = \frac{\overline{w} - w^r}{\overline{w} - \underline{w}} = \psi.$$

The job-acceptance rate is unaffected by the shift in the wage offer distribution. This implies that the average duration of unemployment and the unemployment rate are also unchanged!

While this result is certainly a striking one, it is perhaps not so surprising. It essentially says that if the costs and benefits of searching for a job all go up by the same proportion, then the reservation wage will increase by the same proportion and unemployment will be unaffected. Consider an example where we simply measure the wage offer distribution and unemployment compensation in cents instead of in dollars. Clearly we would expect the reservation wage to increase from \$737.62 to 73,762 cents, with unemployment duration and rates unaffected.

Now consider a second case. Suppose that unemployment compensation does not increase with the wage distribution. Figure 6 shows that in this case the reservation wage increases less than proportionally with the wage offer distribution ($w^{r\prime}/(1+\lambda) < w^r$). This implies that the job-acceptance rate increases, unemployment duration falls, and the unemployment rate declines. The relative cost of searching increases since unemployment compensation, which serves as a subsidy to searching, does not increase with the wage distribution.

As an example, consider a 5 percent increase in the wage distribution, so that λ equals 0.05. For this case the reservation wage increases by 4.9 percent to \$773.96, and the unemployment rate falls slightly, from 4.13 percent to 4.09 percent. For this particular example, the 5 percent increase in the wage offer distribution has little impact on unemployment.

FIGURE 7A

Effect of Changes in the Variance of Wage Distribution

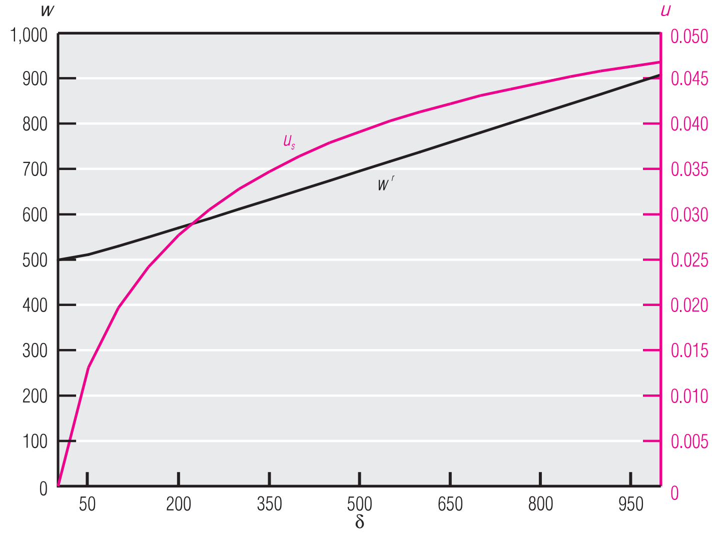
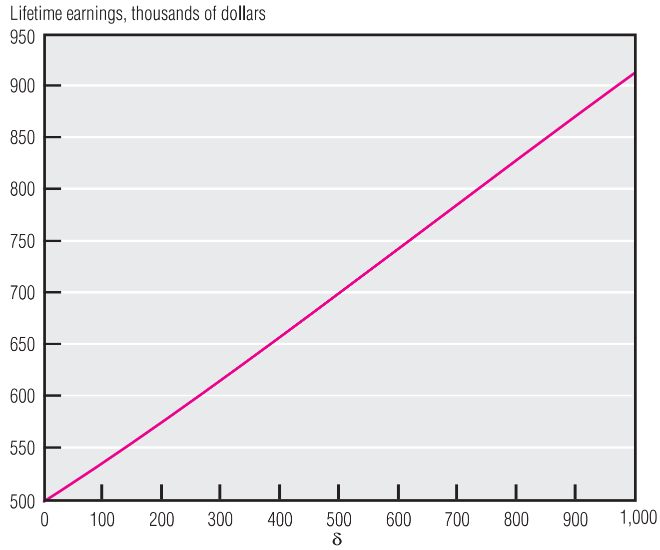


FIGURE 7B

Effect of Changes in Wage Riskiness on Expected Lifetime Earnings



Changes in the Riskiness of the Distribution

Next I examine the effect of a change in the "riskiness'' of the wage offer distribution. To do this, I must first clarify what I mean by riskiness. I define riskiness as the difference between the highest and lowest possible wage offers, ($\bar{w} - \underline{w}$). An increase (decrease) in riskiness will be defined as an increase (decrease) in this spread which does not affect the mean. Define the lower and upper bounds on wage offers to be $500 - \delta/2$ and $500 + \delta/2$, where $0 \leq \delta \leq$ \$1000. Thus δ is the measure of riskiness since ($\delta = \bar{w} - \underline{w}$), and was set equal to 600 in the baseline numerical example. Notice that the mean of the distribution is 500 regardless of the value of δ. When δ is set to zero, there is no riskiness in wage offers in the sense that all wage offers are exactly \$500.

What happens to the reservation wage and the unemployment rate as the riskiness of the distribution changes? Figure 7a shows that the reservation wage increases with wage offer riskiness. This is not too surprising, given that the spread of the distribution is increasing. Furthermore, the job-acceptance rate decreases as riskiness increases, which implies that the unemployment rate rises. This seems to bear out the intuition that riskiness is bad for workers.

Before reaching that conclusion, however, consider the case in which there is no riskiness ($\delta = 0$). Since there is no uncertainty in wage offers, there is no reason to search. Each job pays \$500, and workers who are unemployed at the beginning of the week always accept the offer. The steady state unemployment rate in this case is 0. But is an unemployed worker better off?

Let's compare the expected discounted lifetime earnings, Ev^{offer}, for an unemployed worker first in the model with no riskiness, and then in the baseline numerical example with riskiness ($\delta = 600$). In the case with no riskiness, Ev^{offer} is slightly less than \$500,000, the present value of \$500 per week forever (with no unemployment spells). But in the case with riskiness, Ev^{offer} is \$740,086.

At first blush it may seem surprising that an unemployed worker in the model with wage riskiness and higher unemployment has substantially higher expected lifetime earnings than an unemployed worker in the model with no riskiness and no unemployment. But it should not be. Given that the average duration of a job is almost four years, an unemployed worker would be much better off spending more time searching for a relatively high-paying job than she would be in a world where all jobs paid the average wage. Recall that the reservation wage in our numerical example was \$737.62, which is almost 50 percent higher than the average wage offer of \$500. Figure 7b shows that expected lifetime earnings steadily increase as the spread in the wage distribution increases. Here,

FIGURE 8A

Effect of Higher Unemployment Compensation

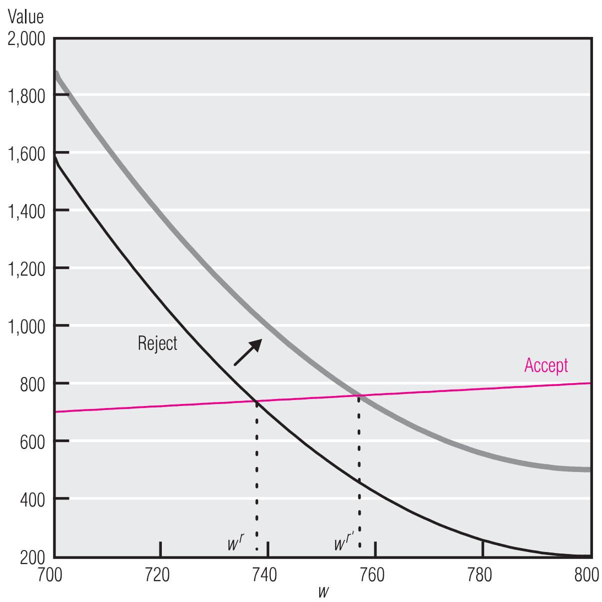
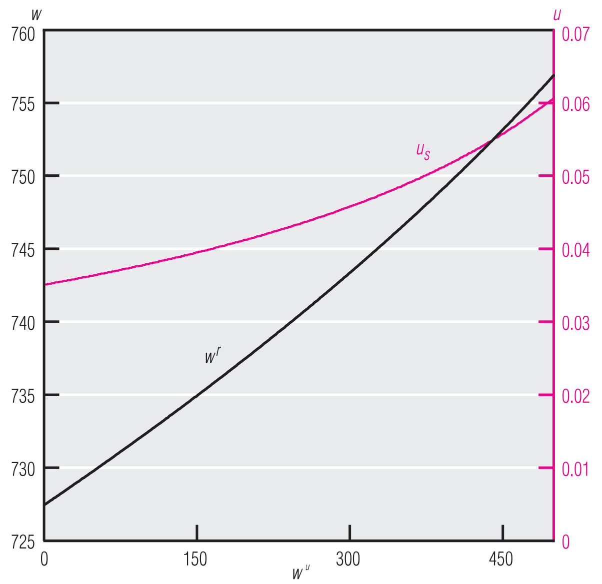


FIGURE 8B

Effect of Changes in Unemployment Compensation



riskiness is good. Again, note that versions of the model with low unemployment are not necessarily the environments which benefit the workers most.

Changes in Unemployment Compensation

Finally, consider what happens when unemployment compensation is increased. Figure 8a shows that an increase in unemployment compensation causes the reject curve to shift outward, implying an increase in the reservation wage. This is not surprising: Since unemployment compensation acts as a subsidy to searching, the worker is willing to wait longer for a high-paying job and thus increases her reservation wage.

The higher reservation wage implies a lower job-acceptance rate, an increase in the average duration of unemployment, and an increase in the unemployment rate. Figure 8b shows that the reservation wage and the unemployment rate increase steadily with increases in unemployment compensation. In the extreme case where w^u is set to 800, it is clear that unemployment will be 100 percent since no job pays better than collecting unemployment compensation.

Consider an increase in w^u from \$200 to \$300 per week. In this case the reservation wage increases from \$737.62 to \$743.35, average unemployment duration increases from 8.6 weeks to 9.6 weeks, and the unemployment rate increases from 4.1 percent to 4.6 percent.

There is a great deal of empirical evidence which supports the finding that increases in unemployment compensation result in higher unemployment. This does not imply that unemployment insurance necessarily makes workers worse off in the real world. The findings do suggest, though, that a tension exists between maintaining low unemployment rates and providing insurance for the unemployed.

III. Concluding Remarks

Search models of unemployment provide a valuable tool for understanding the factors which determine the unemployment rate and the impact of labor market policies and regulations on unemployment. Furthermore, search theory provides an alternative perspective to the view that unemployment represents idle resources. In this theory unemployed workers are not idle, but instead are engaging in the socially beneficial activity of finding a productive job match. The simple version presented in this paper illustrates how search models can be used to examine the influence of elements of the economic environment on the unemployment rate.

The search model discussed here is often referred to as a one-sided search model because it focuses solely on the job decisions of unemployed workers and abstracts from the search decisions of firms. More complex two-sided search models examine the optimizing decisions of workers and firms simultaneously. In addition, these models have incorporated a variety of other considerations which are abstracted from in the simple model, and they have proven to be capable of explaining many features of unemployment data within and across countries. This process of building better theories is perhaps the most important step in designing good economic policies, and search theory is playing a critical role in that process.

Appendix

Solving the Model

In this appendix I lay out the basic mathematical structure of the model and describe a strategy for solving it. The wage offers in each period are drawn from the same wage distribution $F(w)$, where F denotes the cumulative distribution function. That is, $F(\hat{w}) = \text{prob}(w \leq \hat{w})$. The definition of Ev^{offer}, the expected value of the v^{offer} value function, is

$$\text{(A1)} \quad Ev^{offer} = \int_{\underline{w}}^{\overline{w}} v^{offer}(w')dF(w').$$

The Bellman functional equation for v^{offer} is written

$$\text{(A2)} \quad v^{offer}(w) = \max\left\{ w^u + \beta\int_{\underline{w}}^{\overline{w}} v^{offer}(w')dF(w'), \; \frac{w + \beta\alpha\int_{\underline{w}}^{\overline{w}} v^{offer}(w')dF(w')}{1-\beta(1-\alpha)} \right\},$$

where the first term is the value of waiting and the second term is the value of accepting the wage offer. The equation determining the reservation wage w^r is

$$\text{(A3)} \quad \frac{w^r + \beta\alpha Ev^{offer}(w^r)}{1-\beta(1-\alpha)} = w^u + \beta Ev^{offer}(w^r),$$

which can be rewritten

$$\text{(A4)} \quad w^r = c[1-\beta(1-\alpha)] + [\beta(1-\beta)(1-\alpha)Ev^{offer}(w^r)].$$

Assuming a uniform distribution for F makes it possible to obtain a closed form solution for the integral expression that defines Ev^{offer}. This integral can be rewritten

$$\text{(A5)} \quad Ev^{offer} = \int_{\underline{w}}^{\overline{w}} v^{offer}(w')dF(w') = \frac{1}{\overline{w}-\underline{w}}\int_{\underline{w}}^{\overline{w}} v^{offer}(w')dw'$$

using the fact that the density function for a uniform distribution on $[\underline{w}, \overline{w}]$ is $1/(\overline{w}-\underline{w})$. This latter integral is simply the area under the v^{offer} curve, whose shape is illustrated in figure 1. This integral can be written

$$\text{(A6)} \quad \int_{\underline{w}}^{\overline{w}} v^{offer}(w')dw' = v^{wait}(\overline{w}-\underline{w}) + \frac{1}{2}(\overline{w}-w^r)s(\overline{w}-w^r)$$

where $s = 1/(1-\beta(1-\alpha))$ is the slope of v^{accept}. The first term is the area of the rectangle with width $(\overline{w}-\underline{w})$ and height v^{wait}, and the second term is the area of the triangle with width $(\overline{w}-w^r)$ and height $s(\overline{w}-w^r)$. Note, however, that this expression is still a function of Ev^{offer} since v^{wait} equals $(w^u + \beta Ev^{offer})$.

Substituting equation (A6) and the definition of v^{wait} into equation (A5), one obtains

$$\text{(A7)} \quad Ev^{offer}(w^r) = \left(\frac{1}{\overline{w}-\underline{w}}\right) \times \left[(w^u + \beta Ev^{offer}(w^r))(\overline{w}-\underline{w}) + \frac{1}{2}(\overline{w}-w^r)s(\overline{w}-w^r)\right] = w^u + \beta Ev^{offer}(w^r) + \frac{s(\overline{w}-w^r)^2}{2(\overline{w}-\underline{w})}.$$

This expression can be rewritten to obtain Ev^{offer} as a function of w^r

$$\text{(A8)} \quad Ev^{offer}(w^r) = \left(\frac{1}{1-\beta}\right)\left[w^u + \frac{s(\overline{w}-w^r)^2}{2(\overline{w}-\underline{w})}\right].$$

Finally, this expression for Ev^{offer} can be combined with (A4) to obtain an equation in w^r alone:

$$\text{(A9)} \quad w^r = w^u + \left(\frac{\beta(1-\alpha)}{1-\beta(1-\alpha)}\right)\left(\frac{s(\overline{w}-w^r)^2}{2(\overline{w}-\underline{w})}\right).$$

This is a quadratic equation in w^r. It can be shown that the smaller of the two roots for this expression is the equilibrium reservation wage if the solution is interior ($\underline{w} < w < \overline{w}$). Given w^r, equation (A8) can be used to obtain Ev^{offer}, which in turn can be used to obtain v^{wait}, v^{accept}, and v^{offer} using equations (2), (4), and (5) in the text.

References

Rogerson, Richard. "Theory Ahead of Language in the Economics of Unemployment," *Journal of Economic Perspectives*, vol. 11, no. 1 (Winter 1997), pp. 73–92.

Sargent, Thomas J. *Dynamic Macroeconomic Theory*. Cambridge, Mass.: Harvard University Press, 1987.