Sharing with a Risk-Neutral Agent

by Joseph G. Haubrich

Joseph G. Haubrich is an economist and consultant at the Federal Reserve Bank of Cleveland.
He thanks Patricia Beeson and Yukiko Hirao for helpful comments.

Introduction

In the classical principal–agent problem, a risk-neutral agent bears all the risk. This particular solution, while acknowledged as a special case, is prominent in the minds of economists because the more general risk-averse case does not easily yield numerical results. For example, Jensen and Murphy (1990) find a divergence between the risk actually borne by chief executive officers and the risk-neutral solution, which seems too large to be accounted for by reasonable levels of risk aversion.

Although the standard risk-neutral solution is correct, it is also misleading. Other solutions exist wherein the agent does not bear all the risk, and these may be considered more "natural," since they are limits of the risk-averse case.

Specifically, in Grossman and Hart's (1983) principal–agent problem, where there is a finite number of actions and states, many optimal sharing rules exist; in only one does the agent bear all the risk.[1] With a large enough stake in the project, the agent will not shirk—and with a finite number of states and actions, this stake need not be 100 percent.

Once agents have some risk aversion, the principal–agent problem has a unique solution. For the two-state case, limits can be computed as risk aversion approaches zero. The risk-averse solutions do not converge to the classic risk-neutral solution, however, but to the solution with the lowest risk for the agent. Because less risk makes a risk-averse agent happier, he demands a lower risk premium, in turn making the principal happier. But exceptions occur. There are cases in which the optimal action discretely shifts with an infinitesimal increase in risk aversion. In this case, the sharing rule (and thus the risk borne by the agent) differs substantially when the principal wants to induce distinctly different actions.

By increasing the number of actions, the results reduce to the standard continuous-action principal–agent models (see Holmstrom [1979]). Under reasonable conditions, the set of risk-neutral solutions shrinks to one.

This should introduce a note of caution to applications of the principal–agent model. The simple risk-neutral solution is not a good approximation of the optimal contract, even for arbitrarily low risk aversion. It can be misleading to compare actual contracts in which risk aversion is important—executive compensation, for instance—with the predictions of the risk-neutral principal–agent model. Stated more positively, these results show how principal–agent theory implies the relatively flat sharing rules that are observed in practice.

■ 1 Although Grossman and Hart do not explicitly mention multiple solutions in the risk-neutral case, they are careful in stating their theorems. Hence, this result does not imply any error in their work.

I. Sharing Rules

The Model

First let us quickly review the assumptions, notation, and approach of the Grossman–Hart model. For concreteness, assume the principal owns a firm, but she delegates its management to the agent. There is a finite number of outcomes (gross profit states), $q_1 < q_2 < \ldots < q_n$. The principal, who is risk neutral, cares only about the firm's expected net profit, defined as gross profit minus any payment to the manager.

In managing the firm, the agent takes an action, often thought of as *effort*, which the principal cannot observe. The principal does observe the outcome, however, and, like the agent, knows that different actions determine the probability of the outcome states. Both know $\pi_i(a)$, the probability of outcome q_i given action a. This probabilistic setting means the agent might work hard but still have little output to show for it. In choosing an action, the agent does not know the ultimate result. Conversely, in seeing the outcome, the principal cannot deduce the agent's action.

Actions belong to the finite set $A=\{a_1, a_2, a_3 \ldots a_m\}$, making the principal's expected benefit from an action equal to

$$B(a) = \sum_{i=1}^{n} \pi_i(a) q_i .$$

To avoid the problem of increasingly larger penalties being imposed with progressively smaller probabilities (see Mirrlees [1976]), assume that $\pi_i(a)$ is strictly greater than zero for all states and actions.

The agent likes income, but he dislikes effort. His utility function, $U(a,I)$, depends positively on his income from the principal, I, and negatively on his action, a. Grossman and Hart find it useful to place the following restrictions on $U(a,I)$:

> **Assumption (A1):** $U(a,I)$ has the form $G(a) + K(a)V(I)$, where $V(I)$ is a real-valued, continuous, strictly increasing, concave function with the domain $[\underline{I},\infty]$ and $\lim_{I \to \underline{I}} = -\infty$. G and K are real-valued, continuous functions defined by A, and K is strictly positive. For all a_1, a_2 in A and I, J in (I,∞), $[G(a_1) + K(a_1)V(I)] \geq [G(a_2) + K(a_2)V(I)]$ implies $[G(a_1) + K(a_1)V(J)] \geq [G(a_2) + K(a_2)V(J)]$.

The agent has a reservation utility $\bar{U}$, that is, the expected utility he can achieve working elsewhere. Sometimes this is derived from an outside income $\bar{I}$, so that $\bar{U} = V(\bar{I})$. If the principal does not offer him a contract worth at least $\bar{U}$, the agent will take another job. To make the model at all interesting, some income level should induce the agent to work. Grossman and Hart formalize this as

> **Assumption (A2):** $\{[\bar{U}-G(a)]/K(a)\} \leq V(\infty)$ for all a in A.

To see what happens when this assumption does not hold, consider the negative exponential utility $-e^{-k(I-a)}$ and $\bar{U} = 5$. In this case, even infinite income could not make the agent work.

If the principal could observe actions, it would be straightforward to determine how much she pays the agent for each action. Call this the first-best cost, or $C_{FB}(a)$:

$$U[a, C_{FB}(a)] = \bar{U}, \text{ or}$$
$$C_{FB}(a) = h\{[\bar{U}-G(a)]/K(a)\},$$

where $h = V^{-1}$.

As Grossman and Hart put it, "$C_{FB}(a)$ is simply the agent's reservation price for picking action a." Given this cost, the first-best optimal action maximizes the principal's net benefit, $B(a)-C_{FB}(a)$.

Of course, the principal cannot observe the agent's actions, nor can she directly base pay on effort. Instead, she chooses an incentive scheme, $I_i = \{I_1, I_2, \ldots I_n\}$, wherein payment I_i depends on the observed final state, q_i. Given this, the agent will choose the action that maximizes his expected utility. Knowing how the agent will react, the principal now can break her problem into two parts. For each action, she calculates the least costly incentive scheme that will induce the agent to choose that course. This gives her the expected cost of motivating the agent to perform a particular action a, $C(a) = \sum_{i=1}^{n} \pi_i(a) I_i$. She then chooses the action with the highest net benefit—that is, the one that maximizes $B(a)-C(a)$.

Multiple Solutions

The possibility of multiple solutions arises from looking at the mathematics of the agent's problem. With risk neutrality, the concave programming problem with a unique solution becomes a linear programming problem with multiple solutions. With a risk-averse agent, the principal minimizes the agent's risk, subject to meeting the incentive constraints. For a risk-neutral agent, only the incentives matter, and any incentive-compatible risk configuration will work. When the principal is not indifferent between the two most desirable actions, multiple equilibria can result. The larger the gap between the actions, the more risk the agent can bear.

The traditional solution assigns all risk to the agent, who delivers a fixed payment to the principal.

The agent, then, receives

$$I_i = q_i - [B(a^*) - C_{FB}(a^*)].$$

The agent bears all the risk for shortfalls in q, and the principal gets her expected benefit.

$$q_i - I_i = B(a^*) - C_{FB}(a^*).$$

Now, suppose the agent bears less risk and takes only a fraction of the shortfall in q. Income in state i becomes

$$(1) \quad I_i = \tau q_i - t\,[B(a^*) - C_{FB}(a^*)],$$

where t is a constant and τ measures the fraction of risk borne by the agent. Proposition 1 gives sufficient conditions for τ being less than one:

Proposition 1: Assume (A1)–(A2) and a risk-neutral agent. If

$$\tau \in \bigcap_{a:a\neq a^* \text{ and } \tau(a)\leq 1} [\tau(a), 1]$$

where

$$\tau(a) = \left(\frac{1}{\beta}\right)\left(\frac{1}{B(a) - B(a^*)}\right)$$
$$\left\{(\alpha + \beta \bar{I})\left[\frac{1}{K(a)} - \frac{1}{K(a^*)}\right] + \frac{G(a^*)}{K(a^*)} - \frac{G(a)}{K(a)}\right\},$$

then an optimal contract exists that pays the agent $I_i = \tau q_i - t\,[B(a^*) - C_{FB}(a^*)]$ for some value of t. This somewhat complicated condition guarantees there is a "gap" or "jump" between the principal's payoff in different states.

The proof is straightforward and revealing. To emphasize the underlying logic, I have made two simplifying assumptions about utility, both of which are easily generalized. First, I have specialized the risk-neutral income utility to $V(I) = I$, rather than to $V(I) = \alpha + \beta I$. Second, I have used the additively separable form of utility, setting $U(a,I)$ equal to $G(a) + V(I)$ or, here, to $G(a) + I$.

Proof: For the optimal action, the principal calculates the least costly method of getting the agent to choose action a^*. The incentive scheme must minimize the principal's expected payment to the agent while still inducing him to act. This is a programming problem, including individual rationality (IR), incentive compatibility (IC), and feasibility constraints (FEAS).

$$\text{(P1)} \qquad \text{MIN} \sum_{i=1}^{n} \pi_i(a^*),$$

subject to

$$\text{(IR)} \qquad \sum_{i=1}^{n} \pi_i(a^*)\,[G(a^*) + I_i] \geq U,$$

$$\text{(IC)} \qquad \sum_{i=1}^{n} \pi_i(a^*)\,[G(a^*) + I_i] \geq \sum_{i=1}^{n} \pi_i(a)\,[G(a^*) + I_i] \text{ for } a{:}a\neq a^*$$

$$\text{(FEAS)} \qquad I_i \leq \infty \text{ for all } i.$$

We now must determine the value of τ in equation (1) that will satisfy these conditions. This means choosing τ to satisfy

$$\sum_{i=1}^{n} \pi_i(a^*) I_i = \sum_{i=1}^{n} \pi_i(a^*)\,\{\tau q_i - t[B(a^*) - C_{FB}(a^*)]\}$$
$$= C_{FB}(a^*),$$

resulting in

$$(2) \quad t = \frac{\tau B(a^*) - C_{FB}(a^*)}{B(a^*) - C_{FB}(a^*)}.$$

By construction, values between 0 and 1 satisfy the individual-rationality constraint. Some values of τ also satisfy the incentive-compatibility constraint, as I will now show. Substituting equation (2) into equation (1), the incentive scheme becomes

$$(3) \quad I_i = \tau q_i - \tau B(a^*) + C_{FB}(a^*).$$

This makes the incentive-compatibility constraint

$$(4) \quad G(a^*) + \sum_{i=1}^{n} \pi_i(a^*)[\tau q_i - \tau B(a^*) - C_{FB}(a^*)] \geq$$
$$G(a^*) + \sum_{i=1}^{n} \pi_i(a)\,[\tau q_i - \tau B(a^*) - C_{FB}(a^*),$$

which simplifies to

$$(5) \quad G(a^*) - G(a) \geq \tau\,[B(a) - B(a^*)].$$

Whether a risk-neutral agent bears all the risk depends on whether there is a gap between $G(a^*)$–$G(a)$ and $B(a)$–$B(a^*)$.[2] But this gap is not solely a matter of chance: The principal chooses a^* to maximize $B(a)$–$C(a)$ or, in the risk-neutral case, $B(a)$–$C_{FB}(a)$. Because a^* is the optimal action, it satisfies $B(a^*) - C_{FB}(a^*) \geq B(a) - C_{FB}(a)$. Rearranging and using the definition of C_{FB}, we have

$$(6) \quad G(a^*) - G(a) \geq B(a) - B(a^*)\ \forall a \in A.$$

2 Haubrich (1994) provides several numerical examples of problems of this type, showing that solutions do exist and the theorem is not vacuous.

If the inequality in equation (6) is strict, τ can be less than one, meaning the agent does not assume all the risk. There are three cases to consider, depending on the sign of each side of equation (6).

(i) Both $G(a^*)–G(a)$ and $B(a)–B(a^*)$ are positive. In this case, a has the larger gross payoff but is more costly to implement than a^*. Clearly, if equation (6) holds, any τ in the relevant range of [0,1] will satisfy equation (5).

(ii) If $G(a^*)–G(a)$ is positive and $B(a)–B(a^*)$ is negative, any τ works. In this case, the less costly action, a^*, also has the better payoff.

(iii) Both $G(a^*)–G(a)$ and $B(a)–B(a^*)$ are negative. In this case, a^* is more costly but has a better payoff. We usually think of this as the "normal" case. With negative numbers, division reverses signs, so equation (5) implies that τ, the fraction of risk borne by the agent, can fall anywhere in the interval

$$\tau \in \left[\frac{G(a^*) - G(a)}{B(a) - B(a^*)}, 1 \right].$$

With a more general utility function, this becomes the condition stated in the proposition:

$$(7) \quad \tau(a) = \left(\frac{1}{\beta}\right)\left(\frac{1}{B(a)-B(a^*)}\right) \left\{ (\alpha + \beta\bar{I})\left[\frac{1}{K(a^*)} - \frac{1}{K(a)}\right] + \frac{G(a^*)}{K(a^*)} - \frac{G(a)}{K(a)} \right\}.$$

Even equation (7) understates the full range of incentive schemes wherein the principal bears risk. With more than two states, the sharing rule need not be linear, and a single-parameter τ will not capture all possible deviations from the classic case. In general, the solution set will be the convex hull of extreme points, a multidimensional "flat" or "face" of the constraint set for the linear programming problem (P1).

II. Convergence

Solutions in which the principal assumes some risk are more than curiosities. As risk aversion approaches zero, the risk borne by the agent converges to a number less than one. The traditional solution offers a poor approximation of this, even near zero.

The convergence results are for the two-state case—the sole case with closed-form solutions for the risk-averse problem. Answering convergence questions usually requires strong assumptions. For instance, Grossman and Hart assume only two states, or negative exponential utility. Without strong restrictions, odd things can occur in the model: The individual-rationality constraint may not bind, higher profits may mean less money for the agent, or the agent may get more money for less effort.

Limiting Cases

With only two states, the single-parameter τ fully describes how much risk the agent bears. Usually, the risk-averse solutions converge to the solution with the smallest value (rather than the classic solution of $\tau=1$). Some exceptions exist because the optimal action can switch at zero, which in turn causes a discrete jump in the risk burden.

To explore convergence, we must first make sure the utility functions do, in fact, converge. If we index the income utility function by risk aversion γ, (γ, I), we embody this convergence as a new assumption.

Assumption (A3): As γ approaches 0, $V(\gamma,I)$ converges uniformly to $\alpha+\beta I$, $(\alpha,\beta \neq 0)$, on the interval $[-q_n, q_n]$.

Although it is natural, this assumption does restrict utility functions. For example, the negative exponential function $-e^{-\gamma(I-a)}$ converges to zero, a constant function that is inadmissible by assumption (A1).

The statement of proposition 2 requires a little groundwork. First, the proof uses the closed-form solution for the two-state case found by Grossman and Hart:

$$(8) \quad v_1 = \frac{\pi_2(a_j)[\bar{U} - G(a_k)] - \pi_2(a_k)[\bar{U} - G(a_j)]/K(a_j)}{\pi_1(a_k) - \pi_1(a_j)}$$

$$(9) \quad v_2 = \frac{\pi_2(a_j)[\bar{U} - G(a_k)] - \pi_2(a_k)[\bar{U} - G(a_j)]/K(a_j)}{\pi_2(a_k) - \pi_2(a_j)}$$

The derivation of these formulas depends crucially on Grossman and Hart's proposition 6, which proves the agent is indifferent between the optimal action a^* and some less costly action. The existence of two possibilities makes convergence problematic. As risk aversion falls, either the optimal action or the less costly action may change. A change in the optimal action matters for the convergence result, but it is not clear whether a change in the less costly action makes a difference. I have produced neither a proof nor a counterexample for this case. Thus, the statement of proposition 2 reflects these two possibilities.

The unique profit share for a given utility function and risk aversion is defined as $\tau(V,\gamma)$, and the the minimum in equation (7) is defined as $\tau_{\min}$.

Proposition 2: Given assumptions (A1)–(A3), if the optimal action and the indifferent alternative action do not change for risk aversion in the neighborhood of zero, then

$$\lim_{\gamma \to 0} \tau(V,\gamma) = \tau_{\min}.$$

To ease the notational burden and to emphasize the logic, I present the proof for the additively separable case. The generalization to other utility functions is straightforward.

Proof: In the risk-neutral case, we know from equation (6) that

$$\tau_{\min} = \frac{G(a_k) - G(a_i)}{B(a_i) - B(a_k)},$$

which clearly depends on the optimal action a_k and a particular alternative a_i. This implies an income difference between states of

$$(10)\quad I_2 - I_1 = G(a_j) - G(a_k) / \pi_1(a_j) - \pi_1(a_k).$$

In the limit of the risk-averse case, optimal incomes are given by the limits of equations (8) and (9).

$$(11)\quad I_1 = v_1 = \frac{\pi_2(a_j)[\bar{I} - G(a_k)] - \pi_2[\bar{I} - G(a_j)]}{\pi_1(a_k) - \pi_1(a_j)}$$

$$(12)\quad I_2 = v_2 = \frac{\pi_2(a_j)[\bar{I} - G(a_k)] - \pi_1[\bar{I} - G(a_j)]}{\pi_2(a_k) - \pi_2(a_j)}.$$

Because $\pi_1(a_j) + \pi_2(a_j) = 1$, the probabilities can be expressed in terms of $\pi_1(\bullet)$. Making this substitution and collecting terms yields

$$I_1 = \{[\pi_1(a_k) - \pi_1(a_j)]\,\bar{I} + [1 - \pi^1(a_k)]\,G(a_j) - [1 - p_1(a_j)]G(a_k)\}\,\{\pi_1(a_k) - \pi_1(a_j)\}^{-1}$$

$$I_2 = \{[\pi_1(a_j) - \pi_1(a_k)]\,\bar{I} + \pi_1(a_k)G(a_j) - \pi_1(a_k)G(a_k)\}\,\{\pi_1(a_j) - \pi_1(a_k)\}^{-1}$$

Taking the difference and simplifying, we find

$$(13)\quad I_2 - I_1 = G(a_j) - G(a_k) / \pi_1(a_j) - \pi_1(a_k),$$

which matches equation (10).

The equality between equations (10) and (13) depends on the constancy of both the optimal action and the alternative action. I conjecture that even if the alternative action switches in the neighborhood of zero, the equality (and thus the proposition) still holds.

Action Shifts

Proposition 2 does not hold when the optimal action shifts at zero. Suppose one action is best at a risk aversion of zero and another at a risk aversion greater than zero. As the action changes, so does the sharing rule. The best way to illustrate this is a simple two-act example. Here, the principal induces the better action at zero risk aversion, but pays a flat fee and accepts the lower action for risk aversion greater than zero.

We begin with $B(a^*)–C(a^*)=B(a)–C(a)$, or indifference between the two actions, so that the switch occurs at zero. This sets τ equal to one, meaning the agent bears all the risk. We next want $B(a_2)–C(a_2) < B(a_1)–C(a_1)$, making the lower action preferred for $\gamma > 0$. To do this, set $V(I) = I - \gamma I_2$. Then, $h(v) = \sqrt{[1 + (1 - 4\gamma v)]}\,/2\gamma$. With $h(v)$ in hand, we can assess the second-best costs once we have calculated v_1 and v_2. The goal is to show that, in some cases, $\partial C(a_2)/\partial\gamma > 0$. If this is true, an increase in leads the principal to prefer action a_1, since the cost of action a_2 increases while the rest of the variables, $B(a_2)$, $B(a_1)$, and $C(a_1)$, remain unchanged. ($C\,[a_I]$ fixed payment independent of state.)

Simplifying v_1 and v_2 from equations (8) and (9), we have:

$$v_1 = \bar{I} - \gamma\bar{I}^2 + [G(a_1) - G(a_2) + \pi_1(a_1)G(a_2) - \pi_1(a_2)G(a_1)]\,[\pi_1(a_2) - \pi_1(a_1)]^{-1}$$

$$v_2 = \bar{I} - \gamma\bar{I}^2 + \frac{\pi_1(a_2)G(a_1) - \pi_1(a_1)G(a_2)}{\pi_1(a_2) - \pi_1(a_1)}$$

This represents a shift in the optimal action induced by the principal. Both actions remain feasible. The last terms in each of these expressions are constant with respect to γ, so we may rewrite them as

$$v_1 = \bar{I} - \gamma\bar{I}^2 + P$$
$$v_2 = \bar{I} - \gamma\bar{I}^2 + Q.$$

and solve for I_1, I_2, and $C(a_2)$:

$$I_1 = \frac{1}{2\gamma} - \frac{1}{2\gamma}\left[1 - 4\gamma(\bar{I} - g\bar{I}^2 + P)\right]^{\frac{1}{2}}$$

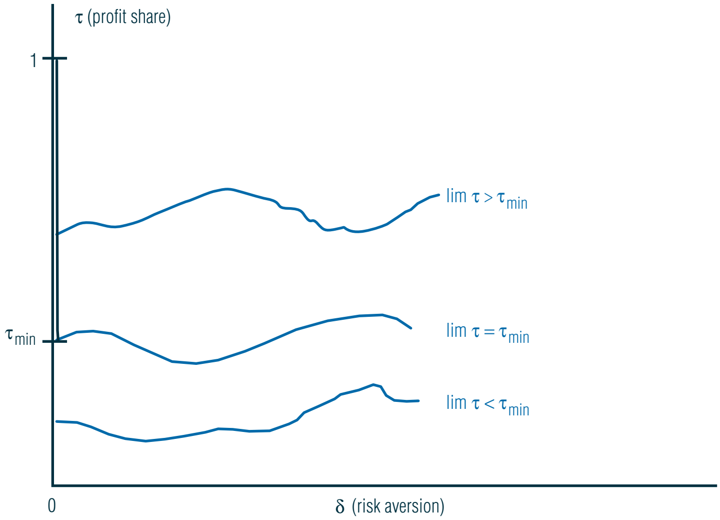
$$I_2 = \frac{1}{2\gamma} - \frac{1}{2\gamma}\left[1 - 4\gamma(\bar{I} - g\bar{I}^2 + P)\right]^{\frac{1}{2}}.$$

Notice that $\partial I_1/\partial\gamma$ and $\partial I_2/\partial\gamma$ have the same sign, matching $\partial C(a_2)/\partial\gamma$. Explicitly calculating the first of these derivatives, we have

$$\partial I_1 / \partial\gamma = \frac{1}{\gamma}\,[1 - 4\,\gamma(v_1)]^{\frac{1}{2}}\left[v_1 - \bar{I}^2\right].$$

FIGURE 1

Convergence of the Sharing Rule



SOURCE: Author's calculations.

The first two terms are positive, while the last can be rewritten as $[\bar{I} - (1+\gamma)\bar{I}^2 + P]$. As $\gamma \to 0$, the last term approaches $\bar{I} - \bar{I}^2 + P$. For values of $\bar{I}$ that are not too large, that term is positive, and we have the counterexample.

In this counterexample, the agent bears all the risk if he is risk neutral, but assumes none if he is even slightly risk averse. In other words, convergence fails in a spectacular way. But it may fail in more prosaic fashions as well. The limit of the risk-averse case may be higher or lower than τ_{min}. Figure 1 schematically illustrates these possibilities.

Mathematically, convergence fails because of a difference between min for the risk-neutral case and for the limit of the risk-averse case. This difference is

$$(14)\quad \tau_{min} - \lim_{\gamma \to 0} \tau = [G(a_1) - G(a_j)] \,/\, [B(a_j) - B(a_1)] - [G(a_k) - G(a_j)] \,/\, [B(a_j) - B(a_k)].$$

Indifference at zero risk aversion implies $G(a_1) + B(a_1) = G(a_k) + B(a_k)$, creating two distinct possibilities: Either a_k or a_1 can be the high-cost, high-benefit action. If $B(a_k) > B(a_1)$, then $G(a_k) < G(a_1)$, and vice versa. The sign of equation (14), then, can go either way.

Despite the myriad possibilities, nonconvergence remains a special case. To start with, the principal must be indifferent between two different actions of the risk-neutral agent, and she must strictly prefer one action for arbitrarily small levels of risk aversion.

Increasing the Number of Actions

The lowest share of risk the agent can take, τ_{min}, is decreasing in the gap in the principal's payoff between the chosen and the indifferent act, $[G(a_k) - G(a_j)]/[B(a_j) - B(a_k)]$. It seems intuitive that as the number of actions increases, the gap decreases and min moves toward one, its value in the continuous-action case. But it is possible to work the convergence so that exceptions occur. If B and G are continuous functions, some condition on the difference (such as $\lim |a_k - a_{k+1}| = 0$) would ensure the result.

III. Conclusion

The traditional solution to the risk-neutral principal–agent problem is misleading. With finite states and finite actions, many solutions exist, and in all but one of these the *principal* bears the risk. The traditional solution cannot even claim to be the limiting case as risk aversion decreases: In fact, it is the solution farthest away from the limit.

These results have two main consequences. First, they caution us against using the traditional solution as an approximation of the less tractable risk-averse case. This may explain why Jensen and Murphy (1990) found CEOs bearing a surprisingly low amount of risk. It also explains, in part, why the numerical calculations of CEO risk in Haubrich (1994) were so small, even for very low levels of risk aversion. Second, they illustrate the range, power, and tractability of Grossman and Hart's version of the principal–agent model.

Nevertheless, the results presented here should be taken as preliminary—brief observations of a rare nocturnal animal. Proposition 1 provides sufficient, but not necessary, conditions for multiple solutions and does not characterize all possible solutions. The convergence results require even stronger restrictions and depend on the two-act case. Still, I believe the scattered sightings reported here show a surprising—and noteworthy—aspect of the principal–agent model.

References

Grossman, Sanford J., and Oliver D. Hart. "An Analysis of the Principal–Agent Problem," *Econometrica*, vol. 51, no. 1 (January 1983), pp. 7–45.

Haubrich, Joseph G. "Risk Aversion, Performance Pay, and the Principal–Agent Problem," *Journal of Political Economy,* vol. 102, no. 2 (April 1994), pp. 258–76.

Holmstrom, Bengt. "Moral Hazard and Observability," *Bell Journal of Economics*, vol. 10, no. 1 (Spring 1979), pp. 74–91.

Jensen, Michael C., and Kevin J. Murphy. "Performance Pay and Top-Management Incentives," *Journal of Political Economy*, vol. 98, no. 2 (April 1990), pp. 225–64.

Mirrlees, James A. "The Optimal Structure of Incentives and Authority within an Organization," *Bell Journal of Economics,* vol. 7, no. 1 (Spring 1976), pp. 105–31.