Loss Characteristics of Commercial Real Estate Loan Portfolios

A White Paper by the staff
of the Board of Governors
of the Federal Reserve System

Prepared as Background for Public Comments
on the forthcoming
Advance Notice of Proposed Rulemaking
on the Proposed New Basel Accord

June 2003

Bradford Case
Economist
Board of Governors of the Federal Reserve System
Mail Stop 153
Washington, DC 20551
202/452-2442 (voice)
202/452-5295 (fax)
bradford.case@frb.gov

Foreword

This White Paper presents the data and analysis reviewed by the staff of the Board of Governors to develop the proposals for applying capital requirements to commercial real estate (CRE) loans in the United States under the Advanced Internal Ratings-Based approach of Basel II. It is being released before the supervisors' Advance Notice of Proposed Rulemaking (ANPR) on Basel II, expected to be available in early July, in order to provide interested parties additional time to examine and evaluate the staff's analysis.

The Board of Governers, Office of the Comptroller of the Currency, and Federal Deposit Insurance Corporation are particularly interested in additional data or alternative empirical analyses for establishing capital requirements for CRE exposures. We are also interested in alternative analytical frameworks that would be useful in evaluating the issues discussed in this White Paper. Comments, evaluations, and criticisms that address the asset correlation evidence and analysis for individual property types would be particularly useful.

Commenters should feel free to contact the principal researcher, Bradford Case, whose contact information is shown on the cover sheet. Comments on the aforementioned ANPR on Basel II should, however, be sent no later than the end of the comment period to the addresses detailed in the ANPR.

Roger W. Ferguson, Jr.
Vice Chairman
Board of Governors of the Federal Reserve System

I. Introduction and Summary

On April 29, 2003, the Basel Committee on Banking Supervision issued its third consultative paper (CP3) seeking public comment on a proposed new framework (termed "Basel II") for setting minimum capital requirements at large, internationally active banks. The centerpiece of this new framework is the Internal Ratings-Based (IRB) approach, which is designed to make minimum capital requirements for credit risk much more risk sensitive than under the 1988 Basel Accord. In developing the proposed IRB approach, the Basel Committee has relied heavily on results from empirical studies undertaken to quantify the key drivers of portfolio credit risk for various lending activities. The comment process provides an important opportunity for the banking industry, as well as the public generally, to assess the reasonableness of the proposals and their empirical underpinnings and, where warranted, to suggest possible improvements.

The IRB treatment of commercial real estate (CRE) lending is an area in which constructive public feedback is critical.[1] In comparison with some other lending activities, the empirical research available to the Basel Committee for use in calibrating CRE risk parameters has been much more limited. An especially challenging limitation has been the paucity of loan-level data covering multiple CRE credit cycles. The principal objective of this paper is to summarize the current research, most of which has been conducted by the staff of the Federal Reserve Board. In so doing, we hope to stimulate informed, broad-based public comments in

[1]Within CP3, a CRE exposure is defined as a loan or other financing for the purpose of funding construction or acquisition of commercial real estate (such as offices, retail space, multifamily residential buildings, industrial or warehouse space, hotels, etc.) where the prospects for repayment and recovery depend primarily on the cash flows generated by that asset.

this area and to learn of any additional research or data that might be helpful in shaping the IRB approach.

A. The IRB Approaches to CRE Lending

Within the United States, the banking agencies are proposing to implement Basel II by allowing two methods for calculating capital requirements for CRE loan portfolios. The first method, termed the Advanced Internal Ratings-Based (A-IRB) Approach, would be the same as that used for all other loan portfolios under the U.S. implementation of the new accord. A second method, termed the supervisory slotting criteria approach, or the Basic IRB Approach, would be available only for CRE loans.

Under both the Advanced and Basic approaches, a CRE loan, like credits in other portfolios, would incur a minimum capital requirement per dollar of exposure (a "capital charge") based on estimates of the loan's one-year probability of default (PD), loss rate given default (LGD), and effective maturity (M). The two approaches differ primarily in how these loan-specific risk parameters are determined.

Under the Advanced Approach, the estimation of these parameters is done by the bank subject to supervisory review. The estimates are then converted into capital charges by substituting them into one of two regulatory capital functions. One capital function (termed the HVCRE function[2]) would apply to CRE portfolios with relatively high asset correlations (that is

[2] HVCRE stands for "high volatility commercial real estate," the term employed by the Basel Committee to describe high-asset-correlation CRE portfolios.

with a tendency for defaults of the loans in the portfolio to occur in "clumps"), while the other (termed the IPRE function[3]) would apply to CRE portfolios with relatively low asset correlations. For a given PD, LGD, and M, a loan in a high-asset-correlation portfolio would incur a higher capital charge than would a loan in a low-asset-correlation portfolio. Under Basel II, subject to certain exceptions discussed below, all lending to finance acquisition, development, and construction (ADC) must be assigned to the high-asset-correlation (HVCRE) category. However, the U.S. banking agencies have sole responsibility for determining which, if any, types of CRE lending that finances in-place U.S. commercial real estate properties should receive the high asset correlation treatment.

The Basic Approach must be employed by banks that are unable to reliably estimate the risk parameters (PD, LGD, and M). Capital charges are determined through a two-step process, which begins with the bank using slotting criteria established by the Committee to assign the CRE loans in each of its internal risk-rating grades to one of five supervisory risk-rating grades. Each supervisory grade is then associated with a specific capital charge calibrated to be consistent with the same capital functions that are explicitly applied under the Advanced Approach and with supervisory values of the loan-level risk parameters (PD, LGD, and M). Because the Basic Approach is based on the same capital functions, it too would imply higher capital charges for high asset correlation CRE loans, all other things equal. Thus, the definition of high-asset-correlation CRE portfolios is relevant not only for the Advanced Approach, but for the Basic Approach as well.

---

[3]IPRE stands for "income producing real estate," the term employed by the Basel Committee to describe low-asset-correlation CRE portfolios.

These proposals raise two broad sets of policy questions on which the U.S. regulatory agencies will be seeking input from the public:

1. *What types of in-place CRE properties, if any, should be treated as HVCRE?*
2. *What should be the capital charges under the Basic Approach?*

In large measure these questions are empirical. To provide an objective basis for decision-making, we have carried out empirical studies aimed at quantifying the extent to which the marginal contribution of a CRE loan to a bank's overall portfolio credit risk depends on the type of property being financed, after controlling for the other IRB inputs, PD, LGD, and M. In addition, we have reviewed published studies summarizing the historical loss characteristics of CRE portfolios, such as loss rates on defaulted CRE loans. The findings from these analyses are the subject of this white paper.

B. What Types of In-Place CRE in the U.S. Should Be Treated as HVCRE?

To address this question, we have carried out research using standard techniques of credit-risk modeling that attempt to quantify the relationships between (1) a CRE loan's marginal contribution to portfolio credit risk, (2) the loan's stand-alone risk parameters (PD, LGD, and M), and (3) the type of property being financed. Modern portfolio theory implies that capital charges should reflect not only a loan's stand-alone credit worthiness but also the correlation between potential credit losses on that loan and credit losses on the rest of the portfolio. For a given exposure amount, such correlations may arise through two channels: (1) correlations among defaults on individual loans and (2) correlations among loss severities when defaults occur.

The IRB framework captures the first channel, correlations among defaults, through a portfolio-level parameter, termed the portfolio's *asset correlation*. Conceptually, this parameter represents the average correlation among future returns on the assets (e.g., the commercial properties) whose cash flows support the individual loans. Asset correlations are used to quantify the tendency for individual loans to default in groups or clumps, if they default at all. For a given level of expected defaults, a loan portfolio with a higher asset correlation is likely to exhibit greater variability in aggregate default rates, and hence require a higher capital charge, compared with a portfolio with a lower asset correlation. Within the IRB framework, the Basel Committee has specified for each broad portfolio type (e.g., commercial and industrial or C&I, HVCRE, IPRE) an assumed relationship determining each loan's asset correlations as a function of its PD. In turn, the assumed asset correlation determines the relationship, reflected in the regulatory capital formula for that portfolio, between the PD, LGD, and M for each loan of that type and its associated capital charge.

The second channel, involving correlations among the loss severities of defaulting loans, also is an important concern, particularly within the Basic Approach. Available evidence indicates that loss severities on CRE loans are highly cyclical: Loss severities tend to be relatively high when default rates are relatively high, and low when default rates are low. Since the role of minimum regulatory capital requirements is to provide a cushion sufficient to absorb losses during periods of economic stress (i.e., high default rates), CRE capital charges should incorporate reasonable assumptions concerning the loss severities likely to prevail during such periods.

To account for cyclicality in LGDs, some banks adjust the assumed asset correlation for CRE lending upward from where it might have been set in the absence of LGD cyclicality. Under this treatment, LGDs for CRE loans are measured as the default-weighted average loss severities of such exposures after taking into account relevant factors such as loan-to-value (LTV) ratios. A recent survey by the Risk Management Association (RMA [2003]), suggests that banks as a whole tend to set the asset correlations for CRE lending materially higher than those for C&I lending, in part to reflect cyclicality in LGDs.

Within the A-IRB framework, cyclicality in LGDs is addressed in a more direct manner. Specifically, banks are required to measure the LGD input for each loan as the loss severity expected to prevail during periods of relatively high defaults rates ("adjusted LGD").[4] This method of dealing with LGD cyclicality has several conceptual advantages over the asset-correlation adjustments applied by many banks. First, it is more consistent with the analytic framework underlying the IRB approach.[5] Second, the A-IRB framework affords banks greater opportunity to use their own internal models for determining the degree to which cyclicality in LGDs varies across different types of loans; the alternative would be a more or less one-size-fits-all supervisory formula for determining a loan's adjusted LGD as a function of its average LGD. Lastly, separating the calibration of asset correlations and adjusted LGDs may be viewed as desirable from the standpoint of transparency and of focusing research on the relevant empirical issues.

[4]For defaulted loans, the LGD input is measured as the expected loss rate on that exposure.
[5]See Gordy [forthcoming].

Research to quantify asset correlations for different types of CRE lending has proceeded along two tracks. One track addresses whether the asset correlations characterizing well-diversified portfolios of CRE loans are materially different from those associated with well-diversified portfolios of C&I loans, the other whether asset correlations for CRE lending are substantially different depending on the types of properties being financed.

Within their own internal economic capital systems, some banks employ asset-correlation assumptions for CRE that are similar to those they use for C&I lending. If supported empirically, a similar treatment under Basel II would permit the same regulatory capital functions to be employed for both CRE and C&I lending, thus simplifying the IRB framework. Within a diversified CRE portfolio, such an approach would presume that CRE loans displaying relatively high asset correlations would be approximately balanced by CRE loans having relatively low asset correlations.[6]

To date, this research has been hampered severely by a paucity of relevant historical data. Detailed information on the historical performance of CRE lending by banks and thrifts has been restricted to sample periods covering the 1990s and early 2000s -- not even a complete economic cycle for the CRE sector. On balance, estimates of asset correlations based on historical bank and thrift charge-off statistics suggest that CRE lending as a whole has exhibited asset correlations well above those for C&I lending, with the largest asset correlations attributable to CRE loans financing ADC for other than single-family homes.

---

[6]Obviously, this treatment would not address issues relating to a bank having a concentration in portfolios with high asset correlations. However, a basic premise of Basel II is that portfolio concentration issues are not treated explicitly within the regulatory capital setting (Pillar 1) but instead should be addressed through supervisory channels (Pillar 2).

More recently, however, we have conducted comparative analyses of asset correlations for C&I using data on default rates for externally rated corporate bond issuers and estimates for CRE loans using data on CRE foreclosure rates at life insurance companies. The findings derived from these new data sources conflict with the earlier results, a difference at least partly reflecting the much longer time period covered by the more recent analysis, 1970-2001. Over the entire sample period, the new data suggest that asset correlations for CRE lending as a whole are about the same as, or only slightly above, those for C&I lending. In general, the more the sample period is weighted toward the experience of the early 1990s -- by eliminating data from prior years -- the greater the estimated asset correlations for CRE lending compared with C&I lending. When using the same sample period (1990-2001), the findings based on external rating agency and life insurance company data are qualitatively similar to the earlier findings based on bank and thrift data.

Although one may have concerns about whether CRE portfolios held by life insurance companies are representative of bank and thrift portfolios, these conflicting results raise questions about asset correlation estimates drawn from the 1990s. More generally, they suggest a need for caution regarding the interpretation of asset-correlation estimates that are based on the historical performance of portfolios whose composition and risk characteristics vary substantially over time. Although supervisory judgment and experience supports the high asset-correlation assumption, the banking agencies have concluded that there is not yet a strong enough empirical basis for including any U.S. in-place property types in Basel II's HVCRE category. An Advance Notice of Proposed Rulemaking (ANPR) to be issued by the U.S. regulatory agencies in early July will invite comments on the reasonableness of this proposal and will solicit additional research findings and data that may be relevant to this topic.

C. What Should Be the Capital Charges under the Basic Approach?

As noted above, capital charges under the Basic Approach are calibrated to be broadly consistent with the capital functions employed in the Advanced Approach using supervisory values of PD, LGD, and M for each supervisory rating grade. Thus, given the explicit capital function for each type of CRE lending, determined from asset correlation assumptions set by the Basel Committee, the capital charges under the Basic Approach flow directly from the assumed PD, LGD, and M parameters for each supervisory grade.

Section V below discusses for each supervisory grade the risk parameter assumptions underlying the proposed supervisory capital charges under the Basic Approach. As described in that section, the slotting criteria for each supervisory grade are presumed to be consistent with external ratings falling within an associated band. The implicit PD for each supervisory grade is based on historical one-year default rates for firms having external ratings at the lower end of the associated band. The average effective maturity is assumed to be five years for CRE loans financing in-place properties and one year for ADC lending. Drawing from published studies of historical loss severities for CRE loans, the Basic Approach assumes adjusted LGDs in the vicinity of 35 to 43 percent for loans financing in-place properties, and roughly 55 percent for loans financing ADC.

An important question that will be posed in the ANPR is whether the extant research in this area can, or should, be used as a predictor of future loss severities on CRE loans during cyclical downturns. The adjusted LGD assumptions cited above are drawn mainly from the recovery experience for CRE loans defaulting in the early 1990s. In light of widespread improvements in the CRE loan underwriting and risk management practices by many lenders following the CRE problems of the late 1980s and early 1990s, some observers have suggested that historical loss severities on CRE loans may be a misleading indicator of future performance. Also, virtually all publicly available evidence on loss severities for CRE loans is based on the performance of life insurance companies rather than of depository institutions.

D. Organization of Paper

The remainder of this paper is organized as follows. Section II reviews in detail the proposed IRB treatments of CRE loan portfolios under Basel II. Section III describes the conceptual underpinnings of the IRB capital functions, with special emphasis on the role played by asset correlation assumptions in linking capital charges to estimates of PD, LGD, and M. Section IV summarizes research undertaken to estimate asset correlations for CRE loan portfolios. Section V summarizes empirical evidence underpinning the calibration of capital charges within the Basic approach. Concluding remarks are presented in Section VI.

II. Proposed IRB Treatments of CRE Loan Portfolios

As noted, the proposal for Basel II implementation in the United States will provide two general methods for determining minimum capital requirements against CRE loans, depending on a bank's ability to provide reliable estimates of the PD, LGD, and M for each exposure. Under the Advanced IRB Approach, the bank itself will estimate these parameters, and the capital charge will be calculated by inserting these estimates into either of two regulatory capital functions, one applicable to high-asset-correlation CRE portfolios (HVCRE, as defined below) and the second applicable to low-asset-correlation CRE portfolios (IPRE). For loans that have defaulted, the LGD will be estimated as the expected loss rate on those loans, while for other loans the LGD will be estimated as the expected loss severity when default rates are relatively high ("adjusted LGD"). Appendix A displays these regulatory capital functions.

The Basic Approach is used when a bank is unable to estimate the above risk parameters reliably. Under this treatment, a bank must use slotting criteria established by the Basel Committee to assign each CRE loan to one of five supervisory rating grades (Strong, Good, Satisfactory, Weak, and Default). To aid in the assignment of supervisory rating grades, each grade is associated with an explicit range of external ratings. In addition to the five supervisory rating grades, at national discretion the supervisory agencies may also permit banks to apply preferential capital charges to a "strong" or "good" loan having either (1) a remaining maturity of less than 2.5 years or (2) underwriting criteria and other risk characteristics substantially stronger than implied by the slotting criteria for its supervisory rating grade. The latter is tantamount to having an additional "very strong" supervisory risk-rating category. For a given supervisory rating grade, the capital charge would again depend on whether the loan is categorized as part of a high- or low-asset-correlation CRE portfolio.

Specifically what constitutes a high-asset-correlation CRE portfolio in each country is subject to national discretion. Each national supervisor is expected to identify which, if any, commercial property types within its jurisdiction tend to be associated with relatively high asset correlations. All CRE loans backed by what are deemed to be high asset correlation property types, including ADC loans for such property types, are to be treated as HVCRE loans.[7] In addition, other ADC lending must be treated as HVCRE if the future sale proceeds or cash flows from the property are substantially uncertain, unless the borrower has substantial equity at risk.

For reference, Table 1 below summarizes for each supervisory rating category the associated ranges of external ratings and IRB capital charges.[8] This table treats the overall effect of the preferential treatment for certain CRE loans as equivalent to permitting a "very strong" rating grade. Note that loans slotted as Very Strong, Strong, and even Good could have capital charges below or equal to the current 8 percent level, while others would have higher capital charges in keeping with the greater risk sensitivity of the Basel II proposal.

---

[7]The same definition of HVCRE -- as established by each national supervisor for properties located within its jurisdiction -- must be used both by domestic and foreign banks when making CRE loans secured by such properties in the country of that national supervisor.

[8]The capital charges are expressed in terms of total regulatory capital. The Tier 1 capital charge is one-half the total capital charge.

Table 1
Summary of Basic IRB Approach for CRE Exposures

Supervisory Rating Grade*	Associated Range of External Ratings	Total Regulatory Capital Charge	
		HVCRE	IPRE
Very Strong/ Preferential	BBB+ or better	6%	4%
Strong	BBB or BBB-	8%	6%
Good	BB+ or BB	10%	8%
Satisfactory	BB- or B+	14%	12%
Weak	B to C-	28%	28%
Default		50%	50%

*The range of external ratings for each grade, and the associated capital charges, incorporate the effect of the preferential capital treatment afforded certain CRE loans whose remaining maturities or other risk characteristics imply levels of credit quality substantially better than normally associated with supervisory grades of strong or good. The table assumes that, inclusive of the preferential treatment, the overall thrust of the proposal is to attribute capital charges for HVCRE and IPRE loans equal to 8 percent and 6 percent (6 percent and 4 percent), respectively, when these loans have credit quality between BBB- or BBB (BBB+ or better).

To facilitate comparisons between the Advanced and Basic approaches, Table 2 shows the risk weights that would be applied to high-asset-correlation (HVCRE) and low-asset-correlation (IPRE) CRE portfolios over ranges of values for PD, LGD, and M. In most cases, for a given PD, LGD, and M, capital charges are lower under the Advanced Approach compared with the Basic Approach, to provide an incentive for banks to develop advanced risk management capabilities for their CRE portfolios over time.

Table 2
Comparison of Capital Charges under the Advanced and Basic IRB Approaches

Advanced IRB Approach						Basic IRB Approach		
Reference Rating	PD	LGD	M	HVCRE	IPRE	Supervisory Risk-Rating	HVCRE	IPRE
BBB+	0.19%	35%	5	5.4%	4.2%	Very Strong/ Preferential	6%	4%
BBB+	0.19%	55%	5	8.5%	6.5%			
BBB+	0.19%	35%	1	2.5%	1.9%			
BBB+	0.19%	55%	1	3.9%	2.9%			
BBB	0.30%	35%	5	6.5%	5.1%	Strong	8%	6%
BBB	0.30%	55%	5	10.2%	7.9%			
BBB	0.30%	35%	1	3.2%	2.5%			
BBB	0.30%	55%	1	5.1%	4.0%			
BBB-	0.34%	35%	5	6.8%	5.3%			
BBB-	0.34%	55%	5	10.7%	8.4%			
BBB-	0.34%	35%	1	3.5%	2.7%			
BBB-	0.34%	55%	1	5.5%	4.3%			
BB+	0.55%	35%	5	8.1%	6.5%	Good	10%	8%
BB+	0.55%	55%	5	12.7%	10.2%			
BB+	0.55%	35%	1	4.5%	3.6%			
BB+	0.55%	55%	1	7.1%	5.7%			
BB	1.13%	35%	5	9.8%	11.5%			
BB	1.13%	55%	5	15.4%	13.0%			
BB	1.13%	35%	1	6.1%	5.2%			
BB	1.13%	55%	1	9.6%	8.2%			
BB-	2.06%	35%	5	11.0%	9.8%	Satisfactory	14%	12%
BB-	2.06%	55%	5	17.2%	15.5%			
BB-	2.06%	35%	1	7.5%	6.7%			
BB-	2.06%	55%	1	11.8%	10.6%			
B+	3.53%	35%	5	12.1%	11.5%			
B+	3.53%	55%	5	19.1%	18.1%			
B+	3.53%	35%	1	8.9%	8.5%			
B+	3.53%	55%	1	14.0%	13.3%			
B	10.78%	35%	5	18.1%	18.1%	Weak	28%	28%
B	10.78%	55%	5	28.4%	28.4%			
B	10.78%	35%	1	15.1%	15.1%			

B	10.78%	55%	1	23.7%	23.7%			

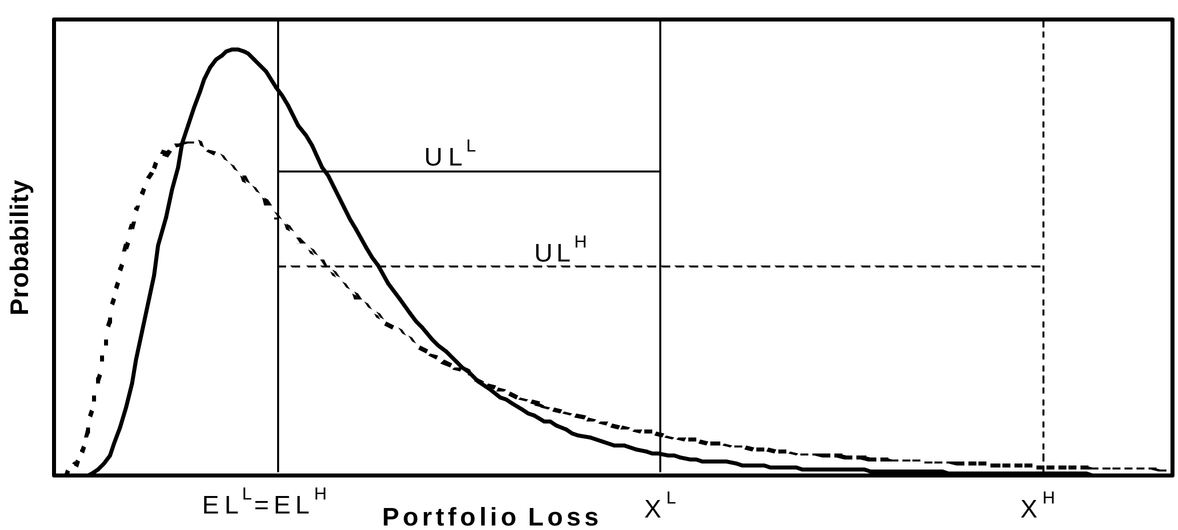
III. Analytic Underpinnings of the IRB Framework

The IRB framework is essentially a regulatory economic capital model for credit risk. In principle, IRB capital charges are calibrated to cover a portfolio's economic credit losses over the next year with a relatively high (99.9 percent) degree of confidence, termed the "loss coverage target." This section describes the analytic basis for the unexpected-loss concept and how it is implemented within the IRB framework, with particular emphasis on the role of estimated asset correlations for various loan types.

A. The Relationship Between Unexpected Losses and Asset Correlations

The chart below shows two portfolio loss distributions with the same *expected loss*, marked $EL^L = EL^H$. The two distributions differ in their *asset correlations*, with the dashed line

Graph 1: Portfolio Loss Distribution



representing a distribution with a larger asset correlation. The skew in each distribution indicates that relatively small portfolio losses are much more likely than are relatively large losses, as shown by the height of the distribution. The points X^L and X^H are selected so that the area under the corresponding distribution to the left of this point is equal to the loss coverage target (e.g., 99.9 percent).

The economic capital systems used for internal management purposes by many banks generally presume that expected losses (EL^L=EL^H) are covered by reserves and/or margin income. This implies that economic capital is needed to cover only unexpected losses (UL^L or UL^H), measured as the difference between X^L or X^H and EL^L=EL^H. Consistent with this methodology, banks typically measure their actual maintained economic capital as Tier 1 or equity capital, with perhaps an adjustment for any surplus or shortfall in the level of loan loss reserves. Because the Basel Committee's total regulatory capital measure is defined inclusive of general loan loss reserves[9], minimum capital requirements under the IRB framework are calculated *inclusive* of a measure of expected losses. Specifically, the capital requirement is determined as (1) the portfolio's estimated UL *plus* (2) the sum across individual credit facilities of PD x LGD x EAD, where EAD is the facility's exposure at default.

The above chart is drawn to highlight the fact that two portfolios with identical expected loss rates may nevertheless exhibit very dissimilar unexpected losses owing to different underlying asset correlations. Within industry-standard credit risk models, the asset correlation applicable to a given loan portfolio is a measure of the average correlation among the returns on the assets (e.g., individual commercial properties) supporting these loans. The asset correlation

---

[9]General loan loss reserves may be included in total regulatory capital up to 1.25 percent (15.625 percent) of a bank's total risk-weighted assets (minimum required total capital).

is a key determinant of the extent to which individual loans are likely to default together: a small asset correlation indicates that loans in that portfolio tend to default independently of each other, and a large asset correlation indicates that loans tend to default in clumps. When borrower defaults are largely independent of one another, the ratio of unexpected losses to expected losses (UL/EL) at the portfolio level would be relatively low, because it is unlikely that enough defaults would occur at the same time to produce a large portfolio loss. On the other hand, when defaults tend to occur in clumps, then the UL/EL ratio would be relatively high because it is more likely that many loans could default together and produce a large portfolio loss.

Appendix B provides a technical discussion of how asset correlations are used within the IRB framework to translate the loan-specific risk parameters (PD, LGD, and M) into an estimate of a loan's contribution to the portfolio's overall UL. The appendix shows that for given values of these parameters, a loan's marginal contribution to the portfolio's estimated UL is an increasing function of the asset correlation applicable to that type of loan. That is, a higher asset correlation leads to a higher capital charge, other things the same.

B. Determinants of Asset Correlations

The factors influencing a CRE loan portfolio's asset correlation are generally not the same as those influencing an individual loan's expected loss. Numerous empirical studies suggest that the most important drivers of loan-level risk are a bank's underwriting standards, such as loan-to-value ratio (LTV) and debt-service-coverage ratio (DSCR).[10] In contrast, as noted above, the asset correlation measures the average correlation among the future rates of

[10]See, for example, the studies summarized in appendix E.

return on the properties supporting the individual CRE loans. Conceptually, the asset correlation will be larger (smaller) the greater (lesser) the extent to which the rates of return on different properties are likely to respond similarly to a common set of economic risk factors, such as overall economic activity, inflation, tax code changes, etc.

This observation has an important implication for the empirical analysis of asset correlations. It has been noted that in the aftermath of the CRE problems of the late 1980s and early 1990s, many CRE lenders strengthened their underwriting standards. Obviously, such improvements would lower PDs and LGDs, thereby reducing capital charges, all other things being equal. However, CRE asset correlations would not necessarily decline because an asset correlation is fundamentally a measure of the degree to which the net operating income and market values of individual commercial properties that serve as the source of the loan's repayment move together over time; it is not a measure of the credit quality of the individual loans used to finance those properties.

The asset correlation for a given CRE loan portfolio is likely to depend significantly on geographic concentrations within the portfolio.[11] Commercial real estate markets tend to be dominated by demand and supply conditions that are regional or local in nature. Properties located within the same geographic region, therefore, are likely to be influenced by similar economic factors, and their returns are likely to be more highly correlated with each other than with properties located elsewhere. Geographically concentrated CRE portfolios will thus tend to exhibit higher asset correlations than portfolios that are less concentrated. Under the IRB

---

[11]The IRB framework abstracts from *granularity* issues--the effect of the number of loans in the portfolio--by assuming implicitly that each bank has effective procedures to limit exposure concentrations to single borrowers.

framework, the asset correlation assumptions for CRE loans are intended to reflect a geographically well-diversified portfolio.

C. Summary

Within the A-IRB framework, the asset correlations attributed to CRE and other portfolios are the primary factors linking capital charges to bank-supplied estimates of each loan's PD, LGD, and M. Other things equal, portfolios with larger asset correlations can be expected to experience greater variability in aggregate default rates and, hence, are subject to higher IRB capital charges. Importantly, asset correlations measure the tendency for the economic returns of the assets supporting the loans in a portfolio (e.g., the CRE properties) to move together over time; they are not a measure of an individual bank's underwriting standards nor of the expected losses inherent in its portfolio. Indeed, depending on underwriting criteria, such as LTVs, loans financing assets with high asset correlations could have high or low levels of expected losses. Similarly, loans financing assets with low asset correlations also could have high or low expected losses. The next section summarizes research undertaken to quantify the asset correlations of CRE loan portfolios.

IV. Empirical Evidence on CRE Asset Correlations

The numerical inputs required to determine minimum regulatory capital requirements under any version of the IRB approach include the estimated PD, LGD, and M of each loan as well as the assumed asset correlation of the loan portfolio. Under both the Advanced and the Basic versions of the IRB approach, the asset correlation parameter is specified by the Basel Committee. This section summarizes the available empirical evidence regarding asset correlations for CRE loan portfolios.

The presentation and discussion in this section are organized around two key questions:

1. Are the asset correlations characterizing well-diversified CRE loan portfolios materially greater than, about equal to, or less than those characterizing well-diversified C&I loan portfolios?

2. Are asset correlations for certain sub-portfolios of CRE loans materially larger than asset correlations for other sub-portfolios?

The second question is motivated by the Basel Committee's decision to establish separate capital treatments for CRE loan portfolios designated as having a high asset correlation (HVCRE) and a low asset correlation (IPRE). Consistent with the proposal described in CP3, the discussion in this section presumes that LGD inputs (determined by banks under the Advanced treatment and by supervisory values under the Basic treatment) are estimated as the loss severities expected to prevail during unfavorable periods of the credit cycle, when CRE default rates are relatively high.[12] Also, this discussion presumes that a bank's CRE portfolio is well diversified geographically and that concerns regarding lack of diversification will be treated within the supervisory process (*i.e.*, under Pillar 2) rather than through adjustments to regulatory capital charges.

1. Are Asset Correlations Materially Different for CRE and C&I Loan Portfolios?

Broadly, two empirically based methods are available for calibrating the asset correlation for a given loan portfolio, depending on available sources of data. One method employs historical data on the default rates of portfolios similar to the portfolios of interest; the other employs data on the market values of the assets whose cash flows support these loans.

Intuitively, for a well-diversified portfolio of loans having a given PD, a higher asset correlation implies greater uncertainty regarding the actual frequency of defaults over the next year. This insight suggests that an estimate of the asset correlation can be inferred from the imprecision (*e.g.*, the variance) surrounding predictions of one-year-ahead default rates. Two alternative techniques are employed in practice for estimating asset correlations from historical default rates: (1) a maximum likelihood estimator and (2) a method-of-moments estimator.[13] Appendix C gives a brief summary of these methods.

---

[12] This discussion pertains only to CRE loans that have not yet defaulted. As noted, within the IRB proposal the LGDs for defaulted loans are always to be measured as the loss rates that the bank actually expects to incur as those loans are worked out. It is only for nondefaulted loans that loss severities are to be measured as stress LGDs.

[13] The method of moments estimator was introduced in Gordy [2000]. Gordy & Heitfield [2002] show that this estimator is biased downward when the available data are restricted to a small number of years and that the maximum likelihood estimator has better performance characteristics.

Asset correlations can also be estimated using data on the market values of the firms or properties whose cash flows support the loans in the portfolio,[14] a technique that is also summarized in appendix C. This methodology has been employed with considerable success to calibrate the IRB asset correlations for C&I loans (see Lopez [2002]), and work to apply it to CRE portfolios is ongoing, as described below. Thus, the following empirical summary focuses entirely on analyses based on historical default rates.

Estimates Based on Bank Charge-off Rates

Since 1991, the quarterly Statements of Condition and Income ("Call Reports") submitted by U.S. banks have included detailed disclosures of charge-offs by broad categories of loans including loans secured by multifamily residential properties, loans secured by nonfarm nonresidential properties, ADC loans, and C&I loans not secured by real estate. For each loan type, dollar-weighted default rates were estimated for each of the 20 largest U.S. banks, and for all U.S. banks together, as the annual net charge-off rate divided by an assumed loss severity for that year.[15]

The LGD for CRE loans was estimated from annual net charge-off rates using the relationship LGD = 23.9 percent + 0.32*default frequency. This yields a time-varying LGD reflecting cyclicality in CRE loan recovery rates. This estimated relationship was developed on

[14]In the context of C&I loan portfolios, KMV Corporation was a pioneer in implementing this methodology, which is based on the seminal work of Merton.
[15]Net charge-offs were estimated as gross charge-offs less average recoveries (recoveries as a percent of gross charge-offs, averaged for each bank over the entire time period). The net charge-off rate was given by total estimated net charge-offs for the year divided by average loan volume for the year. Bank Call Reports also disclose data on loans past due and loans in nonaccrual status, which could be used in the same way as data on loan charge-offs to form a proxy measure of annual loan default rates. Staff estimated asset correlations using these two alternative data series, but the results were qualitatively identical and quantitatively very similar to the results based on charge-off data, and so they are not reported in this paper.

the basis of data on PD and LGD for CRE loans submitted by three large U.S. banks as part of the 2001 data collection exercise known as QIS 2.5[16] and is described in appendix E.

For C&I loans, historical data were not available permitting LGDs to be linked to observed default rates. Rather, a constant LGD of 45 percent was assumed. To the extent that loss severities on C&I loans increase during periods of high default rates, this constant LGD assumption may bias upward the estimated asset correlations for C&I loans relative to those for CRE loans by attributing too much of the volatility in historical charge-off rates to the volatility of underlying default rates.

With these proxies for annual loan default rates, asset correlations were estimated for CRE and C&I loan portfolios using both the maximum likelihood and method-of-moments estimators. Before discussing the empirical results of these analyses, however, it is useful to detail several important shortcomings of these methods. First, there are potentially severe measurement errors in attempting to infer default rates from reported charge-off data. Potential sources of error include the assumed loss severities for each year as well as the likelihood that net charge-offs reported in any calendar year reflect defaults not only from that year but from previous years as well. Second, the available data span only 12 years, which is less than a full CRE credit cycle. Estimates of asset correlations based on historical default rates over short time spans are biased downward and subject to relatively large estimation errors.[17] Lastly, the data clearly violate a key assumption underlying standard applications of both the maximum likelihood and method-of-moments techniques, namely that the default rates pertain to *homogeneous* portfolios having the *same PD* at the beginning of each year. Potentially, this

[16]The necessary data were not collected as part of QIS3.
[17]Gordy & Heitfield [2002].

could produce a bias in either direction. In short, the results of either empirical method must be interpreted with caution.

Table 3 summarizes the CRE and C&I asset correlations estimated using the two empirical methods on the basis of bank Call Report data for 1991-2002. Two sets of estimates are reported for each empirical technique: the median of the estimates for the 20 largest U.S. banks using bank-level data and the estimate derived from aggregate data for all U.S. banks combined. These asset correlation estimates may be biased.[18] To the extent that this bias is consistent across loan types, though, it is unimportant because the question at hand is simply whether the CRE asset correlation is *equal to* (or less than) the C&I asset correlation or whether it is materially *greater than* the C&I asset correlation. For this purpose it is most useful to compare the ratios of asset correlations computed for CRE portfolios to asset correlations computed for C&I portfolios. As the bottom row of Table 3 shows, as applied to these data both methods suggest much higher asset correlations for CRE loan portfolios than for C&I portfolios: roughly 2.5 times as high at the median using bank-level data for the largest U.S. banks, and nearly 4 times as high using aggregate data for the U.S. banking industry.

[18]In addition to the potential biases arising from the violation of the assumptions that default rates pertain to homogeneous portfolios having the same PD at the beginning of each year, under the loan type definitions used in the Call Reports the nonfarm nonresidential, multifamily, and ADC categories include loans that would be characterized under Basel II as C&I loans secured by real estate, rather than as CRE loans. To the extent that the correct underlying asset correlation for C&I loans secured by real estate is lower than that for CRE loans, this implies that the asset correlation estimated using Call Report data is biased downward for CRE loans. (Whether the estimated C&I asset correlation is biased downward or upward depends on whether the correct underlying asset correlation for C&I loans secured by real estate is higher or lower than that for other C&I loans.)

Table 3: CRE and C&I Asset Correlations Estimated from Bank Call Report Data

	Median Bank-Level Estimate		Estimate from Aggregate Data	
	Maximum Likelihood	Method of Moments	Maximum Likelihood	Method of Moments
CRE	18.9%	16.6%	29.5%	23.7%
C&I	7.0%	5.6%	7.4%	6.3%
Ratio*	2.63	2.34	3.96	3.75

*The ratio of median values is not equal to the median ratio, partly because if the asset correlations could be computed for one but not the other of CRE and C&I exposure types then the one estimate was included in the computation of the median asset correlation for that exposure type but not in the computation of the median ratio.

Estimates Based on Thrift Charge-off Rates

Charge-off data similar to those provided in bank Call Reports are also available for U.S. thrift institutions through the quarterly Statements of Condition and Operations collected by the Office of Thrift Supervision. It is important to point out that these data are weaker than bank Call Report data for the purpose of evaluating whether asset correlations for bank CRE and C&I portfolios are materially different: they reflect loans held in both portfolios by thrifts, rather than banks, and the two types of institutions may well differ systematically in terms of the typical composition of either CRE or C&I portfolios, if not both. For this reason, empirical findings based on thrift Call Report data should be considered as of secondary importance.

Table 4 summarizes CRE and C&I asset correlation estimates based on thrift Call Report data for 1990-2002. Again, two sets of estimates are reported for each empirical technique: the median of the estimates for the 20 largest U.S. thrifts using thrift-level data, and the estimate derived from aggregate data for all U.S. thrifts combined. For reasons that are not well understood, the method-of-moments estimator applied to thrift data generates results that conflict sharply depending on whether thrift-level or industry-wide data are used. The maximum likelihood estimator, however, suggests the same qualitative conclusion advanced by the bank Call Report data: that asset correlations for CRE loan portfolios are much greater than for C&I portfolios, on the order of 20 percent higher at the median using thrift-level data for the largest U.S. thrifts, and more than twice as high using aggregate data for the U.S. thrift industry.

Table 4: CRE and C&I Asset Correlations Estimated from Thrift Call Report Data

	Median Bank-Level Estimate		Estimate from Aggregate Data	
	Maximum Likelihood	Method of Moments	Maximum Likelihood	Method of Moments
CRE	30.1%	23.4%	29.0%	75.9%
C&I	23.2%	35.7%	13.8%	40.3%
Ratio*	1.22	0.52	2.09	1.89

*The ratio of median values is not equal to the median ratio, partly because if the asset correlations could be computed for one but not the other of CRE and C&I exposure types then the one estimate was included in the computation of the median asset correlation for that exposure type but not in the computation of the median ratio.

Estimates Based on Insurance Company Foreclosure Rates

The strengths of the bank Call Report data are that (1) they reflect data for actual portfolios of U.S. banks[19] and (2) they permit the estimation of both CRE and C&I asset correlations from the same data source. (Thrift Call Report data share only the second of these advantages.) As noted, however, considerable shortcomings are associated with both sets of

[19]Except for the fact that C&I loans collateralized by CRE are included with CRE data rather than with C&I data, as discussed in note 18.

data, such as the difficulty of inferring default rates from charge-off data and the short sample periods (12 years for bank data, 13 for thrift data).

To mitigate these problems (albeit at some cost, as discussed below) ERisk, a risk-management consulting firm, estimated CRE and C&I asset correlations using different data sources that are available over substantially longer time periods. In particular, ERisk estimated asset correlations for CRE loan portfolios using, as a potentially closer proxy for annual loan default rates, data collected by the American Council of Life Insurers (ACLI) on the percentage of loans that are in process of foreclosure at the end of each year.[20] For estimating C&I asset correlations, ERisk used annual default rates on speculative-grade corporate loans as reported by Moody's. Compared with bank and thrift Call Report data, both data sources used by ERisk are available over a much longer time series: since 1965 for ACLI data and since 1970 for Moody's data. Taken together, these data sources enable estimation of asset correlations for both CRE and C&I loan portfolios over a common 32-year sample period, nearly three times as long as the Call Report data series.

On the other hand, the use of different data sources (ACLI does not report default rates for C&I loan portfolios) means that this important advantage comes at a cost in terms of comparability. In addition, ACLI data are not necessarily representative of bank CRE portfolios, partly because the ACLI data are aggregated over several very large companies and not available for individual institutions, and partly because the typical life insurance company CRE portfolio

[20]A potential concern with the use of data on loans in process of foreclosure to measure default rates is that the foreclosure rate excludes loans that defaulted but were charged off or restructured (perhaps at a loss) within the year. To assess the potential significance for estimating asset correlations, Board staff repeated the analysis described below using data on loans in delinquency status at the end of each year as an alternative proxy for default rates within the year. The results are qualitatively identical.

(like the typical thrift portfolio) may differ systematically from the typical bank CRE portfolio. In particular, ACLI data can be viewed as the equivalent of a CRE portfolio that may well be more broadly diversified than most bank CRE portfolios. Because of this, estimated asset correlations based on ACLI data may be biased downward as an estimate of the asset correlation applicable to bank CRE portfolios. The same problem affects the estimation of C&I asset correlations using Moody's or S&P data, but the bias is likely to be far more modest.

Table 5 summarizes the estimated asset correlations based on the data sources suggested by ERisk as well as on an alternative series of annual default rates for speculative-grade C&I loans published by S&P that is available only since 1981.[21] To facilitate comparison with estimates from bank Call Report data, the table presents results based on three data periods: 1970-2001 (common years for ACLI and Moody's data), 1981-2001 (common years for ACLI and S&P data), and 1991-2001 (available years for bank Call Report data).

Three conclusions are suggested by the estimates presented in Table 5:

- First, as applied to these data, the method-of-moments estimator produces smaller estimated ratios of CRE to C&I asset correlations than does the maximum likelihood estimator.
- Second, estimates based on recent data are quite similar to the estimates, presented earlier, that are based on bank and thrift Call Report data: CRE asset correlations are estimated to be sharply higher than C&I asset correlations.

[21]Figures presented in this table do not match those presented by ERisk because ERisk chose to use data from the end of the second quarter (June 30) of each year, while the figures presented here are based on data from the end of each year (December 31). Qualitatively, however, the results are quite similar.

Table 5: CRE and C&I Asset Correlations Estimated from ACLI, Moody's, and S&P Data

		1970-2001		1981-2001		1991-2001	
		Maximum Likelihood	Method of Moments	Maximum Likelihood	Method of Moments	Maximum Likelihood	Method of Moments
CRE		12.1%	9.6%	14.1%	9.1%	19.5%	9.6%
Moody	C&I	11.5%	10.4%	7.6%	7.1%	8.0%	8.2%
	Ratio	1.05	0.93	1.84	1.51	2.44	1.17
S&P	C&I	na	na	7.8%	6.9%	8.6%	8.9%
	Ratio	na	na	1.80	1.55	2.28	1.09

*ACLI asset correlations are computed from data on the percentage of portfolio assets in process of foreclosure as of the end of each year. This implicitly assumes that (1) all defaults resolve in foreclosure and (2) all foreclosures are resolved in exactly one year. Alternatively, asset correlations could be computed from data on percentage of portfolio assets in delinquency status as of the end of each year. This would implicitly assume that (1) all delinquencies become defaults and (2) all delinquencies are resolved in exactly one year. The estimated asset correlations using delinquency data are 12.2%, 9.2%, 15.1%, 10.7%, 22.6%, and 13.5%. These imply ratios of 1.06, 0.88, 1.98, 1.51, 2.83, and 1.64 relative to C&I estimates based on Moody's data, and 1.93, 1.55, 2.64, and 1.53 relative to estimates based on S&P data.

- Third, and perhaps most striking, over the longest available time period, 1970-2001, the CRE asset correlation estimated using the maximum likelihood method is only 5 percent higher than the estimated C&I asset correlation, and, using the method of moments, the CRE asset correlation is actually estimated to be less than the C&I asset correlation.

The fact that asset correlations are estimated to be much higher for CRE than for C&I portfolios using data for the periods 1991-2001 and 1981-2001 but not for the period 1970-2001 raises the question of whether something about the longer period is driving the discrepancy in empirical findings. To investigate this, CRE and C&I asset correlations were estimated using the maximum likelihood method and data periods that started in successively later years.

Table 6: Asset Correlations Estimated Using ACLI Data over Different Time Series

Time Series	CRE AC (ACLI Data)	C&I AC (Moody's)	CRE/Moody Ratio	C&I AC (S&P Data)	CRE/S&P Ratio
1965-2001	11.8%	-	-	-	-
1966-2001	12.1%	-	-	-	-
1967-2001	12.3%	-	-	-	-
1968-2001	12.4%	-	-	-	-
1969-2001	12.4%	-	-	-	-
1970-2001	12.1%	11.5%	1.05	-	-
1971-2001	11.8%	11.0%	1.08	-	-
1972-2001	11.8%	10.9%	1.08	-	-
1973-2001	12.0%	11.2%	1.07	-	-
1974-2001	12.1%	11.2%	1.09	-	-
1975-2001	12.5%	10.8%	1.16	-	-
1976-2001	12.8%	11.0%	1.17	-	-
1977-2001	13.1%	10.4%	1.26	-	-
1978-2001	13.5%	10.3%	1.32	-	-
1979-2001	14.0%	10.4%	1.35	-	-
1980-2001	14.1%	7.9%	1.78	-	-
1981-2001	14.1%	7.6%	1.84	7.8%	1.80
1982-2001	14.2%	5.6%	2.55	5.8%	2.47
1983-2001	14.5%	5.8%	2.50	6.1%	2.40
1984-2001	14.9%	6.1%	2.44	6.2%	2.41
1985-2001	14.5%	6.4%	2.28	6.4%	2.26
1986-2001	14.8%	6.8%	2.19	6.8%	2.16
1987-2001	15.7%	7.1%	2.19	7.2%	2.19
1988-2001	16.6%	7.6%	2.17	7.4%	2.23
1989-2001	17.6%	8.1%	2.17	8.0%	2.20
1990-2001	18.7%	8.7%	2.16	8.7%	2.15
1991-2001	19.5%	8.0%	2.44	8.6%	2.28

*Asset correlations are computed from data on loans in process of foreclosure, as described in the note on Table 5.

As Table 6 shows, the inclusion of data from the early 1970s makes a material difference in the qualitative interpretation of the empirical results. In particular, the CRE asset correlations estimated using data periods that started in 1976 or earlier are all less than 13 percent and are the lowest shown on the table: the estimated CRE asset correlation increases steadily with later starting periods. At the same time, the C&I asset correlations estimated using the same data samples starting in pre-1977 years are all close to or exceeding 11 percent and are the highest shown on the table. There are two alternative interpretations of this evidence: Either the ratio of

CRE to C&I estimated asset correlations is peculiarly low for data periods starting pre-1977 (and can be expected to return to large disparities), or it is peculiarly high for data periods starting post-1976 (and can be expected to return to near-parity). Of course it is difficult to make a compelling case for excluding data when they are available: generally it should be done only when one is confident that the retained data are representative and the excluded data are not. For this reason it would be very helpful to understand which of the two possible interpretations is more likely to be accurate.

Evidence from Credit Support for Commercial-Mortgage-Backed Securities

Commercial mortgages originated by banks are frequently pooled and securitized with an explicit rating from one of the major credit rating agencies (Fitch/ICBA, Moody's, and Standard & Poor's). In general the rating agencies require a specified amount of credit support for the commercial-mortgage-backed security (CMBS) to be sold with a given rating. The amount of credit support required depends on the agency's evaluation of the probability that credit losses on loans in that pool will be severe enough to cause a default on the cash flows promised by the security issuer. In particular, the desired rating represents the estimated probability that credit losses will exceed the required credit support.

The amount of credit support for a CMBS backed by a sufficiently large number of loans is thus analogous to the amount of capital held against a loan portfolio, with the PD associated with the desired rating corresponding to the "loss coverage target" (see section III). This analogy suggests that the credit support required by rating agencies for securities backed by pools of commercial mortgages can provide a rough idea of the amount of capital that one should expect banks to apply against CRE portfolios. Moreover, the widely held perception is that the average loan-level riskiness (EL) is smaller for securitized CRE loans than for CRE loans held in portfolio, in which case the credit support required by rating agencies can be interpreted generally as the minimum capital that one would expect banks to apply.

Federal Reserve Board staff reviewed the credit support required for several CMBS issuances to achieve a rating of BBB+ or A- (Moody's Baa1 or A3), which approximates the loss coverage target that the Basel Committee has assumed in calibrating the proposed new Capital Accord. For 40 recent CMBS issuances backed by at least 70 properties each[22] and having tranches rated BBB+ or A-, the required credit support for the BBB+ tranche was between 6.75 percent and 14.25 percent,[23] and most of the pools required credit support in the range of 9.75 percent to 12.25 percent.

Variation in required credit support across CMBS issuances can be attributed to several factors, including the share of loans that finance what the agencies consider "more volatile" property types (*e.g.*, hotel, office, assisted living, and skilled nursing) versus the share financing "less volatile" types (*e.g.*, multifamily, industrial, and anchored retail) as well as portfolio average debt-service-coverage ratio (DSCR) and loan-to-value ratio (LTV), geographic concentration, seasoning, amortization structure, and other risk factors. In general one would expect the economic capital applied to the CRE portfolios of different banks to vary for the same reasons. It is useful, however, to consider the range of credit-support requirements--especially the inter-quartile range of 9.75 percent to 12.25 percent--as a rough indicator of the capital that

[22]Most of the securities were backed by more than 156 properties.

[23]Staff also reviewed two issuances with required credit support at 21.5 percent and 33.5 percent, but both of those pools comprised loans with extraordinary characteristics (*e.g.*, loans that had been removed from previous securitizations) that made them unsuitable for comparison with bank portfolios.

one would expect most banks to apply to their CRE portfolios. Relative to the base capital requirement of 8 percent, this range implies a range of portfolio average risk weights between 122 percent and 153 percent.

Appendix E describes how the distribution of loans in bank CRE portfolios by default probability can be estimated from data collected from large U.S. banks as part of the data collection efforts known as QIS 2.5 and QIS3. Although this estimated portfolio distribution can be regarded only as a rough approximation, the analysis suggests that, under the CP3 proposal, the average risk weight for low-asset-correlation CRE portfolios during unfavorable periods in the credit cycle would be about 133 percent, and the average for high-asset-correlation CRE portfolios during unfavorable periods would be about 152 percent. These figures are very close to the bounds of the inter-quartile range suggested by the credit support required for the BBB+ and A- tranches of recent CMBS pools. Of course, the average risk weight for both low- and high-asset-correlation CRE portfolios can be expected to decline during more favorable periods of the credit cycle, so the portfolio average risk weight under both proposed Basel II treatments (high and low asset correlation) would likely be lower than the credit support required for most CMBS issuances.

Evidence from a Survey of Large U.S. Banks

Ten large U.S. banks recently participated in a survey conducted by the Risk Management Association (RMA [2003]) regarding economic capital allocations for commercial real estate and project finance portfolios. Among the questions addressed in this survey was whether banks use the same asset (or default) correlation for SL (specialized lending, including

CRE) as for their corporate loan portfolios. Three banks responded that they use the same correlations, while two others use a higher asset correlation for SL than for C&I; the other four banks noted that their correlation assumptions differed by geography and industry, so that the question could not easily be answered.

A second question asked whether average risk weights for SL loans are higher than average risk weights for C&I loans. Again, three banks responded that they are approximately the same. One bank responded that SL loans have smaller average risk weights than C&I loans because they have lower PDs and LGDs, which more than offset the higher assumed asset correlation assumption. Three other banks agreed that SL portfolios have higher asset correlations and that SL loans have lower PDs and LGDs on average, but they also reported that average SL risk weights were slightly higher than average C&I risk weights as a result.

As part of the RMA survey, six banks provided estimates of the economic capital (EC) that would be generated by their internal EC systems for hypothetical CRE loans with given PDs and LGDs. RMA staff then computed the asset correlations that were implied by these EC estimates and reported the median values across the six reporting banks. Although this exercise was not performed separately for low-asset-correlation (IPRE) and high-asset-correlation (HVCRE) portfolios, the median implied asset correlations were generally between the asset correlations incorporated into those two separate risk weight functions. The median implied correlations, however, decline much less sharply as PD increases than do the Basel II asset correlations for both IPRE and HVCRE, and as a result the median implied correlations for high-risk CRE loans are substantially greater than the Basel II correlations for high-risk loans in either IPRE or HVCRE portfolios (Table 7).

Table 7: Asset Correlations by Rating Grade--RMA, IPRE, and HVCRE

Supervisory Risk-Rating Category	Implied PD	Implied Value of Assumed Asset Correlation		
		RMA*	IPRE	HVCRE
Very strong/pref	0.19%	26.6%	22.9%	28.4%
Strong**	0.34%	26.5%	22.1%	27.2%
Good	1.13%	26.4%	18.8%	22.2%
Satisfactory	3.53%	26.0%	14.1%	15.1%
Weak	10.8%+	24.7%-	12.1%-	12.1%-

*Asset correlation predicted from a linear regression of median implied asset correlations from RMA [2003] on the midpoint of the PD range corresponding to each figure.

**Good loans eligible for preferential risk weights (Good/pref) are assumed to have the same PD, and therefore the same asset correlation, as Strong loans.

Have Asset Correlations Declined Over the Last Decade?

Bank specialists in CRE and credit risk have raised the concern that using data that includes the last serious downturn in CRE loan performance (*e.g.*, including 1991 in bank Call Report data) would produce estimated asset correlations that are systematically larger than current actual asset correlations because of improvements in underwriting criteria that banks have implemented more recently. That is, banks have argued that estimated asset correlations should be based only on recent data because changed conditions in the CRE lending industry have rendered older data obsolete.

In particular, staff from U.S. Bancorp[24] estimated the asset correlation implied by aggregated charge-off data for the 100 largest U.S. banks for two time periods (1991:Q1-

[24]Analysis provided by e-mail from Kevin Storm (U.S. Bancorp) to Roger Tufts (Office of the Comptroller of the Currency), October 21, 2002.

2002:Q2 and 1995:Q1-2002:Q2). They found that the ratio between the asset correlations for CRE and C&I loan types declined from 1.5 to 0.3. Board staff repeated the U.S. Bancorp analysis using bank-level Call Report data for several time series (and using the two empirical approaches outlined above) and found a qualitatively similar result (Table 8).

Table 8: Median Bank-Level Asset Correlations Estimated Using Different Time Series

		1991-2002	1992-2002	1993-2002	1994-2002	1995-2002	1996-2002
Maximum Likelihood	CRE	18.9%	16.7%	11.5%	6.8%	5.1%	3.9%
	C&I	7.0%	5.5%	4.7%	5.0%	5.6%	6.0%
	Ratio*	2.63	2.83	2.26	1.47	0.73	0.70
Method of Moments	CRE	16.6%	15.4%	12.8%	6.2%	3.1%	3.1%
	C&I	5.6%	5.1%	4.9%	5.4%	6.2%	6.0%
	Ratio*	2.34	2.09	1.65	1.15	0.71	0.79

*The ratio of median values is not equal to the median ratio, partly because if the asset correlations could be computed for one but not the other of CRE and C&I exposure types then the one estimate was included in the computation of the median asset correlation for that exposure type but not in the computation of the median ratio.

Board staff then repeated this analysis using thrift-level Call Report data but found a somewhat different pattern, as shown (Table 9).

Table 9: Median Thrift-Level Asset Correlations Estimated Using Different Time Series

		1990-2002	1991-2002	1992-2002	1993-2002	1994-2002	1995-2002	1996-2002
Maximum Likelihood	CRE	30.1%	30.6%	26.7%	27.4%	25.2%	23.7%	24.5%
	C&I	23.2%	25.0%	22.4%	23.8%	14.1%	13.8%	12.1%
	Ratio*	1.22	1.16	1.03	1.00	1.20	1.69	1.75
Method of Moments	CRE	21.3%	19.6%	14.2%	13.7%	10.8%	12.3%	15.6%
	C&I	24.4%	26.5%	21.3%	19.3%	15.1%	13.6%	14.6%
	Ratio*	0.52	0.50	0.48	0.57	0.55	0.76	1.12

*The ratio of median values is not equal to the median ratio, partly because if the asset correlations could be computed for one but not the other of CRE and C&I exposure types then the one estimate was included in the computation of the median asset correlation for that exposure type but not in the computation of the median ratio.

ACLI data offer a particularly rich source for investigating time-related changes in estimates of the asset correlation, as was done above in the analysis summarized in Table 6. To address the concerns raised by U.S. Bancorp and others, Board staff extended the earlier analysis by comparing the estimated asset correlations derived from the maximum likelihood method with ACLI, Moody's, and S&P data for data periods starting in 1991 and later (Table 10).

Table 10: Asset Correlations Using ACLI, Moody's, and S&P Data over Different Time Series

Time Series	CRE AC (ACLI Data)	C&I AC (Moody's)	CRE/Moody Ratio	C&I AC (S&P Data)	CRE/S&P Ratio
1991-2001	19.5%	8.0%	2.44	8.6%	2.28
1992-2001	18.9%	6.5%	2.91	6.5%	2.88
1993-2001	17.0%	7.0%	2.41	6.8%	2.49
1994-2001	15.6%	8.0%	1.96	7.3%	2.13
1995-2001	13.8%	8.0%	1.72	7.3%	1.90
1996-2001	11.1%	9.3%	1.19	8.5%	1.32

*Asset correlations are computed from data on loans in process of foreclosure, as described in the note on Table 5.

Over the 1990s, the results based on ACLI data suggest the same pattern observed for bank and thrift Call Report data: declining CRE asset correlations and (though this was not true for thrift data) declining ratios of CRE to C&I asset correlations. However, the results provide no evidence that the CRE asset correlation has declined to any value less than 20 percent larger than the C&I asset correlation, and most of the estimates (including all of those based on S&P data) suggest a CRE/C&I ratio well in excess of that minimum.

The concern expressed by staff of U.S. Bancorp and other large banks is a valid one: If the asset correlation for bank CRE portfolios has in fact declined relative to the asset correlation for bank C&I portfolios, and if it can be expected to remain low into the foreseeable future, then the reduced current asset correlation should be incorporated into the proposed Basel II treatment of CRE loans. However, as noted in section III, it is difficult to see why changes in underwriting criteria should have reduced the asset correlation. Underwriting criteria affect the probability that a *loan* secured by a given property will default, while the asset correlation describes co-movements among the returns to the *assets* securing loans in that portfolio. It is easy to see how co-movements in asset values affect co-movements in default probabilities--this, after all, is why the asset correlation is a critical parameter in estimating portfolio default risk. But it does not follow that co-movements in default probabilities *that are caused by portfolio-wide changes in underwriting criteria* should affect co-movements in asset values. Ultimately, the question of whether CRE asset correlations have declined over the past decade--and will remain low over the foreseeable future--is an empirical issue that cannot adequately be answered until additional data become available.

Conclusion: Are CRE and C&I Asset Correlations Materially Different?

Although several of the empirical analyses presented above suggest that the asset correlation for CRE portfolios is larger than that for C&I portfolios, the empirical estimates were not consistent: Over some time periods, for example, and using some empirical methods, the CRE asset correlation was estimated to be smaller than the C&I asset correlation. Moreover, several known problems with both the available data sources and the empirical methodologies were noted. Finally, the most recent available empirical estimates (eliminating data extending back to the high-default period of the late 1980s and early 1990s) raise the question of whether CRE asset correlations may have declined recently relative to C&I asset correlations; if they have, then the relatively low ratio of CRE to C&I asset correlations could be expected to persist.

On the basis of the uncertainty regarding the difference between CRE and C&I asset correlations, the Federal Reserve Board staff believes that no adequate empirical basis currently exists for requiring banks to calculate minimum regulatory capital using a risk weight function (explicit or implicit) that differs from that used to establish C&I capital requirements.

2. Are Asset Correlations for Certain Sub-Portfolios of CRE Loans Materially Larger than Asset Correlations for Other Sub-Portfolios?

The second question addressed in developing the current proposed treatment of CRE loan portfolios was whether asset correlations of some CRE sub-portfolios differ a great deal from those of other CRE sub-portfolios. To address this question, Board staff estimated the relative asset correlations of different CRE portfolios using the data sources and empirical methods described above as well as additional data sources and empirical methods that were appropriate only for this question. In particular, data on historical rates of return for commercial properties--

collected on a confidential basis by the National Council of Real Estate Investment Fiduciaries (NCREIF)--was analyzed by Dr. Jeffrey Fisher, Director of the Center for Real Estate Studies at the Indiana University School of Business, using an empirical methodology that is described in detail in appendix C.

Table 11 summarizes the available empirical evidence regarding the relative asset correlations of different CRE sub-portfolios. As noted above, the empirical estimates may be subject to bias;[25] for this reason, and because the issue is simply the *relative* asset correlations of different sub-portfolios, the table presents the ratio between the asset correlation estimate for each sub-portfolio and the estimate for all loans on in-place properties (*i.e.* CRE mortgages). The exception is the CMBS accounting data analysis, which does not permit the estimation of an asset correlation for all CRE mortgages together.

[25]In particular, in addition to the methodological bias associated with the maximum likelihood and method-of-moments estimators using time series on default probabilities, the NCREIF property values regression analysis is subject to bias because many of the observations on property values are appraised values rather than market transaction values and so may include considerable measurement error. This will tend to bias downward the estimated asset correlation.

Table 11: Asset Correlation Estimates by CRE Sub-Portfolio

	CMBS	NCREIF	ACLI		Call Reports				OTS			
			1988-2001		Bank-Level*		Aggregate		Thrift-Level*		Aggregate	
			ML	MM	ML	MM	ML	MM	ML	MM	ML	MM
Multifamily	--**	0.75	1.46	1.59	1.57	1.47	1.06	1.25	0.98	1.08	1.17	0.95
Non-Residen					1.06	1.01	1.09	1.04	0.97	1.05	0.92	1.10
Hotel	46.0		1.62	1.54								
Office	--**	1.53	1.29	1.16								
Industrial	23.5	1.21	1.32	1.23								
Retail	22.5	0.52	0.73	0.74								
Mixed			1.01	2.08								
Other	38.5		1.01	0.95								
Constr & Land					1.55	2.19	1.32	1.68	0.97	1.65	0.85	1.17
Construction									1.00	1.44	0.30	1.18
Single-Family									0.68	1.20	0.39	0.86
Multifamily									0.74	0.30	1.18	1.28
Non-Residen									1.14	1.19	1.66	--
Land									0.57	0.77	1.13	1.26

*Median ratio of sub-portfolio to CRE mortgage estimates for 20 largest banks or thrifts for which both could be estimated.

**Asset correlations could not be estimated for multifamily or office properties using CMBS data.

The data quality and methodological problems are reflected in the fact that relative asset correlation estimates are in many cases inconsistent; however, the estimates shown in Table 11 suggest general qualitative conclusions. Regarding loans on in-place commercial properties, the estimated asset correlation for hotel mortgages is relatively high in all three analyses shown. The evidence regarding office, multifamily, and industrial mortgages is mixed, while retail mortgages have relatively low estimated asset correlations in all four analyses shown.

These general conclusions for in-place commercial properties are broadly consistent with the conclusions reached by credit rating agencies. Moody's, for example, characterizes multifamily (including manufactured housing), industrial/warehouse, self-storage, and anchored retail (including regional malls) as "less volatile" property types, while unanchored retail, office, hotel, assisted living, and skilled nursing facilities are characterized as "more volatile."

Most of the analyses produced relatively high estimated asset correlations for portfolios of construction and land loans; however, the results based on thrift Call Report data suggest that construction loans may be responsible for most of this effect, as estimated asset correlations for land loans are relatively high only when estimated using aggregate data for the U.S. thrift industry. Considering types of properties financed by construction loans, nonresidential property construction loans appear to have relatively high asset correlations, while some evidence suggests lower relative asset correlations for single-family and multifamily residential construction loans.

The treatment of loans financing construction of single-family properties was left unclear in the third consultative paper (CP3), but the results shown in Table 11 suggest that the asset correlation for single-family construction loans may be smaller than the asset correlation for other construction loans. While the treatment of single-family residential construction loans will have to be clarified in further discussions among U.S. regulatory agencies and international Basel II negotiating partners, the available evidence suggests that perhaps these loans should be classified as having low (IPRE) rather than high (HVCRE) asset correlation.

Evidence from Credit Support for Commercial Mortgage Backed Securities

As noted, the general conclusions for in-place commercial properties are broadly consistent with the conclusions reached by credit rating agencies. In addition, the credit support levels required by rating agencies for CMBS issuances can be analyzed for supplemental empirical evidence on what the market considers to be the relative asset correlations of different property types. In particular, the credit enhancement levels required by Moody's for the BBB+ or AA- tranche of 78 CMBS pools rated during 1999-2002 were regressed on several pool characteristics, including the percentage of pool assets represented by properties of different types as well as the percentage in different LTV ratio categories. This regression analysis is described in appendix D.

The estimated regression coefficients on property types suggested that, holding other factors constant, Moody's requires the greatest credit enhancement on single-purpose properties (including hotels, assisted living, skilled nursing, and other "special purpose" uses), mixed-use properties, and office properties; requires moderate credit enhancement on multifamily properties (including manufactured housing); and requires the least credit enhancement on retail properties and industrial/warehouse properties (including self-storage).[26]

More specifically, the regression model can be used to predict the credit enhancement that would be required by Moody's for hypothetical homogeneous portfolios of loans of a given property type and given LTV ratio that would be slotted into a given supervisory risk-rating category. While the regression model is not estimated strongly enough to produce reliable point

[26]Unanchored retail could not be separated from anchored retail (including regional malls) for the purpose of this analysis.

estimates, the relative levels of predicted credit enhancement suggest the differences in portfolio-level risk (that is, asset correlation) across property types.[27]

For a hypothetical portfolio of loans whose LTV ratios would put them roughly in the "Satisfactory" supervisory risk-rating category (Table 12), the model would predict the lowest required credit support if the portfolio is composed of industrial or retail loans (7.3 percent and 9.4 percent respectively), medium credit support if the portfolio is composed of multifamily loans (16.3 percent), and relatively high credit support if the portfolio is composed of office or hotel/single-use loans (18.6 percent and 20.2 percent respectively).[28]

Table 12: Predicted Credit Support by Property Type and Risk Category

Property Type	Preferential	Strong	Good	Satisfactory
Industrial	.0	.0	4.8%	7.3%
Retail	.0	.0	7.0%	9.4%
Multifamily	4.3%	5.1%	13.9%	16.3%
Office	6.5%	7.4%	16.1%	18.6%
Hotel/Single-Use	N/A	10.6%	11.4%	20.2%

Conclusion: What are the Relative Asset Correlations of Different CRE Sub-Portfolios?

[27]These results suggest that the implied effect of asset correlation on portfolio-level default probability (and hence on required credit enhancement) is in addition to the use of different capitalization rates for different property types. See, for example, Rubin & Levidy (Moody's) [2000] for assumed capitalization rates by property type, which notes that "Moody's captures the risks inherent in various asset classes...in the utilization of different capitalization rates for different property types."

[28]As noted, the model is not estimated with precision. In addition, the values shown in Table 12 are predictions for hypothetical homogeneous portfolios in which all loans are secured by properties of the same type and in which the LTV ratios of all loans are in the same range that--in combination with other underwriting data--would cause them all to be slotted in the same supervisory risk-rating category. For these reasons, the predicted values should, of course, be interpreted as only a general guide to the relative level of credit support.

As noted, CP3 permits national regulatory agencies in each country to designate certain types of CRE loans as sharing relatively high asset correlations, and therefore as subject to minimum regulatory capital under the high-asset-correlation (HVCRE) treatment. Although the text noted several qualitative conclusions that appear to be suggested by the empirical evidence on asset correlations for different CRE loan types, it was noted that the different data sources and empirical methodologies produced sometimes conflicting results. Moreover, as noted above, the banking agencies believe that the empirical evidence is not yet sufficient to conclude that the average asset correlation for well-diversified portfolios composed of all CRE loan types differs materially from the average asset correlation for portfolios of C&I loans. Thus, despite the evidence described above regarding the asset correlations for portfolios of CRE loans secured by different property types, Federal Reserve Board staff proposes not to designate any property types as sharing relatively high asset correlations, a suggestion that means that all SL loans secured by in-place commercial property, regardless of type, would be treated under the proposed low-asset-correlation (IPRE) treatment.

The Basel II proposal specifies that ADC loans for low-asset-correlation property types will be subject to the high-asset-correlation (HVCRE) treatment unless the borrower has substantial equity or the source of repayment is substantially certain (*i.e.* the property is pre-sold or substantially pre-leased). Although the empirical evidence summarized above is not adequate to directly address the impact of borrower equity or repayment uncertainty, the Federal Reserve Board staff has had a longstanding supervisory concern that CRE lending to finance speculative construction and development is highly vulnerable to speculative swings in CRE markets and may contribute to such swings, especially when there is little borrower equity at risk. In recognition of this concern, the Board staff suggests implementing the proposed treatment of

ADC lending as a high-asset-correlation portfolio[29] and defining the "substantial equity" and "pre-sold/pre-leased" exceptions partly on the basis of comments received in response to this White Paper and to the U.S. regulatory agencies' forthcoming Advance Notice of Proposed Rulemaking.

[29] Except that, as noted, the Board staff proposes to encourage the Basel Committee to consider specifying single-family residential construction loans as a low-asset-correlation (IPRE) portfolio.

V. Empirical Evidence Underpinning the Basic IRB Approach

As noted in the introduction, the Basic IRB Approach is quite similar in key respects to the Advanced IRB Approach. Under the Advanced Approach each bank estimates the three key parameters (PD, LGD, and M) applicable to a given loan, and then determines the risk weight by substituting these values into the appropriate risk weight function. The Basic Approach has been developed for use by banks that are unable to estimate PD, LGD, or M. Under the Basic Approach each bank uses "slotting criteria" to assign the loans in each of its own internal risk-rating categories to one of the supervisory risk-rating categories and then applies the supervisory risk weight associated with that category. Importantly, however, these supervisory risk weights are based on the same risk weight functions that are explicit in the Advanced Approach. In essence, the risk weight for each rating category was developed by assigning supervisory estimates of PD, LGD, and M to each rating category and then substituting these values into the same risk-weight functions applied under the Advanced Approach.

Table 13 summarizes the risk parameter assumptions (PD, LGD, M, and asset correlations) underlying the Basic IRB Approach. The remainder of this section describes how these parameters were determined.

Table 13: Risk Parameters Implicit in the Basic IRB Approach

		Very Strong/ Preferential	Strong	Good	Satisfactory	Weak
PD		0.19%	0.46%	1.26%	3.63%	18.3%
LGD	In-Place	34.6%	34.7%	35.1%	36.2%	42.9%
	Construction	54.9%				
M	In-Place	5 years				
	Construction	1 year				
Asset Correlation	HVCRE	27.5%	26.3%	21.6%	14.9%	12.0%
	IPRE	22.9%	21.5%	18.4%	14.0%	12.0%
Risk Weight	HVCRE	75%	100%	125%	175%	350%
	IPRE	50%	75%	100%	150%	350%

A. Basic Approach: Assumptions regarding PD

The value of PD attributed to each supervisory risk-rating category was developed from the range of external rating grades associated with that category. As noted in section II, the slotting criteria associated with each supervisory rating category are intended to ensure that CRE loans assigned to that category would have average one-year default rates comparable to those of corporate bond issuers falling within a given range of external ratings. The Very Strong/Preferential category is intended to correspond to external ratings of BBB+ or better, the Strong category to ratings of BBB- to BBB, the Good category to BB+ or BB, the Satisfactory category to BB- to B+, and the Weak category to B or worse.

The estimated PD for each supervisory category (except Weak) is estimated as the average one-year default frequency for the lowest external rating grade associated with that category, a measure based on historical default statistics for corporate bond issuers rated by Standard and Poor's. For the Weak category, the associated PD is benchmarked to the historical default rate for corporate bonds rated B-.

Table 12 of Brady, Vazza & Bos (S&P) [2003] gives the average one-year default frequency over the period 1981-2002 for loans in each external risk-rating category that were still rated at the end of the year. As the authors point out, however, this represents an underestimate of the default frequency because loans from which ratings were withdrawn by the end of the year are generally more likely to have defaulted. Because of this, the PD assumptions underlying the Basic Approach are adjusted for ratings withdrawals.[30] Table 14 shows the impact of these adjustments on the assumed PD level for each supervisory rating category. Except for the Weak category, the adjustments for rating withdrawals were relatively small.[31]

Table 14: Implicit Estimate of PD Associated with Each Supervisory Rating Grade

	Very Strong/ Preferential*	Strong	Good	Satisfactory	Weak
External Risk-Rating Equivalent	BBB+	BBB-	BB	B+	B-
Unadjusted PD	0.18%	0.43%	1.16%	3.29%	13.15%
Adjusted PD	0.19%	0.46%	1.26%	3.63%	15.26%

*Based on data from Brady & Bos (S&P) [2002].

[30]For unmodified ratings (*i.e.* AAA, AA, A, BBB, BB, B, CCC), Table 8 of Brady, Vazza & Bos (S&P) [2003] reports average one-year default frequencies that have been adjusted for ratings withdrawals. These reported data were used to interpolate adjustments for modified ratings (e.g, BBB+) using a quadratic regression of adjusted one-year default frequencies (Brady, Vazza & Bos, Table 8) on unadjusted default frequencies (Brady, Vazza & Bos, Table 7), and applying the estimated regression coefficients to the unadjusted one-year default frequencies by rating modifier (Brady, Vazza & Bos, Table 13).

[31]The supervisory risk weight for the Weak category is consistent with a PD of 25.25 percent, slightly smaller than the adjusted one-year default frequency for bonds with S&P's CCC rating shown in the table.

B. Basic Approach: Assumptions Regarding M

No empirical evidence appears to be available regarding the average remaining maturity of loans in bank CRE portfolios. The supervisory risk weights were selected, largely on the basis of anecdotal information, to be consistent with an assumed effective maturity of five years for mortgage loans secured by in-place properties[32] and one year for ADC loans.

C. Basic Approach: Assumptions Regarding LGD

Appendix E presents empirical data on several loan-level risk parameters, including the average observed loss severity on defaulted CRE loans and on the historical relationship between observed default frequencies and observed losses on defaulted loans. As the appendix indicates, the best available empirical evidence suggests that the average observed loss rate on defaulted CRE mortgage loans during stress periods is perhaps 36.8 percent, the figure suggested by a published study conducted by Esaki, L'Heureux & Snyderman [1999]. (The Committee has not found any data on the average observed loss on defaulted ADC loans, but anecdotal evidence suggests that it is sharply higher than the average observed loss on defaulted mortgage loans secured by in-place properties.) The empirical evidence also suggests a strong positive relationship between observed default frequencies and observed loss severities. On balance, taking account of the available evidence, the Committee selected supervisory risk weights that are consistent with average LGDs in the range of 35 to 43 percent for mortgages secured by in-place properties, and around 55 percent (or roughly 1½ times the figure suggested by Esaki, L'Heureux & Snyderman [1999]) for ADC loans.

[32] Under the IRB approach, M is capped at 5 years. Thus, even if the actual effective maturity of a loan was, say, 20 years, for IRB purposes M would be measured as 5 years.

D. Basic Approach: Assumptions Regarding Asset Correlation

As already noted, the assumed asset correlations for the Basic Approach are identical to those for the Advanced Approach. Specifically, asset correlations are assumed to be a declining function of PD, ranging from 30 percent at the lowest PDs for loans in high-asset-correlation portfolios (24 percent for loans in low-asset-correlation portfolios) to 12 percent for very large PDs for both high and low-asset-correlation portfolios. The asset correlation assumption for low-asset-correlation CRE portfolios is identical to that for C&I lending.

E. Summary

The risk weight for each supervisory rating category under the Basic Approach is consistent with the same risk weight functions used in the Advanced Approach, with supervisory values of PD, LGD, and M associated with each category. For PDs and LGDs, these parameter settings are based on published historical default and loss severity studies. However, implicit in this approach are two key assumptions: (1) that the Basic Approach's slotting criteria are consistent with the external rating band associated with each supervisory rating category and (2) that historical loss severities during the early 1990s provide reasonable estimates of those likely to prevail in future periods of high default rates. An important issue is whether these assumptions are valid, especially given improvements over the past decade at many banks in the underwriting and risk management of CRE loan portfolios.

VI. Concluding Remarks

As noted in section IV, several of the empirical analyses conducted by staff of the Federal Reserve Board suggest that the asset correlation for CRE portfolios is larger than that for C&I portfolios. Considering the inconsistencies in the empirical results, however, as well as the known problems with the available data sources and the empirical methods, the banking agencies believe that the empirical basis is not yet adequate for employing a higher asset correlation in computing capital requirements for CRE loans secured by in-place properties than is used in computing C&I capital requirements. The same problems with data and empirical methods also generate uncertainty regarding the relative asset correlation for loans financing land acquisition, development, and construction (ADC), but the Federal Reserve Board staff has long been concerned that lending to finance speculative construction and development is particularly vulnerable to swings in CRE markets. For this reason, the Board staff proposes that the high-asset-correlation (HVCRE) treatment apply to all ADC lending--with the exception of loans financing the construction of single-family residential properties--except when substantial borrower equity is at risk or the properties have been pre-sold or substantially pre-leased.

Given the uncertainty that remains, however, regarding the relative asset correlations of C&I, in-place CRE, and ADC loan portfolios, the staff of the Federal Reserve Board intends to continue conducting empirical research into the issues raised in this paper. The formal ANPR process will solicit comments on the reasonableness of the proposed treatment of CRE loan portfolios. As noted in the introduction, however, the Board staff particularly hopes that this paper will stimulate active participation by banks and other interested parties in the process of resolving the remaining uncertainty regarding asset correlations. For example, banks may participate by conducting additional research, by making available additional data sources, or by furthering the development and assessment of empirical methodologies. This work will continue to advance the goal of making regulatory capital requirements more sensitive to the level of credit risk represented in the loan portfolios held by U.S. financial institutions.

APPENDIX A: REGULATORY CAPITAL FUNCTIONS FOR CRE UNDER ADVANCED IRB

I. Low-Asset-Correlation CRE portfolios (IPRE)

The formulas for determining regulatory capital under the Advanced IRB Approach for low asset correlation CRE loan portfolios (HVCRE) are given in ¶241 of the third consultative paper:

Capital requirement:

$$K = LGD \times N[(1-R)^{-0.5} \times G(PD) + (\frac{R}{1-R})^{0.5} \times G(0.999)]$$
$$\times (1-1.5 \times b(PD))^{-1} \times (1+(M-2.5) \times b(PD))$$

Asset correlation:

$$R = 0.12 \times \frac{1-\exp(-50 \times PD)}{1-\exp(-50)}$$
$$+0.24 \times 1 - \frac{1-\exp(-50 \times PD)}{1-\exp(-50)}$$

Maturity adjustment: $b = (0.08451 - 0.05898 \times \log(PD))^2$

Risk-weighted assets: $RWA = K \times 12.50 \times EAD$

where PD and LGD are measured as decimals, M is measured as years, and EAD is measured as dollars.

II. High-Asset-Correlation CRE portfolios (HVCRE)

The formulas for determining regulatory capital under the Advanced IRB Approach for high asset correlation CRE loan portfolios (HVCRE) are identical to those for low-asset-correlation (IPRE) portfolios except for the asset correlation function, which takes on a maximum value of 30 percent rather than 24 percent, as noted in ¶252 of the third consultative paper:

Capital requirement:

$$K = LGD \times N[(1-R)^{-0.5} \times G(PD) + (\frac{R}{1-R})^{0.5} \times G(0.999)] \times (1-1.5 \times b(PD))^{-1} \times (1+(M-2.5) \times b(PD))$$

Asset correlation:

$$R = 0.12 \times \frac{1-\exp(-50 \times PD)}{1-\exp(-50)} + 0.30 \times 1 - \frac{1-\exp(-50 \times PD)}{1-\exp(-50)}$$

Maturity adjustment: $b = (0.08451 - 0.05898 \times \log(PD))^2$

Risk-weighted assets: $RWA = K \times 12.50 \times EAD$

where PD and LGD are measured as decimals, M is measured as years, and EAD is measured as dollars.

APPENDIX B: THE STRUCTURE OF IRB CAPITAL CHARGES

Under the Basel II IRB framework, the capital charge for a corporate, bank, sovereign, or specialized lending exposure ("loan") is based solely on loan-specific information. Inputs supplied by the bank include the exposure at default (EAD), the probability of default (PD), the loss rate given default (LGD), and the effective remaining maturity (M). Given these inputs, the IRB capital charge is computed under the assumption that a single risk factor drives correlations in losses across loans, and that a bank's loan portfolio is well diversified. Gordy [forthcoming] shows that this Asymptotic Single Risk Factor (ASRF) framework allows one to calculate capital charges on a decentralized loan-by-loan basis which nonetheless aggregate up to satisfy a portfolio-wide Value-at-Risk (VaR) target.

The IRB framework attempts to set the total capital charge for credit risk at a level sufficient to cover economic credit losses over the next year with probability q, currently set at 99.9 percent. This quantity is approximated as the sum of (a) the portfolio's expected credit losses attributable to defaults occurring over the next year (EL), and (b) the portfolio's Unexpected Loss at the q-percent confidence level (UL^q). Let i index the obligors represented in a bank's credit portfolio at the beginning of the one-year analysis horizon. The portfolio's expected loss is estimated as

(1) $$EL = \sum_i EAD_i \cdot PD_i \cdot LGD_i \ ,$$

while its unexpected loss is calculated as

(2) $$UL^q = L^q - \overline{L}$$

where L^q and $\bar{L}$, respectively, denote the q-th percentile level and the expected level of portfolio economic credit losses over the next year. In the above expression, EL and $\bar{L}$ may differ for M>1 since the former reflects only those losses arising from defaults that are realized over the next year, whereas the latter reflects economic or mark-to-market losses arising from all deteriorations in obligor credit quality, some of which may not lead to defaults within the next year.

To operationalize this framework, it is necessary to estimate UL^q or, equivalently, L^q and $\bar{L}$. While a number of standard credit risk models could accomplish this task, the IRB approach employs the so-called Vasicek framework, which is a special case of the Merton class of credit risk models. The RiskMetrics Group's CreditMetrics and KMV's Portfolio Manager are well-known examples of the broader Merton class. Under this approach, it is assumed that each obligor *i* in year *t* has associated with it a so-called latent normal random variable, Y_{it}, representing the normalized one-year economic rate of return on the obligor's assets. Equivalently, Y_{it} can be viewed simply as a reduced-form index of the obligor's credit quality. Obligor *i* is presumed to default during year *t* if Y_{it} falls below a critical threshold g_i, termed the *default cut-off*. Under this specification, obligor *i*'s *unconditional probability of default* (*i.e.*, PD) is

(3) $$\Pr[Y_{it} < g_i] = \Phi(g_i)$$

where $\Phi(z)$ is the standard normal cumulative density function.

The Vasicek framework assumes that Y_{it} is determined by a systematic risk factor X_t and an idiosyncratic risk factor E_{it}. Both are assumed to be standard normal and are independent of all other risk factors. The relationship between Y_{it}, X_t, and E_{it} is parameterized as follows:

(4) $$Y_{it} = -X_t\sqrt{r_i} + E_{it}\sqrt{1-r_i}\ .$$

Since the systematic risk factor X_t is common across all obligors, it induces cross-obligor correlations in credit quality. The relative importance of systematic risk for obligor *i* is determined by the *asset correlation* parameter r_i. From equation (4), the correlation between Y_{it} and Y_{jt} for the pair of obligors *i* and *j* is

(5) $$Cor[Y_{it}, Y_{jt}] = \sqrt{r_j r_i}\ .$$

Since Y_{it} and Y_{jt} are jointly normally distributed, the probability that both obligor *i* and obligor *j* default at the same time is

(6) $$\Pr[Y_{it} < g_i \cap Y_{jt} < g_j] = F(g_i, g_j; \sqrt{r_i r_j})$$

where $F(z_i, z_j; r)$ is the CDF for a bivariate normal vector with zero mean, unit variance, and correlation *r*. Note that the joint default probability is an increasing function of both obligors' asset correlation parameters.

Conditional on X_t, the probability that obligor *i* defaults is

(7) $$p(X_t) \equiv \Pr[Y_{it} < g_i | X_t] = \Phi\left(\frac{g_i + X_t\sqrt{r_i}}{\sqrt{1-r_i}}\right).$$

As shown in Appendix C, equations (6) and (7) can be used to estimate asset correlation parameters from observed data on loss or default rates.

Given an obligor's PD, LGD, M and an estimate of its asset correlation parameter, one can derive the obligor's regulatory capital charge. From the results in Gordy [forthcoming], under the ASRF and Vasicek assumptions the quantity L^q equals the portfolio's expected loss conditional on setting X_t at its q-th percentile, X^q. This implies (from the linearity of expectation operators) that the capital charge per dollar of exposure for any asset *i* can be calculated as $PD_i \cdot LGD_i + (L_i^q - \overline{L})$, where L_i^q denotes the asset's expected economic loss *conditional* on $X = X^q$, and $\overline{L}$ denotes the asset's *unconditional* expected economic loss.

For a loan having a one-year maturity (i.e., M=1), the ASRF/Vasicek model implies an IRB capital charge per dollar of exposure, $k(PD, LGD, M; r)$. When M=1, this is given by $PD \cdot LGD + (p(X^q) \cdot LGD - PD \cdot LGD) = p(X^q) * LGD$, which is equal to

$$(8) \qquad k(PD, LGD, 1; r) = \Phi\left(\frac{\Phi^{-t}(PD) + \Phi^{-t}(0.999)\sqrt{r}}{\sqrt{1-r}}\right) \cdot LGD$$

where $\Phi^{-1}(p)$ is the inverse of the standard normal CDF.

For a loan having a maturity greater than one year (i.e., M>1), the IRB capital charge formula is somewhat more involved because of the need to reflect the possibility that any deterioration in the credit quality of the obligor can cause a loan to lose economic value, even though the deterioration may not be so great as to trigger a default. Based on analyses employing a variety of industry-standard credit risk models, for M > 1, the IRB capital charge per dollar of exposure is calculated as

(9) $$k(PD, LGD, M; r) = k(PD, LGD, 1; r) \cdot [1 + b(PD) \cdot (M - 1)]$$

where $b(PD) = (0.08451 - 0.05898 \ln(PD))^2$.

APPENDIX C: ESTIMATING ASSET CORRELATIONS

As suggested in Appendix B, asset correlations play a critical role within many credit risk models, including that underpinning the IRB framework. This appendix presents a brief summary of empirical methods used to estimate asset correlations in practice.

I. Estimation Based on Historical Default Rate Volatilities

A number of techniques for estimating asset correlations employ historical data on the annual default frequencies of obligors within a particular risk-rating category; for example, obligors having internal risk ratings that imply similar PDs at the beginning of each year. Under the assumption that the rating bucket contains a very large number of homogeneous obligors, the ASRF framework implies that the aggregate default rate in year t, D_t, is given by

$$D_t = \Phi\left(\frac{g + X_t\sqrt{r}}{\sqrt{1-r}}\right) \tag{10}$$

where X_t is a standard normal random variable, the mean default rate is $\Phi(g)$, and the asset correlation is r.[33]

A. Estimation via Maximum Likelihood

Equation (10) can be rewritten as a standard linear regression model

[33]This equation follows from equation 5 of Appendix B and the assumption that, conditional on X, obligor defaults are independent of one another.

(11) $$Z_t \equiv \Phi^{-1}(D_t) = \frac{g}{\sqrt{1-r}} + \frac{\sqrt{r}}{\sqrt{1-r}} \cdot X_t \,.$$

Letting m_Z and s_Z^2 denote the mean and variance of the observed Z_t, the maximum likelihood estimators of g and r are

(12) $$\hat{g} = m_Z \sqrt{1-\hat{r}}$$ and

(13) $$\hat{r} = \frac{s_Z^2}{1+s_Z^2} \,.$$

B. Estimation via Method of Moments

Under the same assumptions given above (*e.g.*, homogeneity of obligors over time), an alternative approach to estimation begins with the observation that the aggregate default rate D_t may be written

(14) $$D_t = (1/N) \cdot \sum_{i=1}^{N} D_t^i$$

where D_t^i is an indicator equal to unity if the *i*-th obligor defaults and zero otherwise, and N is the number of obligors in the rating bucket.

Let the expected default rate for the obligors in the rating bucket be denoted $m_D = E\{D_t^i\}$ and let the second moment be denoted $\Pi_D = E\{(D_t^i)^2\}$. It follows that the variance of D_t is given by

$$
\begin{aligned}
s_D^{\,2} &= E\{(D_t)^2\} - m_D^2 \\
(15) \qquad &= \frac{E\{\sum_i (D_t^i)^2\} + E\{\sum_{i \neq j} D_t^i \cdot D_t^j\}}{N^2} - m_D^2 \\
&= \frac{E\{(D_t^i)^2\}}{N} + \frac{N(N-1)\cdot(r_{Def} \cdot m_D(1-m_D) + m_D^2}{N^2} - m_D^2
\end{aligned}
$$

where

$$
(16) \qquad r_{Def} \equiv \frac{Cov\{D_t^i, D_t^j)}{\sqrt{Var\{D_t^i\}}\sqrt{Var\{D_t^j\}}} = \frac{E\{D_t^i \cdot D_t^j\}}{m_D(1-m_D)} - m_D^2
$$

is the default correlation between any two loans in the bucket.

When the number of obligors in the risk bucket is very large, then the variance of D_t approaches the limiting value

$$
(17) \qquad s_D^{\,2} = r_{Def} \cdot m_D \cdot (1-m_D)\,.
$$

Thus, an estimator for the default correlation is

$$
(18) \qquad \hat{r}_{Def} = \frac{s_D^{\,2}}{m_D \cdot (1-m_D)}\,.
$$

Given an estimate for r_{Def}, an estimator of the asset correlation can be constructed upon noting that $E\{D_t^i \cdot D_t^j\}$ equals the unconditional probability of two defaults among any two loans picked at random from the risk bucket. By definition, this probability equals

$$
(19)\qquad \begin{aligned} E\{D_t^i \cdot D_t^j\} &= P\{Y_{it} < g \;\; and \;\; Y_{jt} < g\} \\ &= F(g, g; r) \end{aligned}
$$

where Y_{it} is defined in Appendix B and $F(g, g; r)$ denotes the cumulative distribution function of two standardized normal random variables having a correlation equal to r .

Upon substituting (19) into (16), an estimate of the asset correlation can be derived from estimates of r_{Def} and g by solving the following nonlinear equation for $\hat{r}$:

$$
(20)\qquad F(\hat{g}, \hat{g}; \hat{r}) = \hat{r}_{Def} \cdot m_D \cdot (1 - m_D) + m_D^2 \, .
$$

II. Estimation Based on Historical Rates of Return

Under the ASRF assumptions set forth in Appendix B, for a homogeneous set of obligors the asset correlation can be interpreted as the R^2 associated with a one-factor model linear regression predicting firms' economic rates of return on assets:[34]

$$
(21)\qquad \begin{aligned} Rate\ of\ return\ on\ assets_{it} &= ROA_{it} \\ &= a + b \cdot (X_t \sqrt{r} + E_{it} \sqrt{1 - r}) \\ &= a + X_t \cdot (b\sqrt{r}) + E_{it} \cdot (b\sqrt{1 - r}) \end{aligned}
$$

$$
\Longrightarrow \quad R^2 = \frac{r b^2}{b^2} = r \, .
$$

[34]Recall from Appendix B that the latent variables Y_{it} are normalized to have a variance of one and a mean of zero. The parameters 'a' and 'b' below reflect the unwinding of this normalization, since the dependent variable in this equation is the un-normalized rate of return. In this re-parameterization, 'a' represents the mean rate of return, while 'b' represents the standard deviation of the rate of return.

This relationship can be employed to estimate the asset correlation when historical economic return-on-asset data are available for a relatively large group of firms having similar underlying assets. Under the ASRF assumptions, the aggregate return on assets for the group as whole, $ROA_t = \sum_i l_i \cdot ROA_{it}$ where $\sum_i l_i = 1$, approximates the systematic component of the ROA for all the obligors:

(22) $$ROA_t = a + X_t(b\sqrt{r}).$$

Thus, the asset correlation can be estimated as the R^2 in an OLS cross-section or panel-data regression in which the dependent variables are the ROAs of the individual firms, ROA_{it}, and the explanatory variable is the aggregate ROA_t:

(23) $$ROA_{it} = w_0 + w_1 ROA_t + residual_{it}.$$

APPENDIX D: REGRESSION EQUATION SUMMARIZING DETERMINANTS OF CREDIT ENHANCEMENT REQUIRED TO ACHIEVE BBB^{+}/A^{-} RATING ON CMBSS

Sample:

The data used for this exercise were extracted from the Moody's Investor Services Pre-Sale Reports for Commercial Mortgage Backed Securities (CMBSs). The sample includes CMBSs issued between the beginning of 1999 and the end of 2002. CMBSs secured by fewer than 75 loans were dropped from the sample as were pools secured by non-U.S. loans. In addition, only CMBSs for which all required data (as described below) were available were included in the sample. After applying these restrictions, the sample contained 77 pools of loans.

Dataset Construction:

General:

The asset pool underlying each CMBS is made up of four major components: the cooperative component, the credit tenant lease (CTL) component, the shadow-rated component, and the conduit. The cooperative portion includes loans that are secured by residential cooperative properties. CTL loans are secured by properties that have a single tenant leasing the entire building. Shadow-rated loans are all loans for which Moody's has provided a shadow rating. The conduit portion includes all loans that do not fall into one of the other three categories.

Using the pre-sale reports, the following variables were created for each CMBS in the sample:

Variable	Description
CE*	Credit enhancement required for a rating between Baa1 and A3
BalTot	Total poo91l balance

BalSh	Balance of shadow-rated component
ShareSh	% of total pool balance in shadow-rated component
AupSh	% of shadow-rated balance rated A3 or higher.
BalCoop	Balance of cooperative component
ShareCoop	% of total pool balance in cooperative component
BalCTL	Balance of CTL component
ShareCTL	% of total pool balance in CTL component
InvCTL	% of CTL balance that is investment grade (Baa3 or higher).
BalCon	Balance of conduit
ShareCon	% of total pool balance in conduit
LTV110	% of Conduit balance with a loan to value ratio (LTV) greater than 110%
LTV100_110	LTV between 100 and 110
LTV90_100	LTV between 90 and 100
LTV80_90	LTV between 80 and 90
LTV70_80	LTV between 70 and 80
LTV60_70	LTV between 60 and 70
LTV60	LTV below 60
Office	% of conduit balance consisting of office properties
Industrial	% consisting of industrial, warehouse, and self-storage
Retail	% consisting of anchored and unanchored Retail
Multi	% consisting of multifamily
SinglePurp	% consisting of single purpose properties (hotels, health care, etc)
Mixed	% consisting of mixed use properties
Other	% consisting of other properties

Additional information regarding the variables included in the analysis is available upon request.

Model and Results

The credit support necessary for a rating between A3 and Baa1 was regressed on various characteristics of the CMBS pools in the following specification:

$$
\begin{aligned}
CE^* = a_0 &+ ShareSh \cdot (b_0 + b_1 \cdot AupSh) \\
&+ ShareCoop \cdot g_0 \\
&+ ShareCTL \cdot (d_0 + d_1 \cdot InvCTL) \\
&+ ShareCon \cdot (\sum_i w_i \cdot LTV_i + \sum_j w_j \cdot Type_j)
\end{aligned}
$$

The results of the OLS regression are displayed in Table D-1.

Table D-1: OLS Estimates of Credit Enhancement Required for BBB+/A- Rating

Regressor	Estimate	Standard Error	t-statistic
Intercept	-0.048	0.079	-0.61
ShareCoop	-0.027	0.097	-0.28
ShareSh	0.158	0.083	1.91
AupSh	-0.115	0.039	-2.96
ShareCTL	0.321	0.119	2.70
InvCTL	-0.125	0.122	-1.03
LTV110	0.344	0.071	4.83
LTV100_110	0.249	0.084	2.96
LTV90_100	0.097	0.069	1.41
LTV80_90	0.121	0.071	1.71
LTV70_80	0.096	0.073	1.32
LTV60_70	0.008	0.135	0.06
Office	0.113	0.042	2.72
Retail	0.021	0.041	0.52
Multi	0.091	0.040	2.27
Single-Purpose	0.154	0.051	3.03
Mixed	0.120	0.081	1.49
Other	0.533	0.274	1.95
N	77	Adj R^2	88%

Omitted categories: Industrial properties, percentage of conduit with LTV below 60 percent, percentage of CTL loans below investment grade, percentage of shadow-rated loans below Baa1, and conduit share of total pool balance.

APPENDIX E: EMPIRICAL EVIDENCE ON LOAN-LEVEL LOSS CHARACTERISTICS

As noted in the text, the asset correlation is the main parameter that was estimated by the Basel Committee in developing the proposed treatment of CRE loan portfolios, and it is important to recall that the asset correlation is a *portfolio-level* loss characteristic. This appendix briefly reviews available evidence on *loan-level* loss characteristics, namely the probability of default (PD) and loss given default (LGD). It is useful to keep in mind that loan-level loss characteristics were not estimated by the Basel Committee for the purpose of developing the Advanced IRB Approach for CRE portfolios, since these parameters will be estimated directly by banks. However, evidence on the average loss rate for defaulted CRE loans was used in establishing supevisory risk weights under the Basic Approach.

Average PD and PD Volatility[35]

Table E-1 and Graph E-1 summarize available empirical evidence on average observed default frequency for CRE portfolios. Several factors help to explain the divergence in default and delinquency rates shown in the table and graph.

- First, some studies (Ciochetti *et al.* [2001], Vandell [1992], and Goldberg & Capone [1998]) define default as the initiation of foreclosure proceedings, while the remaining studies define default as 90+ days late on a loan payment (*i.e.*, 60+ days after expiration of the grace period). Table E-1 identifies which studies use a foreclosure definition of default and which use a delinquency definition.
- Second, the mix of commercial property types varies from one sample to another: for example, Ciochetti *et al*. [2001] use a sample that "is dominated by loans secured by office properties," which tend to have higher default rates than other property types (see Pelletier & Rudenstein (Fitch) [1998] and Gajjar & Monsma (Fitch) [1995]), while Goldberg & Capone use a sample restricted to multifamily properties, which tend to have lower default rates (see the same sources).

[35] More precisely, this section presents evidence on observed default frequency. Even if PD is estimated correctly *ex ante*, it will not generally equal observed default frequencies.

- Third, the time period over which the default rate is estimated varies: for example, the average annual delinquency rate given by Mejia [1999] is higher than that given by Shilton & Teall [1994] from the same source (American Council for Life Insurance semiannual reports) because Mejia averages the annual rate over a later time period (1975-1997) that includes the delinquency peak of 1992-1993, while Shilton & Teall average over an earlier period (1968-1989) that stops three years short of the peak.
- Fourth, loan seasoning varies, which means that studies vary according to whether they measure defaults and delinquencies over the lowest- or highest-risk periods of the life of the loan. For example, Ciochetti *et al*. [2001] use loans that have been seasoned for at least five years, while Freydberg & Lee (Fitch) [1999] have no seasoning requirement: in fact, 48 percent of the loans in their sample are seasoned *less* than one year, and 16 percent less than six months. For CRE, the peak conditional default rate occurs at perhaps six years.

The two sources of estimates that vary annually suggest that observed default frequencies for CRE loans have generally been quite volatile over the commercial property credit cycle. For example, the estimates given by Ciochetti *et al*. [2001] for the frequency of foreclosure in a sample dominated by office properties vary from 0.33 percent in 1999 to 6.99 percent in 1992, with a standard deviation of 1.96 percent and a coefficient of variation of 73 percent around its average of 2.69 percent. Similarly, the estimates given by Goldberg & Capone [1998] for foreclosure frequency in a sample of multifamily properties vary from 0.14 percent in 1986 to 3.05 percent in 1993, with a standard deviation of 0.98 percent and a coefficient of variation of 71 percent around its average of 1.39 percent. In both cases, the maximum annual PD was about 21 times as large as the minimum annual PD during the time period.

Overall, the best source of data for the average default frequency over a full credit cycle for a portfolio of mixed property types is probably Mejia [1999], which estimates it at 2.9 percent. It is important to keep in mind, however, that the average default frequency may well have declined during the last decade as banks strengthened loan underwriting practices in response to the most recent severe CRE downturn.

Distribution of Loans by PD

Evidence of the distribution of loans by PD in CRE loan portfolios is useful in predicting the average total amount of capital that banks will set aside for CRE portfolios under either the Advanced or the Basic Approach. Essentially the only sources of data on the distribution of CRE loans by PD are the two quantitave information surveys known as QIS 2.5, which yielded information for three U.S. banks, and QIS3, which yielded less detailed information for 13 U.S. banks. Although these data are confidential, it is possible to use the QIS 2.5 data to synthesize a representative distribution of CRE loans by PD, from which one can estimate the share of CRE loans that would be slotted into each of the four supervisory rating grades for performing loans envisioned in the proposed Basic IRB Approach. Using the average one-year default probability by rating grade for corporate loans published by Standard & Poor's (see Brady, Vazza & Bos (S&P) [2003])[36] to estimate the boundaries of the supervisory rating grades, this suggests the following distribution of CRE loans:

Table E-2: Estimated Distribution of CRE Loans by External Rating Grade

Supervisory Rating Grade	Reference External Rating Grade	Average PD	Portfolio Share	Average PD	Portfolio Share
Very Strong/Preferential*		0.19%	18%	0.19%	18%
Strong	BBB	0.30%	7%	0.31%	15%
	BBB-	0.46%	8%		
Good	BB+	0.56%	5%	1.03%	48%
	BB	1.26%	42%		
Satisfactory	BB	2.26%	10%	2.67%	16%
	B+	3.63%	6%		
Weak	B to C-	10.7%+	4%	10.7%+	4%

*Loans having "underwriting and other risk characteristics are substantially stronger than specified in the slotting criteria" for Strong loans, and therefore receiving preferential risk weights. These are assumed to have an external risk-rating equivalent of BBB+ or better.

[36] As noted in the S&P report, it is important to take into consideration the decisions by some companies not to maintain ratings, because many of those non-rating decisions are motivated by declines in quality that would translate into rating downgrades. Because the authors do not provide an N.R.-adjusted one-year default probability for all rating grades, Board staff estimated the adjustment using a quadratic regression of adjusted on unadjusted probabilities for selected rating grades (shown in Tables 7 and 8 of Brady, Vazza & Bos (S&P) [2003]).

These figures suggest that the largest share of CRE loans would be rated at the equivalent of S&P's BB grade, and about half would be slotted into the "Good" supervisory rating grade. Almost one-fifth would receive preferential risk weights, another 15 percent would be slotted "Strong," 16 percent would be slotted "Satisfactory," and four percent would be slotted "Weak." On the basis of this estimated distribution, the average PD for a CRE portfolio can be estimated at about 1.17 percent. Assuming this were a good estimate of the portfolio distribution, it suggests that the average risk weight would be about 101 percent for low asset correlation CRE loans and about 117 percent for high asset correlation CRE loans.[37]

It is important to keep in mind, however, that the PD estimates derived from the U.S. QIS data were lower than most of the sources of data on observed default frequency for CRE loans summarized above in Table E-1 and Graph E-1. It is likely that this reflects a recent time period over which default probabilities were probably estimated by the U.S. banks participating in QIS 2.5: as Ciochetti *et al.* [2001] shows (Graph E-1), observed default rates since 1996 have been well below what appears to be their average over the credit cycle.[38] One way to correct for this is to adjust the distribution of default probabilities for each bank submitting U.S. QIS data so that the average PD equals 2.9 percent, the average estimated by Mejia [1999] over an entire credit cycle. This adjustment gives the following estimated distribution of average PDs over the cycle:

---

[37]These average risk weights would correspond to minimum regulatory capital requirements of about 7.9 percent for low-asset-correlation portfolios and about 9.5 percent for high-asset-correlation portfolios. As noted, half of each figure would be minimum Tier I capital and the other half would be Tier II capital.

[38]It is also possible, if not likely, that the QIS banks applied C&I default probabilities to loans in each rating band, rather than developing *ab initio* estimates of PD for CRE loans in each rating band. For this and other reasons, the QIS data are best regarded only as approximate.

Table E-3: Revised Estimated Distributon of CRE Loans by Rating Grade

Supervisory Rating Grade	Reference External Rating Grade	Average PD	Portfolio Share	Average PD	Portfolio Share
Preferential		0.19%	10%	0.19%	10%
Strong	BBB	0.30%	4%	0.31%	9%
	BBB-	0.46%	5%		
Good	BB+	0.56%	3%	1.05%	43%
	BB	1.26%	40%		
Satisfactory	BB-	2.26%	14%	2.73%	24%
	B+	3.63%	10%		
Weak	B to C-	10.7%+	14%	10.7%+	14%

Assuming this were a good estimate of the portfolio distribution of average PDs over the cycle, it suggests that the average risk weight would be about 137 percent for low asset correlation CRE loans and about 151 percent for high asset correlation CRE loans over the cycle.[39]

The more recent quantitative information survey known as QIS 3 yielded preliminary data on the proportion of each bank's CRE loan portfolio that would be classified into each of the supervisory risk categories. Although these data, like the QIS 2.5 data, are confidential, Table E-4 presents an estimated distribution of PDs that is based on the median proportion of loans reported by participating banks in each supervisory risk category:

Table E-4: Estimated Distribution of CRE Loans Based on QIS 3 Data

Rating Grade*	Strong	Good	Satisfactory	Weak	Default
Proportion	8.1%	46.7%	41.1%	4.1%	0.1%

*For the purpose of QIS 3 no preferential risk weights were used.

This estimated distribution suggests that the average risk weight would be about 128 percent for low asset correlation CRE loans (corresponding to minimum regulatory capital of 10.2 percent,

[39]These average risk weights would correspond to average (over a cycle) minimum regulatory capital requirements of about 10.7 percent for low-asset-correlation portfolios and about 12.2 percent for high-asset-correlation portfolios; again, half of each figure would be minimum Tier I and the other half would be Tier II.

half of it Tier I) and about 152 percent for high asset correlation CRE loans (corresponding to minimum regulatory capital of 12.2 percent, half of it Tier I).

Relative to the distribution of C&I loans, it appears that a smaller proportion of CRE loans are rated investment grade (BBB- or better) and therefore in the “Strong” supervisory rating grade (including the preferential treatment), while a larger percentage would be slotted into the “Good” and “Satisfactory” supervisory rating grades. Treacy & Carey [1998] presented estimates that imply the following approximate distribution of C&I loans by rating grade:

Table E-5: Estimated Distributon of C&I Loans by Supervisory Rating Grade

<table>
<tr><th>Supervisory Rating Grade</th><th>External Rating Grade</th><th>Portfolio Share</th></tr>
<tr><td>Very Strong/Pref</td><td>BBB+ or better</td><td>20%</td></tr>
<tr><td rowspan="2">Strong</td><td>BBB</td><td rowspan="2">28%</td></tr>
<tr><td>BBB-</td></tr>
<tr><td rowspan="2">Good</td><td>BB+</td><td rowspan="3">40%</td></tr>
<tr><td>BB</td></tr>
<tr><td rowspan="2">Satisfactory</td><td>BB-</td></tr>
<tr><td>B+</td><td rowspan="2">10%</td></tr>
<tr><td rowspan="2">Weak</td><td>B to B-</td></tr>
<tr><td>CCC+ to C-</td><td>2%</td></tr>
</table>

It is possible, however, that the average default probability corresponding to each external rating grade differs somewhat for CRE loans than for C&I loans. For example, the average C&I default probability by rating grade is different for companies in industries related to construction and management of real estate than for all corporations generally:

Table E-6: Estimated Distribution of Real Estate Related C&I Loans by Rating Grade

	External Rating				
	A or better	BBB	BB	B	CCC
Real Estate Related	(too few defaults)	0.63%	1.50%	3.85%	20.83%
All C&I 1981-2001		0.26%	1.22%	5.96%	24.72%

Average LGD[40]

Tables B-7 and B-8, and Graph E-2, summarize empirical evidence on average loss rates on defaulted CRE loans. The studies shown in Table E-7 and Graph E-2 provide enough information that readers can have some confidence in their empirical methodology; in contrast, those summarized in Table E-8 do not give enough detail to assess their methodology. It is important to note that the studies differ according to whether they measure losses associated with all defaulting loans or only loans that went to foreclosure, and also according to whether they measure all economic losses through disposition of the asset or only those losses (often accounting losses rather than economic losses) through the date when the property was transferred from debt to equity (Real Estate Owned, or REO). Each study in Table E-7 is identified accordingly. Moreover, for those studies in Table E-7 that are based only on foreclosed properties and/or track losses only through transfer to REO, conclusions presented in Ciochetti & Shilling [1999] and Ciochetti & Riddiough [2000] were used to impute loss rates that can be compared to those computed over all defaulting loans and through asset disposition. This “comparable loss” is shown in the second column of Table E-7 and in Graph E-2.

Considering the studies in which we can have relatively high confidence (Table E-7 and Graph E-2), there appear to be two groups. One group--Pelletier & Rudenstein (Fitch) [1998]; Esaki, L’Heureux & Snyderman (Fitch) [1999]; Dillon & Belanger (Fitch) [1996]; and Ciochetti & Riddiough [2000]--provide only average loss rates over their entire respective time periods, and find average loss rates in the range of 31 percent to 37 percent.[41] (Snyderman [1991] which suggests average loss rates around 27 percent, uses data preceding the CRE downturn of the early 1990s and should therefore be considered biased downwards as an indicator of average loss rates over an entire credit cycle.) The second group--Ciochetti & Shilling [1999]; Ciochetti [1997]; and Ciochetti & Riddiough [1998]--estimate average loss rates separately for each year during their respective time periods, and find higher average loss rates in the range of 48 percent to 56 percent.

---

[40]As before, to be more precise this section presents evidence on observed loss rates on defaulted loans. Even if LGD is estimated correctly *ex ante*, it will not generally equal observed loss rates.

[41]With the exception of Ciochetti & Riddiough [2000], these studies present figures only for loans that have resulted in foreclosure; the figures given in this paper are imputed, as described above, for all defaulted loans regardless of whether they resulted in foreclosure.

The second group of studies provides more precise information concerning data sources and analytical methods; nevertheless, the current proposed treatment of CRE loans is more consistent with the first group of studies. Of these, Esaki, L'Heureux & Snyderman [1999]--which estimates the average loss rate at 31.7 percent--is perhaps the best source of loss rate estimates over an entire credit cycle. The same study also reports a higher average loss rate of 36.8 percent for loans liquidated during 1992-1997, which can be interpreted as reflecting the difference between average LGDs over an entire credit cycle and average LGDs during a stress period.

Relationship Between PD and LGD

There are at least four potential sources of information on the relationship between PD and LGD for CRE:[42]

(A) During 2001 three large U.S. banks reported loan volumes by PD-LGD cells as part of the quantitative impact survey known as QIS 2.5. In one case the cells were defined as ranges over each of the two dimensions (PD and LGD), and the PD-LGD relationship was analyzed using the midpoint of each range as the inferred point estimates for that cell.

(B) Pelletier & Rudenstein (Fitch) [1998] present annual data on observed loan default frequencies and losses on defaulted loans from a sample of 18,839 loans issued in 33 CMBS transactions between 1991 and 1996; of these loans 3,134

---

[42] As before, more precisely this section presents evidence on the relationship between observed default frequencies and observed loss rates.

Only the first data source shown below (QIS 2.5) purports to represent estimated *ex ante* probability of default (PD) and estimated *ex ante* loss given default (LGD). The other data sources explicitly represent *ex post* observed default frequency and *ex post* observed losses on defaulted loans, and are used under the assumption that the relationship between *ex post* observed default frequencies and *ex post* observed losses on defaulted loans is the same as the relationship between (unobservable) *ex ante* PD and (unobservable) *ex ante* LGD.

Also, two of the data sources (QIS 2.5 and Pelletier & Rudenstein) are cross-sectional, reporting PD (or observed default frequency) and LGD (or observed loss rate on defaulted loans) for different segments of the CRE loan portfolio at a given time; the other two data sources (the two Ciochetti papers and ACLI) are time-series, reporting observed default frequencies and observed losses on defaulted loans for the same CRE loan portfolio in different years. The Basel II proposal implicitly assumes that the cross-sectional relationship between PD and LGD for different loans at a given time is the same as the time-series relationship between PD and LGD for a given loan over time.

Finally, it is useful to note that there may be an empirical relationship between *ex post* observed default frequencies and loss rates on defaulted loans even if there is no relationship between *ex ante* PD and LGD.

experienced a default and 795 were completely resolved and therefore included in the analysis of losses on defaulted loans.

(C) Separate papers by Ciochetti *et al.* [2001] and by Ciochetti & Riddiough [1998] present data on observed loan default frequencies and losses on foreclosed loans by year of foreclosure for what may be the same sample of 2,043 loans originated during the period 1974-1990 by a large insurance company.

(D) Finally, the American Council of Life Insurers publishes annual data on aggregate observed loan default frequencies, as well as losses on foreclosed loans, reported by a group of large life insurance companies for their commercial real estate loan portfolios.

Each of these data sets can be used to infer the PD-LGD relationship by regressing observations for LGD (or observed loss rate on defaulted or foreclosed loans) on corresponding observations for PD (or observed default frequency). This analysis suggests the following relationships between PD and LGD:

(A) LGD_i = 23.9 percent + $0.32*PD_i$

(B) LGD_i = 22.3 percent + $2.09*PD_i$

(C) LGD_t = 18.3 percent + $2.34*PD_t$

(D) LGD_t = 19.8 percent + $3.43*PD_t$

Each of these estimated relationships is shown in Graph E-3, E-4, E-5, or E-6 by a line denoted "non-stress." It is important to note, however, that the four data sources are based on different sets of defaulted loans, use different definitions of "losses," and record data on loans in different stages of the commercial real estate cycle: for example, the QIS 2.5 data reflect a time frame from a period (*c*. 2000) of abnormally low CRE loan default and loss rates. Probably for these reasons, the estimated relationships between PD and LGD imply LGDs that, over the relevant range of PDs, are substantially below the figures reported by the published sources that give enough details to produce confidence in their empirical methodologies (Table E-7).

Because of this, Graphs E-3, E-4, E-5, and E-6 display two additional lines showing estimated relationships between PD and LGD. The dashed line shows an estimated relationship between actual reported PD and an LGD that has been adjusted upward so that a PD of 4.3

percent implies an LGD of 32.9 percent, these two figures being the average observed default frequency and average observed loss rate reported by Pelletier & Rudenstein (Fitch) [1998], the only published study that explicitly and reliably estimates both figures from the same data set.[43] The dotted line shows an estimated relationship that has been further adjusted to reflect stress LGDs rather than average LGD over the economic cycle. This is accomplished by adjusting the relationship so that a PD of 2.9 percent (the average default frequency reported by Mejia [1999] over an entire credit cycle) implies an LGD of 36.8 percent (the average loss rate suggested by the results reported by Esaki, L'Heureux & Snyderman [1999] for loans liquidated during the period 1992-1997).

These adjusted relationships suggest the following estimates of LGD for loans that would be slotted into the four supervisory rating grades in the Basic Approach:

Table E-9: Estimated LGD by Supervisory Rating Grade

Supervisory Rating Grade	Default Probability	Implied Average LGD over Cycle (%)				Implied Stress LGD (%)			
		(A)	(B)	(C)	(D)	(A)	(B)	(C)	(D)
Preferential	0.19%	24.0	22.7	18.7	20.4	35.9	31.1	30.5	27.5
Strong	0.46%	24.1	23.2	19.4	21.3	36.0	31.7	31.1	28.4
Good	1.26%	24.3	24.9	21.2	24.1	36.3	33.4	33.0	31.2
Satisfactory	3.63%	25.1	29.8	26.8	32.2	37.0	38.3	38.5	39.3
Weak	10.7%+	27.4+	44.5+	43.2+	56.3+	39.3+	53.0+	54.9+	63.4+

Ultimately, however, the Basel Committee determined that the relationship between PD and LGD is not yet well enough understood to incorporate it explicitly into the supervisory risk weights established for CRE loans under the Basic Approach. Instead, the Committee established supervisory risk weights that are based on reasonable estimates of the average LGD and average effective maturity for CRE loans at all levels of loan-specific risk. For in-place CRE (that is, mortgages) the Committee selected supervisory risk weights that correspond to LGDs ranging roughly from 35 percent to 43 percent for in-place properties, with an average

[43] Pelletier & Rudenstein (Fitch) [1998] reported figures only for loans that had resulted in foreclosure; as noted above, the reported LGD figures were used to impute a comparable figures for all defaulted loans regardless of whether they had resulted in foreclosure.

maturity of five years, while for construction loans the Committee selected supervisory risk weights that correspond to an average LGD of 55 percent and an average maturity of one year.

Models of PD and LGD

Banks seeking to apply the Advanced IRB approach to determine risk weights for their CRE portfolios will have to demonstrate the capability of estimating both PD and LGD for each of their CRE loans. PD and LGD modeling for CRE loans is still in its early stages, but several researchers have developed theoretical and/or empirical models that suggest the factors most relevant in explaining observed default and loss rates. While none of these models has been estimated using data on loans from an individual bank's CRE loan portfolio, collectively these studies provide a foundation for further efforts by banks to predict PD or LGD at the level of the individual CRE loan in accordance with the requirements of the proposed Advanced IRB treatment of CRE loans.

Most theoretical and empirical modeling of CRE loan losses presents the decision to default on a loan as a *costly decision to exercise the default option*. This option framework suggests that a loan's contemporaneous loan-to-value ratio (LTV) and debt service coverage ratio (DSCR) should be particularly important in predicting both the probability of default (PD) and the loss given default (LGD). Specifically, a decline in the value of the property so that the LTV ratio exceeds 100 percent implies that the default option is "in the money," so that the borrower may benefit financially from defaulting. Nevertheless, the fact that exercise of the option is costly means that default may not occur even if the option is in the money unless the net cash flow generated by the property is failing to cover the borrower's debt obligations (*i.e.*, DSCR is less than 100 percent). Empirical analysis suggests that contemporaneous LTV and DSCR are, in fact, the most important variables (either individually or jointly) in predicting PD and LGD for CRE loans.

Several publicly available studies of the factors explaining default and loss behavior on CRE loans are summarized below. The summaries sketch the salient aspects of the empirical approach used in each study, describe the data set employed, and mention the most important findings. The summaries pay particular attention to the use of debt service coverage ratio

(DSCR) and loan-to-value ratio (LTV) in modeling commercial mortgage default and recovery. It is important to note that the variables that have been included in these PD and LGD models have generally been restricted by data availability: that is, the failure of any of these researchers to include a variable in a PD or LGD model does not necessarily suggest that that variable is not relevant in predicting PD or LGD.

Vandell, Barnes, Hartzell, Kraft & Wendt [1993]. The authors estimate a proportional hazards model in which the dependent variable--the number of months from origination until default, repayment, or the end of the sample period, whichever occurred first--is expressed as a function of:

- the estimated contemporaneous LTV,
- the DSCR at origination,
- the contract interest rate,
- dummy variables representing property type (retail, apartment, hotel, office, industrial, or other),
- dummy variables representing borrower type (individual, partnership, corporation, or other), and
- dummy variables representing loan type (accrual or not, cash flow change or not, amortization or not, and step rate payment or not).

The model is estimated using data on 2,899 complete loan histories provided by a large insurance company for that company's first mortgage liens originated during 1962-1989. "Default" refers to loans which were foreclosed upon by the lender or loans for which payment was made in full after the loan had gone through the foreclosure process (including third-party sales), so again the definition of default is considerably stricter than the definition envisioned in the Basel II proposal. The authors found that the hazard rate increases with LTV and with the contract interest rate; the effect of DSCR was not significantly different from zero, a result that the authors noted may reflect collinearities between DSCR and both LTV and contract interest rate.

Goldberg & Capone [1998]. The authors estimate a logistic model in which the dependent variable--whether or not a multifamily mortgage experienced a default in a given year--is expressed as a function of

- the inverse of the estimated contemporaneous DSCR,
- the estimated contemporaneous LTV,
- a dummy variable representing whether each loan was originated prior to the tightening of underwriting standards by Fannie Mae (in 1988) and Freddie Mac (in 1990),
- an index measuring the present value of depreciation writeoffs to a new investor, and
- the number of years since loan origination in quadratic form.

The model is estimated using annual data on 7,564 multifamily mortgages purchased by Fannie Mae or Freddie Mac during 1983-1995, amounting to 52,222 observations on whether a given loan defaulted in a given year. "Default" refers to the forfeiture of property rights and includes foreclosure, third-party sale, note sale, and short sale; thus the definition of default is considerably stricter than the definition envisioned in the Basel II proposal. The authors found that PD increases with LTV and declines with DSCR, and that the predicted conditional probability of default peaks at about seven years after loan origination.

Mejia [1999]. The author estimates a system of three simultaneous equations. The first equation expresses current commercial mortgage supply (*i.e.*, new mortgage commitments) as a function of:

- future mortgage default rates (as a proxy for current expectations of future mortgage default rates),
- the current average contract interest rate, and
- the current inflation rate.

The second equation expresses future loan defaults as a function of:

- the current commercial mortgage supply,
- the current average contract interest rate,
- the current unemployment rate,
- the current average loan-to-value ratio, and
- the current average debt service coverage ratio.

Finally, the third equation expresses the credit spread as a function of:

- future mortgage default rates (as a proxy for current expectations of future mortgage default rates),

- the current 10-year Treasury rate,
- the current average loan term,
- the current average loan size,
- the current average loan-to-value ratio, and
- the current average debt service coverage ratio.

The model is estimated using semiannual data for the period 1975-1997 from the ACLI's Survey of Mortgage Commitments and Mortgage Delinquency Reports. "Default" refers to loan delinquencies. Although the author was not interested primarily in loan default and few of the regressors in the loan default equation were significantly different from zero, the results generally suggest that the delinquency rate increased with the contract interest rate and declined with increases in DSCR; perversely, however, higher LTVs were generally associated *ceteris paribus* with lower delinquency rates.

Ciochetti, Deng, Gao & Yao [2002]. The authors estimate a competing-risks hazard model with two equations, each expressing the probability of default or prepayment as a function of:

- the estimated contemporaneous LTV in quadratic form,
- the estimated contemporaneous DSCR,
- the DSCR at origination,
- the estimated contemporaneous outstanding loan balance as a percentage of the estimated contemporaneous market value of the loan, in quadratic form,
- dummy variables representing loan size (small, medium, or large),
- dummy variables representing property type (apartment, office, industrial, or retail),
- dummy variables representing borrower type (individual, partnership, corporation, or other),
- dummy variables representing loan type (accrual or not, amortized or not, graduated payment or not, and fixed-rate or not),
- dummy variables representing region (East North Central, Mideast, Southeast, Southwest, Mountain, West North Central, Pacific, or Northeast), and
- a dummy variable indicating whether the loan is within one quarter of the maturity date.

The model is estimated using data on 2,090 complete loan histories provided by a large insurance company for that company's first mortgage liens originated during 1974-1990. "Default" is defined in various ways including loans foreclosed or in process of foreclosure, and loans at least 90 days delinquent. The authors found that the hazard rate increases with contemporaneous LTV and declines with increases in contemporaneous DSCR, while DSCR at origination had no statistically significant effect.

Archer, Elmer, Harrison & Ling [2002]. The authors estimate a logistic model in which the dependent variable--whether or not a given multifamily loan was defaulted during the study interval (extending three, five, or seven years after origination)--is expressed as a function of:

- dummy variables representing the LTV at origination (<70%, 70%-80%, 80%-90%, or 90%-100%),
- dummy variables representing the DSCR at origination (<1.2, 1.2-1.4, 1.4-1.6, or >1.6),
- difference between the contract and current 10-year Treasury rates,
- a dummy variable indicating whether or not the mortgage includes a balloon payment,
- mortgage age at securitization,
- year that the property was constructed,
- number of units in the property,
- value of the property per square foot at origination,
- a dummy variable indicating whether the property is located in a state that requires judicial foreclosure proceedings,
- rate of increase of house prices in the MSA since loan origination,
- rates of growth of population, personal income, employment, and average wages in the MSA since loan origination,
- change in 10-year Treasury rates since loan origination, and
- dummy variables representing year of origination, ZIP code, and originating financial institution.

The model is estimated using monthly loan performance data on 495 fixed-rate multifamily mortgages originated during 1989-1995 and securitized by the Resolution Trust Corporation. "Default" is defined as being at least 90 days delinquent. The authors found (as they expected) that LTV at origination had no statistically significant effect on the mortgage default probability, but that PD is significantly higher for loans with original DSCR less than 1.2.

Ambrose & Sanders [2003]. The authors estimate a competing-risks hazard model with two equations, each expressing the probability of default or prepayment as a function of:

- the LTV at origination,
- a dummy variable indicating whether the property is estimated to have negative equity in each time period,
- the property capitalization rate at origination,
- the difference between 10-year and 1-year Treasury rates (representing the term structure),
- the standard deviation of 10-year Treasure rates over the preceding 24 months (representing interest rate volatility),
- the spread between AAA and BBB rated corporate bonds,
- the standard deviation of spreads over the preceding 24 months,
- the percentage difference between the contract and current 10-year Treasury rates,
- dummy variables indicating whether or not the loan includes prepayment lockout provisions or yield maintenance penalties, as well as whether the lockout provision expired in the previous month,
- the mortgage age, and
- dummy variables representing region (North, South, Midwest, or West).

The model is estimated using loan history data on 4,257 loans originated during 1990-2000 underlying 33 commercial mortgage backed securities. The authors found (as they expected) that LTV at origination had no statistically significant effect on the mortgage default hazard.

Ciochetti, Deng, Lee, Shilling, and Yao [2003]. The authors estimate a proportional hazards model in which each dependent variable--the number of months from origination until default, prepayment, or the end of the sample period, whichever occurred first--is expressed as a function of:

- estimated contemporaneous LTV,
- estimated contemporaneous DSCR,
- DSCR at origination,

- percentage difference between the contract and current 10-year Treasury rates,
- a dummy variable indicating whether prepayment or default occurred within one quarter before the scheduled balloon date,
- dummy variables representing loan size (small, medium, or large),
- dummy variables representing property type (retail, apartment, hotel, office, industrial, or other),
- dummy variables representing borrower type (individual, partnership, corporation, or other), and
- dummy variables representing loan type (accrual, graduated/step payment, amortization, or adjustable-rate).

The model is estimated using data on 2,043 complete loan histories provided by a large insurance company for that company's first mortgage liens originated during 1974-1990. "Default" refers to the onset of foreclosure proceedings. The authors found that the hazard rate increases with LTV, and increases with declines in both origination and contemporaneous DSCR.

Ciochetti & Riddiough [1998]. The authors estimate an OLS regression in which the dependent variable--net recovery on foreclosed loans from default until the property is transferred to real estate owned (REO)--is expressed as a function of:

- LTV at origination,
- DSCR at origination,
- age of the loan at foreclosure,
- contract mortgage interest rate,
- length of the foreclosure period,
- a dummy variable indicating whether the foreclosure occurred in a state requiring judicial foreclosure proceedings,
- dummy variables representing property type (apartment, hotel, industrial, office, or other), and
- dummy variables representing the region in which the property is located (East North Central, Mideast, Mountain, Northeast, Pacific, Southeast, Southwest, or West North Central).

The model is estimated using data on 474 commercial mortgages that were originated by a large insurance company during 1974-1990 and that completed foreclosure during 1985-1995. The authors found that the recovery rate increases with original DSCR, but that the effect of original LTV was not statistically significant.

Ciochetti & Shilling [1999]. The authors estimate an OLS regression in which the dependent variable--net recovery on foreclosed loans from default until the ultimate disposition of the property--is expressed as a function of:

- LTV at origination,
- DSCR at origination,
- age of the loan at foreclosure,
- contract mortgage interest rate,
- length of time from default through property disposition,
- a dummy variable indicating whether the foreclosure occurred in a state requiring judicial foreclosure proceedings,
- dummy variables representing property type (apartment, hotel, industrial, office, or other), and
- dummy variables representing the region in which the property is located (East North Central, Mideast, Mountain, Northeast, Pacific, Southeast, Southwest, or West North Central).

The model is estimated using data on 307 commercial mortgages that were originated by a large insurance company during 1974-1990 and that completed foreclosure and asset disposition during 1986-1996. The authors found that neither original DSCR nor original LTV was statistically significant in explaining recovery rates.

Ciochetti & Riddiough [2000]. The authors estimate an OLS regression in which the dependent variable--net recovery on distressed loans (those that became 90 days or more delinquent on any scheduled mortgage payment) from the onset of distress through ultimate disposition of the property or other resolution of the delinquency--is expressed as a function of:

- LTV at origination,
- DSCR at origination,
- age of the loan at foreclosure,

- contract mortgage interest rate,
- length of time of distress (through property disposition, for foreclosures),
- a dummy variable indicating whether the foreclosure occurred in a state requiring judicial foreclosure proceedings,
- dummy variables representing borrower type (individual, partnership, corporation, or other),
- dummy variables representing property type (apartment, hotel, industrial, office, or other), and
- dummy variables representing the region in which the property is located (East North Central, Mideast, Mountain, Northeast, Pacific, Southeast, Southwest, or West North Central).

The model is estimated using data on 307 commercial mortgages that were originated by a large insurance company during 1974-1990 and that completed foreclosure and asset disposition during 1986-1996. The authors found that the recovery rate increases with original DSCR, but that the effect of original LTV was not statistically significant.

Summary. Collectively these studies illustrate the use of a variety of modeling frameworks that could potentially be adapted by banks to estimate PD and LGD for CRE loans. The functional forms employed range from relatively simple and straightforward (*e.g.*, OLS for LGD modeling) to relatively complex and sophisticated (*e.g.*, competing-risks hazard models for PD modeling). The number and type of regressor variables differ as well, although it is no surprise that loan underwriting data are among the most important predictors of both PD and LGD.

As Archer, Elmer, Harrison & Ling [2002] point out, there is reason to believe that origination LTV should be unrelated to the probability of default, and their research supports this belief. Unfortunately, in general underwriting data are available only as of loan origination, so a variety of methods are used to update LTV and DSCR so that they more accurately represent these measures at different times through the life of the loan. In general these efforts seem to have been successful, as estimated contemporaneous LTV is generally among the most important explanatory variables.

As with any empirical research, none of these studies directly generates a forward-looking prediction for PD or LGD. This is true for several reasons. For one, the conditional default probability for CRE loans seems to increase sharply over the first five or so years of the loan before subsiding after about the sixth year; this means that a forward-looking PD prediction would have to take loan seasoning into account. For another, several of the models employ contemporaneous macroeconomic conditions, which would have to be predicted (either as a point estimate or as a distribution) for forward-looking PD or LGD predictions. And, of course, essentially all of the models are based, at least conceptually, on contemporaneous LTV and contemporaneous DSCR, which would have to be projected using loan payment and amortization schedules as well as projections of changes in property values, operating income, and operating expenses.

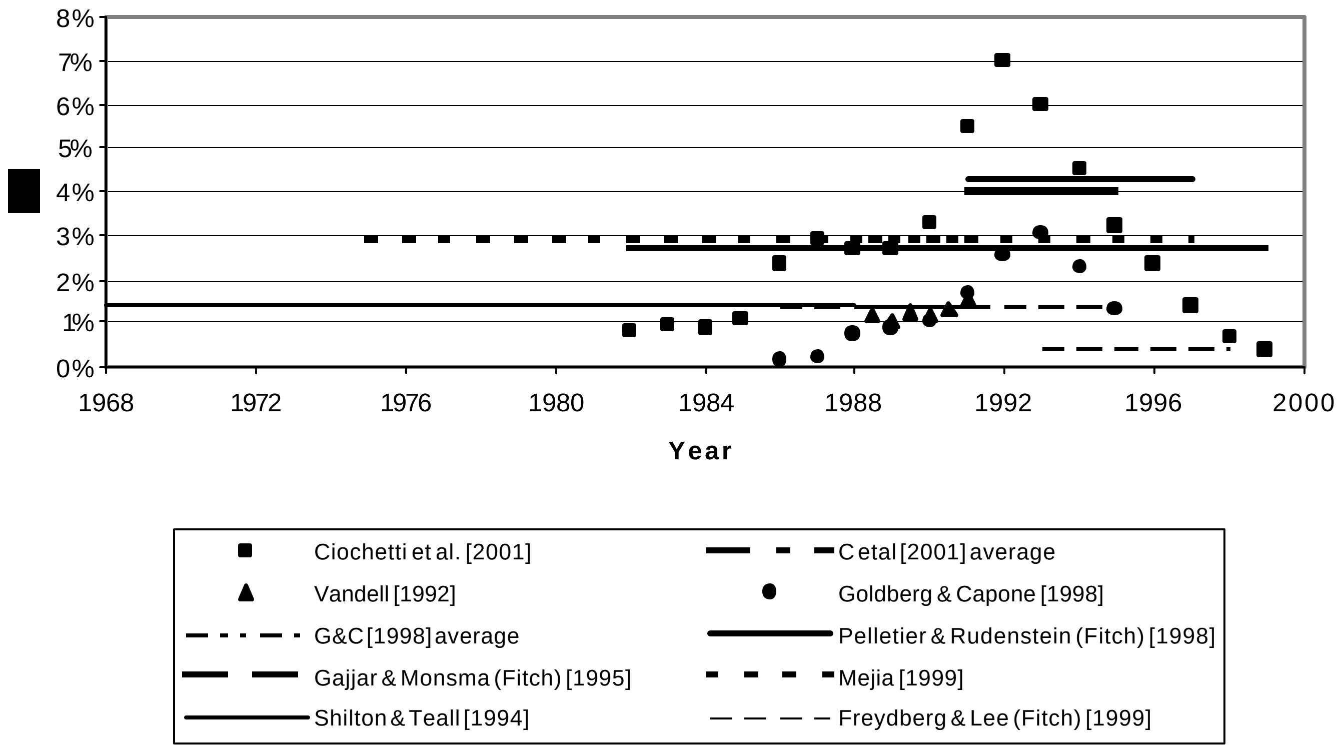
As noted, PD and LGD modeling are still in their early stages, but the variety of empirical techniques and variable definitions used in these published studies may help advance the practice for banks striving to qualify for Advanced IRB treatment of the CRE loan portfolios.

Table E-1: Summary of Evidence on Average PD for Commercial Real Estate

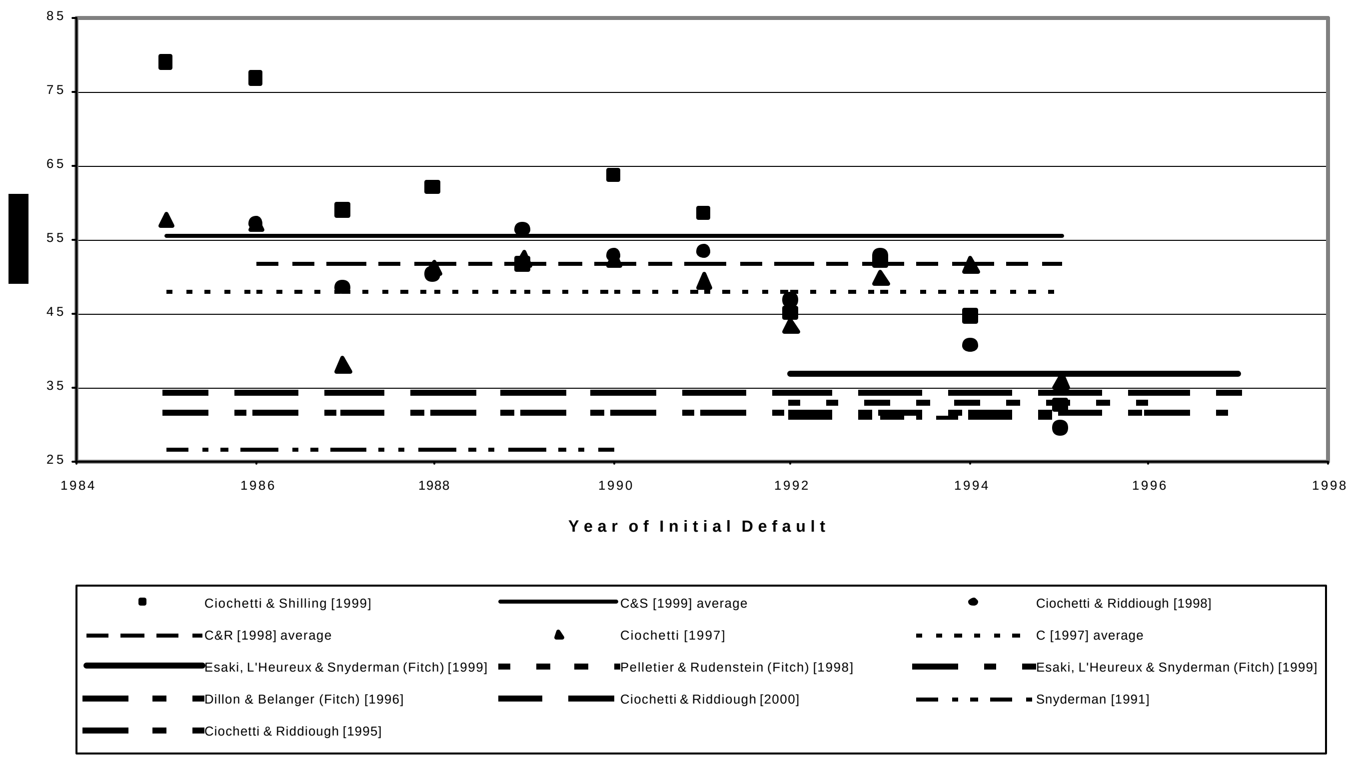
Source	Del/ Fore	Average PD	Comments
Ciochetti *et al.* [2001]	Fore	2.69% average annual default rate for the entire portfolio. By year: 1982 0.79%, 1983 0.94%, 1984 0.87%, 1985 1.06%, 1986 2.34%, 1987 2.90%, 1988 2.72%, 1989 2.68%, 1990 3.27%, 1991 5.47%, 1992 6.99%, 1993 6.00%, 1994 4.48%, 1995 3.21%, 1996 2.31%, 1997 1.36%, 1998 0.68%, 1999 0.33%.	Default defined as "the first event representing foreclosure of the borrower's interest in the property: either onset of the foreclosure process or, in the case where foreclosure occurs rapidly, completion of the process." 2,043 commercial mortgages originated during 1974-1990 by a large multiline insurance company.
Vandell [1992]	Fore	1.24% average annualized default rate for the entire portfolio 1988-1991. Semi-annual default rates by period: 1988-2 0.586%, 1989-1 0.502%, 1989-2 0.611%, 1990-1 0.595%, 1990-2 0.671%, 1991-1 0.763%.	Default defined as foreclosed. ACLI Survey of Mortgage Loan Delinquencies and Foreclosures.
Goldberg & Capone [1998]	Fore	1.39% average annual default rate for the entire portfolio 1986-1995. By year: 1986 0.14%, 1987 0.21%, 1988 0.75%, 1989 0.84%, 1990 1.05%, 1991 1.70%, 1992 2.57%, 1993 3.05%, 1994 2.22%, 1995 1.34%.	"Default in this study refers to a forfeiture of property rights, so it includes foreclosure, third-party sale, note sale, and short sale events." 7,564 multifamily mortgages purchased by Fannie Mae and Freddie Mac in 1983-1995.
Pelletier & Rudenstein (Fitch) [1998]	Del	4.3% average annual default rate for the entire portfolio; 4.37% for RTC loans, 1.97% for conduit loans. By property type (entire portfolio): office 4.8%, retail 4.7%, industrial 4.6%, lodging 4.2%, "other" 4.1%, nursing home 4.0%, multifamily 3.9%, warehouse 2.5%.	Default defined as "60 days or more past due on a debt service payment or 90 days or more past due on a balloon payment." 18,839 loans in 33 Fitch-rated transactions issued 1991-1996, seasoned at least one year.
Gajjar & Monsma (Fitch) [1995]	Del	4.01% average annual default rate for the entire portfolio 1991-1995. By property type: lodging 5.1%, retail 4.5%, office 4.3%, industrial 3.8%, multifamily 3.5%, warehouse 2.0%.	Default defined as "60+ days past due on a scheduled debt service payment or a matured balloon payment." 12,313 loans in nine Fitch-rated transactions originated prior to 1994 that had more than 100 loans. Average seasoning 2.9 years.
Mejia [1999]	Del	2.9% average delinquency rate for aggregate data 1975-1997. Range is from 0.8% (second half of 1979) to 7.3% (first half of 1992).	Delinquency defined as "when the equity holder has failed to make two consecutive monthly mortgage payments." ACLI Mortgage Delinquency Reports.
Shilton & Teall [1994]	Del	1.39% average delinquency rate for aggregate data 1968-1988.	Delinquency defined as "when the equity holder has failed to make two consecutive monthly mortgage payments." ACLI Mortgage Delinquency Reports.
Freydberg & Lee (Fitch) [1999]	Del	0.42% average annual default rate. By property type: hotel 0.8%, multifamily 0.6%, industrial 0.3%, office 0.3%, retail 0.2%, "other/mixed" 0.2%.	Default defined as "60 days or more past due on its debt service payment." 9,264 Fitch-rated CMBS deals. No seasoning requirement.

Dillon & Belanger (Fitch) [1996]	Del	4.3% average annual default rate. By property type: lodging 5.6%, retail 5.1%, nursing home 5.0%, office 4.7%, industrial 4.4%, multifamily 3.9%, warehouse 1.9%.	17,702 loans in 22 Fitch-rated transactions issued 1991-1995.
MacNeill (Fitch) [1998]	Del	Delinquency rates ranged from 0.47% to 5.93% per year over 1965-1991. "The rate for most 'good' years was under 1.00% and from 2.0%-3.0% for 'bad,' but not the 'worst' years." Highest for hotels, lowest for retail.	ACLI data for nine life insurance companies.

Graph E-1: Estimates of PD for CRE

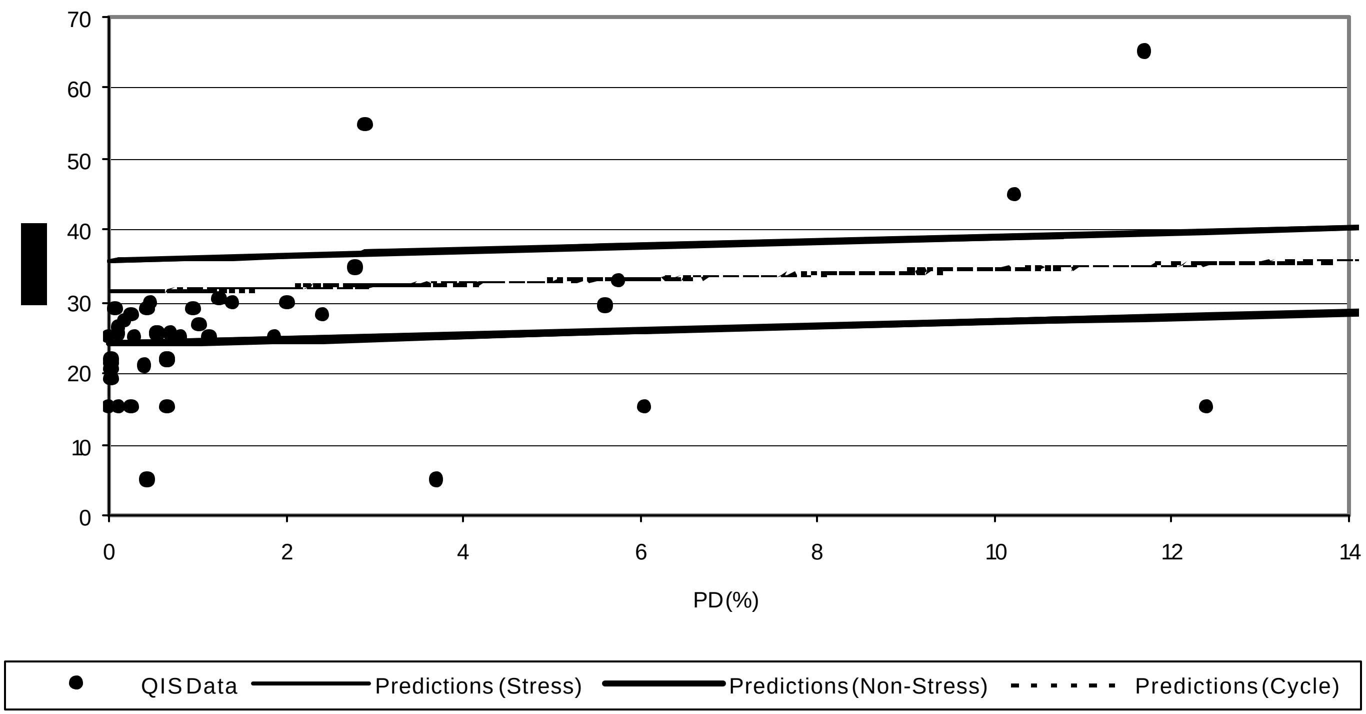


Graph E-2: Estimates of LGD for CRE

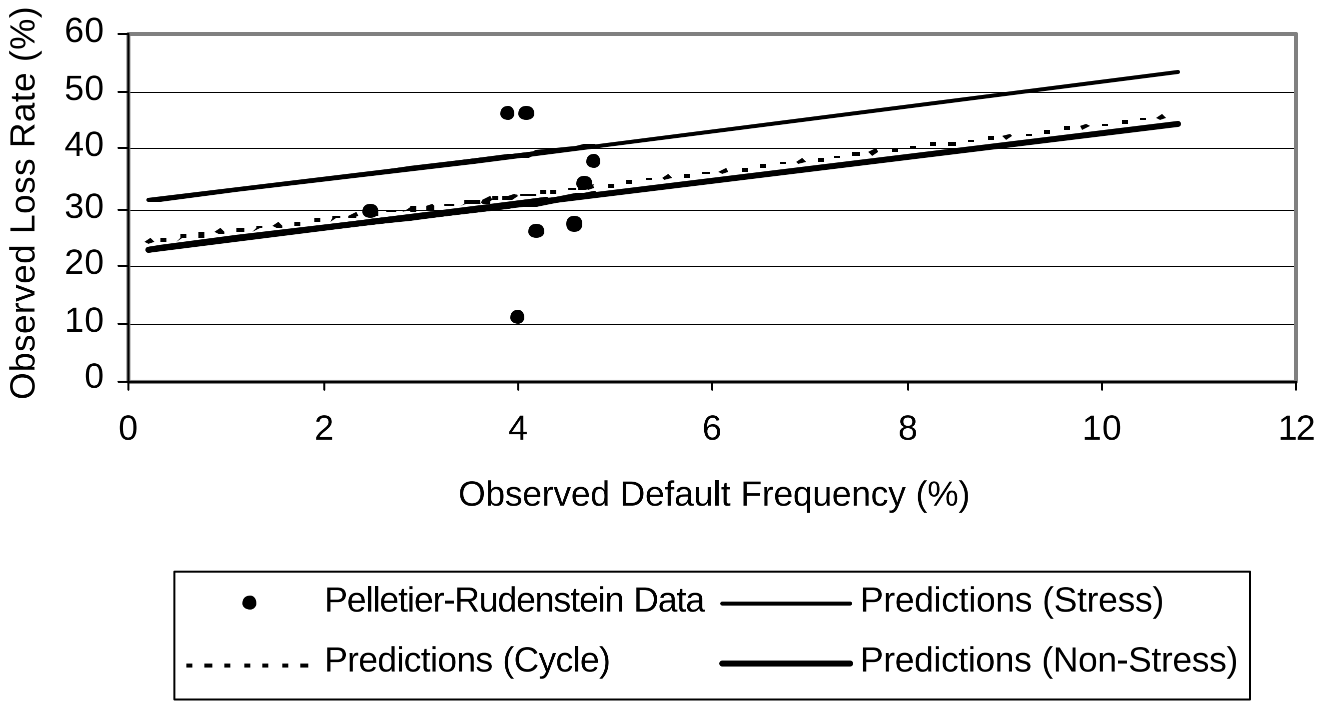


Graph E-3: Relationship Between PD and LGD

Estimated from QIS Cross-Section Data

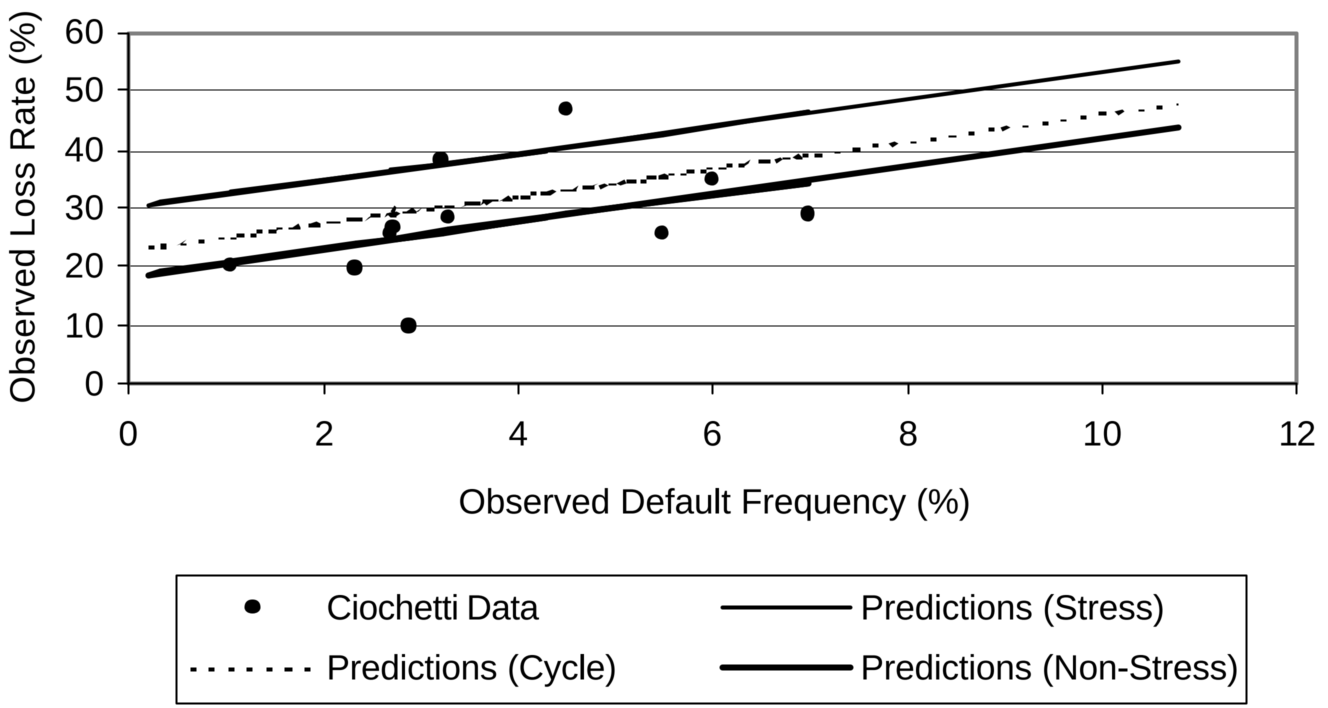


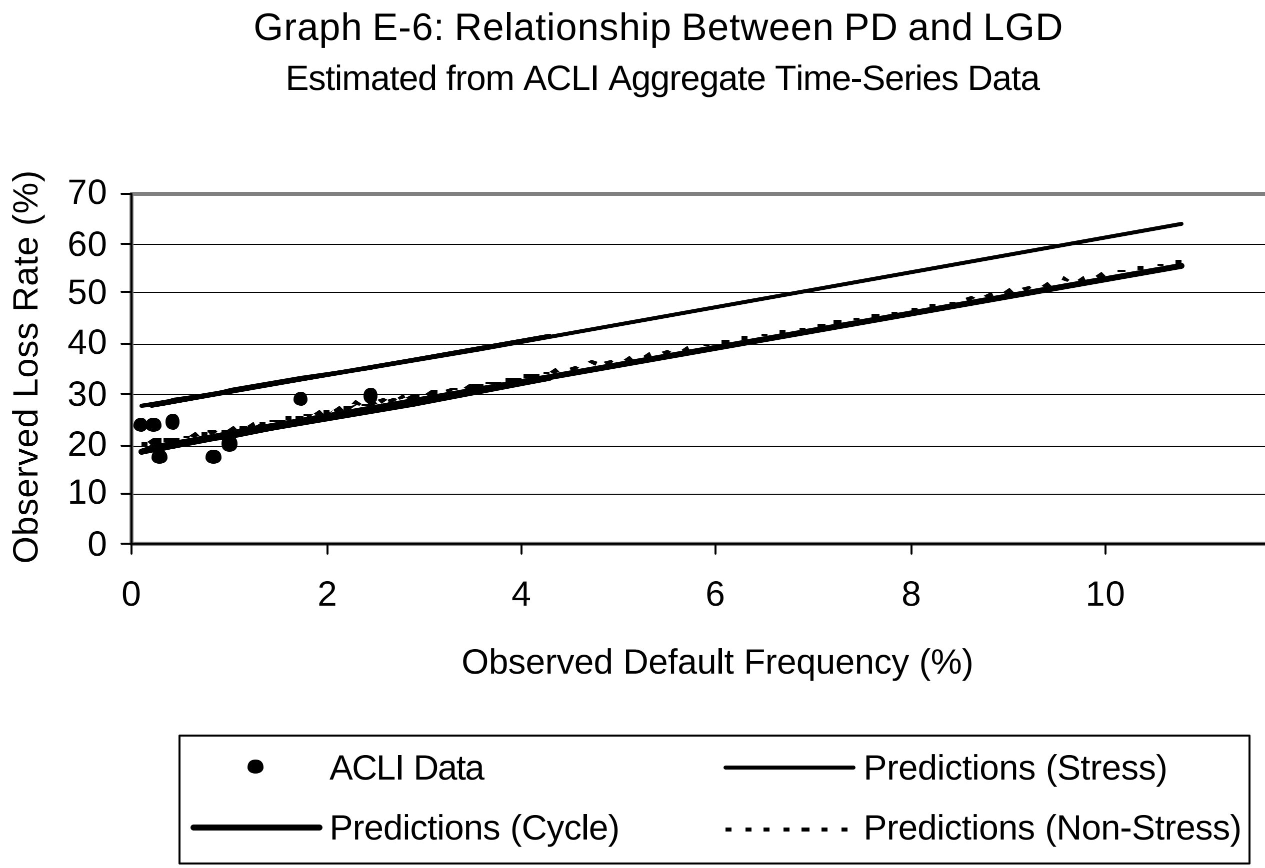
Graph E-4: Relationship Between PD and LGD
Estimated from Pelleter-Rudenstein [1998] Cross-Section Data



Graph E-5: Relationship Between PD and LGD

Estimated from Ciochetti Time-Series Data





Graph E-6: Relationship Between PD and LGD
Estimated from ACLI Aggregate Time-Series Data
Observed Loss Rate (%)
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
Observed Default Frequency (%)
ACLI Data
Predictions (Stress)
Predictions (Cycle)
Predictions (Non-Stress)

Table E-7: Summary of Evidence on Average LGD for Commercial Real Estate--Relatively High Confidence

Source	Comp Loss	Del/ Fore	Disp/ REO	Summary of LGD Findings	Comments
Ciochetti & Shilling [1999]	55.5	Fore	Disp	66.1% on liquidated loans (combined debt and equity periods). By year: 1985 93.8%, 1986 91.1%, 1987 70.0%, 1988 73.6%, 1989 61.2%, 1990 75.9%, 1991 69.6%, 1992 53.6%, 1993 61.8%, 1994 52.9%, 1995 38.6%. By property type: hotel 80.8%, office 78.3%, "other" 70.9%, retail 66.5%, industrial 62.2%, multifamily 39.7%. Equity ownership period only: 48.5%.	308 financially distressed commercial mortgages originated 1974-1990 by a large life insurance company, foreclosed 1985-1995, sold 1986-1996.
Ciochetti & Riddiough [1998]	52.1	Fore	REO	26.7% on foreclosed loans, 30.1% on foreclosed loans never modified, 31.7% on foreclosed loans ever modified, 0% on loans that go through foreclosure process but lender paid in full just prior to title transfer. By property type (all loans): hotel 42.3%, office 33.4%, "other" 32.3%, industrial 25.9%, retail 24.7%, multifamily 16.2%.	474 commercial mortgages originated 1974-1990 by a large life insurance company that completed the foreclosure process in 1985-1995. Losses and recoveries tracked only through transfer to REO.
Ciochetti [1997]	48.1	Fore	REO	30.6% on foreclosed loans including foregone interest (10%) and expenses (3%). Losses and recoveries tracked only through transfer to REO.	2,013 commercial mortgages foreclosed by 14 life insurance companies 1986-1995.
Pelletier & Rudenstein (Fitch) 1998	32.9	Fore	Disp	39.1% of loan balance at securitization on liquidated loans. By property type: multifamily 46%, "other" 46%, office 38%, retail 34%, warehouse 29%, industrial 27%, lodging 26%, nursing home 11%.	Default=60+ days overdue. 18,839 completely resolved loans in 33 transactions issued 1991-1996.
Esaki, L'Heureux & Snyderman [1999]	31.7 36.8	Fore	Disp	37.7% on liquidated loans, including foregone interest and expenses. LGD rises to 43.8% on loans liquidated during 1992-1997.	Default=90+ days overdue. 15,109 liquidated commercial mortgages originated 1972-1992 by eight large life insurance companies.
Dillon & Belanger (Fitch) [1996]	31.0	Fore	Disp	36.9% of loan balance at securitization on liquidated loans. By property type: multifamily 45%, office 35%, retail 33%, warehouse 31%, lodging 22%, nursing home 19%, industrial 19%.	Default=60+ days overdue. 17,702 completely resolved loans in 22 transactions issued 1991-1995.

Ciochetti & Riddiough [2000]	34.2	Del	Disp	Through Loan Termination: All Distressed 23.3% Straight Foreclosure 28.2% Straight Deed in Lieu 25.2% Restructured Foreclosure 37.6% Restructured Deed in Lieu 34.4%	Through Asset Sale: All Distressed 34.2% Straight Foreclosure 59.5% Straight Deed in Lieu 61.3% Restructured Foreclosure 56.4% Restructured Deed in Lieu 64.4%	807 commercial mortgages, originated 1974-1990 and held by a large multi-line insurance company, that became delinquent 90 days or more at any time during the life of the loan.

				Loans with No Asset Sale: Restructured Loans Not Foreclosed 20.8% Loan Sale 26.6% All Other Distressed Loans 8.8% (inc. ongoing)	
Snyderman [1991]	26.9	Fore	Disp	32% on liquidated loans including foregone interest and expenses.	Default=90+ days overdue. 7,205 liquidated commercial mortgages originated 1972-1984 by seven large life insurance companies.

Table E-8: Summary of Evidence on Average LGD for Commercial Real Estate--Less Confidence

Source	Average LGD	Comments
Price (Fitch) [1997]	65% on French commercial mortgages during 1990-1997.	Based on Immo Presse data on loss provisions and reports related to 1996 and 1997 bulk sales of loans and real estate.
Freddie Mac	45%-60% for multifamily mortgages	cited in MacNeill (Fitch) [1998]
RTC bulk asset sales	32%-57.5%	cited in MacNeill (Fitch) [1998]
MacNeill (Fitch) [1998]	40%-50%. Based on other sources shown in this table: 21% for commercial properties sold from REO by a large Midwestern life insurance company 1988-1998 (thought to be biased downward); 32%-57.5% for RTC bulk asset sales in 1990-1991; 39% from GAO study of RTC asset sales; 60% for Freddie Mac; 25%-30% for Fannie Mae (thought to be biased downward).	
Vrchota & Kendra (Fitch) [2001]	30%-52.5%	Not explained.
Large U.S. Bank	22% senior liens, 40% junior liens.	Based on data from 1987-1995.
Resolution Trust Corp.	39%	General Accounting Office study of RTC asset sales.
Somerville & Maggard (Fitch) [1998]	30% for Canadian commercial mortgages.	Based on average loss for the four worst years in which to resolve a defaulted loan (1993-1996).
Fannie Mae	25%-30% for multifamily properties.	MacNeill (Fitch) [1998] believes this to be biased downward.
Large U.S. Bank	25.5% on all defaults, 50% on charged-off loans. Does not include asset write-downs, OREO net expense, or OREO gains/losses.	442 loans defaulted 1986-1993 and charged off 1986-1995. Bank notes that "economic loss rates in actuality were deemed much higher than what was recorded here."
Large Midwestern life insurance company	21% for commercial properties sold from REO 1988-1998.	MacNeill (Fitch) [1998] believes this to be biased downward.

References

Ambrose & Sanders [2001] Brent Ambrose and Anthony B. Sanders, "Commercial Mortgage-Backed Securities: Prepayment and Default," working paper (August 2001).

Archer, Elmer, Harrison & Ling [2002] Wayne R. Archer, Peter J. Elmer, David M. Harrison, and David C. Ling, "Determinants of Multifamily Mortgage Default,"*Real Estate Economics* 30:3 (Fall 2002) p.445-473.

Brady & Bos [2002] Brooks Brady and Roger J. Bos, "Record Defaults in 2001 the Result of Poor Credit Quality and a Weak Economy," S&P, February 2002.

Brady, Vazza & Bos (S&P) [2003] Brooks Brady, Diane Vazza, and Roger J. Bos, "Corporate Defaults Peak in 2002 Amid Record Amounts of Defaults and Declining Credit Quality,"S&P, February 2003.

Ciochetti [1997] Brian C. Ciochetti, "Loss Characteristics of Commercial Mortgage Foreclosures," *Real Estate Finance* (1997) p.53-69.

Ciochetti & Riddiough [2000] Brian A. Ciochetti and Timothy J. Riddiough, "The Impact of Financial Distress on the Performance of Commercial Mortgages," working paper (June 2000).

Ciochetti & Riddiough [1998] Brian A. Ciochetti and Timothy J. Riddiough, "Timing, Loss Recovery and Economic Performance of Foreclosed Commercial Mortgages," working paper (June 1998).

Ciochetti & Shilling [1999] Brian A. Ciochetti and James D. Shilling, "Default Losses and Commercial Mortgages," working paper (October 1999).

Ciochetti, Deng, Gao & Yao [2002] Brian A. Ciochetti, Yongheng Deng, Bin Gao, and Rui Yao, "The Termination of Commercial Mortgage Contracts through Prepayment and Default: A Proportional Hazards Approach with Competing Risks," *Real Estate Economics* 30:4 (Winter 2002) p.595-633.

Ciochetti, Deng, Lee, Shilling & Yao [2001] Brian A. Ciochetti, Rui Yao, Yongheng Deng, Gail Lee, and James D. Shilling, "A Proportional Hazards Model of Commercial Mortgage Default with Originator Bias," draft manuscript (October 2001).

Corcoran & Iwai (JPMorgan) [2002] Patrick J. Corcoran & Yuriko Iwai, "CMBS Loan Loss Severity: Better or Worse than 36%?" J.P. Morgan Securities, Inc., November 15, 2002.

Credit Suisse Financial Products [1997] Credit Suisse Financial Products, "CreditRisk+: A Credit Risk Management Framework" (1997).

Dillon & Belanger (Fitch) [1996] Pamela T. Dillon and Donald S. Belanger, "Trends in Commercial Mortgage Default Rates and Loss Severity," Fitch IBCA, Inc., November 11, 1996.

Esaki, L'Heureux & Snyderman [1999] Howard Esaki, Steven L'Heureux, and Mark Snyderman, "Commercial Mortgage Defaults: An Update," *Real Estate Finance* 16:1 (1999) p.80-86.

Freydberg & Lee (Fitch) [1999] Mary Stuart G. Freydberg and Keith Lee, "CMBS Conduit Loan Defaults by Property Type," Fitch IBCA, Inc., April 6, 1999.

Gajjar & Monsma (Fitch) [1995] Anand N. Gajjar and J. Zachary Monsma, "Commercial Mortgage Default Study," Fitch IBCA, Inc., November 6, 1995.

Goldberg & Capone [1998] Lawrence Goldberg and Charles A. Capone, Jr., "Multifamily Mortgage Credit Risk: Lessons from Recent History," *Cityscape: A Journal of Policy Development and Research* 4:1 (1998) p.93-113.

Gordy [2000] Michael B. Gordy, "A Comparative Anatomy of Credit Risk Models," *Journal of Banking & Finance* 24 (2000) p.119-149.

Gordy [forthcoming] Michael B. Gordy, "A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules, *Journal of Financial Intermediation* (forthcoming).

Gordy & Heitfield [2002] Michael B. Gordy and Erik Heitfield, "Estimating Default Correlations from Short Panels of Credit Rating Performance Data" (2002).

Gupton, Finger & Bhatia (Morgan) [1997] Greg M. Gupton, Christopher C. Finger, and Mickey Bhatia, "CreditMetrics--Technical Document," J.P. Morgan & Co. Incorporated (1997).

MacNeill (Fitch) [1998] Mary MacNeill, "Commercial Mortgage Stress Test Research," Fitch IBCA, Inc., October 23, 1998.

Malysa (Fitch) [2001] John Malysa, "Rating Single-Borrower Commercial Mortgage Transactions," Fitch IBCA, Inc., December 27, 2001.

Mejia [1999] Luis C. Mejia, "Availability of Credit and Loan Default: A Look at the Commercial Mortgage Supply Cycle," *Journal of Real Estate Research* 18:1 (1999) p.175-196.

Pelletier & Bonfiglio (Fitch) [1999] Rodney Pelletier and John Bonfiglio, "Property Market Metric: A New Look at Property Type and Market Volatility," Fitch IBCA, Inc., December 10, 1999.

Pelletier & Rudenstein (Fitch) [1998] Rodney Pelletier and Andrew Rudenstein, "Trends in Commercial Mortgage Default Rates and Loss Severity--1997 Update," Fitch IBCA, Inc., July 20, 1998.

Price (Fitch) [1997] Janet G. Price, "French Real Estate Market and Commercial Mortgage Default Study," Fitch IBCA, Inc., March 3, 1997.

RMA [2003] RMA - The Risk Management Association, "Measuring Credit Risk and Economic Capital in Specialized Lending Activities - Best Practices," unpublished (March 2003).

Rubin & Levidy (Moody's) [2000] Stewart M. Rubin and Nicholas J. Levidy, "CMBS: Moody's Approach to Rating U.S. Conduit Transactions," Moody's Investors Service, September 15, 2000.

Shilton & Teall [1994] Leon Shilton & John Teall, "Option-Based Prediction of Commercial Mortgage Defaults," *Journal of Real Estate Research* 9:2 (1994) p.219-236.

Snyderman [1991] Mark P. Snyderman, "Commercial Mortgages: Default Occurrence and Estimated Yield Impact," *Journal of Portfolio Management* (1991) p.82-87.

Somerville & Maggard (Fitch) [1998] Huxley Somerville and Jeffrey Maggard, "Canadian Commercial Mortgage Default Study," Fitch IBCA, Inc., October 21, 1998.

Treacy & Carey [1998] William F. Treacy and Mark S. Carey, "Credit Risk Rating at Large U.S. Banks," *Federal Reserve Bulletin* 84 (November 1998) p.897-921.

Vandell [1992] Kerry D. Vandell, "Predicting Commercial Mortgage Foreclosure Experience," *Journal of the American Real Estate and Urban Economics Association* 20:1 (1992) p.55-88.

Vandell, Barnes, Hartzell, Kraft & Wendt [1993] Kerry D. Vandell, Walter Barnes, David Hartzell, Dennis Kraft, and William Wendt, "Commercial Mortgage Defaults: Proportional Hazards Estimation Using Individual Loan Histories," *Journal of the American Real Estate and Urban Economics Association* 21:4 (1993) p.451-480.

Vrchota & Kendra (Fitch) [2001] Robert Vrchota and Kevin Kendra, “Rating Performing Loan Pools,” Fitch IBCA, Inc., February 2, 2001.