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Abstract

This paper derives one-month ahead forecasts of the money (M1) multiplier using the Multi-State Kalman Filter and Box-Jenkins ARIMA methods. A comparison of the forecasts for the period 1980-82 reveals that the Multi-State Kalman Filter procedure was generally superior to the ARIMA procedure in terms of most summary statistics. The superiority is traced to the turbulent period of 1980-81. This paper also compares aggregate and component forecasts of the multiplier. The aggregate Multi-State Kalman Filter was the most accurate in predicting the one-month ahead money multiplier.

Comparing Multi-State Kalman Filter and ARIMA Forecasts: An Application to the Money Multiplier

There has been much empirical work done in assessing the ability to forecast the money multiplier. This is because many monetary economists have argued that the most appropriate procedure to achieve desired monetary growth targets is for the monetary authority to inject an amount of monetary base consistent with a money multiplier forecast. Under such a policy, any error in forecasting the money multiplier would immediately be translated into a deviation of the money stock from its target level.[1] Following Bomhoff (1977), recent studies have identified and estimated ARIMA models to forecast the money multiplier.[2]

Our primary purpose in this study is to evaluate an alternative forecasting methodology: the Multi-State Kalman Filter (MSKF) procedure. This is done by directly comparing one-step-ahead monthly forecasts from this procedure with those of the Box-Jenkins ARIMA procedure. MSKF and ARIMA models are developed for the aggregate multiplier. Also, following the work of Johannes and Rasche (1979) the money multiplier is decomposed into individual ratio components, reflecting different behavior. MSKF and ARIMA models are developed for each of these ratios and one-step-ahead monthly forecasts are compared. In this way, further evidence is presented on the efficacy of the components forecasting method.[3]

Out-of-sample forecasts are generated for the period January 1980 through December 1982. The period is chosen because it is a very turbulent one with regard to money market conditions. It is, therefore, a natural setting in which to ask whether the MSKF procedure provides more accurate forecasts than the oft used ARIMA procedure.

The format of the paper is as follows: In section 2 we describe the money multiplier and the component ratios. The ARIMA and MSKF models are presented and the correspondence between the models is detailed. Section 3 presents and discusses the forecasting results obtained using each of these forecasting techniques, for both the aggregate and component approach. Section 4 closes the paper with concluding remarks.

2. Money Multiplier Models

In the Brunner-Meltzer (1964) formulation, the aggregate money multiplier's value (m) at any point in time (t) is simply the ratio of money (M) to monetary base (MB):

$$(1) \qquad m_t = (M_t/MB_t).$$

In our analysis, M is defined as not seasonally adjusted M1 -- currency in the hand of the public plus traveler's checks plus total checkable deposits -- and not seasonally adjusted MB is defined as the St. Louis adjusted monetary base.[4]

Alternatively, the multiplier can be written in terms of its component parts. Because the multiplier reflects actions by the Federal Reserve, the government, depository institutions and the public, a clearer picture of why the multiplier changes may be obtained from the components of the multiplier which reflect these separate actions. Thus, in terms of its component parts, the multiplier can be written as

(2) $m_t = (1 + k + tc)/(r + 1)(1 + t_1 + t_2 + g + z) + k$

where

m = M1 multiplier,

$k = C/D$,

$t_c = TC/D$,

$t_1 = (M2 - M1)/D$,

$t_2 = (M3 - M2)/D$,

$g = (G/D)$,

$z = (Z/D)$, and

$(r + 1) = (SB + RAM - C)/[M2 - C - TC + (M3 - M2) + G + Z]$.

The mnemonics used in defining the above ratios are:

C = currency in the hands of the public,

D = total checkable deposits,

G = government demand deposits,

Z = all deposits at banks due to foreign commercial banks and official institutions,

TC = traveler's checks,

SB = source base,

RAM = reserve adjustment magnitude, and

M1, M2 and M3 are alternative measures of the money stock (see Hafer(1980)).

In terms of the following analysis, our concern is to forecast m_t, whether defined as in equation (1) or equation (2).

2.1 Aggregate ARIMA model

Examination of the autocorrelation function of the logarithm of the aggregate M1 money multiplier (equation (1)) predating 1980 indicates that first differencing and seasonal differencing are required to achieve stationarity.[5] The autocorrelations and partial autocorrelations for the appropriately transformed series suggest a relatively high-order moving average model. Estimation of such a model for the sample period January 1959 to December 1979 yields the following (standard errors appear in parentheses):

$$(3)\quad (1-B)(1-B^{12})\ln m_t = (1 - \underset{(0.050)}{0.284B} - \underset{(0.048)}{0.151B^7} - \underset{(0.051)}{0.514B^{12}})\, a_t$$

$$Q(12) = 12.69 \qquad SE = 0.4697 \times 10^{-2}.$$

The estimated coefficients are significantly different from zero at the 5 percent level. The Q-statistic does not allow rejection of the hypothesis of white noise residuals at the 5 percent level. In section 3, therefore, equation (3) is used to generate the aggregate ARIMA multiplier forecast. The data set is sequentially updated to include the most recent observations when next month's multiplier is forecast.

2.2 Aggregate Multi-State Kalman Filter Model

Because the MSKF procedure does not directly handle variables with seasonal variation, two alternative methods were used: First, a multiplicative seasonal factor in the multiplier was forecast completely outside the MSKF framework. The exogenously calculated seasonal is then used as an input in the MSKF model to predict the unadjusted multiplier. The seasonal factor was calculated by using a truncated ratio-to-moving average approach, which makes use of only past multiplier data. The alternative method incorporated the seasonal adjustment process into the MSKF algorithm, with the seasonal element modeled as a separate component of the multiplier. We report only the results for the exogenous seasonal factor forecast, since the results of the two procedures were not appreciably different.[6]

The MSKF model used to describe the behavior of the money multiplier is a four-state model of the form

$$(4) \qquad m_t = s_t \mu_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, V_\varepsilon)$$

and

$$(5) \qquad \mu_t = \mu_{t-1} + \gamma_t, \qquad \gamma_t \sim N(0, V_{\gamma_t})$$

where m_t is the money multiplier, s_t is the exogenous seasonal factor, μ_t is the permanent level of the multiplier, and ε_t and γ_t are independent random disturbances.[7]

This model assumes that the multiplier is subject to two different types of shocks: one is a temporary level shock, represented by ε_t, the other a permanent level shock, captured by γ_t. Equation (4) states that, ignoring the seasonal factor, the multiplier will fluctuate randomly about some permanent value μ_t. This permanent value then represents the optimum forecast. The random walk character of the permanent value μ_t is displayed in equation (5).

There is a close correspondence between the representation captured in equations (4) and (5) and ARIMA models such as equation (3). Note that combining equations (4) and (5) yield the model

(6) $\Delta m_t = \varepsilon_t - \varepsilon_{t-1} + \gamma_t,$

which is equivalent to the ARIMA (0,1,1) model

(7) $\Delta m_t = (1 - \phi B)\alpha_t.$

This illustrates that an equivalence in functional form exists between the ARIMA (0,1,1) model that is used in the Box-Jenkins framework and the model used in the Kalman filter algorithm. In the estimation and forecasting methodology, however, there is a substantial difference in the two approaches.

The application of the Box-Jenkins approach to equation (7) reduces to estimating ϕ and the variance of α_t, both of which are assumed to be constant over the sample period. Even a recursive Box-Jenkins technique would not substantially alter the rigid character of this methodology. In contrast, the MSKF approach allows for feedback from the data to the forecasting procedure. In other words, the MSKF allows for a changing mixture of permanent and transitory shocks by changing the probabilities associated with these shocks.

Four separate representations of equations (4) and (5) are used in our application of the MSKF methodology. For each model, the ratio $V_{\varepsilon_t}/V_{\gamma_t}$ is specified _a priori_. This can be thought of as specifying the parameter ϕ in equation (7) _a priori_. In the MSKF estimation, however, the level of the variance of ε_t and γ_t is computed adaptively, which is equivalent to estimating the variance of α_t in (7) adaptively.

Table 1 presents the correspondence between the MSKF framework, equation (6), and the ARIMA (0,1,1) model, equation (7). As shown in the table, each of the four states is viewed as having a fixed parameter (ϕ) corresponding to a certain time series process. To see this, using equation (7), the expectation of m_t at t-1 can be written as

$$(8) \qquad E_{t-1}(m_t) = m_{t-1} - \phi[(m_{t-1} - E_{t-2}(m_{t-1})]$$

$$= (1 - \phi)m_{t-1} + E_{t-2}(m_{t-1}).$$

The forecast of next period's multiplier ($E_{t-1}(m_t)$) is a weighted average of this period's multiplier (m_{t-1}) and last period's prediction of this period's multiplier ($E_{t-2}(m_{t-1})$). The lower the value of ϕ, the more weight given to the observed value m_{t-1} and the more probable the difference between m_{t-1} and its prior prediction is caused by a permanent shift: the second and fourth model in Table 1 illustrate this case. A high value of ϕ, on the other hand, indicates that there is a high probability that the difference between m_{t-1} and $E_{t-2}(m_{t-1})$ is of a temporary nature. In this case, little weight is given to the prediction error: the first and third model in Table 1 illustrate this case. Finally, the first two models cope with small prediction errors while the third and fourth models are designed for outlier shocks.

The use of four different ARIMA (0,1,1) models with fixed parameters may not seem a great improvement over the usual Box-Jenkins methodology. The scope for improvement, however, comes from the fact that the weights attached to each of the individual models are allowed to change. These weights are determined by a Bayesian procedure and are equal to the (posterior) probability that the multiplier process at this point in time is indeed described by that model. In this manner, the MSKF forecast is akin to the forecast of a single ARIMA model with the parameter allowed to change from period to period. The use of four fixed models, however, increases the flexibility of the model in describing the multiplier process. The gain is most obvious when the underlying series can be characterized by an ARIMA (0,1,1) process with a time-varying parameter . When the model is more complex, as in the case of the estimated aggregate money multiplier, the gain is not obvious and becomes more of an empirical issue.

Unlike the Box-Jenkins procedure, the MSKF technique tries to identify the nature of the different shocks affecting the multiplier; that is, the ε_t and γ_t. This information then is used in forecasting. Given this period's prediction error and the state of the system represented by all former information, the MSKF algorithm determines the probability that the observed shock was large or small, and what portion of the error should be viewed as temporary or permanent.[8] In this way, the MSKF method reassesses the structure of the forecasting model as new data becomes available.

This methodology is applied to the aggregate money multiplier to develop one-month ahead forecasts for the period January 1980 to December 1982. The data set employed is exactly the same as that employed in the Box-Jenkins ARIMA analysis. The results are summarized in section 3.

2.3 Component Multiplier ARIMA Models

The different component ratios of the money multiplier are estimated using data from the sample period January 1959 to December 1979. In three instances, however, the sample period was shorter. Because large time deposits did not exist prior to January 1961, the sample period for the t_2 ratio does not start until then. Moreover, to avoid any effects stemming from the switch to lagged reserve accounting in October 1968, the reserve ratio (r+1) was estimated with data beginning in October 1968. Finally, the traveler's check component (tc) is modeled using data beginning in January 1969. As noted by Johannes and Rasche (1981c), the time series of tc indicates a break in the late 1960's. They suggest this break may be due to changes in the quality of data collected. Thus, we, too, model our tc component using data starting in January 1969.

The ARIMA models estimated for the individual components are presented in Table 2. As indicated, it was necessary to first-difference the seasonal difference $[(1-B)(1-B^{12})]$ of the logarithm of the ratio to identify and estimate the models. The reported Q-statistics reveal that the residuals have been reduced to white noise at reasonable levels of significance.[9]

2.4 Component Multi-State Kalman Filter Models.

Again the two different procedures for handling the seasonal variation were applied to the various ratios within the MSKF framework. Examination of the autocorrelation function for the residuals indicated that the method of exogenously determining the seasonal factors yielded slightly better results than the endogenous seasonal adjustment process. As a result, we continue to report only the results for the exogenously determined seasonal factors.

After careful inspection of the individual series for data ending in December 1979 the previously described four-state Kalman filter model was chosen for the traveler's check ratio (tc) and the government deposit ratio (g). For the remaining ratios k, z, t_1, t_2 and r+1, however, a six-state MSKF model was found to be necessary. This six-state MSKF model extends the four-state model by allowing for innovations in the permanent growth rate, as well as the permanent level.[10] It can be described as follows:

$$\begin{aligned} x_t &= s_t \mu_t + \varepsilon_t & \varepsilon_t &\sim N(0, V_\varepsilon) \\ \mu_t &= \mu_{t-1} + \beta_t + \gamma_t & \gamma_t &\sim N(0, V_\gamma) \\ \beta_t &= \beta_{t-1} + \rho_t & \rho_t &\sim N(0, V_\rho), \end{aligned}$$

where x_t is the level of the series, st is the seasonal factor, μ_t is the permanent level of the series, β_t is permanent growth rate of the series and ε_t, γ_t and ρ_t are random disturbances which are assumed to be mutually independent and serially uncorrelated error terms.

The six-state MSKF allows for transitory shocks to the level (ε_t), permanent shocks to the level (i.e., transitory shocks to the growth rate, γ_t) and permanent shocks to the growth rate (ρ_t). The MSKF algorithm determines the probability that the different shocks were large or small and the portion of this forecast error which should be viewed as permanent or temporary. Once this evaluation is made, the probabilities associated with the six different states are revised, allowing the forecaster to reassess the structure of the forecasting model as new data become available.

3. Forecast Summary

3.1 Aggregate Multiplier Forecasts

Chart 1 plots the forecast errors for the ARIMA and MSKF aggregate multiplier models. The pattern of the forecast errors from the two different methodologies is remarkably similar. With the exception of four isolated instances, the sign on the forecast errors for any particular date is the same for both procedures. Chart 1 also indicates that, in general, the forecast errors are smaller for the MSKF procedure. The superiority of the MSKF is especially evident in 1981, when the MSKF forecast errors are almost always smaller than the corresponding ARIMA forecast errors.

To further assess the relative forecasting capabilities, summary forecast statistics of the two procedures are presented in Table 3. Turning first to the full period results, the visual notion that the MSKF procedure produces better forecast results is corroborated statistically. For example, the root-mean-squared error (RMSE) is 20 percent smaller for this procedure relative to the ARIMA model. In fact, every summary statistic favors the MSKF procedure for the full period.

Gauged on a year-by-year basis, the superiority of the MSKF procedure is linked to the turbulence of events affecting the money market. For example, the MSKF procedure does much better than the ARIMA procedure in 1980, when the credit controls were first implemented and then removed. It also does better again in 1981 when NOW accounts were legalized nationwide.[11] In 1982, a year with relatively tranquil money market conditions, the superiority of the MSKF is markedly reduced.

As far as using money multiplier forecasts in implementing monetary policy is concerned, the issue of bias is of crucial concern.[12] It is encouraging that the bias in the forecasts for either procedure is relatively minor, even during the turbulent periods. Again, however, the MSKF procedure is superior based on this criterion. This is especially true in 1981 when the fraction of error due to bias is only about 6 percent for the MSKF procedure. In contrast, it is over 15 percent for the ARIMA procedure.

This evidence leads us to conclude that the MSKF procedure yields superior forecasts of the aggregate multiplier measure relative to the ARIMA procedure. This improvement stems from the fact that the MSKF procedure readily adapts to changing conditions.

3.2 Component Multiplier Forecasts

Chart 2 illustrates the forecast errors for the money multiplier, where the multiplier forecasts represents a combination of the individual ratios that make-up the multiplier. Summary statistics for the individual ratio forecasts are presented in the appendix. Since the primary concern for implementing a money stock targeting procedure is the aggregate multiplier forecast, attention here is focused on the ability to forecast the aggregate multiplier. Again, the pattern of the forecast errors from the ARIMA and MSKF methodologies is similar. Relative to the aggregate results, however, the superiority of the MSKF procedure is not obvious: in about half of the 36 monthly forecasts, the absolute value of the forecast error appears to be smaller for the ARIMA procedure.

Table 4 further documents the relative forecasting performance of the two different forecasting methodologies. For the full period, the MSKF procedure again has a slightly superior forecast record, but in this case the difference in the two procedures is reduced substantially. The full sample RMSE for the MSKF procedure, for example, is only 3 percent smaller than that of the ARIMA procedure.

On an annual basis, the MSKF procedure continues to perform relatively better in 1980 and 1981. For 1982, however, the ARIMA procedure produces superior summary statistics. This is the only period for which the ARIMA procedure has a smaller RMSE and a smaller fraction of error due to bias. All in all, the record from forecasting the multiplier's component parts to derive a multiplier forecast continues to suggest a slight statistical superiority in the MSKF forecasting methodology.

3.3 Aggregate vs. Component Forecasts

Hafer and Hein (1984) have shown that there is little statistical gain in forecasting the aggregate multiplier by modeling the individual ratios when the ARIMA procedure is used.[13] This result continues to hold for both modeling techniques. The evidence derived from the ARIMA model's forecasts presented in Table 4 indicates that the RMSE is reduced by only 0.0002 when the component approach is used: the aggregate ARIMA procedure's RMSE (Table 3) is 0.0225, while the component ARIMA procedure RMSE is 0.0223.

The evidence in Table 4 also indicates that when the MSKF procedure is employed, there is actually a loss associated with adopting the component approach.[14] For the full sample period, the RMSE increases by about 16 percent from 0.0180 to 0.0216. This relative deterioration also appears for each individual year. In no case does moving from an aggregate to a components approach improve the forecasting record when the MSKF procedure is employed. Thus, with regard to forecasting the money multiplier, there is no statistical gain associated with forecasting the individual ratios first.

4. Summary and Conclusion

This paper has examined two different forecasting methodologies: the Box-Jenkins ARIMA procedure and a Multi-State Kalman Filter forecast procedure. These methodologies have been used to develop one-month ahead forecasts for the M1 money multiplier over the period 1980-1982. Forecasts of the multiplier were developed based on the time series of the aggregate multiplier -- "the aggregate approach" -- and on its seven component ratios -- "the components approach".

Using a number of summary measures, the aggregate MSKF approach was most accurate in predicting the money multiplier month-to-month. This approach had the smallest mean absolute error and root-mean-squared error for the full period. In addition, the important problem of forecast bias, which pertains directly to the success of money stock targeting, was almost nonexistant with this procedure.

Two observations can be made based on this evidence. First, it was generally observed that, the MSKF procedure was superior to the ARIMA procedure. Second, as far as forecasting the multiplier is concerned, there is little gain associated with forecasting the individual ratios and then combining those ratio forecasts to derive a multiplier forecast. Although there may be important reasons for forecasting the individual ratios our evidence indicates that forecasting accuracy is not one of them.

Footnotes

1. This view, of course, presumes that the monetary authorities possess a large amount of discretionary control over changes in the monetary base. For a discussion of this issue, see Balbach (1981).

2. Other analyses using the ARIMA methodology to forecast the multiplier include Buttler, et al. (1979), Fratianni and Nabli (1979) and Hafer and Hein (1983).

3. For recent comparison based solely on ARIMA procedures, see Hafer and Hein (1984).

4. A discussion of the adjusted monetary base, see Gilbert (1980). This series is composed of the source base and an adjustment for changes in reserve requirements.

   Hafer, Hein and Kool (1983) examine the aggregate multiplier forecasts on a seasonally adjusted basis.

5. The models estimated here are based on data through December 1979. Ending the sample period at this point is done to allow post-sample forecasting for the January 1980 to December 1982 sample.

6. Summary statistics for the endogenous seasonal adjustment procedure are available from the authors upon request.

7. For a detailed treatment of the methodology and estimation technique of the MSKF approach, see Harrison and Stevens (1971, 1976), Lawson (1980), Kool (1982) and Bomhoff (1983).

8. This opens the possibility of viewing all small errors as completely temporary and all large errors as completely permanent or vice versa. This is in marked contrast to the ARIMA approach which treats each error whether large or small, as a fixed proportion of temporary and permanent shocks.

9. The Q-statistics reported in Table 2 are distributed as χ^2 with 12 degrees of freedom. The critical 5 percent value is 21.0. The Q-statistics for the g-ratio and the t_2-ratio exceed this value. If we test for white noise residuals for slightly longer lag length (30), however, we cannot reject the hypothesis of white noise residuals. The Q-statistic for the g-ratio becomes 34.45 and, for the t_2-ratio, 37.09, both less than the 5 percent critical value (43.77). Rather than trying to identify other superior models, we continue to use these representations because of their use in the work of Johannes and Rasche.

10. Except for using a six-state model for some of the component series to allow for secular trend movements, no other transformations have been performed on the various series. In the MSKF framework all series start in January 1959. The nature of the algorithm is such that changes in definitions or other structural changes will result in large forecast errors for a short period of time, after which the algorithm will have adjusted the probability structure.

11. This analysis of money market conditions presumes that the introduction of nationwide NOW accounts contained no new information that would allow one to improve the forecast of the money multiplier. This may not be the case. In fact, Johannes and Rasche (1981, 1982) employ a "ramp adjustment" to their forecasts in early 1981 to allow for portfolio shifts brought about by the nationwide NOW account introduction. In this regard, the forecast errors in 1981 should be viewed as upper bound estimates since no explicit account is taken of the nationwide NOW account introduction.

12. This concern stems from the fact that one-sided forecast errors will induce the monetary authority to continually over- or under-supply reserves to the banking system. If so, the difficulty in achieving a desired, long-run target for money growth is heightened.

13. This statement presumes that forecasting accuracy is the goal. If, however, one wishes to locate the source of the forecast error, then, obviously, the component procedure is better suited for this purpose.

14. An investigation of the covariance structure of the prediction errors of the different components may improve to component procedure. This information has not been exploited in the present analysis.

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Table 1

Model Specifications

Model Disturbance	ϕ	$Var(\varepsilon_t)$	$Var(\gamma_t)$	$Var(\alpha_t)$
Small temporary	0.95	0.95	0.0025	1
Small permanent	0.05	0.05	0.9025	1
Large temporary	0.99	15.84	0.0016	16
Large permanent	0.01	0.16	15.6816	16

Table 2
Multiplier Component Models
In-Sample Estimates
(all-data not seasonally adjusted; standard errors in parentheses)

Ratio	Model
k	$(1-B)(1-B^3)(1-B^{12})\ln k_t = (1-\underset{(0.045)}{0.7397}B^3)(1-\underset{(0.058)}{0.6239}B^{12})a_t$
	Q(12) = 15.60 SE = 0.567×10^{-2} Sample: 59.1-79.12
g	$(1-B)(1-B^{12})\ln g_t = (1-\underset{(0.064)}{0.4143}B)(1-\underset{(0.073)}{0.1313}B^2)(1-\underset{(0.053)}{0.6318}B^{12})a_t$
	Q(12) = 23.14 SE = 0.1996 Sample: 59.1-79.12
z	$(1-\underset{(0.062)}{0.3526}B)(1-B)(1-B^{12})\ln z_t = (1-\underset{(0.048)}{0.6947}B^{12})a_t$
	Q(12) = 15.76 SE = 0.270×10^{-1} Sample: 59.1-79.12
t_1	$(1-B)(1-B^3)(1-B^{12})\ln t_{1t} = (1-\underset{(0.049)}{0.6536}B^3)(1-\underset{(0.060)}{0.5692}B^{12})a_t$
	Q(12) = 8.24 SE = 0.545×10^{-2} Sample: 59.1-79.12
t_2	$(1-B^{12})[(1-B)\ln t_{2t} + \underset{(0.018)}{0.012}\,D1 + \underset{(0.012)}{0.041}\,D2 - \underset{(0.017)}{0.088}\,D3]$
	$= (1-\underset{(0.062)}{0.4421}B)^{-1} \quad (1-\underset{(0.054)}{0.6522}B^{12})a_t$
	Q(12) = 21.98 SE = 0.320×10^{-2} Sample: 61.1-79.12
r+ℓ	$(1-B)(1-B^{12})\ln(r+\ell)_t = (1-\underset{(0.081)}{0.6966}B + \underset{(0.082)}{0.2568}B^2 - \underset{(0.068)}{0.3623}B^{12})a_t$
	Q(12) = 12.60 SE = 0.963×10^{-2} Sample: 68.10-79.12
tc	$(1+\underset{(0.074)}{0.4306}B^{12})(1-B)(1-B^{12})\ln tc_t = (1-\underset{(0.074)}{0.6465}B)a_t$
	Q(12) = 13.73 SE = 0.6110×10^{-1} Sample: 69.1-79.12

Table 3

Summary Statistics for Aggregate M1 Multiplier Forecasts
One-Step-Ahead Predictions: ARIMA vs. MSKF
January 1980 - December 1982

Summary Statistic[a]	1980.1-1982.12		1980.1-1980.12		1981.1-1981.12		1982.1-1982.12	
	ARIMA	MSKF	ARIMA	MSKF	ARIMA	MSKF	ARIMA	MSKF
MAE	0.0185	0.0145	0.0199	0.0160	0.0195	0.0129	0.0161	0.0146
MSE	0.0005	0.0003	0.0006	0.0004	0.0006	0.0003	0.0004	0.0004
RMSE	0.0225	0.0179	0.0234	0.0190	0.0245	0.0159	0.0192	0.0187
U	0.0087	0.0070	0.0091	0.0074	0.0095	0.0062	0.0074	0.0073
B	0.0339	0.0014	0.0042	0.0347	0.1551	0.0611	0.0496	0.0072
V	0.0177	0.0222	0.0356	0.0461	0.0275	0.0418	0.1290	0.1258
CV	0.9484	0.9764	0.9602	0.9193	0.8174	0.8970	0.8214	0.8670

[a]MAE is the mean absolute error; MSE is the mean-squared error; RMSE is the root-mean-squared error; U is the Theil inequality coefficient; B, V, and CV represent the portion of the forecast error due to bias, unequal variation and unequal covariation between the actual and forecasted series, respectively. See Theil (1971).

Table 4

Summary Statistics for Component M1 Multiplier Forecasts
One-Step-Ahead Predictions: ARIMA vs. MSKF
January 1980 - December 1982

Summary Statistic[a]	1980.1-1982.12		1980.1-1980.12		1981.1-1981.12		1982.1-1982.12	
	ARIMA	MSKF	ARIMA	MSKF	ARIMA	MSKF	ARIMA	MSKF
MAE	0.0186	0.0180	0.0195	0.0186	0.0209	0.0189	0.0154	0.0164
MSE	0.0005	0.0005	0.0005	0.0005	0.0006	0.0005	0.0004	0.0004
RMSE	0.0223	0.0215	0.0233	0.0219	0.0242	0.0223	0.0190	0.0204
U	0.0086	0.0084	0.0091	0.0085	0.0094	0.0086	0.0074	0.0079
B	0.0125	0.0001	0.0045	0.0432	0.1526	0.1008	0.0005	0.0232
V	0.0081	0.0654	0.1698	0.2386	0.0014	0.0253	0.0042	0.0444
CV	0.9795	0.9345	0.8257	0.7181	0.8460	0.8738	0.9953	0.9324

[a]MAE is the mean absolute error; MSE is the mean-squared error; RMSE is the root-mean-squared error; U is the Theil inequality coefficient; B, V, and CV represent the portion of the forecast error due to bias, unequal variation and unequal covariation between the actual and forecasted series, respectively. See Theil (1971).

Appendix
Summary Forecast Statistics for Ratios

k

Summary Statistic[a]	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.00373	0.00353	0.00456	0.00421	0.00342	0.00316	0.00323	0.00322
MSE	0.00002	0.00002	0.00003	0.00002	0.00002	0.00001	0.00001	0.00001
RMSE	0.00433	0.00415	0.00520	0.00493	0.00403	0.00386	0.00361	0.00355
U	0.01102	0.01058	0.01329	0.01260	0.01027	0.00984	0.00913	0.00898
B	0.00728	0.00001	0.00098	0.02273	0.03310	0.01985	0.02229	0.00424
V	0.00220	0.03290	0.14370	0.11733	0.12210	0.02005	0.04561	0.02455
CV	0.99052	0.96710	0.85532	0.85993	0.84480	0.96009	0.93211	0.97122

g

	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.00652	0.00799	0.00557	0.00687	0.00633	0.00646	0.00766	0.01065
MSE	0.00007	0.00011	0.00004	0.00008	0.00005	0.00006	0.00012	0.00018
RMSE	0.00837	0.01026	0.00602	0.00870	0.00725	0.00742	0.01102	0.01358
U	0.22426	0.27475	0.16902	0.24432	0.20803	0.21342	0.26754	0.32992
B	0.00022	0.00077	0.00431	0.01757	0.00205	0.00468	0.00076	0.00354
V	0.06061	0.00509	0.28978	0.11712	0.00004	0.00028	0.08213	0.00305
CV	0.93917	0.99415	0.70591	0.86531	0.99790	0.99504	0.91711	0.99340

z

	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.00270	0.00302	0.00247	0.00308	0.00335	0.00308	0.00228	0.00291
MSE	0.00001	0.00001	0.00001	0.00001	0.00002	0.00001	0.00001	0.00001
RMSE	0.00328	0.00367	0.00304	0.00385	0.00392	0.00340	0.00276	0.00376
U	0.04057	0.04547	0.03290	0.04166	0.04670	0.04053	0.04359	0.05935
B	0.02832	0.00884	0.00452	0.01154	0.08530	0.06571	0.06696	0.00439
V	0.00033	0.03120	0.12274	0.31237	0.00282	0.00059	0.01744	0.04553
CV	0.97135	0.95996	0.87273	0.67609	0.91188	0.93370	0.91559	0.95008

t_1

	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.05321	0.03488	0.04590	0.03919	0.04208	0.02913	0.04166	0.03631
MSE	0.00285	0.00177	0.00287	0.00214	0.00225	0.00149	0.00342	0.00166
RMSE	0.05366	0.04201	0.05361	0.04631	0.04739	0.03863	0.05850	0.04073
U	0.01264	0.00995	0.01313	0.01134	0.01126	0.00918	0.01338	0.00932
B	0.00808	0.00112	0.01133	0.00987	0.00538	0.01170	0.16238	0.01302
V	0.03690	0.02069	0.16283	0.20433	0.11472	0.05120	0.01291	0.00105
CV	0.95502	0.97818	0.82585	0.78579	0.87990	0.93709	0.82471	0.98593

Appendix (Cont.)

t_2

Summary Statistic[a]	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.01649	0.01686	0.01332	0.01448	0.01816	0.02021	0.01799	0.01589
MSE	0.00040	0.00047	0.00027	0.00031	0.00046	0.00067	0.00046	0.00042
RMSE	0.01996	0.02167	0.01653	0.01773	0.02153	0.02588	0.02140	0.02059
U	0.01784	0.01937	0.01683	0.01806	0.01910	0.02295	0.01738	0.01672
B	0.01466	0.00401	0.00514	0.05684	0.00880	0.04336	0.08986	0.02059
V	0.03671	0.06488	0.00850	0.07786	0.04283	0.09947	0.00588	0.12672
CV	0.94863	0.93111	0.98637	0.86530	0.94838	0.85717	0.90426	0.85269

$r + \ell$

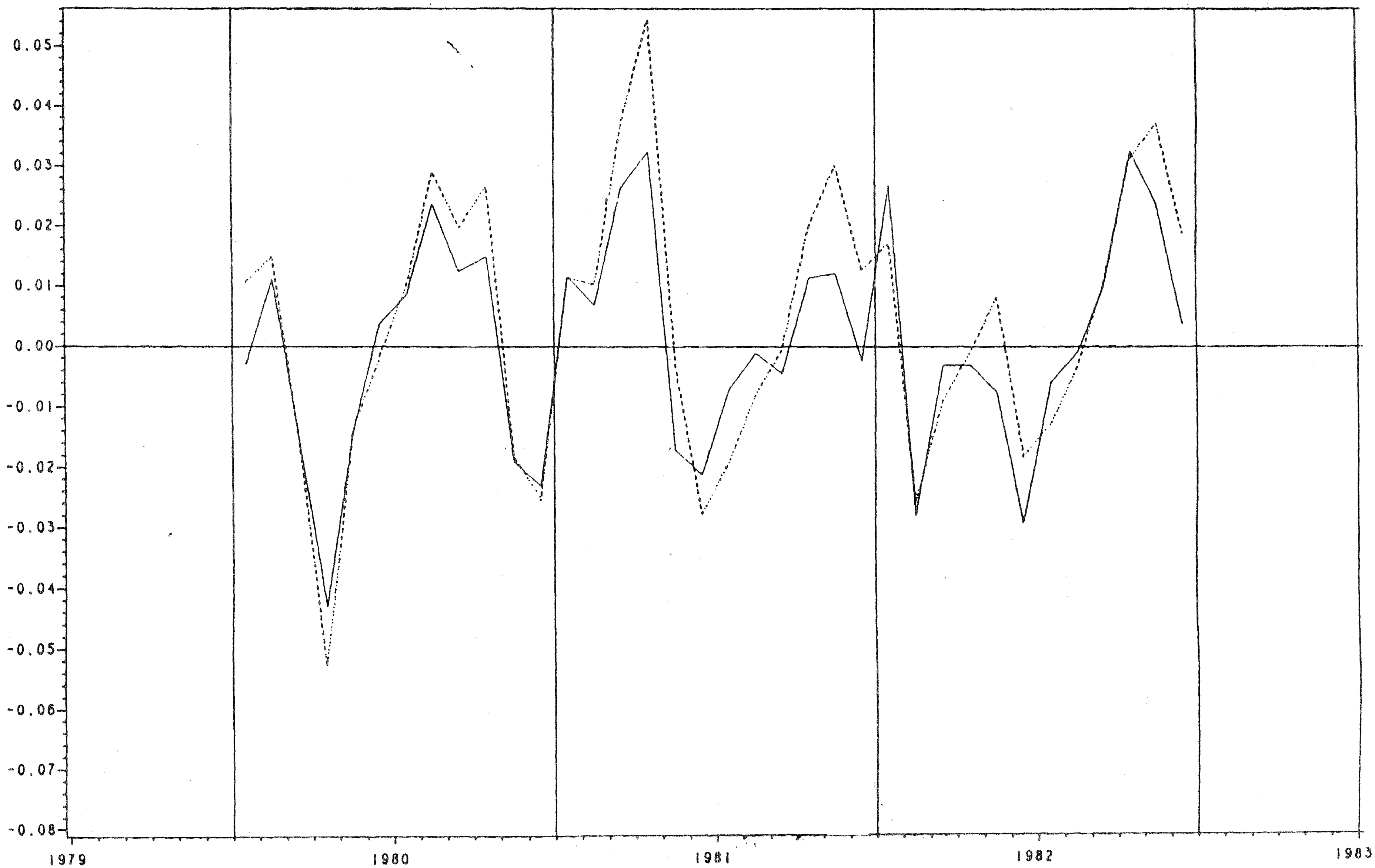
	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.00023	0.00019	0.00021	0.00016	0.00029	0.00021	0.00019	0.00020
MSE	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
RMSE	0.00030	0.00025	0.00027	0.00023	0.00035	0.00027	0.00026	0.00026
U	0.01256	0.01054	0.01081	0.00901	0.01493	0.01131	0.01171	0.01142
B	0.01053	0.00082	0.00000	0.03057	0.45798	0.29911	0.30961	0.24754
V	0.03685	0.02661	0.13625	0.00064	0.00046	0.12055	0.04566	0.00624
CV	0.95262	0.97257	0.86375	0.96878	0.54156	0.58034	0.64474	0.74622

tc

	1980.1 - 1982.12 ARIMA	1980.1 - 1982.12 MSKF	1980.1 - 1980.12 ARIMA	1980.1 - 1980.12 MSKF	1981.1 - 1981.12 ARIMA	1981.1 - 1981.12 MSKF	1982.1 - 1982.12 ARIMA	1982.1 - 1982.12 MSKF
MAE	0.00030	0.00030	0.00029	0.00034	0.00021	0.00024	0.00041	0.00033
MSE	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
RMSE	0.00042	0.00038	0.00039	0.00045	0.00033	0.00028	0.00051	0.00039
U	0.03021	0.02735	0.02848	0.03276	0.02370	0.01964	0.03737	0.02835
B	0.08782	0.01826	0.00616	0.07191	0.00243	0.00013	0.57764	0.49631
V	0.01895	0.00244	0.00476	0.17201	0.02944	0.06735	0.04455	0.04357
CV	0.89323	0.97929	0.98908	0.75609	0.96914	0.093252	0.37780	0.46012

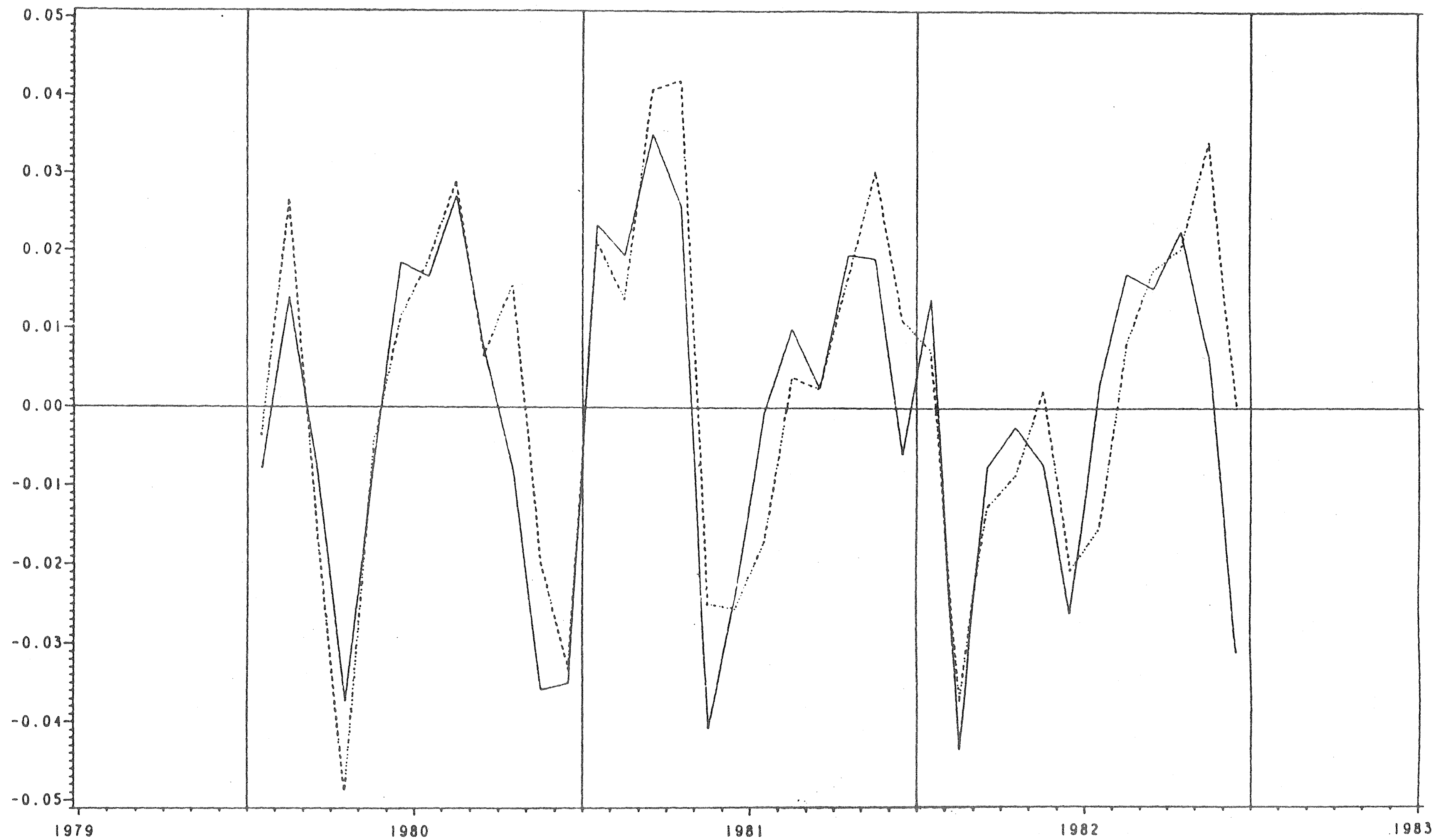
[a]MAE is the mean absolute error; MSE is the mean-squared error; RMSE is the root-mean-squared error; U is the Theil inequality coefficient; B, V, and CV represent the portion of the forecast error due to bias, unequal variation and unequal covariation between the actual and forecasted series, respectively. See Theil (1971).

Chart 1
AGGREGATE MODEL ERRORS: M1 MULTIPLIER



KALMAN FILTER –   BOX–JENKINS – –

Chart 2
COMPONENT MODEL ERRORS: M1 MULTIPLIER



KALMAN FILTER –   BOX–JENKINS ––