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To Chain or Not to Chain Trade-Weighted Exchange Rate Indexes

Cletus C. Coughlin
Patricia S. Pollard
Jerram C. Betts

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FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

ABSTRACT

To Chain or Not to Chain Trade-Weighted Exchange Rate Indexes

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With the advent of chain calculations for the U.S. national income and product accounts, it seems reasonable to contemplate using the chain approach for other indexes, such as trade-weighted exchange rates (TWEXs). A fundamental criticism of measuring the growth of gross domestic product by a fixed-base-year method is that the estimates are highly sensitive, especially when the economy's structure is changing dramatically, to the arbitrary choice of the base year. Such a criticism can be levied against TWEXs. In fact, even TWEXs constructed using a Paasche index rather than a Laspeyres index have problems related to base periods. We examine theoretically and empirically the use of a chain TWEX in relation to two well-known TWEX indexes: the Federal Reserve Bank of Atlanta index, which uses a Laspeyres index, and the Federal Reserve Bank of Dallas index, which uses a Paasche index. The choice of base year alters the behavior of the dollar in these two indexes. We contrast this result with the behavior of the dollar in comparable chain TWEXs, where the base year sensitivity is absent. Our results indicate that developers of TWEXs, as well as those revising TWEXs, should consider a chain approach. Furthermore, users need to be aware of the sensitivity of TWEXs to changes in either the base period for trade weights or the reference base period for exchange rates.

KEYWORDS: Trade-weighted exchange rates, Fisher-chain index

JEL CLASSIFICATION: F31, C43

Cletus C. Coughlin
Vice President
coughlin@stls.frb.org
Federal Reserve Bank of St. Louis
411 Locust Street
St. Louis, MO 63102

Patricia S. Pollard
Senior Economist
pollard@stls.frb.org
FRB St. Louis
411 Locust Street
St. Louis, MO 63102

Jerram C. Betts
Graduate Student
betts@umr.edu
Univ. of Missouri
302 Holtman Hall
Rolla, MO 65401

I. Introduction

With the advent of chain calculations for the U.S. national income and product accounts, it seems reasonable to contemplate using the chain approach for other indexes, such as trade-weighted exchange rates (TWEXs). A fundamental criticism of measuring the growth of real gross domestic product by a fixed-base-year method is that the estimates are highly sensitive, especially in times when the economy's structure is changing dramatically, to the arbitrary choice of the base year. This paper investigates whether such a criticism can also be levied against TWEX indexes.

The breakdown of the Bretton Woods system of fixed exchange rates spurred the development of TWEX indexes, which are simply an average of bilateral exchange rates where the average is determined by the countries included and the weights given to each currency.[1] TWEXs are used by policymakers, market analysts and the media to give a picture of changes in the average foreign exchange value of a currency over time. TWEXs, particularly real TWEXs, are also used by researchers in empirical trade studies. TWEX indexes have been used in studies analyzing the effect of exchange rate changes on a country's trade balance. The persistence of trade imbalances in the face of flexible exchange rates led to a further use of TWEX indexes: to study the effect of exchange rate changes on traded-goods prices.[2]

Trade-weighted exchange rate indexes are produced by various private and public organizations throughout the world. These indexes vary by the currencies included, the method

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[1] See Hirsch and Higgins (1970) for a seminal discussion of the construction of a TWEX index and Coughlin and Pollard (1996) for a recent overview of TWEX indexes.

[2] See Antzoulatos and Yang (1996) for a recent pass-through study and Menon (1995) for a survey of this literature.

for determining the weights given to each currency and the frequency of updating these weights. Several studies have investigated the importance of the choice of a TWEX for the outcome of an empirical study.[3] In choosing among TWEX indexes, researchers need to be aware of the appropriateness of the index for the question under study.

All common TWEXs are based on either a Laspeyres or Paasche price index. We focus on an issue that affects these indexes regardless of the currencies included or method for calculating weights: the choice of base periods. Actually, two interrelated base period decisions are relevant. First, a decision is required as to the base period for the trade weights. Analogous to measuring the growth of gross domestic product by a fixed-base-year method, a major concern with fixed trade weights is that over time the weights are less likely to reflect the existing pattern of trade. For example, as U.S. trade has shifted toward Asia and to selected developing countries, the fixed trade weights in a dollar index may be producing a biased picture of the TWEX. On the other hand, if the base period for trade weights is altered, the economic history described by the index is likely to change. An annual updating of the trade weights, however, does not eliminate all the problems related to base periods.

Second, in any TWEX index the changes in the bilateral exchange rates are calculated relative to exchange rates in a reference period. Ideally this reference period should reflect a period of equilibrium in the exchange rates. Given the difficulty of finding such a period, particularly when a large number of currencies are included in the TWEX, the reference period is

[3] See Batten and Belongia (1987), Feinberg (1991), and Pauls and Helkie (1987).

often chosen because it marks some important event in exchange rate history.[4] For TWEXs based on a Paasche index, the economic history described by the TWEX likely changes as the reference period for the exchange rate is altered.

We begin our study by examining the theoretical problems associated with the choice of a base year in Laspeyres and Paasche indexes and the solution offered by a chain index.[5] Next, we examine the empirical importance of these problems by focusing on two well-known TWEX indexes: the index produced by the Federal Reserve Bank of Atlanta (hereafter Atlanta), based on a Laspeyres index; and, the index produced by the Federal Reserve Bank of Dallas (hereafter Dallas), based on a Paasche index.[6] Finally, we produce a Fisher-chain version of these two TWEX indexes and compare these to the original versions. In the conclusion we summarize the theoretical and empirical justification for a chain TWEX.

II. Problems with Laspeyres and Paasche Indexes

Price indexes, such as a TWEX index, are generally constructed as either a Laspeyres or Paasche index. A Laspeyres index is characterized by fixed weights while a Paasche index is

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[4] For example, the Federal Reserve Bank of Dallas currently uses the first quarter of 1985 as the reference period for the exchange rates in constructing its TWEX index, while the Board of Governors of the Federal Reserve System uses March 1973.

[5] Our analysis of these TWEX indexes uses the nominal rather than the real versions of these indexes. This choice should not be interpreted as suggesting that the nominal versions are more useful than the real versions. Our decision stemmed from the fact that the nominal versions are easier to calculate and that our fundamental points are the same regardless of whether real or nominal exchange rates are used.

[6] Our focus on the TWEX indexes produced by Federal Reserve Banks is not meant to indicate that these are any better or worse than TWEX indexes produced by other organizations. Our choice was driven by the availability of data.

characterized by current weights.[7] The general formulas for Laspeyres, I_t^L, and Paasche, I_t^P, TWEX indexes are given below.

$$I_t^L = 100 * \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,R}} \right)^{w_{i,B}} \quad \textbf{(1)}$$

$$I_t^P = 100 * \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,R}} \right)^{w_{i,t}} \quad \textbf{(2)}$$

The TWEX index for any currency is the geometric average of the exchange rates, with respect to that currency, of n countries at time t, $e_{i,t,}$ relative to their respective exchange rates at some reference period R, $e_{i,R,}$ where the exchange rates are weighted by each country's trade share, $w_{i,B}$ or $w_{i,t}$.

The two key elements of these indexes for our purposes are the base period for the trade weights and the reference base for the exchange rates. In the Laspeyres index, as shown in equation (1), the trade weights are fixed at the period B trade shares. As trade patterns shift over time the weights may become less accurate, which may lead producers to update the weights to reflect more recent trade patterns. Updating the weights, however, changes the history of the index. In addition, while the new weights may be more relevant for recent periods they are less relevant for previous periods. The following example illustrates these problems.

Assume for simplicity there are only 3 currencies in the world: currencies A, B and C. Table 1 provides the information required to construct a TWEX index for currency A. Column 2

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[7] See Allen (1974) for a thorough discussion of Laspeyres and Paasche indexes.

shows the units of currency B required to purchase a unit of currency A in each of the 14 years listed, and column 4 shows the units of currency C required to purchase a unit of currency A. Columns 3 and 5 give the trade shares for country B and country C, respectively, in each of the 14 years. If the trade shares in year 2 (.62, .38) are used as the base weights, the trade-weighted value of currency A rises between years 1 and 7 and falls between years 7 and 14, as shown in Table 2. If the trade shares in year 12 (.45, .55) are used for the base weights a similar pattern is observed: currency A appreciates between years 1 and 6 and depreciates between years 6 and 14. However, the magnitudes of the appreciations and deprecations differ substantially across the two constructed indexes. Using year 2 as the base year for the weights the index shows a 43 percent appreciation for currency A between years 1 and 7, while using year 12 as the base year for the weights currency A shows only a 20 percent appreciation.[8] Likewise, the former index shows a 22 percent depreciation of currency A between years 7 and 14 while the latter index shows a 44 percent depreciation of currency A. Thus, using year 2 as the weights base the effective value of currency A is 21 percent higher in year 14 than in year 1 while using year 12 as the weights base the effective value of currency A is 24 percent lower in year 14 than in year 1.

Examining the correlation between the two indexes further illustrates these results. The correlation between the year-to-year percent changes is .95 indicating that in general the value of currency A moves in the same direction in the two indexes. The correlation between the levels of the two indexes is lower, .56, indicating the divergence in the two indexes over time.

[8] All percentages changes in this paper all calculated using log changes.

A key difference between TWEXs based on a Laspeyres formula and a Paasche formula is that in the Paasche-based index the weights vary from year to year.[9] Thus the value of the index in year t depends on the weights assigned to each currency in year t. This weighting method eliminates the rewriting of economic history caused by updating the weights base. Before concluding that the Paasche index is the better method for calculating a TWEX index, we need to consider the choice of the reference base for the bilateral exchange rates.

As shown in equations (1) and (2) TWEXs based on either a Laspeyres or a Paasche index require that a base period be chosen for the bilateral exchange rates, e_{iR}. With a Laspeyres index the choice of a reference base period for the exchange rates does not affect the behavior of the index, but the behavior of the Paasche index is sensitive to this choice. These results are shown formally in the appendix and can be illustrated using the data in Table 1. Two Laspeyres and two Paasche indexes are constructed from these data. Both Laspeyres indexes use the trade weights in year 1 (.60, .40). First, Laspeyres and Paasche indexes are constructed using the bilateral exchange rates in year 2 as the reference exchange rates. Next, the indexes are recalculated using the bilateral exchange rates in year 12 as the reference exchange rates. Table 3 shows the value of the indexes in each year and the year-to-year percent changes in the indexes.

First, consider the two Laspeyres indexes. The level of the two indexes in any year differs. However, the index based on the year 2 exchange rates can be rescaled by dividing the value of the index in each year by the value of the index in year 12 as follows:

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[9] While TWEX indexes are updated as frequently as monthly, the trade weights are generally updated annually.

$$100 * \left[\frac{I^L_{2,t}}{100 \ * \ \prod_{i=1}^{n} \left(\frac{e_{i,12}}{e_{i,2}} \right)^{w_{i,B}}} \right] = 100 * \left[\frac{100 \ * \ \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,2}} \right)^{w_{i,B}}}{100 \ * \ \prod_{i=1}^{n} \left(\frac{e_{i,12}}{e_{i,2}} \right)^{w_{i,B}}} \right]$$

$$= 100 * \left[\prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,2}} \right)^{w_{i,B}} * \left(\frac{e_{i,2}}{e_{i,12}} \right)^{w_{i,B}} \right]$$

$$= 100 * \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,12}} \right)^{w_{i,B}}$$

$$= I^L_{12,t}$$

This rescaling creates an index identical to the index based on the year 12 exchange rates. This ability to transform the reference base for the index explains why the year-to-year percent changes in the two Laspeyres indexes are identical.

Next, consider the two Paasche indexes. In this case neither the levels nor the year-to-year changes in the indexes are identical. Both Paasche indexes display a similar pattern over time: currency A appreciates between years 1 and 7 and depreciates between years 7 and 14. However, the magnitudes of the movements in the indexes differ. The index using year 2 as the reference base shows currency A appreciating by 51 percent through year 7, while using year 12 as a reference base the appreciation is 30 percent. Between years 7 and 14 the former index shows currency A depreciating by 82 percent while the latter index shows currency A depreciating by only 20 percent. Over the entire period the effective value of currency A declined by 31 percent when calculated using year 2 as the reference base but rose by 10 percent when calculated using year 12 as the reference base.

What explains the different effects of the reference base on the Laspeyres and Paasche indexes? The difference arises from the existence of fixed weights in the Laspeyres index and the varying weights in the Paasche index. While rescaling the Laspeyres index using a year 2 reference base can transform it into an index using year 12 as the reference base, the same cannot be accomplished with the Paasche index. Rescaling the Paasche index using year 2 as the reference base, by dividing the value of this index in each year by the value in year 12, does not result in an index identical to the index with year 12 as the reference base. This fact is demonstrated below:

$$100 * \left[\frac{I^P_{2,t}}{100 * \prod_{i=1}^{n} \left(\frac{e_{i,12}}{e_{i,2}} \right)^{w_{i,12}}} \right] = 100 * \left[\frac{100 * \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,2}} \right)^{w_{i,t}}}{100 * \prod_{i=1}^{n} \left(\frac{e_{i,12}}{e_{i,2}} \right)^{w_{i,12}}} \right]$$

$$= 100 * \left[\prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,2}} \right)^{w_{i,t}} * \left(\frac{e_{i,2}}{e_{i,12}} \right)^{w_{i,12}} \right]$$

$$\neq I^P_{12,t}$$

Now it is not so clear that the Paasche index is a better choice than the Laspeyres index.

III. The Chain Solution

Changing the base period for the trade weights in a Laspeyres index alters the history of the index, while changing the reference base period for the exchange rates in the Paasche index alters the behavior of the index. Technically, comparisons between years in a Laspeyres index are only valid between the year of the base period for the trade weights and all other years. Thus, a calculation of the dollar appreciation (depreciation) between two years, neither of which contains the base period for the trade weights, is inappropriate. A similar conclusion pertains to comparisons in a Paasche index for years not coinciding with the reference base period for the exchange rates. Despite being technically inappropriate, such calculations are common.

It is, however, rather easy to produce a measure that allows for an appropriate calculation of the change in the average value of the dollar relative to periods other than the base. One way to eliminate the problems inherent in the construction of each index is to eliminate the need for a base period. This can be done by constructing chain versions of the indexes. A chain index links together the exchange rates and trade weights from year-to-year. Equations (3) and (4) present the formulas for the chain versions of the Laspeyres and Paasche indexes, respectively.

$$I_t^{LC} = \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,t-1}} \right)^{w_{i,t-1}} * I_{t-1}^{LC} \tag{3}$$

$$I_t^{PC} = \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,t-1}} \right)^{w_{i,t}} * I_{t-1}^{PC} \tag{4}$$

A further advantage of a chain index is that it makes use of all available information. This is because the value of the index in any period t incorporates the value of the index at all previous periods.

The only difference between a Laspeyres chain and a Paasche chain is that the former uses weights from the previous period, while the latter uses weights from the current period. How then does one choose between the chain versions of the Laspeyres and Paasche indexes?

The relationship between Laspeyres and Paasche indexes can be shown by looking at differences in the price elasticities. For a Laspeyres index the price elasticity is:

$$\frac{\delta \ln I_t}{\delta \ln e_{i,t}} = w_{i,B} \tag{5a}$$

and for a Paasche index the price elasticity is:

$$\frac{\delta \ln I_t}{\delta \ln e_{i,t}} = w_{i,t} \tag{5b}$$

From equations (5a) and (5b) it is clear that if the exchange rate for currency i rises (holding all other exchange rates in the index constant) the value of the Laspeyres (Paasche) index will exceed the value of the Paasche (Laspeyres) index if the price elasticity of that currency in the Laspeyres (Paasche) index is relatively higher, i.e., $w_{iB}>w_{it}$ ($w_{iB}<w_{it}$). When more than one exchange rate changes, the differences in price elasticities of each currency in the two indexes

must be weighted by the change in each exchange rate (relative to the reference base year) producing the following relationship:

$$I_t^L > (<)\ I_t^P \ \ if \ \sum_{i=1}^{n} [s_{i,t} * (w_{i,t} - w_{i,B})] < (>)\ 0 \quad where \quad s_{i,t} = \ln\left(\frac{e_{i,t}}{e_{i,R}}\right) \tag{6}$$

The preceding formula can be illustrated using the data in Table 1. Using year 2 as the reference period for exchange rates, the formula implies that in years 1, 2 and 10 the Laspeyres and Paasche indexes should give identical readings, in years 3-9 the Laspeyres index should be less than the Paasche index, and in years 11-14 the Laspeyres index should be greater than the Paasche index. The values for the Laspeyres and Paasche indexes in Table 3 show the predicted relationship between the indexes.[10]

The Laspeyres and Paasche chains can be viewed as providing the bounds for a realistic measure of an exchange rate index. When comparing adjacent periods, the Laspeyres chain uses weights based on the prior period, while the Paasche chain uses weights based on the current period. The appropriate weights for a TWEX are uncertain, but a straightforward solution is to combine the two by taking their geometric average. This is known as a Fisher chain:

[10] In a chain index the relationship between the corresponding Laspeyres and Paasche indexes is complicated by the inclusion of the value of the lagged index in the index itself.

$$I_t^{FC} = \left[\prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,t-1}} \right)^{w_{i,t-1}} * \left(\frac{e_{i,t}}{e_{i,t-1}} \right)^{w_{i,t}} \right]^{\frac{1}{2}} * I_{t-1}^{FC}$$

$$= \left[\prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,t-1}} \right)^{w_{i,t-1} + w_{i,t}} \right]^{\frac{1}{2}} * I_{t-1}^{FC} \qquad \textbf{(7)}$$

Because the Fisher chain is the average of the Laspeyres and Paasche chains it will lie between the two, as shown in Table 4 and Figure 1.

IV. Empirical Analysis

The above analysis suggests some solid reasons to consider using a chain rather than either the Laspeyres or Paasche indexes to create TWEX indexes. This section examines two current indexes to determine the empirical relevance of the problems associated with the simple Laspeyres or Paasche indexes highlighted in the previous section. The dollar TWEX index produced by the Federal Reserve Bank of Atlanta is a Laspeyres index, while the dollar TWEX index produced by the Federal Reserve Bank of Dallas is based on a Paasche index.

The trade weights in the Atlanta TWEX are derived from the 1984 trade shares of the 18 countries whose currencies are included in the index.[11] The bilateral exchange rates in each period are calculated relative to the respective rates in 1980. However, as noted previously, the reference period for the exchange rates does not affect the behavior of a Laspeyres index.

The trade weights in the Dallas index are updated yearly and are based on a three-year moving average of the trade shares of the 128 countries currently included in the index.[12] For example, the value of the index in 1994 is calculated using average trade shares from 1991-93, while the value of the index in 1995 used trade shares from 1992-94. The Dallas index currently uses the first quarter of 1985 as the reference period for the exchange rates.

To determine the sensitivity of these indexes to the choice of the base period we recalculated each index using each possible base year over the sample period 1976-95.[13] Specifically we calculated 20 versions of the Atlanta index each using a different base year for the trade weights in the index, and 20 versions of the Dallas index each using a different

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[11] The countries whose currencies are included in the Atlanta index are: Australia, Belgium, Canada, China, France, Germany, Hong Kong, South Korea, Italy, Japan, Netherlands, Saudi Arabia, Singapore, Spain, Sweden, Switzerland, Taiwan and the United Kingdom. See Rosenweig (1986a) and (1986b) for details on the construction of the Atlanta index.

[12] This procedure for calculating trade shares departs from the standard Paasche formula in equation (2), where the weights are based on the current year's trade shares. This difference however does not affect the nature of our empirical results. See Cox (1986) for a description of the creation of the Dallas index. The formula for the Dallas index is

$$I_t = 100 * \prod_{i=1}^{n} \left(\frac{e_{i,t}}{e_{i,R}} \right)^{[w_{i,t-1} + w_{i,t-2} + w_{i,t-3}]/3} .$$

[13] This period was chosen due to the availability of data for the Dallas index.

reference year for the exchange rates in the index.[14] Then we used measures to determine if there were significant differences in these versions of each index.

First, for each version of the two indexes we calculated the percentage change in the trade-weighted value of the dollar over three periods: 1976-95, 1976-85 and 1985-95. These results are shown in Table 5. Depending upon the choice of the base for the weights, the Atlanta index shows a depreciation of the dollar between 4 and 17 percent over the period 1976 to 1995. For the sub-period 1976-1985 the dollar appreciated between 23 and 29 percent depending upon the choice of a base year for the trade weights; whereas between 1985 and 1995 the dollar depreciated in the range of 25 to 33 percent. The differences in the calculated change in the value of the dollar are even greater using the Dallas index.[15] Depending upon the choice of a reference period for the exchange rates, the Dallas index shows an appreciation of the dollar between 260 and 424 percent over the period 1976 to 1995. For the period 1976 to 1985 the dollar appreciated anywhere from 101 to 164 percent and between 72 and 97 percent from 1985 to 1995.[16]

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[14] We concentrate on base years even in the Dallas index to limit the number of possible reference periods. For the Dallas index we make comparisons relative to an index using 1985 as the reference base year.

[15] The Atlanta and Dallas indexes are not directly comparable given differences in the choice of currencies included in each index. Moreover, it is not surprising that the variation among the calculated versions of the Dallas index are greater than among the versions of the Atlanta index since the year to year variation in trade weights are less than the year-to-year variations in the exchange rates. See Coughlin and Pollard (1996) for a discussion of the importance of currency choice for explaining differences among TWEX indexes.

[16] In August 1991 Dallas changed the reference base period used to calculate its TWEX index from the first quarter of 1973 to the first quarter of 1985. Using the former reference base the calculated appreciation of the dollar between January 1976 and December 1985 was 67 percent while using the later reference base period indicated a 75 percent rise over this period. Using annual data, as in our study, the dollar appreciated by 73 percent using the first quarter

Among the versions of the Atlanta index, over the period 1976 to 1995 the measured depreciation of the dollar is greatest if trade weights from the mid-1980s to the early 1990s are used and lowest if trade weights from the mid-1970s are used. Furthermore, the magnitude of the depreciation of the dollar using the current base year for weights in the Atlanta index, 1984, is larger than the average of the calculated depreciations.

With respect to the versions of the Dallas index, over the period 1976 to 1995, the measured appreciation of the dollar is greatest if a more recent year is chosen for the exchange rate reference period. Interestingly, the variation in the behavior of the dollar has increased with the inclusion of recent reference base years. For example, if 1976 rather than 1985 were chosen as the reference base year the calculated appreciation over the sample period would have been 7 percentage points lower. However, if the reference base year were 1995 instead of 1985 the calculated appreciation of the dollar would have been 150 percentage points higher. The appreciation of the dollar using the current choice of the base year, 1985, is in the lower one-half of the range of appreciations over the sample period.

Next we examined the year-to-year percentage changes in the value of the dollar using all variations on the base year. These results are presented in Table 6. The choice of a base year for weights in the Atlanta index does not have a large effect on the percentage change in the value of the dollar from year-to-year. For the Dallas index the choice of the base year has greater effects. For example, the appreciation of the dollar between 1984 and 1985 ranged from 11.5 to 20.6 percent depending upon the base year. Furthermore, in 1978, 1986 and 1987 the determination whether the dollar appreciated or depreciated is affected by the choice of a reference base year.

1973 as a base period and 85 percent using the first quarter 1985 as a base period.

For the Dallas indexes, while behavior in individual years may vary greatly, overall the indexes and the year-to-year changes in the indexes are highly correlated with the index using 1985 as the base year. Similarly the Atlanta indexes are highly correlated with the indexes using 1984 as the base year. To examine further the substitutability of reference base years in the two indexes we used orthogonal least squares. In contrast to ordinary least squares, the fitted line in orthogonal regression is the one that minimizes the mean square of the perpendicular rather than the vertical deviation of the sample points from the fitted line.[17] For interchangeability the estimate of the slope coefficient of a regression of the Atlanta index using the 1984 weights on the values of the Atlanta index using a different base year must be one. Similarly for the versions of the Dallas index, the estimate of the slope coefficient of a regression of the index using the exchange rates in 1985 as the reference period on the values of the Dallas index using a different base year must be one.[18]

Table 7 shows the orthogonal least squares results when the indexes are expressed in levels and natural logarithms.[19] Using levels of the Atlanta index, the results reveal that in

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[17] See Malinvaud (1980) for a thorough discussion of the differences between orthogonal and ordinary least squares regression.

[18] Thus, for interchangeability the measures must not only be highly correlated, but they must consistently differ by a constant. If this criterion holds, then the alternative measures will yield virtually identical results in econometric studies. For an elementary introduction to interchangeability, and orthogonal least squares, see appendix A in Coughlin and Mandelbaum (1991).

[19] We included the natural log version of the indexes because of its use in empirical work.

12 of the 19 cases the hypothesis that the slope coefficient equals one can be rejected at the 0.05 significance level.[20] Using natural logarithms, in 9 of the 19 cases the hypothesis that the slope coefficient equals one could be rejected. The results using natural logarithms indicate that choosing any year from 1976 trough 1983 as the base year for the weights produces an index that is interchangeable with the Atlanta index using 1984 as the weight base. With the exception of 1987 and 1988, using any year after 1984 as the base year for the weights produces an index that is not interchangeable with the current Atlanta index. The results using the levels of the TWEX index do not provide as clear a break in the pattern of interchangeable years.

For the Dallas index, using levels, the results show that in 16 of the 19 cases the hypothesis that the slope coefficient equals one can be rejected at the 0.05 significance level. Only the indexes using 1986,1987 or 1988 as the base year are interchangeable with the Dallas index using 1985 as the base year. Using natural logarithms, in 9 of the 19 cases the hypothesis that the slope coefficient equals one could be rejected. Using any year from 1988 through 1995 produces an index that is not interchangeable with the current Dallas index.

In sum, our analysis shows that the choice of the base year does affect the behavior of a Laspeyres TWEX index and a Paasche TWEX index. In the Atlanta index, the magnitude of the dollar's nominal deprecation over the period 1976-95 may vary by over 9 percentage points if the 1984 base year for the weights is replaced by another year. In the Dallas index, the magnitude of the dollar's nominal appreciation over the period 1976-95 may vary by 156 percentage points if the 1985 base year for the exchange rates is replaced by another year.

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[20] The results for 1984 are not considered because the slope coefficient is necessarily equal to one. Likewise for the Dallas comparisons the results for 1985 are not considered.

Furthermore, while the versions of each index are highly correlated regardless of the reference base year chosen, there is some evidence from orthogonal least squares that they are not substitutable.

V. Comparing Chain Indexes with Current Indexes

Given the evidence that the choice of a base year alters the behavior of TWEX indexes, we created Fisher chain indexes for the two TWEX indexes discussed in this paper.[21] Figures 2a and 2b compare the Fisher chain versions of the Atlanta and Dallas indexes to the indexes created by each of these organizations. As these Figures indicate, the Fisher chain versions of the indexes present similar views of the trade-weighted behavior of the dollar as do the original versions of the indexes. However, there is some evidence, provided by the orthogonal least squares results in Table 8, that the Fisher chain version and the original version of each index are not interchangeable. For example, using levels of the Atlanta index, the hypothesis that the slope coefficient equals one can be rejected; however, using natural logarithms, the null hypothesis cannot be rejected. Using either the levels or natural logarithms of the Dallas index, the null hypothesis can be rejected.[22]

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[21] Our Fisher chain version of the Dallas index uses the current and preceding year's trade weights as in equation (7) rather than a three year moving average.

[22] We also calculated a "Dallas-type" index using the current year's trade weights (a standard Paasche index) and found that our Fisher chain version and this index were not interchangeable.

VI. Conclusion

In the nearly 25 years since the collapse of the fixed exchange rate system TWEX indexes have been constructed by central banks, governments, international organizations and private institutions. All these indexes differ at least slightly as a result of differences in terms of the currencies used, method of calculating weights, and the frequency of updating the weights. The importance of these factors in accounting for differences among TWEX indexes has been studied by various researchers.[23]

In contrast to previous studies that focus on explaining differences across TWEX indexes, this paper focuses on an issue that is common to all of the TWEX indexes -- the dependence of the behavior of the index on the base year. The behavior of Laspeyres TWEX indexes are affected by the base year(s) chosen for the weights given to the bilateral exchange rates. The behavior of Paasche TWEX indexes are affected by the reference base period chosen for the exchange rates.

Using the Laspeyres index constructed by the Federal Reserve Bank of Atlanta and the Paasche index created by the Federal Reserve Bank of Dallas, we examined the sensitivity of the TWEX indexes to changes in the base year. We found that over a 20 year period (1976-1995) the change in the value of the dollar could differ significantly as a result of the base year choice. For example, the overall change in the value for the dollar using the Dallas index differed by up to 164 percentage points over the period 1976-1995. Based on orthogonal least squares, our

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[23] Many researchers, including Coughlin and Pollard (1996), Pauls (1987) and Rosensweig (1987) have examined the importance of the specific decisions made to construct TWEXs.

results indicated that one base year could not be randomly substituted for another in either the Laspeyres or Paasche TWEX index studied.

Given the sensitivity of the indexes to the base year and that Laspeyres and Paasche chain indexes can be viewed as providing reasonable bounds for exchange rate indexes, we created Fisher chain versions of the two indexes. An eyeball comparison of our Fisher chain versions of the two indexes with their original version does not yield obvious differences; however, a closer statistical examination does yield some potentially noteworthy differences. More importantly, the Fisher chain versions are not dependent on a base period for the trade weights or a reference base period for the exchange rates and avoid the potential problems stemming from such a dependence. The reference base for an exchange rate index is at times revised and these revisions can produce significant changes in the behavior of the index. At a minimum, our results suggest that users of current exchange rate indexes be aware of the importance of the base period in determining changes in the index.

Our results also suggest that developers of TWEXs, as well as those revising existing TWEXs, should consider a chain approach. We have not demonstrated the superiority of the chain approach, but we have provided solid reasons to view a chain approach as a reasonable alternative to current approaches. For example, the chain approach allows for the calculation of changes in the average value of the dollar relative to periods other than the base. Despite the fact that such calculations are performed using indexes that are not based on the chain approach, they are not theoretically justified. Of course, while this paper highlights the usefulness of the chain approach, future research should examine the importance of base year changes in empirical trade studies.

References

Allen, R.G.D., 1975. *Index Numbers in Theory and Practice,* Chicago: Aldine Publishing Company.

Antzoulatos, Angelos A., and Jiawen Yang, 1996. "Exchange Rate Pass-Through in U.S. Manufacturing Industries: A Demand-Side Story," *International Trade Journal*, Vol. 10., No. 3, Fall, pp. 325-352.

Batten, Dallas S., and Michael T. Belongia, 1987. "Do the New Exchange Rate Indexes Offer Better Answers to Old Questions?" Federal Reserve Bank of St. Louis *Review,* Vol. 69, No. 5, May, pp. 5-17.

Coughlin, Cletus, C., and Thomas Mandelbaum, 1991. "Measuring State Exports: Is There a Better Way?" Federal Reserve Bank of St. Louis *Review,* Vol. 73, No. 4, July/August, pp. 65-79.

Coughlin, Cletus C., and Patricia S. Pollard, 1996. "A Question of Measurement: Is the Dollar Rising or Falling?" Federal Reserve Bank of St. Louis *Review*, Vol. 78, No. 4, July/August, pp. 3-18.

Cox, W. Michael, 1986. "A New Alternative Trade-Weighted Dollar Exchange Rate Index," Federal Reserve Bank of Dallas *Economic Review,* September, pp. 20-28.

Feinberg, Robert M., 1991. "The Choice of Exchange Rate Index and Domestic Price Pass-through," *Journal of Industrial Economics*, Vol. 39, No. 4, June, pp. 409-20.

Hirsch, Fred and Higgins, Isle, 1970. "An Indicator of Effective Exchange Rates," *IMF Staff Papers,* Vol. 17, No. 1, March, pp. 453-87.

Malinvaud, Edmond, 1980. *Statistical Methods of Econometrics,* Amsterdam: North-Holland.

Menon, Jayant, 1995. "Exchange Rate Pass-through," *Journal of Economic Surveys,* Vol. 9, No. 2, June, pp. 197-232.

Pauls, B. Dianne, 1987. "Measuring the Foreign-Exchange Value of the Dollar," *Federal Reserve Bulletin*, Vol. 73, No. 6, June, pp. 411-422.

Pauls, B. Dianne and William L. Helkie, 1987. "A Reassessment of Measures of the Dollar's Effective Exchange Value," Board of Governors of the Federal Reserve System, *International Finance Discussion Papers* Number 306, April.

Rosensweig, Jeffrey A., 1986a. "A New Dollar: Capturing a More Global Perspective," Federal Reserve Bank of Atlanta *Economic Review,* Vol. 71, No. 6, June/July, pp. 12-22.

________, 1986b. "The Atlanta Fed Dollar Index and its Component Sub-Indexes," Federal Reserve Bank of Atlanta *Working Paper* #86-7, August.

________, 1987. "Constructing and Using Exchange Rate Indexes," Federal Reserve Bank of Atlanta *Economic Review,* Vol. 72, No. 3/4, Summer, pp. 4-16.

Appendix

Proposition 1:

In a Laspeyres TWEX index the choice of the reference base period does not affect the percentage change in the index between any two periods.

Proof:

The Laspeyres TWEX index for the time periods t and t-1 can be written as follows:

$$I_t^L = 100 \prod_{i=1}^{n} \left(\frac{e_{it}}{e_{iR}} \right)^{w_{iB}} \qquad \text{(A1)}$$

$$I_{t-1}^L = 100 \prod_{i=1}^{n} \left(\frac{e_{it-1}}{e_{iR}} \right)^{w_{iB}} \qquad \text{(A2)}$$

The percentage change in the index between t and t-1 is:

$$\left(\frac{I_t}{I_{t-1}} - 1 \right) 100 = \left(\frac{100 \prod_{i=1}^{n} \left(\frac{e_{it}}{e_{iR}} \right)^{w_{iB}}}{100 \prod_{i=1}^{n} \left(\frac{e_{it-1}}{e_{iR}} \right)^{w_{iB}}} - 1 \right) 100$$

$$= \left(\prod_{i=1}^{n} \left(\frac{e_{it}}{e_{i_{it-1}}} \right)^{w_{iB}} - 1 \right) 100 \qquad \text{(A3)}$$

As is clear from equation (A3) the percentage change in the index between t and t-1 does not depend on e_{ib}. Thus, the base year does not affect the calculation of percentage changes in a trade-weighted exchange index when the trade weights are constant over time.

Proposition 2:

In a Paasche TWEX index the choice of the reference base period does affect the percentage change in the index between any two periods.

Proof:

The Paasche TWEX indexes for periods t and t-1 can be written as follows:

$$I_t = 100 \prod_{i=1}^{n} \left(\frac{e_{it}}{e_{iR}} \right)^{w_{it}} \tag{A4}$$

$$I_{t-1} = 100 \prod_{i=1}^{n} \left(\frac{e_{it-1}}{e_{iR}} \right)^{w_{it-1}} \tag{A5}$$

The percentage change in the index between t and t-1 is:

$$\left(\frac{I_t}{I_{t-1}} - 1 \right) 100 = \left(\frac{100 \prod_{i=1}^{n} \left(\frac{e_{it}}{e_{iR}} \right)^{w_{it}}}{100 \prod_{i=1}^{n} \left(\frac{e_{it-1}}{e_{iR}} \right)^{w_{it-1}}} - 1 \right) 100$$

$$= \left(\prod_{i=1}^{n} \left(\frac{e_{it}}{e_{iR}} \right)^{w_{it}} \left(\frac{e_{iR}}{e_{it-1}} \right)^{w_{it-1}} - 1 \right) 100 \tag{A6}$$

$$= \left(\prod_{i=1}^{n} (e_{iR})^{w_{it-1} - w_{it}} \frac{(e_{it})^{w_{it}}}{(e_{it-1})^{w_{it-1}}} - 1 \right) 100$$

If $w_{i,t-1} - w_{i,t} = 0 \; \forall \; i$ then (A6) reduces to

$$\left(\frac{I_t}{I_{t-1}} - 1 \right) 100 = \left(\prod_{i=1}^{n} \left(\frac{e_{it}}{e_{it-1}} \right)^{w_i} - 1 \right) 100 \tag{A7}$$

which is simply the case of fixed trade weights, a Laspeyres index. When the trade weights vary over time, the calculation of percentage changes is affected by the choice of the base year for the exchange rate.

Table 1 Exchange Rates and Trade Weights for Countries B and C Relative to A				
	Country B		Country C	
Year	Exchange Rate	Trade Weight	Exchange Rate	Trade Weight
1	25	.60	55	.40
2	32	.62	50	.38
3	39	.64	48	.36
4	49	.66	45	.34
5	61	.68	39	.32
6	61	.69	39	.31
7	65	.70	36	.30
8	68	.68	28	.32
9	72	.65	25	.35
10	75	.60	22	.40
11	78	.50	17	.50
12	80	.45	16	.55
13	82	.42	15	.58
14	85	.40	13	.60
Note: The exchange rate is the number of units of the currency of country B(C) per unit of the currency of country A.				

Table 2 Laspeyres Exchange Rate Indexes for Country A Varying Base Year for Weights				
	Indexes		Percent changes	
Year	Year 2 weights	Year 12 weights	Year 2 weights	Year 12 weights
1	100.0	100.0	--	--
2	112.4	106.0	11.7%	5.9%
3	125.1	113.3	10.7	6.7
4	140.6	121.2	11.7	6.7
5	152.6	123.7	8.1	2.0
6	152.6	123.7	0.0	0.0
7	153.9	121.8	0.9	-1.5
8	143.9	108.2	-6.8	-11.8
9	142.8	104.3	-0.8	-3.7
10	139.5	99.0	-2.3	-5.2
11	129.6	87.5	-7.4	-12.4
12	128.7	85.6	-0.7	-2.2
13	127.5	83.5	-0.9	-2.4
14	123.4	78.5	-3.2	-6.3
Note: Percentage changes are calculated on a logarithmic basis from the preceding to the current year.				

Table 3 Exchange Rate Indexes for Country A for Different Exchange Rate Reference Years								
	Level Laspeyres Indexes		Level Paasche Indexes		Percent Change Laspeyres		Percent Change Paasche	
	Reference Base		Reference Base		Reference Base		Reference Base	
Year	Year 2	Year 12	Year 2	Year 12	Year 2	Year 12	Year 2	Year 12
1	89.6	81.5	89.6	81.5	--	--	--	--
2	100.0	91.0	100.0	87.4	11.0%	11.0%	11.0%	6.9%
3	110.8	100.8	111.8	93.8	10.2	10.2	11.2	7.1
4	123.8	112.7	127.8	102.8	11.1	11.1	13.4	9.2
5	133.3	121.4	143.2	110.6	7.4	7.4	11.4	7.3
6	133.3	121.4	144.5	109.3	0.0	0.0	0.9	-1.2
7	134.2	122.1	148.8	110.3	0.6	0.6	2.9	0.9
8	124.6	113.5	138.7	107.1	-7.3	-7.3	-7.1	-2.9
9	123.3	112.2	132.9	109.2	-1.1	-1.1	-4.3	1.9
10	120.0	109.3	120.0	109.3	-2.7	-2.7	-10.2	0.1
11	110.9	100.9	91.0	101.8	-8.0	-8.0	-27.7	-7.1
12	109.9	100.0	80.7	100.0	-0.9	-0.9	-12.0	-1.8
13	108.7	98.9	73.9	97.3	-1.1	-1.1	-8.9	-2.7
14	104.8	95.4	65.9	90.5	-3.6	-3.6	-11.4	-7.3
Note: Percentage changes are calculated on a logarithmic basis from the preceding to the current year.								

Table 4 Chain TWEX Indexes			
Year	Laspeyres Chain	Paasche Chain	Fisher Chain
1	100.0	100.0	100.0
2	111.6	112.4	112.0
3	124.3	125.7	125.0
4	140.5	143.0	141.7
5	154.6	158.5	156.6
6	154.6	158.5	156.6
7	157.6	161.8	159.7
8	150.8	153.9	152.4
9	151.2	153.5	152.4
10	148.5	149.5	149.0
11	137.1	134.0	135.6
12	134.7	131.1	132.9
13	131.5	127.6	129.6
14	122.9	118.8	120.8

Table 5
Percentage Changes for Periods Using Different Base Years

	Atlanta Indexes			Dallas Indexes		
Base Year	1976-1995	1976-1985	1985-1995	1976-1995	1976-1985	1985-1995
1976	-4%	29%	-26%	261%	101%	80%
1977	-5	28	-26	260	101	79
1978	-9	27	-28	265	103	80
1979	-8	27	-28	268	104	81
1980	-9	26	-28	269	104	81
1981	-4	29	-26	274	106	81
1982	-9	26	-28	264	104	78
1983	-8	26	-27	262	104	77
1984	-13	25	-30	268	107	78
1985	-14	25	-31	268	109	76
1986	-17	23	-33	261	108	73
1987	-15	24	-31	261	110	72
1988	-14	25	-31	270	114	73
1989	-13	25	-30	295	121	78
1990	-12	25	-29	310	126	81
1991	-11	25	-29	327	130	85
1992	-9	25	-27	347	136	90
1993	-9	24	-27	401	157	95
1994	-7	25	-26	424	164	99
1995	-6	26	-25	418	163	97

Note: Percentage changes are calculated on a logarithmic basis from the preceding to the current year.

Table 6 Year-to-Year Percentage Changes in Indexes Using All Base Years		
	Atlanta Indexes	Dallas Indexes
Year	Range of Percentage Changes	Range of Percentage Changes
1977	-1.0% — 0.3%	3.7% — 6.4%
1978	-7.8 — -5.6	-2.2 — 6.5
1979	-1.5 — -0.8	1.1 — 9.0
1980	0.4 — 1.3	0.4 — 2.8
1981	7.6 — 8.9	3.6 — 10.8
1982	8.6 — 9.4	11.3 — 17.2
1983	3.2 — 4.1	14.1 — 16.5
1984	5.8 — 6.6	14.6 — 23.9
1985	4.0 — 5.1	11.5 — 20.6
1986	-14.4 — -11.2	-3.0 — 2.2
1987	-10.4 — -8.9	-1.9 — 5.6
1988	-7.0 —-6.1	0.9 — 6.9
1989	1.3 — 2.1	10.3 — 14.8
1990	-3.3 — -1.3	7.7 — 9.4
1991	-1.2 — -0.6	5.3 — 7.2
1992	-1.4 — -0.3	4.6 — 6.8
1993	2.7 — 4.2	9.2 — 13.7
1994	-0.6 — 1.8	7.3 — 11.0
1995	-4.8 — -4.1	2.6 — 7.9

Table 7
Orthogonal Least Squares Regression Coefficients

	Atlanta Indexes		Dallas Indexes	
Base Year	Using Levels	Using Log Levels	Using Levels	Using Log Levels
1976	1.12	0.88	1.98*	0.98
1977	1.10	0.88	1.91*	0.98
1978	1.24*	0.94	2.00*	1.00
1979	1.26*	0.94	1.99*	1.01
1980	1.25*	0.94	1.93*	1.00
1981	1.09	0.88	1.76*	1.01
1982	1.04	0.92	1.47*	0.99
1983	0.98	0.91	1.29*	0.99*
1984	1.00	1.00	1.14*	1.00
1985	0.98*	1.03*	1.00	1.00
1986	1.21*	1.09*	1.01	1.01
1987	1.28*	1.04*	1.00	1.03
1988	1.34*	1.02*	0.98	1.06*
1989	1.28*	0.99	0.90*	1.11*
1990	1.29*	0.97	0.85*	1.14*
1991	1.26*	0.95*	0.82*	1.18*
1992	1.22*	0.91*	0.79*	1.21*
1993	1.15*	0.89*	0.75*	1.31*
1994	1.09	0.86*	0.70*	1.34*
1995	1.12	0.85*	0.68*	1.34*

* Indicates a rejection that the coefficient equals 1.00 at the 0.05 significance level.

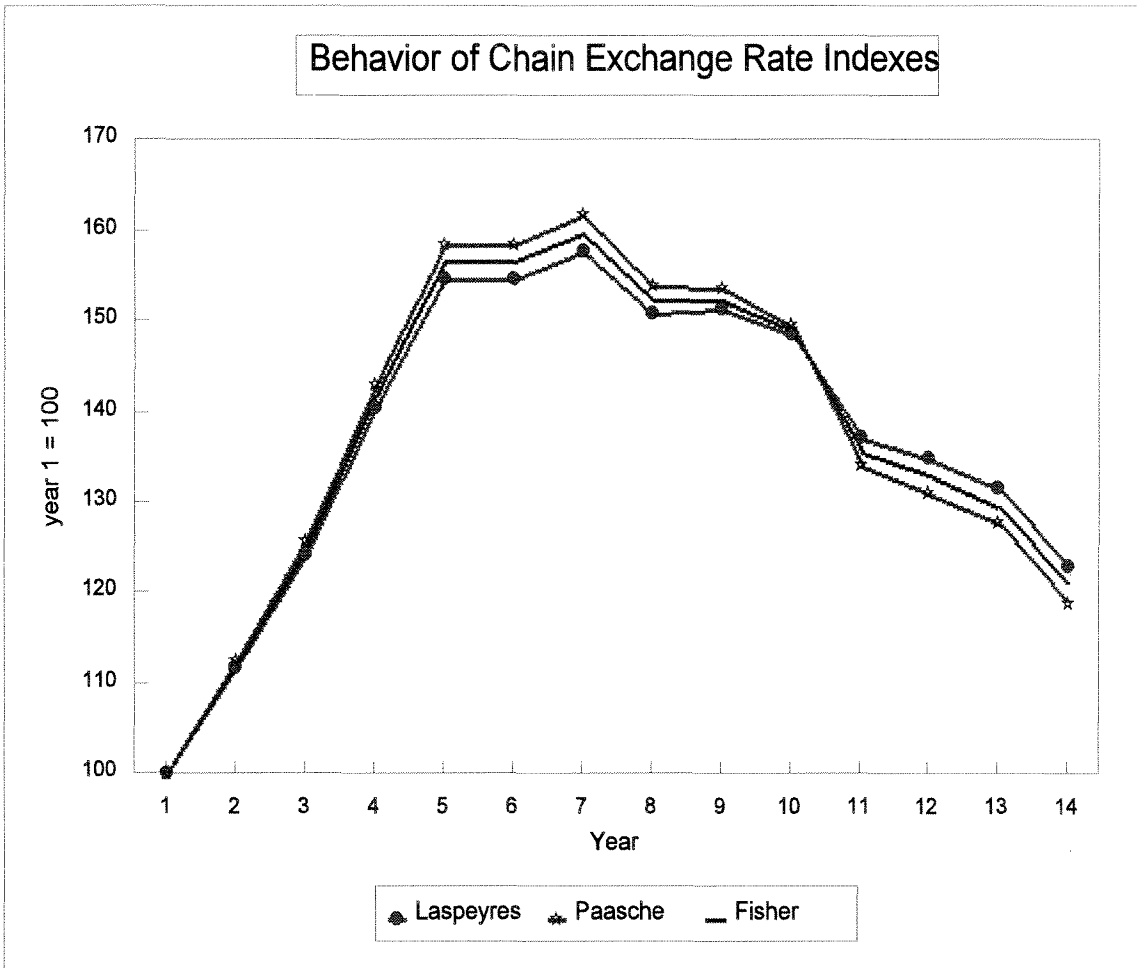
Table 8

Orthogonal Least Squares Regression Coefficients Using Fisher Chain

Atlanta Indexes		Dallas Indexes	
Using Levels	Using Log Levels	Using Levels	Using Log Levels
0.97*	0.98	1.10*	1.05*

*Indicates a rejection that the coefficient equals 1.00 at the 0.05 significance level.

Figure 1



Behavior of Chain Exchange Rate Indexes
170
160
150
140
130
120
110
100
year 1 = 100
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Year
Laspeyres
Paasche
Fisher

Figure 2a

ATLANTA: Chain and Laspeyres (1980=100)

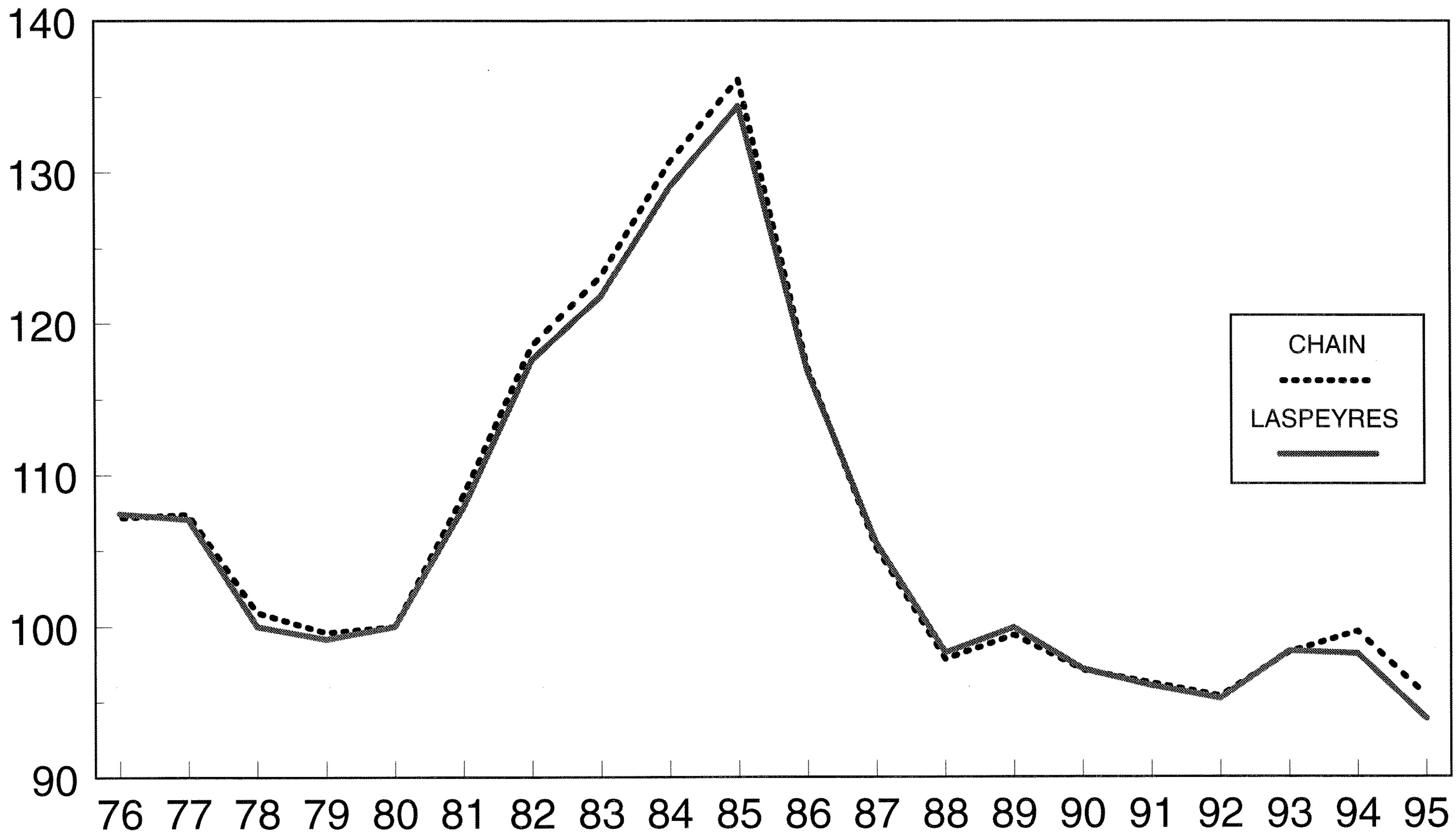


Figure 2b

Dallas: Chain and Paasche (1985=100)