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International Evidence**

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# Duration Dependence in Monetary Policy: International Evidence <sup>\*</sup>

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## Abstract

We study the duration of monetary regimes in a simple neo-classical Phillips curve model. The model is an extension of Owyang (2001) and Owyang and Ramey (2001). In this paper, we consider the role of the duration of inflationary regimes on the average inflation rate in an international cross-section. We find that inflationary regimes in certain countries are duration dependent but anti-inflationary regimes are not. In addition, we find that countries with high central banker turnover switch from inflationary to anti-inflationary with lower probability.

## 1 Introduction

Previous literature on the differences between central banks across countries has identified that a central bank's independence from influence by other branches of the government can affect its conduct and performance (Cukierman (1992),

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etc.). Essentially, countries with lower mean levels of inflation tend to have more independent central banks. The intuition is simple: central bankers with more freedom from political influence have a higher relative preference for fighting inflation than for raising output and lowering unemployment. Perhaps they have a longer decision horizon and believe that output and unemployment has a cyclical effect. Maybe they are wary of building inflationary expectations into the economy. Regardless of the monetary policymaker's motives, removing the political influence from the central bank's decision-making seems to reduce its degree of inflationary accommodation.

This paper builds on Owyang (2001) and Owyang and Ramey (2001) using a model in which the policymaker is faced with an inflation-unemployment trade-off in the form of an expectations-augmented neoclassical Phillips curve model. The Phillips curve is subject to two shocks, a permanent shift governed by a Markov chain and a white noise shock. A switching process also governs the policymaker's relative disutility for inflation versus unemployment. Unlike Owyang (2001) and Owyang and Ramey (2001), however, the two processes are not necessarily independent.

Romer and Romer (1990) and Shapiro (1994) find increases in U.S. inflation are driven by exogenous economic events such as wars or oil crises. However, they find decreases in inflation are a result of contractionary monetary policy. Obviously, the policymaker must have, at some point, accommodated the adverse economic shock, leading to an increase in inflation. What, then, causes him to implement a contractionary policy? A number of possibilities exist. In the previous papers, the policymaker's preferences are driven by an exogenous, stochastic process (Owyang and Ramey (2001) and Sims (1999)). An alternative is that the policymaker's preferences could be tied to the timing of the economic shock and the duration of high inflation.

What influences the central bank's decision making? Cukierman (1992) considers the effect of central bank independence on the level of inflation. He argues that the more independent central banks are more forward looking and therefore are able to sustain a lower level of inflation and resist the temptation to lower unemployment in the short run.

This paper develops a model of monetary policy in which the central bank's disinflation decision is tied to the duration of the high inflation episode. The policymaker's preferences are governed by a switching process in which the transition probabilities are an increasing function of the duration of the high inflation episode, i.e., the longer the preferences reside in an inflation tolerant, the more likely it is that the policymaker will switch to the anti-inflationary regime. In a subsequent section, we consider the case that the transition kernel is dependent on the magnitude of the inflation episode. The model is then estimated using Gibbs sampling, a Markov-chain Monte Carlo technique. The estimation will produce, among other parameters and the posterior densities for the states, a measure of the policymaker's desired inflation-unemployment trade-off, as well as its dependence on the duration of the current inflationary episode.

Estimation of this model will answer the following four questions. Are countries' disinflation decisions dependent on the duration of the inflationary episodes? Do countries with high mean levels of inflation or dependent central banks experience longer inflationary episodes? Do these same countries possess policymakers that have a preference for higher inflation relative to unemployment? Are low inflation episodes also duration dependent?

This investigation proceeds as follows: Section 2 outlines a model of duration dependent Markov-switching policymaker preferences and reviews the time-series phenomena accompanying the addition of the switching technology.

Section 3 reviews the econometric method, Gibbs sampling, and discusses its specific application to this model. Section 4 reviews the results for four countries and compares the findings to Cukierman’s notion of central bank independence. Section 5 concludes and proposes extensions.

## 2 A Duration Dependent Model

Consider the following model of the economy, in which the policymaker is faced with a trade-off between inflation and unemployment in the form of an expectations-augmented Phillips curve, defined by

$$u_t = k(\pi_t^e - \pi_t) + \eta_t + \varepsilon_{1t}, \quad (1)$$

where  $\pi_t$  and  $u_t$  are the t-period inflation and unemployment rates, respectively.  $\pi_t^e$  is the agents’ expectation of inflation.  $\eta_t$  is a persistent shock to the natural rate that follows a Markov process with non-absorbing state  $S_t$ , values  $\vec{h} = (h_1, h_2)$ , and transition matrix  $T^\eta$ . The policymaker is assumed to observe both the timing and magnitude of  $\eta_t$ .  $\varepsilon_{1t}$  is a white noise shock.

The policymaker’s current period loss function is defined in the following manner:

$$L_t(u_t, \pi_t) = \alpha_t (u_t - u^*)^2 + (\pi_t - \pi^*)^2, \quad (2)$$

where  $\pi_t$  and  $u_t$  are the period  $t$  inflation and unemployment rates, respectively, and  $\pi^*$  and  $u^*$  are inflation and unemployment objectives.<sup>1</sup> The parameter  $\alpha_t$  determines the policymaker’s trade-off between inflation and unemployment.

For this application, assume that the policymaker is myopic and minimizes

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<sup>1</sup>While it is possible to estimate the inflation and unemployment targets, we will assume that the policymaker dislikes all forms of inflation and unemployment and set them equal to zero. In principal, this implies the existence of an underlying constant in the estimates of the state values of  $\eta_t$  and  $\alpha_t$ .

only the expectations of the current period loss.<sup>2</sup> The policymaker’s problem, then, is to minimize the expected value of (2) conditional on the policymaker’s information set and the Phillips curve, (1).

We assume that  $\alpha_t$  is governed by a process with states  $Z_t$  in which the timing of the switches is dependent on the duration of the episodes. Specifically, the longer the policymaker resides in a regime, the higher the probability that the policymaker will switch out of that regime. This does not imply that a policymaker faced with an enduring high inflation has increasing disutility of inflation each period. For a given state of the world  $\eta_t$  and for any given  $\alpha_t$ , the policymaker’s inflation target remains invariant; however, the likelihood that the policymaker switches to a lower—a less inflation tolerant preference regime—increases over time.

The duration dependent process governing the policymaker’s inflation preferences has values  $\vec{a} = (a_1, a_2)$  and path-dependent transition probability

$$T_i(\tilde{y}_{t-1}, \tilde{S}_{t-1}, \tilde{Z}_{t-1}) = \Pr[\alpha_t = a_i | \tilde{y}_{t-1}, \tilde{S}_{t-1}, \tilde{Z}_{t-1}]. \quad (3)$$

$\vec{a}$  are the values for the possible states for the variable and reflect the degrees of monetary tightness.  $\tilde{S}_t = (S_1, S_2, \dots, S_t)$  and  $\tilde{Z}_t = (Z_1, Z_2, \dots, Z_t)$  indicate the realized states of  $\eta_t$  and  $\alpha_t$ , respectively. A lower  $\alpha_t$  indicates a tighter policymaker. In a previous paper (Owyang and Ramey, 2001), the transition probabilities are assumed to be constant over time. Here, the transition probabilities are dependent on the number of periods in the current inflationary episode. Following Durland and McCurdy (1994) and Filardo and Gordon (1993), the transition probability for the policymaker’s preference can be defined as a function of a latent state variable  $Z_t^* = g(\tilde{y}_{t-1}, \tilde{S}_{t-1}, \tilde{Z}_{t-1})$ , where  $Z_t^* > 0 \implies \alpha_t = a_2$ . Specifically, define the latent state variable, in the

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<sup>2</sup> Estimates of the degree of forward-looking by the policymaker indicate that the policymaker maximizes over a short horizon (Boivin and Giannoni, 2001).

following manner

$$Z_t^* = \lambda Z_{t-1} + v_t, \quad (4)$$

where

$$\lambda = \left[ \lambda_1 + \psi_1 d_1 f_1(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1}), \lambda_2 + \psi_2 d_2 f_2(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1}) \right]. \quad (5)$$

The path-dependent transition probability then can be defined as

$$\Pr[\alpha_t = a_2 | \tilde{y}_{t-1}, \tilde{S}_{t-1}, Z_{t-1}] = \Pr[Z_t^* > 0 | \tilde{y}_{t-1}, \tilde{S}_{t-1}, Z_{t-1}]. \quad (6)$$

Increasing  $\lambda_i$  increases the unconditional likelihood that  $\alpha_t = a_i$ . Each  $d_i$  is an parameter summarizing different specifications. Essentially, (5) allows for four possible alternate models.  $d_i = 1$  implies that the state  $\alpha_t = a_i$  is duration dependent. Each function  $f_i(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$  maps the history of the economy into the latent state variable  $Z_t^*$ . The parameter  $\psi_{i1}$  is simply a weight on  $f_i(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$ . This formulation allows the possibility that one of the regimes not be time or duration dependent, i.e., the dove's transition probabilities might be invariant to path while the hawk's transition probabilities are duration dependent. Thus, we substitute for the function  $f_i(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$  a measure of the time since the last switch

$$f_i(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1}) = N_i(Z_{t-1}), \quad (7)$$

where  $N_i(Z_{t-1})$  counts the number of periods that the economy has remained in the  $a_i$  state.

The policymaker sets an inflation target  $\bar{\pi}_t$  each period, but the realized

inflation rate is subject to white noise,  $\varepsilon_{3t}$ . The realized rate of inflation can then be written

$$\pi_t = \bar{\pi}_t + \varepsilon_{3t}, \quad (8)$$

where  $\varepsilon_{3t} \sim N(0, \sigma_{\varepsilon_3}^2)$ .

Given the Phillips curve, (1), and the policymaker's loss function, (3), the policymaker's current-period, expected loss-minimizing inflation decision is

$$\bar{\pi}_t = \chi_t(k\pi_t^e + \eta_t) \quad (9)$$

and the realized inflation rate is

$$\pi_t = \chi_t(k\pi_t^e + \eta_t) + \varepsilon_{3t} \quad (10)$$

where  $\chi_t = \frac{k\alpha_t}{1 + k^2\alpha_t}$ . The inflation dynamics are obvious. First, a tighter policymaker will set a lower inflation target. Second, a switch in the underlying state of the economy affects the policymaker's estimate of the economy. For a given policymaker type, as the economy worsens (i.e., the natural rate rises), the policymaker will set a higher inflation target.

The policymaker's inflation target depends on the agents' inflation expectations. Agents form expectations according to

$$\pi_t^e = \gamma_1\pi_{t-1} + \gamma_2\pi_{t-2} + \varepsilon_{2t}, \quad (11)$$

where  $\varepsilon_{2t} \sim N(0, \sigma_{\varepsilon_2}^2)$ .<sup>3</sup> It is assumed that the policymaker can observe the agents' expectations but not the method by which they are determined.

The implication of adding the time varying transition probabilities to the model is that the switch between regimes in the policymaker's preferences can

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<sup>3</sup>It is possible to modify (11) to allow for forward-looking expectations.



respond to the conditions in the economy.

## 3 Results

### 3.1 U.S. Results

Employing the estimation technique outlined in the appendix, estimates of the parameters governing the model (1), (4), (5), (6), (11), and (10) were obtained. These results are compared to the results of Owyang and Ramey (2001) in which a similar structural model is employed with constant transition probabilities for the policymaker's relative tolerances for inflation versus unemployment. Table 1 displays the resulting parameter estimates.

The values  $a_1$  and  $a_2$  implied by the estimates of  $k$ ,  $x_1$ , and  $x_2$  are  $a_1 = 118.15$  and  $a_2 = 330.71$ . The larger values of  $a_1$  and  $a_2$  relative to those of Owyang and Ramey (2001) are due to the lower estimate of  $k$ .<sup>4</sup> This indicates that the accommodative policymaker is willing to set an inflation target three times higher than the inflation hawk. The parameters  $h_1$  and  $h_2$  are the steady state unemployment rates implied by the model.<sup>5</sup> The adverse shock tends to increase the expected unemployment rate about 2.3%. The sample period shows increases in excess of 2.3%, explained by the model as temporary shocks. Some increases are large but they are not persistent. The Markov process governing the Phillips curve explains the persistent change in unemployment caused by, for example, the long-lasting effects of an oil shock. The transitory effects are attributed to  $\varepsilon_t$  shocks.

Note also that the estimated value of  $k$  is smaller for the DD specification than for a NDD specification. There are two implications of this finding. First, the effect of a monetary surprise on the unemployment rate is smaller. Second,

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<sup>4</sup> Maybe it's necessary to calibrate  $k$ ?

<sup>5</sup> Compare the results with Koenig (2001) and Gordon (1997).

the implied cost of disinflation is smaller. Since policy has almost no effect on the unemployment rate, the policymaker can lower the inflation rate, even if agents are unwilling or unable to anticipate the reduction, without significantly affecting employment and output.

One important result of the DD specification is that the dove regime exhibits duration dependence while the hawk regime does not. Recall that, for a given unemployment shock, the dove regime will be associated with high inflation targets than the hawk regime. This result implies that U.S. policymakers dislike extended periods of high inflation. In Section 2, it is assumed that the policymaker optimizes myopically, taking into account only the current period's losses. The duration dependence allows the policymaker's inflation target to exhibit dynamic properties, i.e., to have a modicum of path dependence. This duration dependence does not, however, extend to the hawk regime. The probability of switching out of the hawk regime does not increase the longer the policymaker stays in that state. This is as expected; the policymaker incurs some losses while residing for long periods in a high inflation equilibrium but does not incur the same losses for residing for long periods in a low inflation equilibrium.

It is important to note that  $\psi_2$  does not pass the test for significance from zero. However, the estimated value for  $d_2$  reveals the presence of duration dependence. One possible explanation for the relative insignificance of the estimated  $\psi_2$  is that the function  $f_2$  is misspecified.

Figure 1 compares the estimated posterior probabilities for the states of  $\eta_t = h_2$ , the natural rate of unemployment shock, for both the DD model and the NDD model. The duration dependent model finds more switches in the natural rate because of the increasing probability of the policymaker switching to a less inflation-tolerant state. Note that the estimated values for the two

structural states shows higher steady state unemployment for the DD model. This might indicate that three or more structural states are necessary to identify all of the persistent states in the economy. The DD model does identify the 1974:1 oil shock more readily than the model without duration dependence. However, it does not capture the earlier oil dates.

Figure 2 shows the estimated posterior probability of the policymaker's preference regime in the accommodative state,  $\alpha_t = a_2$ . The estimated accommodative periods for the duration dependent model seem to coincide with that of standard model with the exception that the duration dependent model identifies a switch around 1990. In the previous model, the high inflation rate is attributed to a bad structural variable. Without the presence of duration dependence, the model is unable to identify short-lived switches in the preference process because the switching probabilities are constant (and close to zero). In the DD model, it is policy that creates the higher inflation, allowing the switch to a lower natural rate over the same period.

### 3.2 International Results

The results for the U.S. indicate that there is a tendency for the policymaker to increase his likelihood of switching from the accommodative regime to the inflation hawk regime the longer the accommodative regime endures. This tendency reflects the Federal Reserve's innate desire to keep inflation low. But do other countries share this desire? This section reports the results for the comparison of the international estimation for the model from Section 2. Of particular interest are the estimates of the parameters governing the policymaker's preferences ( $\vec{a}$ ), the parameter governing the existence of duration dependence in each state ( $d_i$ ), and the degree of duration dependence ( $\psi_i$ ). Table 2 compares these parameters for six countries.

A cursory comparison of the results of Table 2 reveal a few interesting features based on the limited sample of countries.  $\sigma_\pi$  seems to be inversely correlated with the tenure of the central bank chair. Additionally  $\sigma_\pi$  is related to the magnitudes and, especially, the ratio of the policymaker preference states. That is, the greater the difference between the accommodative regime and the inflation hawk regime for a given country, the higher is that country's  $\sigma_\pi$ . Consequently, there seems to be a relationship between tenure and the divergence amongst the policymaker's preferences. Whether this occurs because the shorter tenures tend to produce more distinct preferences as new policymakers enter office or because a single policymaker must resort to more extreme policy in order to be reappointed is unknown and could be the subject of further investigation.

Consider Figures 3 – 5, the posterior probabilities for the Markov variable for the Phillips curve. Each country experiences a high unemployment episode. Only Japan does not exhibit a recovery following the shift occurring in the early 1970's. Canada appears to recover from the 1974 shock but does not recover by the end of the data period. The shorter dataset available for the U.K. begins in a period with a high natural rate, but recovers by the late 1980's.

Figure 6 shows the estimated process for the structural variable for each of the countries. Notice that each country for which data was available experiences a bad shock around the beginning of 1974, when a significant oil shock struck the world economy. Likewise, both the U.S. and the U.K. appear to recover from recessions during the late 80's. It is reassuring that the model is able to capture significant events that impact the world economy by estimating the model independently for each country. Had the model failed to identify the major, world-wide oil shock, one might suspect that the model is only able to identify idiosyncratic effects for each country. This would be contrary to how one might typically think of a shift in regime of the economic variable—a large

magnitude, persistent change in the structure of the economy produced by a significant event.

Each country also experiences at least one major change in policymaker preferences over the data period, shown individually in Figures 7 – 9. For the most part, each country appears to maintain a low inflation stance and experience switches to persistent accommodative regimes. Even Japan, which experiences the longest accommodative regime, has two possible switches—one in the mid 1960's and again in the early 1970's, observable as small dips in 8—that the model might pick up if the number of possible states were increased.

Now consider Figure 10, the probabilities for all four countries. Specifically notice that some changes in regime seem to occur simultaneously for multiple countries. For example, the upward shifts of the late 1980's in the U.S. appears to occur in the U.K. as well. This verifies the global economic dependence of the these four countries. Perhaps this also is a verification of the U.S.'s influence in global economic policy.

## **4 The Role of Central Bank Independence**

Central Bank Independence (CBI) is measured by the central bank's ability to maintain the long term objective of price stability, even at the cost of other short term goals, such as unemployment. The formal degree of CBI, independence conferred on the bank by law, is not always the same as the actual degree of CBI. The turnover rate, how often a new chief executive officer is appointed, for example, is not necessarily the same as the legal term of office. Cukierman (1992) points out that while Argentina's legal term is four years, the actual average number of years in office was less than one during the 1980's. It is a widely believed that the turnover rate of the CEO of a central bank is negatively related to its independence. This is because a CEO term longer than that in the executive

branch allows the central bank to be more forward-looking in its policymaking, thereby enabling it to fight off inflation.

Cukierman (1992) developed a proxy of legal CBI. The legal variables he uses are divided into four groups: (1) variables regarding the appointment, dismissal, and term of office of the bank CEO, (2) variables regarding the resolution of conflicts between the executive branch and the central bank, as well as the role of the central bank in the formulation of monetary policy and the budgetary process, (3) the charter's statement of the central bank's final objectives, and (4) legal restrictions on the ability of the public sector to borrow from the central bank. Cukierman first calculates the degree of CBI within each group of characteristics and then aggregates them to achieve a single index of legal CBI in each country. He calls this measure LVAU, an aggregate unweighted measure of legal CBI. Table 2 lists these measures for Canada, Japan, the U.S., and the U.K. The correlation that exists between  $\psi_2$  and LVAU is that a value of LVAU closer to zero indicates a higher degree of CBI. This is not true in the case of Japan, which may be because Japan's legal degree of CBI is much smaller than it actually is in the real world.

There appears to be a correlation between the legal measure of CBI and the rate at which the policymaker's accommodative retention probability decays. Higher legal independence seems to be associated, in this limited sample, with the shorter-lived accommodative regimes. The U.S., for example, has the highest level of legal CBI (at 0.51), and the U.K., which has a slightly lower level of legal CBI (at 0.31), each spent only 23 percent of the sample accommodating inflation. This compares to Japan and Canada, which both have a legal CBI measure of 0.16, spending 29 percent and 49 percent of the sample in an inflation-tolerant state, respectively. The U.S. and U.K. experience relatively short inflation accommodative regimes compared to Canada and Japan, as seen in Figure 10.

This results in a lower average inflation rate over the sample period. These short regimes may be a reflection of the U.S. and U.K.'s relatively independent central banks.

Lastly, CBI, proxied by turnover, seems to be correlated with the cost of disinflation through  $k$ . Recall that  $k$  determines the magnitude of the effect of the policy surprise on unemployment. In cases of high turnover, the value of  $k$  is small. Perhaps the high turnover makes the economy more resilient to shifts in policymaker objectives. For economies that have long tenured central bank officers, the relative cost of changing the policymaker objectives is higher. Certainly, the country's central bank officers would have a significant effect on the country's economic well-being.

## 5 Conclusion

This paper reformulates the model used in Owyang and Ramey (2001) to measure monetary policy. We also shows that the model can be extended to other countries to measure policy in cases in which the central bank notes are unavailable or untrustworthy. The model also verifies, using independent estimation over a few countries, that the economic shocks to each countries Phillips curve represent global shocks incident on a number of countries at once.

We also investigates the duration dependence characteristics of the U.S. and other countries' inflation and unemployment time series. A number of papers have analyzed duration dependence in the business cycle. This paper focuses on the whether the policymaker's preference exhibit the same kind of duration dependence. Estimation of the model for the four countries finds evidence that a specification that supports a duration dependent inflation dove but not a duration dependent inflation hawk might best fit the data.

Certainly the parameters of this model can be estimated for more countries.

Of particular interest are countries that have significantly different structures to their central banks or adopt dissimilar central bank policy to the OECD countries surveyed here. Specifically, Latin American and other developing countries which have less stable and more dependent central banks might exhibit different duration dependence characteristics, as policymakers may have alternate long term objectives. It is reasonable to conjecture that the less independent the central bank, the less duration dependence the policymaker's preference would exhibit.



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## 7 Appendix

### 7.1 Gibbs Sampling

This appendix provides details of the estimation procedure. Define state vectors  $S_t$  and  $Z_t$  such that  $\eta_t = \vec{h} S_t$  and  $\alpha_t = \vec{a} Z_t$ . Let period  $t$  data be denoted by

$$y_t = \begin{bmatrix} \pi_t \\ u_t \end{bmatrix}.$$

$\tilde{S}_T$  defines the vector of states  $\tilde{S}_T = (S_1, S_2, \dots, S_T)$ , where  $T$  indicates the length of the sample, and  $\tilde{Z}_T$ ,  $\tilde{\pi}_T^e$  and  $\tilde{y}_T$  are defined similarly. The vector of model parameters is given by  $\tilde{\varphi} = (a_1, a_2, h_1, h_2, \sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2, \sigma_{\varepsilon_3}^3, k, \gamma_1, \gamma_2)$ .

It is convenient to rewrite the model (1), (4), (5), (6), (11), and (10) in a

state space representation. Redefine  $y_t$  as

$$y_t = \begin{bmatrix} u_t + k\pi_t - \eta_t \\ \pi_t - \alpha_t\eta_t \end{bmatrix}.$$

The model may be rewritten as follows:

$$y_t = H_t\pi_t^e + e_t,$$

$$\pi_t^e = F\pi_{t-1}^e + \mu_t + \varepsilon_{2t},$$

where

$$H_t = \begin{bmatrix} k \\ k\alpha_t \end{bmatrix}, \quad e_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{3t} \end{bmatrix},$$

$F = 0$  and  $\mu_t = \gamma_1\pi_{t-1} + \gamma_2\pi_{t-2}$ .

The objective of the Gibbs sampler is to characterize the joint density  $p(\tilde{S}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi}|\tilde{y}_T)$ , using the ergotic distribution of a Markov simulation of the following conditional joint densities that are generated iteratively:

$$p(\tilde{\pi}_T^e|\tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\varphi}),$$

$$p(\tilde{Z}_T|\tilde{y}_T, \tilde{S}_T, \tilde{\pi}_T^e, \tilde{\varphi}),$$

$$p(\tilde{S}_T|\tilde{y}_T, \tilde{\pi}_T^e, \tilde{Z}_T, \tilde{\varphi}),$$

$$p(\tilde{\varphi}|\tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\pi}_T^e).$$

Samples from these densities are drawn at each step and use to generate the other densities, constituting a Markov chain. After an appropriate number of iterations, the ergotic distribution of this chain of conditional densities is the joint density  $p(\tilde{S}_T, \tilde{Z}_T, \tilde{\pi}_t^e, \tilde{\varphi}|\tilde{y}_T)$ .

In order to incorporate the feedback, assume that the switching process

governing the policymaker's preferences is dependent on the duration of the inflationary episode. Specifically, define the latent state variable as

$$Z_t^* = \lambda Z_{t-1} + v_t,$$

where  $\lambda = [\lambda_1 + \psi_1 d_1 f_1(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1}), \lambda_2 + \psi_2 d_2 f_2(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})]$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $\psi_1$ , and  $\psi_2$  are parameters and  $v_t \sim N(0, 1)$ .  $d_1$  that  $d_2$  and are indicator variables that determine whether the transition probabilities are indeed duration dependent.  $f_1(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$  and  $f_2(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$  are functions that indicate the state of the economy. The transition probabilities are then defined by

$$\Pr[\alpha_t = a_2 | \tilde{y}_{t-1}, \tilde{S}_{t-1}, Z_{t-1}] = \Pr[Z_t^* > 0 | \tilde{y}_{t-1}, \tilde{S}_{t-1}, Z_{t-1}].$$

The algorithm is an application similar to that of Filardo and Gordon (1993) and Kim and Nelson (1998).

## 7.2 *Conditional Density of Inflation Expectations*

The conditional density  $p(\tilde{\pi}_T^e | \tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\varphi})$  can be obtained by applying a Kalman filter modified for the presence of two Markov processes that govern  $\eta_t$  and  $\alpha_t$ . The Kalman filter produces the densities  $p(\pi_t^e | \tilde{y}_t, \tilde{S}_t, \tilde{Z}_t, \tilde{\varphi})$ . Given some initial conditions  $\pi_{t-1|t-1}^e$  and  $V_{t-1|t-1}$ , the filter generates, for all  $t$ :

$$\pi_{t|t-1}^e = F \pi_{t-1|t-1}^e + \mu_t,$$

$$V_{t|t-1} = F V_{t-1|t-1} F' + \sigma_{\varepsilon_2}^2,$$

$$\pi_{t|t}^e = \pi_{t|t-1}^e + (H_t V_{t|t-1} H_t' + R)^{-1} V_{t|t-1} H_t' (y_t - H_t \pi_{t|t-1}^e),$$

$$V_{t|t} = V_{t|t-1} (I - (H_t V_{t|t-1} H_t' + R)^{-1} V_{t|t-1} H_t' H_t),$$

where

$$R = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & 0 \\ 0 & \sigma_{\varepsilon_3}^2 \end{bmatrix}.$$

Then rewrite  $p(\tilde{\pi}_T^e | \tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\varphi})$  as

$$p(\tilde{\pi}_T^e | \tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\varphi}) = p(\pi_T^e | \tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\varphi}) \prod_{t=1}^{T-1} p(\pi_t^e | \tilde{y}_t, \tilde{S}_t, \tilde{Z}_t, \tilde{\varphi}, \pi_{t+1}^e).$$

The final iteration of the Kalman filter provides the first term. Elements of the second term are determined by the results of the Kalman filter and the following recursive conditional densities:

$$\pi_{t|t, \pi_{t+1}^e} = \pi_{t|t}^e + V_{t|t} F (\pi_{t+1}^e - F \pi_{t|t}^e - \mu_t) (F V_{t|t} F' + \sigma_{\varepsilon_2}^2)^{-1},$$

$$V_{t|t, \pi_{t+1}^e} = V_{t|t} - (F V_{t|t} F' + Q)^{-1} V_{t|t} F V_{t|t},$$

where  $F$  and  $\mu_t$  are given above,  $Q = \sigma_{\varepsilon_2}^2$ ,  $\pi_{t|t}^e = p(\pi_t^e | \tilde{y}_t, \tilde{S}_t, \tilde{Z}_t, \tilde{\varphi})$ , and  $V_{t|t}$  is the conditional variance as determined by the Kalman filter. Given the model (1), (11) and (10), the filtering algorithm simplifies to

$$\pi_{t|t-1}^e = \mu_t,$$

$$V_{t|t-1} = \sigma_{\varepsilon_2}^2,$$

$$\pi_{t|t}^e = \pi_{t|t-1}^e + (H_t V_{t|t-1} H_t' + R)^{-1} V_{t|t-1} H_t' (y_t - H_t \pi_{t|t-1}^e),$$

$$V_{t|t} = V_{t|t-1} (I - (H_t V_{t|t-1} H_t' + R)^{-1} V_{t|t-1} H_t' H_t),$$

$$\pi_{t|t, \pi_{t+1}^e} = \pi_{t|t}^e,$$

$$V_{t|t, \pi_{t+1}^e} = V_{t|t}.$$

### 7.3 Conditional Densities of Structural States

Recall that, conditional on  $\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e$  and  $\tilde{\varphi}$ , (1) is linear in  $\tilde{S}_T$ . Given  $p(S_0|\tilde{y}_0)$ , a prior probability for the initial state, the Hamilton (1989) filter generates the conditional density  $p(S_T|\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi})$ . Then, following Carter and Kohn (1994) and Kim and Nelson (1998), the density  $p(\tilde{S}_T|\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi})$  is obtained from

$$p(\tilde{S}_T|\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi}) = p(S_T|\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi}) \prod_{t=1}^{T-1} p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi}, Z_{t+1}). \quad (12)$$

Each density  $p(S_t|\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi}, S_{t+1})$  is generated from a filtering algorithm and Bayes' Law:

$$\begin{aligned} p(S_t|\tilde{y}_T, \tilde{Z}_T, \tilde{\pi}_T^e, \tilde{\varphi}, S_{t+1}) &= \frac{p(S_{t+1}|\tilde{y}_t, S_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}{p(S_{t+1}|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})} \\ &= \frac{p(S_{t+1}|S_t)p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}{\sum_{Z_t} p(S_{t+1}|\tilde{y}_t, S_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})} \\ &= \frac{p(S_{t+1}|S_t)p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}{\sum_{Z_t} p(S_{t+1}|S_t)p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}, \end{aligned}$$

where  $p(S_{t+1}|S_t)$  is the transition probability and the filter determines the density  $p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})$ . The first inequality is simply an application of Bayes' Law. The final two inequalities arise from the Markov property of  $S_t$ : in determining the density for  $S_{t+1}$ , the only relevant information in the available set is the previous state  $S_t$ . The numerator in (12) is calculated from the Hamilton filter as

$$p(S_t|\tilde{y}_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi}) = \frac{f(y_t|\tilde{y}_{t-1}, S_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})p(S_t|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}{f(y_t|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}$$



$$= \frac{f(y_t|\tilde{y}_{t-1}, S_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})p(S_t|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})}{\sum_{Z_t} f(y_t|\tilde{y}_{t-1}, S_t, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})p(S_t|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})},$$

where

$$p(S_t|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi}) = \sum_{Z_{t-1}} p(S_t|S_{t-1})p(S_{t-1}|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi}).$$

The density  $p(S_{t-1}|\tilde{y}_{t-1}, \tilde{Z}_t, \tilde{\pi}_t^e, \tilde{\varphi})$  is taken from the previous iteration.

#### 7.4 Conditional Density of the Parameter Vector.

The conditional densities for the elements of the parameter vector are generated by employing Bayesian OLS. The Bayesian posterior distribution for each element of  $\tilde{\varphi}$ , conditional on all other elements of  $\tilde{\varphi}$ , can be determined given a prior distribution. If model parameters, excluding the variances, have prior distributions of the form  $p_0(\varphi_i) \sim N(a_i, A_i\sigma^2)$ , then their corresponding posterior conditional distributions are given by

$$p(\varphi_i|\varphi_{-i}, \tilde{y}_T, \tilde{S}_T, \tilde{Z}_T, \tilde{\pi}_T^e) \sim N(a_i^*, A_i^*\sigma^2), \quad (13)$$

where  $a_i^* = (A_i^{-1} + X_i'X_i)^{-1}(A_i^{-1}a_i + X_i'Y_i)$ ,  $A_i^* = (A_i^{-1} + X_i'X_i)^{-1}$ ,  $X_i$  is the appropriate regressor, and  $Y_i$  is a forecast error. For example, consider estimating the values of the vector  $\vec{a}$  in (1). In this case, define  $X_\alpha = B\hat{Z}_T$  and  $Y_\alpha = \tilde{\pi}_T$ , where the row  $t$  on-diagonal element of  $B$  is  $\vec{\eta}'T^\eta S_{t-1} + k\pi_t^e$  and the off-diagonal elements are zero,  $\hat{Z}_T = [\tilde{z}_{1T}, \tilde{z}_{2T}]$ , and  $\tilde{z}_{iT}$  is a  $T \times 1$  vector with representative element  $z_{it} = 1$  iff  $Z_i = i$ . Other elements of the parameter vector can be estimated similarly. The values for the vector  $\vec{h}$  can be generated

from a posterior normal similar to (13) in which

$$X_\eta = \begin{bmatrix} \widehat{S}_T \\ \vec{a} \widetilde{Z}_T \widehat{S}_T \end{bmatrix}, \quad Y_\eta = \begin{bmatrix} \widetilde{u}_T - k(\widetilde{\pi}_T^e - \widetilde{\pi}_T) \\ \widetilde{\pi}_T - k \vec{a} \widetilde{Z}_T \widetilde{\pi}_T^e \end{bmatrix},$$

where  $\widehat{S}_T$  is defined similarly to  $\widehat{Z}_T$ .

Given conditional priors for the variances of the form  $\sigma_{\varepsilon_1}^2 \sim IG(\frac{\rho_0}{2}, \frac{R_0}{2})$  and  $\sigma_{\varepsilon_i}^2 \sim IG(\frac{\lambda_0}{2}, \frac{L_0}{2})$ ,  $i = 2, 3$ , the posterior conditional probabilities are given by

$$\sigma_{\varepsilon_1}^2 | \widetilde{y}_T, \widetilde{\pi}_T^e, \widetilde{S}_T, \widetilde{Z}_T, \widetilde{\varphi} \sim IG\left(\frac{\rho_0 + T}{2}, \frac{R_0 + \delta_1}{2}\right),$$

where

$$\delta_1 = (\widetilde{u}_T - k(\widetilde{\pi}_T^e - \widetilde{\pi}_T) - \vec{h} \widetilde{S}_T)' (\widetilde{u}_T - k(\widetilde{\pi}_T^e - \widetilde{\pi}_T) - \vec{h} \widetilde{S}_T);$$

and, for  $i = 2, 3$ :

$$\sigma_{\varepsilon_i}^2 | \widetilde{y}_T, \widetilde{\pi}_T^e, \widetilde{S}_T, \widetilde{Z}_T, \widetilde{\varphi} \sim IG\left(\frac{\lambda_0 + T}{2}, \frac{L_0 + \delta_i}{2}\right),$$

where

$$\delta_i = (\widetilde{\pi}_t - \vec{a} \widetilde{Z}_t (\vec{h} \widetilde{S}_t + k \widetilde{\pi}_t^e))' (\widetilde{\pi}_t - \vec{a} \widetilde{Z}_t (\vec{h} \widetilde{S}_t + k \widetilde{\pi}_t^e)).$$

## 7.5 Transition Probabilities

Given a prior probability distribution for  $T_{ii}^a$  of the form  $T_{ii}^a \sim \beta(u_{ii}, u_{ji})$  for  $j \neq i$ , the distribution for the transition probabilities  $T_{ii}^a$  is determined by

$$T_{ii}^a = \Pr[\alpha_t = a_i | \alpha_{t-1} = a_i] \sim \beta(u_{ii} + n_{ii}, u_{ji} + n_{ji}),$$

where  $n_{ii}$  is the number of periods that  $\alpha_t$  remained in state  $i$ , and  $n_{ji}$  is the number of periods that  $\alpha_t$  switched to state  $j \neq i$  after beginning in state  $i$ . Then the other elements of  $T^\alpha$  can be determined by  $T_{ji}^\alpha = 1 - T_{ii}^\alpha$ . A similar procedure is used to generate the elements of  $T^\eta$ .

$\lambda_1$  and  $\lambda_2$ . The posterior conditional density from which the parameters  $\lambda_1$  and  $\lambda_2$  can be generated is defined as follows. Given the prior density

$$\lambda^* = (\lambda_1, \lambda_2) \sim N(l_0, \Lambda_0)$$

the posterior density is given by

$$\lambda^* | Z_T^*, \psi_1, \psi_2, d_1, d_2 = 1, \tilde{S}_T, \tilde{Z}_T, \tilde{y}_T, \tilde{\pi}_T^e, \tilde{\phi} \sim N(l, \Lambda)$$

where

$$l = (\Lambda_0^{-1} + X'X)^{-1}(\Lambda_0^{-1}l_0 + X'Y)$$

and

$$\Lambda = (\Lambda_0^{-1} + X'X)^{-1}$$

In this case, the  $i$ -th elements of the  $T$ -row vectors  $X$  and  $Y$  are defined as

$$X = Z_{t-1}$$

and

$$Y = Z_t^* - \varphi Z_{t-1}^*$$

where  $Z_t^*$  was computed in the previous iteration and  $\varphi = (\psi_1 d_1 f_1, \psi_2 d_2 f_2)$ .

$\psi_i$  conditional on  $d_i = 1$ . The posterior conditional density is  $p(\psi_i | Z_T^*, \psi_{-i}, d_{-i}, d_i = 1, \tilde{S}_T, \tilde{Z}_T, \tilde{y}_T, \tilde{\pi}_T^e, \tilde{\phi})$

$$\psi_i | Z_T^*, \psi_{-i}, d_{-i}, d_i = 1, \tilde{S}_T, \tilde{Z}_T, \tilde{y}_T, \tilde{\pi}_T^e, \tilde{\phi} \sim N(\bar{w}_i, \bar{W}_i)$$

where

$$\bar{w}_i = (W_{0,i}^{-1} + X'X)^{-1}(W_{0,i}^{-1}w_{0,i} + X'Y),$$

$$\bar{W}_i = (W_{0,i}^{-1} + X'X)^{-1},$$

and  $w_{0,i}$  and  $W_{0,i}$  are parameters take from the prior distribution

$$\psi_i \sim N(w_0, W_0).$$

The  $i$ -th elements of the T-row vectors  $X$  and  $Y$  are defined by

$$X = \rho Z_{t-1}$$

and

$$Y = Z_t^* - \varphi Z_{t-1}^*$$

In this case,  $\varphi = (\varphi_1, \varphi_2)$ , where

$$\varphi_{-i} = \lambda_{-i} + \psi_{-i} d_{-i} f_{-i}(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$$

and

$$\varphi_i = \lambda_i$$

and  $\rho = (\rho_1, \rho_2)$ , where

$$\rho_i = f_i(\tilde{\pi}_{t-1}, \tilde{u}_{t-1}, \tilde{Z}_{t-1})$$

and

$$\rho_{-i} = 0.$$

## 7.6 *Generating the $d_i$ 's.*

Recall that the posterior probability of each  $d_i$  represents the posterior probability of time varying transition probabilities or duration dependence for the state  $i$ . Given that the prior probability of time varying transition probabilities in state is  $p_i$ , Bayes Law gives the posterior probability as

$$p(d_i | Z_T^*, \psi_i, \psi_{-i}, d_{-i}, \tilde{S}_T, \tilde{Z}_T, \tilde{y}_T, \tilde{\pi}_T^e, \tilde{\phi}) = \frac{p_i}{p_i + \theta_i(1 - p_i)},$$

where

$$\theta_i = 2 \exp \left\{ \frac{\bar{w}_i}{2\bar{W}_i} - \frac{w_{0,i}}{2W_{0,i}} \right\} \sqrt{\frac{\bar{W}_i}{W_{0,i}}} \left[ 1 - \Phi \left( -\frac{\bar{w}_i}{\sqrt{\bar{W}_i}} \right) \right].$$

## 7.7 *Generating the posterior probabilities for $\tilde{Z}_T$ .*

In this model, the transition matrix that governs the policymaker's preferences is dependent on the history  $\tilde{Z}_{-t} = (Z_1, Z_2, \dots, Z_{t-1}, Z_{t+1}, \dots, Z_T)$ . Since each state is generated conditional on its history, each  $Z_t$  must be generated individually. Suppressing the conditioning on  $\tilde{S}_T$ ,  $\tilde{\phi}$ , and the parameters in (12), the posterior probability for  $Z_t$  conditional on the set of observables  $\tilde{y}_T = (y_1, y_2, \dots, y_T)$  and  $\tilde{Z}_{-t} = (Z_1, Z_2, \dots, Z_{t-1}, Z_{t+1}, \dots, Z_T)$  can be written as

$$\begin{aligned}
\Pr[Z_t|\tilde{y}_T, \tilde{Z}_{-t}] &= \Pr[Z_t|\tilde{y}_t, \tilde{Z}_{-t}] \\
&= \frac{\Pr[Z_t, y_t, Z_{t+1}, \dots, Z_T|\tilde{y}_{t-1}, \tilde{Z}_{t-1}]}{\Pr[y_t, Z_{t+1}, \dots, Z_T|\tilde{y}_{t-1}, \tilde{Z}_{t-1}]} \\
&= \frac{\Pr[Z_{t+1}|Z_t, \tilde{y}_{t-1}] \Pr[y_t, Z_{t+1}, \dots, Z_T|Z_t, \tilde{y}_{t-1}, \tilde{Z}_{t-1}]}{\Pr[y_t, Z_{t+1}, \dots, Z_T|\tilde{y}_{t-1}, \tilde{Z}_{t-1}]} \\
&= \frac{\Pr[Z_{t+1}|Z_t] \Pr[y_t, Z_{t+1}, \dots, Z_T|Z_t, \tilde{y}_{t-1}, \tilde{Z}_{t-1}]}{\Pr[y_t, Z_{t+1}, \dots, Z_T|\tilde{y}_{t-1}, \tilde{Z}_{t-1}]} \\
&= \frac{\Pr[Z_{t+1}|Z_t] \Pr[y_t|Z_t, \tilde{y}_{t-1}] \Pr[Z_t|Z_{t-1}] \Pr[Z_{t+2}, \dots, Z_T|\tilde{y}_t, \tilde{Z}_{t+1}]}{\Pr[y_t, Z_{t+1}, \dots, Z_T|\tilde{y}_{t-1}, \tilde{Z}_{t-1}]}
\end{aligned}$$

which simplifies to

$$\Pr[Z_t|\tilde{y}_T, \tilde{Z}_{-t}, \tilde{S}_T, \tilde{\phi}] = \frac{\Pr[Z_{t+1}|Z_t] \Pr[y_t|Z_t, \tilde{y}_{t-1}]}{\sum_{Z_t} \Pr[Z_{t+1}|Z_t] \Pr[y_t|Z_t, \tilde{y}_{t-1}] \Pr[Z_t|Z_{t-1}]}$$

where  $\Pr[Z_{t+1}|Z_t]$  and  $\Pr[Z_t|Z_{t-1}]$  are transition probabilities generated from  $Z_t^*$  and  $\Pr[Z_t|Z_{t-1}]$  is the conditional forecast error. The first equality arises from the fact that data past period  $t$  is non-informative. The fourth equality comes from the Markov properties of  $Z_t$ .

**Table 1:**  
**Estimated Parameters For Duration Dependence**  
(U.S. Time Series<sup>1</sup>)

Parameter	Est. Value <sup>2</sup>	Parameter	Est. Value
$x_1$	0.589 (0.017)	$h_1$	4.81 (0.078)
$x_2$	1.64 (0.043)	$h_2$	7.14 (0.106)
$\gamma_1$	0.992 (0.018)	$\gamma_2$	0.006 (0.015)
$\lambda_1$	-1.99 (0.20)	$\lambda_2$	1.78 (0.55)
$d_1$	0.514 (0.500)	$d_2$	0.904 (0.295)
$\psi_1$	0.001 (0.001)	$\psi_2$	-0.022 (0.0152)
$\sigma_{v1}^2$	6.99 (0.632)	$k$	0.005 (0.0046)
$\sigma_{v2}^2$	0.244 (0.285)	$\sigma_\varepsilon^2$	0.993 (0.078)
$\Pr[\eta_t = h_1   \eta_{t-1} = h_1]$	0.989 (0.00745)	$\Pr[\eta_t = h_2   \eta_{t-1} = h_2]$	0.989 (0.00745)

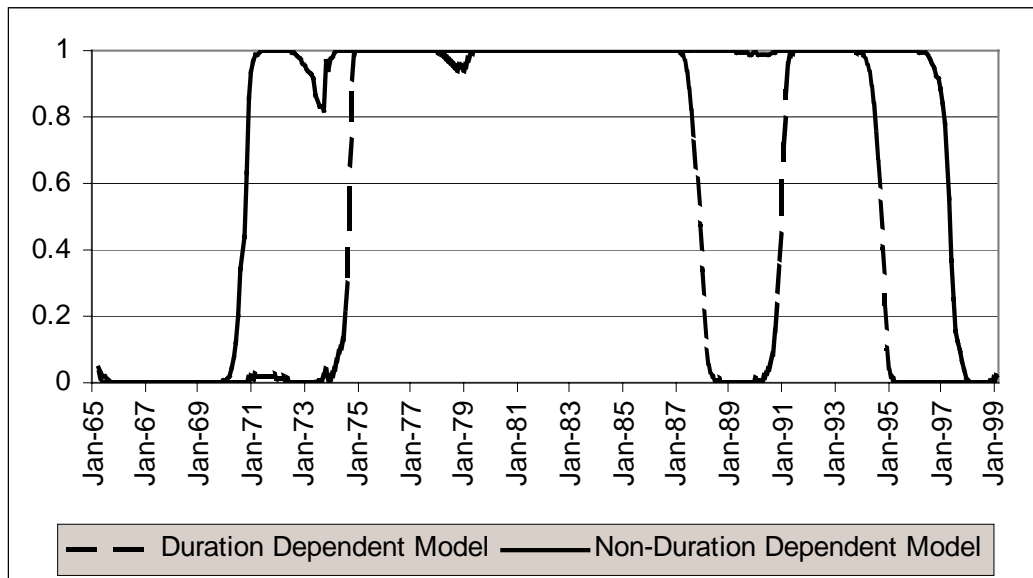
Notes: 1 Data taken from Citibase.  
2 Standard deviations across simulations are given in parentheses.

**Table 2:**  
**Parameter Estimates for International Duration Dependence**

<b>Parameter</b>	<b>U.S.</b>	<b>U.K.</b>	<b>Japan</b>	<b>Canada</b>
Period	1965:3-1999:2	1982:1-1996:9	1960:1-1996:6	1960:1-1996:8
$x_1$	0.589 (0.017)	0.248 (0.102)	0.786 (0.217)	0.397 (0.111)
$x_2$	1.643 (0.043)	0.726 (1.102)	4.190 (1.589)	1.105 (0.528)
$k$	0.005 (0.005)	1.927 (1.208)	0.081 (0.133)	0.588 (1.078)
$\lambda_1$	-1.99 (0.20)	-2.07 (0.654)	-2.31 (0.251)	-2.42 (0.246)
$\lambda_2$	1.78 (0.55)	2.19 (0.250)	2.81 (0.435)	4.61 (2.02)
$d_1$	0.514 (0.500)	0.491 (0.500)	0.562 (0.496)	0.466 (0.499)
$\psi_1$	0.0011 (0.0010)	0.002 (0.002)	0.0006 (0.0006)	0.0006 (0.0006)
$d_2$	0.904 (0.295)	0.910 (0.286)	0.949 (0.220)	0.981 (0.137)
$\psi_2$	-0.022 (0.0152)	-0.031 (0.022)	-0.004 (0.003)	-0.023 (0.0165)
$mean \pi^1$	5.07	4.94	4.93	4.94
$\sigma_\pi$	3.90	2.42	4.43	3.33
$LVAU^2$	0.51	0.31	0.16	0.46
Turnover <sup>3</sup>	0.13	0.10	0.20	0.10

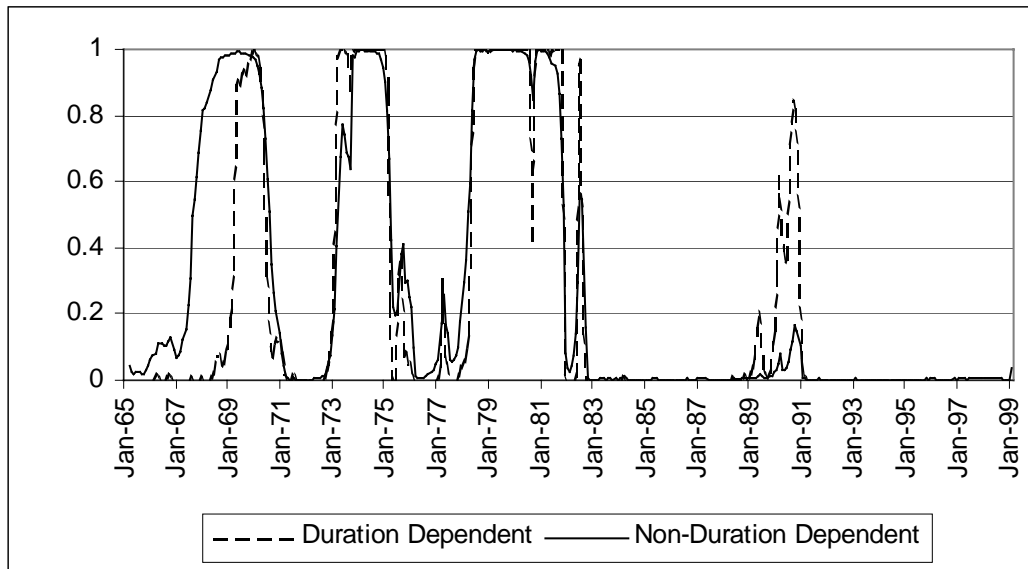
Notes: 1 Taken as the mean inflation rate for the sample period indicated.  
2 Taken from Cukierman (1992). An unweighted, aggregate measure based on legal Central Bank Independence.  
3 Taken from Cukierman (1992). Measures the turnover of the Central Bank CEO.





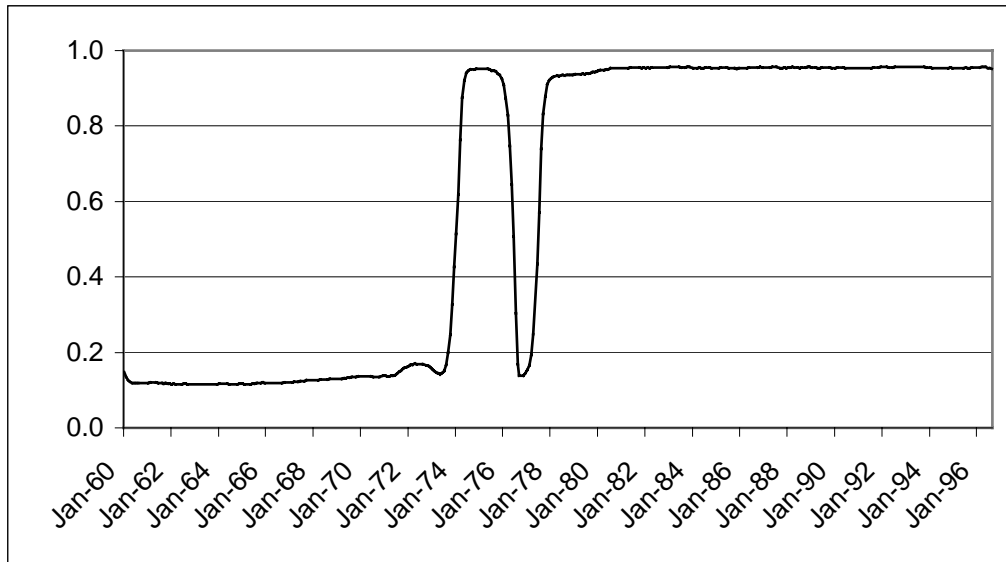
**Figure 1:**

**Posterior Phillips Curve State Probabilities—United States**



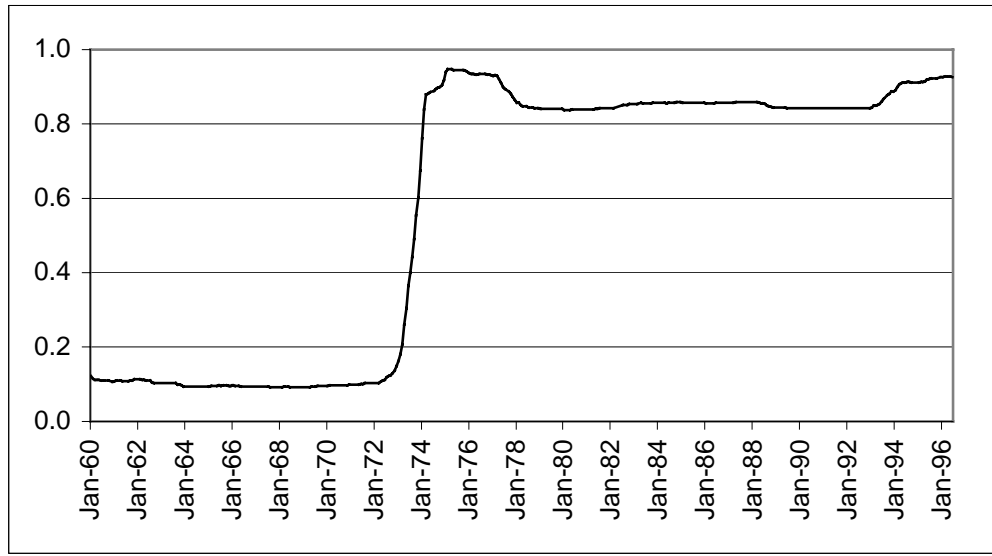
**Figure 2;**

**Posterior Policymaker Preference State Probabilities—United States**



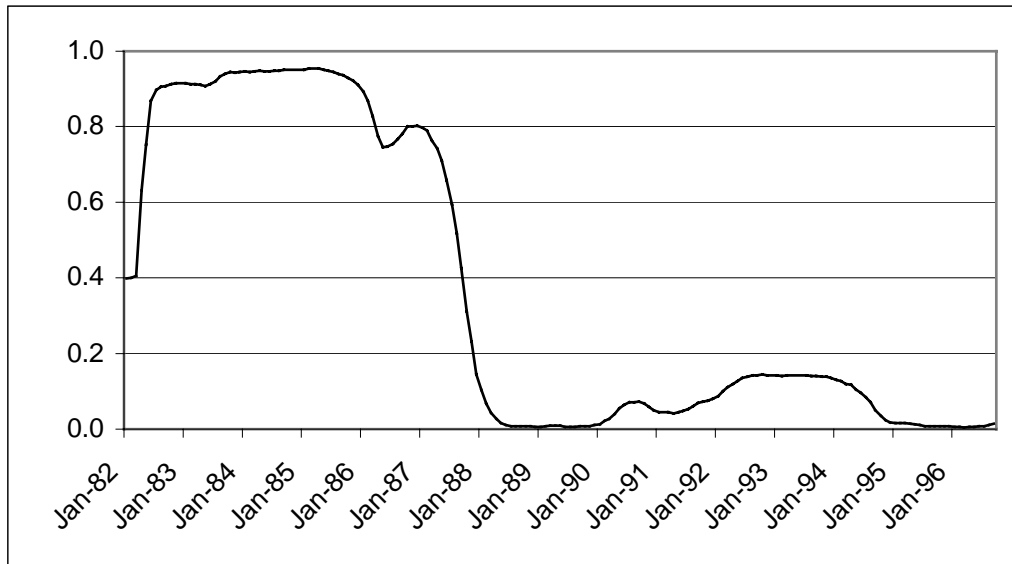
**Figure 3:**

**Posterior Probability for  $\eta_t = h_2$  : Canada**



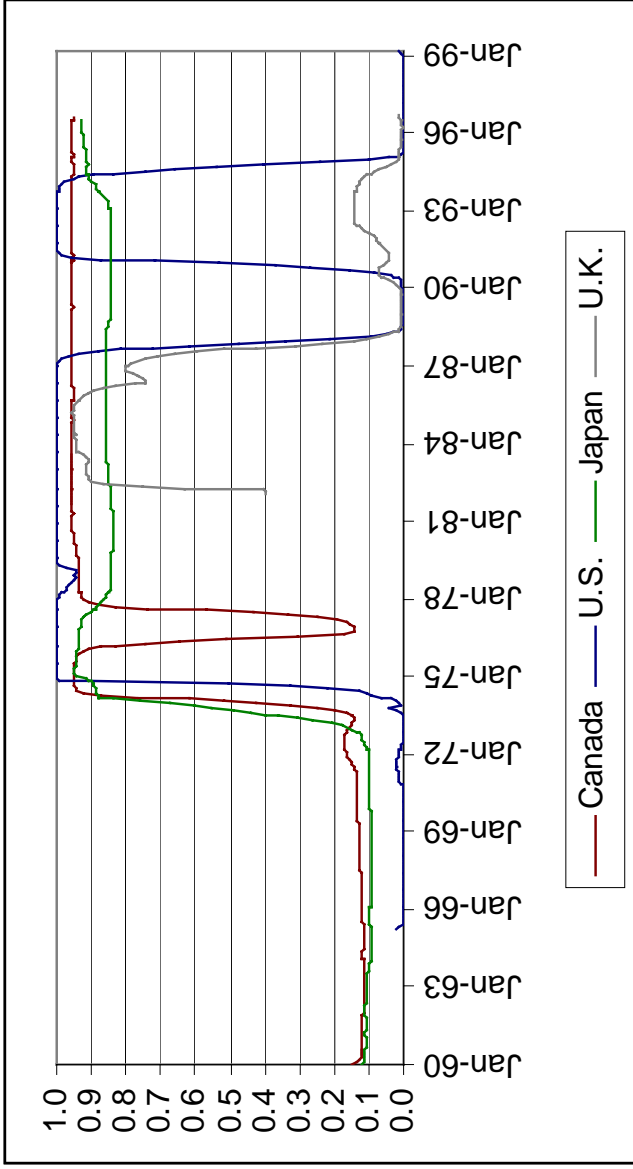
**Figure 4:**

**Posterior Probability for  $\eta_t = h_2$  : Japan**

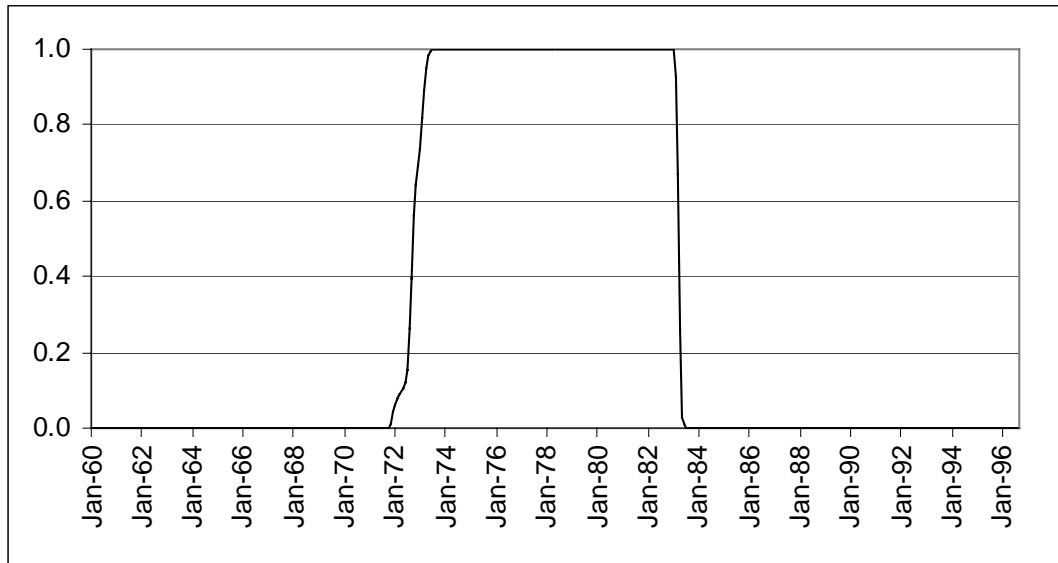


**Figure 5:**

**Posterior Probability for  $\eta_t = h_2$  : United Kingdom**

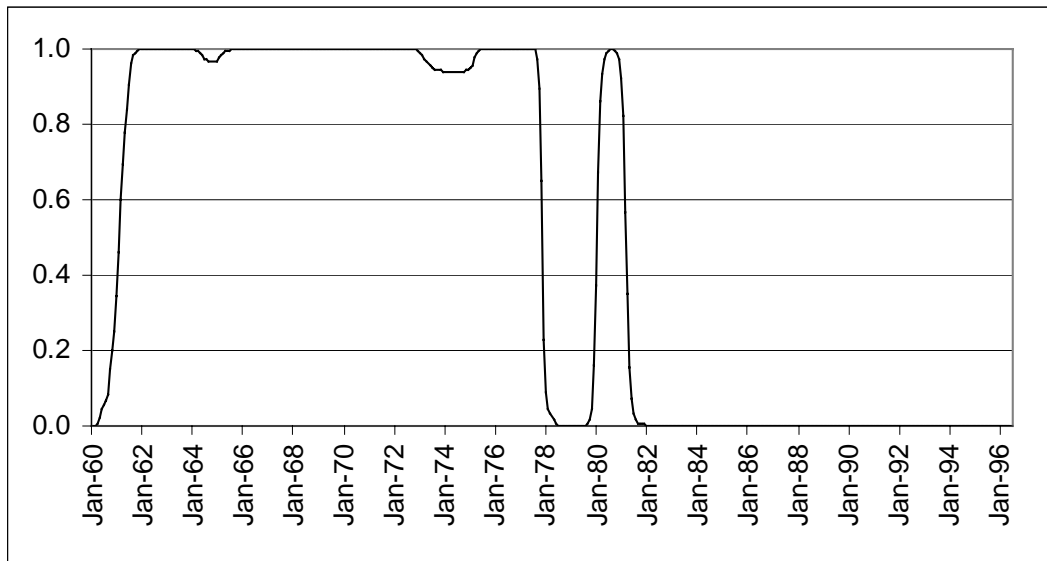


**Figure 6:**  
**Posterior Probability for  $\eta_t = h_2$  : All Countries**



**Figure 7:**

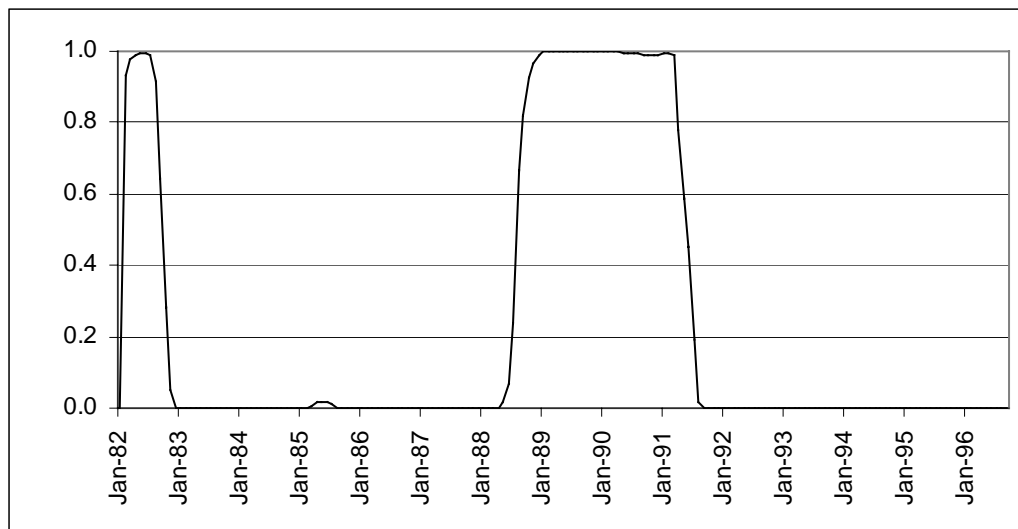
**Posterior Probability for  $\alpha_t = a_2$ : Canada**



**Figure 8:**

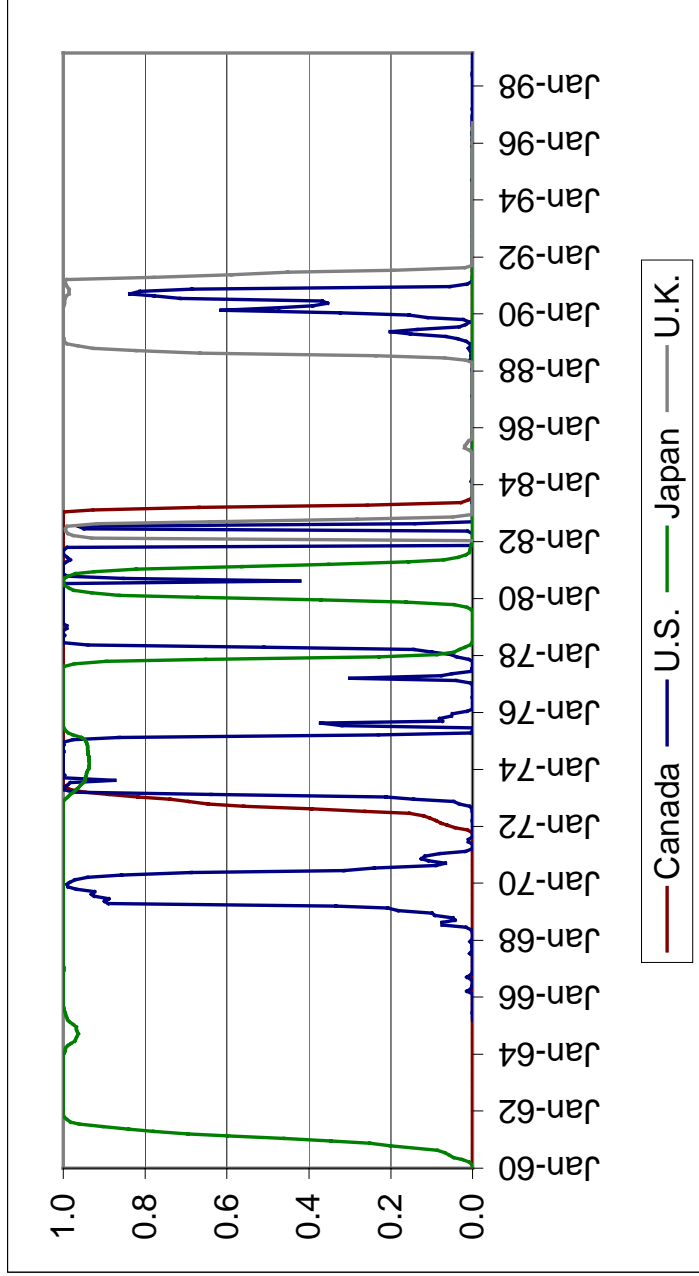
**Posterior Probability for  $\alpha_t = a_2$  : Japan**





**Figure 9:**

**Posterior Probability for  $\alpha_t = a_2$ : United Kingdom**



**Figure 10:**

**Posterior Probability for  $\alpha_i = a_2$ ; All Countries**