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Sticky-Price Models and the Natural Rate Hypothesis

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Abstract

A major criticism of standard specifications of price adjustment in models for monetary policy analysis is that they violate the natural rate hypothesis by allowing output to differ from potential in steady state. In this paper we estimate a dynamic optimizing business cycle model whose price-setting behavior satisfies the natural rate hypothesis. The price-adjustment specifications we consider are the sticky-information specification of Mankiw and Reis (2002) and the indexed contracts of Christiano, Eichenbaum, and Evans (2005). Our empirical estimates of the real side of the economy are similar whichever price adjustment specification is chosen. Consequently, the alternative model specifications deliver similar estimates of the U.S. output gap series, but the empirical behavior of the gap series differs substantially from standard gap estimates.

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1 Introduction

This paper uses estimated, optimizing sticky-price models of the business cycle to explore two issues regarding the "natural rate" concept. The first of these issues is the *natural rate hypothesis.* In a series of papers, McCallum (1982, 1994) has argued that satisfaction of the natural rate hypothesis is a criterion that models used for monetary policy should meet. This hypothesis states that, on average, output should be equal to the level of natural (or "potential") output, regardless of which monetary policy regime is in effect. Our paper will discuss different ways in which the natural rate hypothesis can be imposed on an optimizing model, and will compare the properties of the resulting estimated models.

The second issue that this paper examines is the *behavior of the natural levels of output and interest* in estimated dynamic optimizing models — a topic emphasized in McCallum (2001). Discussions of the natural rate hypothesis must take a stand on the definition of the natural level of output, and McCallum noted that in defining this concept the macroeconomic literature has used "three fundamentally different ones: trend, NAIRU and flexible-price" output. McCallum asks, "Which of the concepts is most appropriate theoretically? From the perspective of dynamic, optimizing analysis, the answer is the flexible-price concept (i.e., the output level that would prevail in the absence of nominal price stickiness)" (2001, p. 261). This was the original natural-rate concept advanced by Friedman (1968), and advocates of optimizing models for monetary policy concur that the flexible-price level of output is indeed the appropriate definition of natural or potential output: see e.g. the discussions in the monographs of Walsh (2003, pp. 241–242) and Woodford (2003, pp. 8, 247–250, 616).[1] The corresponding definition of the natural real rate of interest underpins Woodford's (2003) "neo-Wicksellian approach" to price-level determination.

Our paper proceeds as follows. Section 2 briefly reviews the natural rate hypothesis and explores alternative proposals for imposing the natural-rate restriction on an optimizing model. Section 3 lays out the dynamic, stochastic general equilibrium model that we use in this paper, starting with the common demand side and then detailing the alternative price-adjustment schemes that we consider. Section 4 presents our parameter estimates, and Section 5 examines the behavior of the natural rate of interest and the natural level of output implied by the model estimates. Section 6 concludes, while an Appendix contains detailed derivations and an outline of our computation of natural rates.

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[1] Applications of this concept to quantitative optimizing models include McCallum and Nelson (1999), Rotemberg and Woodford (1999), Edge (2003), Galí, López-Salido, and Vallés (2003), Giammarioli and Valla (2003), Neiss and Nelson (2003), Smets and Wouters (2003), Amato and Laubach (2004), and ourselves (2004). The case of distortions (i.e., inefficient variations in potential output) is discussed in Woodford (2003, Ch. 6).

2   The natural rate hypothesis

The natural rate hypothesis (NRH) states that, on average, and regardless of monetary policy regime, output (in logs, y_t) should be equal to potential output (in logs, y_t^*). That is, the NRH states that the mean of the output gap is zero:

$$E[y_t - y_t^*] = 0, \tag{1}$$

where $E[\bullet]$ denotes the unconditional expectations operator. As discussed in the introduction, the appropriate definition of potential output is the value of output that would prevail under flexible prices. Real business cycle models automatically satisfy the natural rate hypothesis because of their assumption of fully flexible prices—implying that $y_t = y_t^*$ every period, not just in expectation. It follows that the principal element of a dynamic general equilibrium model that will determine whether the natural rate hypothesis is satisfied is the specification of price adjustment.[2] A well-known example is standard Calvo (1983) price setting, as represented by the New Keynesian Phillips curve:

$$\pi_t = b_0 + b_1 E_t \pi_{t+1} + \alpha(y_t - y_t^*), \tag{2}$$

where π_t is quarterly inflation, and the theory implies the restrictions $b_1 < 1$, $\alpha > 0$. Baseline Calvo price setting without indexation implies a zero constant term ($b_0 = 0$), and will not generally satisfy the NRH. In particular, for a positive steady-state inflation rate π^*, the New Keynesian Phillips curve implies $E[y_t - y_t^*] = (1/\alpha)[-b_0 + \pi^*(1 - b_1)] = (1/\alpha)[\pi^*(1 - b_1)]$, i.e. the output gap is nonzero (positive) in steady-state and an increasing function of the inflation rate. McCallum argues that this result is theoretically unappealing because it implies that monetary policy can be used to enrich agents in real terms permanently (relative to an economy where prices are always flexible). This is the basis for what Mankiw and Reis (2002) call the "McCallum critique" of Calvo price setting.[3]

Modifications of this baseline in order to conform to the NRH include the version of Calvo price setting employed in Yun (1996) and Svensson (2003), whereby all contracts are indexed to the steady-state inflation rate. Rotemberg and Woodford (1999, p. 72) also express support for this modification. This variant of Calvo price setting creates a relationship between the constant term b_0 and the parameters b_1 and π^*, that ensures $E[y_t - y_t^*] = 0$. Specifically it implies $b_0 = \pi^*(1 - b_1)$, so that

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[2]Nominal wage stickiness is another form of nominal rigidity that can produce variations in output relative to potential. In this paper, we will limit the discussion to sticky-price models. Also, note that the exposition in this section treats the output gap as the forcing process in the New Keynesian Phillips curve; more generally, log real marginal cost is the appropriate forcing process (and is used in our model below), and is approximately collinear with the output gap under price stickiness (see e.g. Galí, Gertler, and López-Salido, 2001, for a discussion).

[3]Mankiw and Reis cite McCallum (1998), which is closely based on the discussion in McCallum and Nelson (1999).

the inflation terms appear as deviations from the mean inflation rate:

$$(\pi_t - \pi^*) = b_1(E_t\pi_{t+1} - \pi^*) + \alpha(y_t - y_t^*). \tag{3}$$

This price-setting rule corresponds to the case labeled the "static indexing scheme" in Eichenbaum and Fisher (2004), and used in their empirical work. As formulated above, it assumes that the steady-state inflation rate is constant. If—as in U.S. data over the past 20 to 25 years—the unconditional mean of the inflation rate instead appears to decline as the sample period is extended, we can generalize equation (3) to allow for a time-varying inflation target π_t^*:

$$(\pi_t - \pi_t^*) = b_1(E_t\pi_{t+1} - E_t\pi_{t+1}^*) + \alpha(y_t - y_t^*), \tag{4}$$

where π_t^* might exhibit discrete breaks in mean in specified periods, or might instead follow a time trend. We will operationalize the concept of a time-varying inflation target in our empirical work in Section 5.

A disadvantage of formulation (3) or (4) is that the long-run independence of the output gap of the inflation target π_t^* is ensured only via a restriction on the intercept term. The natural-rate property therefore has no effect, for given π_t^*, on the behavior of the log-linearized economy describing fluctuations around the steady state. Consequently, the natural-rate restriction is not permitted to have any bearing on medium-run dynamics. For example, the impulse responses of the output gap and other variables, to highly persistent but not permanent shocks, are identical regardless of whether the Phillips curve takes the form (2), (3) or (4). This goes against the tradition of regarding the natural rate hypothesis as implying slope restrictions on the Phillips curve, and so affecting medium-run dynamics (e.g. Sargent, 1971).

A re-specification of the Phillips curve that is also based on indexation of Calvo contracts, but which allows the natural-rate restriction to affect medium-run dynamics, is the following:

$$(\pi_t - \delta\pi_{t-1} - (1-\delta)\pi_t^*) = b_1(E_t\pi_{t+1} - \delta\pi_t - (1-\delta)E_t\pi_{t+1}^*) + \alpha(y_t - y_t^*), \tag{5}$$

According to this formulation, price-setters that are not permitted to reoptimize in period t instead have their prices automatically changed by a percentage equal to a linear combination of π_{t-1} and π_t^*. In the case $\delta = 1$, as in Christiano, Eichenbaum, and Evans (CEE) (2005), all the indexation is to lagged inflation ("dynamic indexation" in the terminology of Eichenbaum and Fisher, 2004). The Phillips curve may then be written entirely as an expectational difference equation for the change in inflation — a forward-looking version of the "accelerationist" Phillips curve.[4] Indexation to lagged inflation thus introduces into Phillips curve dynamics the "long-run vertical" property associated with the NRH, a fact stressed by Smets and Wouters

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[4]The intermediate case $\delta \in [0, 1]$ has been considered in Giannoni and Woodford (2005), although the point estimate they obtain is $\delta = 1.0$.

(2004). We will reexamine this specification below, with system estimates that allow for variation in π_t^*.

Outside the Calvo framework, a specification of nominal rigidity that imposes the natural rate hypothesis is the proposed price-adjustment scheme of Mankiw and Reis (2002), which they label "sticky information." This specification delivers a form of price adjustment similar to Fischer (1977) contracts. As in the Fischer-contract setup, quantities and prices in the current period correspond to (linear combinations of) prior periods' expectations of the quantities and prices that would prevail under a flexible-price, full-information equilibrium. This specification thus satisfies $E[y_t - y_t^*] = 0$ regardless of the steady-state inflation rate, and indeed regardless of whether the steady-state inflation rate exists. It thus goes beyond the Calvo indexation schemes by ensuring that the NRH is satisfied globally, which is McCallum's (1994) criterion for whether a model satisfies the NRH. In our empirical work, we will consider two variants of Mankiw and Reis' price-adjustment scheme.

We summarize the different implications of alternative price-setting specifications for the natural rate hypothesis in Table 1. The requirement that output be equal to potential on average for a given steady-state inflation rate is how the natural rate hypothesis has been interpreted by Rotemberg and Woodford (1999) and Svensson (2003), and is the weak form of the NRH. This weak version of the NRH can be imposed via a restriction on the constant term, in which case it does not affect the dynamic responses of variables to temporary shocks, or it can be imposed via slope restrictions as in the CEE specification, whereupon medium-term dynamics are affected. McCallum (1994) and McCallum and Nelson (1999) argue for a stricter definition of the NRH, where output equals potential on average for any monetary policy, including policy regimes that deliver nonstationary inflation behavior. We classify this definition as "NRH satisfied globally" in the table. This strict definition is not satisfied by the specifications of price adjustment that augment Calvo price-setting with indexation, but is satisfied by the sticky-information model.

The two basic alternative price-adjustment specifications that we consider that satisfy the natural rate hypothesis are thus Calvo with indexation and Mankiw-Reis' alternative. The choice between these specifications will affect the behavior of the model under sticky prices. But in empirical work, the choice will also affect the model's predictions about the behavior of the natural level of output and the natural rate of interest. In systems estimation, each change in the specification of price adjustment affects the likelihood-maximizing parameter vector, and so the estimates of key production and preference parameters. It is these parameters, together with estimates of the laws of motion for the real shocks, that determine the behavior of the economy under flexible prices, and so natural-output and natural interest-rate behavior. Therefore, a key exercise we carry out in this paper is to examine the sensitivity of natural-rate estimates to price-adjustment specification. Our interest in this issue distinguishes our work from earlier comparisons of the Calvo specification (with and without indexation) and sticky information, such as Keen (2004) and Tra-

bandt (2003), where the real side of the model is held at constant, calibrated values as the price-adjustment specification is varied. In our maximum-likelihood approach, by contrast, estimates of natural-rate dynamics are affected by the specification of nominal rigidity. The resulting empirical behavior of the natural level of output and natural rate of interest is a focus of our paper.

3   The model

In this section we describe the dynamic general equilibrium model that we will estimate on U.S. data. The log-linearized approximation of the model is presented here, with the underlying nonlinear model given in the Appendix. While we will estimate several variants of this model, they have a common demand side, i.e., optimality conditions describing output and portfolio demand. We describe the common elements of the model first, then lay out the various price-adjustment specifications that we will consider.

3.1   Demand Side

The demand side of the model is given by three equations. First the IS equation, relating output and real interest rates,

$$
\begin{aligned}
\widehat{y}_t &= \frac{\phi_1}{\phi_1 + \phi_2}\widehat{y}_{t-1} + \frac{\beta\phi_1 + \phi_2}{\phi_1 + \phi_2}E_t\widehat{y}_{t+1} - \frac{1}{\phi_1 + \phi_2}\left[\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right] \\
&\quad - \frac{\beta\phi_1}{\phi_1 + \phi_2}E_t\widehat{y}_{t+2} + \frac{1 - \beta h\rho_a}{(1 - \beta h)}\frac{(1 - \rho_a)}{\phi_1 + \phi_2}\widehat{a}_t,
\end{aligned} \tag{6}
$$

where β is the household discount factor; $\phi_1 = \frac{(\sigma-1)h}{1-\beta h}$, $\phi_2 = \frac{\sigma+(\sigma-1)\beta h^2-\beta h}{1-\beta h}$, with the parameters σ and h indexing risk-aversion attitudes and (internal) habit formation in consumption, respectively. Here, hatted variables are log-deviations from the model's nonstochastic steady-state, with the log-deviations of output, nominal interest rates and the (gross) inflation rate denoted by $\widehat{y}_t$, $\widehat{r}_t$, and $\widehat{\pi}_t$. The variable $\widehat{a}_t$ is an "IS" (preference) shock whose law of motion will be described below.[5] The intertemporal IS equation (6) arises from the representative household's optimality condition for consumption, where preferences are separable across real balances and consumption, and there is internal habit formation. Notice that as $h \to 0$, expression (6) collapses to the standard Euler equation for consumption under time-separable preferences.

The second element of the demand side of the model is the dynamic money demand

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[5]We will henceforth refer to these variables by the names of the series for which they are log-deviations: $\widehat{y}_t$ by "output", $\widehat{\pi}_t$ by "inflation," etc.

function, relating output, real balances $(\widehat{m}_t)$ and the nominal interest rate,

$$
\begin{aligned}
(1 + \delta_0(1+\beta))\widehat{m}_t \;=\; & \gamma_1\widehat{y}_t - \gamma_2\widehat{r}_t + [\gamma_2(r-1)(h\phi_2 - \phi_1) - h\gamma_1]\,\widehat{y}_{t-1} \\
& - [\gamma_2(r-1)\beta\phi_1]\,E_t\widehat{y}_{t+1} + [\delta_0\beta]\,E_t\widehat{m}_{t+1} + \delta_0\widehat{m}_{t-1} \\
& - (r-1)\beta h(1-\rho_a)\gamma_2\widehat{a}_t + [1 - (r-1)\gamma_2]\,\widehat{e}_t.
\end{aligned} \tag{7}
$$

where the coefficients γ_1 and γ_2 are the long-run real-income and nominal interest-rate response parameters, and δ_0 is a parameter related to the presence of portfolio adjustment costs. The money demand equation also arises from the household's optimization problem, with a money-in-the-utility-function specification (including a shock term $\widehat{e}_t$ to period utility), and the forward-looking terms in real money due to the obstacles to portfolio adjustment.[6] These adjustment costs take the functional form used by Christiano and Gust (1999) but applied to real balances, as discussed in our earlier work.[7] The presence of habits in the utility function introduces further dynamics into the money demand equation, producing terms both in prior and expected future output. In addition, both preference shocks, the IS shocks $\widehat{a}_t$ and the money demand shocks $\widehat{e}_t$, matter for money demand.

The demand side of the model is closed with an identity for real money growth:

$$
\widehat{m}_t - \widehat{m}_{t-1} = \widehat{\mu}_t - \widehat{\pi}_t \tag{8}
$$

We also have an interest-rate reaction function, a Taylor-type rule augmented by smoothing and a response to money growth (see e.g. Ireland, 2004):

$$
\widehat{r}_t = \rho_r\widehat{r}_{t-1} + (1-\rho_r)\rho_\pi\widehat{\pi}_t + (1-\rho_r)\rho_y\widehat{y}_t + (1-\rho_r)\rho_\mu\widehat{\mu}_t + \varepsilon_{r_t} \tag{9}
$$

where ε_{r_t} denotes a monetary policy shock.

3.2   Supply Side

We turn now to the supply side of the model. As noted above, we will not consider a single price-adjustment specification but instead, complete the model with either a "sticky-information" or "Calvo with indexation" specification. We consider two versions of the "sticky information" specification. In each version, price-setters reset their price each period, but they do not automatically update the information set (i.e. their expectations) upon which these price decisions are based. The first of these uses Mankiw and Reis' (2002) proposal that price-setters reset their prices on new information sets which they acquire in a staggered manner analogous to Calvo price-setting; while in the second specification, the acquisition of information is staggered in manner analogous to Taylor (1980) nominal contracts.

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[6] Under no portfolio adjustment costs, $\delta_0 = 0$, i.e. the current money demand decision no longer depends on lagged and expected future real balances.

[7] See our previous paper (ALNS, 2004) for further discussion.

3.2.1 Sticky Information, Staggered *à la* Calvo

In this setup, the representative firm sells its output in a monopolistically competitive market. Each firm deploys new information with a constant probability $(1 - \theta_{MR})$ every period; this probability therefore governs whether the firm will revise its expectations in setting a new price. The probability is independent of the time elapsed since the last acquisition of information. Thus, each period a measure $(1 - \theta_{MR})$ of producers resets prices using newly updated information, while a fraction θ_{MR} simply adjusts prices on the basis of previously used information sets. Let θ_{MR}^k be the fraction of firms that last adjusted their information set—i.e., their expectations—k periods ago. Therefore, the no-new-information firms simply base decisions on expectations calculated k periods ago, so setting the price according to

$$\widetilde{P}_t(j) = E_{t-k}P_t^*(j) \tag{10}$$

Notice that, in the absence of information constraints, the typical firm would set an optimal price according to the rule $P_t^*(j) = \mu_p MC_t(j)$, where $MC_t(j) = \frac{W_t}{\frac{\partial Y_t(j)}{\partial N_t(j)}}$ is nominal marginal cost, $\mu_p \equiv \frac{\varepsilon}{\varepsilon-1}$ is the steady-state price markup, and $\varepsilon > 1$ is the price elasticity of demand. The aggregate price level is defined as the following average,

$$P_t = \left[(1 - \theta_{MR}) \sum_{k=0}^{\infty} \theta_{MR}^k \left(\widetilde{P}_t \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{11}$$

Formally, it can be shown that the Phillips curve for this sticky-information model is given by,

$$\widehat{\pi}_t = \left(\frac{1 - \theta_{MR}}{\theta_{MR}} \right) \widehat{mc}_t + (1 - \theta_{MR}) \sum_{k=0}^{\infty} \theta_{MR}^k E_{t-k-1}\{\widehat{\pi}_t + \Delta\widehat{mc}_t\} \tag{12}$$

where $\Delta\widehat{mc}_t$ represents the change in the deviation of log of real marginal cost from steady state. Mankiw and Reis (2002) derive this representation but have the output gap as the forcing process, in effect imposing collinearity between real marginal cost and the output gap. We use the more general relationship between marginal cost and prices here, allowing the connection between the output gap and marginal cost to arise from the overall equilibrium relations in the model.[8] Notice that the right hand side variables of the Phillips curve are arranged in two portions. The first portion is the evolution of real marginal cost, while the second is related to past expectations of the changes in nominal marginal costs. The latter term reflects the cohorts of firms that were not given the opportunity to update their information set at time t. This term induces some inertia in inflation. An additional feature of this price-adjustment specification is that the elasticity of inflation with respect to real marginal cost is

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[8] In a recent paper, Keen (2004) also uses marginal cost as the forcing process in the sticky-information Phillips curve.

just a function of the sticky information parameter θ_{MR}. In particular, the higher the value of this parameter (i.e., the slower is the process of information-updating by firms in price-setting), the lower is the response of inflation to marginal cost, and so the weaker is the short-run sensitivity of inflation to aggregate demand.

Formally, the expression linking real marginal cost and output in our model is:

$$\widehat{mc}_t = (\chi + \phi_2)\,\widehat{y}_t - \phi_1\widehat{y}_{t-1} - \beta\phi_1 E_t\widehat{y}_{t+1} - \frac{\beta h(1-\rho_a)}{(1-\beta h)}\widehat{a}_t - (1+\chi)\,\widehat{z}_t \qquad (13)$$

where $\chi = \frac{\varphi+\alpha}{1-\alpha}$, $(1-\alpha)$ is the elasticity of output with respect to employment, and φ is the inverse of the Frisch labor supply elasticity. The variable $\widehat{z}_t$ represents a technology shock. This shock corresponds to an exogenous term in a decreasing-returns-to-scale, log-linear production function; we assume a production function where labor is the only endogenous input.[9]

Note from this expression that given their effect on output, IS shocks $(\widehat{a}_t)$ tend to reduce real marginal cost, provided that there is habit formation. This reflects the fact that IS shocks raise potential output when there are habits. Without habit formation, IS shocks cancel from the ratio of marginal utilities of consumption and leisure, and so do not affect the labor-supply condition under flexible prices. With habit formation, they no longer cancel from the ratio, and produce a positive labor-supply response associated with the rise in the natural real interest rate. IS shocks then have a mixed effect on real marginal cost: they stimulate output demand and so have a positive effect via the $\widehat{y}_t$ term; but they also add to aggregate supply and so have a cushioning effect on unit costs. Habit formation also introduces lagged and expected future output into the marginal-cost expression, which not only complicates the effects of IS shocks on costs, but also generates a "contract multiplier" in the model that we discuss in Section 4. Finally, note that substituting out expression (13) into expression (12) yields a Phillips curve relating inflation to lagged expectations of both inflation and output.[10]

This version of the sticky-information specification is a hybrid of Fischer (1977) and Calvo contracts. Mankiw and Reis (2002, p. 1296) motivate their proposal by the observation that "the information about macroeconomic conditions diffuses slowly through the population," and contrast this with the case where agents are forced by contracts to use only old information. As Woodford (2002) notes, Mankiw and Reis' rationalization is problematic because, in dynamic general equilibrium models of this type, a producer acquires information about its own conditions (e.g. developments in

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[9]See the Appendix for details.

[10]The Phillips curve here includes an infinite summation of lagged expectations terms. In order to make the model empirically tractable, we approximate it by truncating it at 3 quarters. Truncation of the lag length in this fashion follows Khan and Zhu (2002). It should be kept in mind that each unit increase in k adds seven extra state variables (i.e., $E_{t-k}\pi_t$, $E_{t-k}y_t$, $E_{t-k}y_{t-1}$, $E_{t-k}y_{t+1}$, $E_{t-k}y_{t-2}$, as well as a_{t-k}, and z_{t-k}). (See the Appendix for details.) We verified that the results we will present in Section 4 are robust to increasing the value of k.

marginal cost of its own product) that would trigger price changes even if it acquired no new macroeconomic information. It is probably more internally consistent simply to interpret the Mankiw-Reis specification as arising from staggered contracts.[11] The same is true of the next version of the sticky-information Phillips curve that we consider.

3.2.2 Sticky Information, Staggered *à la* Taylor

A somewhat different version of the sticky-information Phillips curve arises if the staggering rule for agents' acquisition of information follows Taylor's (1980) approach. In particular, we break with the previous specification by now assuming that the probability of not adjusting expectations is not decreasing in the number of periods since the last adjustment. Let J be the maximum number of periods a firm cannot update their expectations in setting its prices, and γ_j the probability that a firm adjusts its price at time t on the basis of expectations dated $t - j$. Thus, the fraction of firms (ω_j) that charge prices at time t on the basis of expectations dated $t - j$ is given by

$$\omega_j = (1 - \gamma_j)\omega_{j-1}, \text{ for } j = 1, 2, ..J - 1 \tag{14}$$

and $\omega_0 = 1 - \sum_{j=1}^{J-1} \omega_j$. The aggregate price level is now defined as

$$P_t = \left[\sum_{j=0}^{J} \omega_j E_{t-j} \left(P_t^*(j) \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{15}$$

Log-linearizing these equations, the following expression for the aggregate inflation rate can be obtained:

$$\widehat{\pi}_t = \left(\frac{\omega_0}{1 - \omega_0} \right) \widehat{mc}_t + \sum_{j=1}^{J-1} \left(\frac{\omega_j}{1 - \omega_0} \right) E_{t-j} \left(\widehat{\pi}_t + \widehat{mc}_t \right) \tag{16}$$

where $\omega_k = (1 - \gamma_k)\omega_{k-1}$, and so

$$\omega_0 = \left(\frac{1}{1 + \sum_{k=1}^{J-1} \prod_{s=1}^{k}(1 - \gamma_s)} \right)$$

$$\omega_j = \left(\frac{\prod_{s=1}^{j}(1 - \gamma_s)}{1 - \sum_{k=1}^{J-1} \prod_{s=1}^{k}(1 - \gamma_s)} \right)$$

The Phillips curve (16) has some similarities, but also some important differences, relative to the previous expression (12). To see the differences, recall that (12) implied

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[11]Koenig (1997) offered a price-adjustment specification formally similar to that advocated by Mankiw and Reis, but interpreted it as arising from nominal contracts.

the following representation:

$$\widehat{\pi}_t = \left(\frac{\omega_0}{1-\omega_0}\right)\widehat{mc}_t + \left(\frac{\omega_j}{1-\omega_0}\right)\sum_{j=1}^{J-1} E_{t-j}\{\widehat{mc}_{t-1}\} + \sum_{j=1}^{J-1}\left(\frac{\omega_j}{1-\omega_0}\right) E_{t-j}\{\widehat{\pi}_t + \Delta\widehat{mc}_t\}$$

That is, under the Taylor-based distribution of information, the inflation rate depends not only on current real marginal cost and lagged expectations of changes in nominal marginal costs, but also on prior expectations of previous-period real marginal costs, which are not present in the Calvo version.

Finally, for comparability with the other sticky-information specification, in our empirical analysis we set J (i.e., the maximum number of periods a firm cannot update its expectations) equal to 4.

3.2.3   Standard Calvo Price Contracts with Indexation

The alternative price-setting specification we consider that imposes a form of the natural rate hypothesis is that proposed by Christiano, Eichenbaum, and Evans (CEE) (2005). Here, the representative firm sells its output in a monopolistically competitive market and sets nominal prices on a staggered basis, as in Calvo (1983). Each firm resets its price with probability $(1-\theta)$ each period, independently of the time elapsed since the last adjustment. Thus, each period a fraction $(1-\theta)$ of producers reset their prices, while the other fraction θ simply adjusts prices according to an indexation clause. In contrast to baseline Calvo price setting, this setup postulates that those price setters denied the opportunity to reset their prices today have their prices automatically raised by a percentage. As shown by CEE, Smets and Wouters (2003), and Giannoni and Woodford (2005), this leads to an expression for inflation given by,

$$(\widehat{\pi}_t - \eta_t) = \beta E_t(\widehat{\pi}_{t+1} - \eta_{t+1}) + \lambda\widehat{mc}_t \qquad (17)$$

where the slope coefficient $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\xi$, and $\xi = \frac{(1-\alpha)}{1+\alpha(\varepsilon-1)}$, and η_t is a variable determined by the indexation clause in contracts, and is usually related to the inflation rate. Note that under full indexation to inflation, this specification implies that the model satisfies the natural rate hypothesis. We consider several different kind of indexation rules in what follows:

(a) CEE's case. These authors introduce inflation persistence by allowing for *full indexation* of price contracts to lagged inflation, hence

$$\eta_t = \widehat{\pi}_{t-1}$$

This implies that expression (17) takes the more familiar form:

$$\widehat{\pi}_t = \frac{1}{1+\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1} + \frac{\widetilde{\lambda}}{1+\beta}\widehat{mc}_t.$$

Again, notice that the sum of the coefficients on inflation is equal to one.

(b) We can allow for indexation to some target level π_t^*,

$$\eta_t = \pi_t^*$$

The target could be a constant equal to a constant steady-state inflation rate, as in Eichenbaum and Fisher's (2004) static indexation assumption. We consider instead indexation to trend inflation, and for that purpose we approximate π_t^* by a quadratic trend,

$$\eta_t = \pi_t^* = a\,t + b\,t^2$$

in which a and b are parameters to be estimated.[12] It is straightforward to see that it is also possible to allow for intermediate cases between (a) and (b) along the lines of specification (5).

3.3 Shocks

We need to specify laws of motion for the policy shock, demand shocks, as well as the technology shock. We assume that the policy shock is white noise, with the remaining shocks following stationary first-order autoregressive processes,

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_{a_t} \tag{18}$$

$$\widehat{e}_t = \rho_e \widehat{e}_{t-1} + \varepsilon_{e_t} \tag{19}$$

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_{z_t} \tag{20}$$

3.4 Model Estimates

We estimate our model using maximum likelihood methods. Our sample period is 1979.3 to 2003.3, and the data are observations on detrended log per capita output, GDP deflator inflation, the nominal Federal funds rate and the detrended log of per capita domestic base money. All data come from the Federal Reserve Bank of St. Louis.[13]

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[12] An alternative assumption would be that the steady-state inflation rate follows a random walk, as in Smets and Wouters (2004). Smets and Wouters' sample period, however, includes both pre- and post-1979 data, so the random-walk assumption is needed to stand in for changes in the policy rule that are otherwise unmodeled. For the period since 1979, it is more reasonable to treat the policy rule as featuring constant responses, in which case the decline in the inflation rate can be treated as following a more smooth process.

[13] Data on output, the GDP deflator, population, and the funds rate (quarterly average) were obtained from the FRED database. The money series is an updated version of the Anderson-Rasche (2000) domestic monetary base series, subjected to the following adjustments: (1) the bulge in money growth at Y2K was smoothed by interpolating between the actual 12-month growth rates in August 1999 and April 2000, then replacing the actual levels between those months with the levels consistent with the interpolated growth rates; (2) the 9/11 bulge in real balances was smoothed by

The parameter estimates corresponding to the four models discussed in the previous section are listed in Table 2. Both versions of the sticky-information specification produce similar log-likelihood values, which are higher than that under dynamic indexation (CEE price setting). This is despite the fact that (following Khan and Zhu, 2002) we have used a finite-order approximation in estimating the first version of sticky information, and so lost some sources of dynamics in inflation. Thus, sticky information appears to do better than dynamic indexation in describing the data, although it should be stressed that our indexation specification lacks the other sources of nominal rigidity (i.e., those affecting wages) stressed as important for model fit by CEE (2005). The likelihood for the model with trend inflation indexation is not directly comparable with the others, as it incorporates a new variable (π_t^*) into the information set.

Despite the sharp differences in the specification of the supply side across models, we find major similarities in estimates of parameters describing preferences, the policy rule, and shock processes. The intertemporal elasticity of substitution is below 1 in all cases ($\sigma^{-1} \in (0.58, 0.73)$); this value is below the estimate of Rotemberg and Woodford (1999), but is compatible with Amato and Laubach's (2004) demonstration that in the presence of habits a value of σ^{-1} below 1 is needed to make the marginal utility of consumption increasing in prior consumption.

In all four cases there is evidence of a strong habits motive in household preferences, reflecting the degree of output persistence in the data. Throughout, we imposed a parameter value of $h = 0.95$ since our estimates always suggested values very close to 1. This is consistent with Giannoni and Woodford (2005), who obtain $h = 1$ as their point estimate using a somewhat different specification of the period utility function. It is, however, higher than the habit-formation estimates of CEE (2005) obtained under the restriction of log utility ($\sigma = 1$).

The elasticity of labor supply can be derived from our estimates of parameter $\chi = \frac{\varphi + \alpha}{1 - \alpha}$. Assuming that the elasticity of output with respect to hours ($1 - \alpha$) is 0.7, the value of φ is between 0.08 and 0.54, pointing to a labor supply elasticity (φ^{-1}) close to 2. Nevertheless, this result must be taken with some caution since our estimate of χ is not always very precise.

The parameter estimates corresponding to the income elasticity of money demand are very similar across models, with a long-run interest semielasticity of money demand slightly above the one obtained by Lucas (1988). The other money demand parameter, describing portfolio adjustment costs, is highly significant and remarkably similar across models. This component of money demand has been studied at length in ALSN (2004), and implies that money may have a substantial information content regarding the expected future paths of real demand and supply shocks.

The interest-rate rule is estimated with precision in all models, and the parameters

interpolating between August and October 2001 levels of the series. This domestic base series (as a quarterly average) was logged and the logs of both population and prices subtracted. Seasonal effects were then removed, with the effects estimated by regression on seasonal dummies.

are similar to others reported in both single-equation estimates and systems estimates in the literature. There is considerable interest rate smoothing ($\rho_r \in (0.65, 0.78)$), and the interest rate responds to both output and the rate of growth of nominal money in a significant, although not very strong, manner ($\rho_y \in (0.08, 0.18), \rho_\mu \in (0.14, 0.29)$). The Taylor principle is always satisfied, and the nominal-rate reaction to deviations of inflation from its steady-state (target) level is significantly above 1 ($\rho_\pi \in (1.74, 2.01)$). There is substantial persistence in all non-policy shocks, and their estimated variances suggest that these three shocks contribute substantially to the variability of the data.[14]

Let us now consider the different specifications of the price-setting mechanism in the economy. The two sticky-information specifications provide different results in terms of the updating of the information clause. In the Calvo-style model the estimated parameter θ_{MR} leads to an average duration slightly higher than 6 quarters. In the Taylor-type model, 20 per cent of firms use currently available information, while most make decisions on the basis of information gathered between 3 and 4 quarters earlier. It is not easy to compare these two structures: for example, the Calvo structure imposes a exponential decay in the number of firms gathering information at each period, while the Taylor specification allows for a less restrictive distribution. We shall leave the comparison between these two structures to the dynamic analysis below.

The two versions of indexation differ in the estimated degree of nominal rigidity, which is stronger in the model with indexation to trend inflation ($\lambda = 0.03$) than in that with indexation to lagged inflation ($\lambda = 0.87$), although the latter is obtained with a high standard error.

Our overall results indicate that, despite the complex set of highly non–linear cross-equation restrictions imposed by the optimizing model, the vectors of parameter estimates are remarkably similar across models. In a model specified at the level of preferences and technology, this is encouraging, since it means that estimates of what are meant to be deep or structural parameters are not affected to a significant extent by varying the specification of a key portion of the model. Calvo and Taylor updating schemes for expectations produce almost the same fit, although they point towards different dynamic properties, whose implications we consider below. These sticky-information models appear to fit better than that of the Calvo price-setting model with full dynamic indexation, while a direct comparison with the model with trend inflation is not possible, since the parameters indexing variation in the additional variable (π_t^*) are very precisely estimated, thus increasing the value of the likelihood function

We now analyze the dynamic features of the different models as well as the implied natural-rate responses to the real shocks.

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[14]For the money demand shock, this contribution occurs via the presence of money growth in the policy rule. If there were no policy response to money, money demand shocks would matter for variation in real money but not the other variables in the model.

4 Dynamic Properties of the Models

Figure 1 presents responses to a contractionary monetary policy shock for each version of our estimated model. The shock is to the nominal interest-rate rule, and so it produces a sharp rise in the interest rate and a negative response of money growth and real balances. The policy-shock experiment is most comparable across models for the sticky-information and indexation-to-trend inflation cases, because the initial response of the nominal interest rate is almost the same value in those three cases. For the model with CEE price-setting, the policy shock is considerably smaller in its impact on the interest rate (essentially because its standard deviation is lower), and by itself this accounts heavily for the shallower responses of output, prices, and costs for that model in Figure 1. Of the three comparable experiments, we see that the output response is persistent in all cases (in part reflecting habit formation in preferences), but takes longer to wear off under sticky information. This in part reflects the imposition of the NRH on dynamics, as we discuss below.

Another property of Figure 1 is the sharp impact response of inflation, with an immediate response to the monetary policy shock. This is a weakness of the model, since VAR evidence for the U.S., using a similar sample period to our own, suggests that inflation takes five quarters to reach its peak response (Giannoni and Woodford, 2005), while CEE's (2005, p. 34) VAR evidence for 1965–95 suggests the peak response takes even longer—about three years. The rapid response of inflation may appear surprising in light of the fact that Phillips curves with lagged inflation and sticky information are thought of as building in inflation inertia. However, Figure 1 indicates that the absence of inflation inertia comes from the same source identified by Keen (2004) for sticky information and CEE (2005) for dynamic indexation: the sharp response of real marginal cost to monetary policy shocks. The high habit formation and fixed-capital-stock aspects of our model deliver some intrinsic inertia in marginal cost, but evidently these are insufficient to prevent the model from implying a large elasticity of inflation with respect to output. CEE (2005) argue that wage stickiness is a required ingredient to deliver a gradual response of marginal cost and inflation to a policy shock.[15]

The rapid response of marginal cost also occurs in response to the two real shocks in the model (Figures 2A and 2B). Qualitatively, however, the responses to the real shocks are plausible. The responses to a technology shock match some notable features of the estimated VAR impulse responses reported for U.S. data for 1959–2001 by Altig, Christiano, Eichenbaum, and Linde (ACEL) (2005, Figure 2). Their estimated responses are not directly comparable to those in our Figure 2A because we consider persistent but stationary technology shocks, compared to their experiment of a permanent shock. Nevertheless, a qualitative comparison suggests that our model reproduces key aspects of the data responses. In particular, ACEL find that

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[15]CEE use a form of wage stickiness due to Erceg, Henderson, and Levin (2000), but augmented by indexation to prior price inflation.

technology shocks produce a sizable negative inflation response, while the response of growth in nominal cash balances[16] is quite weak and is back to zero within a year. Ou model delivers predictions that give a closer match to these features than ACEL's model. In both our model and ACEL's, technology shocks raise real balances; but in ACEL's model the growth in real balances comes from sharply higher money growth. In Figure 2A, as in the data, price movements do most of the work in raising real balances.

Figure 2B gives responses to an IS shock. The positive response of inflation, output, and nominal interest rates is in keeping with Evans and Marshall's (2001, Figure 2) impulse responses to an identified IS shock from a VAR estimated on U.S. data for 1959–2000.

Figure 3 shows the responses of the natural level of output and the natural real interest rate to the two real shocks in the model. The responses are similar across models, reflecting the similarity of the production and preference parameter estimates.[17] Both real shocks boost potential GDP, but have opposite effects on the natural rate—a technology shock enlarges what can be consumed today relative to future periods and therefore an interest-rate decline is required to boost consumption; an IS or preference shock, on the other hand, raises consumption demand today relative to tomorrow, and the real rate rises both to restrain this increase in demand and to stimulate potential output (via higher labor supply).[18]

Like CEE and Ireland (2004), but unlike much other recent work with sticky-price optimizing models, we include money in the dataset used to obtain our estimates. We argued in ALSN (2004) that the fact that money demand is forward-looking in our model conveys on money extra information about the path of the natural interest rate. The two panels (A and B) of Figure 4 illustrate this principle, again plotting the responses of the natural interest rate, juxtaposed against the responses of the change in real balances with and without portfolio adjustment costs. Real money and the natural rate tend to have an inverse relationship in all cases. In the case of static money demand, this inverse relationship simply reflects the co-movement of today's natural rate and the nominal interest rate. With forward-looking money demand, real balances depend on the whole path of future nominal interest rates, and as future natural rates are a major portion of future nominal rates, money today reflects the expected path of the natural rate. The forward-looking character of money demand also smooths the initial response of money to interest-rate movements, since

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[16]ACEL use "money of zero maturity" as the measure of money in their VAR. This definition of money is more comparable to our monetary base series than are M1 or M2 measures of money because it excludes those deposits which have few characteristics in common with currency.

[17]The natural rates are computed using the procedure proposed by Neiss and Nelson (2003). Our estimated model, and the natural-rate responses, are qualitatively similar to Neiss and Nelson's model without capital.

[18]While high, the habit-formation parameter in our model is sufficiently far below unity to deliver a positive response of the natural real rate to IS shocks. If it is closer to unity, the response turns negative, as discussed in ALSN (2004).

now the criterion for altering money balances is whether the sum of current and expected future rates has changed.

Regardless of price-setting assumption, the two real shocks account for the bulk of the variability in output and inflation, as measured by forecast error decompositions (Table 3). In sticky-price models such as those we use, monetary policy affects the variability of real and nominal variables both by introducing policy shocks into the economy and, via the feedback responses in the interest-rate rule, by shaping the response of endogenous variables to the real shocks. The table indicates that the feedback responses are a far more important means by which monetary policy affects output than the policy-shock terms. This result, which is robust across models, is in accordance with most of the existing empirical VAR literature.

Figure 5 examines the output response to a monetary policy shock in further detail. We compare the response of output under the three models that impose the NRH on dynamics (the two versions of sticky information, plus CEE price-setting) against a parameterization of the model identical to that we obtain under CEE price-setting, but dropping the indexation terms so that price setting corresponds to baseline Calvo—which does not satisfy the NRH. We have computed error bands for the output response for the case of sticky information with contracts *à la* Calvo; these bands are also displayed in the figure.[19]

The precise results are a function of many factors in the model, including the estimated policy rule, but a general impression from Figure 5 is that the models that satisfy the NRH tend to produce a more protracted output response; indeed, even the inner error band for the estimated response under sticky information (with contracts *à la* Calvo) departs further from the origin than the responses from models that do not satisfy the NRH. Thus, even though imposing the NRH restriction by definition makes the steady state of the model correspond to the flexible-price steady state, imposing the NRH appears to generate greater *short-run* deviations of real variables from the flexible-price equilibrium. This can be thought of as a result of the fact that under the NRH, the Phillips curve must, in unconditional expectation, collapse to the condition $E[y_t - y_t^*] = 0$. Unless the NRH is imposed via the constant term, this condition puts restrictions on how variables beside the current output gap $(y_t - y_t^*)$ may appear in the Phillips curve. Lagged expectations and expected future values of the gap may certainly appear (this, in essence, is the sticky-information specification). To the extent that inflation (in prices or wages) appears in the Phillips curve, it must do so in first-difference or expectational-first-difference form so that it is mean zero (i.e., the Phillips curve must be "accelerationist," as it is under CEE price setting). Thus, imposing the NRH tends to build extra dynamics into the Phillips curve and so more persistent short-run deviations of the model from the flexible-price equilibrium.

The protracted deviation of output from potential is especially notable in the case of sticky information based on Taylor contracts. Here nominal contracts expire after

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[19]The error bands correspond to a 95% confidence interval and were computed using 1000 Monte Carlo simulations.

four quarters, so following a monetary policy shock in period 0, real marginal cost returns at the end of the contract period to a flexible-price condition, $E_t \widehat{mc}_{t+4} = 0$. Yet as Figure 5 shows, output does not return to steady state for several further quarters. The reason for this discrepancy is the marginal cost expression (13). Habit formation has the effect of putting not only period-t output but lagged output and expected future output into the expression for current marginal cost.[20] This dynamic relationship means that $E_t \widehat{mc}_{t+4} = 0$ does not imply that $E_t \widehat{y}_{t+4} = 0$; rather, with $E_t \widehat{y}_{t+3}$ nonzero due to nominal rigidities in period $t+3$, $E_t \widehat{y}_{t+4}$ and $E_t \widehat{y}_{t+5}$ will generally be nonzero too, to balance the right-hand-side of (13). A similar argument applies to expectations of real marginal cost in periods $t+5$ and subsequently, so that output effects of monetary policy will dwindle only gradually. Thus, habit formation makes the effects of monetary policy shocks on output exceed the nominal-contract length—a "contract multiplier" in the terminology of Chari, Kehoe, and McGrattan (2000), and an example of what Woodford (2003, Ch. 3) calls "strategic complementarity" in price setting.

Figure 6 plots the output gap series in our estimated models against the CBO output-gap series. The figure is in line with McCallum (2001) and much other evidence to the effect that standard output-gap measures are unreliable because they abstract from the stochastic fluctuations in potential output that optimizing models suggest occur in the wake of preference and productivity shocks.[21] On the other hand, the various models do not differ greatly from one another in the profile of the output gap. Again this reflects the fact that our estimates of utility function parameters and processes for the real shocks are similar regardless of which price-adjustment specification we employ.

5 Conclusion

In this paper we have considered the "McCallum critique" of standard sticky-price specifications—namely, that they do not deliver satisfaction of the natural rate hypothesis. In response to this critique, we have estimated optimizing models of the business cycle on U.S. data using only price-adjustment specifications that satisfy the natural rate hypothesis. The price-adjustment specifications we consider are two variants of the sticky-information specification proposed by Mankiw and Reis (2002), and two versions of Calvo price setting with indexation—the dynamic indexation scheme of Christiano, Eichenbaum, and Evans (2005), as well as indexation to the steady-state inflation rate. Our estimation of the last scheme allows for a trend in steady-state inflation.

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[20]In the case of standard Calvo price setting, expression (13) is the mirror image of Amato and Laubach's (2004) result that, when current real marginal cost is substituted out of the Phillips curve, and real shock terms are combined into a potential-output representation, the forcing process for inflation can be written as a three-period moving average of the output gap.

[21]See the references given in footnote 1.

We found that the dynamic properties of all versions of the models are quite similar, though those specifications that impose the natural-rate hypothesis via dynamic restrictions tend to feature more persistence in output in response to monetary policy shocks. Our empirical estimates of the real side of the economy (preference parameters and laws of motion for the real shocks) are similar whichever price adjustment specification is chosen. Consequently, the alternative model specifications deliver similar estimates of U.S. potential output. The model variants therefore largely agree on the historical behavior of the U.S. output gap. These theory-based gap series, however, bear little resemblance to a standard (CBO) estimate of the U.S. output gap.

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Table 1. Price adjustment and the natural rate hypothesis

	NRH satisfied for given π^*	NRH satisfied globally	NRH affects medium-run dynamics
Calvo, no indexing	No	No	No
Calvo, indexing to steady-state inflation	Yes	No	No
Calvo, indexing to lagged inflation	Yes	No	Yes
Sticky information	Yes	Yes	Yes

Table 2. Maximum Likelihood Estimates

Parameters	Sticky Information Models		Price Contracts *à la* Calvo	
	Calvo	Taylor	π_{-1} Index.	$\overline{\pi}$ Index.
β	0.99	0.99	0.99	0.99
h	0.95	0.95	0.95	0.95
σ^{-1}	0.64 (0.08)	0.58 (0.06)	0.73 (0.08)	0.58 (0.06)
γ_1	0.07 (0.02)	0.11 (0.03)	0.04 (0.04)	0.12 (0.02)
γ_2	2.41 (0.67)	3.47 (0.77)	1.55 (1.64)	3.90 (0.26)
δ	7.20 (0.40)	6.98 (0.53)	5.62 (1.12)	7.85 (0.43)
χ	1.12 (0.29)	0.54 (0.24)	0.88 (0.31)	1.21 (0.18)
λ			0.87 (0.48)	0.03 (0.02)
θ^{MR}	0.85 (0.05)			
θ_0		0.20		
θ_1		0.00 (0.03)		
θ_2		0.18 (0.04)		
θ_3		0.24 (0.06)		
θ_4		0.38 (0.08)		
ρ_r	0.72 (0.05)	0.73 (0.04)	0.65 (0.07)	0.78 (0.02)
ρ_y	0.13 (0.05)	0.14 (0.05)	0.08 (0.04)	0.18 (0.12)
ρ_π	1.74 (0.21)	1.87 (0.21)	2.01 (0.22)	1.80 (0.15)
ρ_μ	0.16 (0.12)	0.29 (0.13)	0.14 (0.09)	0.28 (0.11)
ρ_a	0.79 (0.05)	0.72 (0.06)	0.79 (0.05)	0.73 (0.07)
ρ_z	0.95 (0.02)	0.92 (0.03)	0.93 (0.03)	0.93 (0.04)
ρ_e	0.90 (0.05)	0.86 (0.05)	0.91 (0.05)	0.89 (0.05)
$\sigma_a(\%)$	1.33 (0.38)	1.23 (0.28)	1.03 (0.29)	1.31 (0.27)
$\sigma_z(\%)$	0.98 (0.21)	1.17 (0.27)	0.78 (0.13)	1.01 (0.79)
$\sigma_e(\%)$	3.82 (0.43)	4.12 (0.39)	3.22 (0.58)	4.26 (0.54)
$\sigma_{\varepsilon_r}(\%)$	0.27 (0.02)	0.28 (0.02)	0.30 (0.04)	0.26 (0.02)
Log Likelihood	1565.19	1567.30	1551.25	2328.28

Note: Standard errors are in parentheses.

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Table 3. Forecast Error Variance Decomposition

Fraction of Variances in the Business Cycle Frequencies

	Output Horizon			Inflation Horizon		
	4	12	40	4	12	40
Sticky Info. (Calvo)						
Preferences	86.9	71.8	59.5	23.6	17.0	13.8
Technology	12.1	27.6	40.0	62.9	72.8	77.7
Velocity	0.1	0.0	0.0	1.0	1.4	1.4
Monetary Policy	0.9	0.6	0.5	12.5	8.8	7.1
Sticky Info. (Taylor)						
Preferences	81.5	62.6	55.1	6.4	5.4	5.1
Technology	18.3	37.3	44.8	74.7	79.0	79.7
Velocity	0.0	0.0	0.0	2.8	4.2	4.5
Monetary Policy	0.2	0.1	0.1	16.1	11.4	10.7
CEE						
Preferences	74.6	60.4	53.3	32.4	31.2	30.0
Technology	25.4	39.6	46.7	39.1	42.6	44.6
Velocity	0.0	0.0	0.0	2.0	2.2	2.4
Monetary Policy	0.0	0.0	0.0	26.5	24.0	23.0
CEE (Trend Inflation)						
Preferences	89.9	78.2	73.8	8.0	6.7	6.5
Technology	8.7	20.7	25.2	82.3	83.7	83.5
Velocity	0.3	0.2	0.2	0.4	1.9	2.6
Monetary Policy	1.1	0.9	0.8	9.3	7.7	7.4

Appendix

1. The Model

Let C_t and N_t represent consumption and hours worked by households in period t.[22] Preferences are defined by the discount factor $\beta \in (0,1)$ and a period utility function. These households seek to maximize

$$\max_{C_t,N_t,M_t,B_t,B_{L,t}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t \left[U\left(\frac{C_t}{C_{t-1}^h}\right) + V\left(\frac{M_t}{e_t P_t}\right) - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right] - G(.) \right\} \quad (21)$$

where, in what follows, we specialize the period utility to take the form

$$U(.) = \frac{1}{1-\sigma}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma}, \quad V(.) = \frac{1}{1-\delta}\left(\frac{M_t}{e_t P_t}\right)^{1-\delta}$$

$$G(.) = \frac{d}{2}\left\{ \exp\left[\left\{\frac{M_t/P_t}{M_{t-1}/P_{t-1}} - 1\right\} + \exp\left[-\left\{\frac{M_t/P_t}{M_{t-1}/P_{t-1}} - 1\right\}\right]\right] - 2\right] \right\} \quad (22)$$

where M_t/P_t represents real balances of the household; a_t is a preference shock, and e_t is a shock to the household's demand for real balances. The parameter $\sigma > 0$ governs relative risk aversion, $\varphi \geq 0$ represents the inverse of the Frisch labor supply elasticity, and finally $\delta > 0$, $d > 0$. In addition, we incorporate the presence of portfolio adjustment cost through the function $G(.)$. This functional form for portfolio adjustment costs, used by ALSN (2004), is that of Christiano and Gust (1999), modified to refer to real balances and applied to a model without "limited participation" features.

The budget constraint each period is:

$$\frac{M_{t-1} + B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{\frac{B_t}{r_t} + M_t}{P_t} \quad (23)$$

Households enter period t with money holdings M_{t-1} and maturing one-period riskless bond holdings B_{t-1}. At the beginning of the period, they receive lump-sum nominal transfers T_t, labor income $W_t N_t$, where W_t denotes the nominal wage, and a nominal dividend D_t from the firms. They use some of these funds to purchase new one-period at nominal cost $\frac{B_t}{r_t}$, where r_t denotes the gross nominal interest rate between t and $t+1$. The household carries M_t units of money into period $t+1$.

The first-order conditions for the optimizing consumer's problem can be written as:

$$\Lambda_t = a_t U_{t,C_t} + \beta E_t\{a_{t+1}U_{t+1,C_t}\} \quad (24)$$

$$a_t(N_t)^\varphi = \Lambda_t\left(\frac{W_t}{P_t}\right) \quad (25)$$

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[22]Because there is a continuum of consumption goods available for purchase, C_t is a Dixit-Stiglitz aggregate of consumption goods.

$$\left(\frac{\Lambda_t}{P_t}\right) = \beta r_t E_t \left(\frac{\Lambda_{t+1}}{P_{t+1}}\right) \tag{26}$$

$$a_t V_{t,M_t} - \{G_{t,M_t} + \beta E_t \{G_{t,M_t}\}\} = \left(\frac{1}{P_t}\right) \Lambda_t - \beta E_t \left(\frac{1}{P_{t+1}}\right) \Lambda_{t+1} \tag{27}$$

where $U_{t,C_t} = \frac{\partial U_t}{\partial C_t}$, $U_{t+1,C_t} = \frac{\partial U_{t+1}}{\partial C_t}$, $V_{t,M_t} = \frac{\partial V_t}{\partial M_t}$, $G_{t,M_t} = \frac{\partial G_t}{\partial M_t}$ and $G_{t+1,M_t} = \frac{\partial G_{t+1}}{\partial M_t}$.

Equation (24) is the standard expression for the marginal utility of wealth (i.e., the Lagrange multiplier for the budget constraint), which, in the presence of habit formation, will depend upon both the marginal utility of consumption today and the expected next-period marginal utility of consumption. This relationship is affected by the presence of preference shocks at time t and the expected shocks in time $t+1$. Expression (25) is the labor supply schedule, relating real wages to the marginal rate of substitution between consumption and hours. Expression (26) corresponds to the Euler equation linking the marginal utility of wealth across periods.

The production function for firm j is

$$Y_t(j) = z_t N_t(j)^{1-\alpha}, \tag{28}$$

where $Y_t(j)$ is output, $N_t(j)$ represents the number of work-hours hired from the household (i.e. $N_t = \int_0^1 N_t(j) \, dj$), z_t is a common technology shock and $(1-\alpha)$ parameterizes the technology. Letting $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, the market-clearing condition implies $Y_t = C_t$.

2. Estimation

All the models can be represented by four equations describing the equilibrium conditions for output, real balances, inflation, and the interest rate rule, i.e. $\{y_t, m_t, \pi_t, r_t\}$. In particular, to construct the log-likelihood we first find the unique rational expectations equilibrium. To do so, we follow King and Watson (1998), who cast the linear rational expectations system in the following matrix form

$$A \, E_t y_{t+1} = B \, y_t + C \, v_t \tag{29}$$

where $y_t = \begin{bmatrix} f_t \\ s_t \end{bmatrix}$; s_t are predetermined endogenous variables, and f_t correspond to the jump variables. The size of the vector s_t is equal to n_k, while the size of the vector f_t is n_f (so the overall size of the vector y_t is equal to $n_f + n_k$). The size of v_t is n_z.

Let $s_t = \begin{bmatrix} s_t \\ v_t \end{bmatrix}$, where s_t is $((n_k + n_z), 1)$, then the model solution takes the form:

$$f_t = \Pi \, s_t \tag{30}$$

and

$$s_{t+1} = R \, s_t + W \, \varepsilon_t \tag{31}$$

where $n_u = n_z$ and f_t is $(n_f, 1)$, Π is a $(n_f, (n_k + n_z))$ matrix, s_t is $((n_k + n_z), 1)$, R is a $((n_k + n_z), (n_k + n_z))$ matrix, W is a $((n_k + n_z), n_z)$ matrix, and ε_t is $(n_u, 1)$.

In particular, in the model with sticky information staggered *à la* Taylor, the vector of forward-looking variables is given by $f_t = [y_t \; \mu_t \; r_t \; m_t \; \pi_t \; mc_t \; E_t mc_{t+1}...$ $E_t mc_{t+4} \; E_t\pi_{t+1}... \; E_t\pi_{t+4} \; E_t y_{t+1}]$ and the vector of state variables is $s_t = [y_{t-1} \; r_{t-1} \; m_{t-1}$ $\pi_{t-1} E_{t-1}\pi_t \; E_{t-4}\pi_t \; E_{t-1}\pi_{t+1}...E_{t-1}\pi_{t+3} \; E_{t-2}\pi_{t+1} E_{t-1}mc_t... \; E_{t-4}mc_t E_{t-2}\pi_{t+2} \; E_{t-3}\pi_{t+1}$ $E_{t-1}mc_{t+1}E_{t-1}mc_{t+3} \; E_{t-2}mc_{t+1} \; E_{t-2}mc_{t+2} \; E_{t-3}mc_{t+1}]$. Finally, the vector of shocks is $v_t = [a_t \; z_t \; e_t \; \varepsilon_t]'$. The size of the vector s_t is $n_k = 24$, while the size of the vector f_t is $n_f = 15$, i.e. the size of the vector y_t is equal to $n_f + n_k = 39$. Finally, the size of v_t is $n_z = 4$. In the CEE price setting and trend-indexation models, the vector of forward-looking variables is given by $f_t = [y_t \; \mu_t \; r_t \; m_t \; \pi_t \; mc_t \; E_t y_{t+1}]'$, and the vector of state variables is $s_t = [y_{t-1} \; r_{t-1} \; m_{t-1} \; \pi_{t-1}]'$. Finally, the vector of innovations is $v_t = [a_t \; z_t \; e_t \; \varepsilon_t]'$. The size of the vector s_t is $n_k = 4$, while the size of the vector f_t is $n_f = 7$, so the size of the vector y_t is equal to $n_f + n_k = 11$.

The reduced-form solution of the model forms the basis for likelihood estimation of the structural parameters, as in (e.g.) Ireland (2003, 2004). To that end, let us define the vector of observables, d_t, as follows:

$$d_t = \begin{bmatrix} \widehat{y}_t \\ \widehat{m}_t \\ \widehat{\pi}_t \\ \widehat{r}_t \end{bmatrix} = \begin{bmatrix} \ln(y_t) - \ln(\overline{y}) \\ \ln(m_t) - \ln(\overline{m}) \\ \ln(\pi_t) - \ln(\overline{\pi}) \\ \ln(r_t) - \ln(\overline{r}) \end{bmatrix}$$

Thus, the empirical model is given by the following set of equations

$$d_t = \widetilde{C} s_t$$

$$s_{t+1} = R \, s_t + W \, \varepsilon_t$$

where $\widetilde{C}$ is a matrix that connects observed variables to the vector of state variables, and $\varepsilon_{t+1} = [\varepsilon_{at+1}\varepsilon_{zt+1}\varepsilon_{et+1}\varepsilon_{rt+1} \; \Gamma]' \rightsquigarrow N(0, V)$, with Γ is a vector of zeros whose entries are arranged so that the nonstochastic laws of motion for the states (e.g. identities defining lags) are satisfied. Given that the rank of the matrix $E\varepsilon_t\varepsilon_t' = V$ is the same as that of the matrix of the variance covariance of the data, the construction of the likelihood is relatively simple. The Kalman filter is used to construct an updating procedure of such a likelihood. Given a vector of initial values, we can compute $E(S) = 0 = \widehat{S}_{1|0}$ and $E(SS') = \Sigma_{1|0}^S = (I - \widetilde{A} \otimes \widetilde{A}')^{-1}\widetilde{B}V\widetilde{B}'$. As described in Hamilton (1994, Ch. 13), the procedure for updating the likelihood is:

$$u_t = d_t - \widehat{d}_{t|t-1} = d_t - \widetilde{C} \; \widehat{S}_{t|t-1}$$

$$\Omega_t = E_t \; u_t u_t' = \widetilde{C} \; \Sigma_{t|t-1}^S \; \widetilde{C}'$$

3. Computing the Natural Rates

Here we describe how we compute the natural levels of output and the real interest rate for our estimated models. The method we follow is that proposed by Neiss and Nelson (2003). Potential output is obtained by solving for the decision rules for output under flexible prices. More generally, the solution takes the form:

$$\widehat{y_t^*} = \varkappa_1 \widehat{y_{t-1}^*} + \varkappa_2 \widehat{a}_t + \varkappa_3 \widehat{z}_t$$

The state vector in the flexible-price solution thus consists of lagged potential output (given the existence of habit formation) and the two real shocks. The solution coefficients are functions of the structural parameters in the model, but not the policy rule.

By repeatedly solving out lags of $\widehat{y_t^*}$, the previous expression can be converted into the following infinite-lag-order representation:

$$\widehat{y_t^*} = \vartheta(\widehat{a}_t, \widehat{a}_{t-1}, \widehat{a}_{t-2}, \ldots, \widehat{z}_t, \widehat{z}_{t-1}, \widehat{z}_{t-2}, \ldots)$$

where $\vartheta(\cdot)$ is a linear function. We approximate this infinite-lag function with a representation that has long, but finite, lags. Specifically, we use regression coefficients obtained by projections of output on lags of the real shocks on data generated by simulations of the model under the flexible prices. Using these coefficients, we can generate empirical estimates of potential output from our system estimates of the real shocks.

Figure 1
Comparative Dynamics of the Models (I)
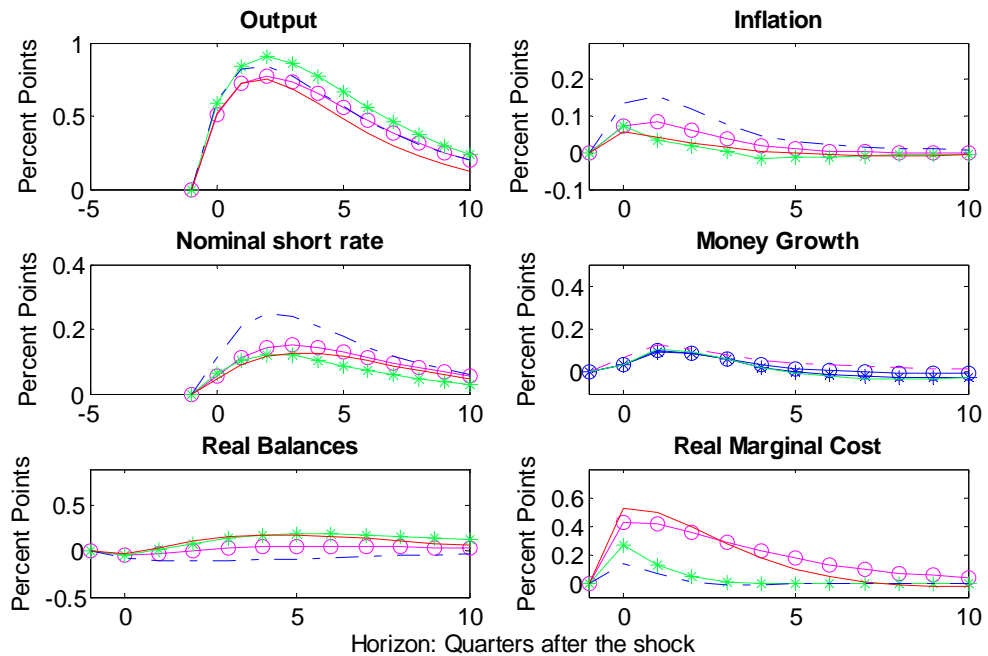### Responses to a Monetary Policy Shock
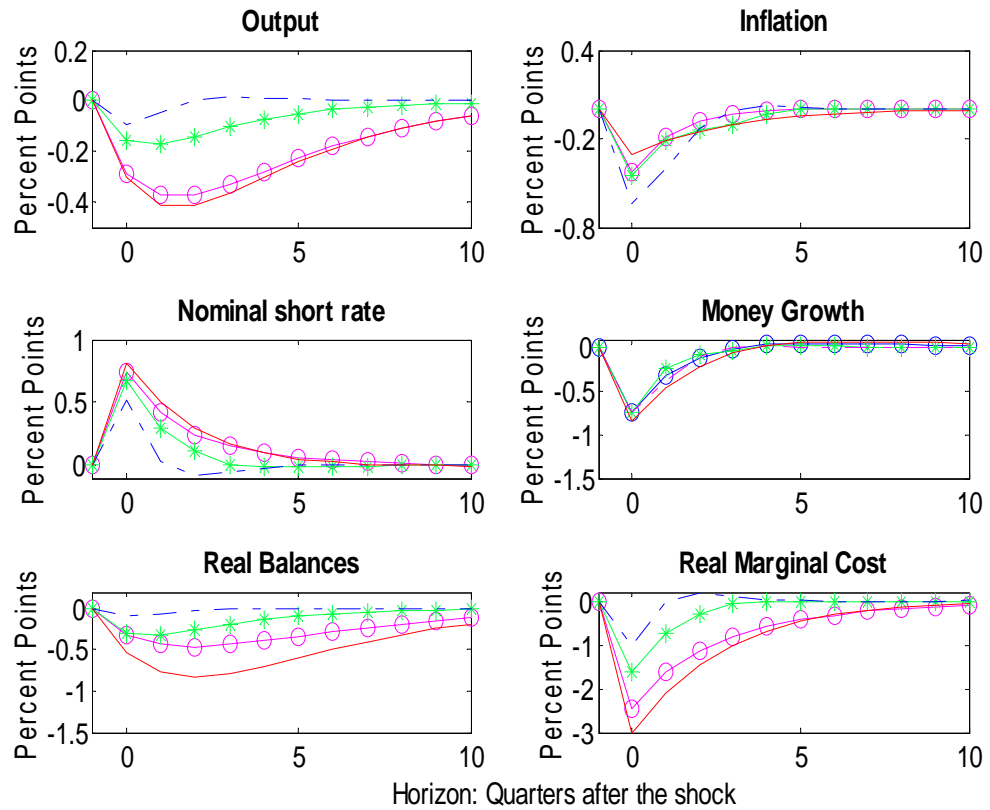
Note: Each panel displays the impulse responses of each variable to a one standard deviation monetary policy shock. Numbers are percent deviations from steady-state values. Circled line: estimated sticky information model *à la* Calvo, starred line: estimated sticky information model *à la* Taylor; dotted line: CEE price setting (dynamic indexation); and continuous line: sticky prices with indexation to trend inflation.

Figure 2
Comparative Dynamics of the Models (II)

A. Responses to a Technology Shock

Horizon: Quarters after the shock
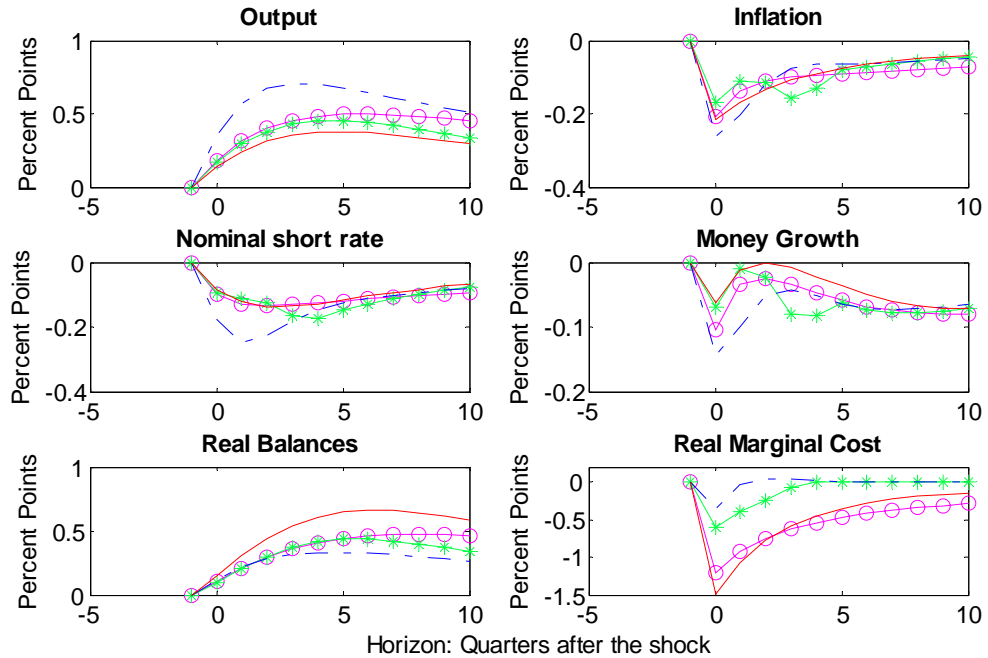
B. Responses to a Preference Shock

Horizon: Quarters after the shock

Note: Each panel plots the impulse responses of one variable to a one standard deviation shock. Numbers are percent deviations from steady-state values. Circled line: estimated sticky information model *à la* Calvo, starred line: estimated sticky information model *à la* Taylor; dotted line: CEE price setting (dynamic indexation); and continuous line: sticky prices with indexation to trend inflation.
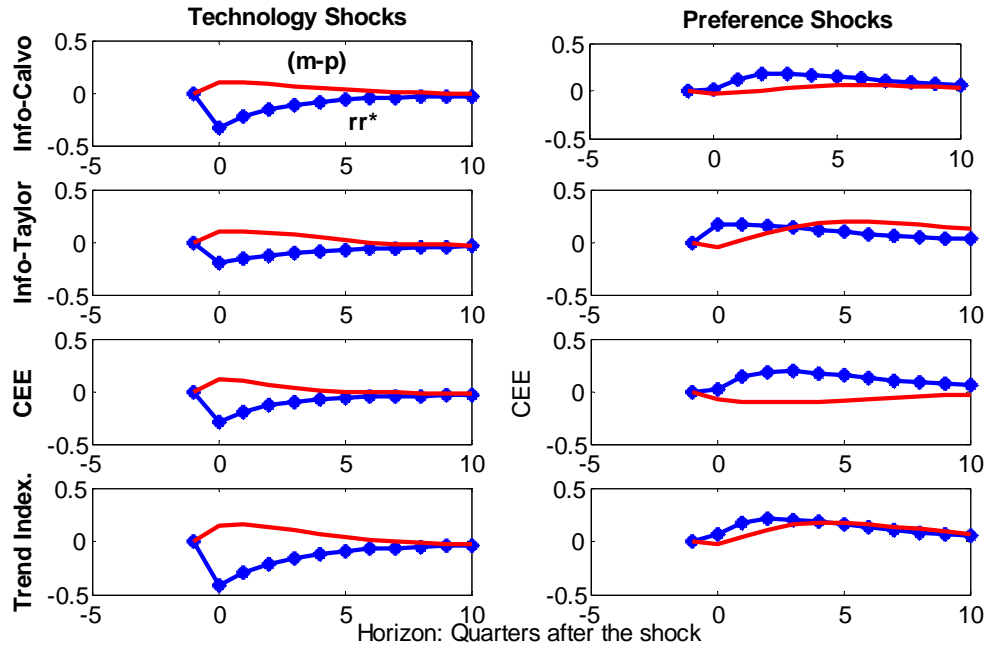
Figure 3
Natural Rate Responses
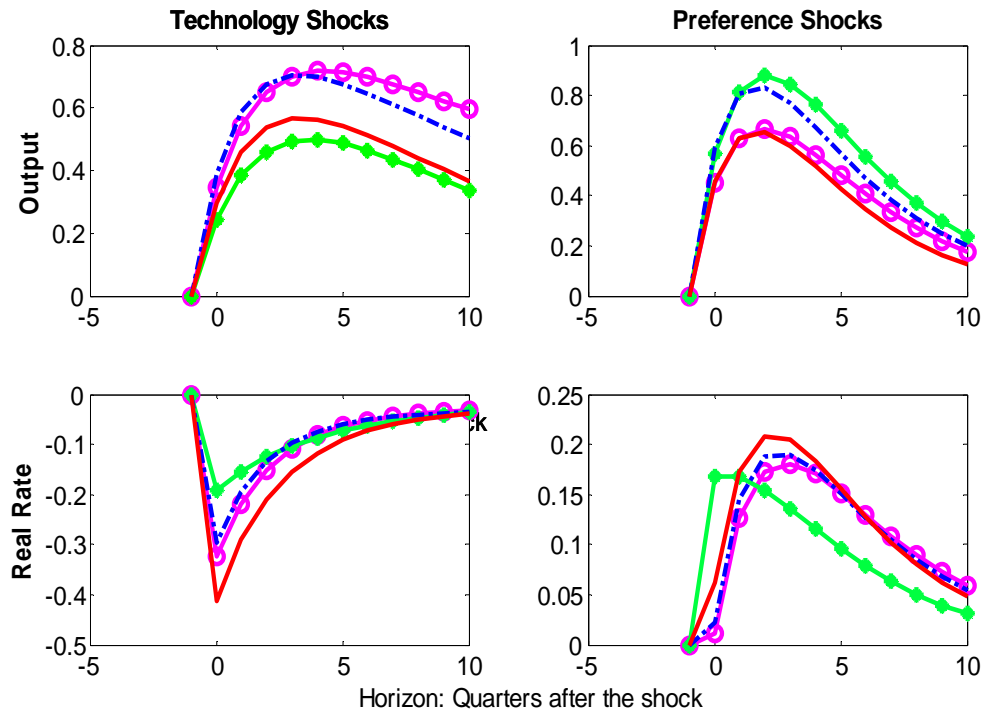
Note: Each panel plots the impulse responses of the natural rate of interest, across models, to a one standard deviation shock. Numbers are percent deviations from steady-state values. Circled line: estimated sticky information model *à la* Calvo, starred line: estimated sticky information model *à la* Taylor; dotted line: CEE price setting model (dynamic indexation); and continuous line: sticky prices with indexation to trend inflation.
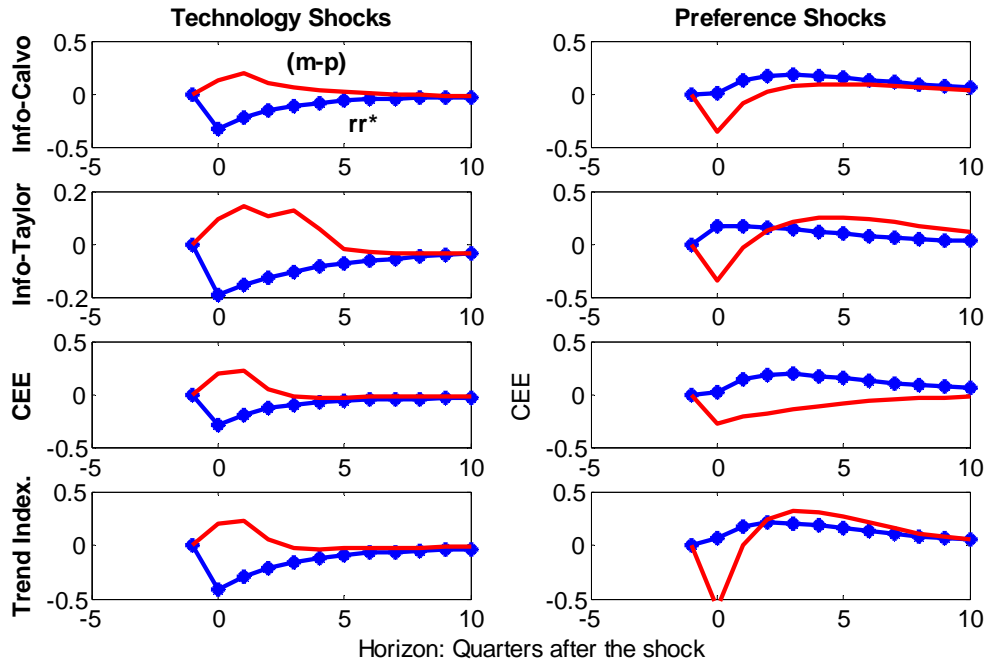
Figure 4
Money and the Natural Interest Rate
A. Money Demand with Portfolio Adjustment costs


B. Money Demand without portfolio adjustment cost

Note: Each panel displays the impulse responses of real money growth and the natural rate of interest to a one standard deviation technology and preference shock. Numbers are percent deviations from steady-state values. Continuous line: real money balances growth, and starred line: responses of the natural rate of interest.

Figure 5
Output Dynamics and the NRH
Responses to a Monetary Policy Shock
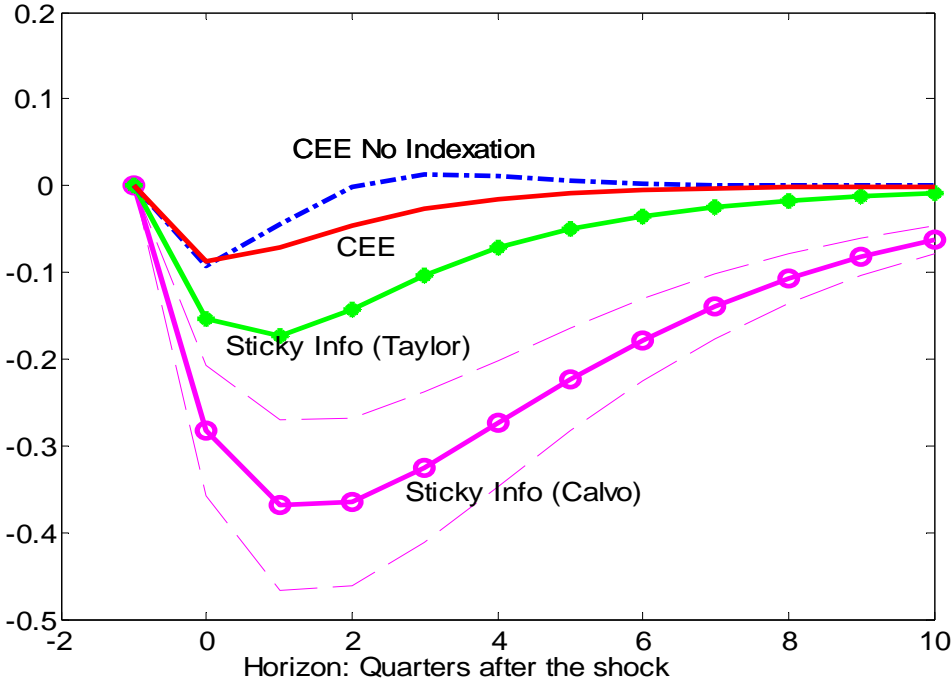
Note: Dashed lines are 95% confidence intervals based upon 1000 MonteCarlo simulations of the model.

Figure 6
Estimated Output Gaps and CBO Gap