Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

Monetary Policy, Determinacy, and Learnability in a Two-Block World Economy

James Bullard
and
Eric Schaling

Working Paper 2006-038A
http://research.stlouisfed.org/wp/2006/2006-038.pdf

May 2006

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

Monetary Policy, Determinacy, and Learnability in a Two-Block World Economy

James Bullard[*] **Eric Schaling**[†]

29 May 2006[‡]

Abstract

We study how determinacy and learnability of worldwide rational expectations equilibrium may be affected by monetary policy in a simple, two country, New Keynesian framework under both fixed and flexible exchange rates. We find that open economy considerations may alter conditions for determinacy and learnability relative to closed economy analyses, and that new concerns can arise in the analysis of classic topics such as the desirability of exchange rate targeting and monetary policy cooperation. *Keywords*: Indeterminacy, learning, monetary policy rules, new open economy macroeconomics, exchange rate regimes, second generation policy coordination. *JEL codes E58, E61, F31, F41.*

[*]Research Department, Federal Reserve Bank of St. Louis, bullard@stls.frb.org. Any views expressed are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

[†]Department of Economics University of Johannesburg, and CentER for Economic Research, Tilburg University, esc@eb.rau.ac.za. This paper was written while Eric Schaling was a visiting scholar at the Research Department of the Federal Reserve Bank of St. Louis.

[‡]This paper was presented at the third conference of the International Research Forum on Monetary Policy at the ECB. The Forum is sponsored by the European Central Bank, the Board of Governors of the Federal Reserve System, the Center for German and European Studies at Georgetown University and the Center for Financial Studies at Goethe University. Earlier versions were also presented at CentER, the University of Johannesburg, Bonn University, the BIS, the Bank of England, De Nederlandsche Bank, the University of Cape Town and the South African Reserve Bank. Helpful comments by Jordi Gali, Martin Ellison, Albert Marcet, Vicente Tuesta and Ed Nelson are gratefully acknowledged.

1 Introduction

1.1 Overview

New Keynesian macroeconomic models have become a workhorse for studying a variety of monetary policy issues in closed economy environments. An important component of this effort has been the development of the idea that equilibrium determinacy and learnability may be significantly influenced by monetary policy choices.[1] Recently, simple extensions of the New Keynesian model to open economy environments have been developed. Our primary concern in this paper is to provide an analysis of the extent to which the findings concerning determinacy and learnability for the closed economy New Keynesian framework may be altered when open economy considerations are brought to bear. Our learnability criterion is that of Evans and Honkapohja (2001).

Our approach to this question is to adopt a simple framework for a two-country world due to Clarida, Gali, and Gertler (2002). This framework provides one straightforward extension of the New Keynesian model to two countries and allows comparison to the more common single country and small open economy analyses as special cases. We focus on the two polar cases of fixed and flexible exchange rates, and ask the question how determinacy and international monetary policy transmission are affected by the exchange rate regime.

1.2 Main findings

The main findings under flexible exchange rates are as follows. Instrument rules which are focussed on domestic inflation and domestic output gaps lead to world determinacy and learnability conditions which must be met in each economy independently of whether they are met in the partner economy. For targeting rules, this result has a natural counterpart when policymakers in

[1] See, for instance, Woodford (2003), Bullard and Mitra (2002), Bullard (2006), Evans and Honkapohja (2003a,b), and Preston (2003).

each country pursue non-cooperative optimal policy under discretion. The choice of how to implement the optimality condition stemming from the minimization problem faced by the monetary authorities can easily be made inappropriately, leading to indeterminacy and expectational instability.

On the other hand, instrument rules which include responses to international economic conditions induce international feedback between the two economies even when there would otherwise be no such feedback. The separability of conditions between countries breaks down. This second result again has a natural counterpart in the case of targeting rules, in the situation where the two countries agree to try to pursue the gains from cooperation which may exist in the model. Implementation will again be an issue.

We also find that, if properly implemented, a flexible exchange rate regime has attractive insulation properties relative to a fixed exchange rate regime (here modelled as an exchange rate peg).

We conclude that determinacy and learnability considerations can alter the evaluation of monetary policy options in an international context.

1.3 Recent related literature

Batini, Levine, Justiniano and Pearlman (2005) study indeterminacy in a two-country New Keynesian model. Their focus is on the relationship between many-period forward-looking inflation forecast rules and indeterminacy conditions. We do not consider rules in this class in this paper. When forward-looking rules are considered here, they arise from the implementation of certain optimality considerations and do not involve forecasts more than one period into the future.

De Fiore and Liu (2005) study indeterminacy in a small open New Keynesian economy. Their model is somewhat different from the one we study. They conclude that whether a given policy rule can deliver determinacy will depend on the degree of openness in the small economy, a result we also obtain.

A number of papers study classic open economy issues in the New Keynesian framework. Pappa (2004) and Benigno and Benigno (2004), for example, study the gains from monetary policy coordination. Corsetti and Pesenti (2005) analyze 'self-oriented' or 'inward-looking' national monetary policies in frameworks related to the one studied here. While touching on some related themes, these papers do not focus on the determinacy and learnability issues we emphasize.

Ellison, Sarno, and Vilmunen (2004) study central bank learning in the two-country world of Aghion, Becchetta, and Banerjee (2001). They allow fundamental parameters in the economy to follow Markov switching processes, and central banks update their inference concerning the current regime via Bayes rule.

Zanna (2004) studies determinacy and learnability in the small open economy case for a model due to Uribe (2003) which is again somewhat different from the one we study.[2] Zanna (2004) contains results on learnable sunspot equilibria under common factor representations, a topic we have not addressed here.

Working in parallel with us, Llosa and Tuesta (2005) study determinacy and learnability in a version of the Clarida, Gali, and Gertler (2002) model we use. Whereas we emphasize the two country model they analyze the model from the point of view of the small open economy. Llosa and Tuesta (2005) study instrument rules more extensively than we do, including different forms of Taylor-type rules as in Bullard and Mitra (2002). The Llosa and Tuesta (2005) discussion of domestic inflation versus consumer price index inflation in the policy rule parallels some of our analysis, and we compare our results to theirs when appropriate.

1.4 Organization

We begin by presenting the basic model environment in the next section. We take up our analysis of the effects of policy on determinacy and learnability by first considering instrument rules under flexible exchange rates, simple descriptions of policy that allow us to develop some basic results

[2] Bullard and Schaling (2006) also discuss purchasing power parity rules.

and intuition, especially concerning "country by country" determinacy and learnability conditions. Policymakers using rules in this class might break the natural separability of country analysis in the model should they decide to react in part to international variables when setting monetary policy, and we develop a version of this situation. We then turn to targeting rules (also under flexible exchange rates), whereby the policy rule is inferred from an optimization exercise undertaken by each monetary authority. The nature of the optimization exercise will be important for our findings. The final portion of the paper takes up certain asymmetric situations associated with fixed exchange rates. One of these is the case of one country pegging its exchange rate to a second country which is following an independent monetary policy. We discuss our findings and directions for future research in the conclusion.

2 A two-country New Keynesian model

2.1 Overview

We employ the two-country model of Clarida, Gali, and Gertler (2002). This is one natural extension of the closed economy New Keynesian model to the open economy case in which two large economies are interacting, and so it provides a good starting point for the analysis of determinacy issues in the open economy. The model has a natural separation between countries that Clarida, Gali, and Gertler (2002) discuss in some detail. Roughly, after making certain adjustments to parameters accounting for the degree of openness of each economy, this version of the open economy New Keynesian model is qualitatively the same as the standard, Clarida, Gali, and Gertler (1999)-style closed economy New Keynesian model. We exploit this feature extensively in this paper.

2.2 Environment

We can provide only a brief discussion of the microfoundations of the model here—interested readers should consult Clarida, Gali, and Gertler (2002).

The two countries are labelled H and F. Preferences and technologies are the same in both countries. Each country has an intermediate goods sector which is subject to a Calvo-style sticky price friction along with a final goods sector which is competitive. Only final goods are traded. Preferences for consumption are defined over an aggregate $C_t = C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}$, with $0 \leq \gamma \leq 1$. The parameter γ is often described as the *degree of openness*, because as $\gamma \to 0$ ($\gamma \to 1$) the foreign (home) economy becomes vanishingly small, and all goods are produced and consumed at home (abroad). The model economy is log-linearized about a steady state and described by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma_o^{-1} \left[r_t - E_t \pi_{t+1} - \overline{rr}_t \right], \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_o \tilde{y}_t + u_t \tag{2}$$

where $\kappa_o = \gamma (\sigma - 1)$, $\sigma_o = \sigma - \kappa_o$, $\kappa = \sigma - \kappa_o + \phi$, $\lambda_o = \delta \kappa$, and $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. The variable $\tilde{y}_t$ represents the domestic output gap, π_t represents domestic producer price inflation, and r_t represents the short term nominal interest rate. The term $\overline{rr}_t$ is the domestic natural real interest rate (conditional on foreign output) given by

$$\overline{rr}_t = \sigma_o E_t \Delta \bar{y}_{t+1} + \kappa_o E_t \Delta y_{t+1}^{\star}$$

where $\Delta \bar{y}_{t+1}$ is the rate of growth of the domestic natural level of output and $\Delta y_{t+1}^{\star}$ is the rate of growth of the level of foreign output. The term u_t follows an $AR(1)$ process given by $u_t = \rho u_{t-1} + \epsilon_{u,t}$,with $0 \leq \rho < 1$, where $\epsilon_{u,t}$ is an $i.i.d.$ stochastic term.[3] The equations (1) and (2) have five fundamental parameters: The household discount factor β, a parameter controlling the curvature in preferences over consumption σ, a parameter controlling the curvature in preferences over leisure ϕ, the mass of agents or degree of openness γ, and the probability that a firm will not be able to change its price today θ, which we sometimes refer to as the degree of price stickiness. The foreign economy is described analogously as

$$\tilde{y}_t^{\star} = E_t \tilde{y}_{t+1}^{\star} - \sigma_o^{\star,-1} \left[r_t^{\star} - E_t \pi_{t+1}^{\star} - \overline{rr}_t^{\star} \right], \tag{3}$$

[3] For simplicity we keep the serial correlation parameter ρ the same for all shocks in the model.

$$\pi_t^\star = \beta E_t \pi_{t+1}^\star + \lambda_o^\star \tilde{y}_t^\star + u_t^\star \tag{4}$$

where $\kappa_o^\star = (1-\gamma)(\sigma-1)$, $\sigma_o^\star = \sigma - \kappa_o^\star$, $\kappa^\star = \sigma - \kappa_o^\star + \phi$, $\lambda_o^\star = \delta\kappa^\star$, and $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. In these equations $\tilde{y}_t^\star$ is the foreign output gap, $\pi_t^\star$ is foreign producer price inflation, and $r_t^\star$ is the foreign nominal interest rate. The term $\overline{rr}_t^\star$ is the foreign natural real interest rate (conditional on domestic output), given by

$$\overline{rr}_t^\star = \sigma_o^\star E_t \Delta \bar{y}_{t+1}^\star + \kappa_o^\star E_t \Delta y_{t+1}$$

where $\Delta \bar{y}_{t+1}^\star$ is the rate of growth of the foreign natural level of output and Δy_{t+1} is the rate of growth of the level of domestic output. The term $u_t^\star$ is analogously assumed to follow an $AR(1)$ process given by $u_t^\star = \rho u_{t-1}^\star + \epsilon_{u,t}^\star$ with $0 \leq \rho < 1$, where $\epsilon_{u,t}^\star$ is an *i.i.d.* stochastic term.

In equations (3) and (4), the fundamental parameters β, σ, ϕ, γ, and θ are all the same as in equations (1) and (2), reflecting the maintained assumption that the preferences and technologies in the two economies are the same. The only difference is that γ in (1) and (2) has been replaced by $1-\gamma$ in (3) and (4).

The nominal exchange rate e_t obeys consumer price index-based, or "aggregate" purchasing power parity, and is given by

$$\begin{aligned} e_t &= (p_{C,t} - p_{C,t}^\star) \\ &= (p_t + \gamma s_t) - (p_t^\star - \{(1-\gamma)s_t\}) \\ &= p_t - p_t^\star + s_t \end{aligned}$$

where p_t is shorthand for the domestic producer price level $p_{H,t}$, $p_t^\star$ is shorthand for the foreign producer price level $p_{F,t}^\star$, and where $p_{C,t}$ and $p_{C,t}^\star$ represent the home and foreign consumer price index, respectively. Finally, a simple expression links the terms of trade to movements in the output gap:

$$\begin{aligned} s_t &= (\tilde{y}_t - \tilde{y}_t^\star) + (\bar{y}_t - \bar{y}_t^\star) \\ &= (\tilde{y}_t - \tilde{y}_t^\star) + \bar{s}_t \end{aligned} \tag{5}$$

where $\bar{s}_t$ is the natural level of the terms of trade.

An advantage of this formulation is that the open economy effects in this model come through the composite parameters κ_o and $\kappa_o^\star$. The special cases where either $\gamma \to 0$ or $\gamma \to 1$ respectively place all the mass of agents in the home or the foreign economy. In these cases, the home or foreign economy behaves as if it were an isolated, closed economy, while the partner behaves as if it were a small open economy.[4] An isolated, closed economy corresponds to the ones that have been extensively analyzed in the New Keynesian literature.

2.3 Determinacy issues

As pointed out by Jensen (2002), since Sargent and Wallace (1975) showed that an interest rate peg rendered the price level indeterminate in a rational expectations IS-LM-AS model, there has been a lot of research in the issue of designing monetary policy in order to secure determinate rational expectations equilibria.

The model above is one where an interest rate peg would also lead to indeterminate equilbrium. To understand some of the intuition for this result, consider a sunspot-driven increase in inflation expectations, $E_t\pi_{t+1}$. As this does not affect the nominal interest rate r_t, the real interest rate falls. This stimulates demand and the output gap via equation (1). Through the interaction of the IS and Phillips curves, this implies an increase in current inflation that is larger than the increase in expected inflation. As the increase in inflation expectations is of arbitrary size, one cannot pin down a unique non-explosive rational expectations equilibrium (REE). The economy is consequently vulnerable to expectations-driven fluctuations, *a.k.a.* sunspot fluctuations.

To ensure determinacy and thus exclude the potential for inefficient, self-fulfilling fluctuations, some restrictions are typically required on the behavior of the nominal interest rate. In the remainder of the paper we will analyze the model under different scenarios for how these interest rates are determined by policymakers. We will begin with a simple specification that produces

[4] Clarida, Gali and Gertler (2001) and Gali and Monacelli (2002) analyze the case of a small open economy using a similar framework.

simple intuition, and later move to more complicated optimal policy specifications under a variety of assumptions on the nature of the optimization policymakers undertake.

3 Flexible exchange rates: instrument rules

3.1 Simple Taylor-type rules

3.1.1 The dynamic system

In this section we simply assume that the policymakers in each country follow Taylor-type policy rules given by

$$r_t = \varphi_\pi \pi_t + \varphi_y \tilde{y}_t \tag{6}$$

for the domestic economy, and by

$$r_t^\star = \varphi_\pi^\star \pi_t^\star + \varphi_y^\star \tilde{y}_t^\star \tag{7}$$

for the foreign economy, allowing the exchange rate to float. Importantly, the inflation terms in these rules refer to domestic producer price inflation (we discuss other possibilities below). By substituting (6) and (7) into equations (1) and (3), we can write the four equation system as follows. First, define $\mathcal{Z}_t = [\tilde{y}_t, \pi_t, \tilde{y}_t^\star, \pi_t^\star]'$ along with $\mathcal{V}_t = [\overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star]'$. Then write the system as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B} E_t \mathcal{Z}_{t+1} + \mathcal{X} \mathcal{V}_t \tag{8}$$

where $\mathcal{A}_0 = 0$,[5]

$$\mathcal{B} = \begin{bmatrix} B_{11} & \mathbf{0} \\ \mathbf{0} & B_{22} \end{bmatrix},$$

$$B_{11} = \frac{1}{\sigma_o + \varphi_y + \lambda_o \varphi_\pi} \begin{bmatrix} \sigma_o & 1 - \beta \varphi_\pi \\ \lambda_o \sigma_o & \lambda_o + \beta \left(\sigma_o + \varphi_y\right) \end{bmatrix},$$

$$B_{22} = \frac{1}{\sigma_o^\star + \varphi_y^\star + \lambda_o^\star \varphi_\pi^\star} \begin{bmatrix} \sigma_o^\star & 1 - \beta \varphi_\pi^\star \\ \lambda_o^\star \sigma_o^\star & \lambda_o^\star + \beta \left(\sigma_o^\star + \varphi_y^\star\right) \end{bmatrix},$$

[5] We stay consistent with Bullard and Mitra (2002) in allowing for constant terms.

where $\mathbf{0}$ is a 2×2 matrix of zeroes, and

$$\mathcal{X} = \begin{bmatrix} X_{11} & \mathbf{0} \\ \mathbf{0} & X_{22} \end{bmatrix},$$

with

$$X_{11} = \begin{bmatrix} \sigma_o^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$X_{22} = \begin{bmatrix} \sigma_o^{\star,-1} & 0 \\ 0 & 1 \end{bmatrix},$$

and where $\mathcal{V}_t$ follows a vector $AR(1)$ process with serial correlation given by the scalar ρ.

3.1.2 Determinacy

Because the four variables in this system are free in the terminology of Blanchard and Kahn (1980), we require all eigenvalues of $\mathcal{B}$ to be inside the unit circle for determinacy. Since $\mathcal{B}$ is block diagonal, this requirement means that the eigenvalues of B_{11} and B_{22} must be inside the unit circle. From a version of Proposition 1 in Bullard and Mitra (2002), this implies that the following two conditions must hold for determinacy in this system:

$$\lambda_o (\varphi_\pi - 1) + (1 - \beta) \varphi_y > 0 \tag{9}$$

and

$$\lambda_o^\star (\varphi_\pi^\star - 1) + (1 - \beta) \varphi_y^\star > 0. \tag{10}$$

These conditions are versions of the Taylor principle[6] for each country and depend on the household discount factor β, on the policy parameters in the Taylor-type rules in the two countries, and on the composite parameters λ_o and $\lambda_o^\star$. We can write the composite parameters as

$$\begin{aligned} \lambda_o &= \delta [\sigma + \phi - \gamma (\sigma - 1)], \\ \lambda_o^\star &= \delta [\sigma + \phi - (1 - \gamma)(\sigma - 1)]. \end{aligned}$$

[6] See Woodford (2001) for a discussion.

Thus the conditions (9) and (10) can be written as

$$\delta\left[\sigma+\phi-\gamma\left(\sigma-1\right)\right]\left(\varphi_{\pi}-1\right)+\left(1-\beta\right)\varphi_{y}>0 \tag{11}$$

and

$$\delta\left[\sigma+\phi-\left(1-\gamma\right)\left(\sigma-1\right)\right]\left(\varphi_{\pi}^{\star}-1\right)+\left(1-\beta\right)\varphi_{y}^{\star}>0. \tag{12}$$

The term in brackets is positive, so that if $\varphi_y = \varphi_y^{\star} = 0$, the conditions state that each central bank has to move nominal interest rates more than one-for-one in response to deviations of inflation from target. We have several remarks on conditions (11) and (12).

First, the conditions for the two economies are not the same except in the special case where policies are identical (in the sense that $\varphi_\pi = \varphi_\pi^{\star}$ and $\varphi_y = \varphi_y^{\star}$) and $\gamma = 1/2$, which would be interpreted as the case that the two economies are equally open.[7] Otherwise, the degree of openness differs and this translates into a difference in the two conditions. This means in particular that identical policy in the two countries, in the sense of identical values for the Taylor-type policy rule coefficients, may be enough to meet one determinacy condition but not the other.

Second, the policy parameters from a single country can only influence one of the two conditions. Thus policymakers from each country must separately meet conditions for determinacy: Determinacy conditions for worldwide rational expectations equilibrium are met, in some sense, "country by country."

We interpret these findings as follows. If the home country policymaker obeys the Taylor principle while the foreign policymaker does not, worldwide equilibrium will be indeterminate. Should a sunspot variable begin to influence expectations, then the foreign economy will endure endogenous volatility, but the home country will not due to the block diagonality of $\mathcal{B}$ which indicates that there is no feedback between the two economies. The intuition is that any international CPI inflation differential will cause the nominal exchange rate to adjust, exactly offsetting the foreign inflation problem, and

[7] If $\gamma = \frac{1}{2}$, there is no home bias in consumption.

exactly insulating the home country. This result relies heavily on the idea that the two Taylor rules react to domestic producer price inflation, which has no imported component, as opposed to consumer price inflation, which does have an imported component. With CPI inflation in the policy rules, or with a fixed exchange rate, this will no longer be the case. We discuss these possibilities below.

3.1.3 Learnability

We now turn to the learnability of rational expectations equilibrium for cases where that equilibrium is unique. We allow the expectations in equation (8) to initially be different from rational expectations.[8] The MSV solution of equation (8) is given by

$$\mathcal{Z}_t = \bar{\mathcal{A}} + \bar{\mathcal{C}}\mathcal{V}_t$$

where the conformable matrix $\bar{\mathcal{A}}$ is null and

$$\bar{\mathcal{C}} = (I - \rho\mathcal{B})^{-1}\mathcal{X}.$$

We endow agents with a *perceived law of motion*

$$\mathcal{Z}_t = \mathcal{A} + \mathcal{C}\mathcal{V}_t \tag{13}$$

where $\mathcal{A}$ and $\mathcal{C}$ are conformable. Using this perceived law of motion and assuming time t information $(1, \overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star)'$ we can calculate

$$E_t\mathcal{Z}_{t+1} = \mathcal{A} + \mathcal{C}\rho\mathcal{V}_t.$$

Substituting this into equation (8) yields the *actual law of motion*

$$\begin{aligned} \mathcal{Z}_t &= \mathcal{B}(\mathcal{A} + \mathcal{C}\rho\mathcal{V}_t) + \mathcal{X}\mathcal{V}_t \\ &= \mathcal{B}\mathcal{A} + (\mathcal{B}\mathcal{C}\rho + \mathcal{X})\mathcal{V}_t. \end{aligned}$$

[8] Preston (2003) considers deriving the fundamental equations of models in this class assuming agents are learning. Under his interpretation of the microfoundations, the equations are altered and long-horizon forecasts matter. We think it would be interesting to carry out an analysis of this type for the open economy case.

We then define a map T from the perceived law of motion to the actual law of motion as

$$T(\mathcal{A},\mathcal{C}) = (\mathcal{B}\mathcal{A},\mathcal{B}\mathcal{C}\rho+\mathcal{X}).$$

Expectational stability is attained if the differential equation

$$\frac{d}{d\tau}(\mathcal{A},\mathcal{C}) = T(\mathcal{A},\mathcal{C}) - (\mathcal{A},\mathcal{C})$$

is locally asymptotically stable at $(\bar{\mathcal{A}},\bar{\mathcal{C}})$. Results in Evans and Honkapohja (2001) establish that under weak conditions, expectational stability governs stability in the real-time learning dynamics.

We use Proposition 10.3 in Evans and Honkapohja (2001) to calculate the condition for expectational stability. According to the proposition, the condition for expectational stability is that the real parts of the eigenvalues of the matrices $\mathcal{B}$ and $\rho\mathcal{B}$ are less than unity. Because $0 \leq \rho < 1$, we need only check the real parts of the eigenvalues of $\mathcal{B}$. Also, because of the block diagonality of $\mathcal{B}$, the expectational stability condition can be calculated country by country, that is, via B_{11} and B_{22}, and by a version of Proposition 2 in Bullard and Mitra (2002) yields conditions (11) and (12). This means that both countries must meet the open economy version of the Taylor principle in order for the world equilibrium to be learnable. It also means that the conditions for determinacy are the same as the conditions for learnability in the special case where both countries follow simple Taylor-type instrument rules. This is known not to be true in general in models in this class with alternative instrument rules, but it provides a good benchmark.[9]

3.1.4 Quantitative effects

As stressed by Clarida, Gali, and Gertler (2001, 2002), the nature of the policy problem faced by each country in this open economy framework is isomorphic to the closed economy case, but there are nevertheless quantitative

[9] An example of a case in which determinacy and learnability conditions do not coincide is when the policy authorities use a Taylor-type policy rule but react to lagged information on inflation and the output gap. See Bullard and Mitra (2002). For a wider variety of Taylor-type instrument rules in a similar model, see Llosa and Tuesta (2005).

consequences. Figure 1 illustrates. Here the calibration has been chosen so that the domestic economy collapses to the one studied by Woodford (2003) when the openness parameter $\gamma \rightarrow 0$. Woodford's (2003) values have been widely used and provide a simple benchmark. The discount factor $\beta = 0.99$. When $\gamma \rightarrow 0$, $\sigma_o \rightarrow \sigma$ and we set this to Woodford's value of $\sigma = 0.157$. The coefficient λ_o would correspond to a value of $\lambda_o = 0.024$ in the Woodford calibration. When $\gamma \rightarrow 0$, $\kappa_o \rightarrow 0$ so that $\kappa \rightarrow \sigma + \phi$, and $\lambda_o = \delta\left(\sigma + \phi\right)$, with $\delta = \left[\left(1-\theta\right)\left(1-\beta\theta\right)\right]/\theta$. We follow Woodford (2003) and set ϕ to the nearly linear value 0.11. Given other parameters, this means that $\theta = 0.745$ to obtain $\lambda_o = 0.024$.

The figure plots (11) as a function of φ_π and φ_y using this calibration for values of γ between zero and unity. Since (12) is the same condition for the foreign country with γ replaced by $1-\gamma$, one can view the lines in Figure 1 as representing this condition as well. The line labelled $\gamma = 0$ represents the case when the home country is closed and corresponds to the condition from Bullard and Mitra (2002, their Figure 1). The determinacy and learnability condition for the foreign country would then correspond to the line labelled $\gamma = 1$ (that is, $1-\gamma$ would equal one if $\gamma = 0$).[10] Thus the small open foreign economy would have to choose its Taylor rule coefficients to the northeast of this line in the figure, while the large closed home economy would only have to choose its Taylor rule coefficients to the northeast of the line labelled $\gamma = 0$. Failure of either country to abide by its condition would produce indeterminacy and the possibility of sunspot fluctuations in the world equilibrium. These lines are closer together if the degree of openness γ is intermediate between zero and one, as illustrated by the lines labelled $\gamma = 1/3$ and $\gamma = 2/3$. For $\gamma = 1/2$, the conditions for determinacy and learnability in the two countries would be identical.

One of the main implications of Figure 1 is that the open economy lines with γ greater than zero all lie above the closed economy line, so that con-

[10] Llosa and Tuesta (2005) focus on the small open economy and depict the same line in their Figure 1 for their domestic inflation rule, which would correspond to this case. Their model is similar to the one used here and has a similar calibration.

Figure 1

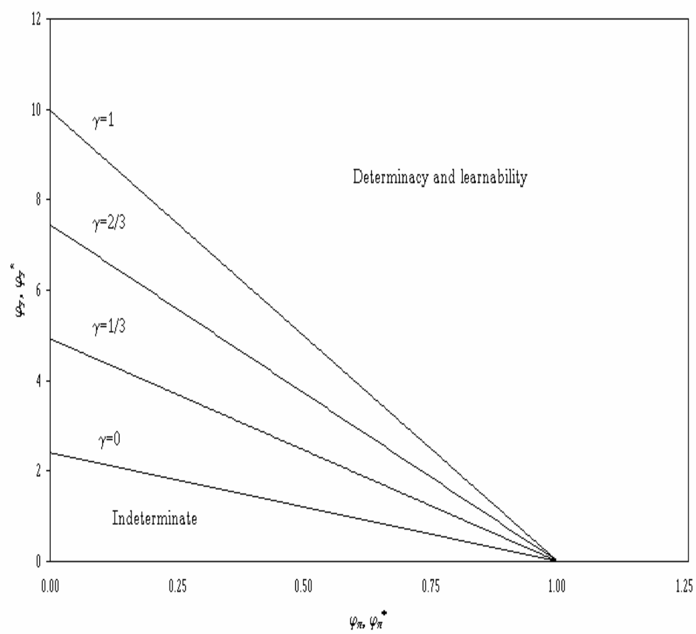

Figure 1: The conditions for determinacy and learnability when each monetary authority uses a simple contemporaneous data Taylor rule. The more open economy will have a steeper tradeoff in the Figure.

ditions for determinacy and learnability become more stringent when open economy considerations are introduced. A central bank analyzing its economy as if it were closed might mistakenly chose Taylor rule coefficients that are too small to deliver determinacy and learnability of equilibrium.

3.2 Instrument rules with international variables

3.2.1 Consumer versus producer price inflation

In Bullard and Schaling (2006) we show that instrument rules which include responses to international economic conditions induce international feedback between the two countries where there would otherwise be no such feedback. In this section we briefly summarize this argument. We begin by supposing that each country pursues a Taylor-type rule featuring consumer price index, or CPI, inflation instead of domestic producer price inflation.[11] This is intuitively plausible as in an open economy CPI inflation, not domestic producer price inflation, is often the variable of interest for the monetary authority. The monetary policy rule in the home country is given by

$$r_t = \varphi_\pi \pi_{C,t} + \varphi_y \tilde{y}_t, \tag{14}$$

and the monetary authority in the foreign economy pursues

$$r_t^\star = \varphi_\pi^\star \pi_{C,t}^\star + \varphi_y^\star \tilde{y}_t^\star \tag{15}$$

where

$$\pi_{C,t} = \pi_t + \gamma s_t \tag{16}$$

is home CPI inflation, and

$$\pi_{C,t}^\star = \pi_t^\star - (1-\gamma) s_t \tag{17}$$

is foreign CPI inflation, and where s_t is the terms of trade.[12] The inflation targets of the monetary authorities implicit in these specifications would then

[11] This is also the second rule analyzed by Llosa and Tuesta (2005) for their small open economy analysis.

[12] In our two-country model following Clarida, Gali and Gertler (2002, p. 882) the home consumption price index can be written as $P_{C,t} = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma}$. Using the definition of

be in terms of CPI inflation. Thus, responding to CPI inflation is equivalent to having a conventional Taylor-type rule augmented by a third term which is the *terms of trade*. The terms of trade is in turn a reflection of foreign output via equation (5). The policymakers in each country are naturally reacting to developments in the partner economy because some fraction of the goods being consumed at home are being produced abroad. Thus the policymaker reaction to CPI inflation introduces international feedback between the two countries that does not exist under Taylor-type rules with domestic producer price inflation.

As a consequence the key matrix $\mathcal{B}$ is no longer block diagonal (B_{12} and B_{21} are no longer null). We stress that the loss of the block diagonality of this matrix is induced by policy alone. Policymakers are reacting to consumer rather than producer prices and this is creating international linkages that would otherwise not exist. This means, in principle, that policy parameters in one country will influence all aspects[13] of worldwide conditions for determinacy and learnability. The separability of these conditions across borders breaks down—in spite of flexible exchange rates and PPP—because the policymakers are reacting to variables that have foreign components. In section 5 we consider the case where the home country irrevocably pegs its exchange rate to the foreign currency, which is another rule that responds to international variables and changes the implied form of the key matrix $\mathcal{B}$.

---

the terms of trade $S_t = P_{F,t}/P_{H,t}$ this equation can be written as $P_{C,t} = k^{-1}P_{H,t}S_t^{\gamma}$, where $k = (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$. For the foreign country one gets $P^*_{C,t} = k^{-1}(P^*_{H,t})^{1-\gamma}(P^*_{F,t})^{\gamma} = k^{-1}P^*_{F,t}\left(\frac{1}{S_t}\right)^{1-\gamma}$. Taking logs of these equations yields $p_{C,t} = p_t + \gamma s_t$ and $p^*_{C,t} = p^*_t - (1-\gamma)\, s_t$. Taking first differences and normalizing the initial $(t-1)$ price levels to zero, these equations can then be rewritten as the home and foreign CPI inflation equations in the main text.

[13] That is, all four eigenvalues.

4 Flexible exchange rates: targeting rules

4.1 Overview

In this section we assume that the central bank sets policy optimally. This means that the nominal interest rate is set according to a rule inferred from an explicit optimization exercise.[14] We investigate the benchmark case of discretion[15] and consider two implementation strategies of the first-order condition along the lines of Evans and Honkapohja (2003b) (hereafter EH). The various implementation strategies may or may not provide determinacy and learnability of rational expectations equilibrium. In the next sub-section we focus on the non-cooperative case in which each policymaker sets monetary policy autonomously. We will turn to the cooperative case in Section 4.3.

4.2 Non-cooperative discretionary policy

4.2.1 The policy problem

Importantly, as Clarida, Gali, and Gertler (2002) mention, the correct inflation variable for the policymaker following a non-cooperative discretionary policy is domestic producer price inflation. This means π_t will enter into the objective for the domestic policymaker.[16] Under discretion the monetary authority will choose a sequence of current and future short-term nominal interest rates to minimize loss defined by

$$L = (1-\gamma)\,\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{2}[\left(\pi_\tau - \pi^T\right)^2 + \alpha_o \left(\tilde{y} - \tilde{y}^T\right)^2] \tag{18}$$

[14] For a recent discussion about targeting versus instrument rules see Svensson (2003), McCallum and Nelson (2005), as well as Svensson (2005).

[15] For a discussion of determinacy issues for optimal rules in a closed economy where the timing protocol is commitment, see Giannoni and Woodford (2002a,b).

[16] The reason is that by targeting a combination of PPI inflation and the domestic output gap (in line with Clarida, Gali and Gertler 2002) – where the weight on the latter, α_0 , is not free but given by $\alpha_0 = \frac{\delta\kappa}{\xi} = \frac{\lambda_0}{\xi}$, the policymaker actually mimics targeting CPI inflation – which in turn has its micro foundations in social welfare.

with $\Lambda \equiv \frac{\xi}{\delta}$ and $\alpha_o \equiv \frac{\delta\kappa}{\xi} = \frac{\lambda_o}{\xi}$, and where π^T and $\tilde{y}^T$ are target values which we will often view as being zero. The parameter ξ represents the price elasticity of demand for intermediate goods in Clarida, Gali, and Gertler (2002). The minimization is subject to

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma_o^{-1}(r_t - E_t\pi_{t+1} - \overline{rr}_t), \tag{19}$$

and

$$\pi_t = \beta E_t\pi_{t+1} + \lambda_o\tilde{y}_t + u_t, \tag{20}$$

where

$$\overline{rr}_t = \sigma_o E_t\Delta\bar{y}_{t+1} + \kappa_o E_t\Delta y^{\star}_{t+1},$$

and

$$u_t = \rho u_{t-1} + \epsilon_t.$$

We can reformulate the problem above as choosing the indirect control variable $\{\tilde{y}_\tau\}_{\tau=t}^{\infty}$ to minimize (18) where the central bank treats $E_t\pi_{t+1}$ as given. We write the central bank's Lagrangian as[17]

$$\mathcal{L} = E_t\sum_{\tau=t}^{\infty} -\frac{\beta^{\tau-t}}{2}\Lambda(1-\gamma)\times \left[\left(\pi_\tau - \pi^T\right)^2 + \alpha_o\left(\tilde{y}_\tau - \tilde{y}^T\right)^2 - \beta^{\tau-t}\mu_\tau\left(\pi_\tau - \beta E_\tau\pi_{\tau+1} - \lambda_o\tilde{y}_\tau - u_\tau\right)\right]$$

where π_t and $E_t\pi_{t+1}$ are state variables. The first order conditions are

$$\frac{\partial\mathcal{L}}{\partial\tilde{y}_t} = -\alpha_o\Lambda(1-\gamma)\left(\tilde{y}_t - \tilde{y}^T\right) + \mu_t\lambda_o = 0, \tag{21}$$

$$\frac{\partial\mathcal{L}}{\partial\pi_t} = -\Lambda(1-\gamma)\left(\pi_t - \pi^T\right) - \mu_t = 0. \tag{22}$$

From equation (21) we have $\mu_t = \frac{\alpha_o\Lambda(1-\gamma)}{\lambda_o}\left(\tilde{y}_t - \tilde{y}^T\right)$. Using this result in (22) yields

$$\tilde{y}_t - \tilde{y}^T = -\frac{\lambda_o}{\alpha_o}\left(\pi_t - \pi^T\right). \tag{23}$$

[17] For a discussion of the relative merits of dynamic programming and the Lagrange method see Schaling (2001). For applications of the latter to a non-linear optimization problem, and a regime switching model see Schaling (2004) and Bullard and Schaling (2001), respectively.

It is well-known in the closed economy literature that there are a variety of strategies for implementing conditions like (23), and that these strategies can have differing implications for determinacy and learnability. We now turn to two implementations for the open economy model in order to see how these results may or may not be altered.

4.2.2 An open economy expectations-based optimal rule

Combining the first-order condition (23) with equation (19) we obtain

$$E_t\tilde{y}_{t+1} - \sigma_o^{-1}(rr_t - \overline{rr}_t) - \tilde{y}^T = -\frac{\lambda_o}{\alpha_o}\left(\pi_t - \pi^T\right)$$

(where rr_t is the ex ante real interest rate $r_t - E_t\pi_{t+1}$). This can be written as

$$rr_t - \overline{rr}_t = \frac{\lambda_o\sigma_o}{\alpha_o}\left(\pi_t - \pi^T\right) + \sigma_o\left(E_t\tilde{y}_{t+1} - \tilde{y}^T\right).$$

Substituting for π_t from equation (20) we obtain

$$rr_t - \overline{rr}_t = \frac{\lambda_o\sigma_o}{\alpha_o}\left(\beta E_t\pi_{t+1} + \lambda_o\tilde{y}_t + u_t - \pi^T\right) + \sigma_o\left(E_t\tilde{y}_{t+1} - \tilde{y}^T\right).$$

Eliminating output via (19) yields

$$r_t - \overline{rr}_t = \delta_{0,0} + \delta_{\pi,0}E_t\pi_{t+1} + \delta_{y,0}E_t\tilde{y}_{t+1} + \delta_{u,0}u_t \tag{24}$$

where the coefficients are given by

$$\delta_{0,0} = -\frac{\sigma_o\left(\lambda_o\pi^T + \alpha_o\tilde{y}^T\right)}{\alpha_o + \lambda_o^2}, \tag{25}$$

$$\delta_{\pi,0} = \frac{\alpha_o + \lambda_o^2 + \sigma_o\lambda_o\beta}{\alpha_o + \lambda_o^2}, \tag{26}$$

$$\delta_{y,0} = \sigma_o, \tag{27}$$

$$\delta_{u,0} = \frac{\lambda_o\sigma_o}{\alpha_o + \lambda_o^2}. \tag{28}$$

Equation (24) is an example of a targeting rule, as discussed for example in Woodford (2003, pp. 290-295). This rule is an open economy version of what Evans and Honkapohja (2003b) call an *expectations-based optimal rule.* By construction, it implements what Evans and Honkapohja label 'optimal discretionary policy' in every period and for all values of private expectations. If $\gamma \rightarrow 0$ our (more general) open economy rule collapses to their version.

Setting the targets π^T and y^T to zero, the world economy can be written as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B}E_t\mathcal{Z}_{t+1} + \mathcal{X}\mathcal{V}_t,$$

where $\mathcal{Z}_t = [\tilde{y}_t, \pi_t, \tilde{y}_t^\star, \pi_t^\star]'$, and the key matrix $\mathcal{B}$ is given by

$$\mathcal{B} = \begin{bmatrix} B_{11} & \mathbf{0} \\ \mathbf{0} & B_{22} \end{bmatrix},$$

where

$$B_{11} = \begin{bmatrix} 0 & \frac{-\beta\lambda_o}{\alpha_o+\lambda_o^2} \\ 0 & \frac{\alpha_o\beta}{\alpha_o+\lambda_o^2} \end{bmatrix}$$

and

$$B_{22} = \begin{bmatrix} 0 & \frac{-\beta\lambda_o^\star}{\alpha_o^\star+(\lambda_o^\star)^2} \\ 0 & \frac{\alpha_o^\star\beta}{\alpha_o^\star+(\lambda_o^\star)^2} \end{bmatrix}.$$

Because $\mathcal{B}$ is block diagonal, determinacy conditions will be have to be met country by country. A unique rational expectations equilibrium exists since

$$0 < \frac{\alpha_o\beta}{\alpha_o + \lambda_o^2} < 1$$

for the domestic economy, and

$$0 < \frac{\alpha_o^\star\beta}{\alpha_o^\star + (\lambda_o^\star)^2} < 1$$

for the foreign economy. What we have in this section is different from section 3.1.2 on determinacy of instrument rules. Although in both cases we have a $\mathcal{B}$-matrix that is block diagonal, under an Evans and Honkapohja (2003b) style expectations-based optimal rule (adhered to in both countries), we have unconditional determinacy of the two-country world economy. There is no possibility of indeterminacy of the 'world equilibrium' like we had in section

3.1.2. So, there is nothing like a Taylor principle that needs to be adhered to.

Next, we turn to the learnability of the rational expectations equilibrium. Because of the block diagonality of $\mathcal{B}$, the expectational stability condition can be calculated country by country, that is, via B_{11} and B_{22}. By a version Proposition 3 in Evans and Honkapohja (2003b), we find that for all parameter values the REE of the two-country world economy under world-wide adherence to open economy expectations-based optimal rules is stable under least squares learning by private agents. So, we find that the EH (2003b) result that incorporation of observed private sector expectations into the policymaker's optimal rule can overcome expectational stability problems carries over to the two-country environment of Clarida, Gali and Gertler (2002) if the relevant rules are modified to take due recognition of open economy effects.

Other implementations of (23) are known to have poor properties with respect to learnability and determinacy, however, and we now turn to this case.

4.2.3 An open economy fundamentals-based optimal rule

A fundamentals-based policy rule implementing (23) generates a different reduced form. To obtain an optimal interest rate rule under rational expectations conjecture a solution of the form

$$\begin{aligned} \tilde{y}_t &= a_1 + d_1 u_t, \\ \pi_t &= a_2 + d_2 u_t, \end{aligned}$$

for the domestic economy, with an analogous conjectured solution for the foreign economy. The MSV solution has

$$\begin{aligned}\bar{a}_1 &= \frac{-\delta_{0,0} - (\delta_{\pi,0} - 1)\,\bar{a}_2}{\sigma_o},\\ \bar{d}_1 &= \frac{-\rho\bar{d}_2\,(\delta_{\pi,0} - 1) - \delta_{u,0}}{\sigma_o},\\ \bar{a}_2 &= \frac{-\delta_{0,0}\lambda_o}{\sigma_o\,(1-\beta\rho) + \rho\lambda_o\,(\delta_{\pi,0} - 1)},\\ \bar{d}_2 &= \frac{\sigma_o - \lambda_o\delta_{u,0}}{\sigma_o\,(1-\beta\rho) + \rho\lambda_o\,(\delta_{\pi,0} - 1)}.\end{aligned}$$

where $\delta_{0,0},\ \delta_{\pi,0},\ \delta_{y,0}, \delta_{u,0}$ are given by (25) through (28) respectively. The policy feedback rule is then

$$r_t = \psi_0 + \psi_u u_t + \overline{rr}_t, \tag{29}$$

with

$$\psi_0 = \delta_{0,0} + \delta_{\pi,0}\bar{a}_2 + \delta_{y,0}\bar{a}_1$$

and

$$\psi_u = \rho\left(\delta_{\pi,0}\bar{d}_2 + \delta_{y,0}\bar{d}_1\right) + \delta_{u,0}.$$

This is sometimes called the fundamentals form of the RE-optimal policy rule. It is known that this interest rate rule is associated with indeterminacy in the closed economy case.[18]

The world economy can be written as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B}E_t\mathcal{Z}_{t+1} + \mathcal{X}\mathcal{V}_t,$$

with $\mathcal{B}$ block diagonal,

$$B_{11} = \begin{bmatrix} 1 & \sigma_o^{-1} \\ \lambda_o & \beta + \lambda_o\sigma_o^{-1} \end{bmatrix},$$

and

$$B_{22} = \begin{bmatrix} 1 & \sigma_o^{\star,-1} \\ \lambda_o^{\star} & \beta + \lambda_o^{\star}\sigma_o^{\star,-1} \end{bmatrix}.$$

[18] See for instance Woodford (1999, 2003) and Svensson and Woodford (2003).

Determinacy requires $|a_0| < 1$ and $|a_1| < 1 + a_0$ in $v^2 + a_1 v + a_0 = 0$, the characteristic equation for B_{11} and B_{22}, respectively. For the domestic economy (and analogously for the foreign economy),

$$a_1 = \frac{-(\lambda_o + \sigma_o + \beta\sigma_o)}{\sigma_o}$$

and $a_0 = \beta$. The condition $|a_1| < 1 + a_0$ is never met under maintained assumptions and so worldwide equilibrium is indeterminate, as in the domestic economy case discussed by Evans and Honkapohja (2003). The MSV solution will also be unstable in the learning dynamics. We conclude that the method of implementing (23) will matter in the open economy case just as it does in the closed economy.

4.3 Cooperative discretionary policy

4.3.1 Overview

As we have seen, block diagonality breaks down if policymakers put weight on international variables in their policy rules, or, in a targeting approach, in their objective function. That is exactly what happens should policymakers in each country attempt to pursue the gains to cooperation which normally exist in this model. We now turn to this issue.

Clarida, Gali and Gertler (2002) study cooperation in the context of their New Keynesian model and are thus part of what Canzoneri, Cumby and Diba (2004) call second generation models of policy coordination. Canzoneri, *et al.*, state that the gains from coordination are larger in second generation models than in first generation models.[19]

Clarida, Gali, and Gertler (2002) show in their Proposition 3 that gains to international policy cooperation will accrue to both countries when $\sigma > 1$ and each country follows a rule dictated by the solution to a joint optimization problem. We now follow Clarida, Gali, and Gertler (2002) and discuss the

[19] For a survey of the lessons from the first generation literature see Nolan and Schaling (1996).

prospects for determinacy and learnability if each country attempts to pursue the gains from cooperation.

4.3.2 The policy problem

Clarida, Gali, and Gertler (2002) define cooperation to mean that the two central banks in the model agree to maximize a weighted average of the utility of the home and foreign households under discretion. The weights are naturally γ and $1-\gamma$. Both governments refrain from creating a surprise appreciation, and hand out employment subsidies that just offset the monopolistic competition distortion. The monetary authorities jointly maximize an approximation to weighted household utility given by

$$L = -\frac{1}{2}\Lambda E_0 \sum_{t=0}^{\infty} \beta^t \times \left[(1-\gamma)\left(\pi_t^2 + \alpha\left(\tilde{y}_t^C\right)^2\right) + \gamma\left(\left(\pi_t^{\star}\right)^2 + \alpha^{\star}\left(\tilde{y}_t^{\star,C}\right)^2\right) - 2\Phi \tilde{y}_t^C \tilde{y}_t^{\star,C} \right],$$

where $\Lambda = \xi/\delta$, $\alpha_o = \lambda_o/\xi$, $\alpha_o^{\star} = \lambda_o^{\star}/\xi$,

$$\Phi \equiv \frac{\delta(1-\sigma)\gamma(1-\gamma)}{\xi},$$

and $\tilde{y}_t^C$ and $\tilde{y}_t^{\star,C}$ are the output gaps defined under cooperation as the deviation, in percent, of output from the cooperative steady state level for the domestic and foreign economy, respectively.[20]

The first order conditions for this problem can then be written in terms of standard output gaps as

$$\begin{aligned} \tilde{y}_t &= -\xi\left(\pi_t + \frac{\kappa_o}{\kappa}\pi_t^{\star}\right), \\ \tilde{y}_t^{\star} &= -\xi\left(\pi_t^{\star} + \frac{\kappa_o^{\star}}{\kappa^{\star}}\pi_t\right). \end{aligned}$$

[20] For more details see Clarida, Gali, and Gertler (2002).

4.3.3 One implementation

Combining these conditions with (1) and (3) gives optimal cooperative policy rules

$$
\begin{aligned}
r_t &= \vartheta E_t \pi_{t+1} + \frac{\kappa_o}{\kappa}(\vartheta - 1) E_t \pi^\star_{t+1} + \overline{rr}_t, \\
r^\star_t &= \vartheta^\star E_t \pi^\star_{t+1} + \frac{\kappa^\star_o}{\kappa^\star}(\vartheta^\star - 1) E_t \pi^\star_{t+1} + \overline{rr}^\star_t,
\end{aligned}
$$

where

$$
\begin{aligned}
\vartheta &= 1 + \frac{\xi \sigma_o (1-\rho)}{\rho}, \\
\vartheta^\star &= 1 + \frac{\xi \sigma^\star_o (1-\rho)}{\rho}.
\end{aligned}
$$

The dynamic system, determinacy, and learnability The world economy can be written as

$$
\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B} E_t \mathcal{Z}_{t+1} + \mathcal{X}\mathcal{V}_t,
$$

where the key matrix $\mathcal{B}$ is

$$
\mathcal{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}
$$

and

$$
\begin{aligned}
B_{11} &= \begin{bmatrix} 1 & -\sigma_o^{-1}(\vartheta - 1) \\ \lambda_o & \beta - \sigma_o^{-1}\lambda_o(\vartheta - 1) \end{bmatrix}, \\
B_{12} &= \begin{bmatrix} 0 & -\sigma_o^{-1}\kappa_o\kappa^{-1}(\vartheta - 1) \\ 0 & -\sigma_o^{-1}\lambda_o\kappa_o\kappa^{-1}(\vartheta - 1) \end{bmatrix}, \\
B_{22} &= \begin{bmatrix} 1 & -\sigma_o^{\star,-1}(\vartheta^\star - 1) \\ \lambda^\star_o & \beta - \sigma_o^{\star,-1}\lambda^\star_o(\vartheta^\star - 1) \end{bmatrix},
\end{aligned}
$$

and

$$
B_{21} = \begin{bmatrix} 0 & -\sigma_o^{\star,-1}\kappa^\star_o\kappa^{\star,-1}(\vartheta^\star - 1) \\ 0 & -\sigma_o^{\star,-1}\lambda^\star_o\kappa^\star_o\kappa^{\star,-1}(\vartheta^\star - 1) \end{bmatrix}.
$$

Determinacy properties will again depend on the eigenvalues of the matrix $\mathcal{B}$. The lack of block diagonality indicates that policy in each country will influence determinacy properties. The four eigenvalues of $\mathcal{B}$ are given by

$$v_{1,\pm} = \frac{(1+\beta)\,\rho + \delta\,(1+\phi)\,(\rho-1)\,\xi}{2\rho} \pm \frac{\left[(\delta\,[1+\phi]\,\xi - \rho\,[1+\beta+\delta\,(1+\phi)\,\xi])^2 - 4\beta\rho^2\right]^{1/2}}{2\rho}$$

and

$$v_{2,\pm} = \frac{(1+\beta)\,\rho + \delta\,(\sigma+\phi)\,(\rho-1)\,\xi}{2\rho} \pm \frac{\left[(\delta\,[\sigma+\phi]\,\xi - \rho\,[1+\beta+\delta\,(\sigma+\phi)\,\xi])^2 - 4\beta\rho^2\right]^{1/2}}{2\rho}.$$

These eigenvalues are independent of γ, the degree of openness. This is because the two economies are following a cooperative policy which takes the size of each economy into account. Determinacy does not always hold. In particular,

$$\begin{aligned} \lim_{\rho\to 0} v_{1,-} &= -\infty, \\ \lim_{\rho\to 0} v_{2,-} &= -\infty. \end{aligned}$$

Unless the serial correlation in the shock is sufficiently large, this cooperative policy will generate indeterminacy.[21]

We use the baseline calibration with the addition of $\xi = 7.88$ implying a markup of about 15 percent, and we report results for values of ρ. The cutoff value for the serial correlation parameter is $\rho_c \approx 0.165$.[22] Values less than this will create indeterminacy given the baseline calibration. Should the shock process become something more like white noise, optimal policy cooperation implemented in this way will be associated with indeterminacy.

[21] This is a version of a similar result for the closed economy in Evans and Honkapohja (2003b).

[22] For $\sigma = 2$, $\rho_c \approx 0.28$.

For determinate cases, we verified numerically at baseline parameter values that expectational stability holds.

One might wonder if full cooperation is really a good positive model for world monetary policy. In the international policy arena, we seem to observe a variety of strategies in play. So far in the paper we have only considered certain types of symmetry in policy, but there are also interesting asymmetric situations. We now turn to one of these.

5 Fixed exchange rates: asymmetry in monetary policy

5.1 An exchange rate peg

5.1.1 Overview

In this section we suppose the home country targets its nominal exchange rate e *vis-a-vis* the foreign country. We assume the foreign economy sets its monetary policy based on its own domestic considerations. The home country gives up its domestic monetary autonomy in return for "importing monetary stability" from the foreign, anchor country.

This is a leading example of an *asymmetric* exchange rate regime, as only the anchor country's variables matter for its interest rate (depending on the nature of the policy adopted there), and the home country simply sets its interest rate to ensure it realizes a fixed exchange rate. The home country in setting policy takes foreign monetary conditions into account, but the foreign country need not incorporate the home country's conditions in its own monetary policy stance. This arrangement is similar to the regimes adopted by some European countries prior to economic and monetary union and to the present peg of the Chinese renminbi to the U.S. dollar.

5.1.2 The policy problem

The home country minimizes

$$(1-\gamma)\,\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\frac{1}{2}[\left(e_\tau - e^T\right)]^2. \tag{30}$$

The minimization is subject to

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma_o^{-1}\left[r_t - E_t\pi_{t+1} - \overline{rr}_t\right], \tag{31}$$

$$\pi_t = \beta E_t\pi_{t+1} + \lambda_o\tilde{y}_t + u_t, \tag{32}$$

$$\bar{r}r_t = \sigma_o E_t\Delta\bar{y}_{t+1} + \kappa_o E_t\Delta y^\star_{t+1},$$

$$u_t = \rho u_{t-1} + \epsilon_t,$$

$$e_t = e_{t-1} + s_t - s_{t-1} + \pi_t - \pi^\star_t,$$

and

$$s_t = (\tilde{y}_t - \tilde{y}^*_t) + \bar{s}_t.$$

For ease of exposition we normalize the initial levels of the nominal exchange rate and terms of trade at zero ($e_{t-1} = s_{t-1} = 0$), so that

$$e_t = s_t + \pi_t - \pi^\star_t. \tag{33}$$

In what follows we normalize the exchange rate target at zero ($e^T = 0$). From (30) the first-order condition then becomes $e_t = 0$, which combined with (33) implies

$$s_t = -(\pi_t - \pi^\star_t). \tag{34}$$

The intuition behind (34) is the following. The nominal exchange rate obeys CPI-based purchasing power parity and, after appropriate normalization, is given by $e_t = \pi_t - \pi^\star_t + s_t$. In order to prevent fluctuations in e_t, the home central bank should manipulate the terms of trade s_t, which it can affect via the domestic output gap, in such a way as to offset the GDP deflator-based inflation differential. Thus we have (34).

Since the terms of trade can be affected by the domestic output gap, which in turn is affected by the home nominal interest rate, the home central bank should try to achieve a level of the home output gap given by

$$\tilde{y}_t = -(\pi_t - \pi_t^{\star}) + \tilde{y}_t^{\star} - \bar{s}_t. \tag{35}$$

Equation (35) is obtained by substituting the expression for the terms of trade into the first-order condition and rearranging.

5.1.3 The policy rule

Substituting (32) into (35), we obtain the home country's optimal monetary policy rule in terms of its indirect control $\tilde{y}_t$

$$\tilde{y}_t = -\frac{\beta}{1+\lambda_o} E_t \pi_{t+1} + \frac{1}{1+\lambda_o}(\pi_t^{\star} + \tilde{y}_t^{\star}) - \frac{1}{1+\lambda_o}(\bar{s}_t + u_t). \tag{36}$$

The home interest rate reaction function can be obtained by combining (36) with (31) to obtain

$$r_t - \overline{rr}_t = \delta'_{\pi,0} E_t \pi_{t+1} + \delta'_{y,0} E_t \tilde{y}_{t+1} + \delta'_{\star,0} \pi_t^{\star} + \delta'_{\star,0} \tilde{y}_t^{\star} + \delta'_{u,0} \left(u_t + \bar{s}_t\right), \tag{37}$$

where the coefficients are given by

$$\delta'_{\pi,0} = \frac{(1+\lambda_o) + \sigma_o \beta}{1+\lambda_o}, \tag{38}$$

$$\delta'_{y,0} = \sigma_o, \tag{39}$$

and

$$\delta'_{\star,0} = -\delta'_{u,0} = -\frac{\sigma_o}{1+\lambda_o}. \tag{40}$$

The rule (37) describes the optimal home monetary reaction function that implements its monetary policy of pegging the exchange rate to the foreign anchor country.[23]

We substitute the home country's policy rule (37) into (31). This implies

$$\tilde{y}_t = \sigma_o^{-1}(1 - \delta'_{\pi,0}) E_t \pi_{t+1} - \sigma_o^{-1} \delta'_{\star,0} (\pi_t^{\star} + \tilde{y}_t^{\star}) - \sigma_o^{-1} \delta'_{u,0} \left(u_t + \bar{s}_t\right), \tag{41}$$

Here the dependence of home's economic outcomes on the foreign macroeconomy is evident from the presence of the terms $\pi_t^{\star}$ and $\tilde{y}_t^{\star}$.

[23] We stress that there may be other ways to implement the first order condition for the fixed exchange rate.

5.1.4 The dynamic system, determinacy, and learnability

Whether or not a fixed exchange rate regime is compatible with determinacy of worldwide rational expectations equilibrium depends on how the foreign, anchor country implements monetary policy, and on any international spillover effects on the home country. We make the assumption that the foreign, anchor country is inward-looking, and concerned only about reacting to developments in its own economy. We proceed with the most straightforward assumption, namely that the foreign inflation country follows a simple Taylor-type policy rule. This allows us to easily study cases where the foreign, anchor monetary authorities are pursuing policies either consistent or inconsistent with determinacy and learnability of worldwide rational expectations equilibrium.

The world economy can again be written in standard form. The matrix $\mathcal{B}$ is given by

$$\mathcal{B} = \begin{bmatrix} B_{11} & B_{12} \\ \mathbf{0} & B_{22} \end{bmatrix}$$

where B_{22} is the matrix associated with a simple Taylor rule in use in the foreign country. The eigenvalues there will depend on whether the foreign country is following the open economy version of the Taylor principle or not, as discussed earlier in the paper. The eigenvalues of B_{11} will also have to be less than unity for determinacy. This matrix is given by

$$B_{11} = \begin{bmatrix} 0 & \sigma_o^{-1}\left(1 - \delta'_{\pi,0}\right) \\ 0 & \beta + \sigma_o^{-1}\lambda_o\left(1 - \delta'_{\pi,0}\right) \end{bmatrix}.$$

The eigenvalues are zero and

$$v = \frac{\beta}{1 + \lambda_o} < 1.$$

We conclude that determinacy holds under maintained assumptions provided the foreign, anchor monetary authorities are following the Taylor principle. Learnability holds under the same conditions.

One may be able to imagine scenarios under which this result would break down, if the foreign, anchor economy had some other policy. But this result suggests there need not be anything intrinsically unstable in the use of an exchange rate peg.

6 Conclusion

We have developed results on determinacy and expectational stability for a simple open economy New Keynesian model due to Clarida, Gali, and Gertler (2002). We used this model with an eye toward comparing the open economy findings to known results for closed economies under similar assumptions.

We have shown that even for simple Taylor-type policy rules, open economy considerations will have quantitative effects on determinacy and learnability conditions. Closed economy analyses tend to understate the degree of aggressiveness the policymaker must adopt to avoid indeterminacy and expectational instability. Quantitative differences of this type are alluded to by Clarida, Gali, and Gertler (2002) and are in accord with the findings of Llosa and Tuesta (2005).

When central banks are inward-looking—reacting to domestic variables in their policy rules—and when exchange rates are floating, our results indicate that determinacy and learnability conditions for worldwide equilibrium must be met country by country. This is true whether we are considering inward-looking instrument rules or targeting rules which are implied by non-cooperative policy objectives. Optimal policy will require an implementation, but the natural implementations suggested in the closed economy literature imply the separability of determinacy and learnability conditions across economies. We interpret this finding as follows. If one country out of many adopts an instrument rule that is inconsistent with determinacy and learnability, or one country out of many adopts an implementation of an optimal policy which is inconsistent with determinacy and learnability, then worldwide equilibrium will be indeterminate and expectationally unstable. The remaining countries, even if they attempt to be very aggressive in promoting determinacy and learnability, will not have an impact on this facet of the world equilibrium. This might be viewed as an undesirable aspect of inward-looking policies, even if they are judged 'optimal' on other grounds.

When monetary authorities are actively responding to international variables, our results indicate that determinacy and learnability conditions for worldwide equilibrium are met by something akin to an average of world monetary policy. This also occurs for targeting rules where monetary authorities are attempting to pursue cooperative policies to achieve the available gains. Optimal cooperative policy will also require an implementation, and the baseline implementation from the literature may not be consistent with determinacy and learnability. Still, inclusion of reactions to international variables allows the monetary authorities from a sufficiently large economy to mitigate the threats of indeterminacy and expectational instability posed by a partner country that is pursuing a poor policy, either through an ad hoc policy or through an inadvertently bad implementation of an optimal policy. The ability to influence these conditions may be viewed as a desirable aspect of monetary policy in an open economy context.

References

[1] Aghion, P., P. Bacchetta, and A. Banerjee. 2001. "A Simple Model of Monetary Policy and Currency Crises." *European Economic Review* 44: 728-738.

[2] Benhabib, J., and R. Farmer. 1999. "Indeterminacy and Sunspots in Macroeconomics." Chapter 6: 387-448 in Taylor, J. B., and M. Woodford (Eds.). *Handbook of Macroeconomics, Volume 1 A.* Elsevier.

[3] Batini, N., P. Levine, A. Justiniano and J. Pearlman. 2005. "Model Uncertainty and the Gains from Coordinating Monetary Rules." Paper Presented at the Third Conference of the International Research Forum on Monetary Policy, ECB, 20-21 May.

[4] Benigno, G., and P. Benigno. 2004. "Designing Targeting Rules for International Monetary Policy Cooperation." European Central Bank, Working Paper #279.

[5] Blanchard, O., and C. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations," *Econometrica* 48(5): 1305-1312.

[6] Bullard, J. 2006. "The Learnability Criterion and Monetary Policy." *Federal Reserve Bank of St. Louis Review* 88(3): 202-217.

[7] Bullard, J., and E. Schaling. 2001. "New Economy-New Policy Rules?" *Federal Reserve Bank of St. Louis Review* 83(5): 57-66.

[8] Bullard, J., and E. Schaling. 2006. "Monetary Policy, Determinacy, and Learnability in the Open Economy" *European Central Bank Working Paper*, No 611, April.

[9] Bullard, J., and K. Mitra. 2002. "Learning About Monetary Policy Rules." *Journal of Monetary Economics* 49: 1105-1129.

[10] Canzoneri., M., R. Cumby, and B. Diba 2004. "The Need for International Policy Coordination: What's Old, What's New, What's Yet to Come." *Journal of International Economics,* forthcoming.

[11] Clarida, R., J. Gali, and M. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37(4): 1661-1707.

[12] Clarida, R., J. Gali, and M. Gertler. 2001. "Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach." *American Economic Review Papers and Proceedings* 91: 253-257.

[13] Clarida, R., J. Gali, and M. Gertler. 2002. "A Simple Framework for International Monetary Policy Analysis." *Journal of Monetary Economics* 49: 879-904.

[14] Corsetti, G., and P. Pesenti. 2005. "International Dimensions of Optimal Monetary Policy." *Journal of Monetary Economics* 52(2): 281-306.

[15] De Fiore, F., and Z. Lui. “Does Trade Openness Matter for Aggregate Instability?” *Journal of Economic Dynamics and Control* 29(7): 1165-1192.

[16] Ellison, M., L. Sarno, and J. Vilmunen. 2004. “Caution or Activism? Monetary Policy Strategies in an Open Economy.” Manuscript, University of Warwick.

[17] Evans, G., and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton University Press.

[18] Evans, G., and S. Honkapohja. 2003a. “Adaptive Learning and Monetary Policy Design.” *Journal of Money, Credit, and Banking* 35(6): 1045-1073.

[19] Evans, G., and S. Honkapohja. 2003b. “Expectations and the Stability Problem for Optimal Monetary Policies.” *Review of Economic Studies* 70: 807-824.

[20] Gali and Monacelli. 2002. “Monetary Policy and Exchange Rate Volatility in a Small Open Economy.” NBER Working Paper #8905.

[21] Giannoni, M., and M. Woodford. 2002a. “Optimal Interest Rate Rules: I. General Theory.” NBER Working Paper #9419.

[22] Giannoni, M., and M. Woodford. 2002b. “Optimal Interest Rate Rules: II. Applications.” NBER Working Paper #9420.

[23] Jensen, H. 2002. "Monetary Policy Frameworks and Real Equilibrium Determinacy." Mimeo University of Copenhagen, February.

[24] Llosa, G., and V. Tuesta. 2005. “Learning About Monetary Policy Rules in Small Open Economies.” Manuscript, Central Reserve Bank of Peru.

[25] McCallum, B, and E. Nelson. 2005. "Targeting vs. Instrument Rules for Monetary Policy." Federal Reserve Bank of St. Louis *Review* 87(5): 597-612.

[26] Monacelli, T. 2005. "Monetary Policy in a Low Pass-Through Environment." *Journal of Money, Credit and Banking*, forthcoming.

[27] Nolan, C. and E. Schaling. 1996. "International Monetary Policy Coordination: Some Lessons from the Literature." *Bank of England Quarterly Bulletin* 36 (4): 412-417.

[28] Pappa, E. 2004. "Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World." *Journal of Monetary Economics* 51: 753-779.

[29] Preston, B. 2003. "Learning About Monetary Policy Rules When Long-Horizon Forecasts Matter." Working Paper, Columbia University.

[30] Sargent, T and N. Wallace. 1975. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule.", *Journal of Political Economy* 83, 241-254.

[31] Schaling, E.2001. "Dynamic Programming versus Lagrange: Is there a Horse Race in Dynamic Optimization?" in Klok, H., T. van Schaik and S. Smulders (eds), *Economologues - Liber Amicorum for Theo van de Klundert*, Tilburg University Press, pp. 334-349.

[32] Schaling, E. 2004. "The Non-Linear Phillips Curve and Inflation Forecast Targeting: Symmetric vs. Asymmetric Monetary Policy Rules." *Journal of Money, Credit and Banking* 36(3): 361-386.

[33] Svensson, L. 2003. "What's Wrong with Taylor Rules? Using Judgment in Monetary Policy Through Targeting Rules." *Journal of Economic Literature* 41: 426-477.

[34] Svensson, L. 2005. "Targeting Rules Versus Instrument Rules for Monetary Policy: What is Wrong With McCallum and Nelson?" Federal Reserve Bank of St. Louis *Review* 87(5): 613-626.

[35] Svensson, L and M. Woodford. 2003. "Implementing Optimal Policy Through Inflation-Forecast Targeting," in B. Bernanke and M. Woodford (eds.), *Inflation Targeting*, Chicago: University of Chicago Press.

[36] Taylor, J. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

[37] Uribe, M. 2003. "Real Exchange Rate Targeting and Macroeconomic Instability." *Journal of International Economics*, forthcoming.

[38] Woodford, M. 1999. "Optimal Monetary Policy Inertia." NBER Working Paper #7261.

[39] Woodford, M. 2001. "The Taylor Rule and Optimal Monetary Policy." *American Economic Review Papers and Proceedings* 91(2): 232-237.

[40] Woodford, M. 2003. *Interest and Prices*. Princeton University Press.

[41] Zanna, L. 2004. "PPP Rules, Macroeconomic (In) stability and Learning." Board of Governors of the Federal Reserve System, International Finance Discussion Papers, #814, August.