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THE INFLATIONARY EFFECTS OF THE USE OF RESERVE RATIO REDUCTIONS, OR OPEN MARKET PURCHASES, TO REDUCE MARKET INTEREST RATES: A THEORETICAL COMPARISON

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I. Introduction

The level of market interest rates has served as the principal intermediate target of Federal Reserve monetary policy throughout most of the System's history. Concern about the behavior of monetary aggregates began in the early 1950s, and explicit aggregate targeting in the early 1970s; for several years beginning in the late 1970s, the growth rates of various monetary aggregates largely supplanted interest rates as policy targets. Recently, however, the System has drawn back from rigid adherence to aggregate targeting, broadening its target ranges and allowing a wider variety of considerations to influence the conduct of policy. While interest rates have not been explicitly targeted, the System has adopted operating procedures which give it tighter control over rates, and has intervened on more than one occasion to change their average levels.[1]

The devices according to which the System has attempted to influence interest rates have varied greatly over time. Initially the discount rate was simply set at the target rate, and the Reserve Banks lent (discounted) relatively freely at that rate. In the aftermath of the Great Depression, and particularly with the outbreak of the Second World War, the System shifted to a policy of influencing rates indirectly through open market operations.

Another outgrowth of the Depression was acquisition by the System of the authority to set and vary the required reserve ratio imposed on member banks. During 1936-1937 the System used this authority in dramatic fashion, doubling required reserve ratios in a series of steps in an attempt to absorb and sterilize the large excess reserves then held by member banks. This action was followed immediately by, and is widely believed to have precipitated, the sharp recession of 1937-1938.[2] Since this episode the System has adjusted reserve ratios infrequently, and has not used changes in the ratio as a short- or intermediate-term tool of policy.

During the 1950s the reluctance of the System to vary the reserve ratio countercyclically, and in particular its tendency to reduce the ratio from time to time without subsequently increasing it, attracted some criticism from Congress. Kareken (1961) argued that that the justifications offered by the System in defense of this behavior did not stand up under careful scrutiny. He did not, however, attempt to provide a positive rationale for a more active policy with regard to ratio changes.[*]

One of the reasons the System has avoided frequent reserve ratio changes has undoubtedly been the desire to avoid a repetition of events like those of of 1936-1938. Prior to 1980, another source of restraint was the desire to prevent the financial burden of reserve ratio maintenance from driving member banks out of the System.[3] This danger existed because the reserve ratios imposed by state banking agencies were typically lower and less variable than those imposed by the System. A related source of restraint has been the use of lagged reserve accounting, which reduces the short-run effectiveness of policy interventions which rely directly or indirectly on the required reserve

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[*] Kareken observed (p. 65) that "the economic effects produced by a change in reserve ratios differ significantly from those produced by an equivalent open market operation," but did not elaborate. He devoted the bulk of his paper to arguing that "official reasons for making day-to-day adjustments in member banks' reserve positions by means of open market sales and purchases will not stand up under close scrutiny, and the same can be said of the post-Accord record on reserve ratios." His analysis included an attack on what remain the most frequently-cited reasons for the failure of the Federal Reserve System to use the required reserve ratio as an active policy instrument -- that significant changes in the ratio have large, discontinuous effects, and that frequent small changes would unduly complicate banks' reserve management problem.

ratio. The shift to lagged accounting, which occurred in 1968, was designed to reduce member banks' costs of compliance with System reserve requirements.[4]

The Monetary Control Act of 1980 gave the Federal Reserve System power to set required reserve ratios for a broader range of transactions accounts, and for all depository institutions. In addition, in 1984 the System returned to contemporaneous reserve accounting. These legislative and regulatory changes have created conditions under which the required reserve ratio could acquire a more active role as an instrument of monetary policy.[5]

Until recently, academic analyses of monetary policy have paid scant attention to role of Federal Reserve base money creation in helping finance federal government budget deficits. Since reserve requirememts are an important source of demand for base money, this omission may detract substantially from our understanding of both the economic impact of changes in reserve ratios and the formulation of policy regarding reserve requirements. Goodfriend and Hargraves (1983) point out that during recent decades revenue considerations have exerted considerable influence on government (Congressional and Treasury, as well as Federal Reserve System) behavior regarding reserve requirements.

One of the reasons the financing implications of reserve requirement policy have so often been ignored is that models capable of describing them convincingly have not been available. The development of dynamic general equilibrium monetary models has gone far towards solving this problem. In recent years authors such as Wallace (1984), Romer (1985), and Freeman (1987) have used different specifications of the overlapping generations model to examine the interrelationships between the required reserve ratio, other monetary policy instruments, government budget deficits, and the levels of endogenous real and nominal variables.[6]

The purpose of our paper is to examine the implications of use of the required reserve ratio as an active policy instrument in a model in which (1) monetary policy decisions are constrained by the requirements of deficit finance and (2) the monetary authority targets interest rates. In particular, we are interested in contrasting the effects of changes in the required reserve ratio with those of open market purchases or sales which achieve the same rate targets.

Our analysis is conducted in the context of stylized policy situations (initial settings, problems, and goals) which we view as analogous to actual situations frequently confronted by the Federal Reserve System. We assume that the initial level of real (or nominal) interest rates is viewed by the monetary authorities as "too high," and that they consequently select a lower target. We assume that the authorities can choose between two policy instruments: the required reserve ratio, which can be changed directly, and the ratio of nominal bonds to nominal money, which can be changed through open market operations. (We do not examine policy experiments involving changes in both instruments.) Finally, we assume that the authority seeks the instrument which will permit the target interest rate to be achieved at the lowest rate of inflation.*

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*We make this assumption because it seems clear that the System continues to regard a higher rate of inflation as the principal "cost" of policy interventions designed to reduce market interest rates (and vice-versa). See for example, the recent Summary Report of the Federal Reserve Board (dated 20 February 1990), which surveyed monetary policy during 1989:

(p. 10) In the opening months of the year, the Federal Open Market Committee extended the move toward restraint which began almost a year earlier, seeking to counter a disquieting intensification of inflationary pressures. Policy actions in January and February, restraining credit availability and increasing the discount rate, The model we use for this purpose is borrowed from Wallace (1984), who used it to examine the effects of open market operations. He assumes throughout his analysis that a binding reserve ratio is imposed, and comments in a closing footnote that his model can also be used to study the effects of changes in reserve requirements. Our extension is conducted along these lines, though unlike Wallace we are not primarily concerned with welfare comparisons.

Our use of this model in an attempt to gain insights into the effects of practical policy experiments raises a potentially troubling issue which we need to discuss before proceeding. Wallace found that the effects of open market operations were often "perverse" -- that while a reduction in the bonds/currency ratio (an open market purchase) always produced a lower real interest rate, it often resulted in a lower rather than higher rate of inflation. Since reducing the required reserve ratio (the other device for reducing the real rate) always produces a higher rate of inflation, under "perverse" circumstances our monetary authority will always prefer open market purchases.

In practice it is clear that the Federal Reserve System does not see itself as facing perverse circumstances, and believes instead that any move to ease credit conditions creates inflationary pressures. Since we are

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prompted a further 3/4 percentage point increase in short-term market interest rates.

(p. 3) In June, the FOMC began a series of steps, undertaken with care to avoid excessive inflationary stimulus, that trimmed 1 1/2 percentage points from short-term interest rates by yearend.

interested in giving "advice" (of an admittedly abstract sort) to the System, we confine ourselves to studying circumstances under which this is the case. (We refer to these circumstances as those in which the "conventional wisdom" holds true.) We conclude the introduction to this paper with a brief discussion of the empirical plausibility of these circumstances.

In his 1984 paper Wallace shows that in a stationary economy, a sufficient condition for "perverse" results is that the real interest rate on government bonds exceeds the real growth rate of the economy.[7] The logic behind this result is quite simple. In Wallace's model the entire real deficit is "pure" in the sense of being uncovered by future surpluses. As a result, in equilibrium the real deficit must be covered by a combination of currency and bond seigniorage. When real interest rates exceed real growth rates, bond seigniorage is negative. An increase in the ratio of bonds to money (an open market sale) increases the losses on bond seigniorage, and thus forces an increase in currency seigniorage -- which is to say a higher rate of inflation.

At the time Wallace was writing the U.S. was in the midst of a lengthy period during which real interest rates almost certainly exceeded real growth rates. Darby (1984), commenting on a earlier (1981) paper by Sargent and Wallace which obtained similarly "perverse" results, shows that historically this inequality has typically been reversed. During the years since Wallace wrote the relationship between these variables has been much closer to historical form: the real growth rate exceeded the real interest rate for four of the six years beginning in 1984, and the average growth rate for this period exceeded the average real interest rate by approximately one half of one percent.[8] Thus both trend and historical experience suggest that if we have not already returned to a situation of the sort necessary for the conventional wisdom to hold, we may soon do so.

Real growth rates in excess of real interest rates are not <u>sufficient</u> for the conventional wisdom, however. Under the assumptions described in the next section the conventional wisdom will hold whenever a decrease in the bond/currency ratio reduces total revenue from bond seigniorage. Now a decrease in the ratio reduces the real value of bonds, which tends to reduce bond seigniorage revenue; it also reduces the real interest rate on bonds, which has the opposite tendency. When the latter effect dominates, as is often the case, an open market purchase permits a reduction in currency seigniorage revenues, which is to say a lower inflation rate.*

It turns out that under the aforementioned assumptions the initial bond/currency ratio must be relatively low for the conventional wisdom to hold. This is true in part because low bond/currency ratios tend to be associated with low real interest rates, and in part because when the initial volume of bonds is low the negative effect on bond seigniorage of a reduction in bond volume is more easily offset by the positive effect of higher unit revenues.

Our analytical need for a low bond/currency ratio creates a potential conflict with the empirical facts concerning the ratio. At the time Wallace was writing its value for the U.S. was on the order of 7; it has since

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*The assumption that gross saving is fixed, so that real currency balances are a positive multiple of the required reserve ratio, is critical to the simplictiy of the analysis just presented. Were gross saving increasing in the real interest rate (for example) an open market purchase might, by reducing gross saving, reduce real currency balances as well as the real volume of bonds. This would make a perverse inflation response less likely.

increased, and now approaches 10.[9] In simple specifications of the type presented in the text of the paper, ratios such as these are far too high to support the conventional wisdom, given plausible initial values for the real interest rate and the inflation rate. Fortunately, we have recently succeeded in constructing specifications of a slightly more general type for which the conventional wisdom holds at bonds/currency ratios consistent with the empirical values just cited, as well as plausible initial real interest and inflation rates. One specification of this type is described in the appendix to this paper.

II. The Model

As noted above, our model is borrowed from Wallace (1984). At each discrete date $t \geq 1$ a total of N two-period lived agents (the members of generation t) are born. The preferences of each of these agents are representable by the utility function $U(c_1(t), c_2(t+1))$, where $c_i(t)$ represents the amount of the single consumption good consumed at date t. We assume that $U(\cdot,\cdot)$ is twice continuously differentiable, strictly quasiconcave, and strictly increasing in each argument. We also assume that it satisfies the "Inada conditions"

$$\lim_{\frac{c_1}{c_2} \to 0} \frac{U_1}{U_2} = \infty , \qquad \lim_{\frac{c_1}{c_2} \to 0} \frac{U_1}{U_2} = 0 .$$

The members of generation 0 (the "initial old") simply maximize their consumption of date 1 good.

Each generation $t \geq 1$ includes two groups of agents: savers and borrowers. These agents receive intertemporal endowments $(w_1^i(t), w_2^i(t+1))$, i = s,b, where s and b represent savers and borrowers, respectively. Their intertemporal budget constraints are

$$c_1^i(t) + \frac{c_2^i(t+1)}{R_i(t)} = w_1^i(t) + \frac{w_2^i(t+1)}{R_i(t)} , \quad i = s,b,$$

where $R_b(t)$ will henceforth be denoted $R(t)$, and $R_s(t)$ will be denoted $R_d(t)$. Borrowers' aggregate savings function is $D(R(t)) = N[w_1^b(t) - c_1^b(R(t))]$; we require $D(R(t)) < 0$ for relevant values of $R(t)$. Savers' aggregate savings function is $S(R_a(t)) = N[w_1^s(t) - c_1^s(R_d(t))]$; we assume $S(R(t)) > 0$ for $R(t) > 0$. The $c_1^i(R_i(t))$ represent optimal levels of first period consumption for agents endowed $(w_1^i(t), w_2^i(t+1))$ and confronted by interest rates $R_i(t)$, $i = s,b$.

The government must finance a real deficit of G at each date $t \geq 1$. It may do so by issuing fiat currency in the amount $H(t)$ or bonds in the amount $B(t)$. Bonds issued at date t are payable in fiat currency at date t+1; $B(t)$ represents the face value at date t+1 of the bonds issued at date t. Let $p(t)$ represent the price of a unit of fiat currency in units of the consumption good at date t, and $P_b(t)$ the date t price, in units of fiat currency, of a bond with a face value of one unit of fiat currency. The government's budget constraint is then given by

$$(1) \quad G = p(t)[H(t)-H(t-1)] + p(t)[P_b(t)B(t)-B(t-1)] \quad , \quad t \geq 1 \; .$$

The initial quantities $H(0)$ and $B(0)$ constitute the aggregate endowment of the members of generation 0.

Any borrowing or lending which may occur in this economy is assumed to be intermediated by commercial banks which operate competitively and costlessly. These banks are required to hold fiat currency reserves equal at minimum to a positive fraction λ of their total liabilities (deposits). The banks lend to borrowers and/or the government at rate $R(t)$, and borrow (accept deposits) from savers at rate $R_d(t)$. Since the banks must earn zero profits, we have

$$(2) \quad R_d(t) = \lambda R(t) + (1-\lambda)R_m(t)$$

as a condition of equilibrium, where $R_m(t) \equiv \frac{p(t+1)}{p(t)}$. $R_m(t)$ is the (real) rate of return on fiat currency held from date t to date t+1.

A perfect foresight competitive equilibrium with a binding required reserve ratio consists of a positive constant λ, and a set of nonnegative sequences for $H(t)$, $B(t)$, $R_m(t)$, $R_d(t)$, $R(t)$, $p(t)$, and $P_b(t)$, for all $t \geq 1$, which satisfy conditions (1) and (2), as well as

$$(3) \quad \lambda S(R_d(t)) = p(t)M(t)$$

$$(4) \quad S(R_d(t)) + D(R(t)) = p(t)[H(t)+P_b(t)B(t)]$$

$$(5) \quad R(t) = \frac{R_m(t)}{P_b(t)} , \quad \text{with} \quad P_b(t) < 1 .$$

Condition (3) ensures that banks comply with the reserve requirement, and that there are no excess reserves. Condition (4) guarantees that the credit market clears -- that the excess of private saving over private borrowing is absorbed by private purchases of bonds and fiat currency. Condition (5) ensures that government bonds and private securities are perfect substitutes for banks, and that nonbank agents do not choose to hold fiat currency.

Unfortunately, analysis of the general version of this model gets quite complicated. In order to simplify the analysis we make the following sets of additional assumptions:

Set 1: Regularity of savings behavior.

We assume that gross savings are fixed -- invariant, in particular, to changes in the rate of return on deposits. That is, $S(R_d) = S > 0$ for all $R_d > 0$. In addition, we assume that the gross borrowing function $D(R)$ satisfies $D'(R) > 0$ and $D''(R) < 0$ for all $R > 0$.

Set 2: Existence of a pure currency seigniorage ($\beta = 0$) equilibrium with a binding reserve ratio and a gross real interest rate less than unity.

We assume that the demand functions are specified, and the reserve ratio λ selected, so that when $\beta = 0$ there exists a rate of return vector $\hat{\mathbf{R}}(\lambda) = (\hat{R}(\lambda), \hat{R}_m(\lambda))$ which satisfies the binding reserve ratio equilibrium conditions as well as $\hat{R}(\lambda) < 1$. Stated differently, we assume that there exists $\hat{\mathbf{R}}(\lambda)$ such that $1 > \hat{R}(\lambda) > \hat{R}_m(\lambda) > 0$, $[1-\hat{R}_m(\lambda)]\lambda S = G$, and $(1-\lambda)S + D(\hat{R}(\lambda)) = 0$.

Notice that $G < \lambda S$ is an immediate implication of the second set of assumptions.

Assumption set #2 is fairly strong; it is easy to specify plausible demand functions, and values of λ, so that $\hat{R}(\lambda) > 1$. It turns out, however, that in specifications of this type the "conventional wisdom" <u>never</u> holds.

One simple assumption which is sufficient (but <u>not</u> necessary -- see Example 1) to ensure that $\hat{R}(\lambda) < 1$ for at least some binding values of λ is that the deficit G can be financed without reserve requirements: that is, that there exists $0 < R^* < 1$ such that $(1-R^*)[S+D(R^*)] = G$. Having made this assumption, define the "threshold" reserve ratio -- the highest nonbinding ratio -- by $\underline{\lambda} \equiv \frac{S+D(R^*)}{S}$, and define $\bar{\lambda} \equiv \frac{S+D(1)}{S}$; $\bar{\lambda}$ is the reserve ratio which generates $\hat{R}(\lambda) = 1$. Since $S+D(R^*) < \lambda S$ whenever λ is binding, we must have $G < \lambda S$, which in turn implies $\frac{\partial R}{\partial \lambda} > 0$ (see pp. 17-18 below). Consequently $\underline{\lambda} < \bar{\lambda}$, and $\hat{R}(\lambda) < 1$ whenever $\lambda \in (\underline{\lambda}, \bar{\lambda})$.

In what follows we also confine our attention to <u>stationary</u> equilibria. A stationary equilibrium can be defined as consisting of a positive constant λ, and nonnegative values R, R_d, R_m, p_1, P_bB, and M which satisfy

$$(1') \quad G = (1-R_m)M + (1-R)P_bB$$

$$(2') \quad R_d = (1-\lambda)R + \lambda R_m$$

$$(3') \quad \lambda S = M$$

$$(4') \quad S + D(R) = M + P_bB$$

$$(5') \quad P_b = \frac{R_m}{R} < 1$$

where $M = p(t)H(t)$ and $B = p(t)B(t)$.

III. Open Market Operations

Define $\beta = \frac{B(t)}{H(t)}$. Following Wallace (1984) we view β, the nominal bonds/currency ratio, as a policy instrument which can be varied through open market operations. We will assume henceforth that $\beta \geq 0$.

Notice that (1′) and (4′) can now be rewritten

$$(1'') \quad G = M[(1-R_m) + (1-R)\beta P_b]$$

$$(4'') \quad S + D(R) = M[1 + \beta P_b] \ .$$

Given the definition of β, conditions (3′) and (5′) are readily seen to imply $P_bB = \beta\lambda S\frac{R_m}{R}$. This identity, along with condition (3′), allows us to rewrite conditions (1′′) and (4′′) as

$$(6) \quad [1-\lambda(1+\beta\frac{R_m}{R})]S + D(R) = 0 \ , \quad \text{and}$$

$$(7) \quad G = \lambda S[1 + R_m\{\beta(\frac{1}{R}-1) - 1\}] \ .$$

In Sections III and IV we examine the properties of the functions $R(\beta,\lambda)$ and $R_m(\beta,\lambda)$ which are implicitly defined by equations (6) and (7). We begin by applying the Implicit Function Theorem to verify the existence of these functions. Define $\mathbf{R} \equiv (R,R_m)$, and the vector-valued function $g(\mathbf{R};\beta,\lambda)$ by

$$g_1(\mathbf{R};\beta,\lambda) \equiv [1-\lambda(1+\beta\frac{R_m}{R})]S + D(R)$$

$$g_2(\mathbf{R};\beta,\lambda) \equiv \lambda S[1 + R_m\{\beta(\frac{1}{R}-1) - 1\}] - G \ .$$

Equilibrium functions $R(\beta,\lambda)$ and $R_m(\beta,\lambda)$ exist in an open neighborhood around a given (β^*,λ^*) so long as there exists an equilibrium vector $(\mathbf{R}^*;\beta^*,\lambda^*)$ and the derivative matrix $D_{\mathbf{R}}g(\mathbf{R}^*;\beta^*,\lambda^*)$ is not singular. Differentiation reveals that $R^*(1+\beta^*)-\beta^* > 0$ is sufficient to ensure that $D_{\mathbf{R}}g(\mathbf{R}^*;\beta^*,\lambda^*)$ has a nonzero determinant.

Lemma. $R(1+\beta)-\beta > 0$ in any stationary equilibrium with a binding reserve ratio.

Proof:

Recall [equilibrium condition (1′)] that $G = (1-R_m)M + (1-R)P_bB$, where $M = \lambda S$ and $P_bB = \lambda S\frac{R_m}{R}\beta$. Thus we can write $G = \lambda S[(1-R_m)+(1-R)\frac{R_m}{R}\beta]$. Now suppose $R \leq \frac{\beta}{1+\beta} \rightarrow \frac{1}{R} \geq \frac{1+\beta}{\beta}$. This implies

$$G \geq \lambda S[(1-R_m)+(1-R)(1+\beta)R_m]$$

$$= \lambda S[1+R_m(\beta-R(1+\beta))] \ .$$

Since $R \le \frac{\beta}{1+\beta} \rightarrow \beta - R(1+\beta) \ge 0$, we have $G \ge \lambda S$. Thus $G < \lambda S$ (an implication of assumption set #2 -- see above) implies $R(1+\beta)-\beta > 0$.

Equations (6) and (7) can now be differentiated in order to obtain the following expressions for $\frac{\partial R}{\partial \beta}$ and $\frac{\partial R_m}{\partial \beta}$:

$$\frac{\partial R}{\partial \beta} = \frac{RP_bB}{\beta\{D'(R)[R(1+\beta)-\beta] + P_bB(1+\beta)\}}$$

$$\frac{\partial R_m}{\partial \beta} = \frac{RB}{\lambda S\beta} \frac{D'(R)(1-R) - P_bB}{D'(R)[R(1+\beta)-\beta] + P_bB(1+\beta)} .$$

The Lemma ensures that $\frac{\partial R}{\partial \beta} > 0$, and that the denominator of $\frac{\partial R_m}{\partial \beta}$ is positive. Consequently positivity of the numerator is necessary and sufficient for $\frac{\partial R_m}{\partial \beta} > 0$. Notice that $R \ge 1$ suffices to ensure that the numerator is nonpositive. Thus since $\frac{\partial R}{\partial \beta} > 0$, $\hat{R}(\lambda) < 1$ is necessary for the conventional wisdom to hold for any positive value of β (see above). Notice also that since $P_bB = (1-\lambda)S + D(R)$, the numerator is positive whenever $(1-R)D'(R) - D(R) > (1-\lambda)S$.

Proposition 1 demonstrates that the conventional wisdom $(\frac{\partial R_m}{\partial \beta} > 0)$ holds whenever β is sufficiently low -- as we suggested in Section I.

Proposition 1. Let λ represent a required reserve ratio which is binding when $\beta = 0$. Then there exists $\tilde{\beta} > 0$ such that $\beta \in [0,\tilde{\beta})$, which is to say $R \in [\hat{R}(\lambda),R(\tilde{\beta},\lambda))$, implies $\frac{\partial R_m}{\partial \beta}(\beta,\lambda) > 0$.

Proof:

Define $F(\beta,\lambda) = [1-R(\beta,\lambda)]D'(R(\beta,\lambda))-D(R(\beta,\lambda))$. For $\frac{\partial R_m}{\partial\beta} > 0$ we need $F(\beta,\lambda) > (1-\lambda)S$. Recall that $F(\beta,\lambda) > (1-\lambda)S$ is equivalent to $(1-R)D'(R) > (1-\lambda)S + D(R) = P_bB$, and that $P_bB = \beta\lambda S\frac{R_m}{R}$. Since $\hat{R}(\lambda) < 1$ by hypothesis, we have $(1-\hat{R}(\lambda))D'(\hat{R}(\lambda)) > 0$. Thus $\frac{\partial R_m}{\partial\beta} > 0$ at $\beta = 0$.

$$\begin{aligned} \text{Now} \quad \frac{\partial F}{\partial\beta} &= D''(R)\frac{\partial R}{\partial\beta}(1-R) - 2\frac{\partial R}{\partial\beta}D'(R) \\ &= \frac{\partial R}{\partial\beta}[D''(R)(1-R) - 2D'(R)] . \end{aligned}$$

Since $\frac{\partial R}{\partial\beta} > 0$, assumption set #1 implies that $\frac{\partial F}{\partial\beta} < 0$ so long as $R \leq 1$. Equations (6) and (7) can be used to show that $R(\beta) = 1$ is solved by $\beta' \equiv \frac{(1-\lambda)S + D(1)}{\lambda S-G} > 0$. Since $F(\beta') = -D'(1) < 0 < (1-\lambda)S$, and we have seen that $F(0) > (1-\lambda)S$, we know that there exists a unique $\tilde{\beta} \in (0,\beta')$ such that $F(\tilde{\beta}) = 0$, and that $F(\beta) > 0$ for $0 \leq \beta < \tilde{\beta}$. Define $\tilde{R} \equiv R(\tilde{\beta})$. Since $\frac{\partial R}{\partial\beta} > 0$ we know that $\hat{R}(\lambda) < \tilde{R} < 1$, and that $(1-R)D'(R) > (1-\lambda)S$ for $R \in [\hat{R}(\lambda),\tilde{R})$.

An example which establishes that this proposition is not vacuous appears below.

Example 1.

Let $S = \frac{N}{2}$, $D(R) = -\frac{2N}{5R}$, $\lambda = \frac{1}{10}$, and $G = \frac{N}{100}$. [These demand functions will arise if each generation $t \geq 1$ consists of N "savers" with preferences $U(c_1,c_2) = \ln c_1 + \ln c_2$ and endowments $(w_1^S,w_2^S) = (1,0)$, and N "borrowers" with identical preferences and endowments $(w_1^S,w_2^S) = (0,\frac{4}{5})$.]

Since $(1-R)[S+D(R)] = G$ does not have real solutions any positive reserve ratio is binding; the smallest feasible ratio is $\frac{1}{50}$ and necessitates $R_m = 0$. Nevertheless, $\hat{R}(\frac{1}{10}) = \frac{8}{9} < 1$. The value of β' is $\frac{5}{4}$, and the value of $\tilde{\beta}$ is approximately 0.5851 ; $R(\tilde{\beta},\frac{1}{10}) \cong 0.9428$. A plot of $F(\beta,\frac{1}{10})$ over the domain $[0,\tilde{\beta}]$ is displayed as Figure 1 .

IV. Changes in the Required Reserve Ratio

We have seen that an open market purchase (a reduction in the bonds/currency ratio) always has the effect of "loosening" (reducing real interest rates). Under the conditions of Proposition 1, moreover, tightening is "inflationary" (reduces the rate of return on currency). These results conform to the "conventional wisdom" regarding the impact of open market operations. It is useful to establish analogous results for changes in the required reserve ratio. A reduction in this ratio always "loosens" in the sense of reducing the real interest rate. And whenever loosening via open market purchases is "inflationary," loosening via reserve ratio reductions is also inflationary.

Equations (6) and (7) can be differentiated with respect to λ instead of β , and solved for $\frac{\partial R}{\partial \lambda}$ and $\frac{\partial R_m}{\partial \lambda}$. When this is done it is readily seen that

$$\frac{\partial R}{\partial \lambda} = \frac{\beta G + [S+D(R)][R(1+\beta)-\beta]}{\lambda\{D'(R)[R(1+\beta)-\beta] + P_b B(1+\beta)\}}$$

$$\frac{\partial R_m}{\partial \lambda} = \frac{\frac{RG}{\lambda S} D'(R) - R_m P_b B(1+\beta)}{\lambda\{D'(R)[R(1+\beta)-\beta] + P_b B(1+\beta)\}} .$$

Since $S+D(R) = M + P_bB > 0$, Lemma 1 ensures that $\frac{\partial R}{\partial \lambda} > 0$.

Proposition 2. In any stationary equilibrium in which λ is binding at β , $\frac{\partial R_m}{\partial \beta}(\beta,\lambda) > 0$ implies $\frac{\partial R_m}{\partial \lambda}(\beta,\lambda) > 0$.

Proof:

Since the Lemma ensures that the denominator of $\frac{\partial R_m}{\partial \lambda}$ is positive, $\frac{\partial R_m}{\partial \lambda} > 0$ whenever $\frac{RG}{\lambda S} D'(R) - R_mP_bB(1+\beta) > 0$. Given that $P_bB = \beta\lambda S\frac{R_m}{R}$, multiplying both sides of this condition by $\frac{\beta\lambda S}{R}$ yields the equivalent condition $\beta G\ D'(R) - (P_bB)^2(1+\beta) > 0$. The latter condition is equivalent in turn to $\frac{\beta}{1+\beta}\frac{G}{P_bB} D'(R) > P_bB$.

Since $\frac{\partial R_m}{\partial \beta} > 0$ implies $P_bB < (1-R)D'(R)$, $\frac{\beta}{1+\beta}\frac{G}{P_bB} > (1-R)$ is sufficient for $\frac{\partial R_m}{\partial \lambda} > 0$. And since $G = \lambda S[(1-R_m) + (1-R)\frac{R_m}{R}\beta]$ and $P_bB = \lambda S\frac{R_m}{R}\beta$, we have $\frac{G}{P_bB} = \frac{R}{\beta R_m}(1-R_m) + (1-R)$. Consequently we need $\frac{\beta}{1+\beta}[\frac{R}{\beta R_m}(1-R_m) + (1-R)] > (1-R) \leftrightarrow \frac{1}{1+\beta}\frac{R}{R_m}(1-R_m) > \frac{1}{1+\beta}(1-R) \leftrightarrow$ $\frac{R}{R_m}(1-R_m) > (1-R) \leftrightarrow \frac{R}{R_m} > 1$. This is true whenever λ is binding.

V. Real Interest Rate-Pegging Experiments

In this section we analyze the inflationary implications of the use of open market purchases, or reductions in the required reserve ratio, to reduce the level of real interest rates. We attempt in particular to derive conditions under which a decline in the real interest rate engineered via open market operations results in a larger (or smaller) increase in the inflation rate than a decline brought about by a reduction in the reserve ratio.

It may be helpful to begin by describing our "rate-pegging experiments" in terms of discrete changes in policy instruments and interest rates. Suppose we begin with a particular specification of the model, and a particular setting of the policy vector (β,λ); we will call the latter $(\bar{\beta},\bar{\lambda})$. If $\bar{\lambda}$ is binding at $\bar{\beta}$ this policy vector generates a unique real interest rate $\bar{R} \equiv R(\bar{\beta},\bar{\lambda})$, and a unique currency rate of return $\bar{R}_m \equiv R_m(\bar{\beta},\bar{\lambda})$.

Now suppose the monetary authority attempts to reduce the real interest rate by reducing the bonds/currency ratio, holding the required reserve ratio fixed. In particular, suppose that the authority chooses $\hat{\beta}$ with $0 < \hat{\beta} < \bar{\beta}$, generating $\hat{R} \equiv R(\hat{\beta},\bar{\lambda})$ and $\hat{R}_m \equiv R_m(\hat{\beta},\bar{\lambda})$. We know that $\hat{R} < \bar{R}$; under assumptions described in Proposition 1, we also know that $\hat{R}_m < \bar{R}_m$.

Suppose instead that the authority attempts to achieve the same reduction in the real interest rate by holding the bonds/currency ratio fixed and reducing the required reserve ratio. That is, suppose there exists a binding reserve ratio $\tilde{\lambda} < \bar{\lambda}$ such that $R(\bar{\beta},\tilde{\lambda}) = \hat{R}$. We are interested in the relative magnitudes of $R_m(\hat{\beta},\bar{\lambda}) \equiv \hat{R}_m$ and $R_m(\bar{\beta},\tilde{\lambda}) \equiv \tilde{R}_m$.

Unfortunately, we know of no easy way to provide a general characterization of the results of discrete rate pegging experiments. We opt instead for an approach which does not (at least in the first instance) involve discrete changes, and allows us to use calculus techniques.

We begin by reinterpeting our model by treating the real interest rate R as a parameter, along with the reserve ratio λ, and treating the bonds/currency ratio β as an endogenous variable. We then compute the derivative of R_m, the rate of return on currency, with respect to R -- holding the reserve ratio λ fixed. This derivative describes the inflationary implications of a marginal reduction in R engineered by a reduction in β.

Next we reinterpret the model by treating R as a parameter, along with the bonds/currency ratio β, and treating the required reserve ratio λ as an endogenous variable. Again we compute the derivative of R_m with respect to R -- this time holding β fixed. The derivative in question describes the inflationary implications of a marginal reduction in R engineered by a reduction in λ. Finally, we evaluate this derivative at the value of β associated with the initial levels of R and λ from the previous experiment. This choice of β ensures that each of the two experiments start at the same equilibrium position, and permits a fair comparison of the induced increases in the inflation rate.

A. Endogenization of β

Notice that equilibrium conditions (1′) and (3′) imply $G = \lambda S(1-R_m) + P_b B(1-R)$. And since conditions (3′) and (4′) imply $P_b B = (1-\lambda)S + D(R)$, we can write $G = \lambda S(1-R_m) + (1-R)[(1-\lambda)S + D(R)]$. This equation can be solved for R_m, yielding

$$R_m(R,\lambda) = \frac{(\lambda S-G) + (1-R)[(1-\lambda)S + D(R)]}{\lambda S} .$$

Notice that

$$\frac{\partial R_m}{\partial R}(R,\lambda) = \frac{(1-R)D'(R) - [(1-\lambda)S + D(R)]}{\lambda S} = \frac{(1-R)D'(R) - P_b B}{\lambda S} .$$

Thus $\frac{\partial R_m}{\partial R}(R,\lambda) > 0$ whenever the conventional wisdom holds.

We can also use (3′), (4′), and (5′) to obtain the equation $(1-\lambda)S + D(R) = \beta\lambda S\frac{R_m}{R}$. Substituting $R_m(R,\lambda)$ into this expression and then solving it for β yields

$$\beta(R,\lambda) = \frac{R[(1-\lambda)S + D(R)]}{(\lambda S-G) + (1-R)[(1-\lambda)S + D(R)]} .$$

B. Endogenization of λ

Equation (7) can be solved for R_m, yielding $R_m = \frac{\lambda S-G}{\lambda S[1 - \frac{1-R}{R}\beta]}$. This expression for R_m can be substituted into equation (6), and the latter solved for λ. This procedure yields

$$\lambda(R,\beta) = \frac{[R(1+\beta)-\beta][S+D(R)] + \beta G}{R(1+\beta)S} .$$

Equation (6) can also be solved for R_m, yielding $R_m = \frac{R}{\beta}[\frac{S+D(R)}{\lambda S} - 1]$. Substituting $\lambda(R,\beta)$ into this expression generates

$$R_m(R,\beta) = \frac{R}{\beta}\left[\frac{R(1+\beta)[S+D(R)]}{[R(1+\beta)-\beta][S+D(R)] + \beta G}\right] .$$

Differentiation then produces

$$\frac{\partial R_m}{\partial R}(R,\beta) = \frac{(1+\beta)R^2 GD'(R) - \beta[S+D(R)-G]^2}{\{[R(1+\beta)-\beta][S+D(R)] + \beta G\}^2}$$

$$= \frac{\beta[R^2GD'(R) - (S+D(R)-G)^2] + R^2GD'(R)}{\{\beta[R(S+D(R)) - (S+D(R)-G)] + R(S+D(R))\}^2} .$$

C. Derivative comparisons

We wish to compare the magnitudes of $\frac{\partial R_m}{\partial R}(R,\lambda)$ and $\frac{\partial R_m}{\partial R}(R,\beta)$. These derivatives describe the declines in the rate of return on currency associated with marginal declines in the real interest rate which are caused by reductions in the bonds/currency ratio or reductions in the required reserve ratio, respectively. Stated differently, we wish to determine the relative magnitudes of the inflation rate increases associated with attempts to use open market purchases, or reductions in the required reserve ratio, to peg interest rates at levels marginally lower than their current levels. When we do this we must ensure that each of the two rate-pegging experiments whose results are compared begins at the same equilibrium position; that is, that the values of β and λ associated with $\frac{\partial R_m}{\partial R}(R,\lambda)$ are identical to those associated with $\frac{\partial R_m}{\partial R}(R,\beta)$. We accomplish this by evaluating $\frac{\partial R_m}{\partial R}(R,\beta)$ at $\beta(R,\lambda)$ and forming the function

$$\phi(R,\lambda) \equiv \frac{\partial R_m}{\partial R}(R,\lambda) - \frac{\partial R_m}{\partial R}(R,\beta(R,\lambda)) .$$

Substituting the expression for $\beta(R,\lambda)$ into the second expression for $\frac{\partial R_m}{\partial R}(R,\beta)$ yields (after considerable simplification)

$$\frac{\partial R_m}{\partial R}(R,\beta(R,\lambda)) = \frac{[(S+D(R)-G) - RP_bB][RGD'(R) - P_bB(S+D(R)-G)]}{R[S+D(R)-G]\ (\lambda S)^2}$$

where as always $P_bB = (1-\lambda)S + D(R)$.

Proposition 3 demonstrates that reducing the real interest rate by cutting the required reserve ratio is always more inflationary than reducing the rate through open market purchases.

Proposition 3. Assumption sets #1 and #2 imply $\phi(R,\lambda) < 0$.

Proof:

We have seen that

$$\phi(R,\lambda) = \frac{(1-R)D'(R) - P_bB}{\lambda S} - \frac{[(S+D(R)-G) - RP_bB][RGD'(R) - P_bB(S+D(R)-G)]}{R[S+D(R)-G]\ (\lambda S)^2} .$$

It can ultimately be shown that $\phi(R,\lambda) \gtrless 0$ as

$$- \frac{RD'(R)[\lambda S-G]}{S+D(R)-G} \{G - (1-R)[S+D(R)]\} \gtrless P_bB \{G - (1-R)[S+D(R)]\} .$$

Condition (1′) requires $G = (1-R_m)M + (1-R)P_bB$. Now $(1-R_m)M + (1-R)P_bB > (1-R)[M + P_bB]$, since condition (5′) requires $R_m < R$. Since condition (4′) requires $S + D(R) = M + P_bB$, we have $G > (1-R)[S+D(R)]$. Consequently

$$\phi(R,\lambda) \gtrless 0 \quad \text{as} \quad - \frac{RD'(R)[\lambda S-G]}{S+D(R)-G} \gtrless P_bB .$$

Since $R > 0$, we know $G > S+D(R)$. Assumption set #1 gives us $D'(R) > 0$, and set #2 gives us $\lambda S > G$. Consequently the left-hand-side term above is strictly negative. As the right hand-side-term is nonnegative, we have $\phi(R,\lambda) < 0$.

VI. Nominal Interest Rates

The direct observability of nominal interest rates makes them convenient short-run targets for monetary policy.[10] It seems therefore of interest to determine the effects of changes in β and λ on the level of nominal interest rates, and to determine the effects on inflation rates and real interest rates of attempts to use changes in these parameters to reduce nominal rates.

We begin by confirming the existence of a function $R_{nom}(\beta,\lambda)$ which links the nominal interest rate to the levels of the policy parameters in an open neighborhood around a given parameter setting. Since $R_{nom} = \frac{R}{R_m}$, we can rewrite (6) and (7) as

$$(\hat{6}) \quad S + D(R) = \lambda S[1 + \frac{\beta}{R_{nom}}]$$

$$(\hat{7}) \quad G = \lambda S[(1 - \frac{R}{R_{nom}}) + \frac{\beta(1-R)}{R_{nom}}] .$$

Now define $\hat{\mathbf{r}} = (R, R_{nom})$, and the vector-valued function $\hat{g}(\hat{\mathbf{r}}, \beta, \lambda)$ by

$$\hat{g}_1(\hat{\mathbf{r}}; \beta, \lambda) \equiv S + D(R) - \lambda S[1 + \frac{\beta}{R_{nom}}]$$

$$\hat{g}_2(\hat{\mathbf{r}}; \beta, \lambda) \equiv \lambda S[(1 - \frac{R}{R_{nom}}) + \frac{\beta(1-R)}{R_{nom}}] - G .$$

Equilibrium functions $R(\beta, \lambda)$ and $R_{nom}(\beta, \lambda)$ exist in an open neighborhood around a given (β^*, λ^*) so long as there exists an equilibrium vector $(\hat{\mathbf{r}}^*; \beta^*, \lambda^*)$ and the derivative matrix $D_{\hat{\mathbf{r}}}\hat{g}(\hat{\mathbf{r}}^*; \beta^*, \lambda^*)$ is not singular. Differentiation reveals that $R^*(1+\beta^*) - \beta^* > 0$ is sufficient to ensure that $D_{\hat{\mathbf{r}}}\hat{g}(\hat{\mathbf{r}}^*; \beta^*, \lambda^*)$ has a nonzero determinant.

Notice that since $R_{nom} = \frac{R}{R_m}$, we have

$$\frac{\partial R_{nom}}{\partial \beta} = \frac{R_m \frac{\partial R}{\partial \beta} - R \frac{\partial R_m}{\partial \beta}}{R_m^2} = \frac{R_m \lambda S - R[(1-R)D'(R) - P_b B]}{R_m\{D'(R)[R(1+\beta) - \beta] + P_b B(1+\beta)\}}$$

Alternatively, conditions $(\hat{6})$ and $(\hat{7})$ can be differentiated with respect to β , and the resulting equations solved for $\frac{\partial R}{\partial \beta}$ and $\frac{\partial R_{nom}}{\partial \beta}$. This procedure yields

$$\frac{\partial R_{nom}}{\partial \beta} = \frac{R_{nom}[(1+\beta)\lambda S - R_{nom} D'(R)(1-R)]}{\beta(1+\beta)\lambda S + R_{nom} D'(R)[R(1+\beta) - \beta]} .$$

In either case, it is readily seen that $\frac{\partial R_{nom}}{\partial \beta} \gtrless 0$ as $\frac{1+\beta}{\beta} P_b B \gtrless (1-R)D'(R)$, or equivalently as $\hat{F}(\beta, \lambda) \equiv \frac{\beta}{1+\beta} (1-R)D'(R) - D(R) \lessgtr (1-\lambda)S$. Notice that $\hat{F}(\tilde{\beta}, \lambda) < (1-\lambda)S$ [since $F(\tilde{\beta}, \lambda) = (1-\lambda)S$] . Thus there exists an open interval around $\tilde{\beta}$ on which $\frac{\partial R_{nom}}{\partial\beta} > 0$. Notice also that $\hat{F}(0,\lambda) = (1-\lambda)S$ [since at $\beta = 0$ we have $(1-\lambda)S + D(\hat{R}) = 0$]; it can be shown that $\hat{F}'(0,\lambda) \gtrless 0$ as $\hat{R}(1-\hat{R})D'(\hat{R}) \gtrless \lambda S-G$. Since $\hat{R} < 1$ by assumption, when G is sufficiently large (close to λS) there is an open interval to the right of (but not including) $\beta = 0$ on which $\frac{\partial R_{nom}}{\partial\beta} < 0$. It is possible, however, to construct examples in which $\left.\frac{\partial R_{nom}}{\partial\beta}\right|_{\beta=0} > 0$, and indeed in which $\frac{\partial R_{nom}}{\partial\beta} > 0$ for all $\beta > 0$.

Unfortunately, we cannot make the conditions under which $\frac{\partial R_{nom}}{\partial\beta} > 0$ more precise than those just described. This is disappointing, since one might well regard $\frac{\partial R_{nom}}{\partial\beta} > 0$ as part and parcel of the "conventional wisdom": certainly the monetary authorities seem to believe that open market purchases drive down nominal interest rates. What we can say is contained in

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Proposition 4.

(1) Let $\underline{G}$ solve $\hat{R}(1-\hat{R})D'(\hat{R}) = \lambda S-G$, with the understanding that $\underline{G} = 0$ if this equation has no positive solution for G . Then if $G \in (\underline{G},\lambda S)$, there exists a range of nonnegative (but relatively small) values of β at which the the conventional wisdom holds for real rates but not nominal rates. This range is bounded below by, but does not include, $\beta = 0$.

(2) There exists a range of positive (and relatively large) values of β at which the conventional wisdom holds for both nominal and real rates. This range includes, and is bounded above by, $\beta = \tilde{\beta}$.

Example 2 below describes the simplest situation of this type. In this example there exists $\beta' \in (0,\tilde{\beta})$ such that $\frac{\partial R_{nom}}{\partial \beta} < 0$ whenever $\beta \in (0,\beta')$, and $\frac{\partial R_{nom}}{\partial \beta} > 0$ whenever $\beta \in (\beta',\tilde{\beta}]$. [Note that the existence of at least one $\beta \in (0,\tilde{\beta})$ such that $\frac{\partial R_{nom}}{\partial \beta}(\beta,\lambda) = 0$ is guaranteed by Proposition 4.]

Example 2.

This economy is identical to that of Example 1. The value of β' (see above) is approximately $0.1110 < \tilde{\beta} \cong 0.5851$. A plot of the function $F(\beta,\frac{1}{10})$ over the domain $(0,\tilde{\beta})$ is presented in Figure 2.

The procedure for obtaining $\frac{\partial R_{nom}}{\partial \lambda}$ is essentially similar to that for obtaining $\frac{\partial R_{nom}}{\partial \beta}$. Implementing it yields

$$\frac{\partial R_{nom}}{\partial \lambda} = \frac{1}{R_m^2 \beta \lambda^2 S} \left[\frac{\{\beta G + A[S+D(R)]\} \{R_m \beta \lambda S - R[P_b B + RD'(R)\}}{A} + \frac{R^2 [S+D(R)]}{\beta} \right]$$

where $A \equiv \beta\{D'(R)[R(1+\beta)-\beta] + (1+\beta)P_b B\}$. It can then be shown that

$$\frac{\partial R_{nom}}{\partial \lambda} \gtrless 0 \quad \text{as} \quad \frac{1+\beta}{\beta} P_b B \gtrless \frac{G}{S+D(R)} D'(R) .$$

Since $G > (1-R)[S+D(R)]$, $\frac{\partial R_{nom}}{\partial \lambda} > 0$ implies $\frac{\partial R_{nom}}{\partial \beta} > 0$, and $\frac{\partial R_{nom}}{\partial \beta} < 0$ implies $\frac{\partial R_{nom}}{\partial \lambda} < 0$. Thus $\frac{\partial R_{nom}}{\partial \lambda} < 0$ for $\beta \in (0,\beta']$, and indeed for some distance to the right of β', in the example above. Stated differently, we can use specifications which conform to the hypotheses of Proposition 1 to construct "perverse" examples in which $\frac{\partial R_m}{\partial \beta} > 0$ and $\frac{\partial R_{nom}}{\partial \beta} > 0$, but $\frac{\partial R_{nom}}{\partial \lambda} < 0$. In these examples a marginal reduction in the nominal interest rate can be effected either by a reduction in the bonds/currency ratio or by an increase in the required reserve ratio. Since $\frac{\partial R_m}{\partial \lambda} > 0$ whenever the conventional wisdom holds, in "perverse" examples a reserve ratio adjustment which reduces the nominal interest rate will also reduce the inflation rate. Clearly, open market purchases are not always the inflation-minimizing method of reducing the nominal rate; stated differently, we will not be able to obtain an analogue of Proposition 3 for nominal interest rate-pegging. Notice, however, that in "perverse" examples reserve ratio adjustments that bring down the nominal rate also causes the real rate to increase. Thus in economies like these reserve ratio changes are of doubtful utility to policymakers whose ultimate objective is to ease real credit conditions.

At $\tilde{\beta}$ we have $P_b B = (1-R)D'(R)$, so $\frac{\partial R_{nom}}{\partial \lambda} \gtrless 0$ as $(1-\tilde{R})[S+D(\tilde{R})] \gtrless \frac{\tilde{\beta}}{1+\tilde{\beta}}G$. Thus we may not always be able to construct "thoroughly conventional" examples in which $\frac{\partial R_m}{\partial \beta} > 0$, $\frac{\partial R_{nom}}{\partial \beta} > 0$, and $\frac{\partial R_{nom}}{\partial \lambda} > 0$. Example 3 describes an economy in which it is possible to construct such examples. In fact, in this economy the interval $(0,\tilde{\beta})$ on which the "conventional wisdom" holds can be partitioned into the following disjoint and consecutive subintervals:

(1) a "thoroughly perverse" interval on which $\frac{\partial R_{nom}}{\partial \beta} < 0$ and $\frac{\partial R_{nom}}{\partial \lambda} < 0$,
(2) a "perverse" interval on which $\frac{\partial R_{nom}}{\partial \beta} > 0$ but $\frac{\partial R_{nom}}{\partial \lambda} < 0$, and
(3) a "thoroughly conventional" interval on which $\frac{\partial R_{nom}}{\partial \beta} > 0$ and $\frac{\partial R_{nom}}{\partial \lambda} > 0$.

Example 3.

This economy is also identical to that of Example 1. The lower bound of the "thoroughly conventional" interval is $\bar{\beta} \cong 0.4660 < \tilde{\beta} \cong 0.5851$. A plot of the function $\bar{F}(\beta,\frac{1}{10}) \equiv \frac{\beta}{1+\beta} \frac{G}{S+D(R(\beta,\frac{1}{10}))} D'(R(\beta,\frac{1}{10})) - D(R(\beta,\frac{1}{10}))$ over the domain $(0,\tilde{\beta})$ appears in Figure 2. Notice that the $\bar{F}$ curve crosses the horizontal line $(1-\lambda)S$ at $\bar{\beta}$. When the curve lies above the line $\frac{\partial R_{nom}}{\partial \lambda} < 0$; when it lies below the line $\frac{\partial R_{nom}}{\partial \lambda} > 0$

In this example the "thoroughly conventional" interval is short relative to the "perverse" interval. Both the existence and length of this interval are quite sensitive to changes in the specification, however. Increasing G to 0.0125 eliminates it entirely; reducing G to 0.0075 makes it longer than the perverse interval.

The existence of "thoroughly conventional" intervals suggests the possibility that a reduction in the nominal interest rate engineered via a change in the required reserve ratio can (1) produce a lower inflation rate than an equal nominal rate reduction engineered by open market purchases , while (2) causing a <u>decline</u> in the real interest rate -- stated differently, that there might be circumstances under which an inflation-averse policymaker who desired to reduce the nominal interest rate without tightening real credit conditions would prefer reducing the required reserve ratio to conducting open market purchases. So far we have succeeded neither in constructing examples in which this is the case nor in proving that no such examples exist. We <u>can</u> show that any example of this type which may exist will involve a tradeoff between inflation rates and real interest rates: that is, that whenever a nominal interest rate decline engineered by a reduction in the required reserve ratio produces a lower inflation rate than an equal decline brought about by an open market purchase, the reserve ratio reduction leaves the real interest rate higher than does the open market purchase.

Proposition 5. Let $(\bar{\beta},\bar{\lambda})$ represent an initial policy setting, and $(\bar{R}_m,\bar{R},\bar{R}_{nom})$ the associated initial rates of return. Let $\hat{R}_{nom} = R_{nom}(\hat{\beta},\bar{\lambda})$ for some $\hat{\beta} < \bar{\beta}$, where $\hat{R}_{nom} < \bar{R}_{nom}$. Suppose that there exists $\hat{\lambda} < \bar{\lambda}$ such that $R_{nom}(\bar{\beta},\hat{\lambda}) = \hat{R}_{nom}$, and that $R_m(\bar{\beta},\hat{\lambda}) > R_m(\hat{\beta},\bar{\lambda})$. Finally, suppose that the "conventional wisdom" (in the sense of Proposition 1) holds for each $(\beta,\lambda) \in [\hat{\beta},\bar{\beta}] \times [\hat{\lambda},\bar{\lambda}]$, that $\frac{\partial R_{nom}}{\partial \beta}(\beta,\bar{\lambda}) > 0$ for $\beta \in [\hat{\beta},\bar{\beta}]$, and that $\frac{\partial R_{nom}}{\partial \lambda}(\bar{\beta},\lambda) > 0$ for $\lambda \in [\hat{\lambda},\bar{\lambda}]$. Then $R(\bar{\beta},\hat{\lambda}) > R(\hat{\beta},\bar{\lambda})$.

Proof:

Suppose $R(\bar{\beta},\hat{\lambda}) \leq R(\hat{\beta},\bar{\lambda})$. Then since $\frac{\partial R}{\partial \lambda}(\bar{\beta},\lambda) > 0$ throughout the relevant domain, there exists $\tilde{\lambda} \in [\hat{\lambda},\bar{\lambda})$ such that $R(\bar{\beta},\tilde{\lambda}) = R(\hat{\beta},\bar{\lambda})$. Now Proposition 2 ensures that $\frac{\partial R_m}{\partial \lambda}(\bar{\beta},\lambda) > 0$ throughout the relevant domain. Consequently we must have $R_m(\bar{\beta},\tilde{\lambda}) \geq R_m(\bar{\beta},\hat{\lambda}) > R_m(\hat{\beta},\bar{\lambda})$, which is to say $R_m(\bar{\beta},\tilde{\lambda}) > R_m(\hat{\beta},\bar{\lambda})$. This inequality contradicts Proposition 1. Thus $R(\bar{\beta},\hat{\lambda}) > R(\hat{\beta},\bar{\lambda})$.

VII. Concluding Remarks

The purpose of this paper was to use a general equilibrium model of deficit finance to compare the relative magnitudes of the increases in the rate of inflation associated with attempts to reduce market interest rates through open market purchases, or reductions in the required reserve ratio. We have identified conditions under which changes in both policy instruments have the "conventional" effects just described -- conditions under which both open market purchases and reserve ratio reductions tend to drive down the real interest rate while driving up the inflation rate. We have shown that under these circumstances a given reduction in the real interest rate engineered through open market purchases always yields a smaller increase in the rate of inflation than an equal reduction brought about by a decrease in the required reserve ratio. Thus in our model an inflation-averse policymaker who desires to reduce the real interest rate should prefer an open market purchase over a reserve ratio reduction.

We have also succeeded, though somewhat less completely, in identifying conditions under which the nominal-rate analogue of the aforementioned "conventional wisdom" holds true. We have also shown that there are circumstances, consistent with the real-rate version of the conventional wisdom, under which the nominal-rate version holds for open market purchases but not reserve ratio changes -- circumstances in which a reduction in the nominal interest rate engineered through a change in the required reserve ratio requires an increase in the ratio. Under these circumstances an inflation-averse who desires to reduce the nominal rate will prefer to increase the reserve ratio rather than carry out the open market purchase which generates the same rate.

While we have not been able to rule out the possibility that there are model specifications/policy settings, consistent with the conventional wisdom, under which the aforementioned nominal rate-reducer might prefer reserve ratio <u>reductions</u> to open market purchases, we have been unable to find such specifications/settings. Finally, we can show that if situations of this type do arise the decline in the real interest rate caused by the reserve ratio reduction will be smaller than the decline caused by the open market purchase which produces the same nominal interst rate. Thus whenever reserve ratio changes are the least inflationary tool for nominal rate-pegging, they are also the tool which which produces the least improvement in real credit conditions.

Footnotes

1. For a description of current Federal Reserve System monetary policy operating procedures, and the sense in which they are similar to (and different from) the interest rate-targeting procedures characteristic of the years prior to 1979, see Wallich (1984), esp. pp. 21-22, 26-27, and Gilbert (1985), esp. pp. 19-21. For an example of a recent policy intervention apparently designed to reduce interest rates, see note 8 below.

2. For a detailed account of the history and impact of the 1936-1937 reserve ratio increases see Friedman and Schwartz (1963), pp. 520-532.

3. This source of restraint is emphasized by Toma (1988), who argues that it prevented the System from increasing reserve ratios in order to enhance its seigniorage revenues. However, Haslag and Hein (1989) conclude that the reserve requirement burden has not in fact increased during the years since the passage of the Monetary Control Act. See also Sellon (1984), esp. p. 12.

4. See Goodfriend and Hargraves (1983), esp. pp. 15-16, Goodfriend (1984), and Sellon (1984), esp. p. 11.

5. This point is made by Toma (1988); see note 3 above. See also Sellon (1984), esp. pp 15-16; Sellon emphasizes the ptential usefulness of manipulating the differential reserve ratios on various classes of managed liabilities as a device to improve control of the broader monetary aggregates.

6. Our focus on overlapping generations models is not intended to imply that analysis of the sort we conduct cannot be, or has not been, conducted using dynamic general equilibrium models of other types.

7. Strictly speaking, Wallace showed that in a stationary economy without population or per capita output growth (with a gross total output growth rate of unity) the conventional wisdom must fail whenever the gross real interest rate on bonds exceeds unity (see p. 15 below). The assertion contained in the text is a straightforward extension of this result to the case of a constant output growth rate.

8. The real growth rates cited are average quarterly percent changes in real GNP, seasonally adjusted annual rates (source: U.S. Commerce Department). The real interest rate cited is the difference between the average annual interest rate on 90-day Treasury bills (source: Federal Reserve System) and the average annual percent change in the all-items Consumer Price Index (source: U.S. Department of Labor).

9. The figures cited are ratios of the annual average U.S. national debt (source: U.S. Treasury Department) to the annual average U.S. monetary base (source: Federal Reserve System).

10. Walsh (1983) considers, but ultimately rejects, the notion that the Federal Reserve System should attempt to target the real interest rate. His rejection is based in part on the difficulty of usefully defining and/or accurately measuring this rate.

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Appendix

An economy in which the conventional wisdom holds $\left[\frac{\partial R_m}{\partial \beta}(\beta,\lambda) > 0\right]$ for plausible initial values of λ, R, and R_m, and relatively large values of β:

Let $U(c_1,c_2) = \ln c_1 + \ln c_2$, as in Examples 1-3. Each generation $t \geq 1$ consists of N savers endowed $(w_1^s,w_2^s) = (1,0)$ and N borrowers endowed $(w_1^b,w_2^b) = (40,39)$; consequently $S = \frac{N}{2}$ and $D = N(20-\frac{39}{2R})$. Finally, let $G = 0.015$ and $\lambda = 0.1$. Since $R(0,\frac{1}{10}) \cong 0.9535 > R_m(0,\frac{1}{10}) = 0.7$, this value of λ is binding for all $\beta \geq 0$. Since we require $D(R) < 0$, which is to say $R < \frac{39}{40} = 0.975$, the range of relevant β-values must be confined to those consistent with $R(\beta,\frac{1}{10}) < \frac{39}{40}$; the β-range this condition generates has the form $\beta \in (0,\hat{\beta})$, with $\hat{\beta} \cong 9.49$. Analysis of $\frac{\partial R_m}{\partial \beta}(\beta,\frac{1}{10})$ reveals that $\tilde{\beta} \cong 10.14 > \hat{\beta}$, so the conventional wisdom holds for all relevant values of β. When $\beta = 9$, for example, $R \cong 0.974$, $R_m \cong 0.923$, and $\frac{\partial R_m}{\partial \beta} \cong 0.00527$. A plot of $\frac{\partial R_m}{\partial \beta}(\beta,\frac{1}{10})$ is displayed in Figure 3.

Figure 1

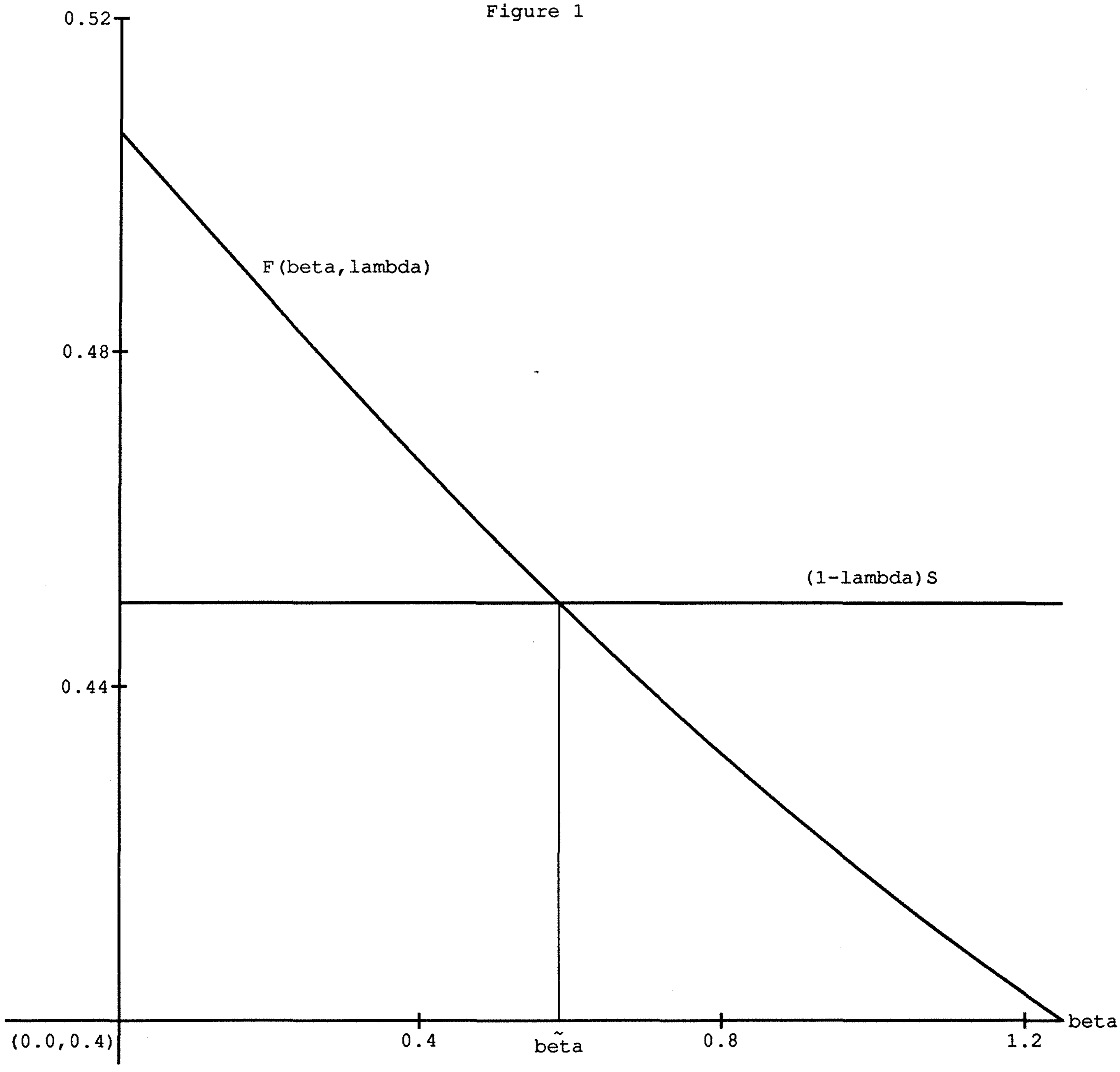
0.52
0.48
0.44
F(beta,lambda)
(1-lambda)S
(0.0,0.4)
0.4
$\tilde{\text{beta}}$
0.8
1.2
beta

Figure 2

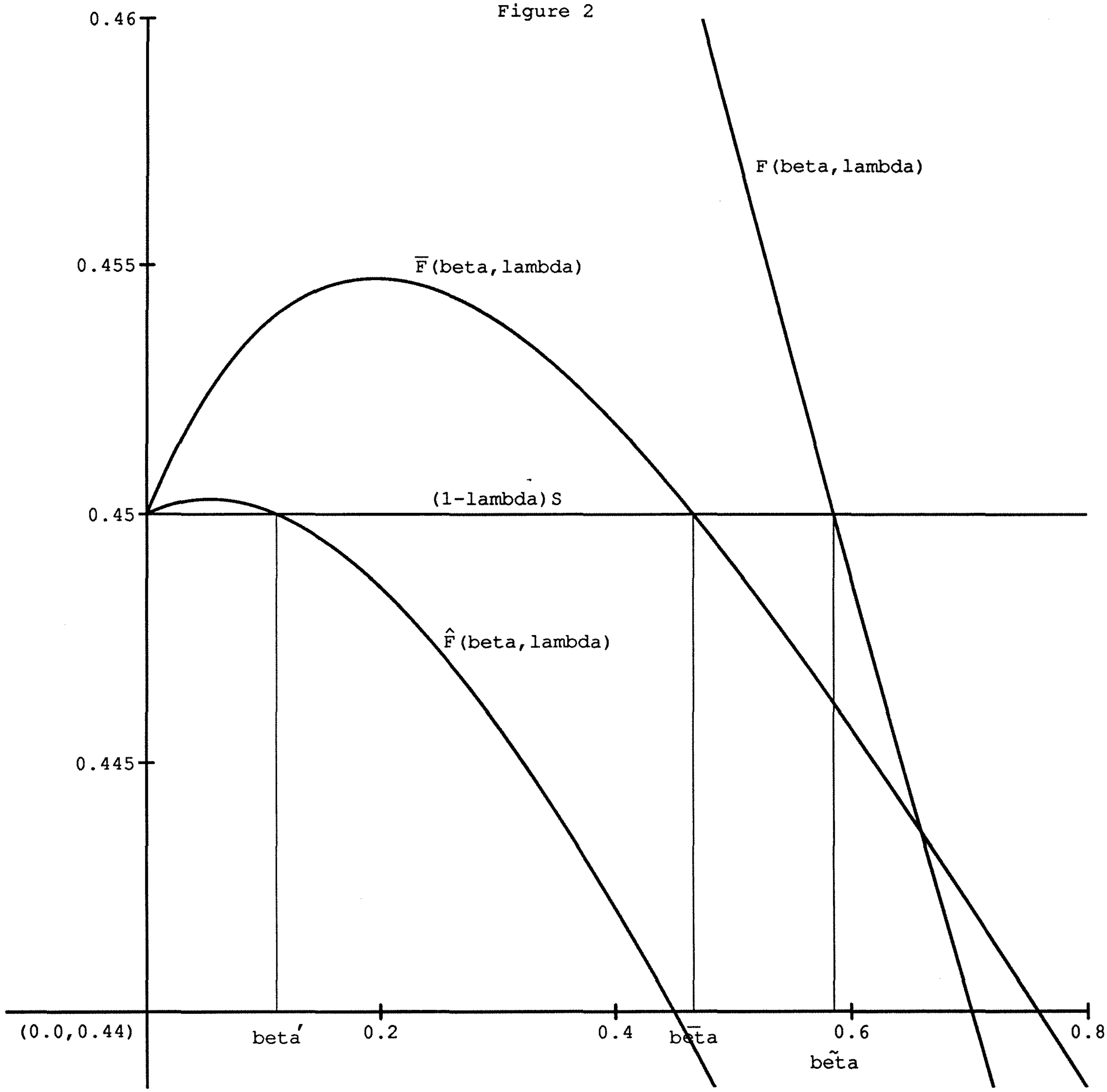
0.46
0.455
0.45
0.445
F(beta,lambda)
F̄(beta,lambda)
(1-lambda)S
F̂(beta,lambda)
(0.0,0.44)
beta′
0.2
0.4
bēta
0.6
0.8
bẽta

Figure 3