Inflation Risk and Optimal Monetary Policy

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ABSTRACT

This paper shows that the optimal monetary policies recommended by New Keynesian models still imply a large amount of inflation risk. We calculate the term structure of inflation uncertainty in New Keynesian models when the monetary authority adopts the optimal policy. When the monetary policy rules are modified to include some weight on a price path, the economy achieves equilibria with substantially lower long-run inflation risk. With either sticky prices or sticky wages, a price path target reduces the variance of inflation by an order of magnitude more than it increases the variability of the output gap.

## I.	Introduction

Significant progress has been made in adapting modern macroeconomic models for use in policy analysis. Monetary economists have recently focused on determining the optimal monetary policy in New Keynesian models. This paper shows that optimal monetary policy in New Keynesian models produces a great deal of uncertainty about inflation at medium to long horizons. We show, however, that including some weight on a price path in the monetary policy rule substantially reduces long-run inflation variability regardless of the types of nominal rigidities. The source of nominal frictions does influence the impact that a price-path target has on output stabilization. In a New Keynesian model, however, a price-path target reduces long-run inflation uncertainty by an order of magnitude more that it increases the variability of the output gap.

An important distinction between New Keynesian models and reality is the relative importance of long-run inflation uncertainty. In New Keynesian models, long-run inflation uncertainty is typically not, by itself, a source of welfare loss. Policymakers, on the other hand, closely monitor long-term interest rates because they reflect expectations about future policy. Investors pay to insure against long-run risks. Although the fundamental risk that concerns policymakers and investors may be about long-run real growth—as in Orphanides and Williams (2002), Bansal and Yaron (2004), or Ries (2005)—uncertainty about long-run inflation also matters.

Uncertainty about long-run inflation is revealed in the variability of the term structure of interest rates. Gallmeyer et al. (2007) present evidence about the volatility of yields on U.S. Treasury securities across a term structure from 1 quarter to 10 years. They report that the standard deviation of yields at the short end of the term structure, for the first 4 or 5 quarters, is

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about 1.9 percent at an annual rate since 1990. The standard deviation of the 10-year Treasury

yield is about 1.1 percent. Using U.S. data from the indexed bond market from 1997 to 2000,

McCulloch and Kochin (2000) estimate that the volatility of the inflation premium is roughly

twice that of the real interest rate across the term structure. The purpose of our paper is not to

model the reason why this long-run inflation uncertainty matters. Rather, it is to evaluate the

relative effects of alternative monetary policy rules on uncertainty.

The remainder of the paper proceeds as follows. Section II describes a general model.

Section III discusses the calibration assumptions that are common to all the models and the

assumptions about nominal rigidities that distinguish among them. Section IV shows how the

New Keynesian models behave as policymakers approach the optimal policy. We also examine

the effect of the policy rules on long-run uncertainty about expected inflation. In Section V, we

analyze the consequences of price-path targeting for short-run output and inflation variability as

well as for long-run inflation uncertainty. Section VI concludes.

II.   The Model

A New Keynesian model is developed in this paper that nests the three alternative

versions: a flexible model, a sticky-price model, and a sticky-wage model. The three versions

share many common features: the utility function, specification for money, and policy rule. They

all have distortions associated with monopolistic competition in both the goods and labor

markets. Modest investment adjustment costs are included to keep the real effects of policy from

having too large an impact in the sticky-price and sticky-wage models. Our model includes

exogenous disturbances to the monetary policy rule, preferences, and technology. As in Ireland

(2005), the monetary policy shocks comprise both highly persistent shocks to the inflation target

and less persistent liquidity shocks to the short-term interest rate. This section outlines a dynamic stochastic general equilibrium model with both sticky prices and sticky wages, but then indicates the calibrations that convert the price- and wage-setting rules into flexible specifications.

Households: Each household is an infinitely lived agent who participates in state-contingent securities markets. That assumption enables households to be homogenous with respect to consumption investment, capital, money, and bonds. Household h values consumption, c_t, and real money balances, (M_t/P_t), but dislikes labor. Those preferences are summarized by the following expected utility function,

$$E_t\left[\sum_{i=0}^{\infty}\beta^i a_{t+i}\left(\ln(c_{t+i}^*) - \kappa\frac{n_{h,t+i}^{1+\sigma_1}}{1+\sigma_1}\right)\right],$$

where the consumption bundle is an aggregate of the consumption good and real balances,

$$c_t^* = \left((c_t)^{(\sigma_2-1)/\sigma_2} + b\left(\frac{M_t}{P_t}\right)^{(\sigma_2-1)/\sigma_2}\right)^{\sigma_2/(\sigma_2-1)}.$$

E_t is the conditional expectation at time t and β is the discount factor. The preference parameter, a_t, resembles an aggregate demand shock and evolves such that

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \upsilon_{a,t},$$

where $0 \leq \rho_a < 1$ and $\upsilon_{a,t} \sim N(0, \sigma_a^2)$.

Households are monopolistically competitive suppliers of differentiated labor services to firms. Total labor hours utilized by the firms, n_t, is calculated as a Dixit and Stiglitz (1977) continuum of labor hours, $n_{t,h}$, supplied by each household, $h \in [0,1]$:

$$n_t = \left[\int_0^1 (n_{h,t})^{\varepsilon_w/(\varepsilon_w-1)} dh\right]^{(\varepsilon_w-1)/\varepsilon_w},$$

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where $-\varepsilon_w$ is the wage elasticity of demand for household h's labor services. Cost minimization by the firms yields the demand equation for household h's labor services:

$$n_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\varepsilon_w} n_t,$$

where $W_{h,t}$ is the nominal wage rate of household h and W_t is interpreted as the aggregate nominal wage rate:

$$W_t = \left[\int_0^1 (W_{h,t})^{(1-\varepsilon_w)} dh\right]^{1/(1-\varepsilon_w)}.$$

Households own the capital, k_t, in this economy and supply it to the firms. Every period, household h chooses a level of investment, i_t, such that:

$$k_{t+1} - k_t = \phi(i_t / k_t)k_t - \delta k_t,$$

where $\phi(\cdot)$ is a functional form for capital adjustment costs and δ is the depreciation rate. The functional form, $\phi(\cdot)$, represents capital adjustment costs in the manner of Hayashi (1982), where the resources lost in the conversion of investment to capital equals $i_t - \phi(i_t / k_t)k_t$. Those lost resources are an increasing and convex function of the steady-state investment-to-capital ratio, such that $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$.

Household h begins each period with an initial level of nominal money balances, M_{t-1}, and receives a payment, $R_{t-1}B_{t-1}$, from its nominal bond holdings, B_{t-1}, where R_t is the gross nominal interest rate earned on bonds during period t. During the period, household h receives labor income, $W_{h,t}n_{h,t}$; dividends from the firms, D_t; a lump-sum transfer from the monetary authority, T_t; a payment from the state contingent securities markets, $A_{h,t}$; and rental income from capital, $P_t q_t k_t$, where q_t is the real rental rate of capital. Those resources then are used to fund

consumption and investment purchases and end-of-the-period bond, B_t, and money, M_t, holdings.
The budget constraint for household h is represented as follows:

$$B_t + P_t(c_t + i_t) + M_t = W_{h,t} n_{h,t} + P_t q_t k_t + D_t + R_{t-1} B_{t-1} + T_t + M_{t-1} + A_{h,t}.$$

Finally, household h chooses a level of c_t, i_t, k_t, B_t, and M_t that maximizes its expected utility
subject to its capital accumulation and budget constraint equations.

Wage contracts between households and firms can last for multiple periods. As a result,
household h must determine each period whether or not an opportunity exists to negotiate a new
nominal wage, $W_{h,t}$, for its labor services, $n_{h,t}$. Using the Calvo (1983) model of random
adjustment, the probability that household h can set a new nominal wage, W^*_t, is η_w, and the
probability that its nominal wage can rise only by the steady-state inflation rate, π, is $(1 - \eta_w)$.
The nominal wage is perfectly flexible in this specification when η_w is set equal to 1. When
household h has a wage-adjustment opportunity, it selects a nominal wage which maximizes its
utility given the firms' demand for its labor:

$$W^*_t = \left[\left(\frac{\chi \varepsilon_w}{\varepsilon_w - 1} \right) \frac{E_t \left[\sum_{i=0}^{\infty} \beta^i \left(1 - \eta_w\right)^i \left(W_{t+i} \pi^{-i}\right)^{\varepsilon_w (1+\sigma_1)} \left(n_{t+i}\right)^{1+\sigma_1} \right]}{E_t \left[\sum_{i=0}^{\infty} \beta^i \left(1 - \eta_w\right)^i \left(P_{t+i} \pi^{-i}\right)^{-1} \left(W_{t+i} \pi^{-i}\right)^{\varepsilon_w} n_{t+i} \left(c^*_{t+i}\right)^{(1-\sigma_2)/\sigma_2} \left(c_{t+i}\right)^{-1/\sigma_2} \right]} \right]^{\frac{1}{\varepsilon_w \sigma_1 + 1}},$$

where $(1 - \eta_w)^i$ is the probability that household h will not have an opportunity to negotiate a
new nominal wage in the subsequent i periods.

Firms: Firms, owned by the households, are monopolistically competitive producers of
differentiated goods. Firm f hires labor, $n_{f,t}$, and rents capital, $k_{f,t}$, from the households to produce
its output, $y_{f,t}$, according to a Cobb-Douglas production function:

$$y_{f,t} = Z_t (k_{f,t})^\alpha (n_{f,t})^{(1-\alpha)},$$

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where Z_t is an economy-wide productivity factor and $0 \le \alpha \le 1$. The productivity factor, Z_t, evolves such that

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + (1 - \rho_Z) \ln(Z) + \upsilon_{Z,t},$$

where $0 \le \rho_Z < 1$, Z is the steady-state value of Z_t, and $\upsilon_{Z,t} \sim N(0, \sigma^2_Z)$.

Aggregate output, y_t, is a Dixit and Stiglitz (1977) continuum of differentiated products:

$$y_t = \left[\int_0^1 (y_{f,t})^{\varepsilon_p / (\varepsilon_p - 1)} df \right]^{(\varepsilon_p - 1)/\varepsilon_p},$$

where $-\varepsilon_p$ is the price elasticity of demand for good f. Cost minimization by households yields the demand equation for firm f's good:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\varepsilon_p} y_t,$$

where $P_{f,t}$ is the price charged by firm f and P_t is a nonlinear price index:

$$P_t = \left[\int_0^1 (P_{f,t})^{(1-\varepsilon_p)} df \right]^{1/(1-\varepsilon_p)}.$$

Every period, firm f utilizes the combination of labor and capital that minimizes its production costs, $w_t n_{f,t} + q_t k_{f,t}$, given the production function. The first-order conditions from firm f's cost minimization yield the following factor demand equations:

$$q_t = \psi_t \alpha Z_t (n_{f,t} / k_{f,t})^{(1-\alpha)}$$

$$W_t / P_t = \psi_t (1-\alpha) Z_t (k_{f,t} / n_{f,t})^{\alpha},$$

where ψ_t is the Lagrange multiplier on the cost minimization problem and is interpreted as the real marginal cost of producing an additional unit of output. Furthermore, ψ_t is identical for all firms because every firm pays the same per unit capital and labor costs and has an equal measure of productivity, Z_t.

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Firm f also determines each period whether or not the price, $P_{f,t}$, for its product, $y_{f,t}$, can be adjusted. Using the Calvo (1983) model of random adjustment, the probability that firm f can set a new price, P^*_t, is η_p, and the probability it can only adjust its price by the steady-state inflation rate, π, is $(1 - \eta_p)$. Prices in this specification become perfectly flexible when η_p is set equal to 1. When a firm can set a new price, it selects a price that maximizes the discounted value of its expected current and future profits subject to its factor demand and product demand equations:

$$P^*_t = \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \right) \frac{E_t \left[\sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \pi^i \left(1 - \eta_p\right)^i \left(P_{t+i} \pi^{-i}\right)^{1+\varepsilon_p} \psi_{t+i} y_{t+i} \right]}{E_t \left[\sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \pi^i \left(1 - \eta_p\right)^i \left(P_{t+i} \pi^{-i}\right)^{\varepsilon_p} y_{t+i} \right]},$$

where $\beta^i \lambda_{t+i} \pi^i$ is the households' real value in period t of an additional unit of profits in period $t+i$ and $(1 - \eta_p)^i$ is the probability that the firm will not have an opportunity to set a new price in the subsequent i periods.

The Monetary Authority: Monetary policy operates with an interest rate rule. Specifically, the monetary authority utilizes a generalized Taylor (1993) rule. The policy rule is defined by the following equations:

$$\ln(R_t / R) = \ln(\pi^*_t / \pi) + \theta_\pi \ln(\pi_t / \pi^*_t) + \theta_{dW} \ln(dW_t / dW^*_t) + \theta_P \ln(P_t / P) + \varepsilon_{R,t},$$

where π^*_t is the target inflation rate, dW_t is the nominal wage inflation rate, dW^*_t is the nominal wage-inflation target, and $\varepsilon_{R,t}$ is a transitory monetary policy shock in which $\varepsilon_{R,t} = \rho_R \varepsilon_{R,t-1} + v_{R,t}$, $v_{R,t} \sim N(0, \sigma^2_R)$. Variables without time subscripts are steady-state values. The steady-state price inflation is equal to the steady-state wage inflation because there is no trend in the real wage. Whether the central bank has a target for price inflation or wage inflation, we assume a common

functional form for the target-setting process (i.e., $dW_t^* = \pi_t^*$), so that we limit the discussion to the target-setting process for price inflation. The target follows a common stochastic AR(1) process such that:

$$\ln(\pi_t^* / \pi) = \rho_\pi \ln(\pi_{t-1}^* / \pi) + v_{\pi,t},$$

where $v_{\pi,t} \sim N(0, \sigma_\pi^2)$. Assuming a common stochastic process for the price- and wage-inflation targets allows us to model policy uncertainty in a symmetric manner.

The inflation target shock and the transitory monetary policy shock have similar effects on the nominal interest rate, except for the differences in their persistence.[1] The target shock is to the long-run inflation objective, while the transitory shock is to short-run liquidity. The monetary authority in the model has a credible long-run inflation target equal to the steady-state inflation rate, but the actual target rate may deviate from the steady state for an extended period.

In our model, the monetary authority has full credibility, which enables agents to distinguish between persistent shocks to the inflation target and temporary shocks to the liquidity position. That assumption is clearly realistic for cases such as the Fed's short-run responses to the 1987 stock market crash and the September 11 attacks. The absence of an explicit inflation target in the United States, however, likely causes some confusion between these two shocks, especially in the Fed's reaction to news about the state of the economy.

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[1] The equivalence of the inflation target shock and the liquidity shock depends on the calibrated values of θ_π or θ_{dW}. In general, if the weight on inflation is equal to 2 and both shocks have the same persistence, then an expansionary shock to the inflation policy has the same effect as an expansionary shock to liquidity.

III. Calibrating the Model

Most of the parameter calibrations that we use are taken from characteristics of the U.S. data and/or have been widely used in the literature. In the utility function, the value of σ_1 is set at 1/3 implying that the elasticity of the labor supply with respect to the real wage is equal to 3. The value of σ_2 is set at 1/2, implying that the interest elasticity of money demand is equal to $-1/2$. The steady-state labor share is 0.3 and the discount factor is 0.99. The capital share of output is set to 0.33 and the capital stock depreciates at 2.5 percent per quarter. The average and marginal capital adjustment costs around the steady state are zero (i.e., $\phi(\cdot) = i/k$ and $\phi'(\cdot) = 1$). The elasticity of the investment-to-capital ratio with respect to Tobin's q, $\chi = [(i/k)\phi''(\cdot)/\phi'(\cdot)]^{-1}$, is set to 5.

Large autocorrelation coefficients for the technology and preference shocks ($\rho_Z = 0.95$ and $\rho_a = 0.90$) imply a high level of persistence. The standard deviation of the technology shock, σ_Z, is set equal to 0.005, which is consistent with the lower volatility of output observed since 1984. The standard deviation of the preference shock, σ_a, is set equal to 0.01, which is consistent with Ireland (2005), who estimates that the preference shock is twice as large (in standard deviation) as the technology shock.

The parameters determining the effect of nominal rigidities are consistent with previous studies by Rotemberg and Woodford (1992), Erceg, Henderson, and Levin (2000), Keen (2004), Christiano, Eichenbaum, and Evans (2005), Ireland (2005), and Levin et al. (2005). The price elasticity of demand is set equal to 6, implying a steady-state markup of 20 percent. This assumption is consistent with Rotemberg and Woodford (1992) and Christiano, Eichenbaum, and Evans (2005). We set the probability of price adjustment equal to 1 for the flexible-price case and equal to 0.25 for the sticky-price case which implies that firms change prices on average

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once a year. We follow Levin et al. (2005) in setting the wage elasticity of labor demand equal to 6, implying a steady-state markup of 20 percent.[2] The probability of price adjustment is set equal to 1 for the flexible wage case and equal to 0.25 for the sticky-wage case.

In our view, there are two important sources of uncertainty in U.S. policymaking that correspond to liquidity and inflation target shocks. We calibrate the liquidity shock to follow an AR(1) process with a first-order autocorrelation coefficient of 0.3 and a standard deviation of 0.002. This uncertainty may be due to the Fed's practice of making discrete changes in the interest rate. That practice causes a stochastic difference between the model's implied target and the actual target. Dueker (2000), in a study of the prime rate, used a dynamic ordered probit model to estimate the spread between the banking system's desired prime rate and the actual prime rate when changes are discrete. Using data from 1974-1999, Dueker estimates that the spread follows a first-order AR(1) process with an autocorrelation coefficient equal to 0.37 and a standard deviation of 26 basis points. Since the prime rate has been closely linked to the federal funds rate, the difference between the prime rate and this latent variable should be a good proxy for the error induced into policy by the FOMC's practice of making discrete policy changes.

In recent years, FOMC members have stated inflation target preferences ranging from 1 to 3 percent. The lack of an explicit target suggests that the perceived target inflation rate is stochastic. We set the steady-state annual inflation rate at 2 percent and assume that a shock to the target inflation rate has a first-order autocorrelation coefficient equal to 0.95 and a quarterly standard deviation of 0.125 percent. Under that specification, a 95 percent confidence interval approximates an informal annual target range of 0.5 to 3.5 percent in the flexible price model.

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[2] Christiano, Eichenbaum, and Evans (2005), on the other hand, estimate a much smaller wage markup.

IV.   Evaluating Monetary Policy

The optimal monetary policy in New Keynesian models depends on the source of the nominal rigidity. King and Wolman (1999) and Woodford (2003) show that targeting price inflation is optimal monetary policy in a sticky-price framework. Canzoneri, Cumby, and Diba (2004), on the other hand, find that monetary policy should target wage inflation when wages are sticky. Given those results, we evaluate both a price-inflation-targeting rule in a sticky-price model and a wage-inflation-targeting rule in sticky-wage model. We begin by examining the effect of alternative policies on the variability of the output gap and inflation.

Optimal Policy with No Policy Shocks. Levin et al. (2005) maintain that policy shocks to the inflation target or the liquidity position are not beneficial and, as a result, the monetary authority eliminates them when conducting optimal policy. Beginning with that assumption, we set the variances of the policy shocks to zero. The weights on inflation deviations from target range from 1.1 to 5. To determine the standard deviation of the output gap and the annual inflation rate, we perform 5,000 simulations on a sample set of 260 quarters of data where the first 100 quarters are discarded to randomize the initial conditions.

We assume that the monetary authority follows a price-inflation-targeting rule in the sticky-price model and a wage-inflation targeting rule in the sticky-wage model. Figure 1 shows the variance of the output gap (upper panels) and the annual inflation rate (lower panels) as the weight on price inflation, θ_π, changes in the sticky-price model (left column) and as the weight on wage inflation, θ_{dW}, varies in the sticky-wage model (right column). The results without policy shocks are shown with a solid line. The figure illustrates that, in the absence of policy shocks, the variance of the output gap and inflation is nearly zero in the sticky-price model when

θ_π is above 3. When θ_{dW} equals 3 or more in the sticky-wage model, the variance of the output gap is less than 0.5 percent and the variance of inflation is slightly above 3.5 percent at an annual rate.[3]

Optimal Policy with Policy Shocks. We now modify our analysis by adding the assumption that both inflation target shocks and liquidity shocks are present in the economy. In the United States, the Federal Reserve follows a strategy that leads to both the inflation target shocks and liquidity shocks. The absence of a numerical inflation objective creates uncertainty about the Fed's inflation target.[4] Gurkaynak, Sack, and Swanson (2003) show that the inflation premium in forward interest rates responds significantly to macroeconomic news, which they attribute to the absence of an explicit inflation target. Using a learning model, Orphanides and Williams (2005) show that uncertainty about the inflation objective can lead to sizable fluctuations in the long-term inflation outcome. Hence, inflation target shocks are the unintentional consequence of having a vague inflation objective. Liquidity shocks are also inadvertently introduced into the U.S. economy by the FOMC's practice of adjusting the federal funds rate in 25-basis-point increments.

The dashed lines in Figure 1 replicate the experiments described in the previous subsection—this time including policy shocks. The introduction of policy shocks raises the variability of the output gap and inflation in both models. In a sticky-price economy, the variance of the output gap increases from near zero to 0.2 percent and the variance of the annual inflation rate climbs from near zero to 2.0 percent when θ_π is 3 or more. Similarly, the variance of the output gap rises from less than 0.1 percent to 0.3 percent and the variance of the annual inflation

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[3] We also experimented with an output term in the policy rule. In general, the optimal weight on output is zero if the reaction coefficient on price or wage inflation is 3 or more.
[4] See Ireland (2005) for a historical analysis of the Fed's unobserved inflation objective. Also, see Kozicki and Tinsley (2005) for estimates of these policy shocks.

rate escalates by more than 2 percentage points to 6.0 percent when wages are sticky and θ_{dW} is 3 or higher.

The Term Structure of Uncertainty with Inflation Targeting. Monetary policymakers are concerned about the consequences of their policy actions on expectations embedded in the longer-term bond market. Although our model does not contain the features needed to examine this issue fully, we report the impact of alternative policy rules on the volatility of inflation and the output gap over a term structure ranging from one to ten years. We do not consider the covariance between inflation and the output gap. In Figure 2, the coefficient on the price-inflation target or wage-inflation target is set to 3. By starting at the steady state and shocking the exogenous variables, we simulate 10 years of data over 5,000 times to obtain a distribution of the average annual inflation rates and output gaps over 1- through 10-year horizons. This experiment enables us to address the question, "How does the monetary policy rule affect the distribution of future inflation and the output gap over the next 10 years?"

The upper panels in Figure 2 show the distribution of the standard deviation of the average annual inflation rates over 1- through 10-year horizons in both models with and without the policy shocks. The results indicate that policy shocks are an important source of inflation uncertainty. In the sticky-price model, the standard deviation of the annual inflation rate is near 0.15 percent at all forecast horizons without policy shocks. Adding policy shocks to that model raises the standard deviation of inflation by about 0.5 percentage points at a 1-year forecast horizon and about 0.75 percentage points after 5 years. In the sticky-wage model, the non-policy shocks create more inflation uncertainty than in the sticky-price specification, especially at short-term horizons. The standard deviation of inflation is 0.88 percent at a 1-year forecast horizon but falls to 0.30 percent at 10-year horizon. Policy shocks create additional inflation uncertainty at

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all horizons in the sticky-wage specification. They increase the standard deviation of inflation by 0.25 percentage points at a 1- year horizon and by 0.7 percentage points at a 10-year horizon.

The lower panels in Figure 2 show the distribution of the standard deviation of the average output gap over 1- through 10-year horizons in both models with and without the policy shocks. The results indicate that policy shocks are an important source of uncertainty in the output gap in the near term, but the long-term uncertainty, as measured in Figure 2, reflects the averaging of the higher output gap variability in the first few years. In the sticky-price model, the standard deviation of the output gap is less than .025 percent at all forecast horizons without policy shocks. Adding policy shocks to this model raises the standard deviation of the output gap by a factor of 5 at all horizons. In the sticky-wage model, the non-policy shocks create more variability in the output gap than in the sticky-price specification, but the addition of policy shocks only increases the relative variability of the output gap by a factor of about 1-1/2 at all horizons.

V.   Price-path targeting

Several authors, beginning with Svensson (1999), have shown that price-path targeting by the monetary authority improves the tradeoff between output and inflation variability. The term "price-path targeting" rather than "price-level targeting" recognizes the possibility that the price target can grow over time. The critical point is that the price level follows a deterministic, not a stochastic, trend. Our price-path target is equivalent to imposing a long-run inflation target on the monetary policy rule. Furthermore, Dittmar and Gavin (2005) find that a small weight on the price level (i.e., $\theta_P > 0$) enables a model to generate a unique solution with calibrated values of less than one for the price-inflation target, θ_π, and wage-inflation target, θ_{dW}.

Figure 3 displays the optimal weight on the price-path target, θ_P, given two baseline

weights on the the price-inflation target, θ_π, and wage-inflation target, θ_{dW}. We set θ_π to 3, which

is the value used in Figure 2, and θ_{dW} equal to1, which is a value that we found to work well

when a price-path target is included in the policy rule. The upper panels show how the variance

of the output gap changes as θ_P is increased from 0.01 to 0.5. When prices are sticky, a higher

value for θ_π generates lower output gap variance when θ_P is less than 0.4. In the sticky wage

model, a policy rule with θ_{dW} equal to 3 produces a relatively high, but fairly constant output

variability over the range of values for θ_P. The overall variability in the output gap, however, is

lower in the sticky wage model when θ_{dW} is set to 1 and its minimum variance occurs when θ_P

equals 0.1. The lower panels of Figure 3 show how the variance of inflation depends on θ_P. In

both policy rules for both models, the variability of inflation declines monotonically as θ_P

increases, but models with lower values for θ_π and θ_{dW} generate less output gap movement if θ_P

is 0.05 or larger.

The optimal monetary policy rule depends on the source or sources of nominal rigidities

and the policymaker's preferences. If the policymaker either believes that the economy is

characterized by sticky prices and not sticky wages, or only cares about inflation, then the

optimal policy in our model would be to put an infinite weight on the price level, θ_P. If, on the

other hand, the nominal rigidity is in wage setting, then a higher value for θ_P generates a lower

variance in inflation but a higher variance in the output gap.

Figure 4 examines the variances of the output gap and the annual inflation rate when a

price-path target is combined with a price- or wage-inflation target. For these simulations the

weight on the price path, θ_P, is set to 0.2. This choice balances concern about the effect on the

output gap when wages are sticky with concern about inflation variability in all cases.[5] We restrict our analysis to models with policy shocks. For the purpose of comparison, we reproduce the case with policy shocks and no price-path target in Figure 4. Following the computation procedure from Figure 1, the variances of the output gap and inflation are calculated for values of θ_π and θ_{dW} between 0.1 and 5 in the sticky-price model and the sticky-wage model, respectively. The left column of Figure 4 shows that the introduction of a price-path target in the sticky-price specification substantially reduces the variability of inflation while only slightly increasing movements in the output gap. The variance of inflation is minimized at only 0.14 percent when θ_π equals 1. That inflation variance is considerably smaller than the 2 percent generated when θ_π is 3 or more and the price path is not targeted. The output gap, on the other hand, is only 0.08 percentage points higher when the price path is targeted and θ_π is set to 1, than it is with no price-path target and a value of θ_π equal to 3.

The right column of Figure 4 reveals that a price-path target in the sticky-wage model also reduces inflation movements but increases variability in the output gap. Those results are qualitatively the same as in the sticky-price model. Inflation variance is minimized with a value of θ_{dW} below 1, but the output gap variance still falls rather sharply until θ_{dW} goes above 1. Setting θ_{dW} equal to 1 generates a variance of about 2 percent in the annual inflation rate which is about 4 percentage points lower than the minimum variance without a price-path target. This policy setting generates a variance of the output gap that is about 0.2 percentage points higher than the case with no price-path target and a value of θ_{dW} greater than 3. Thus, introducing a price-path target in a sticky-wage model also generates a tradeoff between lower inflation variability and higher output gap variability.

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[5] Our choice would be optimal in the loss function presented later in this section if the weight on the output gap were 3 to 4 times the weight on inflation. As the weight on the output gap is reduced, the optimal value of θ_P rises.

Understanding the policy implications of this tradeoff between output and inflation variability requires using a welfare metric to evaluate the alternative policy rules. We know from Canzoneri, Cumby, and Diba (2004), that optimal policy in our particular versions of the New Keynesian model can be achieved by eliminating policy shocks and putting an infinite weight on the inflation target. We are addressing the case where such a policy is not feasible. Walsh (1998), Clarida, Gali, and Gertler (1999), Woodford (2002), and others argue that the optimal monetary policy in a New Keynesian model can be approximated by minimizing a loss function (L) that is quadratic in inflation and the output gap. Consider the following form:

$$L = \text{var}(4 \times \hat{\pi}_t) + \lambda \times \text{var}(\hat{y}_t - \hat{y}_t^f),$$

where $\text{var}(4 \times \hat{\pi}_t)$ is the unconditional variance of the annualized inflation rate, $\text{var}(\hat{y}_t - \hat{y}_t^f)$ is the unconditional variance of output from its flexible price and wage equilibrium, and λ measures the weight on the output gap in the loss function such that $\lambda \in [0, \infty)$. Disagreement exists on the appropriate calibration of λ. Rotemberg and Woodford's (1997) estimates imply a value for λ of 0.05, while Rudebusch and Svensson (1999), Williams (2003), and others argue that equal weight on inflation and the output gap (i.e., $\lambda = 1.0$) is consistent with stabilization objectives. Using this loss function, we can rank alternative policy rules and consider differing values of λ.

Our results suggest that targeting the price path and setting θ_π or θ_{dW} equal to 1 is preferable to having no price-path target as long as the loss function parameter λ is not too large. In the sticky-price case, comparing the policy rule $(\theta_\pi, \theta_P) = (3, 0)$ with the policy rule $(\theta_\pi, \theta_P) = (1, 0.2)$, we find that the second rule is preferred for all values of λ less than 24.7. In the case with sticky wages, comparing the policy rule $(\theta_{dW}, \theta_P) = (3, 0)$ with the policy rule

$(\theta_{dW}, \theta_P) = (1, 0.2)$, we find that the rule that puts weight on the price path is preferred for all values of λ less than 21.9.

Svensson (1999) shows that there is less short-run inflation variability with a price level target than with an inflation target if there is moderate persistence in output. The reason is seen by evaluating the determination of inflation variance in each regime. In an inflation targeting regime, the variance of inflation depends on the variance of the output gap. In a price level targeting regime, the variance of inflation depends on the variance of the difference in the output gap. With a realistic calibration of the model, there is a high degree of persistence in output and the variance of the first difference of the output gap is less that the variance of the output gap itself.

The Term Structure of Inflation and Output Gap Expectations with Price-path targeting. The upper panels in Figure 5 display the impact of a price-path target on the standard deviation of the average annual inflation rate across the term structure of maturities from 1 to 10 years. As in Figures 3 and 4, we restrict our investigation to models with policy shocks. In the top left panel of Figure 5, the effect of three alternative monetary policy rules on the variability of expected inflation is examined in a sticky-price specification. The baseline specification with policy shocks from Figure 2 assumes there is no price-path target ($\theta_P = 0$) and θ_π equal to 3. In comparison, adding a price-path target ($\theta_P = 0.2$) to the policy rule greatly reduces the inflation uncertainty caused by the policy shocks. Furthermore, recall from Figure 4 that inflation variability is further reduced when the price path is targeted and θ_π is set closer to 1. This policy rule specification eliminates most of the inflation variability induced by the policy shocks and even removes some of the variation caused by non-policy shocks at both the short and long ends of the term structure.

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The top right panel of Figure 5 displays the effects of the same three alternative monetary policy rules on expected inflation movements in a sticky-wage model. The outcome in this specification is also qualitatively similar to the result in the sticky-price model. The addition of a price-path target ($\theta_P = 0.2$) to a wage-inflation-targeting rule ($\theta_{dW} = 3.0$) basically eliminates the uncertainty about future inflation produced by the policy shocks. By lowering θ_{dW} to 1 with a price-path target, monetary policy can also reduce the inflation uncertainty caused by non-policy shocks over the entire term structure.

The intuition underlying the findings for inflation uncertainty in Figure 5 is straightforward: Adjusting the nominal interest rate target in response to the inflation rate allows the price level to deviate permanently from its expected path. Greater variation in actual inflation pushes the price level further from its expected path and increases long-run inflation uncertainty. A small weight on the price path, however, ensures that the price level will converge toward a deterministic trend over time, which has the effect of lowering the variability of long-run inflation. Although inflation uncertainty is reduced at all horizons in regimes with a target for a price path, Figure 5 shows that the relative reduction in inflation uncertainty in a regime with a price path is much greater at longer horizons than at shorter ones.

The lower panels in Figure 5 display the impact of a price-path target on the standard deviation of the expected output gap across the term structure of maturities from 1 to 10 years. In the bottom left panel, most of the output gap variation in the sticky-price model is concentrated in the first 2 years. The average standard deviation of the output gap over longer horizons declines monotonically and mainly reflects the averaging of the higher variability in the near term. In the bottom right panel, we show that adding a price-path target to the sticky-wage model has the same qualitative impact on output gap movements as in the sticky-price specification, but

its quantitative effect is larger. Intuitively, output gap movements decline over time in both models as more firms and households are able to adjust their prices or wages.

VI. Conclusion

The optimal monetary policy in New Keynesian models depends on the source of nominal rigidity. When prices are sticky, it is optimal to stabilize price inflation, whereas it is optimal to stabilize wage inflation when wages are sticky. This article shows that, in both cases, the optimal policy generates a relatively large amount of uncertainty about long-run inflation.

By placing some weight on a deterministic price path, a central bank can reduce the long-run inflation uncertainty. A price-path target eliminates most variation in long-run inflation caused by monetary policy shocks and even reduces long-run inflation movements generated by non-policy shocks in the sticky-wage specification. This policy induces a small increase in the variability of the output gap but a much larger reduction in inflation variability. Social welfare, as measured by a Woodford (2003) style loss function, rises with a price-path target in the monetary policy rule except when the weight on the output gap in the loss function is extremely high.

Our results suggest two important topics for future research. First, the effect of long-run inflation risk on social welfare needs to be explicitly modeled. Second, we need to understand whether wage stickiness characterizes modern economies and, if it does, why central banks do not target wage inflation as theory suggests.

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Figure 1: Optimal Monetary Policy: The Impact of Policy Shocks

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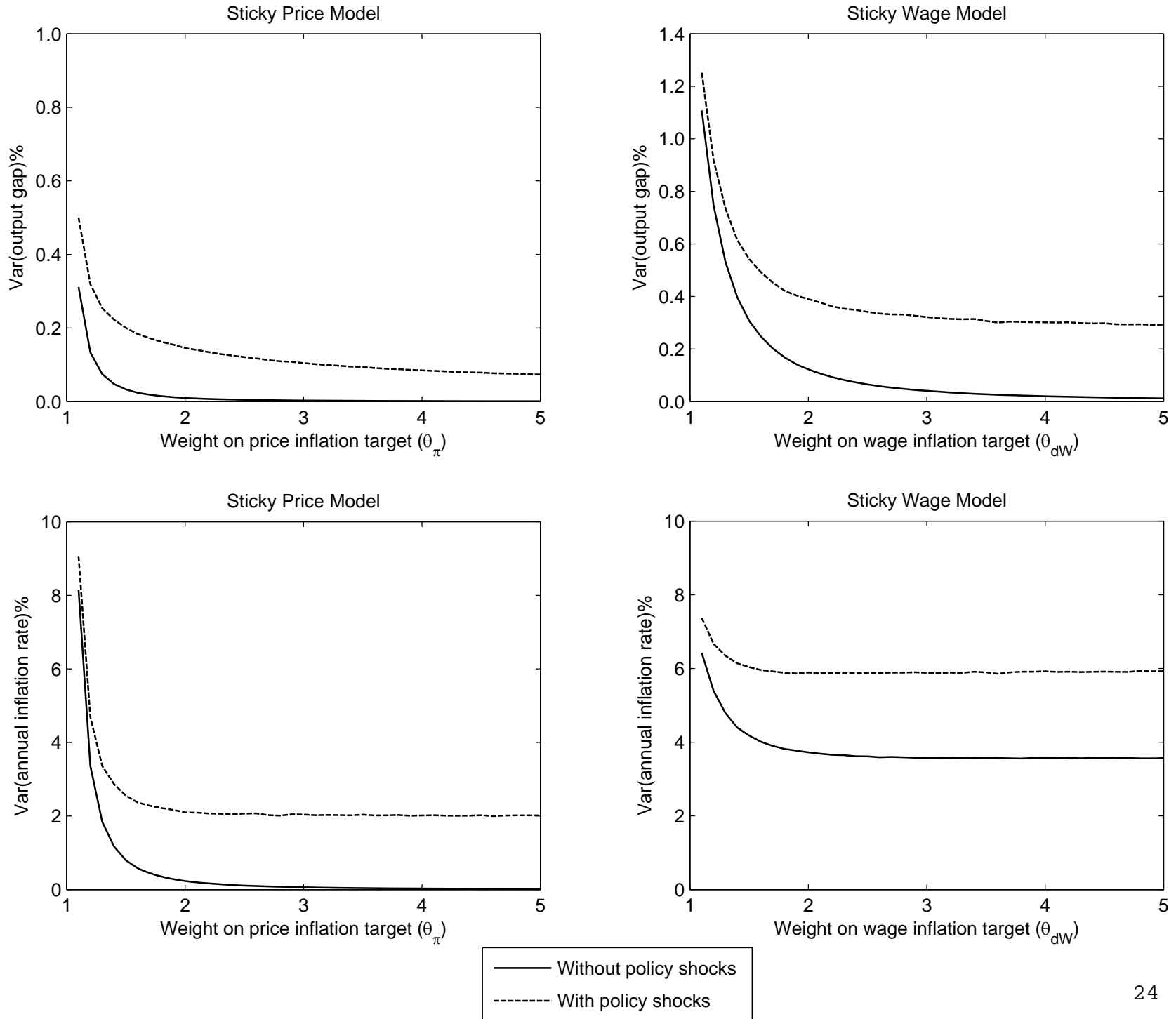
Figure 2: Long-Run Inflation and Output Uncertainty: The Impact of Policy Shocks
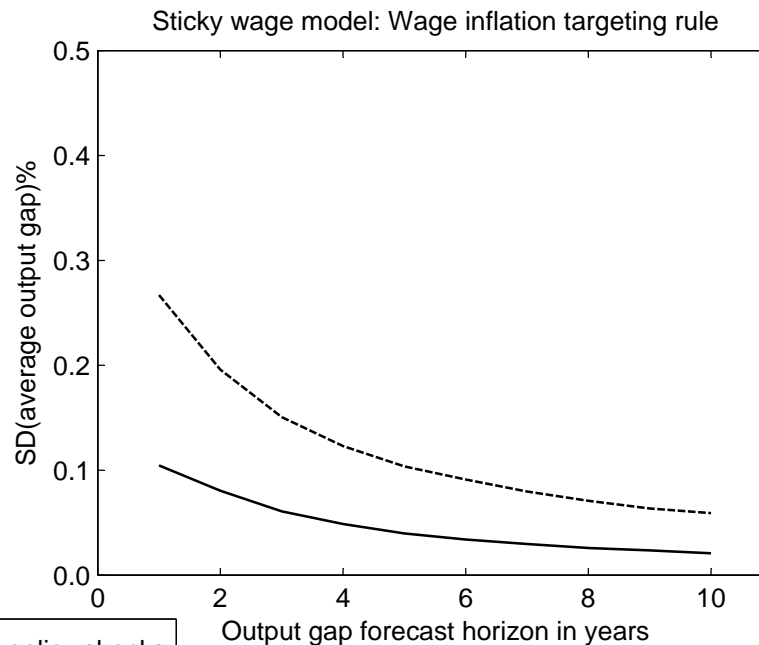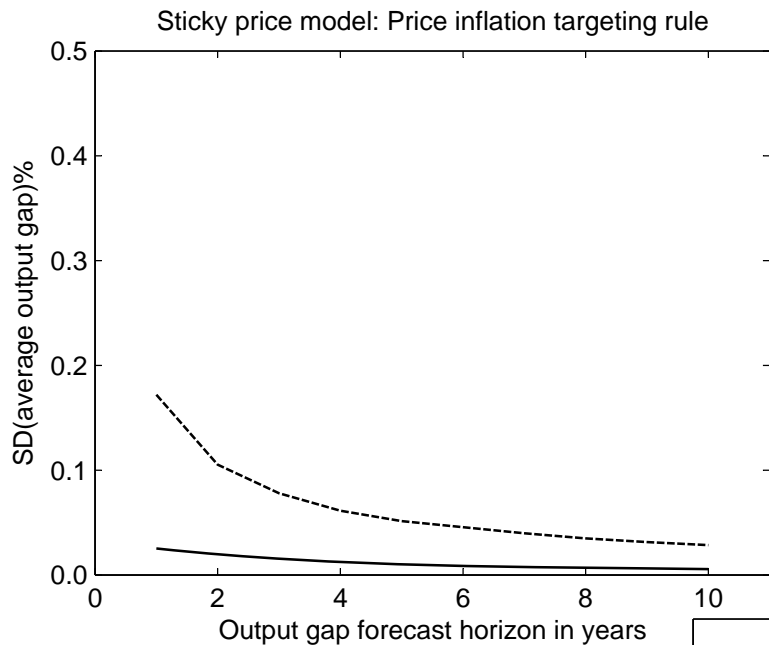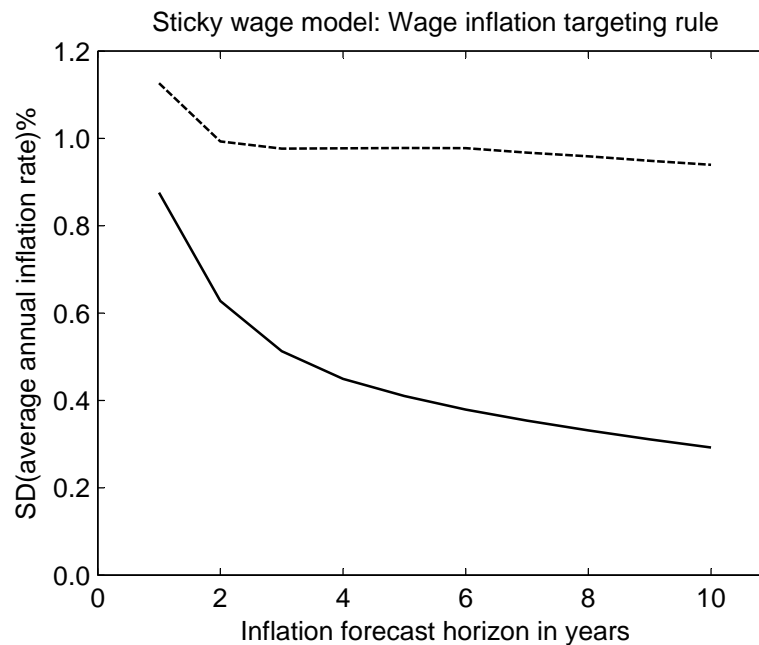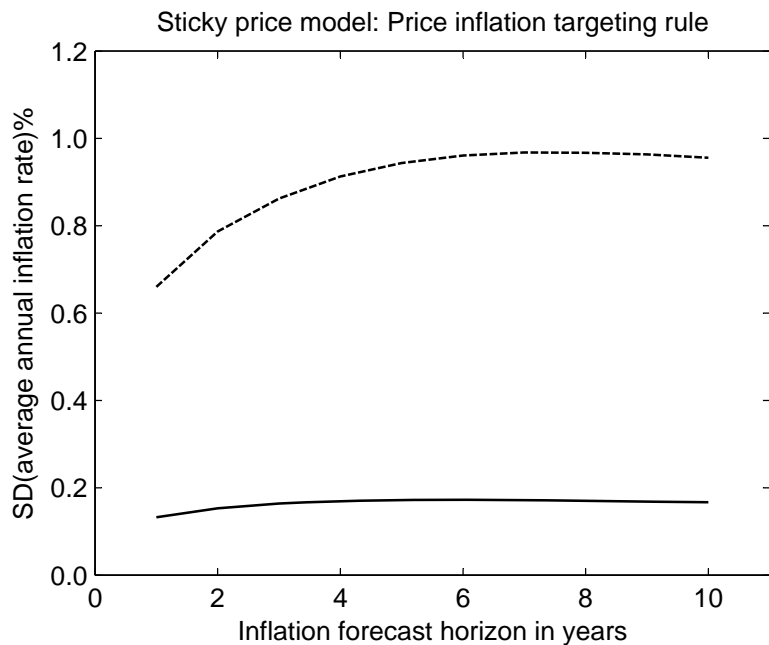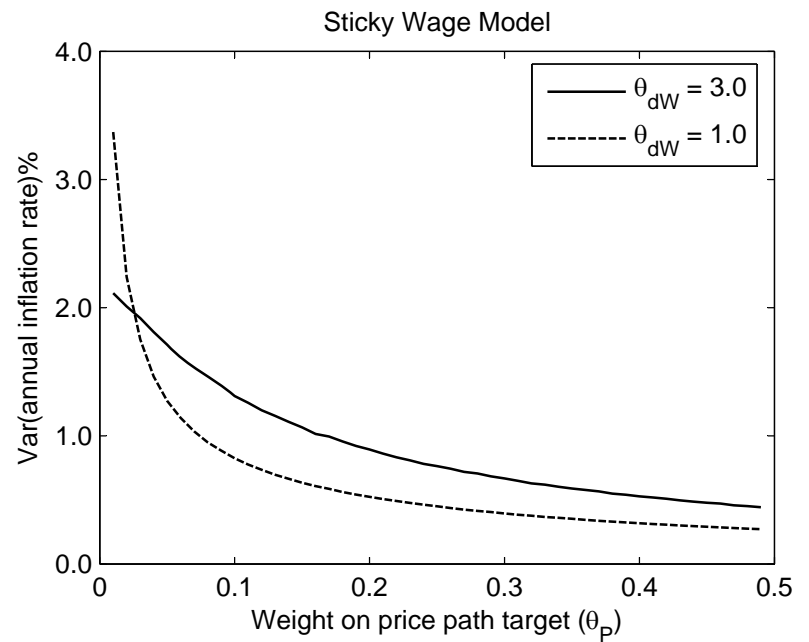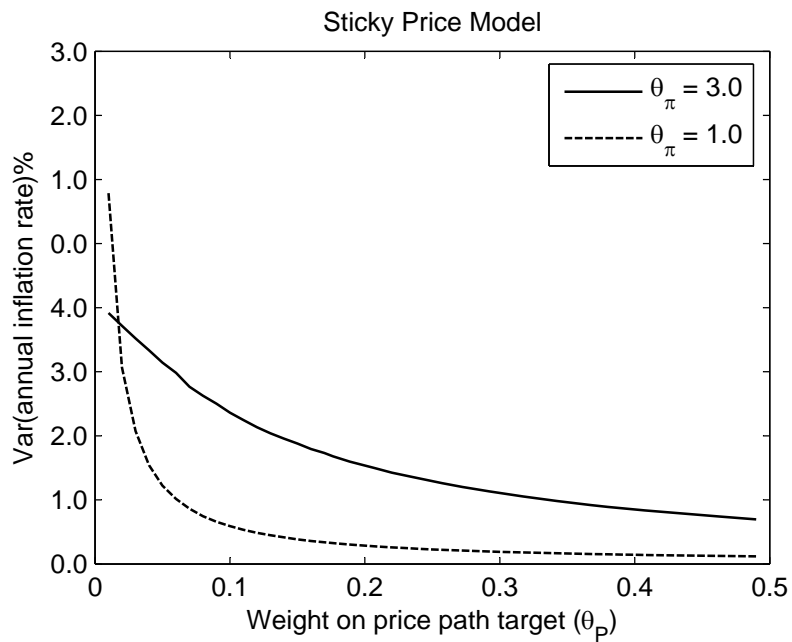
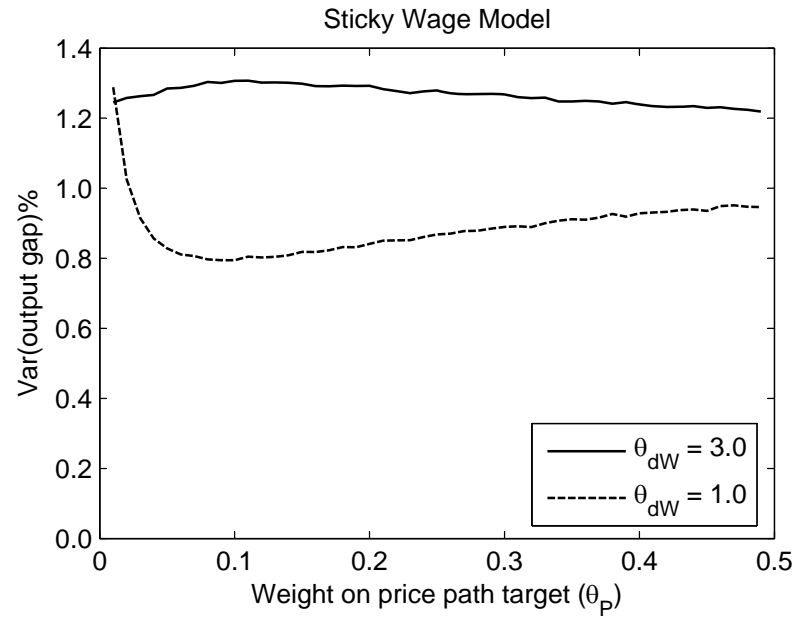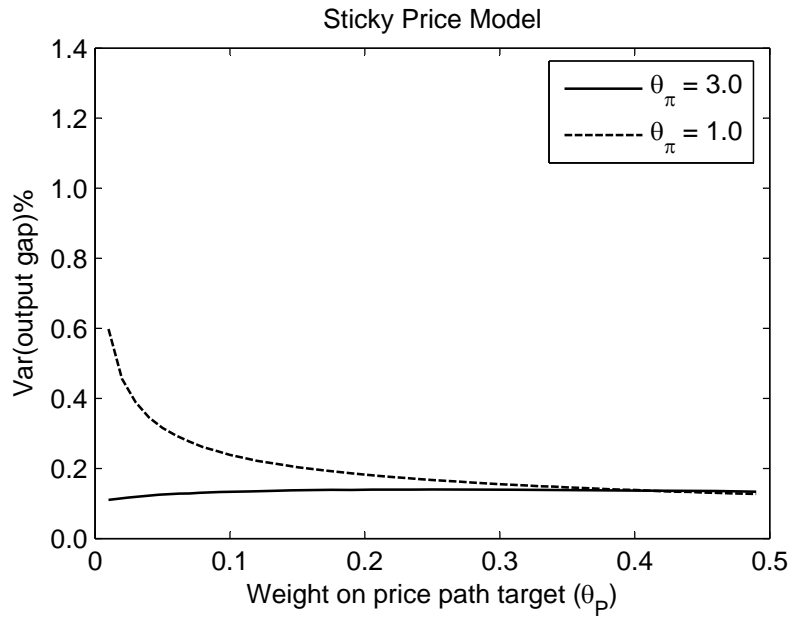# Figure 3: The Optimal Price Path Target

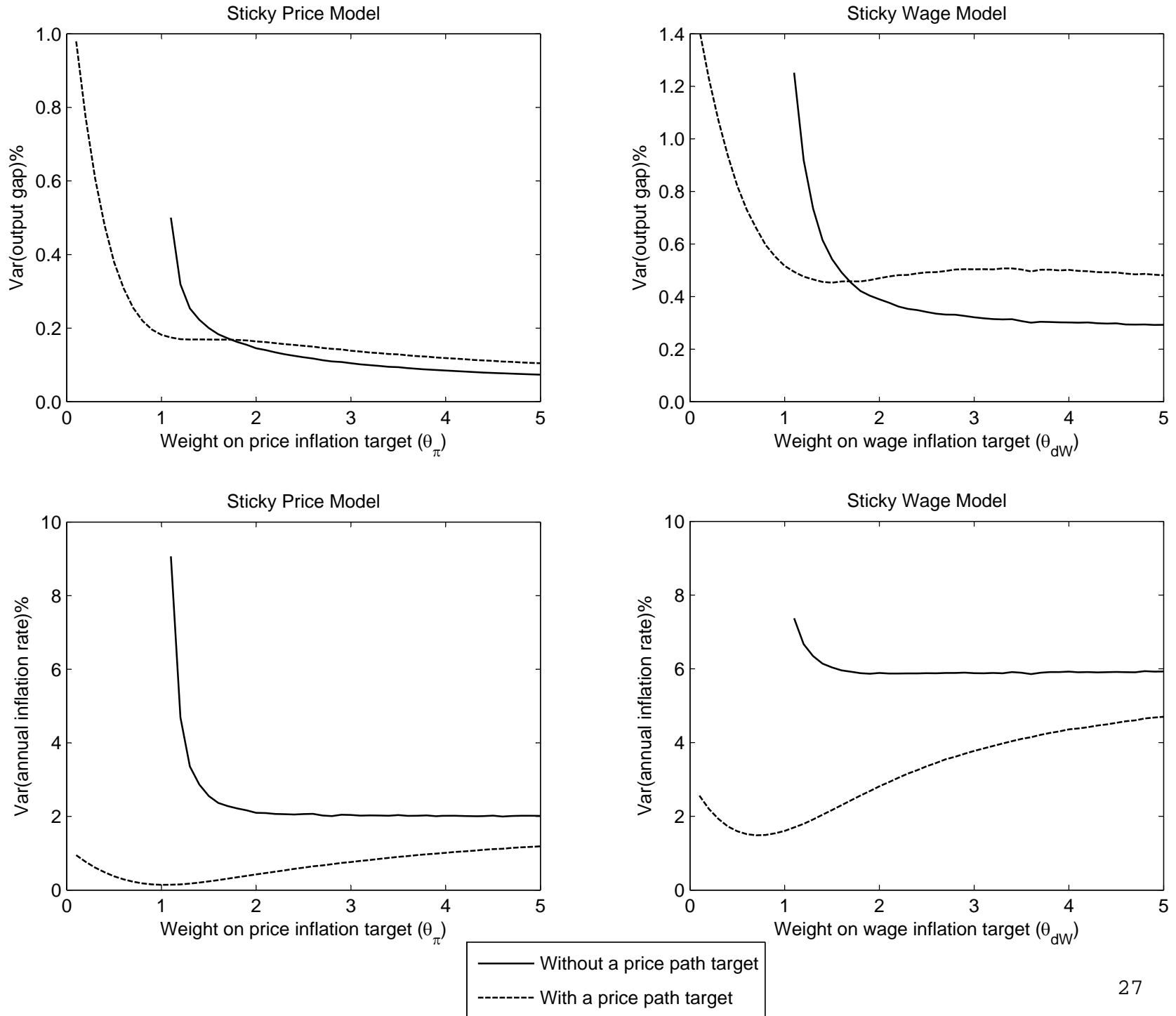Figure 4: Optimal Monetary Policy: The Impact of a Price Path Target

Figure 5: Long-Run Inflation and Output Uncertainty: The Impact of a Price Path Target