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Incomplete Information and Self-fulfilling Prophecies*

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Abstract

This paper shows that incomplete information can lead to self-fulfilling business cycles. This is demonstrated in a standard dynamic general equilibrium model of monopolistic competition à la Dixit-Stiglitz. In the absence of fundamental shocks, the model has a unique certainty (fundamental) equilibrium. But there are also multiple stochastic (sunspots) equilibria that are not mere randomizations over fundamental equilibria. Thus, sunspots can exist in infinite-horizon models with a unique saddle-path steady state. In contrast to the indeterminacy literature following Benhabib and Farmer (1994), sunspots are robust to parameters associated with production technologies and preferences.

Keywords: Sunspots, Self-fulfilling Expectations, Demand Externality, Incomplete Information, Indeterminacy, Forecasting the Forecasts of Others.

JEL codes: E31, E32.

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1 Introduction

Keynes (1936) emphasized the process of expectation formation and argued that expectations may become self-fulfilling when reinforced via iterated speculation under incomplete information. A number of papers, starting with Benhabib and Farmer (1994), have revived the study of expectations-driven fluctuations within the general equilibrium framework of Kydland and Prescott (1982).[1] This literature can be cast in the framework of monopolistic competition *à la* Dixit and Stiglitz (1977).[2] A key feature of this literature is that the steady state in the model is locally indeterminate so that there exit infinitely many equilibrium paths converging to the same steady state. This multiplicity of equilibria can give rise to fluctuations driven by self-fulfilling expectations (or sunspots).[3]

However, sunspot equilibrium based on local indeterminacy is extremely sensitive to structural parameters and, consequently, lack robustness. Local indeterminacy would not be possible in this class of models, for example, if there are small adjustment costs in capital or labor (see Georges 1995, and Wen 1998b). Slight change of parameter values (such as the elasticity of labor supply, rate of capital depreciation, capital's share in total income, degree of returns to scale, and so on) can easily eliminate indeterminacy and insulate the model economy from fluctuations driven by self-fulfilling expectations. In addition, since such models imply indeterminacy in the impulse responses to fundamental shocks, it is difficult to confront data based on VAR analysis without additional assumptions about the set of the indeterminate variables and their initial values.

This paper focuses on a different source of sunspot equilibria within the class of imperfectly competitive DSGE models *à la* Dixit and Stiglitz. This source does not rely on local indeterminacy or the topological properties of the steady state (i.e., the eigenvalues). Instead, it is related to expectation formation under incomplete information. In this regard, sunspot equilibria under our consideration are less sensitive to structural parameters pertaining to production technologies and preferences.

Incomplete or imperfect information as a mechanism of generating expectations-driven business cycles has been emphasized by Townsend (1983) and Sargent (1991). This tradition has recently been revived by a number of people, including Kasa (2000), Woodford (2003), Adam (2007), and Lorenzoni (2008), among others. The recent studies of "news" as a source of the business cycle is also related to this literature (see, e.g., Beaudry and Portier, 2004 and 2007; Jaimovich and

[1] Also see Woodford (1986, 1991), among others. For a literature review, see Benhabib and Farmer (1999). For early contributions to the sunspot literature, see Shell (1977), Azariadis (1981), and Cass and Shell (1983).

[2] See Benhabib and Farmer (1994) and Schmitt-Grohé (1997).

[3] Important recent works following this literature include Benhabib and Farmer (1996), Benhabib and Wen (2004), Christiano and Harrison (1999), Farmer (1999), Farmer and Guo (1994), Gali (1994), Jaimovich (2007, 2008), Pintus (2006, 2007), Weder (1998), and Wen (1998a), among many others.

Rebelo, 2007a and 2007b; and Wang, 2007).[4] This literature, however, does not consider incomplete information as a mechanism of generating self-fulfilling sunspot equilibria. In contrast, we show that incomplete information is itself a natural mechanism of endogenous business cycles in a Dixit-Stiglitz world.

The nature of sunspot equilibria arising under incomplete information differs from that arising under local indeterminacy in three important aspects: 1) sunspots are independent of the eigenvalues of a model; 2) sunspot equilibria are not based on mere randomizations over fundamental equilibria; and 3) sunspots can be serially correlated. Because of property (1), sunspots are more pervasive and easier to occur in a standard economic environment. Because of property (2), sunspots equilibria can exist even in models with a unique saddle-path fundamental equilibrium. Because of property (3), models driven by sunspot shocks have better potential to explain the business cycle than realized in the literature.[5] These properties are in contrast to the recent sunspot literature following Benhabib and Farmer (1994) and constitute an extension of the original insight of Cass and Shell (1983).[6]

DSGE models featuring imperfect competition *à la* Dixit-Stiglitz have been widely used and extensively studied in the literature, but the possibility of sunspot equilibria in this class of models has gone virtually unnoticed because the literature implicitly assumes perfect information – that each firm can perfectly anticipate (or infer) the equilibrium level of aggregate demand when setting its own prices. This assumption of perfect information rules out sunspot-Nash equilibria. However, because firms must each choose a price simultaneously (taking as given the anticipated level of aggregate demand and prices set by other firms) and equilibrium quantities of demand are subsequently determined at these prices, it is only natural to assume that prices are set based on expected demand, not on realized demand. In such a case, expectations can be self-fulfilling when there exist strategic complementarities among firms actions.[7]

Strategic complementarities as a source of multiple equilibria are emphasized by Cooper and John (1988) and arise naturally in models with monopolistic competition *à la* Dixit-Stiglitz because firms' output are imperfect substitutes. However, in the class of dynamic general equilibrium models we study, strategic complementarities are necessary but not sufficient for multiple equilibria. This

[4] "News shocks" in this literature mean signals received today about future fundamental shocks. Since the anticipated future shocks may not realize and the signals may be incorrect or noisy, this literature is a departure from rationality and complete information.

[5] See Schmitt-Grohe (2000) for criticisms of sunspot models.

[6] In the original Cass-Shell paper, sunspot equilibria are not based on randomizations over fundamental equilibria and sunspots can be serially correlated. Hence the existence of multiple fundamental equilibria is not a necessary condition for the existence of sunspots equilibria. In contrast, sunspots equilibria in the recent indeterminacy literature are based on randomizations over fundamental equilibria. Moreover, when sunspots are associated with the forecast errors of the model because of local indeterminacy, they cannot be serially correlated.

[7] For the early literature that links imperfect competition and imperfect information to sunspots equilibria, see Ng (1980, 1992), Chatterjee and Cooper (1989), Chatterjee, Cooper and Ravikumar (1993), Carlstrom and Fuerst (1998a,1998b), Gali (1994), Peck and Shell (1991), and Woodford (1991), among others. For the more recent literature along this line of research, see Jaimovich (2007), Dos Santas Ferreira and Dufourt (2006), and Wang and Wen (2008).

is why the possibility of sunspot equilibria in this class of models has gone unnoticed despite of the popularity of the Dixit-Stiglitz model in the literature.[8,9].

Our findings are important to the literature because DSGE models with monopolistic competition *à la* Dixit-Stiglitz are the workhorse of theoretical and applied macroeconomics in the study of business cycles and monetary policy. The fast growing New Keynesian sticky-price literature is just one of the many noticeable areas that rely on this framework for business-cycle studies and monetary policy analyses. Yet this literature has been assuming unique equilibrium all the way along, while in fact there may be multiple equilibria in such models. In addition, since the mode of Kydland and Prescott (1982) can be cast as a limiting case of the Dixit-Stiglitz imperfect competition model, and since fluctuations driven by technology shocks look similar to those driven by sunspot shocks in this class of models (see the analysis in Section 3 below), the implications of our findings are broader than what we can cover in this paper.

This paper also provides an alternative approach to modeling autonomous movements in the marginal cost (or markup) as a source of the business cycle, which complements the approach of Dos Santas and Dufourt (2006), Jaimovich (2007), and Wang and Wen (2008). These papers consider firms' entry and exit under imperfect competition as a mechanism to generate multiple fundamental equilibria or steady states. Sunspot equilibria in these papers are all based on randomizations over multiple fundamental equilibria. In contrast, sunspot equilibria considered in this paper do not rely on randomizations over fundamental equilibria, exemplifying the original insight of Cass and Shell (1983).[10]

The rest of the paper is organized as follows: Section 2 presents a simple benchmark general equilibrium model of imperfect competition and shows the possibility of stochastic sunspot-Nash equilibria under incomplete information. Section 3 extends the model to a more general setting

[8] An important exception is Carlstrom and Fuerst (1998a,1998b), who show that a combination of sticky prices and monetary policy can lead to global indeterminacy of the marginal cost under imperfect competition *à la* Dixit-Stiglitz. In particular, they showed that firms' marginal costs can be indeterminate in a monetary model with one-period sticky prices if monetary policy can induce zero interest rate so that the cash-in-advance constraint is slack. However, as we will discuss in more details in this paper, the fundamental factor that causes the marginal cost to be indeterminate is neither sticky prices nor monetary policy, but incomplete information. When firms must set prices one period in advance, their information regarding the next period's aggregate demand is bound to be incomplete. It is this incompleteness in information that can generate self-fulfilling expectations. Therefore, we are able to show that self-fulfilling sunspot equilibria can also exist in standard monetary models even if prices are perfectly flexible (regardless of monetary policy).

[9] Blanchard and Kiyotaki (1987) are able to construct multiple fundamental equilibria in a model similar to ours under the additional assumption of menu costs. Kiyotaki (1988) uses a similar set up to generate multiple fundamental equilibria under the additional assumption of increasing returns to scale. Chatterjee and Cooper (1989) prove the existence of multiple fundamental Nash equilibria in similar models under the additional assumption of participation externalities. An important distinction between this literature and our paper is that we do not need to change the physical structure of the standard Dixit-Stiglitz model except relaxing the assumption of complete information. The type of sunspot equilibria we construct are not based on randomizations over fundamental equilibria and they continue to exist in the extended models of Blanchard and Kiyotaki (1987), Kiyotaki (1988), Benhabib and Farmer (1994), Chatterjee and Cooper (1989), and many others.

[10] In another related paper, Wang and Wen (2006) use the same mechanism of this current paper to study the welfare implications of sunspot-driven fluctuations in an endogenous growth model. Wang and Wen (2006) show that the average growth rate and the volatility of output can be negatively related because of sunspots, which is consistent with the empirical evidence found by Ramey and Ramey (1995).

of the information structure and further illustrates the nature of sunspot equilibria based on incomplete information. Serially correlated sunspots are constructed and calibrated business-cycle studies are conducted. Section 4 extends the analysis to monetary models with and without sticky prices. Finally, Section 5 concludes the paper.

2 The Benchmark Model

Suppose there is a continuum of intermediate good producers indexed by $i \,\epsilon[0,1]$, with each producing a single differentiated good $Y(i)$. The price of $Y(i)$ is denoted $P(i)$. These intermediate goods are used as inputs to produce a final good according to the technology,

$$Y = \left(\int_0^1 Y(i)^{\frac{\sigma-1}{\sigma}}(i)di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$ measures the elasticity of substitution among the intermediate goods. The final good industry is assumed to be perfectly competitive. The price of the final good, P, is normalized to one. Profit maximization by the final good producer yields the demand function for intermediate goods:

$$Y(i) = P(i)^{-\sigma} Y. \tag{2}$$

Notice that the demand for good i depends not only on the relative price of the good, but also on aggregate demand Y. There are thus demand externalities in the model as pointed out by Blanchard and Kiyotaki (1987). The demand externalities arise endogenously within the model due to the complementarity of production factors (intermediate goods) in the final good industry, as opposed to being exogenously imposed from outside as in the Benhabib-Farmer model. Substituting the demand functions into the final-good production function (1) gives the aggregate price index, $P(=1) = \left(\int_0^1 P(i)^{1-\sigma}di\right)^{1/(1-\sigma)}$.

For simplicity, the production technology for intermediate goods is given by

$$Y(i) = N(i). \tag{3}$$

Intermediate good producers have monopoly power in the output market but are perfectly competitive in the factor markets. Given the production technology, the cost function of an intermediate good firm can be derived by solving a cost-minimization problem, $\min WN(i)$ subject to $N(i) \geq Y(i)$, where W denotes the real wage. Letting $\phi(i)$ denote the marginal cost of firm i (which is the Lagrangian multiplier for the constraint of the firm's cost minimization problem), we have $\phi(i) = W$ as the unit cost function of the firm. Thus marginal cost is the same across all firms. Since ϕ is the shadow cost of increasing firm i's output by one unit, in general equilibrium its correlation with aggregate demand is nonnegative: $cov(\phi, Y) \geq 0$.

Suppose we close the model by having a representative household maximizing the utility function, $u(C, N) = \log C - N$, subject to the budget constraint, $C \leq WN + \Pi$, where C denotes aggregate consumption, N aggregate labor supply, and Π aggregate profit income. The first-order condition for labor supply gives $C = W = \phi$. In general equilibrium, $C = Y$; hence, the marginal cost is a function of aggregate demand, $\phi(Y_t) = Y_t$. Also, the marginal utility of consumption is given by $\frac{!}{Y}$.

A key feature of the model is that intermediate-goods firms set prices simultaneously while taking as given the anticipated aggregate marginal cost and prices set by other firms, with equilibrium quantities (including the aggregate marginal cost itself) being then determined at these prices. This price-setting game permits self-fulfilling expectations because intermediate-goods firms must decide profit-maximizing prices without knowing the consequent equilibrium aggregate demand and the marginal cost that may prevail in the market. Yet these aggregate economic variables depend crucially on the actions of the other firms over which an individual firm has no influence.

The possibility of multiple sunspot-Nash equilibrium in this class of models has gone largely unnoticed by the existing literature because this literature implicitly assumes that firms are able to perfectly anticipate the equilibrium marginal cost (ϕ) and aggregate demand (Y) when setting prices. If the marginal cost is known, then the level of aggregate demand is known and firms can set price accordingly as a markup over the marginal cost, $\phi(Y)$. In a symmetric equilibrium, $P(i) = 1$; hence, the equilibrium output (Y) is then fully and uniquely determined in general equilibrium, and each firm's output level is also determined according to (2). However, there are no a priori grounds to guarantee that all firms can perfectly anticipate the equilibrium outcomes of the market, as emphasized by Keynes (1936). If each firm must form expectations on the equilibrium outcome, or try to forecast the forecast of others, multiple sunspot-Nash equilibria are possible.

Define $\tilde{\Omega}_t$ as the information set available to price setting firms in period t, which includes the entire history of the economy up to period t except the realizations of sunspots (if any) in period t. Denote Ω_t as the information set that includes $\tilde{\Omega}_t$ and any realization of sunspots in period t. Thus we have $\Omega_t \supseteq \tilde{\Omega}_t \supseteq \Omega_{t-1}$.[11] Based on this definition of information sets, each individual firm i chooses price $P_t(i)$ in each period t to maximize expected profits by solving[12]

[11] Notice that our definition of information sets does not imply sticky prices. Prices respond immediately to any fundamental shocks in the model. That is, the information set $\tilde{\Omega}_t$ can include fundamental shocks realized in period t. As such, prices can respond to money shocks one for one.

[12] The reason that an individual firm needs to form expectations when maximizing profits is because the profits depend on aggregate demand, which is unknown to the firm because it depends on other firms' actions. The marginal utility of income serves as firms' discounting factor, but the results hold regardless.

$$\max E\left\{\frac{1}{Y_t}\left[\left(P_t(i)-\phi_t\right)Y_t(i)\right]|\tilde{\Omega}_t\right\}, \tag{4}$$

subject to the downward sloping demand function (2); where firm's profit is discounted by the marginal utility of the household, $\frac{1}{C}$.[13] Notice that (2) is also the best correspondence of firm i's action given the other firms' actions.

The optimal monopolistic price is given by

$$P_t(i)=\frac{\sigma}{\sigma-1}E\left\{\phi(Y_t)|\tilde{\Omega}_t\right\}, \tag{5}$$

where $\frac{\sigma}{\sigma-1}\geq 1$. In the limiting case where $\sigma\to\infty$, the model converges to a perfectly competitive economy. Our analysis of sunspot equilibria is independent of σ, hence it applies equally to perfectly (or near-perfectly) competitive economies where firms set prices equal to marginal cost with zero markup in the steady-state. The optimal pricing rule (5) shows that an individual firm sets prices according to the expected marginal cost that may prevail in the factor market, which in turn depends on the level of aggregate demand. In a symmetric equilibrium, $P(i)=P=1$, Equation (5) becomes

$$\frac{\sigma}{\sigma-1}E\phi=1. \tag{6}$$

Suppose there is no extrinsic uncertainty (i.e., there is perfect information about the marginal cost or the level of aggregate demand); then Equation (6) implies $\frac{\sigma}{\sigma-1}\phi_t=1$. Hence, a constant marginal cost, $\phi=\frac{\sigma-1}{\sigma}$, is the only fundamental-equilibrium solution to Equation (6). Given ϕ, aggregate demand is then fully determined at the level $Y=\frac{\sigma-1}{\sigma}$. Equation (2) then indicates that all firms produce $Y(i)=\frac{\sigma-1}{\sigma}$.

However, with extrinsic uncertainty or imperfect information, a random process ϕ_t may also constitute an equilibrium. To see this, note Equation (6) can be rewritten as

$$\phi_t=\frac{\sigma-1}{\sigma}\varepsilon_t, \tag{7}$$

where ε_t denotes sunspots. Any random process $\{\phi_t\}$ satisfying $E\varepsilon=1$ may constitute an equilibrium in which the level of aggregate demand is given by $Y_t=\phi_t$.

The intuition is illustrated in Figure 1. Suppose firms have full information about the marginal cost $\phi(Y)$ when setting prices, then the optimal monopoly price is set to $P(i)=\frac{\sigma}{\sigma-1}\phi(Y)$. Hence,

[13] Whether or not to discount firm's profit by household's marginal utility does not affect the existence of sunspots.

equation (2) implies that a firm's best correspondence (or production decision) is given by

$$Y(i) = \left(\frac{\sigma}{\sigma - 1} \phi(Y) \right)^{-\sigma} Y = \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} Y^{1-\sigma}. \tag{8}$$

Since $\sigma > 1$, this best correspondence is shown in Figure 1 as a downward sloping curve intersecting the 45 degree line at the point Y^*. Since $P(i) = 1$ in a symmetric equilibrium, we must have $\phi = \frac{\sigma-1}{\sigma}$ and $Y(i) = Y^* = \frac{\sigma-1}{\sigma}$, which is the unique certainty equilibrium under full information. Notice that, even though equation (2) implies $Y(i) = Y$, there are no other equilibria along the 45 degree line except Y^*. This is so because prices are set according to aggregate demand.

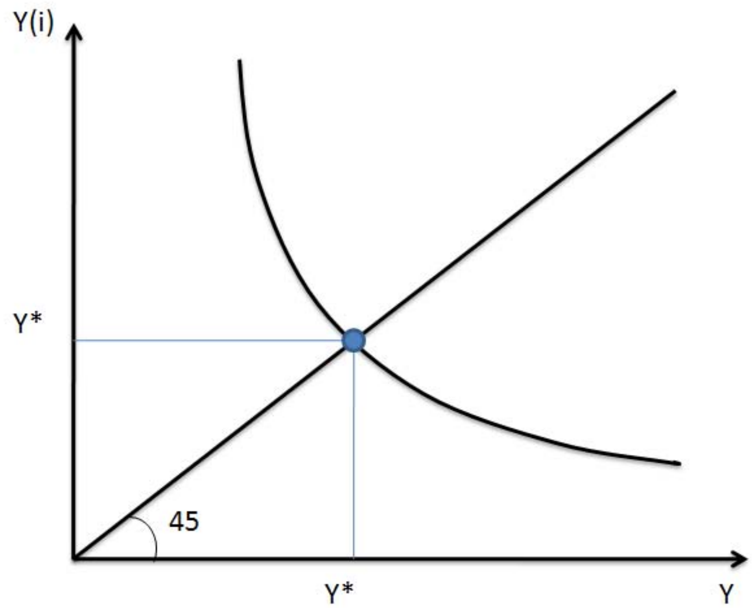

Figure 1. Sunspot Equilibrium.

However, suppose prices are exogenously fixed at $P(i) = 1$ by, say, the government.[14] Then the best correspondence (2) becomes

$$Y(i) = Y, \tag{9}$$

which is the 45 degree line in Figure 1. In such a case, the equilibrium output is completely indeterminate, and the model has infinite certainty Nash equilibria along the 45 degree line as long as $\phi(Y) \leq 1$ so that production is profitable. That is, any point along the 45 degree line below $Y \leq 1$ is a possible equilibrium and the equilibria are Pareto ranked. This case is similar to that analyzed by Copper and John (1988) and it arises because there is nothing to pin down the marginal cost and the demand externalities create a strategic complementarity among firms' actions.

[14] Government can also fix the prices at other values and the arguments are the same, but the discussions are slightly more involved because this involves the profits of the final-good sector.

Now, suppose we let firms endogenously set their prices optimally according to equation (5). Then in a symmetric equilibrium we still have $P(i) = 1$ and the firm's best correspondence is still given by (9). In this case, the only certainty equilibrium is given by $Y(i) = Y^*$ and there do not exist other certainty equilibria along the 45 degree line. To see this, suppose all firms expect the aggregate marginal cost to be lower than $\frac{\sigma-1}{\sigma}$ so that the aggregate demand is at $Y = Y^* - \varepsilon$, where ε is a positive number. Firms would then set prices low accordingly so that $P(i) < 1$. At the low price level we have the best correspondence $Y(i) = P(i)^{-\sigma} [Y^* - \varepsilon] > Y^* - \varepsilon$ for all i. This implies $Y > Y^* - \varepsilon$ in a symmetric equilibrium, which is a contradiction. Hence, a constant output level $Y < Y^*$ cannot be an equilibrium. Similarly, any constant output level $Y > Y^*$ does not constitute an equilibrium either. This explains why the model economy has a unique certainty equilibrium given by Y^* even with imperfect information. However, with imperfect information, there also exist stochastic equilibria that are not mere randomizations over fundamental equilibria. For example, when setting prices, if firms expect the aggregate marginal cost to be high with probability $\pi > 0$ and low with probability $1 - \pi > 0$ such that the implied average marginal cost satisfies the constraint (6); then such an expectation constitutes a self-fulfilling stochastic sunspot equilibrium. In a sunspot equilibrium, the level of aggregate output moves stochastically along the 45 degree line around Y^*. The variance of the stochastic path in this benchmark model is not restricted by the model's structural parameters except by feasibility conditions (e.g., $Y \in (0, \infty)$).

Therefore, a sunspot equilibrium, if it exists, must be stochastic in nature. The source of sunspot equilibria comes from the fact that firms do not know how the other firms will behave when setting prices, and hence must form expectations for the status of the aggregate economy (e.g., the marginal cost or the level of aggregate demand). Due to the endogenous demand externalities among firms' actions, such expectations can be self-fulfilling. Prices in a sunspot equilibrium appear to be "sticky" in the sense that they cannot be adjusted after sunspots are realized. However, since prices can be set after observing fundamental shocks such as monetary shocks, they are not sticky in the conventional sense because they can respond proportionately to aggregate money supply shocks. Figure 2 illustrates the time line of events in a sunspot equilibrium.[15]

[15] In any game where firms must choose prices simultaneously instead of quantities, the optimal prices can be chosen based on expected equilibrium outcomes that may prevail as soon as all parties have set their prices. Does this imply 'sticky" prices? We do not think so. It is only a metaphor when we say that sunspots are realized *after* prices are set but *before* quantities are determined. In fact, all events can take place simultaneously in the price setting game. See the next section for more discussions on this issue using Lucas' (1972) island model in which prices can respond to sunspots. In addition, one may argue that sunspots do not matter if they are realized before firms can set prices. True, but this is not the point. "Sunspots" by its conventional definition of Cass and Shell (1983) can exist anywhere at anytime. Economic agents cannot control where and when sunspots appear. The only important question is whether they matter or not to the economy. Here we show that they do matter.

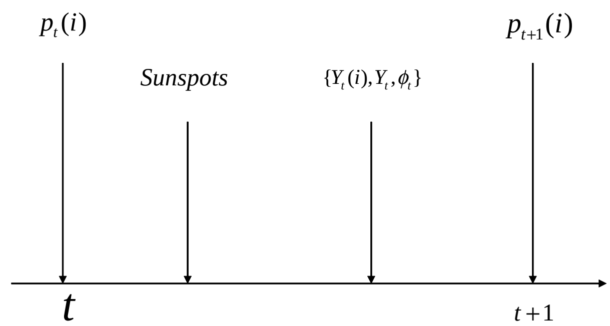

Figure 2. Timing of Sunspots.

3 A More General Approach

To further illustrate that the key condition for sunspot equilibria is not exactly sticky prices per se but speculations based on incomplete information, this section uses an alternative information structure to prove the existence of sunspot equilibrium. This information structure is akin to Lucas' (1972) island economy, is more general, and includes the previous analysis as a special case. We also add capital accumulation into the model to study the robustness and business cycle implications of sunspots.

Suppose firms reside in different islands with only limited information about the aggregate economy. They can be informed about the level of aggregate demand and marginal cost through signals when setting prices. Since signals contain idiosyncratic noises, information about aggregate demand and marginal cost is not perfect.

Let the production technology for intermediate goods be given by

$$Y(i) = A(i)K(i)^{\alpha}N(i)^{1-\alpha}, \tag{10}$$

where $A(i)$ represents idiosyncratic shocks to firm i's productivity (or marginal cost) and $K(i)$ represents the capital stock. Letting $\phi(i)$ denote the marginal cost of firm i, the factor demand functions for labor and capital are then given by $W = (1-\alpha)\phi(i)\frac{Y(i)}{N(i)}$ and $R = \alpha\phi(i)\frac{Y(i)}{K(i)}$, respectively. Hence, we have $\phi(i) = \frac{1}{A(i)}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}$ as the unit cost function of firm i. Thus, the marginal cost of firm i contains both an aggregate component, $\Phi \equiv \left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}$, and an idiosyncratic component $A(i)$. Under the normalization, $\int_0^1 \frac{1}{A(i)}di = 1$, the aggregate component equals the aggregate marginal cost, $\Phi = \int_0^1 \phi(i)di$.

Firms set prices in each period based on imperfect signals of aggregate demand. Define s_t^i as the signal received by firm i, Ω_t^i as the information set of firm i, which includes the entire history of the economy up to the point when s_t^i is received. That is, the information set includes s_t^i. Such an information structure is standard in the incomplete information literature (see, e.g., Lucas 1972; Kasa 1996, Lorenzoni 2006, and Rondina 2007, among others). Based on this definition of information sets, each individual firm i chooses price $P_t(i)$ in period t to maximize expected profits by solving

$$\max_{P(i)} E\left\{\left[\Lambda_t\left(P_t(i)-\phi_t(i)\right)Y_t(i)\right]|\Omega_t^i\right\}, \tag{11}$$

subject to the downward-sloping demand function, $Y(i) = \left(\frac{P(i)}{P}\right)^{-\sigma} Y$; where Λ_t denotes the marginal utility of income and $E\left\{[\cdot]|\Omega_t^i\right\}$ denotes firm i's expectations conditioned on the information set Ω_t^i. Denote $E^i \equiv E\left\{[\cdot]|\Omega_t^i\right\}$ to simplify notations. The optimal monopolistic price is given by

$$P_t(i) = \frac{\sigma}{\sigma-1}\frac{E^i\left\{\Lambda_t\phi_t(i)Y_t\right\}}{E^i\left\{\Lambda_t Y_t\right\}}. \tag{12}$$

Aggregation gives

$$\frac{\sigma}{\sigma-1}\int_0^1 \frac{E^i\left\{\Lambda_t\phi_t(i)Y_t\right\}}{E^i\left\{\Lambda_t Y_t\right\}}di = 1. \tag{13}$$

Equation (13) determines the equilibrium aggregate marginal cost Φ_t.

As in the previous section, the current model has a unique certainty equilibrium featuring $\Phi = \frac{\sigma-1}{\sigma}$. To illustrate the possibility of multiple Nash sunspot equilibria, log-linearizing equation (12) around the unique steady state[16] and using circumflex to denote log-linearized variables gives us

$$\hat{P}_t(i) = E^i\hat{\phi}_t(i) = E\left\{[\hat{\Phi}_t - \hat{A}_t(i)]|\Omega_t^i\right\}. \tag{14}$$

Suppose the signal received by firm i in period t is a (log)linear combination of aggregate demand (measured by the aggregate marginal cost Φ_t) and a noise term (measured by its own productivity shock $A_t(i)$),

$$s_t^i = \mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i), \tag{15}$$

[16] The steady state of individual firm's variables can be defined as the corresponding average value across firms. Such a definition has no first-order effects on our results.

where $\mu \in [0,1]$ and $\hat{A}_t$ is *i.i.d.* Under the assumption of *i.i.d.* shocks, only the most recent signal is useful in forecasting ϕ_t^i. Assuming the optimal forecast is based on least square projection, we have

$$E\left\{[\hat{\Phi}_t - \hat{A}_t(i)]|s_t^i\right\} = \left(\frac{\mu\sigma_\Phi^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\Phi^2 + (1-\mu)^2\sigma_A^2}\right)\left[\mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i)\right]. \tag{16}$$

Since the aggregate price is normalized to one (equation 13), we have $\int \hat{P}(i)di = 0$. Hence, integrating equation (16) over i gives

$$0 = \left(\frac{\mu\sigma_\Phi^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\Phi^2 + (1-\mu)^2\sigma_A^2}\right)\mu\hat{\Phi}_t. \tag{17}$$

The condition in equation (17) holds true in three different cases: (i) $\hat{\Phi} = 0$, (ii) $\sigma_\Phi^2 = \frac{1-\mu}{\mu}\sigma_A^2$, and (iii) $\mu = 0$. Each case corresponds to a particular equilibrium path of the marginal cost. First, if $\hat{\Phi}_t = 0$, then Φ is a constant; hence, $\phi_t(i)$ is orthogonal to the aggregate variables Λ_t and Y_t. Equation (12) implies $p(i) = \frac{\sigma}{\sigma-1}\phi(i)$.[17] Integration implies $\Phi = \frac{\sigma-1}{\sigma}$. Thus, we obtain the fundamental (certainty) equilibrium. Therefore, regardless whether information is incomplete or not, a constant marginal cost ($\hat{\Phi} = 0$) is always an equilibrium and it is the only certainty equilibrium.

However, notice that the certainty equilibrium can also be obtained under the assumption of complete information. Namely, suppose $\mu = 1$; then, in order for Equation (17) to hold, the equilibrium marginal cost must be a constant ($\hat{\Phi}_t = 0$), which is why the existing literature obtains a unique fundamental equilibrium under the implicit assumption of perfect information.

Second, if $\mu < 1$ (incomplete information), we are able to construct sunspot equilibria where the aggregate marginal cost (and hence aggregate output) is stochastic with variance $\sigma_\Phi^2 = \frac{1-\mu}{\mu}\sigma_A^2$ and mean $E\hat{\Phi}_t = 0$. In fact, any stochastic *i.i.d.* process $\{\Phi_t\}$ with variance $\sigma_\Phi^2 = \frac{1-\mu}{\mu}\sigma_A^2$ and mean $\frac{\sigma-1}{\sigma}$ constitutes a sunspot equilibrium. Therefore, with incomplete information, there can exist multiple sunspot equilibria and such equilibria are not mere randomizations over fundamental equilibria.

Finally, in the special case where $\mu = 0$, the signal s_t^i provides no information about period t aggregate demand. Hence, the aggregate marginal cost is completely indeterminate because it is impossible for firms to forecast it. With log linearization, the variance of the marginal cost is completely unrestricted. This is identical to the sunspot equilibrium analyzed in the previous section of this paper.

[17] With Φ being constant, firm i is able to perfectly forecast $\phi(i)$: $E\{\phi(i)|\Omega^i\} = \phi(i)$.

The intuition for the existence of sunspot equilibria with imperfect signals ($0 < \mu < 1$) is as follows. Sunspot equilibrium, by definition, implies coordinated behaviors based on common beliefs among agents. If individuals are largely affected by their own idiosyncratic shocks, then such a coordinated action becomes harder. Hence, sunspot equilibria in our model involves the condition under which all firms behave similarly regardless of their idiosyncratic shocks. Indeed, Equations (16) and (17) imply that this condition for the rise of sunspot equilibrium is such that the least-square projection coefficient is zero, $\left(\frac{\mu\sigma_\Phi^2-(1-\mu)\sigma_A^2}{\mu^2\sigma_\Phi^2+(1-\mu)^2\sigma_A^2}\right) = 0$. This implies that all firms set the same prices (see 14) and produce the same quantities along a sunspot equilibrium path regardless of their idiosyncratic shocks A^i. In other words, in a sunspot equilibrium, individual signals do not matter (but their distribution matters). The way to ensure this is to require the variance of sunspots satisfy $\sigma_\Phi^2 = \frac{1-\mu}{\mu}\sigma_A^2$, which restricts the standard deviation of a stochastic Nash equilibrium path of aggregate demand along the 45 degree line in Figure 1. This variance restriction requires that the variability of sunspots be proportional to that of idiosyncratic noise and the proportionality be a decreasing function of the precision (μ) of the signal. For example, if the variance of the noise is zero, then a sunspot equilibrium must involve a constant path of the aggregate demand. On the other hand, if σ_A^2 is large, then in order to achieve coordination as well as profit maximization, a sunspot equilibrium must involve a large variance of Φ. On the other hand, given σ_A^2, the more information there is in the signal (i.e., a larger μ), the less variable is the sunspot equilibrium path along the 45 degree line.

It is also possible to obtain serially correlated sunspots in this model. For example, assume that each firm's idiosyncratic cost shock is a serially correlated stationary $AR(1)$ process in log,

$$\hat{A}_t(i) = \rho \hat{A}_{t-1}(i) + \varepsilon_t^i, \tag{18}$$

where ε is *i.i.d.* with variance σ_ε^2; and that each firm's information set contains the entire history of the signal $s_t^i = \mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i)$:

$$\Omega_t^i = \{s_t^i, s_{t-1}^i, s_{t-2}^i, ...\}. \tag{19}$$

Such a setup of firm's information set has also been used by Woodford (2003). We can show that there exists correlated sunspot equilibria such that

$$\hat{\Phi}_t = \rho\hat{\Phi}_{t-1} + \xi_t, \tag{20}$$

where ξ_t is *i.i.d.* with variance

$$\sigma_\xi^2 = \frac{1-\mu}{\mu}\sigma_\varepsilon^2. \tag{21}$$

In other words, sunspots follows the same $AR(1)$ process as the idiosyncratic productivity shocks.

The proof is quit straightforward. Given the information set Ω^i, equation (14) becomes

$$\hat{P}_t^i = E\left\{[\hat{\Phi}_t - \hat{A}_t(i)]|[\mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i), \mu\hat{\Phi}_{t-1} + (1-\mu)\hat{A}_{t-1}(i), ...]\right\}. \tag{22}$$

Notice that because both Φ_t and $A_t(i)$ follows the same process by conjecture, we can write

$$s_t^i = \mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i) = \sum_{j=0}^{\infty}\rho^j(\mu\xi_{t-j} + (1-\mu)\varepsilon_{t-j}^i). \tag{23}$$

The information set Ω_t^i is then equivalent to $\tilde{\Omega}_t^i = \{\mu\xi_t + (1-\mu)\varepsilon_t^i, \mu\xi_{t-1} + (1-\mu)\varepsilon_{t-1}^i, ...\}$. So $\hat{P}_t^i$ can be further written as

$$\begin{aligned} \hat{P}_t^i &= E\left\{[\hat{\Phi}_t - \hat{A}_t(i)]|[\mu\xi_t + (1-\mu)\varepsilon_t^i, \mu\xi_{t-1} + (1-\mu)\varepsilon_{t-1}^i, ...]\right\} \qquad (24) \\ &= \sum_{j=0}^{\infty}\rho^j E^i\left\{[\xi_{t-j} - \varepsilon_{t-j}^i]|[\mu\xi_t + (1-\mu)\varepsilon_t^i, \mu\xi_{t-1} + (1-\mu)\varepsilon_{t-1}^i, ...]\right\} \\ &= \sum_{j=0}^{\infty}\rho^j\left\{\frac{\mu\sigma_\xi^2}{\mu^2\sigma_\xi^2 + (1-\mu)^2\sigma_\varepsilon^2}\left[\mu\xi_{t-j} + (1-\mu)\varepsilon_{t-j}^i\right] - \frac{(1-\mu)\sigma_\varepsilon^2}{\mu^2\sigma_\xi^2 + (1-\mu)^2\sigma_\varepsilon^2}\left[\mu\xi_{t-j} + (1-\mu)\varepsilon_{t-j}^i\right]\right\}. \end{aligned}$$

Notice that ε^i is idiosyncratic across firms but ξ is an aggregate innovation. Hence, we have

$$0 = \int_0^1 \hat{P}_t^i di = \left(\frac{\mu\sigma_\xi^2 - (1-\mu)\sigma_\varepsilon^2}{\mu^2\sigma_\xi^2 + (1-\mu)^2\sigma_\varepsilon^2}\right)\mu\sum_{j=0}^{\infty}\rho^j\xi_{t-j} = \left(\frac{\mu\sigma_\xi^2 - (1-\mu)\sigma_\varepsilon^2}{\mu^2\sigma_\xi^2 + (1-\mu)^2\sigma_\varepsilon^2}\right)\mu\hat{\Phi}_t, \tag{25}$$

which implies (21). So a sunspot equilibrium is given by any stochastic marginal cost process $\left\{\hat{\Phi}_t\right\}$ such that equations (20) and (21) are satisfied.

Business Cycle Implications. Sunspots under incomplete information have important implications for understanding the business cycle. To see this, we close the model by having a representative household with period-utility function, $u(C, N) = \log C - a_n\frac{N^{1+\gamma}}{1+\gamma}$, and the budget constraint,

$$C_t + K_{t+1} = W_t N_t + (1 + R_t - \delta)K_t + \Pi_t, \tag{26}$$

where K_t is the household's existing stock of capital, which depreciates at the rate $\delta \in (0, 1]$; $W_t N_t$ and $R_t K_t$ are the household's wage income and rental income, respectively; and Π is the aggregate profit income from firms.

Denote a circumflex variable as $\hat{X}_t(i) \equiv \log X_t(i) - \log \bar{X}$, where $\bar{X}$ denotes the long-run average value of $X(i)$ in a deterministic steady state. Log-linearizing equation (13) and the first-order conditions of the firms and the household around the deterministic steady state give the following system of four equations for the aggregate variables $\{\Phi, C, Y, N\}$:

$$\int_0^1 E^i \left\{ [\hat{\Phi}_t - \hat{A}_t(i)] | s_t^i \right\} di = 0 \tag{27}$$

$$(\alpha + \gamma) \hat{N}_t = \hat{\Phi}_t + \alpha \hat{K}_t - \hat{C}_t \tag{28}$$

$$\hat{C}_t = E_t \hat{C}_{t+1} - (1 - \beta(1-\delta)) E_t \left[\hat{\Phi}_{t+1} + (\alpha - 1) \hat{K}_{t+1} + (1-\alpha) \hat{N}_{t+1} \right] \tag{29}$$

$$(1-s)\hat{C}_t + s \left(\frac{1}{\delta} \hat{K}_{t+1} - \frac{1-\delta}{\delta} \hat{K}_t \right) = \alpha \hat{K}_t + (1-\alpha) \hat{N}_t, \tag{30}$$

where $s = \delta \frac{\beta \alpha \bar{\Phi}}{1 - \beta(1-\delta)}$ is the steady-state saving rate.

Notice that equation (27) determines the aggregate marginal cost $\hat{\Phi}_t$. Once the time path of the marginal cost is given, the model is identical to a standard RBC model with an exogenous forcing variable, $\hat{\Phi}_t$. Let the idiosyncratic productivity shock follow a stationary $AR(1)$ process, $\hat{A}_t^i = \rho \hat{A}_{t-1}^i + \varepsilon_t^i$, and the signal received by firms in each period t be a mixture of an aggregate component and an idiosyncratic component, $\mu \hat{\Phi}_t + (1-\mu) \hat{A}_t^i$. In such a case, as shown previously, a stochastic process $\hat{\Phi}_t$ satisfying (27) is $AR(1)$ with persistence ρ and variance $\sigma_\Phi^2 = \frac{1}{1-\rho^2} \frac{1-\mu}{\mu} \sigma_\varepsilon^2$. Given the process of $\hat{\Phi}_t$, the above system of equations implies the following state-space representation,

$$E_t \begin{pmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = M \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix} + \Gamma \hat{\Phi}_t. \tag{31}$$

The saddle-path property of the model implies that the coefficient matrix M has exactly one explosive eigenvalue and one stable eigenvalue. Hence, the equilibrium consumption path $\left\{ \hat{C}_t \right\}$ can be solved by the method of Blanchard and Kahn (1980). The solution takes the form, $\hat{C}_t = \lambda_k \hat{K}_t + \lambda_\phi \hat{\Phi}_t$, where the coefficients $\{\lambda_k, \lambda_\phi\}$ are functions of the structural parameters of the model.

Following the existing RBC literature (e.g., King, Plosser, and Rebelo, 1988), we calibrate the model as follows: the time period is a quarter, the time discounting factor $\beta = 0.99$, the rate of depreciation $\delta = 0.025$, the inverse labor supply elasticity $\gamma = 0.25$, and capital's share in aggregate output $\alpha = 0.4$. We set the elasticity parameter $\sigma = 10$ (implying a 10% markup for intermediat-goods firms in the steady state) and the persistence parameter $\rho = 0.9$. We take the normalization $\frac{1-\mu}{\mu} \sigma_\varepsilon^2 = 1$.

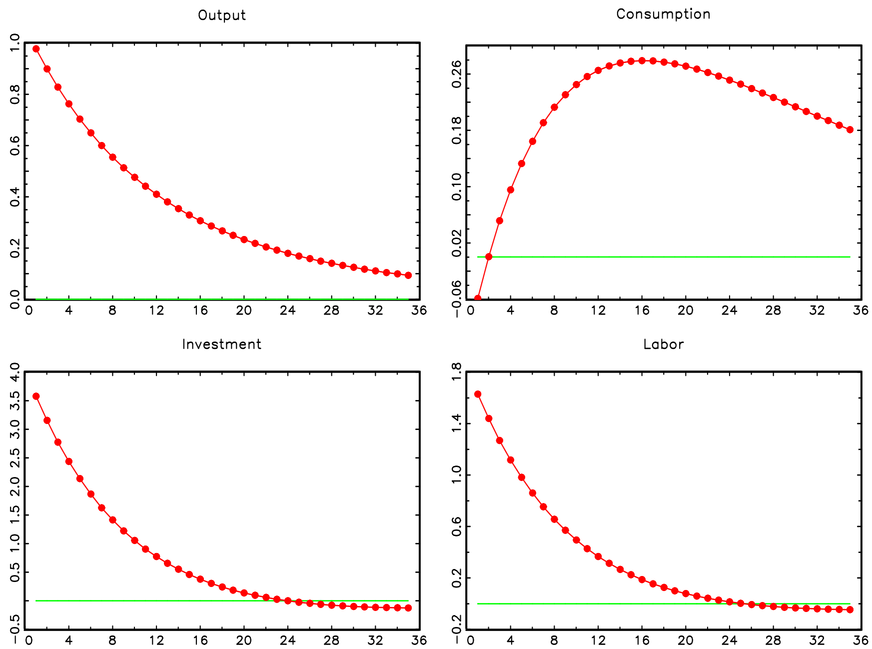

Figure 3. Impulse Responses to a Sunspot Shock.

The impulse responses of the benchmark model to a sunspot shock to the marginal cost (or aggregate demand) are graphed in Figure 3. Notice that a positive one-standard-deviation shock to the marginal cost generates positive responses from employment, output, and investment. Consumption is initially negative but soon turns to positive in the subsequent periods.[18] Investment is far more volatile than output because of the incentive for consumption smoothing. Thus the model is able to explain the stylized business cycle facts emphasized by Kydland and Prescott (1982): the positive comovements among key aggregate variables; the typical volatility orders among consumption, output, and investment; and the persistence of these variables. These business cycle facts are commonly thought to be explainable only by technology shocks. Here we show that they are also explainable by sunspot-driven aggregate demand shocks.

The predicted second moments of the model under $AR(1)$ sunspot shocks and $AR(1)$ aggregate technology shocks with the same persistence are summarized in Table 1. It shows that the model is able to explain the salient features of the business cycle as well as a standard RBC model driven by aggregate technology shocks and that the business-cycle effects of sunspot shocks and technology shocks are very similar.[19] One important exception is hours worked: the volatility of hours worked relative to output is too small under technology shocks but too large under sunspots shocks.[20] The reason is that sunspots shocks do not affect aggregate productivity while technology shocks do. The

[18] The initial consumption is positive if the shocks are less serially correlated.

[19] Notice that our model is identical to a standard RBC model without sunspot shocks. Hence, its dynamics under technology shocks are identical to those of a standard RBC model.

[20] In the U.S. data, hours worked are about as volatile as output.

intuition for the similarity between sunspot-driven business cycle and technology-driven business cycle is that the marginal cost measures the increase in production cost when firms' output demand increases by one unit. As such, sunspots shocks to the marginal cost reflect shocks to expected demand. Because of strategic complementarity under the demand externalities, such demand-side shocks are effectively the same as shocks to firms' marginal revenue or production efficiency. Thus, they look like productivity shocks except that they do not change aggregate productivity.

Table 1. Predicted Second Moments*

	Volatility ($\frac{\sigma_x}{\sigma_y}$)			Correlation with y			Autocorrelation			
	c	i	n	c	i	n	y	c	i	n
Sunspots	0.57	2.88	1.32	0.61	0.92	0.93	0.93	0.99	0.88	0.88
Technology	0.61	2.48	0.51	0.79	0.93	0.81	0.92	0.99	0.88	0.86

*In the table, y denotes output, c consumption, i investment, and n labor.

Discussion. If variable capital utilization rate is allowed in the model as in Greenwood et al. (1988), then shocks to the marginal cost also increase the total factor productivity through their impact on capacity utilization. Hence, the problem of excess volatility of hours relative to output under sunspot shocks can be mitigated. An important implication of sunspots equilibria is that the markup is countercyclical, which is in line with the empirical evidence.[21] In the model, the markup is given by the inverse of the marginal cost, $\frac{1}{\Phi}$. When expected demand is high, firms opt to produce more, and the marginal cost increases, leading to a lower markup. This implication of counter-cyclical markup is in sharp contrast to cases with fundamental shocks. Under fundamental shocks only (i.e., without extrinsic uncertainty), the markup is always constant in the model. More importantly, notice that counter-cyclical markup is obtained regardless of the monopoly power, since the same results hold even as $\sigma \to \infty$. In this case, although the markup is zero in the steady state, it fluctuates under sunspots shocks. Thus, even though the markets are perfectly (or near-perfectly) competitive and firms set prices equal to expected marginal cost, because the expected marginal cost comoves with expected aggregate demand, the markup can be countercyclical during the business cycle, regardless of the degree of imperfect competition.

4 Money and Sticky Prices

Our previous analyses about incomplete information as a new source of sunspots are conducted in real models where firms set real prices. One may argue that in a monetary model (e.g., a model with the cash-in-advance constraint always binding), if nominal prices are sticky, then sunspots do not matter because the CIA constraint pins down the level of aggregate demand given the money

[21] The stylized fact of countercyclical markup has been documented extensively in the empirical literature. See, e.g., Bils (1987), Rotemberg and Woodford (1991,1999), Martins, Scapetta, and Pilat (1996).

supply and the sticky price level. This section shows that such a conclusion is not true in general and our previous analyses carry over to a variety of standard monetary models with and without sticky prices. That is, under a more general specification of incomplete information, there can still exist sunspot equilibria as long as there is a positive measure of firms who can adjust prices in each period.[22] To simplify the analysis, we abstract capital from the models (implying the technology $Y(i) = A_t^i N_t(i)$) and set the household problem identical to that in the benchmark model.

4.1 Exogenous Money Supply

Let money be exogenously supplied and the cash-in-advance (CIA) constraint strictly bind. Denote P as the aggregate nominal price level and $P(i)$ the nominal price charged by firm i. The optimal price of intermediate good i is given by solving

$$P_t(i) \in \arg\max_{P(i)} E\left\{[u'(c_t)\left(P_t(i)/P_t - \phi_t(i)\right)Y_t(i)]|\Omega_t^i\right\} \tag{32}$$

subject to $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\sigma} Y_t$. This yields

$$P_t(i) = \frac{\sigma}{\sigma-1}\frac{E\left\{[u'(c_t)P_t^{\sigma}Y_t\phi_t(i)]|\Omega_t^i\right\}}{E\left\{\left[u'(c_t)P_t^{\sigma-1}Y_t\right]|\Omega_t^i\right\}}. \tag{33}$$

Using lower-case circumflex to denote percentage deviations from the steady state, log-linearizing the price equation gives

$$\hat{P}_t^i = E\left\{[\hat{\Phi}_t - \hat{A}_t^i + \hat{P}_t]|\Omega_t^i\right\}. \tag{34}$$

The aggregate price is given by

$$\hat{P}_t = \int_0^1 \hat{P}_t^i di. \tag{35}$$

Notice that the model involves a form of "forecasting the forecast of others" discussed by Townsend (1983).

Since the real wage equals the marginal cost, the household's optimization leads to a relationship between aggregate income and the real marginal cost around the steady state:

$$\hat{Y}_t = \lambda\hat{\Phi}_t, \tag{36}$$

[22] Since the focus of our paper is not about monetary policies as a source of indeterminacy, in this section we consider only the standard monetary models with standard monetary policies.

where $\lambda > 0$ is a function of structural parameters (e.g., from the utility functions). The system of equations that determine the general equilibrium of the model are given by equations (34)-(36) plus the CIA constraint,

$$\hat{P}_t + \hat{Y}_t = 0. \tag{37}$$

So we have essentially three equations with three unknowns, $\{\hat{Y}_t, \hat{\Phi}_t, \hat{P}_t\}$.

Notice that, under perfect information, equation (34) becomes $\hat{P}^i_t = \hat{\Phi}_t - \hat{A}^i_t + \hat{P}_t$, so by integrating equation (35) we get $\hat{\Phi}_t = 0$. Therefore, under perfect information, $\hat{\Phi}_t = \hat{Y}_t = \hat{P}_t = 0$ is the only equilibrium. So, sunspots do not matter. However, suppose the information set of firm i is given by equation (15), where the idiosyncratic productivity shocks are *i.i.d.* The CIA constraint (37) and the relationship (36) implies

$$\hat{P}_t = -\lambda\hat{\Phi}_t. \tag{38}$$

The pricing equation (34) then becomes

$$\hat{P}^i_t = \left[\frac{(1-\lambda)\mu\sigma^2_\Phi - (1-\mu)\sigma^2_A}{\mu^2\sigma^2_\Phi + (1-\mu)^2\sigma^2_A}\right]\left[\mu\hat{\Phi}_t + (1-\mu)\hat{A}^i_t\right]. \tag{39}$$

The aggregate price level is then

$$\hat{P}_t = \left(\frac{(1-\lambda)\mu\sigma^2_\Phi - (1-\mu)\sigma^2_A}{\mu^2\sigma^2_\Phi + (1-\mu)^2\sigma^2_A}\right)\mu\hat{\Phi}_t. \tag{40}$$

Thus, the system of equations determining the general equilibrium of the model is given by equations (37), (38), and (40), with three unknowns $\{\hat{Y}, \hat{\Phi}, \hat{P}\}$. In order for this system to be indeterminate, the rank of the system has to be less than 3. This suggests that (40) and (38) must be colinear, or

$$\left(\frac{(1-\lambda)\mu\sigma^2_\Phi - (1-\mu)\sigma^2_A}{\mu^2\sigma^2_\Phi + (1-\mu)^2\sigma^2_A}\right)\mu = -\lambda. \tag{41}$$

Equation (41) is satisfied if the variance of the marginal cost satisfies

$$\sigma^2_\Phi = \left(\frac{1-\mu}{\mu}\right)\left[1 - \lambda\left(\frac{1-\mu}{\mu}\right)\right]\sigma^2_A, \tag{42}$$

which is positive if

$$1 > \mu > \frac{\lambda}{1+\lambda}. \tag{43}$$

This is the condition of sunspot equilibria in a monetary model with flexible prices and the CIA constraint always binding. Although the variance of sunspot shocks depends on the model's structural parameters (such as σ_A and λ), sunspots always exist for any finite value of λ as long as the information precision parameter μ satisfies (43).

Notice that if $\mu = 0$, then sunspots do not matter. In this case, the system has full rank and the unique solution of the system is $\hat{\Phi}_t = \hat{Y}_t = \hat{P}_t = 0$. This case is identical to a sticky nominal price model where prices are set one period in advance without any information about the equilibrium aggregate demand that may prevail in the economy.[23]

The intuition behind the sunspot condition (41) is similar to that in the real model, except here individual firms' prices and quantities are no longer the same across firms in a sunspot equilibrium. To see this, substituting (41) into (39) gives

$$\hat{P}_t^i = -\lambda\hat{\Phi}_t - \lambda\frac{(1-\mu)}{\mu}\hat{A}_t^i = \hat{P}_t - \lambda\frac{(1-\mu)}{\mu}\hat{A}_t^i, \tag{44}$$

which indicates that individual firms' prices differ across firms by the idiosyncratic marginal cost term, $-\lambda\frac{(1-\mu)}{\mu}\hat{A}^i$, but share the same systematic component $\hat{P}$ (the aggregate price). Why in a monetary model must firms set different prices along a sunspot path whereas they must choose the same prices in non-monetary models? The answer: With money and the CIA constraint binding, a sunspot equilibrium requires that aggregate price be affected by sunspots (i.e., not perfectly observable). If firms all set the same prices, the aggregate price is then known to the firms. In such a case, sunspots do not matter. However, despite firms setting different prices, these prices must be equal the aggregate price on average in a sunspot equilibrium according to (44). Because firms' prices must be influences by idiosyncratic noise, coordination is more difficult in a monetary model. This is why the variance of sunspots is more restrictive than that in a real model (see 43) in the sense that the information precision of the signal has to be high enough to achieve coordination: $\mu > \frac{\lambda}{1+\lambda}$.

4.2 Taylor Rule

To study the effects of endogenous monetary policy on the possibility of sunspots, consider the Taylor rule,

$$\hat{r}_t = \omega_\pi\hat{\pi}_t + \omega_y\hat{Y}_t, \tag{45}$$

where r denotes the nominal interest rate and π the inflation rate in period t. Assume $\omega_\pi > 1$ so as to ensure that the Taylor rule itself does not cause indeterminacy. With endogenous monetary

[23] But, if the CIA constraint is slack, then there is indeterminacy in P, implying that Φ and Y are also indeterminate. This is akin to the result obtained by Carlstrom and Fuerst (1998b). However, as long as $\mu \in \left(\frac{\lambda}{1+\lambda}, 1\right)$, sunspots always exist whether the CIA constraint binds or not.

policy, the additional new equation is the household's Euler equation for nominal bond holding,

$$\theta \hat{Y}_t + \hat{r}_t = \theta E_t \hat{Y}_{t+1} + E_t \hat{\pi}_{t+1}, \tag{46}$$

where θ is the elasticity of the marginal utility of income. The rest of the equations are the same as (34), (35), (36), and (37).

For simplicity, we focus on *i.i.d.* sunspots only. Hence, $E_t \hat{\Phi}_{t+1} = E_t \hat{\pi}_{t+1} = 0$. Substituting out $\hat{Y}_t$ and $\hat{r}_t$ in the equation (46), we get $(\theta + \omega_y)\lambda \hat{\Phi}_t + \omega_\pi \hat{\pi}_t = 0$, which implies $\hat{\pi}_t = -\frac{(\theta+\omega_y)\lambda}{\omega_\pi} \hat{\Phi}_t \equiv -\tilde{\lambda} \hat{\Phi}_t$. Because $\hat{P}_{t-1}$ is a function of $\hat{\Phi}_{t-1}$ and is known at t, we can subtract $\hat{P}_{t-1}$ from both sides of equation (34) to obtain

$$\hat{P}_t^i - \hat{P}_{t-1} = E^i \left(\hat{\Phi}_t - \hat{A}_t^i + \hat{P}_t - \hat{P}_{t-1} \right). \tag{47}$$

Define $\hat{\pi}_t^i \equiv \hat{P}_t^i - \hat{P}_{t-1}$ and $\hat{\pi}_t \equiv \hat{P}_t - \hat{P}_{t-1}$, the above equation becomes

$$\hat{\pi}_t^i = E^i \left(\hat{\Phi}_t - \hat{A}_t^i + \hat{\pi}_t \right). \tag{48}$$

Notice that $\hat{\pi}_t = \int_0^1 \hat{\pi}_t^i di$. Substituting out the aggregate inflation rate using $\hat{\pi}_t = -\tilde{\lambda} \hat{\Phi}_t$ in equation (48), our previous discussions following equations (34)-(40) show that if $1 > \mu > \frac{\lambda}{1+\lambda}$, there exist sunspot equilibria. Hence, the existence of sunspots dose not depend on the form of monetary policy.

4.3 Sticky Prices

Although we believe our result holds in the more general Calvo-type (1983) sticky price models, to simplify our analysis, we consider only one-period sticky prices in this paper. Suppose there are two type of firms in the economy, with θ fraction of firms set nominal prices (P^1) one period in advance, and $1 - \theta$ fraction of firms set prices (P^2) each period based on the signals s_t as discussed in the previous cases. For firms who set prices one period in advance (type 1 firm), the profit-maximizing nominal price is given by

$$P_t^1(i) = \frac{\sigma}{\sigma - 1} \frac{E_{t-1}[u'(c_t) P_t^\sigma Y_t \phi_t(i)]}{E_{t-1}\left[u'(c_t) P_t^{\sigma-1} Y_t\right]}; \tag{49}$$

and for firms who set their prices in current period t, the optimal price is

$$P_t^2(i) = \frac{\sigma}{\sigma - 1} \frac{E\left\{[u'(c_t) P_t^\sigma Y_t \phi_t(i)] | s_t^i\right\}}{E\left\{\left[u'(c_t) P_t^{\sigma-1} Y_t\right] | s_t^i\right\}}. \tag{50}$$

The log-linearized prices are given, respectively, by

$$\hat{P}_t^1(i) = E_{t-1}[\hat{P}_t + \hat{\Phi}_t - \hat{A}_t(i)], \tag{51}$$

$$\hat{P}_t^2(i) = E[\hat{P}_t + \hat{\Phi}_t - \hat{A}_t(i)]|s_t^i. \tag{52}$$

The log-linearized aggregate price index is given by

$$\hat{P}_t = \theta \int_0^1 \hat{P}_t^1(i)di + (1-\theta) \int_0^1 \hat{P}_t^2(i)di; \tag{53}$$

and the CIA constraint implies

$$\lambda\hat{\Phi}_t + \hat{P}_t = \hat{m}_t, \tag{54}$$

where $\hat{m}_t = 0$ is total money supply. To see the possibility of sunspots in this setup, assume the idiosyncratic shock A^i is *i.i.d.*, and we can conjecture a sunspot equilibrium in which the aggregate marginal cost $\hat{\Phi}_t$ is also an *i.i.d.* process. Then we have $E_{t-1}[\hat{P}_t + \hat{\Phi}_t - \hat{A}_t(i)] = 0$ and

$$\hat{P}_t^2(i) = \left(\frac{(1-\lambda)\mu\sigma_\Phi^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\Phi^2 + (1-\mu)^2\sigma_A^2}\right)[\mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i)]. \tag{55}$$

The aggregate price level then becomes

$$\hat{P}_t = (1-\theta)\left(\frac{(1-\lambda)\mu\sigma_\Phi^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\Phi^2 + (1-\mu)^2\sigma_A^2}\right)\mu\hat{\Phi}_t. \tag{56}$$

To be consistent with the CIA constraint we must have

$$(1-\theta)\left(\frac{(1-\lambda)\mu\sigma_\Phi^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\Phi^2 + (1-\mu)^2\sigma_A^2}\right)\mu = -\lambda \tag{57}$$

The above constraint is satisfied if the variance of the marginal cost satisfies

$$\sigma_\Phi^2 = \frac{(1-\theta)\frac{(1-\mu)}{\mu} - \lambda(\frac{1-\mu}{\mu})^2}{1-\theta+\theta\lambda}\sigma_A^2. \tag{58}$$

A positive variance requires

$$(1-\theta) > \lambda\left(\frac{1-\mu}{\mu}\right). \tag{59}$$

That is, the fraction of flexible-price firms must be large enough. Clearly sunspot equilibria are more difficult to arise in sticky-price models. For example, if all firms set prices one period in advance ($\theta = 1$), the above condition cannot be satisfied and sunspots do not matter.

Now suppose that the money supply is stochastic so that $\hat{m}_t \neq 0$. Since there are one-period sticky prices, monetary effects can only have at most a one-period real effect on the real economy. Without loss of generality, we can focus on $i.i.d$ monetary shocks. With $\hat{m}_t \neq 0$, the two types of firms' monopoly prices become

$$\hat{P}_t^1(i) = E_{t-1}[(1-\lambda)\hat{\Phi}_t + \hat{m}_t - \hat{A}_t(i)] = 0, \tag{60}$$

$$\hat{P}_t^2(i) = E\left\{[(1-\lambda)\hat{\Phi}_t - \hat{A}_t(i) + \hat{m}_t] | [\mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i), \hat{m}_t]\right\}, \tag{61}$$

respectively. There is always a solution for the marginal cost such that $\hat{\Phi}_t = \frac{\theta}{1-\theta+\theta\lambda}\hat{m}_t$ and hence $\hat{P}_t = \frac{1-\theta}{1-\theta+\theta\lambda}\hat{m}_t$. This is the fundamental equilibrium with aggregate monetary shocks. But there also exist sunspot equilibria satisfying

$$\hat{\Phi}_t = \frac{\theta}{1-\theta+\theta\lambda}\hat{m}_t + \hat{\varepsilon}_t, \quad \hat{P}_t = \frac{1-\theta}{1-\theta+\theta\lambda}\hat{m}_t - \lambda\hat{\varepsilon}_t; \tag{62}$$

where $\hat{\varepsilon}_t$ denotes sunspots with variance

$$\sigma_\varepsilon^2 = \frac{(1-\theta)\frac{(1-\mu)}{\mu} - \lambda(\frac{1-\mu}{\mu})^2}{1-\theta+\theta\lambda}\sigma_A^2, \tag{63}$$

where $(1-\theta) > \lambda(\frac{1-\mu}{\mu})$. To prove this, notice that

$$\begin{aligned} \hat{P}_t^2(i) &= E\left\{[(1-\lambda)\hat{\Phi}_t - \hat{A}_t(i) + \hat{m}_t] | [\mu\hat{\Phi}_t + (1-\mu)\hat{A}_t(i), \hat{m}_t]\right\} \\ &= \frac{1}{1-\theta+\theta\lambda}\hat{m}_t + \left(\frac{(1-\lambda)\mu\sigma_\varepsilon^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\varepsilon^2 + (1-\mu)^2\sigma_A^2}\right)[\mu\hat{\varepsilon}_t + (1-\mu)\hat{A}_t(i)]. \end{aligned} \tag{64}$$

It is then straightforward to see that aggregate price satisfies

$$\hat{P}_t = (1-\theta)\int_0^1 \hat{P}_t^2(i)di = \frac{1-\theta}{1-\theta+\theta\lambda}\hat{m}_t + (1-\theta)\left(\frac{(1-\lambda)\mu\sigma_\varepsilon^2 - (1-\mu)\sigma_A^2}{\mu^2\sigma_\varepsilon^2 + (1-\mu)^2\sigma_A^2}\right)\mu\hat{\varepsilon}_t. \tag{65}$$

Comparing (65) with (62) gives the results we need.

Stabilizing Monetary Policy. Since money has real effects under sticky prices, monetary policies can stabilize the economy driven by sunspots. For example, the central bank can decrease money stock if $\hat{\varepsilon}_t$ is high and increase money stock if $\hat{\varepsilon}_t$ is low. This type of endogenous monetary policy requires the central bank be able to observe the aggregate demand. Consider the following counter-cyclical policy,

$$\hat{m}_t = -\pi\hat{\Phi}_t, \tag{66}$$

where π is a Taylor-rule type parameter. Given this monetary policy function, the CIA constraint becomes

$$\lambda\hat{\Phi}_t + \hat{P}_t = -\pi\hat{\Phi}_t, \tag{67}$$

which implies

$$\hat{P}_t = -\left(\lambda + \pi\right)\hat{\Phi}_t, \tag{68}$$

The condition that gives rise to sunspots discussed before (59) now becomes

$$(1-\theta) > (\lambda + \pi)\left(\frac{1-\mu}{\mu}\right). \tag{69}$$

Clearly, the central bank can set the policy parameter π to insulate the economy from sunspot-driven fluctuations. For example, if $\pi > \frac{(1-\theta)\mu}{1-\mu} - \lambda$, then (69) is impossible to satisfy, so sunspot equilibria are completely eliminated. On the other hand, if monetary policies are procyclical ($\pi < 0$), then sunspots-driven fluctuations become much easier to arise. This also suggests that government policy itself can facilitate self-fulfilling fluctuations when it is accommodative.

5 Conclusion

This paper shows that self-fulfilling rational expectations equilibria can arise in standard Dixit-Stiglitz DSGE models with monopolistic competition. Even though the fundamental equilibrium is unique in this class of models, there can exist multiple stochastic sunspot-Nash equilibria that are not mere randomizations over fundamental equilibria. This type of sunspot equilibria is associated with extrinsic uncertainty during the process of expectation formation. A key friction for generating extrinsic uncertainty in our model is that individual firms make price decisions simultaneously without knowing how the other agents in the economy will behave; thus, they each must face an aggregate uncertainty regarding other firms' actions.[24] Given the complementarity among agents' actions, such extrinsic uncertainty can be self-fulfilling. By embedding this insight into DSGE models, our approach provides a new channel to study expectations-driven fluctuations.

Our analyses also show that aggregate fluctuations driven by sunspots are almost indistinguishable from those driven by technology shocks because sunspots affect aggregate demand through the marginal costs. The welfare implications of such sunspot-driven business cycles is carried out in Wang and Wen (2006) to study the interactions between sunspots and endogenous growth, where we show that with extrinsic uncertainty, short-run volatility and long-run growth are negatively related, confirming the empirical findings of Ramey and Ramey (1995).

[24] This type of uncertainty is referred to as market uncertainty by Peck and Shell (1991).

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