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Money and Capital as Competing Media of Exchange in a News Economy*

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Abstract

Conventional theory suggests that fiat money will have value in capital-poor economies. We demonstrate that fiat money may also have value in capital-rich economies, if the price of capital is excessively volatile. Excess asset-price volatility is generated by *news*; information that has no social value, but is privately useful in forming forecasts over the short-run return to capital. One advantage of fiat money is that its expected return is not linked directly to news concerning the prospects of an underlying asset. When money and capital compete as media of exchange, excess volatility in the short-term returns of liquid asset portfolios is mitigated and welfare is improved. A legal restriction that prohibits the use of capital as a payment instrument renders the expected return to money perfectly stable and, as a consequence, may generate an additional welfare benefit. *JEL codes: E4, E5. Keywords: Money, Capital, News, Excess Volatility.*

1 Introduction

When record-keeping and commitment is limited, realizing the gains to intertemporal trade requires a settlement object. In an economy with physical capital, the settlement object may take the form of capital; or, equivalently, financial instruments representing claims against capital. A long-standing question in the theory of money and banking is whether the supply of such settlement objects should remain in the exclusive domain of the private sector; or whether a government intervention is warranted.

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In a large body of the literature devoted to this question, the answer appears to hinge on the productivity of an economy's storage technology. That is, if the storage technology is poor, efficiency dictates the accumulation of a low stock of capital. But if capital also serves as a medium of exchange, this low stock may not be sufficient to serve an economy's liquidity needs. The equilibrium response is an overaccumulation of capital; see, for example, Lagos and Rocheteau [6] and the references cited within.[1] This result continues to hold if capital is replaced by financial claims to capital, or financial claims against other objects (e.g. labor) collateralized by capital; see Ferraris and Watanabe [4]. When this is so, the introduction of a fiat money instrument improves social welfare. Money is a good substitute for poor-return capital as a payment instrument; its introduction expands liquidity and economizes on capital.

In this paper, we explore another possible source of societal value for fiat money; namely, its relative insensitivity to flows of information that cause the price of capital to fluctuate excessively. To formalize this idea, we develop a version of the Lagos and Rocheteau [6] model where the return to capital is subject to aggregate risk. Optimal capital investment is determined solely by long-run expected return. Between the time an investment is made and the time it matures, new information alters the expected short-run return to capital. By construction, "news" confers no social benefit. Nevertheless, news has private value; and as such, it is rapidly incorporated into asset prices. In this way, the interim price of capital exhibits excess volatility; a property that hinders its use as a payment instrument. In particular, a "bad news" event depresses the price of capital in the short-run, leaving some individuals with insufficient purchasing power (they are debt-constrained).

One advantage of fiat money is that its expected return at any frequency is not linked directly to news concerning the prospects of an underlying asset. When money and capital compete as media of exchange, excess volatility in the short-term return of a liquid asset portfolio is thereby mitigated and welfare is increased. Our result is subtle in that it does not rely on money having less risk than capital. Indeed, the result continues to hold if money is risky, or even riskier than capital; when return is measured over longer horizons. The key property is that the expected return to money is less sensitive to news.

The monetary equilibrium of our model economy has the following properties. First, although money is welfare-improving, there is still an overaccumulation of capital (except at the Friedman rule). Second, there is a Tobin effect; in the sense that higher inflation stimulates capital expenditure. Third, although money and capital are equally liquid, capital dominates money in (long-run) expected return. Fourth, the imposition of a "cash-in-advance" constraint has ambiguous welfare consequences; but is generally welfare-improving at low inflation rates.

The first two of these properties are also found in Lagos and Rocheteau

[1] If the capital stock is fixed, then the result is an overvaluation of the asset.

[6]; but the second two are not. In particular, their (liquid) capital earns the same rate of return as money and the imposition of a "cash-in-advance" constraint there is strictly welfare-improving. We elaborate on these results below, following the formal exposition of the model.

2 The Environment

There is a $[0, 1]$ continuum of infinitely-lived individuals. Time is discrete and the horizon is infinite. Each time-period is divided into two subperiods labeled *day* and *night*. Output is produced and consumed in the day and in the night; label this *day-output* and *night-output*, respectively. Economic activity in the day and night is centralized (there are no search frictions).

Let $x_t(i) \in \mathbb{R}$ denote consumption (*viz* production, if negative) of day-output by individual i at date t. Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production, respectively, of night-output by individual i at date t. Individuals have quasi-linear and additively-separable preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t(i) + 0.5u(c_t(i)) - 0.5h(y_t(i)) \right] \tag{1}$$

where $u'' < 0 < u'$, $u'(0) = \infty$, $h' > 0$, $h'' \geq 0$ and $0 < \beta < 1$. The interpretation here is that each individual is subject to an *i.i.d.* shock, realized at the beginning of each night, that determines their *type* for the night. We assume that there are only two types: *consumer* and *producer*; and that the population is divided evenly among each type. A consumer wishes to consume and has no ability to produce; while a producer has no desire to consume and has an ability to produce. Assume that night-output is nonstorable.

Let κ_t denote the aggregate capital stock at date t; with $\kappa_0 \geq 0$ given. This capital produces $z_t f(\kappa_t)$ units of day-output; where $0 < z_t < \infty$ denotes a productivity parameter and $f'' < 0 < f'$, $f'(0) = \infty$, $f'(\kappa)\kappa = \alpha f(\kappa)$ for some $0 < \alpha < 1$. Implicit in this formulation is the existence of a fixed factor; which we interpret as a human capital input, distributed equally among the population.

Capital depreciates fully after use in production; the future capital stock κ_{t+1} is generated entirely by the day-output stored from one day to the next. Capital is not valued as consumption during the night; nor can it be used to augment production at night. Together, these considerations imply the following resource constraints

$$z_t f(\kappa_t) \geq \int x_t(i)di + \kappa_{t+1} \tag{2}$$

$$\int y_t(i)di \geq \int c_t(i)di \tag{3}$$

Productivity evolves randomly over time. This stochastic process is *i.i.d.*

from one day to the next, so that $E_t z_{t+1} = \zeta$ for all t. There is another stochastic process that generates information η_t (*news*) at the beginning of each night. Assume that news can be either *bad* or *good*; so that $\eta_t \in \{b, g\}$. Let $\pi \equiv \Pr[\eta_t = b]$ and $0 < \pi < 1$. News received at night in period t may be useful for the purpose of forecasting productivity the next day. That is, let $G(z^+ \mid \eta) \equiv \Pr[z_{t+1} \leq z^+ \mid \eta_t = \eta]$ and assume that $G(z^+ \mid g) \leq G(z^+ \mid b)$. Define $z(\eta) \equiv \int dG(z^+|\eta)$. Clearly $0 < z(b) \leq \zeta \leq z(g) < \infty$, where $\zeta \equiv \pi z(b) + (1-\pi)z(g)$.

Consider a planner that maximizes a population-weighted sum of (1) subject to (2) and (3); and label the solution to this problem the *first-best allocation.* It should be clear enough that this solution will entail $\kappa_{t+1} = \kappa^*$ for all t; where

$$\beta \zeta f'(\kappa^*) = 1 \tag{4}$$

Moreover, given the properties of u and h, together with a population that is equally divided at night between consumers and producers, it follows that $c_t(i) = y_t(i) = y^*$ for all (t, i); where

$$u'(y^*) = h'(y^*) \tag{5}$$

Given (κ^*, y^*), the allocation $\{x_t^*(i)\}$ is determined residually by the resource constraint (2).[2]

Lemma 1 *The first-best allocation is independent of news.*

The result in Lemma 1 is, of course, an artifact of the manner in which "news" is modeled here. In particular, information useful for forecasting arrives "too late" to impinge on capital investment decisions.

3 Competitive Equilibrium

We begin by examining the competitive equilibrium of this economy absent any government policy. In what follows, we assume that individuals are anonymous; so that private debt cannot exist. As in Lagos and Rocheteau [6], assume that capital can be used as a means of payment in the night-market.[3]

3.1 Decision-Making

Competitive factor markets imply the standard equilibrium pricing functions for capital and the fixed factor (human capital) in the day; i.e.,

$$\begin{aligned} r(\kappa, z) &= z f'(\kappa) & (6) \\ w(\kappa, z) &= z f(\kappa) - z f'(\kappa)\kappa & (7) \end{aligned}$$

[2] Because individuals have preferences that are linear in day-output, they are indifferent between any lottery over $\{x_t(i)\}$ that generates the expected value $z_t f'(\kappa_t) - \kappa_{t+1}$.

[3] Alternatively, one could follow Ferraris and Watanabe [4] and assume that individuals (perhaps through an intermediary) can issue debt that is securitized by physical capital.

Anticipating that κ will remain constant in equilibrium, we suppress the dependence of κ in these pricing functions to ease notation.

At the beginning of the day, an individual will have been either a consumer or producer the previous night; index this history by $j \in \{c, p\}$. An individual who enters the day with capital k_j generates income $r(z)k_j + w(z)$. Let $k_j^+ \geq 0$ denote an individual's capital investment during the day (matures as productive capital the next day). The budget constraint during the day is given by

$$x_j = r(z)k_j + w(z) - k_j^+ \tag{8}$$

A recursive representation of the choice problems follows. The choice problem during the day is given by

$$D(k_j, z) \equiv \max_{k_j^+} \left\{ r(z)k_j + w(z) - k_j^+ + E_\eta N(k_j^+, \eta) \right\}$$

where $N(k_j^+, \eta)$ represents the value of entering the night with future capital k_j^+, conditional on news η. Desired investment is characterized by

$$1 = E_\eta N_1(k_j^+, \eta) \tag{9}$$

Here we have the familiar result (Lagos and Wright [7]) that $k_j^+ = k^+$ is identical across all individuals, regardless of their trading history. By the envelope theorem

$$D_1(k_j, z) = zf'(\kappa) \tag{10}$$

At the beginning of the night, individual types become known and news is revealed. Let $\phi(\eta)$ denote the price of capital measured in units of night-output. Consumers are subject to the debt-constraint $\phi(\eta)k^+ \geq c$ or $k_c^+ \equiv k^+ - c/\phi(\eta) \geq 0$. With this in mind, the consumer choice problem is given by

$$C(k^+, \eta) \equiv \max_{k_c^+} \left\{ u\left(\phi(\eta)(k^+ - k_c^+)\right) + \beta E\left[D(k_c^+, z^+)|\eta\right] + \lambda k_c^+ \right\}$$

Hence

$$\phi(\eta)u'\left(c(\eta)\right) = \beta z(\eta)f'(\kappa^+) + \lambda(\eta) \tag{11}$$

By the envelope theorem

$$C_1(k^+, \eta) = \phi(\eta)u'\left(c(\eta)\right) \tag{12}$$

Similarly, the producer choice problem is

$$P(k^+, \eta) \equiv \max_{k_p^+} \left\{ -h\left(\phi(\eta)(k_p^+ - k^+)\right) + \beta E\left[D(k_p^+, z^+)|\eta\right] \right\}$$

Hence

$$\phi(\eta)h'(y(\eta)) = \beta z(\eta)f'(\kappa) \tag{13}$$

By the envelope theorem

$$P_1(k^+, \eta) = \phi(\eta) h'(y(\eta)) \tag{14}$$

Now form the expression $N \equiv 0.5C + 0.5P$. Using the envelope results (12) and (14), we have

$$N_1(k^+, \eta) \equiv \pi 0.5 \phi(b) [u'(c(b)) + h'(y(b))] + (1-\pi)\phi(g) 0.5 [u'(c(g)) + h'(y(g))]$$

This latter expression, together with (9) implies

$$2 = \pi \phi(b) [u'(c(b)) + h'(y(b))] + (1-\pi)\phi(g) [u'(c(g)) + h'(y(g))] \tag{15}$$

3.2 Market Clearing

The following restrictions must hold

$$\begin{aligned} k^+ &= \kappa^+ \\ c(\eta) &= y(\eta) \\ 0.5k_c^+(\eta) + 0.5k_p^+(\eta) &= \kappa^+ \end{aligned} \tag{16}$$

Apart from the initial date, the capital stock will remain constant over time so that $\kappa = \kappa^+$.

3.3 Equilibrium

Define the object

$$A(y) \equiv 0.5 \left[\frac{u'(y)}{h'(y)} + 1 \right]$$

Note that $A(y)$ is strictly decreasing in y and that $A(y^*) = 1$. Next, invoke the market-clearing condition $c(\eta) = y(\eta)$, and express (15) as

$$1 = \pi \phi(b) h'(y(b)) A(y(b)) + (1-\pi)\phi(g) h'(y(g)) A(y(g)) \tag{17}$$

By condition (13), one can derive an asset-pricing equation

$$\phi(\eta) = \left[\frac{\beta z(\eta) f'(\kappa)}{h'(y(\eta))} \right] \tag{18}$$

which, when substituted into (17) yields

$$1 = \beta \zeta f'(\kappa) \left[\pi \frac{z(b)}{\zeta} A(y(b)) + (1-\pi) \frac{z(g)}{\zeta} A(y(g)) \right] \tag{19}$$

Moreover, the debt-constraints imply $\phi(\eta)\kappa \geq y(\eta)$; or by use of (18)

$$\beta z(\eta) f'(\kappa) \kappa \geq h'(y(\eta)) y(\eta) \tag{20}$$

3.3.1 Equilibrium in a No-News Economy

In a *no news* economy, $z(\eta) = \zeta$; so that $y(\eta) = y$ and $A(y(\eta)) = A(y)$. Assume that the debt constraint does not bind (a conjecture that needs to be verified). It follows that $y = y^*$; in which case $A = 1$. In this case, the restriction (19) reduces to $1 = \beta\zeta f'(\kappa)$; so that $\kappa = \kappa^*$. If the debt constraint is to remain (at least weakly) slack, then condition (20) implies

$$\kappa^* \geq h'(y^*)y^* \tag{21}$$

Whether condition (21) holds or not depends on parameters. It is common in overlapping generations models with capital to assume that the analog to (21) does not hold. Lagos and Rocheteau [6], whose environment is essentially identical to our own absent news, also assume that (21) does not hold; see also Ferraris and Watanabe [4]. When this is so, the equilibrium allocation $\{\kappa_0, y_0\}$ is characterized by

$$\begin{aligned} 1 &= \beta\zeta f'(\kappa_0)A(y_0) \\ \beta\zeta f'(\kappa_0)\kappa_0 &= h'(y_0)y_0 \end{aligned}$$

where clearly, $\kappa_0 > \kappa^*$ and $y_0 < y^*$. This is just the standard "over-accumulation of capital" result that motivates the introduction of a fiat money instrument in much of the literature.

For the remainder of the paper, we assume that condition (21) holds as an equality; i.e.,

[A1] $\kappa^* = h'(y^*)y^*$

We impose [A1] primarily for the purpose of exposition; the main results may continue to hold more generally. When [A1] does hold, it follows that the competitive equilibrium implements the first-best allocation in a no-news economy.

3.3.2 Equilibrium in a News Economy

In a *news* economy, $z(b) < \zeta < z(g)$.

Lemma 2 *In a news economy, it cannot be the case that $\lambda(\eta) = 0$ for $\eta \in \{b, g\}$ (consumer debt constraints cannot remain slack in both news states).*

Proof. If $\lambda(\eta) = 0$, then (11) and (13) imply that $y(\eta) = y^*$. In this case, $A = 1$ and so, by condition (19), $\kappa = \kappa^*$. Moreover, since $y(b) = y^*$, condition (20) implies that $\beta z(b) f'(\kappa^*)\kappa^* \geq h'(y^*)y^*$, which by [A1] simplifies to $z(b) \geq \zeta$; a contradiction. ■

Lemma 3 *In a news economy, it cannot be the case that $\lambda(\eta) > 0$ for $\eta \in \{b, g\}$ (consumer debt constraints cannot bind tightly in both news states).*

Proof. If $\lambda(\eta) > 0$, then (20) implies

$$\begin{aligned} \beta z(b) f'(\kappa)\kappa &= h'(y(b))y(b) \\ \beta z(g) f'(\kappa)\kappa &= h'(y(g))y(g) \end{aligned}$$

These two conditions, together with $\lambda(g) > 0$, imply that $y(b) < y(g) < y^*$. In turn, this implies that $A(y(b)) > A(y(g)) > 1$; so that, by condition (19), $\kappa > \kappa^*$. Observe that the two equalities above imply

$$\beta\zeta f'(\kappa)\kappa = \pi h'(y(b))y(b) + (1-\pi)h'(y(g))y(g)$$

As $\kappa > \kappa^*$, it follows from [A1] and the assumed properties of f that $\beta\zeta f'(\kappa)\kappa > \beta\zeta f'(k^*)k^* = h'(y^*)y^*$. Therefore,

$$\pi h'(y(b))y(b) + (1-\pi)h'(y(g))y(g) > h'(y^*)y^*$$

But this is a contradiction; as $y(b) < y(g) < y^*$. ■

Lemmas 2 and 3 imply that, given [A1], the debt-constraint will bind tightly in one news state and remain slack in the other. It is easy to verify that the constraint will bind in the bad news state. Label this equilibrium allocation $\{y_1(\eta), \kappa_1\}$.

Proposition 1 *In a news economy, the equilibrium allocation satisfies $0 < y_1(b) < y_1(g) = y^*$ and $\kappa_1 > \kappa^*$.*

Proof. As $\lambda(g) = 0$, it follows that $y_1(g) = y^*$ so that $A(y_1(g)) = 1$. As $\lambda(b) > 0$, it follows that $y_1(b) < y^*$ so that $A(y_1(b)) > 1$. By conditions (19) and (20), $y_1(b)$ and κ_1 are jointly determined by

$$\begin{aligned} 1 &= \beta\zeta f'(\kappa_1)\left[\pi\frac{z(b)}{\zeta}A(y_1(b)) + (1-\pi)\frac{z(g)}{\zeta}\right] \\ \beta z(b) f'(\kappa_1)\kappa_1 &= h'(y_1(b))y_1(b) \end{aligned}$$

As $A(y_1(b)) > 1$, it follows from the first of these conditions that $\kappa_1 > \kappa^*$. ■

If the analog to condition [A1] holds in the Lagos and Rocheteau [6] economy, there is no over-accumulation of capital. When [A1] holds in the news-economy, however, there is an over-accumulation of capital. This over-accumulation is directly related to the stochastic news shock; which leads to a binding debt-constraint in bad-news events only. So, even the chance of being debt-constrained generates the same incentive to over-accumulate capital.

Consistent with the efficient-market hypothesis, equilibrium asset-prices $\phi(\eta)$ rapidly adjust to any new and relevant information; in particular,

Lemma 4 *In a news economy, the equilibrium price of capital at night satisfies* $\phi(b) < \phi(g)$.

Proof. In equilibrium, the debt-constraints imply $\phi(\eta)\kappa \geq y(\eta)$. Given Proposition 1, we get

$$\phi(g) \geq \frac{y^*}{\kappa_1} > \frac{y_1(b)}{\kappa_1} = \phi(b)$$

■

One consequence of this "informationally efficient" market structure is that it leads to an allocative inefficiency; see also Andolfatto [1]. A bad-news event here leads individuals to (rationally) revise downward their forecast of the future return to capital. In turn, this leads to a decline in the asset price, leaving consumers with insufficient purchasing power to acquire the first-best level of night-output.

Proposition 2 *In a news economy satisfying [A1], a nondisclosure policy (suppressing the news flow) is welfare-improving (the first-best allocation is implementable).*

If the nondisclosure of news is infeasible, then condition [A1] guarantees that the competitive equilibrium allocation described above is inefficient. This then opens the door to policy interventions that may improve welfare. Naturally, if we endow the government with enough coercive power and tax instruments, the policy-design problem becomes trivial. In what follows, we assume that the government can impose no penalties on individuals. While admittedly extreme, this allows us to focus on Pareto-improving policies that do not rely on any form of coercion. These considerations lead us to examine the role of government debt.

4 A Monetary Economy

The government can issue durable, divisible, and non-counterfeitable tokens that will henceforth be labeled *money*. Let M denote the aggregate supply of money and let $T \equiv M - M^-$ denote new money creation. Assume that the money supply grows at a constant rate $M = \mu M^-$; so that $T = [1 - 1/\mu]M$. New money is injected as a lump-sum transfer at the beginning of each day. As lump-sum taxation is prohibited, we have $\mu \geq 1$.

Let v_1 and v_2 denote the price of money measured in units of output in the day and night, respectively. Individuals enter the day with fiat money m_j; or in real terms $a_j \equiv v_1 m_j$. They leave the day with money m; or in real terms $q \equiv v_1 m$. Let $\tau \equiv v_1 T$.

4.1 Decision-Making

In the day, the choice-problem is described by

$$D(k_j, a_j, z) \equiv \max_{k^+, q} \left\{ r(z)k_j + w(z) + a_j - k^+ - q + \tau + E_\eta N(k^+, q, \eta) \right\}$$

Desired money and capital holdings are characterized by

$$\begin{aligned} 1 &= E_\eta N_1(k^+, q, \eta) && (22) \\ 1 &= E_\eta N_2(k^+, q, \eta) && (23) \end{aligned}$$

By the envelope theorem

$$\begin{aligned} D_1(k_j, a_j, z) &= z f'(\kappa) && (24) \\ D_2(k_j, a_j, z) &= 1 && (25) \end{aligned}$$

At night, the consumer's problem is

$$C(k^+, q, \eta) \equiv \max_{k_c^+, a_c^+} \left\{ u(c) + \beta E\left[D(k_c^+, a_c^+, z^+) | \eta \right] + \lambda k_c^+ + \psi a_c^+ \right\}$$

where

$$c = \phi(\eta)(k^+ - k_c^+) + \frac{v_2(\eta)}{v_1} \left(q - \frac{v_1}{v_1^+} a_c^+ \right)$$

The desired future asset position is characterized by

$$\begin{aligned} \phi(\eta) u'(c(\eta)) &= \beta z(\eta) f'(\kappa) + \lambda(\eta) && (26) \\ v_2(\eta) u'(c(\eta)) &= \beta v_1^+ + \psi(\eta) v_1^+ && (27) \end{aligned}$$

By the envelope theorem

$$\begin{aligned} C_1(k^+, q, \eta) &= \phi(\eta) u'(c(\eta)) && (28) \\ C_2(k^+, q, \eta) &= \frac{v_2(\eta)}{v_1} u'(c(\eta)) && (29) \end{aligned}$$

The producer's choice problem is

$$P(k^+, q, \eta) \equiv \max_{k_p^+, a_p^+} \left\{ -h(y) + \beta E\left[D(k_p^+, a_p^+, z^+) | \eta \right] \right\}$$

where

$$y = \phi(\eta)(k_p^+ - k^+) + \frac{v_2(\eta)}{v_1} \left(\frac{v_1}{v_1^+} a_p^+ - q \right)$$

The desired future asset position is characterized by

$$\begin{aligned}\phi(\eta)h'(y(\eta)) &= \beta z(\eta)f'(\kappa) \quad (30)\\ v_2(\eta)h'(y(\eta)) &= \beta v_1^+ \quad (31)\end{aligned}$$

By the envelope theorem

$$\begin{aligned}P_1(k^+,q,\eta) &= \phi(\eta)h'(y(\eta)) \quad (32)\\ P_2(k^+,q,\eta) &= \frac{v_2(\eta)}{v_1}h'(y(\eta)) \quad (33)\end{aligned}$$

As in an earlier section above, form the expression $N \equiv 0.5C + 0.5P$ and gather restrictions to derive

$$\begin{aligned}2 &= \pi\phi(b)\left[u'(c(b)) + h'(y(b))\right] + (1-\pi)\phi(g)\left[u'(c(g)) + h'(y(g))\right] \quad (34)\\ 2v_1 &= \pi v_2(b)\left[u'(c(b)) + h'(y(b))\right] + (1-\pi)v_2(g)\left[u'(c(g)) + h'(y(g))\right] \quad (35)\end{aligned}$$

Note that (34) is identical to (15) derived earlier.

4.2 Market Clearing

Define $Q \equiv v_1 M$. The government budget constraint is given by

$$\tau = \left[1 - \frac{1}{\mu}\right]Q \quad (36)$$

The market-clearing conditions are given by

$$\begin{aligned}k^+ &= \kappa^+ \quad (37)\\ c(\eta) &= y(\eta)\\ 0.5k_c^+(\eta) + 0.5k_p^+(\eta) &= \kappa^+\\ q &= Q\\ \mu\left[0.5a_c^+(\eta) + 0.5a_p^+(\eta)\right] &= Q^+\end{aligned}$$

4.3 Monetary Equilibrium

In a stationary equilibrium, $\kappa = \kappa^+$ and $Q = Q^+$. It follows that $v_1/v_1^+ = \mu$.

Gathering restrictions, a monetary equilibrium (if it exists) is characterized by

$$1 = \beta\zeta f'(\kappa)\left[\pi\frac{z(b)}{\zeta}A(y(b)) + (1-\pi)\frac{z(g)}{\zeta}A(y(g))\right] \quad (38)$$

$$1 = \left(\frac{\beta}{\mu}\right)\left[\pi A(y(b)) + (1-\pi)A(y(g))\right] \quad (39)$$

$$h'(y(\eta))y(\eta) \leq \beta z(\eta)f'(\kappa)\kappa + \left(\frac{\beta}{\mu}\right)q \quad (40)$$

Note that condition (38) is identical to condition (19). If a monetary equilibrium does not exist, then $q = 0$ and (39) satisfies $1 > \left(\frac{\beta}{\mu}\right)\left[\pi A(y(b)) + (1-\pi)A(y(g))\right]$. The remaining two conditions (38) and (40) are in this latter case equivalent to (19) and (20).

Proposition 3 *In a no-news economy, there does not exist a monetary equilibrium with $\mu \geq 1$.*

Proof. Consider a no-news economy, i.e., $z(\eta) = \zeta$ and suppose a monetary equilibrium exists, i.e., $q > 0$. Note that no news coupled with $\mu \geq 1$ imply $y(b) = y(g) = y < y^*$. Thus, conditions (38) and (39) become $1 = \mu\zeta f'(\kappa)$ and $1 = \frac{\beta}{\mu}A(y)$, respectively. This implies $\kappa > \kappa^*$ and strictly increasing in μ; and $y < y^*$ and strictly decreasing in μ. Condition (40) is held with equality and implies $q = \frac{\mu}{\beta}[h'(y)y - \beta\zeta f'(\kappa)\kappa]$. Mechanically, notice that if $\mu = \beta$, then $\kappa = \kappa^*$, $y = y^*$ and thus, by [A1] $q = 0$. Since κ is strictly increasing in μ and y is strictly decreasing in μ, it follows that $q < 0$ for any $\mu \geq 1$, a contradiction. ■

The intuition for the result above is that in a no news economy, capital as the only means of payment achieves the first-best. Thus, there are no individual gains from acquiring an asset dominated in rate of return, which means fiat money is not valued.

We now analyze the monetary equilibrium in a news economy.

Lemma 5 *In a news economy with money and $\mu \geq 1$, consumer debt-constraints cannot remain slack in both news states.*

Proof. Slackness in both states implies $y(\eta) = y^*$; so that $A(y(\eta)) = 1$. By (39), this can only be possible when $\mu = \beta$; which violates $\mu \geq 1$. ■

Lemma 6 *In a news economy with money, consumer debt-constraints cannot bind tightly in both news states.*

Proof. If debt-constraints bind in both states, then (40) implies

$$\begin{aligned} h'(y(b))y(b) &= \beta z(b)f'(\kappa)\kappa + \left(\frac{\beta}{\mu}\right)q \\ h'(y(g))y(g) &= \beta z(g)f'(\kappa)\kappa + \left(\frac{\beta}{\mu}\right)q \end{aligned}$$

Hence, $y(b) < y(g) < y^*$ and $A(y(b)) > A(y(g)) > 1$. It then follows from (38) that $\kappa > k^*$. Utilizing these latter two expressions, one may derive

$$\pi h'(y(b))y(b) + (1-\pi)h'(y(g))y(g) = \beta\zeta f'(\kappa)\kappa + \left(\frac{\beta}{\mu}\right)q$$

By [A1] and the properties of f, $\beta\zeta f'(\kappa)\kappa > \beta\zeta f'(k^*)k^* = h'(y^*)y^*$. As $q \geq 0$, this implies

$$\pi h'(y(b))y(b) + (1-\pi)h'(y(g))y(g) > h'(y^*)y^*$$

which contradicts the condition $y(b) < y(g) < y^*$. ■

Lemmas 5 and 6 imply that the debt-constraint will bind tightly in one news state and remain slack in the other. It is easy to verify that the constraint will bind in the bad news state. Label this equilibrium allocation $\{y_2(\eta), \kappa_2, q_2\}$. Since the debt-constraint binds in the bad-news state and remains slack in the good-news state, $y(b) < y(g) = y^*$. This, in turn, implies that $A(y(b)) > 1$ and $A(y(g)) = 1$. These conditions, together with (38), (39) and (40), imply that the competitive equilibrium allocation $\{y_2(\eta), \kappa_2, q_2\}$ is characterized by

$$1 = f'(\kappa_2)[\beta\zeta + (\mu - \beta)z(b)] \tag{41}$$

$$1 = \left(\frac{\beta}{\mu}\right)[\pi A(y_2(b)) + 1 - \pi] \tag{42}$$

$$h'(y_2(b))y_2(b) = \beta z(b)f'(\kappa_2)\kappa_2 + \left(\frac{\beta}{\mu}\right)q_2 \tag{43}$$

We need to establish the conditions under which fiat money coexists with capital. To this end, note that condition (42) determines $y_2(b)$ as a function of μ. Observe that $y_2(b)$ is strictly decreasing in μ. Condition (41) determines $\kappa_2 > \kappa^*$, given $\mu > \beta$. Moreover, note that κ_2 is strictly increasing in μ. With $y_2(b)$ and κ_2 so determined, condition (43) determines the demand for real money balances,

$$q_2 = \left(\frac{\mu}{\beta}\right)[h'(y_2(b))y_2(b) - \beta z(b)f'(\kappa_2)\kappa_2] \tag{44}$$

Fiat money coexists with capital if and only if $q_2 > 0$.

Assume momentarily that lump-sum taxation is feasible (so that any $\mu > \beta$ is possible). As $\mu \searrow \beta$, conditions (41) and (42) imply $\kappa_2 \searrow \kappa^*$ and $y_2(b) \nearrow y^*$, respectively; using [A1], condition (44) implies $q_2 \nearrow \kappa^*[1 - z(b)/\zeta]$. In short, if the Friedman rule is feasible, then our economy with outside money implements the first-best allocation and $q_2 > 0$. Now, given that $f'(k)k$ is increasing in k, and that $y_2(b)$ is decreasing while κ_2 increasing in μ, it follows from (44) that q_2 is decreasing in μ.

So, we have to this point established that $q_2 > 0$ in the neighborhood of the Friedman rule and that q_2 declines monotonically in μ. Evidently, there exists a large enough μ, say $\beta < \overline{\mu} \leq \infty$ that will guarantee $q_2 = 0$. The value $\overline{\mu}$ depends on parameters; for example, $\overline{\mu} \nearrow \infty$ as $z(b) \searrow 0$ and $\bar{\mu} \searrow \beta$ as $z(b) \nearrow z(g)$. In general, $\overline{\mu} \gtreqless 1$. When lump-sum taxation is infeasible, as we have assumed here, $\overline{\mu} < 1$ implies that fiat money cannot coexist with capital. In what follows then, we assume a parameterization such that the following is true.

[A2] The minimum (gross) inflation rate that generates a zero demand for real money balances, $\overline{\mu}$, is greater than unity.

Proposition 4 *In a news economy that satisfies [A1] and [A2], fiat money coexists with capital for any $1 \leq \mu < \overline{\mu}$. The monetary equilibrium exhibits the following properties: (i) $y_2(b) < y_2(g) = y^*$; (ii) $\kappa_2 > \kappa^*$; (iii) $y_2(b)$ strictly decreasing in μ and κ_2 strictly increasing in μ; and (iv) $\zeta f'(\kappa_2) > 1/\mu$.*

Proof. Coexistence and properties *(i)*—*(iii)* follow from the discussion preceding the proposition. To prove rate of return dominance, note that $\beta < \mu$ implies $\beta\zeta + (\mu - \beta)z(b) < \zeta\mu$. Thus, (41) implies $1/\mu < \zeta f'(\kappa_2)$. ■

The analysis to this point implies that in the monetary equilibrium: [a] there is an over-accumulation of capital; [b] there is a Tobin effect, in the sense that higher inflation stimulates capital expenditure; [c] both money and capital are equally liquid and used as a means of payment; and [d] capital dominates money in expected rate of return.

In relation to the literature, we make the following observations. Lagos and Rocheteau [6] demonstrate [a]-[c]; but achieve [d] by introducing a separate form of capital that is assumed to be illiquid.[4] Aruoba and Wright [2] demonstrate [d]; but do so with an exogenous restriction that limits the use of capital as a payment instrument. Moreover, in their model, the equilibrium capital stock is independent of the inflation rate.[5] In Stockman [9] and Aruoba, Wright, and Waller [3], inflation acts as a tax on capital accumulation; and rate of return dominance is achieved by imposing exogenous restrictions on the use of capital as a means of payment.

In contrast, we make no restrictions on the type of assets that may be used in making payments. Nevertheless, money in our environment not only coex-

[4] Lagos and Rocheteau [6] assume two storage technologies: one liquid, the other illiquid. Liquid capital earns the same return as fiat money, while illiquid capital dominates money in rate of return. Note that the money growth rate only affects the accumulation of liquid capital.

[5] The steady state capital stock is also invariant to inflation in Sidrauski [8]; and rate of return dominance there is achieved by assuming that money enters the utility function.

ists with a higher expected return asset, its existence is Pareto-improving (see Proposition 6 below). Rate of return dominance is explained by news shocks that make the value of capital (or capital backed assets) vary "too much" relative to what is desirable in a payment instrument. Money, in our model, offers a more stable rate return over the relevant (short-term) time horizon. To make this more clear, consider the following argument.

Define $R(\eta) \equiv v_2(\eta)/v_1$; that is, the *ex post* rate of return on money from day to night. The following proposition establishes some results for the rates of return on capital and money in our environment.

Proposition 5 *In a news economy with money that satisfies [A1] and [A2] with* $1 \leq \mu < \overline{\mu}$*, the monetary equilibrium exhibits the following properties: (i)* $\phi(b) < R(b)$*; (ii)* $\phi(g) > R(g)$*; (iii)* $\phi(b) < \phi(g)$*; (iv)* $R(b) = R(g)$ *if* $h''(y) = 0$ *and* $R(b) > R(g)$ *otherwise; (v)* $\frac{\phi(g)}{\phi(b)} > \frac{R(g)}{R(b)}$*; (vi)* $Cov(\phi(\eta), R(\eta)) = 0$ *if* $h''(y) = 0$ *and* $Cov(\phi(\eta), R(\eta)) < 0$ *otherwise.*

Proof. From conditions (30) and (31) we have

$$\begin{aligned}
\phi(\eta) &= \frac{\beta z(\eta) f'(\kappa)}{h'(y(\eta))} \\
R(\eta) &= \frac{\beta}{\mu h'(y(\eta))}
\end{aligned}$$

Rewrite conditions (38) and (39) as

$$\begin{aligned}
1 &= \beta\left[\pi z(b) f'(\kappa) A(y(b)) + (1-\pi) z(g) f'(\kappa)\right] \\
1 &= \beta\left[\pi \mu^{-1} A(y(b)) + (1-\pi)\mu^{-1}\right]
\end{aligned}$$

It follows that $(1-\pi)\left[z(g)f'(\kappa) - \mu^{-1}\right] = \pi\left[\mu^{-1} - z(b)f'(\kappa)\right] A(y(b))$; and this latter equality requires $z(b)f'(\kappa) < \frac{1}{\mu} < z(g)f'(\kappa)$. Parts *(i)* and *(ii)* follow.

Part *(iii)* follows from the debt-constraints; we have $\phi(g) \geq \frac{1}{\kappa}[y^* - \frac{\beta q}{\mu h'(y^*)}]$ and $\phi(b) = \frac{1}{\kappa}[y(b) - \frac{\beta q}{\mu h'(y(b))}]$. Thus,

$$\phi(g) - \phi(b) \geq \frac{1}{\kappa}\Big[y^* - y(b) + \frac{\beta q}{\mu}\Big(\frac{1}{h'(y(b))} - \frac{1}{h'(y^*)}\Big)\Big] > 0$$

Part *(iv)* follows from $\frac{R(b)}{R(g)} = \frac{h'(y^*)}{h'(y(b))}$. Thus, if $h(y)$ is linear, $R(b) = R(g)$. Otherwise, given $y(b) < y^*$, $R(b) > R(g)$.

Part *(v)* follows from *(i)* and *(ii)*.

For *(vi)*, use $Cov(\phi(\eta), R(\eta)) = E[\phi(\eta)R(\eta)] - E[\phi(\eta)]E[R(\eta)]$. Thus,

$$Cov(\phi(\eta), R(\eta)) = \frac{\beta^2 \pi (1-\pi) f'(\kappa)[h'(y^*) - h'(y(b))][z(b)h'(y^*) - z(g)h'(y(b))]}{\mu h'(y(b))^2 h'(y^*)^2}$$

If $h(y)$ is linear then $h'(y^*) - h'(y(b)) = 0$ and the covariance is zero. Suppose now $h''(y) > 0$. Since $h'(y^*) - h'(y(b)) > 0$, the sign of the covariance is equal to the sign of $z(b)h'(y^*) - z(g)h'(y(b))$. From (4) and [A1] we have $h'(y^*) = \frac{\beta\zeta f'^*)\kappa^*}{y^*}$, and from (43) we have $h'(y(b)) \geq \frac{\beta z(b) f'(\kappa)\kappa}{y(b)}$. Thus,

$$z(b)h'(y^*) - z(g)h'(y(b)) \leq \beta z(b) \left\{ \frac{\zeta f'(\kappa^*)\kappa^*}{y^*} - \frac{z(g) f'(\kappa)\kappa}{y(b)} \right\} < 0$$

■

Results *(i)*—*(iv)* in the proposition above establish the relationship between the *ex post* rates of return on capital and money. As in the case with no fiat money, $\phi(b) < \phi(g)$. This just follows from capital being more valuable when the news is good. In contrast, money features a higher *ex post* return when the news is bad, i.e., $R(b) > R(g)$—unless $h(y)$ is linear, in which case the returns are equal. Fiat money is in higher demand when news is bad.

Result *(v)* shows that, for any parametrization consistent with a monetary equilibrium, the *ex post* rate of return on capital is more "volatile" than the *ex post* return on money. Consider the ratio $\frac{\phi(g)/\phi(b)}{R(g)/R(b)}$, which is a measure of the volatility of the *ex post* return on capital relative to money. After some simple algebra, we get that this ratio is equal to $\frac{z(g)}{z(b)}$. Thus, the relative volatility of the *ex post* returns of the two assets depends only on the distance between $z(b)$ and $z(g)$. Notably, it does not depend on the money growth rate.

Finally, result *(vi)* shows that when $h(y)$ is strictly convex, the covariance of *ex post* returns on capital and money is negative. This result can be understood in terms of the preceding discussion. When $h(y)$ is linear, the *ex post* returns are uncorrelated, since the *ex post* return on money is constant.

5 Welfare

5.1 Inflation

Proposition 6 *In a news economy with money that satisfies [A1] and [A2] with* $1 \leq \mu < \bar{\mu}$*: (i) ex ante welfare is strictly decreasing in* μ*; and (ii) ex ante welfare is higher than in the non-monetary economy.*

Proof. Part *(i)* follows from Proposition 4, part *(iii)*. For part *(ii)*, note that when $\mu = \bar{\mu}$, $q = 0$ and the equilibrium allocation coincides with the non-monetary economy. Given part *(i)*, for $\mu \in [1, \bar{\mu})$, welfare in the monetary equilibrium is strictly higher than in the non-monetary economy. ■

The proposition above implies that the constrained-efficient monetary policy here is to choose a zero (expected) inflation.[6] Furthermore, whenever a

[6] Because preferences are quasilinear, stochastic variation in the rate of money growth has no *ex ante* welfare consequences.

monetary equilibrium with $\mu \geq 1$ exists, the presence of fiat money improves welfare.

5.2 Endogenous Capital Illiquidity

Aruoba and Wright [2] and Aruoba, Wright and Waller [3] assume that capital is illiquid (it cannot be used to make payments at night). This sort of "cash-in-advance" (CIA) constraint is frequently imposed in the literature for the purpose of generating a demand for fiat money that is dominated in rate of return. As pointed out in Lagos and Rocheteau [8], however, a CIA constraint of this form is welfare-improving. Here, this result does not hold in general.

Suppose capital cannot be used as a means of payment at night. The monetary equilibrium is characterized by

$$
\begin{aligned}
1 &= \beta\zeta f'(\kappa) && (45)\\
1 &= \left(\frac{\beta}{\mu}\right)[\pi A(y(b)) + (1-\pi)A(y(g))] && (46)\\
h'(y(\eta))y(\eta) &\leq \left(\frac{\beta}{\mu}\right) q && (47)
\end{aligned}
$$

Note that is this case, a monetary equilibrium exists for any finite μ, i.e., $\bar{\mu} = \infty$.

Lemma 7 *In a news economy with illiquid capital and money with $\mu \geq 1$, $y(b) = y(g) < y^*$ and $\kappa = \kappa^*$.*

Proof. Condition (45) implies $\kappa = \kappa^*$. From (46), we cannot have $y(b) = y(g) = y^*$ for $\mu > \beta$. Suppose that for some η, $y(\eta) = y^*$. Then, for the other state, (47) is held with strict inequality, i.e., the cash constraint is necessarily slack and we get y^* as well, a contradiction. Thus, $y(b) < y^*$ and $y(g) < y^*$. Since (47) is held with equality in both states and $h'(y)y$ is strictly increasing in y, we get $y(b) = y(g)$. ■

If capital cannot be used as a means of payment, then using (45)—(47) and Lemma 7, the equilibrium allocation $\{y_3, \kappa_3, q_3\}$ is characterized by

$$
\begin{aligned}
\kappa_3 &= \kappa^* && (48)\\
\frac{u'(y_3)}{h'(y_3)} &= \frac{\mu}{\beta} - 1 && (49)\\
q_3 &= y_3[u'(y_3) + h'(y_3)] && (50)
\end{aligned}
$$

Proposition 7 *In a news economy that satisfies [A1] and [A2], restricting the use of capital as a means of payment at night has an ambiguous welfare effect.*

Proof. From equations (41), (42), (48) and (49), we have $\kappa_2 > \kappa_3 = \kappa^*$ and $y_2(b) < y_3 < y_2(g) = y^*$. Thus, restricting capital as a means of payment eliminates the overaccumulation of capital (i.e., consumption during the day increases), but has an ambiguous effect on consumption at night. ■

We complete the proof with an example. Due to the linear preferences in the day, we can look at average consumption in the day. Period expected utility is

$$f(\kappa) - \kappa + 0.5[\pi(u(y(b)) - h(y(b)) + (1-\pi)(u(y(g)) - h(y(g))]$$

Assume $u(c) = \ln c$, $h(y) = y^2/2$ and $f(\kappa) = \kappa^\alpha$, which implies $y^* = \kappa^* = 1$, $y_2(b) = \sqrt{\frac{\pi\beta}{\pi\beta+2(\mu-\beta)}}$ and $\kappa_2 = [\alpha\mu z(b) + \alpha\beta(1-\pi)(z(g) - z(b))]^{\frac{1}{1-\alpha}}$.

When both capital and money are used as means of payment, period expected utility is

$$[\alpha\beta(1-\pi)(z(g) - z(b)) + \alpha\mu z(b)]^{\frac{\alpha}{1-\alpha}} - [\alpha\beta(1-\pi)(z(g) - z(b)) + \alpha\mu z(b)]^{\frac{1}{1-\alpha}}$$
$$- \frac{1}{4}\Big\{1 - \pi + \frac{\pi^2\beta}{2\mu - (2-\pi)\beta} - \ln\frac{\pi^2\beta}{2\mu - (2-\pi)\beta}\Big\}$$

When capital cannot be used as a means of payment, the period expected utility is

$$-\frac{1}{4}\Big\{\frac{\beta}{2\mu - \beta} - \ln\beta + \ln(2\mu - \beta)\Big\}$$

Assume $z(g) = z + \epsilon$, $z(b) = z - \epsilon$ and to satisfy [A1], set $z = \frac{1}{\alpha\beta} - \epsilon(1 - 2\pi)$. For parameters, assume $\alpha = 1/3$, $\beta = 0.9$, $\pi = 0.2$ and $\epsilon = 2$. Let $\Delta(\mu)$ be the difference in expected utility between the cases with only fiat money and with money and capital as means of payment, as a function of the money growth rate. For our parametrization, we get $\Delta(1) = 0.0108$ and $\Delta(2.5) = -0.0111$ (note $\bar{\mu} = 2.9634$). In other words, for this example, restricting capital as a means of payment is welfare improving for low money growth rates and welfare reducing for high money growth rates. See Figure 1.

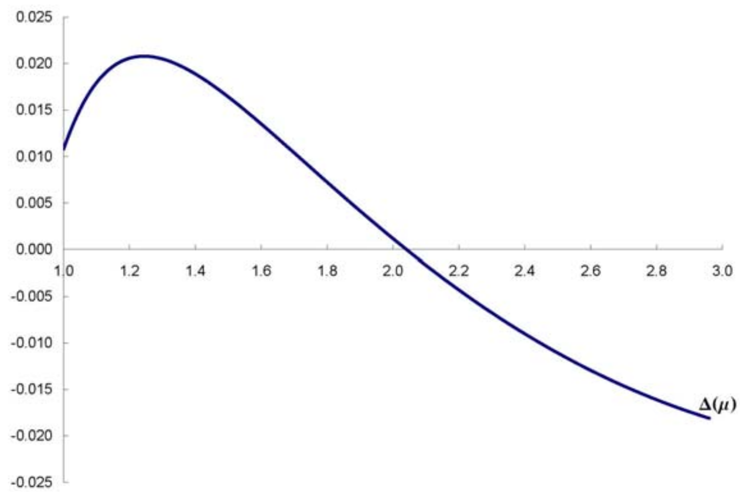
We now rely on numerical methods to provide a better idea of the welfare effects of restricting capital as a means of payment. Using the functional forms from the example above, we conduct the following simulation. First, fix the value of μ. Second, take a random draw for $\{\alpha, \beta, \pi\}$. Third, create a grid for ϵ that takes values between 0 (i.e., no news) and $\frac{1}{2\alpha\beta(1-\pi)}$ (i.e., $z(b) = 0$). We use 102 grid points and drop the first and last elements of the grid. For each of these parameterizations $\{\mu, \alpha, \beta, \pi, \epsilon\}$ we verify if $q > 0$. It it is, then we calculate $\Delta(\mu)$, i.e., the difference in welfare between the cases with only money and with money and capital as means of payment. For the fixed μ, we repeat this sequence until we get $10,000,000$ parameterizations with $q > 0$ and calculate the fraction of cases for which restricting capital as a means of payment is welfare improving (i.e., for which $\Delta(\mu) > 0$).[7] For $\mu = 1$, which is the constrained-efficient policy, restricting capital as means of payment improves welfare in 70% of admissible

[7] The results are not significantly different if we only simulate the model for $100,000$ parameterizations.

parameterizations. Increasing μ lowers this fraction: for μ equal to 1.25, 1.5 and 2, the number is 52%, 36% and 15%, respectively.

FIGURE 1

Welfare gain from restricting the use of capital as a function of μ



The result here is reminiscent of Kocherlakota [5], where restricting the liquidity properties of bonds improves allocative efficiency. In the context of our environment, the result appears related to the claim made in Proposition 2; namely, that restricting the news flow is welfare-improving. Capital constitutes a poor medium of exchange as its value fluctuates at high-frequency in response to news, leaving some individuals debt-constrained when the news is bad. The value of fiat money also varies in response to news; but this is because it must compete with capital as a means of payment at night. This competition can be eliminated by a restriction that prohibits the use of capital as a means of payment. When this is so, the value of money is rendered "informationally insensitive;" its expected return can be stabilized independent of the news flow.

6 Conclusion

Our model highlights an inherent drawback in the use of private securities backed by capital as a medium of exchange. The problem is that if these securities circulate, their values will fluctuate at high-frequency—like any other traded security—in response to news. While this is not necessarily a bad property for any asset to possess, it hinders the value of any such asset as a medium of exchange. This observation may rationalize the widespread use of debt (rather than equity) as a payment instrument. To the extent that the government sector now has an advantage in creating debt that is insensitive to news, our analysis suggests a rationale for the recent emergence of fiat money systems.

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