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Pengfei Wang
and
Yi Wen

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FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

Imperfect Competition and Sunspots*

Pengfei Wang
Department of Economics
Cornell University

Yi Wen
Research Department
Federal Reserve Bank of St. Louis

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Abstract

This paper shows that imperfect competition can be a rich source of sunspots equilibria and coordination failures. This is demonstrated in a dynamic general equilibrium model that has no major distortions except imperfect competition. In the absence of fundamental shocks, the model has a unique certainty (fundamental) equilibrium. But there is also a continuum of stochastic (sunspots) equilibria that are not mere randomizations over fundamental equilibria. Markup is always counter-cyclical in sunspots equilibria, which is consistent with empirical evidence. The paper provides a justification for exogenous variations over time in desired markups, which play an important role as a source of cost-push shocks in the monetary policy literature. We show that fluctuations driven by self-fulfilling expectations (or sunspots) look very similar to fluctuations driven by technology shocks, and we prove that such fluctuations are welfare reducing.

Keywords: Sunspots, Self-fulfilling Expectations, Imperfect Competition, Imperfect Information, Indeterminacy, Real Business Cycles, Marginal Costs, Counter-cyclical Markup.

JEL codes: E31, E32.

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1 Introduction

The work of Benhabib and Farmer (1994) has triggered a fast growing interest in studying expectations-driven fluctuations. The major reason for this is that it makes quantitative analysis of the business-cycle effects of sunspots possible within the popular framework of Kydland and Prescott (1982).[1] The Benhabib-Farmer model can be cast in two versions, one with production externalities or external increasing returns to scale, and one with monopolistic competition with internal increasing returns to scale at the firm level. Benhabib and Farmer show that if the returns to scale are sufficiently larger than one – either at the aggregate level or at the firm level – then the steady state of the model can become indeterminate and there can be infinitely many equilibrium paths converging to the same steady state. This multiplicity of equilibria can give rise to fluctuations driven by purely extrinsic uncertainty (sunspots) or self-fulfilling expectations in this class of dynamic stochastic general equilibrium (DSGE) models.

The Benhabib-Farmer model has been criticized by many as unrealistic because it relies on returns to scale that are empirically implausible. Later developments in this literature, however, have been able to bring the degree of returns-to-scale required for indeterminacy down to an empirically plausible range (see, e.g., Benhabib and Farmer, 1996, and Wen, 1998a). Despite this, indeterminacy in these types of models is still sensitive to the values of other structural parameters, such as the output elasticity of capital and the elasticity of labor supply (e.g., it requires the elasticity of labor supply to be close to infinity). This is so because these structural parameters and returns to scale jointly affect the eigenvalues of the model. Local indeterminacy would not arise in this class of models, for example, if there were adjustment costs in capital or labor (see Georges 1995, and Wen 1998b).[2] Due to these restrictions on structural parameters in the Benhabib-Farmer model, the Keynesian idea of fluctuations driven by animal spirits (or sunspots) has not gained sufficient popularity in the real business cycle (RBC) circle, where technology shocks are still viewed as the dominate force behind economic fluctuations.

This paper shows that there exists an entirely different source of sunspots equilibria in the Benhabib-Farmer model. This new source of sunspots equilibria can arise naturally in standard dynamic stochastic general equilibrium models, and is robust to the choice of parameter values in production technologies and utility functions. In other words, we show that in order to generate sunspots equilibria in the class of infinite-horizon, calibrated RBC (or DSGE) models, returns to scale are completely irrelevant and unnecessary. The key condition required for sunspots equilibria,

[1] For a literature review, see Benhabib and Farmer (1999). For early contributions to the sunspots literature, see Shell (1977), Azariadis (1981), Cass and Shell (1983), and Woodford (1986), among others.

[2] A common feature of the requirements on the parameter values for indeterminacy is that they must facilitate greater flexibility of factor mobility and enhance the temporal/intertemporal substitutability of factors, so that the short-run returns to labor can exceed one. Adjustment costs tend to work against these effects. Gali (1994) presents a different model that does not rely on increasing returns to scale to generate local indeterminacy. However, the parameter region of indeterminacy in Gali's model is also sensitive to the values of structural parameters.

or for expectations to be self-fulfilling, is imperfect competition with differentiated products. As is shown by Blanchard and Kiyotaki (1987), product differentiation can lead to externalities of its own – demand externalities.[3] We show that this type of demand externality is a rich source of sunspots equilibria under imperfect competition. Furthermore, we show that fluctuations driven by sunspots behave very much like those driven by technology shocks.[4]

The nature of the sunspots equilibria arising under the demand externalities differs from that arising under the production externalities studied by Benhabib and Farmer in two important aspects: 1) sunspots are global instead of local, hence they have nothing to do with the topological properties of the steady state, as they do not require the steady state to be a sink as in Benhabib and Farmer (1994); 2) sunspots equilibria under our specification are not based on mere randomizations over fundamental equilibria, in sharp contrast to the sunspots equilibria studied in the literature using the Benhabib-Farmer framework.[5]

Because of property (1), sunspots are more pervasive than is currently known since multiplicity of equilibria no longer depends on the structural parameters that affect the eigenvalues of the model, as in the Benhabib-Farmer model. Consequently, the multiplicity is robust – any economic environment featuring the Dixit-Stiglitz type of imperfect competition is a potential source of sunspots and hence can be subject to sunspots-driven fluctuations, regardless of utility functions and production technologies. Because of property (2), sunspots equilibria can arise in a dynamic model with a unique saddle-path fundamental equilibrium. This property is important because it makes the classic Keynesian notion of animal spirits more convincing. In this regard, our finding extends the classical analysis of sunspots equilibria by Cass and Shell (1983) into a new dimension – the analysis of sunspots-driven fluctuations can be conducted in standard, infinite-horizon RBC/DSGE models with constant returns to scale and a unique fundamental equilibrium.

The intuition behind our findings is simple. When there are multiple firms in the economy and each firm's optimal action depends on other firms' actions, there exists strategic complementarity among firms' behaviors. Such strategic complementarity can lead to multiple equilibria and coordination failures, as is shown by Cooper and John (1988). Strategic complementarity arises naturally in the Dixit-Stiglitz type of imperfect competition models due to the imperfect substitutability among the intermediate goods produced by monopolistic firms, which gives rise to an endogenous demand externality (as argued by Blanchard and Kiyotaki, 1987). The externality is endogenous because it is not imposed from outside as in the Benhabib-Farmer framework. Since

[3] This refers to the case where demand for an individual firm's output depends on demand for other firms' output due to imperfect substitutability among firms' output.

[4] Since perfect competition can be represented as a limiting case of imperfect competition, our results also apply to models with perfect (or near-perfect) competition, as in the set up of Alvarez and Lucas (2004). See Proposition 1 in the text.

[5] See, e.g., Farmer and Guo (1994).

many representative-agent DSGE models can be easily mapped into models with decentralized decision making featuring the Dixit-Stiglitz type of competition among firms,[6] sunspots equilibria can therefore exist in a wide class of real business cycle models as well as in Keynesian sticky-price models, including the monopolistic competition version of the Benhabib-Farmer model (regardless of the production externalities or increasing returns to scale).

The demand externality is a necessary but not sufficient condition for the rise of sunspots equilibria in models with imperfect competition. Another crucial condition needed for sunspots equilibria is imperfect information. That is, individual monopolistic firms make price decisions without knowing the level of aggregate demand – because they do not know how the other firms will set their prices. Hence, firms must each form expectations about the aggregate conditions of the economy when setting prices. Such expectations can be self-fulfilling because of the strategic complementarities among firms' actions. Monopolistic competition has been studied extensively in the literature. The reason that the possibility of sunspots equilibria has gone unnoticed in the literature is that standard DSGE models of imperfect competition always implicitly assume perfect information – that each firm knows precisely the level of aggregate demand when setting its prices. This assumption of perfect information rules out multiple-sunspots Nash equilibria. However, if firms each choose a price (taking as given the prices set by other firms) and quantities of demand are subsequently determined at these prices, then it is more natural to assume that prices are set based on expected demand, not on realized demand.[7]

Our findings can be viewed as a natural extension of the analysis of Cooper and John (1988). Our main contribution in this paper, compared to Cooper and John, is to show the existence of multiple equilibria in calibrated, infinite horizon, DSGE models. This class of models is the workhorse of theoretical and applied macroeconomics in the study of business cycles and monetary policy. Although the Dixit-Stiglitz (1977) imperfect competition model belongs to the class of models considered by Cooper and John in a broad sense, they do not analyze the conditions of multiple equilibria in a dynamic-general-equilibrium framework. The theorems proposed by Cooper and John for multiple equilibria are therefore insufficient for characterizing the existence of multiple equilibria in the class of dynamic models we study. For example, the models we study meet the conditions for multiple fundamental equilibria specified by Cooper and John, yet we show that the fundamental equilibrium is nonetheless unique in our models. The only type of sunspots equilibria that can arise in the models studied by Cooper and John are randomizations over fundamental equilibria, whereas sunspots equilibria studied in this paper are not mere randomizations over fundamental equilibria. As such, Cooper and John do not prove the existence of multiple equilibria

[6] See Proposition 2 in this paper.

[7] We are not the first to link imperfect competition and imperfect information to sunspots equilibria. For the early literature, see Peck and Shell (1991), Woodford (1991), and Gali (1994), among others.

in the class of dynamic models we study and do not show how to construct stochastic sunspots equilibria in an environment where there is a unique fundamental equilibrium.[8]

Our findings provide an interesting contribution to the literature because the Dixit-Stiglitz imperfect competition framework is widely used in economic analysis. The fast growing New Keynesian sticky-price literature is just one of the many noticeable areas that relies on this framework for business-cycle studies and monetary policy analyses. Yet this literature has been assuming unique equilibrium all the way along, while in fact there may be multiple equilibria and severe coordination-failure problems in such models. In addition, since the most celebrated neoclassical business-cycle model of Kydland and Prescott (1982) can be cast as a limiting case of the Dixit-Stiglitz imperfect competition model, and since fluctuations driven by technology shocks look similar to those driven by sunspots shocks, business cycles in general may not be the optimal response of the market economy to exogenous shocks (as accepted by the RBC literature), but instead may be fluctuations driven by self-fulfilling expectations and coordination failures. Such fluctuations are highly inefficient, as we prove in this paper. Hence, designing policies that can dampen fluctuations and prevent the coordination failures is central to business cycle studies, and such analysis can be conducted using the framework we provide in this paper.

Another important contribution of the paper is that it provides a justification for exogenous variations over time in desired markups, which play an important role as a source of cost-push shocks in the monetary policy literature (see, e.g., Clarida, Gali, and Gertler, 1999 and 2001; Walsh, 1999; Steinsson, 2003; Ravenna and Walsh, 2004; and Lane, Devereux, and Xu, 2005, among many others). This literature shows that there exist inflation-output trade-offs for the design of optimal monetary policies when there are independent shocks to firms' marginal costs or markups. To justify autonomous shifts in firms' marginal costs, the literature has been relying mainly on the *ad hoc* assumption that the elasticity of substitution across intermediate inputs in the final-good production technology is random. This paper shows that marginal costs can be driven by purely extrinsic uncertainty or sunspots without changes in the fundamentals, hence providing a justification for the independent random movements in marginal costs.

The rest of the paper is organized as follows. Section 2 presents a general equilibrium model of imperfect competition and proves the uniqueness of the fundamental equilibrium in the model. Section 2 shows the existence of sunspots equilibria, and that fluctuations driven by sunspots behave similar to those driven by technology shocks. Section 4 studies the welfare properties of

[8] A fundamental equilibrium is an equilibrium in the absence of extrinsic uncertainty. Blanchard and Kiyotaki are able to construct multiple fundamental equilibria in a model similar to ours under the additional assumption of menu costs. Kiyotaki (1988) also uses a similar set up with Dixit-Stiglitz imperfect competition to generate multiple fundamental equilibria in a two-period dynamic setting. However, Kiyotaki requires increasing-returns-to-scale as a necessary condition for multiple equilibria. An important distinction between our paper and this literature is that the type of sunspots equilibria we construct are not mere randomizations over fundamental equilibria. There is always a unique fundamental equilibrium in our model and we do not rely on menu costs or increasing returns to scale to construct sunspots equilibria.

sunspots equilibria and proves that sunspots-driven fluctuations are inefficient. Section 5 studies the robustness of sunspots equilibria in models with money and sticky prices. Finally, Section 6 concludes the paper.

2 The Benchmark Model

2.1 Firms

Suppose there is a continuum of intermediate good producers indexed by $i \,\epsilon[0,1]$, with each producing a single differentiated good $Y(i)$. The price of $Y(i)$ is denoted $P(i)$. These intermediate goods are used as inputs to produce a final good according to the technology,

$$Y = \left(\int_0^1 Y(i)^{\frac{\sigma-1}{\sigma}}(i)di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$ measures the elasticity of substitution among the intermediate goods. The final good industry is assumed to be perfectly competitive. The price of the final good, P, is normalized to one. Profit maximization by the final good producer yields the demand function for intermediate goods:

$$Y(i) = P(i)^{-\sigma} Y. \tag{2}$$

Notice that the demand for good i depends not only on the relative price of the good, but also on the aggregate demand Y. There are thus demand externalities in the model as pointed out by Blanchard and Kiyotaki (1987). It is important to emphasize that the demand externality arises endogenously within the model due to the complementarity of production factors (intermediate goods) in the final good industry, as opposed to being exogenously imposed from outside as in the case of the production externalities assumed in the Benhabib-Farmer model. Substituting the demand functions into the final-good production function (1) gives the aggregate price index, $P(=1) = \left(\int_0^1 P(i)^{1-\sigma} di \right)^{1/(1-\sigma)}$.

The production technology for intermediate goods has constant returns to scale and is given by

$$Y(i) = K(i)^{\alpha} N(i)^{1-\alpha}. \tag{3}$$

Intermediate good producers have monopoly power in the output market but are perfectly competitive in the factor markets. Given the production technology, the cost function of an intermediate good firm can be derived by solving a cost-minimization problem, $\min \{WN(i) + RK(i)\}$, subject to $K(i)^{\alpha} N(i)^{1-\alpha} \geq Y(i)$, where W and R denote the real wage and the real rental rate, respectively.

Letting ϕ denote the marginal cost (which is the Lagrangian multiplier for the constraint of the firm's cost minimization problem), we have the factor demand functions for labor and capital for each firm i: $W = (1-\alpha)\phi(i)\frac{Y(i)}{N(i)}$ and $R = \alpha\phi(i)\frac{Y(i)}{K(i)}$. Combining these factor demand functions and the production function (1), we have $\phi = \left(\frac{W}{1-\alpha}\right)^{1-\alpha}\left(\frac{R}{\alpha}\right)^{\alpha}$ as the unit cost function of the firm. This shows that marginal cost is the same across all firms in the model. Since ϕ is the shadow cost of increasing firm i's output by one unit, in general equilibrium its correlation with aggregate demand is nonnegative: $cov(\phi, Y) \geq 0$.

A key feature of the model is that intermediate good firms each choose a price while taking as given the prices set by other firms, with quantities being then determined by demand at these prices in general equilibrium. This sequential feature of the model permits imperfect information. This is so because, in each period t, intermediate good firms must set prices without knowing the aggregate economic conditions (such as aggregate demand) that may prevail in period t. These aggregate economic conditions depend crucially on the actions of the other firms over which an individual firm has no influence. Figure 1 illustrates the sequence of events in the model economy.

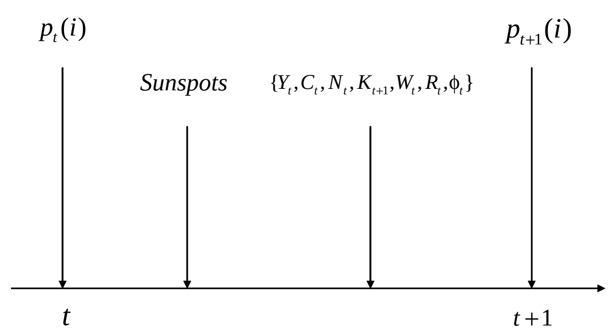

Fig. 1. Timing of Sunspots.

Define $\tilde{\Omega}_t$ as the information set available to price setting firms in period t, which includes the entire history of the economy up to period t except the realizations of sunspots (if any) in period t. Denote Ω_t as the information set that includes $\tilde{\Omega}_t$ and any realization of sunspots in period t. Thus we have $\Omega_t \supseteq \tilde{\Omega}_t \supseteq \Omega_{t-1}$. Since we do not consider fundamental shocks in this paper, we have

$\tilde{\Omega}_t = \Omega_{t-1}$. Extension of the analysis to including fundamental shocks is straightforward.[9] Based on this definition of information sets, each individual firm i chooses price $P_t(i)$ in the beginning of period t to maximize expected profits by solving

$$\max E_{t-1}\left\{(P_t(i) - \phi_t)\, Y_t(i)\right\}, \tag{4}$$

subject to the downward sloping demand function, $Y(i) = \left(\frac{P(i)}{P}\right)^{-\sigma} Y$, where E_{t-1} denotes expectations conditioned on the information set $\Omega_{t-1}(= \tilde{\Omega}_t)$.[10]

The optimal monopolistic price is given by

$$P_t(i) = \frac{\sigma}{\sigma - 1}\frac{E_{t-1}(\phi_t Y_t)}{E_{t-1} Y_t}, \tag{5}$$

where $\frac{\sigma}{\sigma-1} \geq 1$. In the limiting case where $\sigma \to \infty$, the model converges to a perfectly competitive economy. Our analysis of sunspots equilibria is independent of σ, hence it applies equally to perfectly (or near-perfectly) competitive economies where firms set prices equal to marginal cost with zero markup in the steady-state. The optimal pricing rule shows that an individual firm sets prices according the expected aggregate cost-to-revenue ratio, where both the firm's revenue and its costs depend on expected aggregate demand. Equation (5) indicates a potential source of multiple Nash-sunspots equilibria. To see this, suppose we close the model by specifying an aggregate labor supply curve, $N_t = N(W_t)$, which implies that in general equilibrium the aggregate output is a function of the marginal cost (since the real wage depends on the marginal cost), $Y_t = Y(\phi_t, K_t)$. In a symmetric equilibrium, $P(i) = P = 1$, and assuming that the aggregate capital stock is known to firms at the moment of price setting, Equation (1) becomes

$$\frac{\sigma}{\sigma - 1}\frac{E_{t-1}\left[\phi_t Y(\phi_t, K)\right]}{E_{t-1} Y(\phi_t, K)} = 1. \tag{5'}$$

This equation may permit sunspots solutions for the marginal cost, $\phi_t(K)$, which are not mere randomizations of fundamental equilibria. Suppose there is no extrinsic uncertainty (i.e., there is perfect information about the level of aggregate demand, Y); then Equation (5′) implies $\frac{\sigma}{\sigma-1} E_{t-1}\phi_t = 1$. Since by assumption $Y(\phi_t, K)$ is known to the firms at the time of price setting, the marginal cost ϕ_t must also be known to the firms in period t based on the information set Ω_{t-1}. Hence a

[9] Notice that our definition of information sets does not necessarily imply sticky prices. Prices can be allowed to respond immediately to any fundamental shocks in the model. That is, the information set $\tilde{\Omega}_t$ can include fundamental shocks realized in period t. As such, prices can respond to money shocks one for one. However, with respect to sunspots shocks, the price setting behavior in our model is in spirit similar to the sticky price literature (e.g., Yun 1996) and the sticky information literature (e.g., Mankiw and Reis, 2002). In reality, it is often the case that firms set prices before observing demand and demand decisions are often made based on observed prices.

[10] The reason that an individual firm needs to form expectations when maximizing profits is because the profits depend on the aggregate demand, which in turn depends on the prices set by other firms in the economy.

constant marginal cost, $\phi = \frac{\sigma-1}{\sigma}$, is the only fundamental-equilibrium solution to Equation (5′). Given ϕ, aggregate demand is then fully determined in period t at the level $Y(\frac{\sigma-1}{\sigma}, K)$. However, with extrinsic uncertainty or imperfect information, a random process ϕ_t could also constitute a solution for Equation (5′). To see this, let Y be a separable function, $Y(\phi, K) = g(\phi)f(K)$. Then Equation (5′) becomes $\frac{\sigma}{\sigma-1}\frac{E[\phi g(\phi)]}{Eg(\phi)} = 1$. This implies $Eg(\phi)\left[E\phi - \frac{\sigma-1}{\sigma}\right] = -cov(\phi, g(\phi)) \leq 0$.[11] Hence, any random process $\{\phi_t\}$ satisfying $E\phi \leq \frac{\sigma-1}{\sigma}$ may constitute an equilibrium in which the level of aggregate demand is given by $Y(\phi_t, K)$.

The source of the stochastic multiple equilibria for the marginal cost comes from the fact that each individual firm does not know how the other firms will behave, and hence must form expectations for the status of the aggregate economy when making price decisions. Due to the endogenous demand externalities among firms' actions, such expectations can be self-fulfilling. This possibility is analyzed in more detail in what follows.

2.2 Households

Suppose there is a representative household in the economy, whose objective is to maximize its lifetime utility,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \qquad 0 < \beta < 1, \tag{6}$$

where c and l are the household's consumption and leisure time, $u(\cdot, \cdot)$ is an increasing and concave period-utility function, and β is the time discount rate. The household is endowed with one unit of time in each period, so its labor supply in each period is $n = 1 - l$. The household's budget constraint in period t is given by

$$c_t + k_{t+1} = W_t n_t + (1 + R_t - \delta)k_t + D_t, \tag{7}$$

where k_t is the household's existing stock of capital, which depreciates at the rate $\delta \in (0, 1]$, $W_t n_t + R_t k_t$ is the household's real income from labor and renting capital to firms, and D_t is the real profits distributed from firms.

2.3 General Equilibrium

A general equilibrium is defined as the set of prices and quantities, $\{W, R, \phi, P(i), Y(i), N(i), K(i), c, k, n\}$, such that firms maximize profits and the household maximizes utility subject to their

[11] The marginal cost ϕ also measures the efficiency of the aggregate economy. A higher value of ϕ implies less monopolistic distortion to the economy and thus more output. Hence $cov(\phi, Y) \geq 0$, or likewise $\frac{\partial Y}{\partial \phi} \geq 0$.

respective technological and budget constraints, and all markets clear:

$$n_t = N_t \equiv \int N_t(i)di \tag{8}$$

$$k_t = K_t \equiv \int K_t(i)di \tag{9}$$

$$c_t + K_{t+1} - (1-\delta)K_t = Y_t. \tag{10}$$

We analyze symmetric equilibria where all intermediate good firms choose the same prices and produce the same equilibrium quantities. The sunspots equilibria we construct are pure Nash equilibria.

Since all firms face the same problem in each period, they set the same prices according to (5). Hence, in a symmetric equilibrium, we have $P(i) = 1$ and $\frac{\sigma}{\sigma-1}\frac{E_{t-1}(\phi_t Y_t)}{E_{t-1}Y_t} = 1$. Given that $P_t(i) = 1$ in equilibrium, the supply function for firm i becomes

$$Y_t(i) = Y_t, \tag{11}$$

which is the same across all firms. That is, in equilibrium, how much each individual firm produces depends on how much the rest of the economy produces. This type of externality can give rise to multiple equilibria because each firm may opt to produce more if they all expect that the other firms will produce more. That is, expectations can be self-fulfilling.

However, as will become clear shortly, the demand externality is a necessary but not sufficient condition for multiple equilibria in the dynamic general equilibrium model. In order to obtain multiple equilibria in the model, we need another key element – imperfect information. That is, firms determine prices without knowing the prices set by other firms and hence must form expectations for the equilibrium marginal cost and aggregate demand, which serve as indicators for the aggregate economic conditions.

The equilibrium conditions can be summarized by the following seven equations:

$$\frac{\sigma}{\sigma-1}\frac{E_{t-1}\phi_t Y_t}{E_{t-1}Y_t} = 1 \tag{12}$$

$$W_t = (1-\alpha)\phi_t \frac{Y_t}{N_t} \tag{13}$$

$$R_t = \alpha\phi_t \frac{Y_t}{K_t} \tag{14}$$

$$Y_t = K_t^{\alpha} N_t^{1-\alpha} \tag{15}$$

$$W_t = \frac{u_n(c_t, N_t)}{u_c(c_t, N_t)} \tag{16}$$

$$u_c(c_t, N_t) = \beta E_t \left[u_c(c_{t+1}, N_{t+1})(R_{t+1} + 1 - \delta) \right] \tag{17}$$

$$c_t + K_{t+1} - (1 - \delta) K_t = Y_t, \tag{18}$$

plus a standard transversality condition, $\lim_{T \to \infty} \beta^T u_c(c_T, N_T) K_{T+1} = 0$. These seven equations plus the transversality condition together determine the paths of seven aggregate variables $\{\phi_t, W_t, R_t, Y_t, N_t, c_t, K_{t+1}\}_{t=0}^{\infty}$ in general equilibrium, given the initial value of the capital stock, K_0.

Definition A fundamental equilibrium is the set of prices and quantities, $\{\phi, W, R, Y, N, c, K\}$, such that firms' profits and the household's utility are maximized subject to their respective technology and resource constraints when there is no extrinsic uncertainty.

In order to present our analysis in a clear-cut manner, we have assumed away any fundamental shocks in the model. Hence in this paper all fundamental equilibria are certainty equilibria, and all sunspots (nonfundamental) equilibria are stochastic equilibria.

Proposition 1 *There exists a unique fundamental equilibrium in the model.*

Proof. In the absence of extrinsic uncertainty, we can drop all the expectation operators in the model. Equation (12) then implies that the marginal cost is given by $\phi = \frac{\sigma - 1}{\sigma}$. Given that the marginal cost is constant, the rest of the Equations (13)-(18) are the same as in a standard optimal growth model. Hence standard fixed-point theorems can be applied to show that the optimal path of consumption, as well as those of capital, labor, and output, are unique for any given initial value of the capital stock. ■

Proposition 2 *There exists at least one equivalent, perfectly competitive model that can achieve the same set of equilibrium allocations as that in the Dixit-Stiglitz imperfect competition model.*

Proof. See Appendix 1. ■

Proposition 2 implies that it may not be possible to identify whether a particular allocation is generated by a perfectly competitive economy in which firms are price takers, or by an imperfectly competitive economy in which firms are price setters. Hence, there exists a fundamental identification problem in distinguishing between perfectly competitive economies and imperfectly competitive economies.

3 Sunspots Equilibria

3.1 Without Capital

In order to illustrate the existence of sunspots equilibria, consider first a special case where the production function is given by $Y_t = N_t$, (namely, there is no capital in the model, $\alpha = 0$), and the period-utility function is given by $u(c, n) = \log c - N$. Under these simplifying assumptions, the equilibrium conditions (12)-(18) imply the equilibrium relationships, $\phi_t = W_t = c_t = Y_t$, and

$$\frac{\sigma}{\sigma - 1} \frac{E_{t-1} Y_t^2}{E_{t-1} Y_t} = 1. \tag{19}$$

Any value of output that satisfies Equation (19) constitutes an equilibrium. One can verify that there is a unique fundamental equilibrium in which $Y_t = \frac{\sigma - 1}{\sigma}$. But there are also infinitely many stochastic sunspots equilibria in this model. Consider another solution for (19),

$$Y_t = \frac{\sigma - 1}{\sigma} \varepsilon_t, \tag{20}$$

where ε_t denotes sunspots. Then Equation (19) becomes

$$E_{t-1} \varepsilon_t^2 = E_{t-1} \varepsilon_t. \tag{21}$$

Since $E\varepsilon^2 = var(\varepsilon) + (E\varepsilon)^2$, Equation (21) implies

$$var(\varepsilon) = (1 - E_{t-1}\varepsilon)\, E_{t-1}\varepsilon. \tag{22}$$

Since $var(\varepsilon) \geq 0$, any random variable ε with conditional mean satisfying $E_{t-1}\varepsilon \leq 1$ can constitute an equilibrium. For example, consider a random variable with the distribution

$$E\varepsilon_t = \frac{1}{2}, var(\varepsilon_t) = \left(\frac{1}{2}\right)^2. \tag{23}$$

This random variable satisfies Equation (22). The deterministic nonsunspots solution corresponds to the case where $E\varepsilon_t = 1$ and $var(\varepsilon_t) = 0$.

Since the only restriction for sunspots equilibria is $E\varepsilon \leq 1$, any level of aggregate output, $Y \in [0, \frac{\sigma - 1}{\sigma}\bar{\varepsilon}]$, where $\bar{\varepsilon}$ is the upper bound of the support for ε_t, can therefore be a potential equilibrium output in the model. The key feature of a sunspots equilibrium, however, is that the aggregate output level is not a constant. It changes stochastically within the interval $\left[0, \frac{\sigma - 1}{\sigma}\bar{\varepsilon}\right]$. Notice that the value of σ can be arbitrarily large, hence sunspots equilibria can also exist in the limiting case of a perfectly competitive economy (by letting $\sigma \to \infty$).

These equilibria are clearly Pareto ranked since the household's utility function depends monotonically on the aggregate output Y. The maximum expected output is $\frac{\sigma-1}{\sigma}$, which can be achieved only in the certainty (nonsunspots) equilibrium (recall that $E\varepsilon = 1$ implies $var(\varepsilon) = 0$ by Equation 22). As such, sunspots are welfare reducing in this simplified model. A general statement on the welfare properties of sunspots will be provided later.

3.2 With Capital

The existence of multiple equilibria in the model does not depend on the absence or presence of capital. To illustrate, suppose the production function is given by $Y = K^{\alpha} N^{1-\alpha}$ and the period-utility function is the same as in the above example. In order to obtain closed-form solutions, also assume $\delta = 1$. The intertemporal Euler equation for asset accumulation implies

$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \phi_{t+1} \frac{\alpha Y_{t+1}}{K_{t+1}} \right). \tag{24}$$

A particular solution can be found by guessing

$$E_t \phi_{t+1} = \theta \tag{25}$$

and $c_t = (1 - \beta\alpha\theta) Y_t$. Substituting these guess-solutions into (25) gives

$$K_{t+1} = \beta\alpha\theta Y_t. \tag{26}$$

The consumer's labor supply equation implies $(1-\alpha)\frac{Y_t}{N_t}\phi_t = c_t$, so we have

$$N_t = \frac{(1-\alpha)}{1-\beta\alpha\theta} \phi_t. \tag{27}$$

The production function then can be written as

$$Y_t = \left(\frac{(1-\alpha)}{1-\beta\alpha\theta} \phi_t \right)^{1-\alpha} K_t^{\alpha}. \tag{28}$$

Substituting this into $\frac{\sigma}{\sigma-1} \frac{E_{t-1}(\phi_t Y_t)}{E_{t-1} Y_t} = 1$, we have

$$\frac{\sigma}{\sigma-1} \frac{E_{t-1}\phi_t^{2-\alpha}}{E_{t-1}\phi_t^{1-\alpha}} = 1, \tag{29}$$

where we have cancelled out the capital variable from the numerator and denominator since it is assumed to be known to the firms when prices are determined. When $\alpha = 0$, the above equation reduces to the previous model without capital.

A constant marginal cost, $\phi = \frac{\sigma-1}{\sigma}$, is the only fundamental equilibrium in the model. There are also many stochastic sunspots equilibria in the model. Any dynamic process of the marginal cost satisfying (25) and (29) constitutes a rational-expectations sunspots equilibrium. For example, a solution that takes the form

$$\phi_t = \frac{\sigma - 1}{\sigma}\varepsilon_t, \tag{30}$$

with the sunspots variable ε satisfying

$$E_{t-1}\varepsilon_t^{2-\alpha} = E_{t-1}\varepsilon_t^{1-\alpha} \tag{31}$$

and $E_{t-1}\varepsilon_t$ equalling a constant, constitutes an equilibrium. For example, the distribution

$$\varepsilon_t = \begin{cases} 0 & \text{with probability } p \\ 1 & \text{with probability } 1-p \end{cases} \tag{32}$$

will satisfy these conditions for any $p \in [0,1]$. In this case, the equilibrium value of θ is given by $\theta = \frac{\sigma-1}{\sigma}(1-p)$. Notice that the sunspots shock to the marginal cost, ε, represents a shock to the expected aggregate demand because the marginal cost is the Lagrangian multiplier for the constraint, $K^\alpha N^{1-\alpha} \geq Y$, in the cost-minimization problem of the firms. The marginal cost increases if and only if the demand increases.

Since $p \in [0,1]$, we have just constructed a continuum of sunspots equilibria that are not mere randomizations over fundamental equilibria. Each value of $p \in (0,1)$ corresponds to one particular stochastic sunspots equilibrium. The certainty equilibrium corresponds to the case of $p = 0$. There are also other types of sunspots equilibria in the model. For example, we can assume that the probability p is a time-varying random variable with a constant mean ($\bar{p} \in (0,1)$),

$$p_t = \bar{p} + \epsilon_t, \tag{33}$$

where ϵ_t is a stationary mean-reverting random process with zero mean and support $[-\bar{p}, 1-\bar{p}]$. Since ϵ_t can be serially correlated in a complicated manner, the sunspots equilibria can have very complicated dynamic properties.

It is important to emphasize that the type of sunspots equilibria in the model is global and robust. It is global because it is not based on a local linearization, hence it is independent of the topological properties of the steady state. It is robust because it is independent of the model's structural parameters, as opposed to the case analyzed by Benhabib and Farmer. The results hold regardless of the utility functions and the output elasticities of capital and labor. They hold even in the limiting case of (near) perfect competition (i.e., when $\sigma \to \infty$). The real business cycle model of Kydland and Prescott (1982) can be cast into this framework by properly decentralizing it in a way similar to Benhabib and Farmer (1994) and then taking the limit, $\sigma \to \infty$.

3.3 A Calibration Exercise

Let the period-utility function be given by $u(c, N) = \log(c) - a_n \frac{N^{1+\gamma}}{1+\gamma}$. When $\delta \neq 1$ and $\gamma \neq 0$, closed-form solutions are not obtainable, but approximate solutions can be found by linearizing the model around a deterministic steady state where the long-run value of the marginal cost is a constant, $\bar{\phi} = \frac{\sigma-1}{\sigma}$. Denote a circumflex variable as $\hat{x}_t \equiv \log X_t - \log \bar{X}$, where $\bar{X}$ denotes the long-run value of X for a deterministic steady state. Log-linearizing the equilibrium conditions (12)-(18) around the deterministic steady state, after substituting out $\{\hat{w}, \hat{r}, \hat{y}\}$ we have

$$E_{t-1}\hat{\phi}_t = 0 \tag{34}$$

$$(\alpha + \gamma)\,\hat{n}_t = \hat{\phi}_t + \alpha\hat{k}_t - \hat{c}_t \tag{35}$$

$$\hat{c}_t = E_t\hat{c}_{t+1} - (1 - \beta(1-\delta))E_t\left[\hat{\phi}_{t+1} + (\alpha - 1)\,\hat{k}_{t+1} + (1-\alpha)\hat{n}_{t+1}\right] \tag{36}$$

$$(1 - s_i)\hat{c}_t + s_i\left(\frac{1}{\delta}\hat{k}_{t+1} - \frac{1-\delta}{\delta}\hat{k}_t\right) = \alpha\hat{k}_t + (1-\alpha)\hat{n}_t, \tag{37}$$

where $s_i = \delta\frac{\beta\alpha\phi}{1-\beta(1-\delta)}$ is the long-run saving rate in the deterministic steady state.

Notice that the model is reduced to a standard RBC model without sunspots if $\hat{\phi}_t = 0$ for all t (i.e., if ϕ_t is constant). However, ϕ_t does not have to be constant. Firms' price setting behavior only implies that the expected value of ϕ_t is constant in this linearized version of the model (Equation 34). Thus, the above system clearly suggests that we can treat the marginal cost $\hat{\phi}_t$ as an exogenous forcing variable of the model economy. Let $\hat{\phi}_t$ be an *i.i.d.* random variable with mean zero; the above system of equations can be reduced to

$$E_t\begin{bmatrix}\hat{k}_{t+1}\\ \hat{c}_{t+1}\end{bmatrix} = M\begin{bmatrix}\hat{k}_t\\ \hat{c}_t\end{bmatrix} + \Gamma\hat{\phi}_t. \tag{38}$$

The saddle-path property of the model implies that the coefficient matrix M has exactly one explosive eigenvalue and one stable eigenvalue. Hence the optimal consumption level can be solved by the method of Blanchard and Kahn (1980) to get the saddle-path solution, $\hat{c}_t = \gamma_1\hat{k}_t + \gamma_2\hat{\phi}_t$, where

$\{\gamma_1, \gamma_2\}$ denote coefficients. Since the marginal cost $\hat{\phi}_t$ is indeterminate, any *i.i.d.* representation of $\hat{\phi}_t$ with zero conditional mean can constitute an equilibrium path for consumption.[12]

Following the existing RBC literature (e.g., Kydland and Prescott, 1982), we calibrate the model as follows: the time period is a quarter, the time discounting factor $\beta = 0.99$, the rate of depreciation $\delta = 0.025$, the inverse labor supply elasticity $\gamma = 0.25$, and capital's share in aggregate output $\alpha = 0.42$. The impulse responses of the benchmark model to an *i.i.d.* sunspots shock to the marginal cost are graphed in Figure 2. Since the model is linearized around a particular steady state, the impulse responses in Figure 2 can be interpreted as the transitional dynamics around that steady state. Notice that a positive one-standard-deviation shock to the marginal cost generates positive responses from consumption, employment, output, and investment. Also notice that investment is far more volatile than output due to the incentive for consumption smoothing. Thus the model is able to explain the stylized business cycle facts emphasized by Kydland and Prescott (1982): the positive comovements and the volatility orders among consumption, output, and investment. These business cycle facts are commonly thought to be explainable only by technology shocks. Here we show that they are also explainable by sunspots shocks.

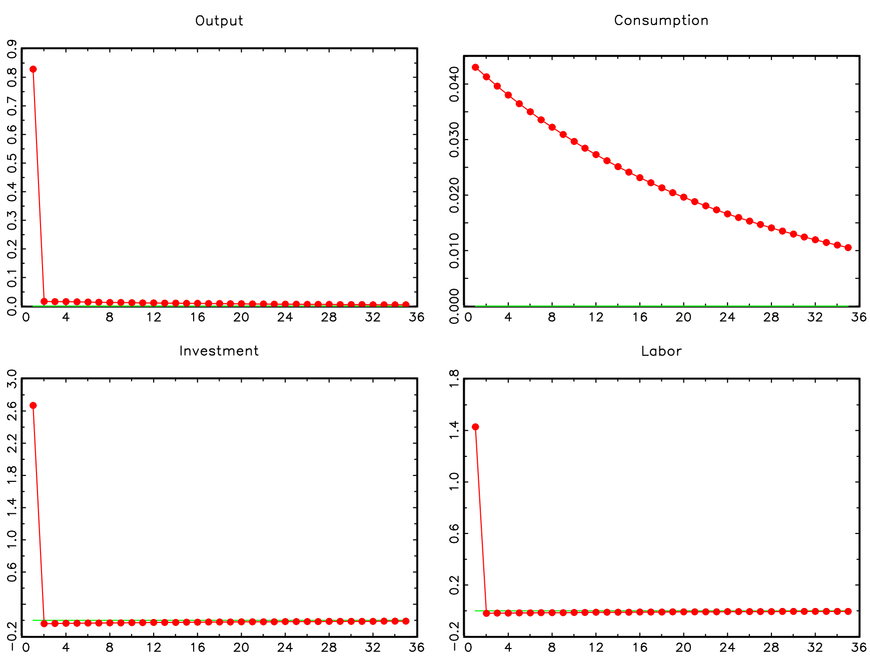

Fig. 2. Impulse Responses to a Sunspots Shock.

The predicted second moments of the model under *i.i.d.* sunspots shocks and *i.i.d.* technology shocks are summarized in Table 1. It shows that the business-cycle effects of sunspots shocks

[12] Notice that in the original nonlinear model, the marginal cost does not have to be *i.i.d.* Also notice that in the original nonlinear model, the marginal cost may not be allowed to be any *i.i.d.* process. These features or (un)restrictions on the distribution of sunspots are lost in the linearized version of the model.

and technology shocks are identical for all of the variables except hours worked: the volatility of hours worked relative to output is smaller under technology shocks than under sunspots shocks. The reason is that sunspots shocks do not affect aggregate productivity while technology shocks do. The intuition for the similarity is that the marginal cost measures the increase in production cost when firms' output demand increases by one unit. As such, sunspots shocks to the marginal cost reflect shocks to expected demand. Due to strategic complementarity under the demand externalities, such demand-side shocks are effectively the same as shocks to individual firm's marginal revenue. Thus, they look like productivity shocks except that they do not change aggregate productivity.[13]

Table 1. Predicted Second Moments*

	Volatility ($\frac{\sigma_x}{\sigma_y}$)			Correlation with y			Autocorrelation			
	c	i	n	c	i	n	y	c	i	n
Sunspots	0.18	3.22	1.72	0.35	0.99	0.99	0.02	0.96	-0.01	-0.02
Technology	0.18	3.22	0.76	0.35	0.99	0.98	0.02	0.96	-0.01	-0.01

*y: output; c: consumption; i: investment; n: labor.

3.4 Discussion

An important implication of sunspots equilibria is that markup is counter-cyclical, which is in line with the empirical evidence.[14] In the model, markup is given by the inverse of the marginal cost, $\frac{1}{\phi}$. When expected demand is high, firms opt to produce more, and the marginal cost increases, leading to lower markup. This implication of counter-cyclical markup is in sharp contrast to the cases with fundamental shocks. Under fundamental shocks only (i.e., without extrinsic uncertainty), the markup is always constant in the model. More importantly, notice that counter-cyclical markup is obtained regardless of the monopoly power, since the same results hold even as $\sigma \to \infty$. In this case, although markup is zero in the steady state, it fluctuates under sunspots shocks due to firms' price-setting behavior. Thus, even though the markets are perfectly (or near-perfectly) competitive and firms set prices to expected marginal cost, because the expected marginal cost co-moves with expected aggregate demand, markup can be countercyclical during the business cycle, regardless of the degree of imperfect competition or market power.

[13] However, if a variable capital utilization rate is allowed in the model, then shocks to the marginal cost also increase the total factor productivity via their impact on capacity utilization. For example, let the production function of intermediate goods be given by $Y(i) = [e(i)K(i)]^{\alpha} N(i)^{1-\alpha}$, where $e(i)$ is the rate of capital utilization that affects the rate of capital depreciation of firm i according to $\delta(i) = \frac{1}{v}e(i)^v$ $(v > 1)$. It can be shown that this leads to a reduced-form production function at the optimal rate of capital utilization,

$$Y(i) = (\alpha\phi(i))^{\frac{\alpha}{v-\alpha}} K(i)^{\alpha\frac{v-1}{v-\alpha}} N(i)^{(1-\alpha)\frac{v}{v-\alpha}}.$$

Hence the marginal cost affects the total factor productivity just like technology shocks in an RBC model.

[14] The stylized fact of counter-cyclical markup has been documented extensively in the empirical literature. See, e.g., Bils (1987), Rotemberg and Woodford (1991,1999), Martins, Scapetta, and Pilat (1996), among others.

4 Welfare Implications of Sunspots

Economic fluctuations driven by sunspots are inefficient in the imperfect competition model. The following proposition shows that sunspots equilibria are dominated by the fundamental equilibrium in terms of welfare.

Proposition 3 *Sunspots equilibria are Pareto inferior to the fundamental equilibrium.*

Proof. See Appendix 2. ■

The intuition behind Proposition 3 can be understood from the firms' pricing rule,

$$E\phi_t y_t = \frac{\sigma - 1}{\sigma} E y_t, \tag{39}$$

where $\phi y = Wn + Rk$ is the household's variable income received from labor supply and capital rental. The variable income can be controlled by the household by changing labor supply and investment, whereas the distributed profit D is a fixed income that is not controlled by the household. Note that $E\phi_t y_t = E\phi_t E y_t + cov(\phi_t, y_t)$. Hence the average variable income can exceed the variable income under an invariant marginal cost, $E\phi_t E y_t$, provided that the covariance between the marginal cost and production, $cov(\phi, y)$, is positive.[15] Hence, depending on the variation in the marginal cost, the expected variable income ($E\phi y$) could be high enough to more than compensate the potential losses in utility due to variations in consumption and hours worked if there were no further restrictions on the variability of the marginal cost. However, the firms' pricing rule effectively imposes a constraint on the level of the variable income $E\phi y$: it cannot exceed $\frac{\sigma-1}{\sigma} E y_t$. Note that if the household's variable income is given by $\frac{\sigma-1}{\sigma} E y$, then there is no gain by varying the production level y since the household is better off keeping production constant under the concavity of the utility function and the production function (given the constant marginal cost $\frac{\sigma-1}{\sigma}$). Therefore, welfare under sunspots is lower than under constant marginal cost simply because of constraint (39), which is the result of imperfect competition.

5 Nominal Price Setting and Sunspots

This section shows that if there is money in the economy and if firms set prices in nominal terms, then money may be able to help eliminate sunspots equilibria. However, we also show that money's effectiveness in serving such a role is model-dependent. In many cases, money is completely ineffective for eliminating sunspots equilibria.

[15] As the proof in Appendix 1 shows, a higher value of ϕ implies higher marginal returns to labor and capital. Thus $cov(\phi, Y) \geq 0$.

When firms set nominal prices instead of real prices to maximize profits, the level of aggregate money supply may be able to help determine the output level by determining aggregate demand, thereby eliminating indeterminacy and sunspots equilibria in the model. This is similar to the traditional Keynesian IS-LM model where the equilibrium conditions from both the goods market (the IS curve) and the money market (the LM curve) are needed in order to uniquely pin down the equilibrium output level.

To illustrate, consider the general model in Section 2 and let intermediate good firms choose nominal prices $P(i)$ to maximize expected profits, $E_{t-1}\left\{\left(\frac{P_t(i)}{P_t}-\phi_t\right)Y_t(i)\right\}$, subject to the downward sloping demand function, $Y(i)=\left(\frac{P(i)}{P}\right)^{-\sigma}Y$. The optimal price is given by

$$\frac{P_t(i)}{P_t}=\frac{\sigma}{\sigma-1}\frac{E_{t-1}\left(\phi_t Y_t\right)}{E_{t-1}Y_t}. \tag{40}$$

Recall that the aggregate price satisfies $P=\left(\int P(i)^{1-\sigma}di\right)^{1/(1-\sigma)}$, hence we have $P=P(i)$ in a symmetric equilibrium. As such, the aggregate price is determined as soon as the intermediate good prices are set, before any realizations of sunspots. Thus, if there is a money demand equation that relates the aggregate price level to aggregate income, then that equation may potentially help pin down the output level in equilibrium, ruling out sunspots. For example, suppose money enters the model via a cash-in-advance constraint on the household and suppose that the constraint binds in equilibrium,

$$Y_t=\frac{\bar{M}}{P_t}. \tag{41}$$

In this case, given that the aggregate price level is determined before the realization of sunspots, aggregate income (Y) cannot be affected by sunspots shocks. That is, sunspots do not matter. However, this result is not general. There also exist economic environments where sunspots cannot be ruled out by money. We provide several examples below to illustrate this point.

5.1 Predetermined Money Demand

Let the household choose money demand before the realization of sunspots in each period.[16] Consider a slightly modified version of the household's objective in (6) with money in the utility. The household chooses money demand, M, consumption, c, the next-period capital stock, k', and labor

[16] This implies that the money market opens before the realizations of sunspots in each period. Thus, if there are money supply shocks in the model, the shocks are observed before sunspots shocks.

supply, n, to solve

$$\max_{\{M_t\}} E_{-1}\left\{\max_{\{c_t,n_t,k_{t+1}\}} E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(\log c_t - a_n\frac{n_t^{1+\gamma}}{1+\gamma} + a_m\log\frac{M_t}{P_t}\right)\right\}\right\} \tag{42}$$

subject to

$$c_t + k_{t+1} + \frac{M_t}{P_t} \leq W_t n_t + (R_t + 1 - \delta)k_t + \frac{M_{t-1}}{P_t} + D_t. \tag{43}$$

Since money demand is chosen before realizations of sunspots, the intertemporal Euler equation for money demand is given by

$$E_{t-1}\frac{1}{c_t} = \beta E_{t-1}\frac{1}{c_{t+1}}\frac{P_t}{P_{t+1}} + \frac{a_m}{m_t}, \tag{44}$$

where m_t is the real money balance in period t. Notice that the aggregate price, $P_t = \left[\int P(i)^{1-\sigma}di\right]^{\frac{1}{\sigma-1}}$, is determined before realizations of sunspots given that firms choose prices based on the information set Ω_{t-1}. Assuming that money supply is constant ($M_t = \bar{M}$), using the real money balance relationship, $P_t = \frac{M}{m_t}$, to replace the prices in the above Euler equation, we have

$$m_t E_{t-1}\frac{1}{c_t} = \beta E_{t-1}\left[m_{t+1}\left(E_t\frac{1}{c_{t+1}}\right)\right] + a_m \tag{45}$$

where the left-hand side has utilized the law of iterated expectations.

The firm's optimization problem is the same as before. Hence all of the first-order conditions are the same as in (12)-(18) except there is an additional money demand function (45). To construct sunspots equilibria, consider the simpler case where there is no capital and let $\gamma = 0$ and $a_n = a_m = 1$. Hence in a symmetric equilibrium we have $Y_t = N_t = W_t = \phi_t = c_t$ and $\frac{\sigma}{\sigma-1}\frac{E_{t-1}\phi_t^2}{E_{t-1}\phi_t} = 1$. Notice that the only fundamental equilibrium is still given by $\phi = \frac{\sigma-1}{\sigma}$. Consider a sunspots equilibrium in which the expected marginal utility is constant, $E_{t-1}\frac{1}{\phi_t} = \theta$. We then have $m_t = \beta E_{t-1}m_{t+1} + \frac{1}{\theta}$. Solving this equation forward gives $m_t = \frac{1}{(1-\beta)\theta}$. Any stochastic process $\{\phi_t\}$ that satisfies $\frac{\sigma}{\sigma-1}\frac{E_{t-1}\phi_t^2}{E_{t-1}\phi_t} = 1$ (with $E_{t-1}\frac{1}{\phi_t}$ constant) constitutes a sunspots equilibrium. For example, let $\phi_t = \frac{\sigma-1}{\sigma}\varepsilon_t$, where ε_t satisfies the distribution,

$$\varepsilon_t = \begin{cases} \varepsilon_1 & \text{with probability } p \\ \varepsilon_2 & \text{with probability } 1-p \end{cases}, \tag{46}$$

where $\varepsilon_1 \in (0,1]$ and $\varepsilon_2 \in [1,\infty)$. In this case, there is a continuum of sunspots equilibria. To see this, note that $\frac{\sigma}{\sigma-1}\frac{E_{t-1}\phi_t^2}{E_{t-1}\phi_t} = 1$ implies

$$\varepsilon_1 - \varepsilon_1^2 = -\frac{(1-p)}{p}\left[\varepsilon_2 - \varepsilon_2^2\right], \tag{47}$$

which implies a solution

$$\varepsilon_2 = \frac{1}{2}\left[1 + \sqrt{1 + 4\frac{p}{1-p}\left(\varepsilon_1 - \varepsilon_1^2\right)}\right]. \tag{48}$$

Notice that for any $\varepsilon_1 \in (0,1]$, we can always find a $\varepsilon_2 \geq 1$ using the above equation for any given value of $p \in [\frac{\varepsilon_2 - 1}{\varepsilon_2 - \varepsilon_1}, 1]$.[17] Hence the set of sunspots equilibria is at least as large as the set $\{\varepsilon : \varepsilon \in (0,1]\}$.

5.2 Without Predetermined Money Demand

Even if the household's money demand decision is made after sunspots are realized, there are still cases where money cannot eliminate sunspots. For example, let the utility function be given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t - \frac{n_t^{1+\gamma}}{1+\gamma}\right) + v\left(\frac{M_t}{P_t}\right)\right] \tag{49}$$

where $u(\cdot)$ is concave and $\gamma > 0$. The Euler equation for money demand is given by

$$u_c'\left(c_t - \frac{n_t^{1+\gamma}}{1+\gamma}\right) = \beta E_t u_c'\left(c_{t+1} - \frac{n_{t+1}^{1+\gamma}}{1+\gamma}\right)\frac{P_t}{P_{t+1}} + v'\left(\frac{M_t}{P_t}\right). \tag{50}$$

It is easy to check that the unique fundamental equilibrium is still given by $\phi_t = \frac{\sigma-1}{\sigma}$ in this model. To construct sunspots equilibria, notice that as long as there exist equilibrium paths of consumption and labor such that the marginal utility, $u'(x)$, is constant, the money demand function (50) cannot provide information to pin down the output level. Hence, money is not effective in ruling out sunspots equilibria. To illustrate this, again consider the case of no capital, $Y = N$, and let $\gamma = 1$. Under these assumptions, the first-order conditions of the firm imply $\phi = W = c = Y = N$ and

$$\frac{\sigma}{\sigma-1}\frac{E_{t-1}Y_t^2}{E_{t-1}Y_t} = 1. \tag{51}$$

The argument in the marginal utility, $u'(x)$, is then given by $x_t = Y_t - \frac{1}{2}Y_t^2$. Consider an equilibrium such that $Y_t = \{Y_1, Y_2\}$ $(Y_1 \neq Y_2)$ with probabilities of $\{p, 1-p\}$, respectively. According to (54),

[17] Since we also require $E\varepsilon \leq 1$, this implies $p \geq \frac{\varepsilon_2 - 1}{\varepsilon_2 - \varepsilon_1}$.

we then have $\frac{\sigma}{\sigma-1}[pY_1^2 p + (1-p)Y_2^2] = pY_1 + (1-p)Y_2$, which implies

$$Y_1 - \frac{\sigma}{\sigma-1}Y_1^2 = -\frac{(1-p)}{p}\left(Y_2 - \frac{\sigma}{\sigma-1}Y_2^2\right) \tag{52}$$

To satisfy the requirement that $u'(x)$ is constant, we also have the following restriction,

$$Y_1 - \frac{Y_1^2}{2} = Y_2 - \frac{Y_2^2}{2}. \tag{53}$$

To ensure positive utility, we also need $Y - \frac{1}{2}Y^2 > 0$ or $Y \in (0,2)$. Any pair of $\{Y_1, Y_2\}$ (with $Y_1 \neq Y_2$) satisfying the above two equations in the domain of $(0,2)$ constitutes a sunspots equilibrium.

Proposition 4 *For each value of $\sigma \in (1,\infty)$, there exists a continuum of sunspots equilibria.*

Proof. Equation (53) implies a solution of

$$Y_2 = 1 - \sqrt{1 - \left(2Y_1 - Y_1^2\right)}. \tag{54}$$

Since the function $2Y_1 - Y_1^2$ has a unique maximum at $Y_1 = 1$ and two zeros at $Y_1 = \{0,2\}$, for any $Y_1 \in (1,2)$ the above solution gives $Y_2 \in (0,1)$. On the other hand, Equation (52) implies a solution of

$$Y_2 = \frac{1}{2}\left(\frac{\sigma-1}{\sigma} + \sqrt{\left(\frac{\sigma-1}{\sigma}\right)^2 + 4\frac{p}{1-p}\left(\frac{\sigma-1}{\sigma}Y_1 - Y_1^2\right)}\right). \tag{55}$$

Since the function $\frac{\sigma-1}{\sigma}Y_1 - Y_1^2$ has a unique maximum at $Y_1 = \frac{1}{2}\frac{\sigma-1}{\sigma}$ and two zeros at $Y_1 = \left\{0, \frac{\sigma-1}{\sigma}\right\}$, for any $Y_1 \in \left(0, \frac{\sigma-1}{\sigma}\right)$, the above solution gives $Y_2 > \frac{\sigma-1}{\sigma}$ for any $p \in (0,1)$. Notice that as $p \to 1$, we have $Y_2 \to \infty$. Thus, there must exist a value $p \in (0,1)$ such that $Y_2 \in (\frac{\sigma-1}{\sigma}, 2)$. Combining Equations (54) and (55) implies that the set of sunspots equilibria is identical to the set $\left\{Y : Y \in \left(0, \frac{\sigma-1}{\sigma}\right)\right\}$, which is a subset of $(0,1)$. ■

5.3 Sunspots under Sticky Prices

In this section, we show that sunspots equilibria are robust to sticky prices. Hence, self-fulfilling business cycles can also be a natural feature of the new Keynesian sticky-price models. For example, in a standard Calvo (1983) type sticky price model, the optimal price set by intermediate good firms (who can adjust their prices in period t) is given by

$$P_t^* = \frac{\sigma \sum_{s=0}^{\infty} (\beta\theta)^s E_{t-1} \Lambda_{t+s} P_{t+s}^{\sigma} Y_{t+s} \phi_{t+s}}{(\sigma - 1) \sum_{s=0}^{\infty} (\beta\theta)^s E_{t-1} \Lambda_{t+s} P_{t+s}^{\sigma-1} Y_{t+s}}, \tag{56}$$

where Λ_{t+s} is the ratio of marginal utilities in period $t+s$ and period t, and θ is the fraction of firms that cannot adjust their prices in period t. This equation is reduced to Equation (5) when $s = 0$.

Consider first the money-in-the-utility model without capital as in the previous section. Notice that the only certainty equilibrium is still given by $Y_t = c_t = N_t = \phi_t = \frac{\sigma-1}{\sigma}$ (also, $m_t = \frac{\sigma-1}{(1-\beta)\sigma}$ and $p_t = \bar{M}\frac{(1-\beta)\sigma}{\sigma-1}$). Consider a sunspots equilibrium in which the sunspots are serially independent. Due to zero serial correlation in sunspots shocks, the expected marginal utility is constant, $E_{t-1}\frac{1}{c_t} = z$. So the real money demand is also a constant, $m_t = \frac{1}{(1-\beta)z}$. Given that the money supply is fixed, this implies that prices are constant: $P_t^* = P_t$. Exploiting this fact, the price equation implies

$$1 = \frac{\sigma \sum_{s=0}^{\infty} (\beta\theta)^s E_{t-1} \phi_t \phi_{t+s}}{(\sigma - 1) \sum_{s=0}^{\infty} (\beta\theta)^s E_{t-1} \phi_t}. \tag{57}$$

Note that for any $s \geq 1$, $E_{t-1}\phi_t\phi_{t+s} = E_{t-1}\phi_t E_{t-1}\phi_{t+s} = (E_{t-1}\phi_t)^2$. The above equation can then be simplified to

$$\frac{\sigma}{\sigma - 1} \left[(1 - \beta\theta) E_{t-1}\phi_t^2 + \beta\theta (E_{t-1}\phi_t)^2 \right] = E_{t-1}\phi_t. \tag{58}$$

As before, let $\phi_t = \frac{\sigma-1}{\sigma}\varepsilon_t$, where ε_t represents sunspots. Using this definition to substitute out ϕ in the above equation, we have

$$(1 - \beta\theta)\, var(\varepsilon) = E_{t-1}\varepsilon_t (1 - E_{t-1}\varepsilon_t). \tag{59}$$

Clearly, any distribution of ε such that $E_{t-1}\varepsilon_t \leq 1$ and $E_{t-1}\varepsilon_t$ is constant constitutes a sunspots equilibrium. For example, the distribution

$$\varepsilon_t = \begin{cases} \frac{1}{4} & \text{with probability } p \\ \frac{1}{2} & \text{with probability } 1 - p \end{cases}, \tag{60}$$

will satisfy these conditions for any $p \in [0,1]$. In this case, $E_{t-1}\phi = \frac{\sigma-1}{\sigma}\frac{2-p}{4} < \frac{\sigma-1}{\sigma}$ and $z \equiv E_{t-1}\frac{1}{c_t} = \frac{\sigma}{\sigma-1}2(1+p)$.

When there is capital in the model, a closed-form solution is no longer possible. Hence we solve the model by a log-linear approximation around the certainty equilibrium. In the linearized model, the firm's optimal price is given by the well-known New-Keynesian Phillips relationship,

$$\pi_t = \beta E_{t-1}\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} E_{t-1}\phi_t, \tag{61}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross inflation rate. Note that $E_{t-1}\pi_t = \pi_t$ since period-t prices are determined before realizations of sunspots in period t. We can denote the sunspots shocks to the marginal cost as $\varepsilon_t = \phi_t - E_{t-1}\phi_t$. The household's problem is the same as problem (42). Details of solving the sticky-price model are provided in Appendix 3.

The impulse responses of the sticky-price model to a sunspots shock are reported in Figure 3. It shows that sunspots have similar effects in the sticky-price model compared to the real model in the impact period: output, consumption, investment, and employment all increase in the initial period. But sunspots have dramatically different effects in the sticky-price model in the following periods. In particular, consumption drops rapidly in the second period after the shock and is hence less smooth in the sticky price model. The intuition is that when an unanticipated sunspots shock is realized, marginal cost increases due to a self-fulfilling expected increase in aggregate demand. Since prices are set in advance before the sunspots shocks and the sunspots are not anticipated, the first period response of the model is similar to that of a real model. After the realization of sunspots, though, prices must drop in order to maintain a high level of real balances that matches the level of consumption. This is not possible due to sticky prices and a fixed money supply, thus forcing consumption demand to decrease sharply. This leads firms to reduce production and demand for capital. Table 2 reports standard business-cycle statistics of the sticky-price model. It shows that the markup, $-\hat{\phi}$, is counter-cyclical and inflation is procyclical. These predictions are qualitatively consistent with the empirical evidence.

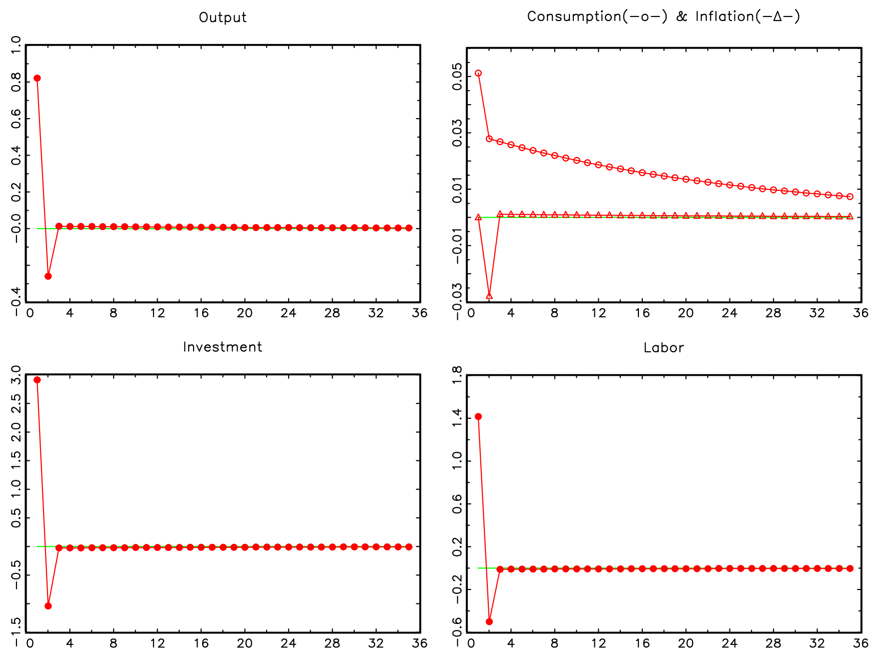

Fig. 3. Impulse Responses to a Sunspots Shock under Sticky Prices.

Table 2. Correlation with Output

	$\hat{c}$	$\hat{\imath}$	$\hat{n}$	$\hat{\pi}$	-$\hat{\phi}$
$cor(\hat{x}, \hat{y})$	0.40	0.99	0.99	0.30	-0.99

6 Conclusion

This paper extends the insight of Cooper and John (1988) by showing that self-fulfilling expectations can be a general feature of dynamic general equilibrium models of imperfect competition. Even though the fundamental equilibrium is unique in this class of models, there are multiple sunspots equilibria that are not based on mere randomizations over fundamental equilibria. In a sunspot equilibrium, markup is always counter-cyclical; consumption, investment, employment, and output always co-move together, in accordance with empirical evidence. A key element for the existence of sunspots equilibria is an information structure where individual firms make decisions without knowing how the other agents in the economy will behave, and thus they each must make choices before aggregate uncertainty is resolved.[18] Given the demand externalities among agents' actions, such extrinsic uncertainties can be self-fulfilling.

We often talk about the microfoundations of the macroeconomy, but seldom discuss the macrofoundations of the microeconomy (as Keynes did in the past). In reality, the outcome of individual agents' optimal plans often depend crucially on macroeconomic conditions over which agents can have significant influence collectively but little influence individually. When agents' optimal deci-

[18] This type of uncertainty is referred to as market uncertainty by Peck and Shell (1991).

sions must be conditioned on their expectations of such macroeconomic conditions, such expectations can be self-fulfilling when synchronized. We show that fluctuations driven by self-fulfilling expectations look very similar to those driven by technology shocks. Given that such fluctuations are welfare reducing (as shown in this paper), interventionary policies are called for. The design of optimal fiscal and monetary policies to counter sunspots-driven fluctuations is a promising topic which can be pursued in future research using the framework provided in this paper. This is particularly relevant to the fast growing New Keynesian literature of monetary policy analysis. This literature shows that cost-push shocks to firms' marginal cost can generate a painful inflation-output trade-off that complicates the design of optimal monetary policies (see, e.g., Clarida, Gali, and Gertler, 1999; Woodford, 2001; and Gali, 2002). We show in this paper that sunspots and self-fulfilling expectations are a natural source of cost-push shocks under imperfect competition.

Appendix 1: Proof of Proposition 2

The equilibrium paths of the marginal cost in the imperfect competition model, $\{\phi_t\}_{t=0}^{\infty}$, are determined by the price setting behavior of the monopolistic firms given in Equation (12). Given a path of ϕ, Equations (13)-(18) (plus the transversality condition) fully determine the equilibrium allocation of the imperfect competition model. Since the utility functions and production functions are concave, a competitive model can achieve the same equilibrium allocation if and only if it has the same set of first-order conditions (but not necessarily the same production functions) as those in the imperfect competition model.

Let the utility functions be the same in both models. Let the production function of intermediate goods in the Dixit-Stiglitz model be denoted by $y = g(k, n)$. Hence Equations (13) and (14) can be expressed as $w = \phi g_n = \phi \epsilon_n \frac{y}{n}$ and $r = \phi g_k = \phi \epsilon_r \frac{y}{k}$, where $\{w, r\}$ denote the real wage and real rental rate, $\{\epsilon_n, \epsilon_k\}$ denote output elasticities of labor and capital, and ϕ denotes the marginal cost. Consider a competitive RBC model featuring a representative firm with the aggregate production technology, $y = f(k, n)$. Profit maximization by the representative firm gives $w = (1 - \tau) f_n = (1 - \tau)\varepsilon_n \frac{y}{n}$ and $r = (1 - \tau) f_k = (1 - \tau)\varepsilon_n \frac{y}{k}$, where $\tau \in [0, 1]$ denotes the income tax rate, and $\{\varepsilon_n, \varepsilon_k\}$ denote the output elasticities of labor and capital in this model. We assume that the government can use lump-sum transfers to redistribute the revenue from the income tax (if any) back to the household, so that the budget constraint of the household in the competitive model is the same as that in the imperfect competitive model (Equation 18).

Given that the utility function and production functions are concave in both models, the two models have the same set of equilibrium allocations if and only if the real wage and real interest rate are the same in both models – namely, if and only if the following conditions hold: $(1 - \tau)\varepsilon_n = \phi \epsilon_n$,

$(1-\tau)\varepsilon_k = \phi\epsilon_k$, and $(1-\tau)(\varepsilon_n+\varepsilon_k) = \phi(\epsilon_n+\epsilon_k)$. For example, if both models have constant returns to scale technologies (i.e., $\varepsilon_n+\varepsilon_k = 1 = \epsilon_n+\epsilon_k$), then $\tau = 1-\phi$ and $f(\cdot) = g(\cdot)$ are the requirements for equivalence. Notice that the equivalence holds for any path of $\{\phi\}$. If ϕ is constant (as in the fundamental equilibrium), it is then also possible to achieve equivalence without an income tax($\tau = 0$) in the competitive model. For example, let the production functions be Cobb-Douglas in both models, and let $\epsilon_n + \epsilon_k = 1$ and $\tau = 0$. In this case, a diminishing returns to scale production technology in the competitive model with $\varepsilon_n = \phi\epsilon_n, \varepsilon_k = \phi\epsilon_k$, and $\varepsilon_n + \varepsilon_k = \phi$ are the requirements for equivalence.[19]■

Appendix 2: Proof of Proposition 3

Since sunspots give rise to allocations that differ across different sunspot states within any period, the key of the proof is to show that in any given period, any state-dependent allocations for consumption, hours worked, the capital stock, and the marginal cost are not optimal. As such, the household prefers the certainty equilibrium to sunspots equilibria in the imperfect competition model.

The first step in the proof is to transform the imperfect competition model to an equivalent, representative-agent model with distortionary income taxation:

$$\max_{\{c_t,n_t,k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) \tag{62}$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t \leq \phi_t y_t + D_t \tag{63}$$

$$E_{t-1}\phi_t y_t = \frac{\sigma-1}{\sigma} E_{t-1} y_t, \tag{64}$$

where $y_t = k_t^{\alpha} n_t^{1-\alpha}$ is the aggregate production technology, ϕ_t is one minus the income tax rate, and $D_t = (1-\phi_t)y_t$ is a lump-sum government transfer which the agent takes as parametric. One can easily verify that this representative-agent model of income taxation is equivalent to the imperfect competition model by comparing the first-order conditions of the two models. This equivalence implies that one minus the marginal cost $(1-\phi)$ in the imperfect competition model serves as a distortionary income tax in the representative agent model. This is intuitive because $1-\phi$ is a measure of the markup in the imperfect competition model.

In the absence of extrinsic uncertainty, constraint (64) implies $\phi = \frac{\sigma-1}{\sigma}$. Given this, the optimal paths for consumption, hours worked, and the capital stock are denoted by $\left\{c_t^*, n_t^*, k_{t+1}^*\right\}_{t=0}^{\infty}$, which is the unique optimal allocation in the certainty equilibrium.

[19] We assume that profits (if any) are redistributed back to the representative household in both models.

Next, consider the case where the agent is given the option of having a random marginal cost that is state dependent, $\phi_t(x)$, at the beginning of any period t and for that period only. We show that the representative agent is worse off in welfare by facing the state-dependent random marginal cost. Let there be a continuum of states, x, in period t. Without loss of generality, assume that x has a uniform distribution over $[0,1]$. Starting at the beginning of period t, the representative agent's problem is to solve

$$\max \sum_{s=0}^{\infty} \beta^s \int u\left(c_{t+s}(x), 1 - n_{t+s}(x)\right) dx \tag{65}$$

subject to

$$\int \left[c_{t+s}(x) + k_{t+s+1}(x) - (1-\delta)k_{t+s}(x)\right] dx \leq \int \left[\phi_{t+s}(x) y_{t+s}(x) + D_{t+s}(x)\right] dx \tag{66}$$

$$\int \left[\phi_{t+s}(x) y_{t+s}(x)\right] dx = \frac{\sigma - 1}{\sigma} \int y_{t+s}(x) dx \tag{67}$$

Notice that constraint (66) is weaker than the constraint

$$c_{t+s}(x) + k_{t+s+1}(x) - (1-\delta)k_{t+s}(x) \leq \phi_{t+s}(x) y_{t+s}(x) + D_{t+s}(x). \tag{66$'$}$$

The household's income is constant (risk free) in constraint (66) while it is stochastic in constraint (66′). However, in a state-independent allocation, these two constraints are the same. Substituting the equality in the second constraint (67) into the first constraint (66), the period-budget constraint becomes

$$\int \left[c_{t+s}(x) + k_{t+s+1}(x) - (1-\delta)k_{t+s}(x)\right] dx \leq \frac{\sigma - 1}{\sigma} \int y_{t+s}(x) dx + \int D_{t+s}(x) dx. \tag{68}$$

In period t, let $c_t = \displaystyle\int_0^1 c_t(x) dx$ and $n_t = \int_0^1 n_t(x) dx$. It is clear that the state-independent allocation $\{c_t, n_t\}$ satisfies the budget constraint (68) in period t:

$$\begin{aligned} c_t + \int k_{t+1}(x) dx - (1-\delta)k_t \quad &\leq \quad \frac{\sigma - 1}{\sigma} \int y_t(x) dx + \int D_t(x) dx & (69) \\ &\leq \quad \frac{\sigma - 1}{\sigma} k_t^{\alpha} n_t^{1-\alpha} + \int D_t(x) dx, & (70) \end{aligned}$$

where k_t is independent of x because it is determined in the last period $t-1$, and the second inequality (70) is based on the concavity of the production function. By the concavity of the utility function, we have

$$\int u(c_t(x), 1 - n_t(x))dx \leq u(c_t, 1 - n_t). \tag{71}$$

Hence, the state-independent allocation $\{c_t, n_t\}$ gives higher welfare by increasing the agent's utility and output in period t. As such, there are no state-dependent allocations for consumption and hours worked in period t that can yield higher welfare than the allocation $\{c_t, n_t\}$. Since the optimal hours worked, n_t, is state-independent, it follows that $y_t(x) = k_t^\alpha n_t^{1-\alpha}$ is also state-independent in period t. Hence $D_t(x) = \frac{1}{\sigma} y_t(x)$ is also state-independent.

In the next period, the concavity of the production function $f(k, n)$ implies that a state-independent allocation, $\{k_{t+1}, n_{t+1}\} = \left\{\int k_{t+1}(x)dx, \int n_{t+1}(x)dx\right\}$, can yield at least as much output as any state-dependent allocation $\{n_{t+1}(x), k_{t+1}(x)\}$:

$$f(k_{t+1}, n_{t+1}) \geq \int f(k_{t+1}(x), n_{t+1}(x))dx. \tag{72}$$

For the same reason, we have $D_{t+1} \geq \int D_{t+1}(x)dx$ in equilibrium. It is also clear that the state-independent capital stock, $k_{t+1} = \int k_{t+1}(x)dx$, can cost no more resources in period t than the expected capital stock $\int k_{t+1}(x)dx$. Hence a state-independent allocation for k_{t+1} is optimal and it satisfies the budget constraints in both period t and period $t+1$ simultaneously. Therefore, the state-independent allocation $\{c_t, n_t, k_{t+1}\}$ is both feasible and optimal in period t. Since the utility function is increasing in c_t, the period-t budget constraint binds with equality under the optimal allocation $\{c_t, n_t, k_{t+1}\}$:

$$c_t + k_{t+1} - (1 - \delta)k_t = \frac{\sigma - 1}{\sigma} y_t + D_t. \tag{73}$$

The same logic applies to periods $t+1, t+2, ...$, and so on, *ad infinitum*. Therefore, the state-independent allocation $\{c_{t+s}, n_{t+s}, k_{t+s+1}\}_{s=0}^{\infty}$ is better than any random allocation. Under the state-independent allocation, the resource constraints (73) for all t are the same as the resource constraints specified in (63) (which hold with equality when $\phi = \frac{\sigma-1}{\sigma}$). Hence, the state-independent allocation is also equivalent to the allocation $\left\{c_t^*, n_t^*, k_{t-1}^*\right\}_{t=0}^{\infty}$ under the certainty equilibrium.■

Appendix 3: Solving the Sticky-Price Model

Using circumflex variables to denote deviations in log, $\hat{x} = \log(x_t/\bar{x})$, the log-linearized first-order conditions of the sticky-price model are summarized below:

$$(\alpha + \gamma)\hat{n}_t = \hat{\phi}_t + \alpha\hat{k}_t - \hat{c}_t \tag{74}$$

$$-\hat{c}_t = -E_t\hat{c}_{t+1} + (1-\beta(1-\delta))E_t[\hat{\phi}_{t+1} + (\alpha-1)\hat{k}_{t+1} + (1-\alpha)\hat{n}_{t+1}] \tag{75}$$

$$(1-s_i)c_t + s_i\left(\frac{1}{\delta}k_{t+1} - \frac{1-\delta}{\delta}k_t\right) = \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \tag{76}$$

$$\pi_t = \beta E_{t-1}\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}E_{t-1}\hat{\phi}_t \tag{77}$$

$$\hat{m}_t - E_{t-1}c_t = -\beta E_{t-1}(\pi_{t+1} + c_{t+1}) - (1-\beta)\hat{m}_t \tag{78}$$

$$\hat{m}_t + \pi_t = \hat{m}_{t-1} \tag{79}$$

where the last equation is based on the law of motion for the money stock, $\frac{M_t}{P_t} = \frac{M_{t-1}}{P_t} = \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}}$.[20]

The model is solved in two steps. In step one, we solve for the policy functions of inflation, real money demand, and expected marginal cost by taking the expectation for the above system of equations based on the lagged information set in period $t-1$. The saddle-path equilibrium solution takes the form,

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{m}_t \\ E_{t-1}\hat{\phi}_t \end{bmatrix} = A \begin{bmatrix} \hat{k}_t \\ \hat{m}_{t-1} \end{bmatrix}, \tag{80}$$

where A is a 3×2 coefficient matrix. In step two, since $\hat{\phi}_t$ can be written as $\hat{\phi}_t = E_{t-1}\hat{\phi}_t + \varepsilon_t$, where $\varepsilon_t \equiv \hat{\phi}_t - E_{t-1}\hat{\phi}_t$ is the sunspots shock to the expectation error of marginal cost, we can eliminate Equations (76)-(78) and substitute out $\{\pi, E_{t-1}\phi\}$ in the rest of the equations to get the following system of equations which are free of the lagged expectation operator E_{t-1}:

$$(\alpha+\gamma_n)\hat{n}_t = (\alpha + A_{3,1})\hat{k}_t + A_{3,2}\hat{m}_{t-1} - c_t + \varepsilon_t \tag{81}$$

$$-\hat{c}_t = -E_t\hat{c}_{t+1} + (1-\beta(1-\delta))E_t[(\alpha-1+A_{3,1})\hat{k}_{t+1} + A_{3,2}\hat{m}_t + (1-\alpha)\hat{n}_{t+1}] \tag{82}$$

$$(1-s_i)c_t + s_i\left(\frac{1}{\delta}k_{t+1} - \frac{1-\delta}{\delta}k_t\right) = \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \tag{83}$$

where $A_{i,j}$ denotes the $(i,j)^{th}$ element in A. These equations can be combined with the solution for real money balances in step one, $\hat{m}_t = A_{2,1}\hat{k}_t + A_{2,2}\hat{m}_{t-1}$, to solve for $\left\{\hat{n}_t, \hat{c}_t, \hat{k}_{t+1}\right\}$ as functions

[20] If there are money supply shocks in the model, we assume that the money supply is realized before the realization of sunspots.

of the states, $\left\{\hat{k}_t, \hat{m}_{t-1}, \varepsilon_t\right\}$. The fact that there is a unique fundamental equilibrium in this class of models implies that there are just enough explosive eigenvalues as control variables to solve for the saddle-path decision rules.

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