Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

Input and Output Inventory Dynamics

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March 14, 2008

Abstract

This paper develops an analytically tractable general equilibrium model of inventory dynamics. Inventories are introduced into a standard RBC model through a precautionary stockout-avoidance motive. Under persistent aggregate demand shocks, the model is broadly consistent with the U.S. business cycle and key features of inventory behavior, including (i) a large inventory stock-to-sales ratio and a small inventory investment-to-sales ratio in the long run, (ii) excess volatility of production relative to sales, (iii) procyclical inventory investment but countercyclical stock-to-sales ratio over the business cycle, and (iv) more volatile input inventories than output inventories. Similar results can also be obtained under persistent aggregate supply shocks.

Keywords: Inventory Behavior, Input and Output Inventory Investment, Aggregate Fluctuations, Stockout Avoidance, Stock-to-Sales Ratio, Business Cycle.

JEL codes: E13, E20, E32.

1 Introduction

Why are there always inventories? If prices are flexible and markets clear quickly, supply should always equal demand. Unlike capital investment, inventory investment generates a negative real rate of return (due to depreciation and storage costs, for example), making it strictly dominated by capital investment in terms of prospective yields. Thus, with regard to resource allocation, inventory investment is highly "inefficient". However, despite this inefficiency, inventory stocks are large; inventory investment is procyclical and accounts for the bulk of fluctuations in GDP (see, e.g., Blinder, 1981; Blinder and Maccini, 1991).

Inventory behavior is so unique and intriguing because of its paradoxical features. For example, the stock of finished goods inventories is about 60% of GDP and 90% of aggregate consumption on average. According to National Income and Product Accounts (NIPA), private inventories are three times larger than final sales of domestic business for the postwar period (1947-2007). On the other hand, inventory investment accounts for less than 1% of GDP on average, suggesting a remarkably low demand for inventory replenishment. However, inventory investment is extremely volatile and procyclical, making it the single largest contributor to the business cycle. For example, aggregate inventory investment is about 20 times more volatile than GDP and can account for up to 87% of the drop in GNP during the average postwar recession in the United States (Blinder and Maccini, 1991). Furthermore, finished goods inventories are procyclical only at the business cycle frequencies, but countercyclical at higher frequencies (Wen, 2005a).[1]

The economy accumulates not only inventories of finished goods, but also a large amount of inventories of intermediate goods (including raw materials and work-in-process). Intermediate goods inventories behave similarly to finished goods inventories over the business cycle, except they are larger in volume and more volatile. In the manufacturing sector, for example, the average inventory-to-sales ratio for intermediate goods is two times larger than that of finished goods, and input inventory investment can be three times more volatile (Humphreys, Maccini, and Schuh, 2001).[2] Input inventories arise whenever the delivery and usage of input materials differ. Because they provide the linchpin across stages of fabrication and between upstream and downstream firms in the chain of the production process,

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[1] Also see Hornstein (1998).

[2] Also see Feldstein and Auerbach (1976).

the dynamic interaction between input and output inventories is emphasized by Humphreys, Maccini, and Schuh (2001) as playing an important role in propagating the business cycle.

Although inventory investment is extremely volatile and strongly procyclical over the business cycle, the ratio of inventory stock to sales is countercyclical.[3] This is puzzling because it suggests that inventory stocks behave sluggishly and fail to keep up with sales. Bils and Kahn (2000) stress the importance of the countercyclical inventory-to-sales ratio in understanding the business cycle. According to Bils and Kahn (2000), the countercyclical inventory-to-sales ratio reflects procyclical marginal costs and countercyclical markups – which prevent production from keeping track of sales in booms.

Despite the importance of inventories in economic activities and for understanding the business cycle, general-equilibrium analysis of inventories has been surprisingly rare.[4] The bulk of the inventory literature uses partial-equilibrium models to analyze inventory behavior, and, in the analyses, interactions between input and output inventories are often neglected (Humphreys, Maccini, and Schuh, 2001).[5] Partial-equilibrium analysis treats prices, marginal costs, and sales as exogenous. Such practice fails to take into account the feedbacks from production and inventory decisions on sales and prices. Increases in inventories may negatively affect prices and stimulate demand. Hence, both prices and the volatility of sales are affected by production and inventory decisions. There have been attempts in the literature to include inventories in general-equilibrium models; however, their role is generally inconsistent with their definition (Kahn and Thomas, 2007a). For example, inventories are treated as a factor of production (similar to capital) by Kydland and Prescott (1982) and Christiano (1988), whereas they are treated as a source of household utility (similar to durable consumption goods) by Kahn, McConnell and Perez-Quiros (2001).

When inventory investment is strictly dominated by capital investment in the rate of return, to induce firms to hold inventories in general equilibrium requires additional frictions/benefits that give inventory investment a positive rate of return. One of the most important and obvious benefits for carrying inventoies is liquidity. As such, the challenge for modeling inventories in general equilibrium is similar to that of modeling money, which is strictly dominated by interest-bearing assets in the rate of return. This is why the classic

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[3]This can happen even if inventory stock itself is procyclical and more volatile than sales if there is a substantial delay in inventory replenishment.

[4]Exceptions include Fisher and Hornstein (2000), Kahn and Thomas (2007a, 2007b) and Wen (2005b).

[5]Important empirical works based on partial-equilibrium analysis include Blanchard (1983), Blinder (1986), Coen-Pirani (2004), Eichenbaum (1989), Haltiwanger and Maccini (1988), Kahn (1992), Ramey (1991), Ramey and West (1999), Wen (2005a), and West (1986), among many others.

money demand models are based on inventory theories (see, e.g., Baumol, 1952; and Tobin, 1956). The existing literature emphasizes two types of frictions to induce inventory holdings: fixed-cost friction and timing (or information) friction.[6] The traditional (S,s) model of inventories stresses the cost friction. According to the (S,s) theory, firms hold inventories because they face fixed costs of ordering inputs. To economize on these costs, firms choose to order infrequently by carrying inventories. The (S,s) theory has the potential to explain aggregate inventory dynamics but is computationally demanding and analytically less tractable.[7]

This paper develops an analytically tractable general equilibrium model of input and output inventories by focusing on the timing/information friction. Namely, inventories are introduced through a precautionary stockout-avoidance motive when stores/firms face delivery/production lags (Kahn, 1987).[8] To avoid losses of opportunity for prospective sales, they have an incentive to order/produce more than the expected sales so as to prevent stockouts. This leads to larger volatility in orders/production (relative to sales) and procyclical inventory investment. The stockout-avoidance motive also leads to an endogenous downward stickiness in prices. Namely, if the realized demand is low, rather than choosing to sell at a price below marginal cost, stores/firms opt to hold the excess supply as inventories, speculating that demand may be stronger in the future. Such rational "speculative" behavior attenuates downward pressure on prices. When realized demand is high, on the other hand, stores/firms draw down their inventories until a stockout occurs and price rises to clear the market. In other words, inventories exist in the economy not because markets "fail" to clear or firms are not able to adjust prices, but because production/delivery lags and rational "speculation" of higher profits from future sales induce firms to hold inventories as a form of liquidity.[9]

Under persistent aggregate demand shocks, the general-equilibrium model with a precautionary stockout-avoidance motive is broadly consistent with the key features of inventory

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[6]The conventional production smoothing theory of inventories is not a complete theory. It argues that firms hold inventories to reduce the volatility of production under demand uncertainty because the production function is concave. This theory fails to take into account the substitutability between inventory investment and capital investment in general equilibrium. Production smoothing can also be achieved by capital investment, which also yields higher rates of return than inventory investment. Hence, without additional frictions in general equilibrium, firms will not hold inventories despite the production-smoothing motive. This is why Kydland and Prescott (1982) put inventories in the production function to avoid the "rate of return dominance" problem.

[7]See, for example, Fisher and Hornstein (2000) and Khan and Thomas (2007a) for a general-equilibrium analysis of the (S,s) inventory model.

[8]Partial-equilibrium analysis of the stockout-avoidance inventory theory also includes Abel (1985), Bils and Kahn (2000), Brown and Haeglerb (2004), Kahn (1992), and Reagan (1982), among many others.

[9]See Samuelson (1971) for the analysis of inventory and speculative price behavior.

behavior, including (i) a large stock-to-sales ratio and a small inventory investment-to-GDP ratio in the steady state, (ii) excess volatility of production relative to sales, (iii) more volatile input inventories than output inventories, and (vi) procyclical inventory investment but countercyclical inventory-to-sales ratio at the business cycle frequencies.

The rest of the paper is organized as follows. To gain intuition, Section 2 presents a simple benchmark general-equilibrium model of inventories by embedding the partial-equilibrium model of Kahn (1987) and Bils and Kahn (2000) into a standard, perfectly competitive, RBC model. A social planner's version of the model is presented and analyzed. The model offers simple explanations as to why the inventory-to-sales ratio can be countercyclical when inventory investment is strongly procyclical. Section 3 extends the simple model by including both input and output inventories. A decentralized version of the model is presented. The model's dynamic properties under different types of aggregate shocks are studied. Section 4 concludes the paper with remarks for future research.

2   A Benchmark Model

The model is similar to a standard representative-agent RBC model with Dixit-Stiglitz production technologies. In this model, a final good is allocated between consumption (C) and capital investment (I) and is produced by the Dixit-Stiglitz aggregation function over intermediate goods, $C + I = \left[\int_0^1 \theta(i) y(i)^\rho di \right]^{\frac{1}{\rho}}$, where $\rho \in (0, 1)$ pertains to the elasticity of substitution across intermediate goods $y(i)$ and $\theta(i)$ represents idiosyncratic shocks that affect the optimal demand of $y(i)$. The distribution of θ is denoted by the CDF $F(\theta)$. The optimal supply of intermediate good i is denoted by $x(i)$. Without inventories, the resource constraint for intermediate good i is given by $y(i) \leq x(i)$. However, if there is inventory accumulation for good i, the resource constraint is given by $y_t(i) + s_t(i) \leq s_{t-1}(i) + x_t(i)$, where $s_t(i) \geq 0$ denotes the inventory stock of good i carried forward to the next period. Without loss of generality, a zero rate of depreciation for inventory stocks is assumed.

Intermediate goods are produced by the technology, $AK^\alpha N^{1-\alpha}$, where A represents aggregate technology shocks with the law of motion, $\log A_t = \log A_{t-1} + \varepsilon_{at}$; K the aggregate capital stock and N the aggregate labor. Intermediate goods are homogenous from the viewpoint of the upstream supplier; hence, the aggregate resource constraint for the supply of intermediate goods is $\int x(i) di \leq AK^\alpha N^{1-\alpha}$. However, these goods are heterogenous from the viewpoint of the downstream because of the idiosyncratic component in their demand

curves, $\theta(i)$, which renders the shadow values of intermediate goods different across i.

To meet the random demand for intermediate good i from the downstream, the amount $x(i)$ must be ordered in advance before $\theta(i)$ is realized in each period. This information lag creates a precautionary stockout-avoidance motive for carrying inventories. The decisions regarding $y(i)$ and $s(i)$ are not subject to this information lag. In addition, aggregate shocks are realized in the beginning of each period before all decisions in the period are made.

A social planner or representative agent in the economy chooses $\{C_t, K_{t+1}, N_t, y_t(i), x_t(i), s_{t+1}(i)\}$ to solve the following program,

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \Theta_t \frac{C_t^{1-\gamma}}{1-\gamma} - a\frac{N_t^{1+\gamma_n}}{1+\gamma_n} \right\}$$

subject to

$$C_t + K_{t+1} - (1-\delta_k)K_t \leq \left[\int_0^1 \theta(i)y(i)^\rho di \right]^{\frac{1}{\rho}}, \tag{μ 1}$$

$$y_t(i) + s_t(i) \leq s_{t-1}(i) + x_t(i), \tag{λ_i 2}$$

$$s_t(i) \geq 0, \tag{π_i 3}$$

$$\int_0^1 x_t(i) di \leq A_t K_t^\alpha N_t^{1-\alpha}; \tag{v 4}$$

where Θ represents aggregate shocks to consumption demand with the law of motion, $\log \Phi_t = \log \Phi_{t-1} + \varepsilon_{\phi t}$.

2.1 First-Order Conditions

Denoting $\tilde{Y} \equiv \left[\int_0^1 \theta(i)y(i)^\rho di \right]^{\frac{1}{\rho}}$, $X \equiv A_t K_t^\alpha N_t^{1-\alpha}$, and $\{\mu, \lambda(i), \pi(i), v\}$ as the non-negative Lagrangian multipliers for the constraints (1)-(4), respectively, the first-order conditions are given by

$$\Theta_t C_t^{-\gamma} = \mu_t \tag{5}$$

$$aN_t^{\gamma_n} = v_t(1-\alpha)\frac{X_t}{N_t} \tag{6}$$

$$\mu_t = \beta(1-\delta_k)E_t\mu_{t+1} + \beta\alpha E_t\left(v_{t+1}\frac{X_{t+1}}{K_{t+1}} \right) \tag{7}$$

6

$$\mu_t \tilde{Y}_t^{1-\rho} \theta_t(i) y_t(i)^{\rho-1} = \lambda_t(i) \tag{8}$$

$$v_t = E_t \lambda_t(i) \tag{9}$$

$$\lambda_t(i) = \beta E_t \lambda_{t+1}(i) + \pi_t(i), \tag{10}$$

plus the transversality conditions, $\lim_{T \to \infty} \beta^T E \mu_T K_{T+1} = 0, \lim_{T \to \infty} \beta^T E \lambda_T(i) s_T(i) = 0$, and the complementary slackness condition, $s_t(i)\pi_t(i) = 0$, for all $i \in [0,1]$.

Notice that equation (9) reflects the information lag in ordering intermediate goods $x(i)$. Without the information lag, equation (9) becomes $v_t = \lambda_t(i)$. Equation (10) then implies $\pi_t(i) = v_t - \beta E_t v_{t+1} > 0$.[10] Hence, it is not optimal to carry inventories when the value of θ is known; thus, $s(i) = 0$ for all i. Given this, we have $y(i) = x(i), \int y(i) di = X$; and equation (8) implies $\mu_t \tilde{Y}_t = v_t X_t$ and $v_t = \bar{\theta} \mu_t$, where the constant coefficient $\bar{\theta}$ is given by $\bar{\theta} \equiv \left[\int_0^1 \theta(i)^{\frac{1}{1-\rho}} di \right]^{\frac{1-\rho}{\rho}} = \left[\int \theta^{\frac{1}{1-\rho}} dF(\theta) \right]^{\frac{1-\rho}{\rho}}$, by the law of large numbers. Consequently, the first-order conditions (6) and (7) become $a N_t^{\gamma_n} = \mu_t (1-\alpha) \frac{\tilde{Y}_t}{N_t}$ and $\mu_t = \beta E_t \mu_{t+1} \left[\alpha \frac{\tilde{Y}_{t+1}}{K_{t+1}} + 1 - \delta_k \right]$, respectively; and the aggregate resource constraint becomes $C + I = \tilde{Y} = \bar{\theta} A K^\alpha N^{1-\alpha}$. Therefore, without the information lag, the model is reduced to a standard one-sector RBC model. Obviously, the model is also reduced to a standard one-sector RBC model if there are no idiosyncratic shocks, $\theta(i) = 1$ for all i. In this case, $\bar{\theta} = 1, y(i) = \tilde{Y} = X$, and $C + I = A K^\alpha N^{1-\alpha}$. However, with idiosyncratic shocks and the information lag, the model is no longer reduceable to a standard one-sector RBC model and inventories will play an important role in aggregate dynamics.

In the above setup, aggregate shocks do not play a role in the existence of inventories.[11] This feature makes the model analytically tractable because the decision rules for inventories can be solved by taking the aggregate variables as given. Then in equilibrium and by the law of large numbers, there is always a positive measure of intermediate goods to be associated with inventories. Hence, the aggregate inventory stock is strictly positive and the log-linearization technique can be applied to analyzing the model's aggregate dynamics.

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[10] Suppose this is not true and $\pi(i) = 0$; then $v_t = \beta E_t v_{t+1}$, which implies $v_t \to 0$ as time increases. Since the utility function is strictly increasing, the resource constraint (4) binds with equality in equilibrium, hence implying $v_t > 0$. This is a contradiction.

[11] This is a consequence of the "rate-of-return dominance" by capital investment and the lack of information friction with respect to aggregate shocks. Introducing information frictions at the aggregate level is possible but it may not have significant value added to the results.

2.2 Decision Rules for Inventories

The key to solving for the decision rules in the intermediate goods sector is to determine the optimal stock, $x_t(i) + s_{t-1}(i)$, based on the distribution of θ. The first-order condition for $x(i)$ is given by (9), which suggests that the optimal level of orders depends on the expected shadow value of inventory, $E_t \lambda_t(i)$. Under the law of iterated expectations, we have $E_t \lambda_{t+1}(i) = E_t v_{t+1}$; hence, equations (9) and (10) imply

$$\lambda_t(i) = \beta E_t v_{t+1} + \pi_t(i). \tag{11}$$

Therefore, the decision rules for the intermediate goods sector are characterized by an optimal cutoff value of the idiosyncratic shock, θ^*, such that the non-negativity constraint (3) on inventory is slack if $\theta(i) \leq \theta^*$, and it binds if $\theta(i) > \theta^*$. Thus, there are two possible cases to consider.

Case A: In the case where $\theta(i) \leq \theta^*$, we have $\pi(i) = 0, s(i) \geq 0$, and $\lambda_t(i) = \beta E_t v_{t+1}$. The resource constraint (2) implies $y(i) \leq x(i) + s_{t-1}(i)$. Since equation (8) implies $y_t(i) = \left[\frac{\mu_t \tilde{Y}_t^{1-\rho} \theta_t(i)}{\beta E_t v_{t+1}} \right]^{\frac{1}{1-\rho}}$, we have $\theta(i) \leq [x(i) + s_{t-1}(i)]^{1-\rho} \left[\frac{\beta E_t v_{t+1}}{\mu_t \tilde{Y}_t^{1-\rho}} \right] \equiv \theta^*$, which defines the optimal cutoff value θ^* and the optimal stock as $x(i) + s_{t-1}(i) \equiv \left[\frac{\mu_t \tilde{Y}_t^{1-\rho} \theta^*}{\beta E_t v_{t+1}} \right]^{\frac{1}{1-\rho}}$.

Case B: In the case where $\theta(i) > \theta^*$, we have $\pi(i) > 0, s(i) = 0$, and $y(i) = x(i) + s_{t-1}(i) \equiv \left[\frac{\mu_t \tilde{Y}_t^{1-\rho} \theta^*}{\beta E_t v_{t+1}} \right]^{\frac{1}{1-\rho}}$. Equation (8) then implies $\lambda_t(i) = \beta E_t v_{t+1} \frac{\theta_t(i)}{\theta^*} > \beta E_t v_{t+1}$.

Given these two possibilities, equation (9) can be written as

$$v_t = \int_{\theta(i) \leq \theta^*} (\beta E_t v_{t+1}) \, dF(\theta) + \int_{\theta(i) > \theta^*} (\beta E_t v_{t+1}) \frac{\theta_t(i)}{\theta^*} dF(\theta), \tag{12}$$

where the LHS is the marginal cost of inventory, the first term on the RHS is the shadow value of inventory when there is excess supply, and the second term is the shadow value of inventory when there is a stockout. Thus, the optimal cutoff value is determined at the point where the marginal cost equals the expected marginal benefit. Since aggregate variables are independent of idiosyncratic shocks, equation (12) can be written as

$$v_t = \beta E_t v_{t+1} R(\theta_t^*), \tag{13}$$

where $R(\theta^*) \equiv F(\theta^*) + \int_{\theta(i) > \theta^*} \frac{\theta(i)}{\theta^*} dF(\theta) > 1$ measures the rate of returns to liquidity or inventory investment. Notice that the optimal cutoff value θ_t^* is time varying and $\frac{dR(\theta^*)}{d\theta^*} < 0$.

The rate of return negatively depends on the cutoff value because a higher cutoff value implies a larger probability of excess supply and a smaller probability of stockout, which lowers the value of inventory. Given aggregate economic conditions, equation (14) solves the optimal cutoff value as $\theta_t^* = R^{-1}\left(v_t/\beta E v_{t+1}\right)$.

The decision rules for the intermediate goods sector are thus given by

$$x_t(i) + s_{t-1}(i) = \tilde{Y}_t \left[\frac{\mu_t \theta_t^*}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{14}$$

$$y_t(i) = \tilde{Y}_t \left[\frac{\mu_t}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}} \times \min\left\{\theta_t(i)^{\frac{1}{1-\rho}}, \theta_t^{*\frac{1}{1-\rho}}\right\}, \tag{15}$$

$$s_t(i) = \tilde{Y}_t \left[\frac{\mu_t}{\beta E_t v_{t+1}}\right]^{\frac{1}{1-\rho}} \times \max\left\{\theta_t^{*\frac{1}{1-\rho}} - \theta_t(i)^{\frac{1}{1-\rho}}, 0\right\}. \tag{16}$$

The shadow price of inventory i is determined by

$$\lambda_t(i) = \beta E_t v_{t+1} \times \max\left\{1, \frac{\theta(i)}{\theta^*}\right\}, \tag{17}$$

which is downward sticky with respect to the demand shock ($\theta(i)$). That is, the price of inventory does not decrease to "clear" the market when demand is low ($\theta \leq \theta^*$). Rather than choosing to sell the good at a price below the shadow value ($\beta E_t v_{t+1}$), firms opt to hold any excess supply as inventories ($s_t(i) > 0$), speculating that demand may be stronger in the future. On the other hand, when demand is high ($\theta > \theta^*$), firms draw down inventories and price rises with θ to clear the market ($\lambda(i) = \beta E_t v_{t+1} \frac{\theta(i)}{\theta^*}$). The optimal cutoff θ^* determines the probability of stockouts and is determined endogenously at the point where the average profit is zero ($E\lambda(i) - v = 0$). The asymmetric price behavior will be averaged out across a large number of firms; hence, it will not show up at the aggregate level in the model.

Notice that $\frac{\mu_t}{\beta E_t v_{t+1}} = \frac{\mu_t}{v_t} \frac{v_t}{\beta E_t v_{t+1}} = \frac{\mu_t}{v_t} R(\theta_t^*)$. Hence, equation (14) shows that the optimal stock of intermediate good i, $x_t(i) + s_{t-1}(i)$, is determined entirely by four aggregate factors: the level of aggregate output ($\tilde{Y}$), the ratio of marginal utility of aggregate output to the marginal cost of aggregate intermediate good ($\frac{\mu}{v}$), the rate of return to inventory investment (R), and the optimal cutoff value (θ^*). The ratio $\frac{\mu}{v}$ can be interpreted as a pseudo measure

9

of aggregate markup for intermediate goods.[12] Such a decomposition is reminiscent of the decomposition of Bils and Kahn (2000). Among the four factors, the cutoff value is clearly countercyclical because a higher demand for intermediate goods increases the marginal cost of production (v), which calls for a higher rate of return to inventory in order to induce firms to order more intermediate goods. Hence, the optimal cutoff value must be lowered according to equation (12), holding expected future marginal cost constant.[13] On the other hand, the pseudo markup and the rate of return are both procyclical; otherwise, there would be no incentive to increase the stock of inventory for any given level of aggregate output.[14] This intuition will be confirmed by impulse response analysis below.

2.3 Aggregate Dynamics

Defining the aggregate variables, $Y \equiv \int y(i)di, S \equiv \int s(i)di$, and aggregating the decision rules (14)-(16) under the law of large numbers gives

$$Y_t = \tilde{Y}_t \left[\frac{\mu_t}{\beta E_t v_{t+1}} \right]^{\frac{1}{1-\rho}} D(\theta_t^*) \tag{17}$$

$$X_t + S_{t-1} = Y_t \frac{D(\theta_t^*) + H(\theta_t^*)}{D(\theta_t^*)} \tag{18}$$

$$S_t = Y_t \frac{H(\theta_t^*)}{D(\theta_t^*)}, \tag{19}$$

and aggregating the first-order condition (8) gives

$$v_t = \mu_t R(\theta_t^*) G(\theta_t^*)^{\frac{1-\rho}{\rho}}, \tag{20}$$

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[12] The model is equivalent to a perfectly competitive economy, and the true measure of gross aggregate markup for intermediate goods is $\frac{E\lambda}{v} = 1$.

[13] The same argument holds as long as changes in the expected future marginal cost are less than that of current marginal cost.

[14] That is, a shock to aggregate demand (Φ) raises the marginal utility of consumption (μ) more than the marginal cost (v) or the expected marginal cost ($\beta E_t v_{t+1}$); otherwise consumption would not react positively.

where

$$D(\theta^*) \equiv \int_{\theta(i) \leq \theta^*} \theta(i)^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta(i) > \theta^*} \theta^{*\frac{1}{1-\rho}} dF(\theta) > 0, \qquad (21)$$

$$H(\theta^*) \equiv \int_{\theta(i) \leq \theta^*} \left[\theta^{*\frac{1}{1-\rho}} - \theta(i)^{\frac{1}{1-\rho}} \right] dF(\theta) > 0,$$

$$\theta^{*\frac{1}{1-\rho}} = D(\theta^*) + H(\theta^*),$$

$$G(\theta^*) \equiv \int_{\theta(i) \leq \theta^*} \theta(i)^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta(i) > \theta^*} \theta(i) \theta^{*\frac{\rho}{1-\rho}} dF(\theta) > D(\theta^*).$$

The aggregate resource constraint (1) can be written as

$$C_t + K_{t+1} - (1 - \delta_k) K_t = P_t \left(A_t K_t^\alpha N_t^{1-\alpha} + S_{t-1} - S_t \right), \qquad (22)$$

where $P \equiv G(\theta^*)^{\frac{1}{\rho}} D(\theta^*)^{-1}$ measures the relative price of intermediate goods with respect to the final good.

Recall that in a standard RBC model without inventories, $v_t = \mu_t$ in the case of $\theta(i) = 1$ and $v_t = \bar{\theta} \mu_t$ in the case of no information lag. In both cases the pseudo measure of markup ($\frac{\mu_t}{v_t} = \left\{ 1, \bar{\theta}^{-1} \right\}$) and the relative price of intermediate goods ($\frac{\tilde{Y}}{Y} = \{ 1, \bar{\theta} \}$) are constant. However, when there are inventories, the pseudo markup is given by $R(\theta_t^*) G(\theta_t^*)^{\frac{1-\rho}{\rho}}$ and the relative price is given by $G(\theta_t^*)^{\frac{1}{\rho}} D(\theta_t^*)^{-1}$, which are no longer constant. Thus, inventories bring about important changes to aggregate dynamics and relative price movements.

By equation (13), the optimal cutoff variable θ_t^* is stationary even under permanent shocks. Hence, the aggregate decision rules (18) and (19) indicate that aggregate inventory stock and sales are cointegrated. The decision rules also show that the aggregate stock-to-sales ratio for intermediate goods exceeds one, $\frac{X_t + S_{t-1}}{Y_t} = \frac{D(\theta^*) + H(\theta^*)}{D(\theta^*)} > 1$, and the aggregate inventory-to-sales ratio is strictly positive, $\frac{S_t}{Y_t} = \frac{H(\theta^*)}{D(\theta^*)} > 0$. Since $\frac{X_t + S_{t-1}}{Y_t} = 1 + \frac{S_t}{Y_t}$, if either one of these ratios is countercyclical, so is the other. These predictions are consistent with the empirical facts.[15] To see the dynamic behavior of $\frac{X_t + S_t}{Y_t}$, notice that $H + D = \theta^{*\frac{1}{1-\rho}}$ and both functions of H and D are increasing in θ^*: $\frac{dH(\theta^*)}{d\theta^*} = \frac{1}{1-\rho} \theta^{*\frac{\rho}{1-\rho}} F(\theta^*) > 0$, and $\frac{dD(\theta^*)}{d\theta^*} = \frac{1}{1-\rho} \theta^{*\frac{\rho}{1-\rho}} [1 - F(\theta^*)] > 0$, where $F(\theta) \equiv \Pr[\theta \leq \theta^*]$.[16] Given a small change in θ^*, the change

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[15] See, e.g., Bils and Kahn (2000) and Kahn (1992).

[16] The function $G(\theta^*)$ also increases with θ^*.

in $\frac{X_t + S_{t-1}}{Y_t}$ is given by $\frac{d}{d\theta^*}\left(\theta^{*\frac{1}{1-\rho}}/D\right) = \frac{1}{D^2}\frac{1}{1-\rho}\theta^{*\frac{\rho}{1-\rho}}\left[D - \theta^{*\frac{1}{1-\rho}}(1-F)\right]$, which is positve if

$D > \theta^{*\frac{1}{1-\rho}}(1-F)$. This is clearly true because $D = \theta^{*\frac{1}{1-\rho}}(1-F) + \int_{\theta(i)\le\theta^*}\theta(i)^{\frac{1}{1-\rho}}dF(\theta)$ by

(21). Hence, the stock-to-sales ratio comoves with the optimal cutoff variable θ_t^*. By equation

(13), θ_t^* is determined completely by movement in the marginal cost and is countercyclical

to the marginal cost (v_t). Thus, if the marginal cost (or its forward growth rate $\frac{v_t}{\beta E v_{t+1}}$) is

procyclical (which is the case under aggregate demand shocks), then the stock-to-sales ratio

will be countercyclical.

Assume $\theta(i)$ follows the Pareto distribution, $F(\theta) = 1 - \left(\frac{1}{\theta}\right)^\sigma$, with support $\theta \in (1,\infty)$

and the shape parameter $\sigma > 1$. With this distribution, closed-form solutions for θ^* and

the other functions in (21) are available. Combinations of the two parameters, $\{\rho,\sigma\}$, can

generate essentially any sensible values for the inventory-to-sales ratio in the steady state.

For example, consider $\rho = 0.1$ and $\sigma = 3$, then equation (13) implies $\theta^* = \left[\frac{\beta}{1-\beta}\frac{1}{\sigma-1}\right]^{\frac{1}{\sigma}}$.[17]

At a quarterly frequency, if $\beta = 0.99$, then $\theta^* = 3.2$, $\frac{S}{Y} = 1.76$, and $\frac{X+S}{Y} = 2.76$.[18] These

numbers suggest that the economy is willing to hold a very large amount of inventories

under the stockout-avoidance motive. On the other hand, the ratio of inventory investment-

to-sales is given by $\delta\frac{S}{Y}$ in the steady state, which approaches zero if the depreciation rate

of inventories (δ) approaches zero. This suggests that a large inventory stock-to-sales ratio

is fully consistent with a small inventory investment-to-sales ratio as long as the rate of

depreciation is small. These predictions are qualitatively consistent with the U.S. data.

Suppose the structural parameters take the following values at a quarterly frequency:

$\alpha = 0.3$, $\beta = 0.99$, $\delta_k = 0.025$, $\delta = 0.015$, $\gamma_n = 0.25$, $\gamma = 1$, $\rho = 0.1$, and $\sigma = 3$.

The impulse responses of inventory investment and the inventory-to-sales ratio $\left(\frac{S_t}{Y_t}\right)$ to one-

standard-deviation aggregate shocks are graphed in Figure 1. The window on the left shows

responses of inventory investment to an aggregate demand shock (circles) and an aggregate

technology shock (triangles). The window on the right shows responses of inventory-to-stock

ratio to a demand shock and a technology shock, respectively. Under aggregate demand

shocks, aggregate inventory investment is procyclical and far more volatile than aggregate

output (Y). However, the inventory-to-sales ratio (as well as the total stock-to-sales ratio,

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[17]An interior solution requires $\theta^* > 1$ so that the cutoff value is within the support of the distribution.
This conditions requires $1 < \sigma < \frac{1}{1-\beta}$.

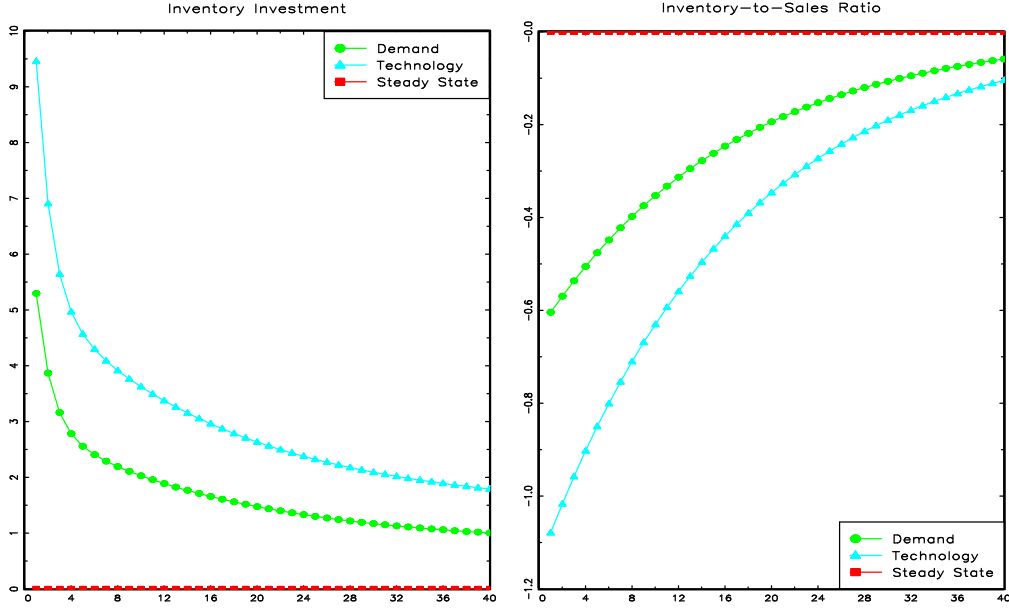[18]These long-run ratios increase with ρ and decrease with σ.

Figure 1: Figure 1. Impulse Responses to Demand & Technology Shocks.

$\frac{X_t + S_{t-1}}{Y_t}$) are countercyclical. In the meantime, the pseudo measure of the markup $\left(\frac{\mu}{v}\right)$ and the rate of return to inventory investment $(R(\theta^*))$ are both procyclical. Interestingly, the same results obtain under aggregate TFP shocks, which is contrary to the arguments of Bils and Kahn (2000), which are based on partial-equilibrium analyses of the stockout-avoidance model.

Under technology shocks, the marginal cost (v_t) is countercyclical. This would imply that θ^* as well as the stock-to-sales ratio are procyclical. However, if the shocks are persistent enough, then the expected future marginal cost can decrease even more than the current marginal cost because of capacity accumulation, rendering the ratio $\frac{v_t}{\beta E v_{t+1}}$ procyclical. Hence, by equation (13), the cutoff variable θ^* and the stock-to-sales ratio can become countercyclical. The partial-equilibrium model of Bils and Kahn (2000) is not able to reveal this property because their analysis is subject to the Lucas (1976) critique.[19]

3   The Full Model

This section enriches the benchmark model in several dimensions so as to explain, among other things, two important stylized facts regarding inventory dynamics. First, input inventories are more volatile than output inventories (Humphreys, Maccini, and Schuh 2001);

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[19]Namely, the distribution of shocks can affect the coefficients in the decision rules.

and second, finished goods inventories are countercyclical at the high frequencies (Hornstein, 1998; Wen, 2005a).

3.1 Household

A representative household has preferences over a spectrum of finished goods indexed by $j \in [0,1]$. From the producer's point of view, these goods are the same (homogenous) because they are produced by the same production technology with the same costs; but they have different colors and yield different utilities to the household. In other words, these goods are not perfect substitutes in the household's utility function. The household purchases these finished goods in different colors in a competitive market and is able to store them in refrigerators if needed (refrigerator j stores good j).[20] The cost for storing goods is the depreciation rate $\delta > 0$ and the discounting of the future. The marginal utility of consumption of good j is subject to idiosyncratic taste shocks, $\theta_1(j)$, with distribution $F(\theta) = \Pr[\theta_1 \leq \theta]$. These taste shocks are not known to the household when orders (purchases) are made.[21] Hence, to cope with the idiosyncratic uncertainty, the household has incentive to store inventories of goods with all colors to avoid stockouts. The problem of the household is to solve

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\Phi_t}{1-\gamma} \left[\int_0^1 \theta_{1t}(j) c_t(j)^\rho dj \right]^{\frac{1-\gamma}{\rho}} - a \frac{N_t^{1+\gamma_n}}{1+\gamma_n} \right\}$$

subject to

$$c_t(j) + s_{1t}(j) \leq (1-\delta)s_{1t-1}(j) + y_t(j) \tag{λ_{1i} 23}$$

$$s_{1t}(j) \geq 0 \tag{π_{1i} 24}$$

$$\int_0^1 y_t(j)dj + W_{t+1} \leq (1+r_t)W_t + w_t N_t + \Pi_t, \tag{μ 25}$$

where $\delta \in [0,1]$ is the depreciation rate of finished goods inventories (s_1), r is the interest rate on aggregate wealth (W), w is the real wage, and Π is total profit income distributed from firms. The parameters in the utility function satisfy standard restrictions: $\rho \in (0,1), \gamma \geq 0$, and $\gamma_n \geq 0$.

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[20] Refrigerators in the model are a metaphor for retail stores in the real world. According to Blinder (1981), most of finished goods inventories are held by the retail sector rather than the manufacturing sector.

[21] For example, the household must go shopping in the morning and idiosyncratic taste shocks arrive at noon.

3.2 Firms

Final Goods. Final goods are produced competitively under the technology

$$\tilde{Y} = AK^\alpha \tilde{M}^{1-\alpha}, \tag{26}$$

where $\tilde{M}$ is a composite of intermediate goods. The price of the composite good is P^m. The problem of final goods firms is to solve

$$\max \left\{ A_t K_t^\alpha \tilde{M}_t^{1-\alpha} - (r_t + \delta_k)K_t - \frac{\xi}{2\bar{K}}(K_t - \bar{K})^2 - P_t^m \tilde{M}_t \right\},$$

where $(r_t + \delta_k)$ is the user's cost of capital with δ_k as the depreciation rate of capital, and $\xi \geq 0$ is the coefficient for a quadradic adjustment cost of capital relative to its steady state $(\bar{K})$.

Intermediate Goods. In this sector a representative firm uses labor to produce intermediate goods $m(i)$. These intermediate goods come with different colors indexed by $i \in [0,1]$. They are used to synthesize the composite good $\tilde{M}$ according to the aggregation technology, $\tilde{M} = \left[\int \theta_2(i)m(i)^\rho di \right]^{\frac{1}{\rho}}$. That is, the marginal revenue product of intemediate goods are subject to idiosyncratic shocks, $\theta_2(i)$, which generate idiosyncratic uncertainty for the demand of intermediate goods of different colors. Assume θ_2 has the same distribution $F(\theta)$. Intermediate goods are produced under identical linear technologies, $Bn(i)$, where B is an aggregate cost shock to labor's productivity. This shock differs from the TFP shock because it does not directly affect the rate of return to capital investment. The labor market is perfectly competitive and the labor used in producing intermediate good i is a perfect substitute for that used in producing other intermediate goods. However, labor must be determined before the idiosyncratic shocks (θ_2) are realized in each period. Therefore, intermediate goods firms have incentive to keep inventories of work-in-process (s_2) in all colors so as to maximize expected profits. The problem of a representative intermediate goods firm is to solve

$$\max E \sum_{t=0}^{\infty} \beta^t \frac{\mu_{t+1}}{\mu_0} \left\{ P_t^m \left[\int \theta_{2t}(i)m_t(i)^\rho di \right]^{\frac{1}{\rho}} - w_t \int n_t(i)di \right\}$$

subject to

$$m_t(i) + s_{2t}(i) \leq (1-\delta)s_{2t-1}(i) + B_t n_t(i), \tag{λ_{2i} 27}$$

$$s_{2t}(i) \geq 0, \tag{π_{2i} 28}$$

15

where μ in the objective function denotes the marginal utility of the final good (i.e., $\frac{\mu_t}{\mu_{t-1}} = 1 + r_t$ is the real interest rate).

3.3   First-Order Conditions

Denoting $\{\lambda_1, \pi_1, \mu\}$ as the Lagrangian multipliers of Equations (23)-(25) for the household, respectively, the first-order conditions of the household are given by

$$aN_t^{\gamma_n} = \mu_t w_t \tag{29}$$

$$\mu_t = \beta E_t \mu_{t+1} (1 + r_{t+1}) \tag{30}$$

$$\Phi_t \tilde{C}_t^{1-\rho-\gamma} \theta_1(j) c(j)^{\rho-1} = \lambda_1(j) \tag{31}$$

$$\mu_t = E\lambda_{1t}(j) \tag{32}$$

$$\lambda_{1t}(j) = \beta(1-\delta)E_t\mu_{t+1} + \pi_{1t}(j), \tag{33}$$

plus the transversality condition $\lim_{T\to\infty} \beta^T E\mu_T W_{T+1} = 0, \lim_{T\to\infty} \beta^T E\lambda_{1T} s_{1T+1} = 0$, and the complementary slackness conditions, $s_{1t}(j)\pi_{1t}(j) = 0$ for all j. Equation (29) determines the optimal labor supply, (30) the optimal wealth accumulation, (31) the optimal level of consumption of color j, (32) the optimal orders of good with color j, and (33) the optimal inventory holdings of color j. Notice that the optimal orders are made before the realization of θ; hence, the household must form expectations regarding the shadow value of the final consumption good.

The first-order conditions for the final good firm are given by

$$r_t + \delta_k + \frac{\xi}{\bar{K}}(K_t - \bar{K}) = \alpha A_1 K_t^{\alpha-1} \tilde{M}_t^{1-\alpha}, \quad P_t^m = (1-\alpha)A_t K_t^\alpha \tilde{M}_t^{-\alpha}. \tag{34}$$

Denoting $\{\lambda_2, \pi_2\}$ as the Lagrangian multipliers for Equations (27) and (28), respectively, for the intermediate goods firm, the first order conditions are given by

$$P_t^m \tilde{M}_t^{1-\rho} \theta_2(i) m(i)^{\rho-1} = \lambda_2(i) \tag{35}$$

$$\frac{w_t}{B_t} = E\lambda_2(i) \tag{36}$$

$$\lambda_{2t} = \beta(1-\delta)E_t \frac{\mu_{t+1}}{\mu_t} \lambda_{2t+1}(i) + \pi_{2t}(i), \tag{37}$$

16

plus a transversality condition, $\lim_{T \to \infty} \beta^T E \lambda_{2T} s_{2T} = 0$, and the complementary slackness conditions, $s_{2t}(i)\pi_t(i) = 0$ for all i. Equation (35) determines the optimal usage of the intermediate good with color i, equation (36) the optimal production of the intermediate good i, and (37) the optimal accumulation of inventories of work-in-process for color i.

3.4 Decision Rules of Inventories

The decision rules associated with inventories are derived in a similar manner as in the benchmark model. The decision rules for finished goods inventories are given by

$$\mu_t = \beta(1 - \delta)E_t\mu_{t+1}R(\theta^*_{1t}), \tag{38}$$

$$y_t(i) + s_{1t-1}(i) = \tilde{C}_t \left[\frac{\Delta_t\theta^*_{1t}}{\beta(1 - \delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{39}$$

$$c_t(i) = \tilde{C}_t \left[\frac{\Delta_t}{\beta(1 - \delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}} \times \min\left\{\theta_{1t}(i)^{\frac{1}{1-\rho}}, \theta^*_{1t}{}^{\frac{1}{1-\rho}}\right\}, \tag{40}$$

$$s_{1t}(i) = \tilde{C}_t \left[\frac{\Delta_t}{\beta(1 - \delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}} \times \max\left\{\theta^*_{1t}{}^{\frac{1}{1-\rho}} - \theta_{1t}(i)^{\frac{1}{1-\rho}}, 0\right\}, \tag{41}$$

where $\Delta \equiv \Phi_t \tilde{C}_t^{-\gamma}$ denotes the marginal utility of the composite consumption good $\tilde{C} \equiv \left[\int \theta c(i)^\rho di\right]^{\frac{1}{\rho}}$, and $R(\theta^*_1)$ the rate of return to inventory investment in finished goods. The optimal cutoff value in the finished goods industry is determined by equation (38).

The decision rules for input inventories are given by

$$\frac{w_t}{B_t} = \beta(1 - \delta)E_t\tilde{\mu}_{t+1}R(\theta^*_{2t}) \tag{42}$$

$$Bn_t(i) + s_{2t-1}(i) = \tilde{M}_t \left[\frac{P_t^m\theta^*_{2t}}{\beta(1 - \delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{43}$$

$$m_t(i) = \tilde{M}_t \left[\frac{P_t^m}{\beta(1 - \delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}} \times \min\left\{\theta^*_{2t}{}^{\frac{1}{1-\rho}}, \theta_{2t}(i)^{\frac{1}{1-\rho}}\right\}, \tag{44}$$

$$s_{2t}(i) = \tilde{M}_t \left[\frac{P_t^m}{\beta(1 - \delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}} \times \max\left\{\theta^*_{2t}{}^{\frac{1}{1-\rho}} - \theta_{2t}(i)^{\frac{1}{1-\rho}}, 0\right\}, \tag{45}$$

where $\tilde{\mu}_{t+1} \equiv \frac{\mu_{t+1}}{\mu_t} \frac{w_{t+1}}{B_{t+1}}$ denotes the next-period marginal cost of labor discounted by the interest rate (the ratio of the marginal utilities of the final good) and $R(\theta_2^*)$ denotes the rate of return to inventory investment in goods-in-process. The optimal cutoff value in the input inventory industry is determined by equation (42). Notice that equations (38) and (42) are analogous to equation (13).

3.5 Aggregate Dynamics

Market clearing in the asset and labor markets imply $W_t = K_t$ and $N_t = \int n(i)di$. Define $C \equiv \int c(i)di, Y = \int y(i)di, S_1 \equiv \int s_1(i)di, S_2 \equiv \int s_2(i)di$, and $M \equiv \int m(i)di$. Aggregating the decision rules (39)-(41) for the finished-good sector under the law of large numbers gives

$$\mu_t = \Delta_t R(\theta_1^*)G(\theta_1^*)^{\frac{1-\rho}{\rho}} \tag{45}$$

$$C_t = \tilde{C}_t D(\theta_1^*)G(\theta_1^*)^{-\frac{1}{\rho}} \tag{46}$$

$$Y_t + (1-\delta)S_{1t-1} = C_t \frac{D(\theta_{1t}^*) + H(\theta_{1t}^*)}{D(\theta_{1t}^*)} \tag{47}$$

$$S_{1t} = C_t \frac{H(\theta_{1t}^*)}{D(\theta_{1t}^*)}, \tag{48}$$

where (45) is analogous to (20) and the rest are analogous to (17)-(19).[22] The functions $\{G(\theta), D(\theta), H(\theta)\}$ are the same as those defined in (21). The aggregate decision rules for the input inventory sector are similarly given by

$$\frac{w_t}{B_t} = P_t^m R(\theta_2^*)G(\theta_2^*)^{\frac{1-\rho}{\rho}} \tag{49}$$

$$M_t = \tilde{M}_t D(\theta_2^*)G(\theta_2^*)^{-\frac{1}{\rho}} \tag{50}$$

$$B_t N_t + (1-\delta)S_{2t-1} = M_t \frac{D(\theta_{2t}^*) + H(\theta_{2t}^*)}{D(\theta_{2t}^*)} \tag{51}$$

$$S_{2t} = M_t \frac{H(\theta_{2t}^*)}{D(\theta_{2t}^*)}. \tag{52}$$

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[22]Note that (45) can also be written as $\mu = \Delta R(\theta)G(\theta)^{\frac{1-\rho}{\rho}}$, and (17) can also be written as $Y = \tilde{Y}D(\theta)G(\theta)^{-\frac{1}{\rho}}$.

Substituting out the factor income and aggregate profits, the aggregate resource constraints (25) can be written as

$$C_t + S_{1t} - (1 - \delta)S_{1t-1} + K_{t+1} - (1 - \delta_k)K_t = A_t K_t^\alpha \tilde{M}_t^{1-\alpha} - \frac{\xi}{2\bar{K}}(K_t - \bar{K})^2, \qquad (53)$$

$$M_t + S_{2t} - (1 - \delta)S_{2t-1} = B_t N_t. \qquad (54)$$

For both types of inventories, the stock-to-sales ratio is determined by the function $\frac{D(\theta^*) + H(\theta^*)}{D(\theta^*)}$, which in turn is a function of the cutoff variable θ_t^*. This suggests that the cyclicality of the stock-to-sales ratio in each sector is determined by the movements of marginal cost of inventories in that sector, as in the benchmark model (see equations 13, 38, and 42). In particular, a rise in the current marginal cost reduces the cutoff value θ_t^*, which in turn lowers the stock-to-sales ratio, $\frac{D+H}{D}$; on the other hand, a rise in the expected future marginal cost increases the cutoff value and raises the stock-to-sales ratio. Therefore, the dynamics of the stock-to-sales ratio depends not only on the source of shocks but also on the persistence of shocks, because the persistence of shocks changes the relative weight of the current versus the future marginal costs.

The aggregate resource constraint in equation (53) suggests that finished goods inventories are a perfect buffer for aggregate consumption and are substitutable for capital investment, whereas the input inventories in (54) are not directly substitutable for either consumption or capital goods. This difference gives rise to different inventory behavior across finished and unfinished goods, especially at the high frequencies.

Structural Parameters. Inventory behavior in the model depends on structural parameters. Although the influence of these parameters on the model are complex and intertwined, their major roles are easy to distinguish. For example, the parameters $\{\rho, \sigma\}$ affect primarily the steady-state stock-to-sales ratio because they influence the variance of sales at the micro level. When ρ is large, there is more substitutability across goods with different colors, making sales of each colorful good more volatile for the same distribution of idiosyncratic shocks. The shape parameter σ in the Pareto distribution is negatively associated with the variance of the distribution. Hence, a smaller σ is associated with more volatile sales. Since a larger variance of sales increases the possibility of stockouts, firms have incentive to keep a larger inventory stock relative to sales for larger ρ and/or smaller σ.

The parameters in the utility function $\{\gamma, \gamma_n\}$ affect inventory behavior by primarily affecting the relative strength of the income effect and the substitution effect. For example,

19

the smaller is γ, the more responsive is aggregate consumption to aggregate shocks. In this case, finished goods inventories are more likely to play the role of buffer stock in the face of consumption changes. Consequently, output inventory investment is more likely to be countercyclical at the high frequencies. On the other hand, larger values of γ or γ_n are more likely to generate negative responses of labor supply to technology shocks because of the increased income effect. Consequently, input inventories are more likely to be countercyclical under TFP shocks.

The adjustment cost parameter, ξ, affects primarily the substitutability between capital investment and inventory investment in finished goods. Hence, as consumption increases under either preference shocks or supply shocks, the effectiveness of buffer-stock roles of capital investment and inventory investment are different. For example, a larger value of ξ tends to attenuate the initial response of capital investment and make finished goods inventory investment more responsive to aggregate shocks on impact. The general dynamic properties of the model can be summarized as follows:

A. Under persistent aggregate demand shocks and with a wide range of parameter values, the model exhibits the following general properties: (i) inventory investment for both finished and intermediate goods are procyclical at the business cycle frequencies; (ii) their respective stock-to-sales ratios are countercyclical; (iii) input inventories are more volatile than output inventories; and (iv) finished goods inventories have a tendency to be countercyclical at high frequencies. By the accounting identity for input and output inventories, production/usage is automatically more volatile than sales/orders because inventory investment is procyclical. These predictions are consistent with the data.

B. Persistent TFP shocks can generate similar results as those under demand shocks, provided that the substitution effect is strong enough (e.g., $\gamma < 1$). Otherwise, input inventory investment is countercyclical because TFP shocks generate a lower demand for intermediate goods when the income effect dominates. However, regardless of the parameter values, input inventories are less volatile than output inventories, which is inconsistent with the data.

C. Under persistent labor cost shocks, the model's dynamics are very similar to those under preference shocks with a wide range of parameter values. Namely, (i) inventory investment for both finished and intermediate goods are procyclical at the business cycle frequencies; (ii) their respective stock-to-sales ratios are countercyclical; (iii) input inventories are more volatile than output inventories; and (iv) finished goods inventories have a tendency to be countercyclical at high frequencies.

The main intuition behind these results can be analyzed using the aggregate resource

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equations (53) and (54), which reveal the demand-supply chain of the production process. First, a permanent aggregate preference shock increases the marginal utilities of consumption not only in the present period but also for the future periods. This encourages the household to accumulate more finished goods inventories and the capital stock. Such an increase in the demand for wealth accumulation raises the shadow price of finished goods and stimulates production; hence, the demand for intermediate goods also increase persistently. This leads to accumulation of intermediate goods inventories and more production of intermediate goods. Therefore, a persistent shock to aggregate consumption demand at the downstream can generate synchronized business cycles over the entire economy. Since an increase in the final demand of finished goods requires more than a one-for-one increase in intermediate goods because of the diminishing marginal product of intermediate goods in producing the final good, production at the upstream must increase more than that at the downstream. This multiplier effect causes input inventory investment to be more volatile than output inventory investment in order to replenish inventories and maintain a desired stock-to-sales ratio at all stages of production. Finally, increases in demand at all stages of the production process raises the marginal costs of production at each stage, making the stock-to-sales ratio countercyclical for both input and output inventories.

The same type of aggregate fluctuations driven by aggregate demand shocks can also be obtained under permanent cost-push shocks. An increase in B_t increases aggregate supply of intermediate goods as well as input inventories. This reduces the shadow price of intermediate goods and encourages production of the finished goods. More supply of finished goods encourages consumption and accumulation of wealth (including capital and finished goods inventories). Also because of the diminishing marginal product of the intermediate goods, an increase in intermediate goods can translate only into less than a one-for-one increase in final goods. Hence, output inventory investment is less volatile than input inventory investment. Finally, since the shock is highly persistent, the decrease in the expected future marginal cost outweighs that of the current marginal cost, leading to countercyclical stock-to-sales ratio in all sectors.

The dynamic effects of TFP shocks are very different from the other two types of shocks. A shock to the TFP serves as a supply-push shock for the final-good sector but a demand-pull shock for the intermediate goods sector. However, the magnitude of the supply-side effect is larger than that of the demand-side effect. A one-unit increase in intermediate good $\tilde{M}$ under a positive TFP shock is just a one-unit increase in demand for intermediate goods, but it represents more than a one-for-one increase in the supply of finished goods because of

the compounded effect from a higher TFP. This explains why input inventory investment is in general less volatile than output inventories under TFP shocks. Also, if the income effect dominates the substitution effect, then a positive shock to TFP leads to a decrease in the demand for intermediate goods, causing input inventory investment to be countercyclical. Hence, the effects of TFP shocks on inventory behavior are more sensitive to structural parameters than those of other shocks.

Finally, since finished goods inventories stored in the refrigerators (i.e., held by retail stores) are a better buffer than capital goods for unexpected increases in consumption needs, finished goods inventories tend to be countercyclical on impact at the high frequencies. On the other hand, since finished goods inventories are substitutable for capital investment, an unexpected rise in the marginal product of capital also tends to crowd out orders of finished goods from the household and reduce inventory investment. Thus, countercyclical inventory investment at the high frequencies can be generated by both aggregate demand shocks and aggregate supply shocks. This is consistent with the stylized fact documented and analyzed by Wen (2005a).

Calibration and Impulse Responses. The time period is a quarter. Following the existing RBC literature, set capital's share of income $\alpha = 0.3$, the time-discounting factor $\beta = 0.99$, the inverse labor supply elasticity parameter $\gamma_n = 0.25$ (which corresponds to a log utility function on leisure), the rate of capital depreciation $\delta_k = 0.025$ (which implies the capital stock depreciates about 10% a year), the rate of inventory depreciation $\delta = 0.015$ (which implies a 6% annual rate of depreciation for inventories),[23] the shape parameter $\sigma = 3$ and the substitution parameter $\rho = 0.1$ (which imply an inventory stock-to-sales ratio of about 1.0 and an inventory investment to GDP ratio of about 1% in the steady state),[24] and the adjustment cost parameter $\xi = 0.1$. The risk aversion parameter γ plays an important role in determining the strength of the substitution effect, it is left free for experiments in the impulse response analysis below.

To get a sense of the adjustment cost parameter ξ, we can estimate the adjustment cost as follows. The ratio of the adjustment cost to aggregate output can be written as

$$\frac{\xi}{2} \frac{\bar{K}}{Y_t} \left(\frac{K_t - \bar{K}}{\bar{K}} \right)^2. \tag{55}$$

Assume that the steady-state annual capital-output ratio $\frac{K}{Y} \approx 2$. The estimated variance of

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[23]Because of wear and tear in use, the capital stock depreciates faster than inventory stocks.

[24]Since the parameters $\{\rho, \sigma\}$ are assumed to be the same for both input and output inventory sectors, the implied steady-state stock-to-sales ratios are the same for both sectors.

the capital stock relative to its HP-filter trend for the manufacturing sector between 1925 and 2002 is roughly $\sigma_k^2 = 0.0013$. Then with $\xi = 0.1$, the steady-state adjustment cost is approximately 0.01% of output a year. Even with $\xi = 5$, it amounts to capital adjustment costs about 0.5% of output. This is a very small number compared with the estimates of Shapiro (1986).[25] Without the adjustment cost, the model can still generate similar inventory dynamics, except the finished goods inventory investment has a higher tendency to be negative on impact. This negative initial response can always be countered by a higher value of γ.
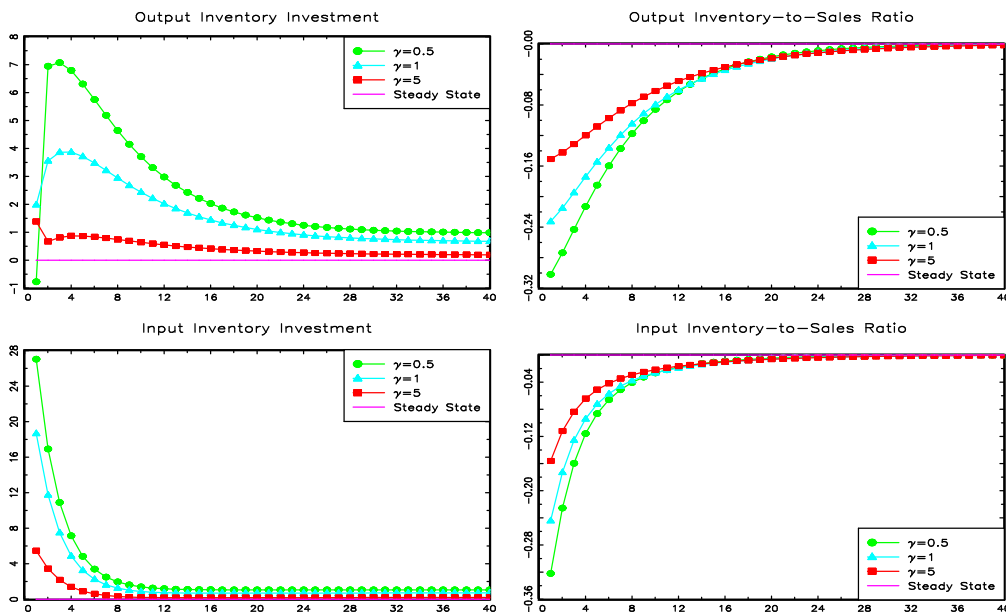

Figure 2. Impulse Responses to Demand Shock.

The impulse responses of the model to a one-standard-deviation shock to aggregate demand are graphed in Figure 2. Different values of γ are used in generating Figure 2 in order to illustrate the sensitivity (robustness) of the model to parameter values. Under the shock, aggregate activities – including total output, consumption, capital investment, labor, and inventory investments – all increase and comove. These predictions are robust to the value of γ, except the initial change in output inventories, which may be negative or positive depending on the value of γ. A lower value of γ makes consumption more responsive on impact because of lower risk aversion, which crowds out inventories in the short run. In the longer run, however, finished goods inventories always comove with final sales because of the desire for replenishment. Also, input inventory investment is at least 4 times more

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[25] Shapiro (1986) estimates the capital investment adjustment costs to be around 0.7% of output for a quarter.

volatile than output inventory investment in both the short and long run, and both are significantly more volatile than their respective sales. In the meantime, both output and input inventory-to-sales ratios are countercyclical despite their large volatilities. These predictions are consistent with the data.
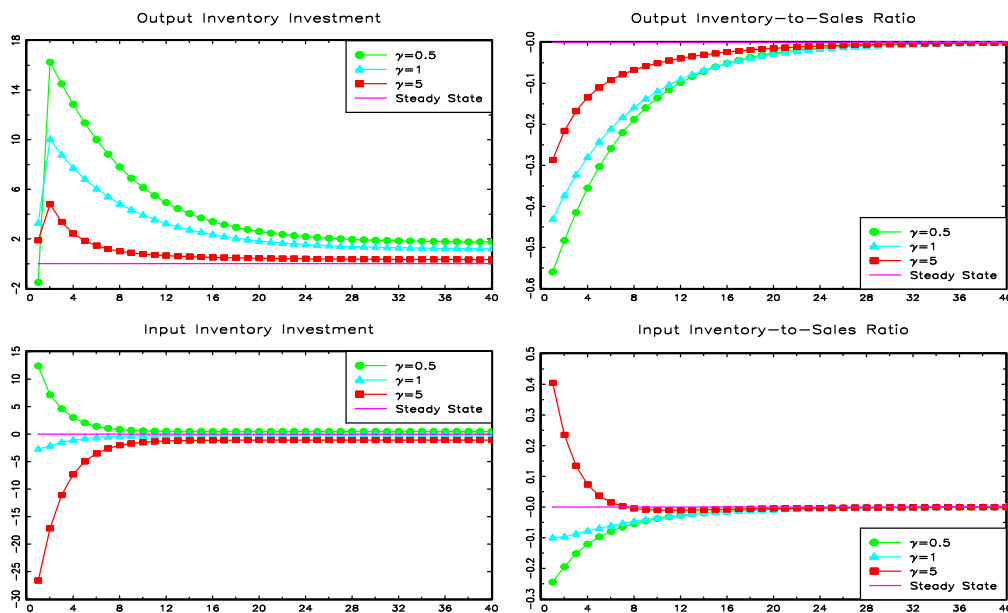

Figure 3. Impulse Responses to TFP Shock.

Under TFP shocks (Figure 3), the predicted inventory dynamics are consistent with the data if γ is sufficiently small (i.e., $\gamma < 1$, e.g., see the lines with circles in Figure 3). In this case, both input and output inventory investment are procyclical and the corresponding inventory-to-sales ratios are countercyclical. However, if γ is large enough (i.e., $\gamma \geq 1$), input inventory investment becomes countercyclical because a large income effect caused by γ decreases the demand for intermediate goods and input inventories under a positive productivity shock.

The impulse responses of the model to a one-standard-deviation labor cost shock is graphed in Figure 4. The predicted dynamics are nearly identical to those under aggregate demand shocks except more volatile.[26] Namely, inventory investment is procyclical in both input and output sectors; the inventory-to-sales ratio is countercyclical; and input inventory investment is more volatile than output inventory investment. This suggests that inventory behavior in the data, especially the countercyclical stock-to-sales ratio, by itself does not

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[26]The exception is labor. Labor is much less volatile relative to output under cost shocks than under demand shocks.

24

indicate which type of shocks are important in driving the business cycle. This is in contrast to the arguments made by Bils and Kahn (2000).[27]
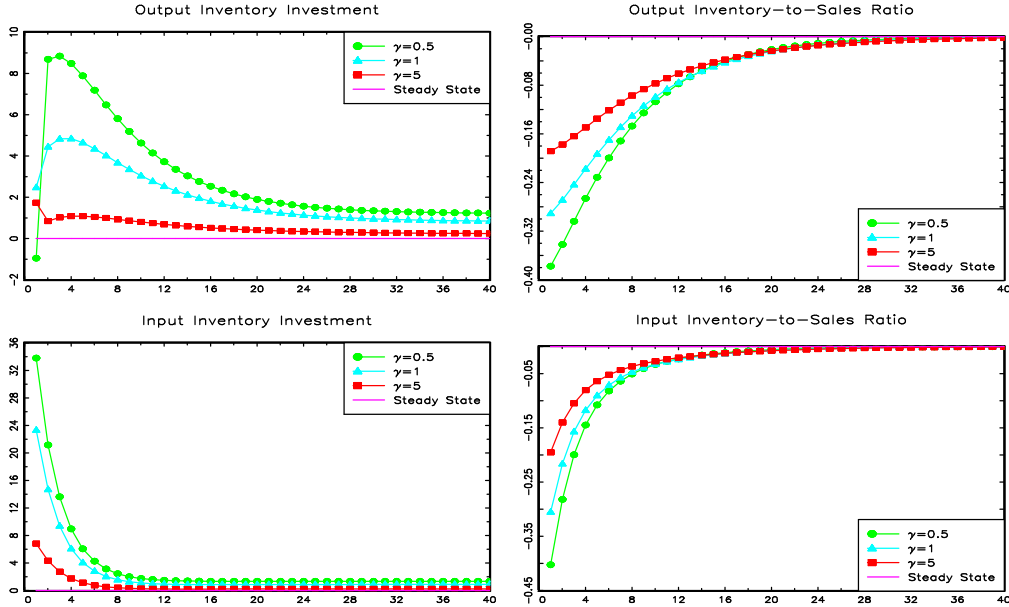

Figure 4. Impulse Responses to Cost Shock.

Matching Data.

The model has no problem matching the long-run ratios of inventory stock to sales and inventory investment to sales by properly choosing the parameter values of $\{\sigma, \rho\}$, as well as the other great ratios of the U.S. economy. This section, therefore, focuses on the ability of the model to match the second moments of the data.

To ensure consistency between the data and the model in the definition of variables, all variables in the data are transformed into percentage deviations from their respective long-run trends, $\hat{X}_t \equiv \log X_t - \log X_t^*$, where the long-run trend (X^*) is defined as the HP trend. This is consistent with the log-linearization solution method of the model. The relationship between a stock variable S and its flow I is defined according to the model as

$$S_t - (1 - \delta)S_{t-1} = I_t. \tag{56}$$

Hence, the log-linearized relationship between stock and flow is given by

$$\hat{S}_t - (1 - \delta)\hat{S}_{t-1} = \delta\hat{I}_t. \tag{57}$$

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[27]Khan and Thomas (2007a) also have similar findings in a general-equilibrium (S,s) model.

Based on this definition, if a flow variable I has both positive and negative entries and cannot be "log-linearized" directly and data on its stock S is not available, then its percentage deviation from trend can be consntructed according to relationship (57). For example, to compute percentage changes of aggregate inventory investment in finished goods $(\hat{I}_t)$, which has non-positive entries sometime, we can first construct the inventory stock variable S_t according to (56) by assuming $\delta = 0.015$. The initial value of S_0 is set such that the imputed stock variable shares a common growth trend with GDP or the stock-to-GDP ratio is stationary over time.[28] The stock variable is then logged and HP filtered, yielding the series $\hat{S}_t$. Using (57), we obtain $\hat{I}_t$.[29]
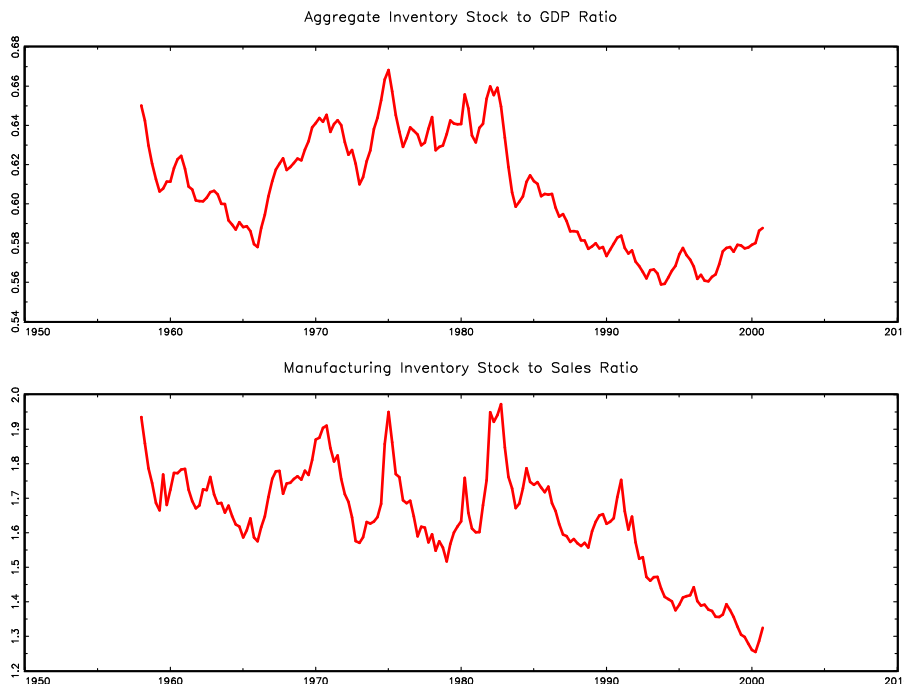

Figure 5. Output and Input Inventory Behavior.

Figure 5 shows the aggregate inventory-to-GDP ratio based on the constructed aggregate inventory stock, along with the inventory stock-to-sales ratio in the manufacturing sector. Clearly, the constructed aggregate inventory stock series mimics that of the manufacturing

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[28] Since the series of inventory stock-to-sales ratio in the manufacturing sector is available, the initial value of S_0 can be further narrowed down by ensuring that the constructed inventory-to-sales ratio of the aggregate finished goods look similar to that of the manufacturing sector. Using this method, the initial value is set at $S_0 = 0.65 GDP_0$, where GDP_0 is the initial value of GDP for our U.S. data sample.

[29] The variance of $\hat{I}_t$ based on this construction is sensitive to the value of δ. To make sure that $\delta = 0.015$ does not exaggerate the variance of inventory investment, we have used this procedure to construct the series of log-linearized fixed capital investment under the value $\delta = 0.015$ and found that the variance of fixed investment is not exaggerated compared with the series under direct log-linearization.

sector very closely over the business cycle. The inventory-to-sales ratio for both types of inventories has exhibited a downward trend since the early 80s, reflecting the great moderation of the U.S. economy. The average inventory stock-to-GDP ratio is 0.61. This value is 0.92 with respect to aggregate consumption. For the manufacturing sector, the average inventory stock-to-sales ratio is 1.64.

Table 1 reports some selected business cycle statistics of the U.S. economy. All data are measured in billions of 2000 dollars. Aggregate consumption (C), fixed capital investment (dK), and inventory investment (dS_1) are from NIPA tables and they correspond to the final-good sector in the model. Since there is no government and international trade in the model, aggregate production is defined as $Y = C + dK + dS_1$ and aggregate sales is defined as $Y - dS_1$.[30] We use data from the manufacturing sector of the U.S. economy as a proxy that corresponds to the intermediate-good sector of the model, where total manufacturing production is denoted by Z, total sales (shipments) by M, and the inventory stock by S_2 (which includes only inventories of raw materials and work-in-process).[31]

Table 1. Business Cycle Statistics (U.S. 1958:1 - 2000:4)

Variables	All Frequencies		8-40 Quarters		2-3 Quarters	
Final Good	$std./y$	$cor./sales$	$std./y$	$cor./sales$	$std./y$	$cor./sales$
Y	1	0.97	1	0.98	1	0.60
C	0.62	0.97	0.60	0.97	0.85	0.94
dK	2.44	0.94	2.44	0.95	2.09	0.70
dS_1	21.6	0.42	17.7	0.62	71.3	-0.36
S_1	0.66	0.35	0.67	0.29	0.57	-0.33
$\frac{S_1}{C}$	0.89	-0.71	0.93	-0.72	1.23	-0.90
Z	1.59	0.57	1.50	0.65	2.72	0.45
Interm. Good	$std./z$	$cor./m$	$std./z$	$cor./m$	$std./z$	$cor./m$
Z	1	0.99	1	0.99	1	0.97
M	0.95	1	0.94	1	0.96	1
dS_2	32.1	0.62	27.5	0.78	72.5	0.22
S_2	1.13	0.32	1.16	0.27	0.58	0.18
$\frac{S_2}{M}$	1.22	-0.48	1.28	-0.49	1.03	-0.83

[30]There are no separate data on consumption good inventories and investment good inventories. Hence, the data and the model's final good sector are not a perfect match because in the model there are only consumption goods inventories.

[31]Data on inventory stocks for the manufacturing sector are available from Haver.

In Table 1, two classes of statistics of each times series are reported, including standard deviation relative to production ($std./prod$) and correlation relative to sales ($cor./sales$). The HP-filtered data correspond to the "All Frequencies" column, movements isolated by the Band-Pass filter at the business cycle frequencies (8-40 quarters per cycle) correspond to the "8-40 Quarters" column, and those at the high frequencies correspond to the "2-3 Quarters" column. For example, standard deviations of the final-good sector relative to production ($std./y$) are reported in the upper panel in the first column under each frequency band, and their correlations with total sales in the final-good sector ($cor./(y-ds)$) are reported in the next column under the same frequency band. Similarly, statistics from the intermediate-good sector are reported in the (lower panel) under each frequency band.

Several stylized facts are worth emphasizing in Table 1. First, inventory investment is extremely volatile and procyclical over the business cycle. For example, over the 8-40 quarters frequency band, its volatility is 17.7 times that of production in the final-good sector and 27.5 times that of production in the intermediate sector; and its correlation with sales is 0.62 in the final-good sector and 0.78 in the intermediate-good sector. Second, despite this, the inventory stock-to-sales ratio is countercyclical. Its correlation with sales is -0.47 in the final-good sector and -0.49 in the other sector. Third, intermediate goods inventories are more than twice as volatile as those for finished goods. To see this, notice that the standard deviation of production in the intermediate-good sector is 1.5 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to the final good production is $27.5 \times 1.5 = 41.25$, which makes it more than twice as large as the volatility of finished goods inventory investment (which is 17.7). Finally, finished goods inventories are counercyclical at high frequencies. For example, their correlation with sales is -0.36 for inventory investment and -0.33 for inventory stock. However, these correlations are positive for intermediate good inventories.

Table 2 reports the business cycle statistics predicted by the model (with $\gamma = 0.5$) under demand shocks (where numbers in parentheses are predictions under TFP shocks).[32] The production in the final-good sector is denoted by $\tilde{Y}$, total sales by C, capital investment by dK, inventory investment by dS_1, and inventory stock-to-sales ratio by $\frac{S_1}{C}$. The production in the intermediate-good sector is denoted by Z, sales by M, inventory by S_2, and stock-to-sales ratio by $\frac{S_2}{M}$.

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[32] The statistics are based on simulated time series with 2000 observations.

Table 2. Model Predictions under Demand (Technology) Shocks

Var.	All Frequencies		8-40 Quarters		2-3 Quarters	
Final	$std./\tilde{y}$	$corr./c$	$std./\tilde{y}$	$corr./c$	$std./\tilde{y}$	$corr./c$
$\tilde{Y}$	1	0.98 (0.97)	1	0.97 (0.97)	1	0.99 (0.99)
C	0.83 (0.81)	1	0.87 (0.85)	1	0.82 (0.72)	1
dK	1.47 (1.60)	0.82 (0.75)	1.28 (1.37)	0.75 (0.71)	2.28 (2.71)	0.98 (0.98)
dS_1	10.3 (10.9)	0.69 (0.71)	9.61 (10.2)	0.84 (0.86)	11.9 (13.8)	−0.85 (-0.80)
S_1	0.51 (0.52)	0.39 (0.46)	0.65 (0.62)	0.58 (0.59)	0.11 (0.12)	−0.76 (-0.86)
$\frac{S_1}{C}$	0.79 (0.73)	−0.79 (-0.77)	0.73 (0.69)	−0.68 (-0.70)	0.91 (0.83)	−0.99 (-0.99)
Z	1.74 (0.53)	0.93 (0.90)	1.61 (0.51)	0.91 (0.88)	2.16 (0.62)	0.99 (0.99)
Interm.	$std./z$	$corr./m$	$std./z$	$corr./m$	$std./z$	$corr./m$
Z	1	0.97 (0.99)	1	0.98 (0.99)	1	0.99 (1.00)
M	0.82 (0.87)	1	0.88 (0.90)	1	0.68 (0.75)	1
dS_2	17.0 (12.5)	0.66 (0.74)	13.9 (10.5)	0.57 (0.67)	22.3 (17.8)	0.99 (0.99)
S_2	0.57 (0.44)	0.89 (0.82)	0.70 (0.52)	0.92 (0.85)	0.18 (0.14)	0.97 (0.98)
$\frac{S_2}{M}$	0.42 (0.56)	−0.75 (−0.90)	0.36 (0.53)	−0.66 (−0.86)	0.51 (0.61)	−0.99 (−0.99)

Under aggregate demand shocks, the model is able to qualitatively replicate the stylized facts in Table 1. Namely, (i) inventory investment is very volatile and procyclical over the business cycle. Over the 8-40 quarters frequency band, its volatility is about 10 times that of production in the final-good sector and 14 times that of production in the intermediate sector; and it is positively correlated with sales in both sectors (the correlation is 0.84 in the final-good sector and 0.57 in the intermediate-good sector). (ii) The inventory stock-to-sales ratio is countercyclical. Its correlation with sales is −0.68 in the final-good sector and −0.66 in the other sector. (iii) Intermediate goods inventories are more than twice as volatile as those for finished goods. The standard deviation of production in the intermediate-good sector is 1.61 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to the final good production is $14 \times 1.6 = 22$, which makes it more than twice as large as the volatility of finished goods inventory investment (which is 9.61). (vi) finished goods inventories are counercyclical at high frequencies. For example, their correlation with sales is −0.85 for inventory investment and −0.76 for inventory stock. In the meantime, the respective correlations are positive for intermediate good inventories, as in the data.

The predictions under cost shocks (B_t) are almost identical to those of aggregate demand shocks; hence, they are not reported. The predictions under TFP shocks are also reported in Table 2 (numbers in parentheses). Most of the predictions are consistent with the data, except the volatility of input inventories relative to output inventories. For example, over the 8-40 quarters frequency band, the standard deviation of production in the intermediate-good

sector is only 0.51 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to the final-good production is $10.5 \times 0.51 = 5.4$, which makes it only half as large as the volatility of finished-good inventory investment (which is 10.2). The reason is precisely the lack of a multiplier effect of TFP shocks on intermediate-good sector relative to the final-good sector. An increase in TFP raises the final-good production (supply) more than the intermediate-good production (demand). That is, the supply-side effect on final good is the combination of changes in TFP and $\tilde{M}$, whereas the demand-side effect on intermediate goods is only changes in $\tilde{M}$. In addition, for the risk aversion parameter γ large enough, the effect on intermediate-good demand is even negative. This problem does not arise for aggregate demand shocks (which originate from the bottom of downstream) or aggregate cost shocks to labor or raw materials (which originate from the top of the production chain upstream).

Finally, notice that the model is qualitatively consistent with the U.S. business cycle along other dimentions. For example, the model is able to explain the procyclical aggregate consumption, capital invesment, and hours across all cyclical freqiencies. The model is also able to explain the stylzed fact that consumption is less volatile but capital investment is more volatile than GDP at different fequency bands.

3.6   Challenges

While the model is broadly successful in explaining the key features of the business cycle and inventory behavior, there are still challenges left for general-equilibrium models of the stockout-avoidance theory of inventories. Most notably, the volatility of inventory investment relative to production in the model is still significantly lower than that of the data. Recalibrating the structural parameters of the model does not solve this problem completely. Also, the model with a single transitory shock is not as successful as that with a single permanent shock in explaining the business cycle and the inventory behavior. For example, under transitory demand shocks, although the inventory-to-sales ratio remains countercyclical and inventory investment remains procyclical, capital investment tends to be countercyclical because a sharp rise in consumption tends to crowd out aggregate savings. This is a typical problem of standard RBC models under demand shocks. Maybe introducing increasing returns to scale can resolve this problem (see, e.g., Benhabib and Wen 2004). Under transitory cost-push shocks, although capital investment as well as inventory investment remain procyclical, the inventory stock-to-sales ratio tends to become procyclical because a

decrease in the current marginal cost relative to expected future marginal costs drives up the stock-to-sales ratio. Under transitory TFP shocks, input inventory investment becomes countercyclical unless the risk aversion parameter γ is further reduced from the benchmark value of 0.5 toward zero. Based on these results, a multiple-shock model with a mixture of permanent and transitory demand and supply shocks may resolve these comovement problems. But this requires careful calibrations of the driving processes and relative variances of different types of shocks.

4   Conclusion

This paper has developed a general-equilibrium model of input and output inventories with the stockout-avoidance motive. Under persistent aggregate shocks, the model is broadly consistent with the stylized inventory behavior of the U.S. economy over the business cycle, such as, among other things, the excess volatility of production relative to sales, procyclical inventory investment and countercyclical inventory-to-sales ratio, and more volatile input inventories than output inventories. Although the model still has shortcomings, the model's analytical tractability makes it easy to introduce inventories into more complicated DSGE models than the one studied in this paper, such as models with imperfect competition, firm entry and exit, money and sticky prices, international trade, and so on. Also, the approach can be used to study durable goods inventory behavior, which is another important long-standing puzzle of the business cycle (see, e.g., Feldstein and Auerbach, 1976). Given the sheer magnitude of inventory stocks in the economy and the large contribution of inventory investment to GDP fluctuations, a business cycle model without inventories is cearly incomplete and unsatisfactory. General-equilibrium analysis of the business cycle with inventories is still in its infant stage. Hopefully this paper will contribute to further research and development in this area.

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