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A State Space Forecasting Model with Fiscal and Monetary Control

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Abstract

In this paper we model the U.S. economy parsimoniously in an atheoretic state space representation. We use monthly data for thirteen macroeconomic variables. We treat the federal deficit as a proxy for fiscal policy and the Fed Funds rate as a proxy for monetary policy and use each of them as control (exogenous) variables, and designate the rest as state variables. The output (measured) variable is the growth rate of quarterly real GDP which we interpolate to obtain a monthly equivalent. We specify a linear relation between state variables and implicitly allow for time variation of the relationship by using a recursive least squares (RLS) with forgetting factor algorithm to estimate the coefficients. The model coefficients are also estimated using ordinary least squares (OLS) and the resulting forecasts (in-sample and out-of-sample) are compared. The RLS algorithm performs better in the out-of-sample forecasts, particularly for those state variables which exhibit the greatest cyclical variations. Variables which had greater stability were forecasted more precisely with OLS estimated parameters.

Keywords: Forecasting, Time-varying parameters, Adaptive Control

JEL Classification: E17, E27

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Introduction

The forecasting arena is a treacherous one. Most forecasters recognize the futility of accurate short run forecasts in the context of stochastic models when random shocks are just that - random. Structural models steeped in the fundamentals of supply and demand equilibria and rational agents provide comfort but no greater accuracy than atheoretic models. Despite clamors to ground macroeconomic models firmly in microeconomic principles, it is a great leap of faith, on a path strewn with assumptions regarding aggregation, to get from microeconomic decisions to the aggregate economy. Equilibrium analysis - a must in the long run - provides limited information on the dynamic path between equilibria in the presence of stochastic disturbances. In particular, several dynamic models may be observationally equivalent in the steady state with radically different transient dynamics. In an environment such as this it is best to tread lightly.

A policy maker must sift current data to determine the long run direction of the economy as a guide to policy. Policy decisions can have the unintended consequence of amplifying the volatility of the economy if implemented injudiciously. As a result, the data must be filtered to decipher whether current conditions reflect natural short run perturbations around the equilibrium path or a permanent deviation from equilibrium. The problem becomes one of signal extraction. The low sampling frequency of macroeconomic data further exacerbates this already tenuous and intractable exercise.

One philosophical approach to the problem is simplicity. Some analysts using very little theory appear to predict financial markets in the short run as well or better than sophisticated models. But danger lies ahead when observed changes in fundamentals go unrecognized because of ignorance of theoretical foundations.

Intuition suggests that in a macroeconomic environment key relationships among aggregate variables change over time for various reasons such as the Lucas critique or compositional effects of heterogeneity. Tracking time-varying systems is a fundamental problem in control engineering and signal processing. One method used in these situations is recursive estimation algorithms which update estimates as new information is received. The optimal algorithm depends on the way in which the system varies over time. Ljung and Gunnarsson (1990) provide a survey and tutorial of recursive identification algorithms.

One such algorithm is the recursive least squares (RLS) with exponential forgetting factor. The effect of this algorithm is to reduce the weight of past errors by a specified discount factor. If the parameters are believed to vary slowly relative to the frequency of the data (as opposed to random switching between states, for example), then the RLS algorithm with exponential forgetting implicitly adjusts over time. We believe that time variation in macro relationships, especially those produced by compositional effects is likely to be slow. In estimating the forecast model we use the RLS with exponential forgetting method and ordinary least squares (OLS) estimation for the parameter estimates and compare the results of out-of-sample forecasts using each estimate.

Our core model is a linear state space representation with control (exogenous) variables. We compare three scenarios: one using the Fed Funds rate set by the Federal Reserve as exogenous, another using the smoothed monthly deficit, and the third using both as exogenous. The results indicate that the out-of-sample forecasts of the state variables using the recursive least squares estimation method had lower root mean squared errors on average than those using the ordinary least squares. In particular, variables with visible business cycle frequency fluctuations were forecasted with greater accuracy using the recursive least squares with a forgetting factor set at business cycle frequency. Forecasts of GDP growth rates using the state space model performed well compared to other popular forecasts. The model performance is especially commendable when we use RLS estimates of the transition equation parameters and OLS estimates of the output equation parameters in obtaining the forecasts.

LITERATURE REVIEW

This paper incorporates several modeling concepts: State space representation, forecasting quarterly variables using monthly data, time varying parameter estimation, and exogenous control variables. The exercise is primarily one of application rather than introduction of new techniques. However a review of some of the literature in each concept is in order.

State space representation:

State space (SS) representation has been widely used in the control theory literature since the 1960s because it lends itself easily to extracting information regarding the stability of the system under study and methods of effecting optimal control. Aoki (1990), and Mittnik (1990), have contributed to increased use of the SS representation in economic analysis. As a point of departure, a review of the basic definitions, taken from Ogata (1970) and consistent with SS representations in all disciplines, are given below.

> **State.** The state of a dynamic system is the smallest set of variables (called state variables) such that the knowledge of these variables at $t = t_0$, together with the input (or control variable -defined below) for $t \geq t_0$,

completely determines the behavior of the system for any time $t \geq t_0$. Thus, the state of a dynamic system at time t is uniquely determined by the state at time t_0 and the input for $t \geq t_0$, and it is independent of the state and input before t_0. Note that, in dealing with linear time-invariant systems, we usually choose the reference time t_0 to be zero.

State variables. The state variables of a dynamic system are the smallest set of variables which determine the state of the dynamic system. If at least n variables $x_1(t), x_2(t), \ldots, x_n(t)$ are needed to completely describe the behavior of a dynamic system (such that once the input is given for $t \geq t_0$ and the initial state at $t = t_0$ is specified, the future state of the system is completely determined), then such n variables $x_1(t), x_2(t), \ldots, x_n(t)$ are a set of state variables. Note that the state variables need not be physically measurable or observable quantities. Practically, however, it is convenient to choose easily measurable quantities for the state variables because optimal control laws will require the feedback of all state variables with suitable weighting.

State vector. If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered to be the n components of a vector $\mathbf{x}(t)$. Such a vector is called a state vector. A state vector is thus a vector which determines uniquely the system state $\mathbf{x}(t)$ for any $t \geq t_0$, once the input $\mathbf{u}(t)$ for $t \geq t_0$, is specified.

State space. The n-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, $\ldots$, x_n axis is called a state space. Any state can be represented by a point in the state space.

Control variable. The input (exogenous variable) to the system is designated as the control variable u and represents the variable that can be chosen by the controller to affect the movement of the state variables. In some stochastic economic models the control variable is assumed to be random innovations.

Output variable. The measured or output variable is designated as y and is related to the state variables. In the model used below, it is assumed that there is no direct interaction between the input and the output variables.

The state space representation of the system used in this paper can then be described by a triplet **A**, **b**, and **c** as in equations (1) and (2) below.[1]

$$X_{t+1} = \boldsymbol{A}\ X_t + \boldsymbol{b}\ u_t \qquad \textbf{(1)}$$

[1] The system described is fully deterministic. For this paper we assume initially a single-input single-output (SISO) system which means that our output and control variables are scalars, then use two control variables. Stochastic errors can be assumed to enter either additively or in the coefficients.

$$y_t = c^T X_t \qquad \textbf{(2)}$$

X is an n x 1 vector of state variables which describes the economy,[2] **A** is an n x n matrix of coefficients, u is a control variable (scalar), **b** is an n x 1 vector of coefficients when u is a scalar or n x m if u is an m x 1 vector, y is the output variable (of interest), **c** is an n x 1 vector of coefficients. In a stochastic environment we assume that the measurements of the variables are noisy and uncertain and the noise components are independent, identical normally distributed disturbances

The popular use of the SS representation in economics treats disturbances as random and state variables as unobservable. The use of the Kalman filter allows estimation of state variables from observation of input and output over time. In the physical sciences, the traditional use of the SS representation involves more precise mathematical models and knowledge of the state variables, even if they are unobservable. In other words, physical laws determine the relationships betweeen the state variables of the system, whether it is an electrical network or a chemical process. In an economy, the relationships are not as easily defined, although a linear stochastic framework is most often used.

In our model the state variables are assumed to be known along with the measured or

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[2] The accepted format is a first order difference equation. If additional lags of particular variables are desired then the list of variables is expanded appropriately by defining lagged values of these variables as X's. It can be shown that an ARMA representation can be modeled by this first order difference equation model.

output variable, and the control variable is assumed to be exogenously determined by the policy maker. For forecasts, the projected values of the control variables are used, allowing the option of evaluating different policy regimes. We assume the economy can be represented by variables measuring consumption, production, investment, employment, interest rate, and money. We also include inventories which serves as a measure of intertemporal transfers of production. Net exports are excluded for parsimony and because of the monthly volatility of the data.

The choice of state variables reflect priors regarding the interaction of various characteristics of the system. Linear approximations of theoretical relationships which hold in an ordinal sense but vary among agents in a cardinal sense, have been used as mainstays in economics. We recognize, for example, that the aggregate "marginal propensity to consume" (MPC) may be a fictional coefficient, comprised of the aggregate results of multiple agents' consumption choices, but for the purposes of monitoring the aggregate economy, the MPC is a useful pedagogical and analytical tool. To the extent that empirical estimates of this coefficient leads to a relatively stable number over time, there should be no need to be more precise. If the marginal propensity to consume varies among different types of agents, then as the number of agents in particular groups change, through changes in income distribution or demographics for example, the aggregate marginal propensity to consume should change. The empirical evidence in the U.S. suggests that this variable is relatively stable. For other linear relationships however, heterogeneity of agents which make up the composite and changes in the deep parameters which affect their decision making process suggests that stable linear relationships are too fictitious for useful analysis.

Time Varying Parameters

There is a large and growing literature on estimating time-varying parameters. Empirical tests have tended to verify that allowing for variation in parameters in models result in improved forecast performance. We will mention a few here. Stock and Watson (1996) address the issue of instability in the relation between macroeconomic variables over time. Their study tries to find out "how generic is instability in multivariate time series relations." Using 76 monthly time series, they first assess the prevalence of parameter instability in economic time series relations, and use the sample to compute empirical distributions of various tests of structural stability. Their tests indicate that instability is widespread. Their results also suggest that in over half the pairs of variables, the adaptive models perform better than fixed-coefficient models, although the gain is very small in most cases; and that the time varying parameter and recursive least squares models are more robust than fixed-coefficient models. They conclude that, 'if the application (of the model) is to forecasting, this instability provides an opportunity to improve on the forecasts of fixed-parameter models.'

Edlund and Sogaard (1993) compare the suitability of fixed versus time varying transfer functions for modeling the relationship between leading economic indicators and business cycles using Swedish data. They look for parameter stability over time in transfer functions, and compare fixed-parameter transfer function models to time-varying transfer function models. Their results support the existence of non-stochastic variation in the relationship between leading indicators and business cycles. They further note that, 'state space formulation provides the appropriate formulation, when explicitly modeling embedded parameter variation.'

Wolff (1987) uses a varying-parameter estimation technique based on Kalman filtering to improve the forecasting performance of a class of monetary exchange rate models. He finds that the out of sample performance of these structural models are improved when time variation is accounted for. This is consistent with his observation that instability in money demand functions, the Lucas critique, and factors leading to changes in the long-run real exchange rate can cause variations in the parameters of the structural models. Swamy, Kennickell and Muehlen (1990) compare forecasts of money demand from fixed and variable coefficient models and also conclude that variable coefficient models are superior.

The finance literature also favors time-varying parameter models in many cases. Chiang and Kahl (1991), for example, use a Kalman filtering technique to develop a time-varying coefficient model and use it to forecast the future spot treasury bill rates.

Quarterly Forecasts Using Monthly Forecasts

Macroeconomic data are gathered at a relatively low sampling frequency. The highest frequency that National Income and Product Accounts data are measured is quarterly. Components of these data are available at monthly frequencies. Despite the higher noise in these monthly data, we believe the dynamics of the economy is better reflected in higher frequency data.

Several researchers have addressed the use of high frequency data to forecast lower frequency variables. Bharat Trehan (1992) uses contemporaneous monthly data on three variables to predict quarterly real GDP. He first makes monthly predictions for the three indicator variables by estimating a Bayesian VAR, and uses these to obtain current quarter real GDP forecast. He finds that real GDP forecasts are improved when current quarter (monthly) forecasts of the indicator variables are included.

Rathjens and Robins (1993) obtain one-step-ahead and multi-step-ahead real GNP forecasts by using high frequency data. They show how to improve quarterly forecasts by using within-quarter variations of monthly data. These gains are shown to diminish rapidly in when going from one-step-ahead to multi-step-ahead forecasts. They construct Industrial Production models for forecasting real GNP growth by using data on industrial production which is released in monthly frequency. Klein and Park (1993) incorporate high-frequency updates of quarterly projections as new information arrives. They use their model to forecast the period around the Gulf War and find that their model/method produced an earlier prediction of the 1990-91 recession than other forecasters did and also find little support for the war as the cause of the recession. The proliferation of high frequency data and the timing of releases suggests that as much information as can be gleaned from new data should be used to update forecasts in a timely manner.

THE MODEL

The model is developed as a state space representation of the economy using equations (1) and (2) in the previous section. The fundamental assumption in the model developed here is that there is a linear relationship between the variables chosen as state vectors. We assume that current period consumption, investment, industrial production, inventory, inventory-to-sales ratio, employment, CPI, M2, and 3 month treasury bill rates are linearly related to last period values of all variables and the last period values of the control variable(s), the federal deficit and/or the Fed Funds rate. Because of the volatility of the monthly federal revenue and expenditure, we smooth the monthly deficit using the Hodrick-Prescott filter. We estimate 11 (12) linear equations of this form and use the coefficients of each equation as the row values of the A matrix and b vector. We then assume that GDP (which we interpolate to get an equivalent "monthly" value) is a linear combination of these variables and estimate the coefficients of this relationship as the **c** vector. At the end of the "in-sample" period, we use the values obtained for the **A**, **b**, **c**, triplet to project the future values of the state variables and the GDP. We estimate **A**, and **b** using the RLS method and the OLS method and compute a RMSE for projections of each variable for the last 60 periods in-sample and for the next twelve months "out-of-sample".

There are two primary differences between the typical Kalman filter approach and the method we use in this forecasting model. The first is that the recursive least squares method is used to estimate the parameters of the transition equation (i.e., **A** and **b** in equation 1) rather than the state variable *X*. The other is that the representation of the system includes an exogenous control variable *u* rather than just noise. We estimate n equations of the form:

$$x_{it} = a_{i1}x_{1t-1} + a_{i2}x_{2t-1} \cdots\cdots + a_{ii}x_{it-1} \cdots\cdots a_{\in}x_{nt-1} + b_i u_{t-1}$$

The a_{ij} 's are the entries of the Matrix **A**, and the b_i 's are the entries of the **b** vector. We estimate them using both OLS and RLS with the forgetting factor.

The "observation" equation expresses the relationship between the output variable of GDP and the state variables. GDP is an identity relating the measures of consumption, investment, government spending, and net exports. Since the GDP identity is time invariant, we assume that the OLS method is more appropriate for estimating the **c** vector. Estimation using the RLS showed higher RMSE, confirming this.

We did not place structural restrictions on the estimation based on theory or intuition. For example, we did not exclude M2 or the 3 month treasury bill rate from the estimation of the relationship between the state variables and GDP. In practice restrictions can be imposed where deemed applicable. We did not extend the model to incorporate additional lags of any state variables or the control variable. Only a single period lag was assumed in keeping with the desire to make the model as parsimonious as possible. We also did not develop an "**A** matrix" using the better of OLS and RLS for a particular variable. Nor did we eliminate coefficients that were not statistically significant in the OLS estimation. These adjustments might conceivably improve the forecasting capability of the model.

RESULTS

The *ex ante* forecasts of quarterly GDP of this state space model using the RLS algorithm compares favorably with those of the Fair Model, which is a structural model, and the median Blue Chip forecasts of the corresponding duration. The RMSE comparisons of RLS estimated coefficients versus OLS estimated coefficients showed promising results for those variables which are cyclical.

State Variables:

Table 1 summarizes the RMSE for the out-of-sample forecast of each of the state variables for three alternate assumptions for the control variables, using both RLS with exponential forgetting factor and OLS estimation of the model. The percentage root mean square errors on these forecasts were compared to check the performance of the model using the two estimation procedures. The case which had the lowest forecast RMSE is shown highlighted. As the results show, RLS performs better in the case of 8 out of the 13 variables. Of particular interest are consumption, investment, industrial production, employment, and CPI. RLS provides better forecasts for investment, industrial production, and employment, while OLS performs better in the case of the other two variables. This is in line with our expectations, because RLS does better where the variables exhibit variations and adaptability is an issue. As figures in the appendix show, the three variables for which RLS does better show more cyclical variations than consumption and CPI.

RLS does better for changes in manufacturing inventories while OLS does better for changes in retail inventories. This is most likely due to the fact that manufacturing inventory to sales ratios have been declining since the recession of 1982, whereas retail inventory-to-sales ratios have not. Thus, time variation over the sample period is more evident in manufacturing inventories.

GDP forecasts:

Forecasts of the output variable (GDP growth rates) are obtained using the forecasts for the state variables, and parameter estimates for **A** and **b** matrices and *c* vector. Once again we consider alternate estimating procedures. As Table 2 reveals, best RMSE[3] results are obtained

[3] The RMSE computations are based on the "monthly" GDP estimates. Later when we compare the results to the Fair model and the Blue Chip forecasts we use the third month of the quarter because the monthly data are interpolated.

when we consider RLS estimates for **A** and **b**, and OLS estimates for ***c***.

This can be explained once again by the role played by variation and the necessity for adaptation. While parameters in matrices **A** and **b** play a role in explaining the relation between the state variables, which are subject to variations depending on the state of the economy, parameters that enter vector ***c***, explain the more fundamental and stable relationship between the state variables and GDP.

Using one step ahead forecasts for **A** and **b** does better as would be expected, since updates are made using the errors of the last period. The performance is better when we have only Fed Funds as control. This could be because in this case, the deficit is included as one of the state variables and it also enters directly into GDP.

Comparison with Blue Chip and Fair model forecasts of GDP growth rates

We compare the GDP forecasts for the four quarters of 1996 to the forecasts from the structural model created by Ray Fair and made available on the Internet and to the median forecasts by the Blue Chip forecasters. Blue Chip forecasts are reported as year to year quarterly rates and forecasts from the Fair model are computed as annualized quarterly growth. Tables 3, 4, and 5 show the forecasted quarterly growths for each scenario at both an annual rate and a year over year rate with the equivalent actual value and the values forecasted by the Fair model (2/29/96) and the median Blue Chip February 1996 forecasts. In general, the RLS model performs on par with both the Fair model and the equivalent median Blue Chip forecasts, with the advantage of simplicity.

When both the fed funds rate and the federal deficit are used as control (exogenous) variables, the SS model performs better than both. The one step ahead looks better than the Fair and Blue Chip forecasts, but this is not a valid comparison because the one step ahead model is updated with actual values after each forecast period. The OLS model forecast performs the worst in predicting GDP. It predicts a falling GDP in each scenario which puts it very far from the actual.

Figures 8, 9, and 10 show graphically the "monthly" GDP forecasted by each scenario and the equivalent "monthly" GDP generated by interpolation. These figures show that, for all three scenarios, forecasts obtained using one-step-ahead RLS estimates provide the best "tracking of the actual annual monthly GDP growth rates. As shown in Table 2, the one step ahead forecast using the fed funds as control tracks the actual results best.

Summary and Conclusions

The most significant finding was that the RLS with forgetting factor algorithm performed better with variables which were "obviously" cyclical at a business cycle frequency or were obviously shifting over time as in the case of manufacturing inventory-to-sales ratio. Stock and Watson (1996) conclude that there is instability in many macro variables. When the source of instability is known, particular methods appropriate to detecting and estimating the time-variation can be applied. In cases where the time variation is unknown but can be assumed to occur slowly relative to the sampling frequency, the use of the RLS with exponential forgetting factor algorithm can improve short run forecasting performance. Enhancements to the basic model could include zero restrictions on some coefficients where intuition or preliminary results indicate, and choice of coefficients based on the better of OLS and RLS with forgetting in predicting particular variables out-of-sample. For example, consumption demonstrated less cyclical variation than say investment and industrial production and therefore may be a candidate for using the OLS estimated coefficients. Where parsimony is desired, a SS formulation using recursive least squares with exponential forgetting appears to be adequate for short-run forecasting.

Appendix

Recursive Least Squares with exponential forgetting:

Recursive Least Squares (RLS) estimation is a special case of the Kalman filter which can be used to avoid the numerical difficulties of matrix inversion present in ordinary least squares (OLS) estimation. OLS is applied to the first k observations of the data sample to determine a starting point for parameter estimates. Each additional observation is used to update coefficient estimates recursively, thus avoiding the need for further matrix inversion. With proper choice of the initial conditions, the final estimator at the end of the sample period is equal to the OLS estimator. Harvey (1993) summarizes the method. The RLS method with exponential forgetting used here modifies the basic RLS updating algorithm to weigh new information more heavily. The method is appealing for cases where time-varying parameters are suspected.

For an equation of the form

$$z(t) = \varphi^T(t)\ \Theta \qquad \textbf{(4)}$$

where Θ is a vector of model parameters and $\varphi(t)$ is a set of explanatory variables, the usual quadratic loss function is replaced by a discounted loss function of the form

$$V(\Theta,t) = 1/2 \sum_{i=1}^{t} \lambda^{t-i} (z(i) - \varphi^T(i)\Theta)^2 \qquad \textbf{(5)}$$

where λ is a number less than or equal to one and is referred to as the forgetting factor.

The recursive algorithm is given by

$$
\begin{aligned}
\hat{\Theta}(t) &= \hat{\Theta}(t-1) + K(t)\,(z(t) - \varphi^T(t)\,\hat{\Theta}(t-1)) \\
K(t) &= P(t)\,\varphi(t) = P(t-1)\,\varphi(t)\,(\lambda I + \varphi^T(t) P(t-1)\,\varphi(t))^{-1} \\
P(t) &= (I - K(t)\varphi^T(t))\,P(t-1)/\lambda
\end{aligned}
\qquad \textbf{(6)}
$$

The essential feature of the algorithm is that t-1 estimates of Θ are adjusted with new information by a transformation of the error in predicting z using Θ_{t-1} and current φ's. The adjustment to the error, K(t), is called the Kalman gain and is a function of the rate of change in the errors and is weighted by the discount factor λ. P(t) is the covariance matrix at time t. Both K(t) and the moment matrix P(t) are updated recursively. In the state space model of equations (1) and (2), z(t) is X_t, φ(t) is X_{t-1} and Θ(t) is A(t), or φ(t) can be [X_t , u_t] and Θ(t) would correspond to [A(t) , b(t)].

There are two primary differences between the typical Kalman filter approach and the method we use in this forecasting model. The first is that the recursive least squares method is used to estimate the parameters of the transition equation (i.e., **A** and **b** in equation 1) rather than the state variable *X*. The other is that the representation of the system includes an exogenous control variable *u* rather than just noise.

For a scalar equation of the form:

$$\hat{\Theta} = (\Phi^T\Phi)^{-1}\Phi^T Y$$

We can estimate theta by treating it as a process:

$$\Theta_{t+1} = \Theta_t \qquad (a)$$

$$y_t = \Phi_t^T \Theta_t + \epsilon_t \qquad (b)$$

and then use the Kalman filter approach to get the least squares estimate of theta recursively. In this case equation (a) is the system equation, and equation (b) is the measurement equation (with c = Φ). So that P*(t) = P(t-1). The equivalent Kalman filter in common notation is:

$$(1) \qquad P_t^* = P_{t-1}$$

$$(2) \qquad K_t = P_{t-1} \; \Phi_t \; (I + \Phi_t^T P_{t-1} \Phi_t)^{-1}$$

$$(3) \qquad \hat{\Theta}_t = \hat{\Theta}_{t-1} + K_t \; [y_t - \Phi_t^T \hat{\Theta}_{t-1}]$$

$$(4) \qquad P_t = [I - K_t \Phi_t^T] P_{t-1}$$

Chapter 3 in Åström and Wittenmark's *Adaptive Control* discusses the method of Recursive Least Squares.

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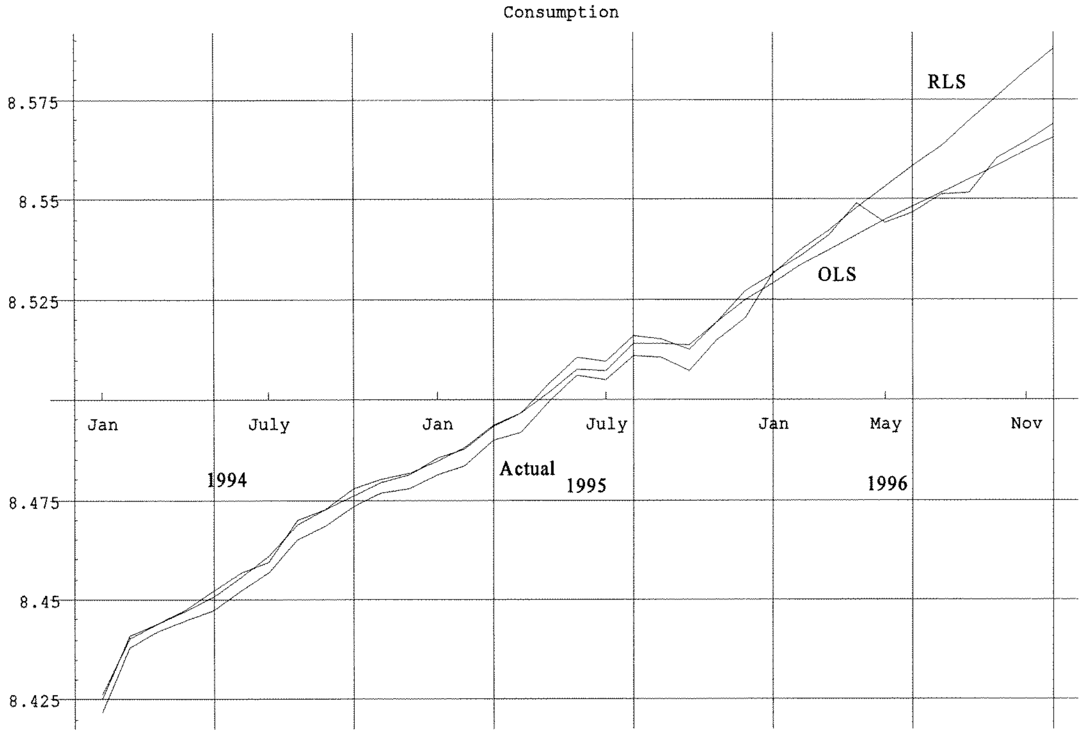

Figure 1 Consumption: In-Sample and Out-of Sample Forecast Using Fed Funds and Federal Deficit as Control Variables

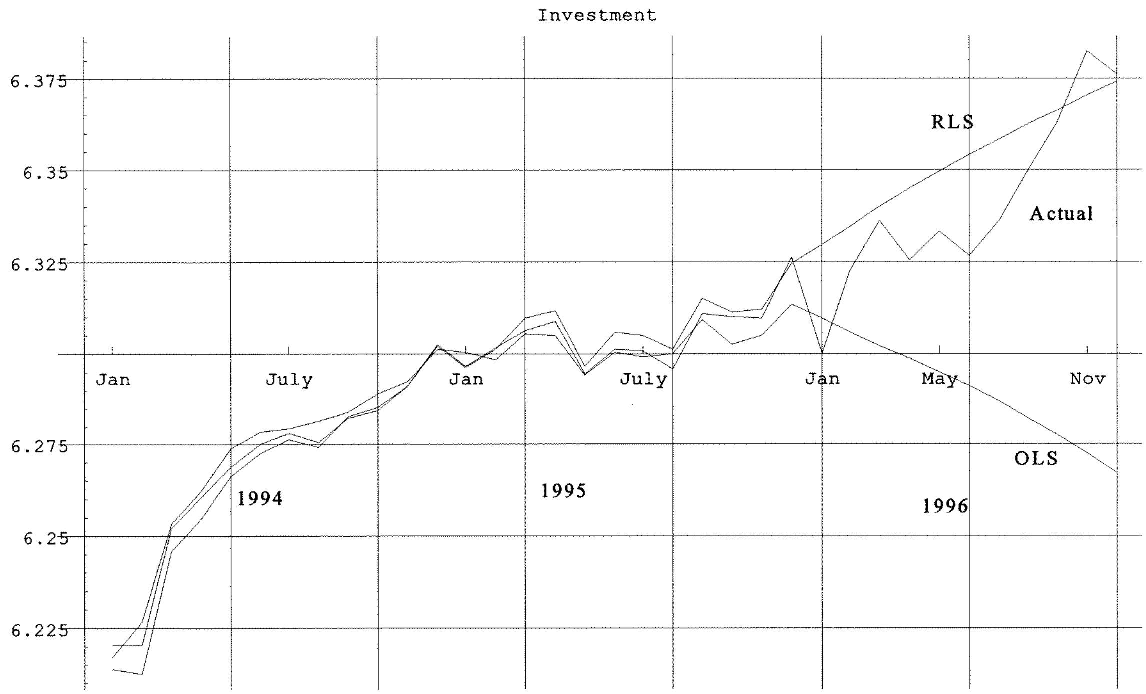

Figure 2 Investment: In-Sample and Out-of-Sample Forecast Using Federal Deficit as Control Variable

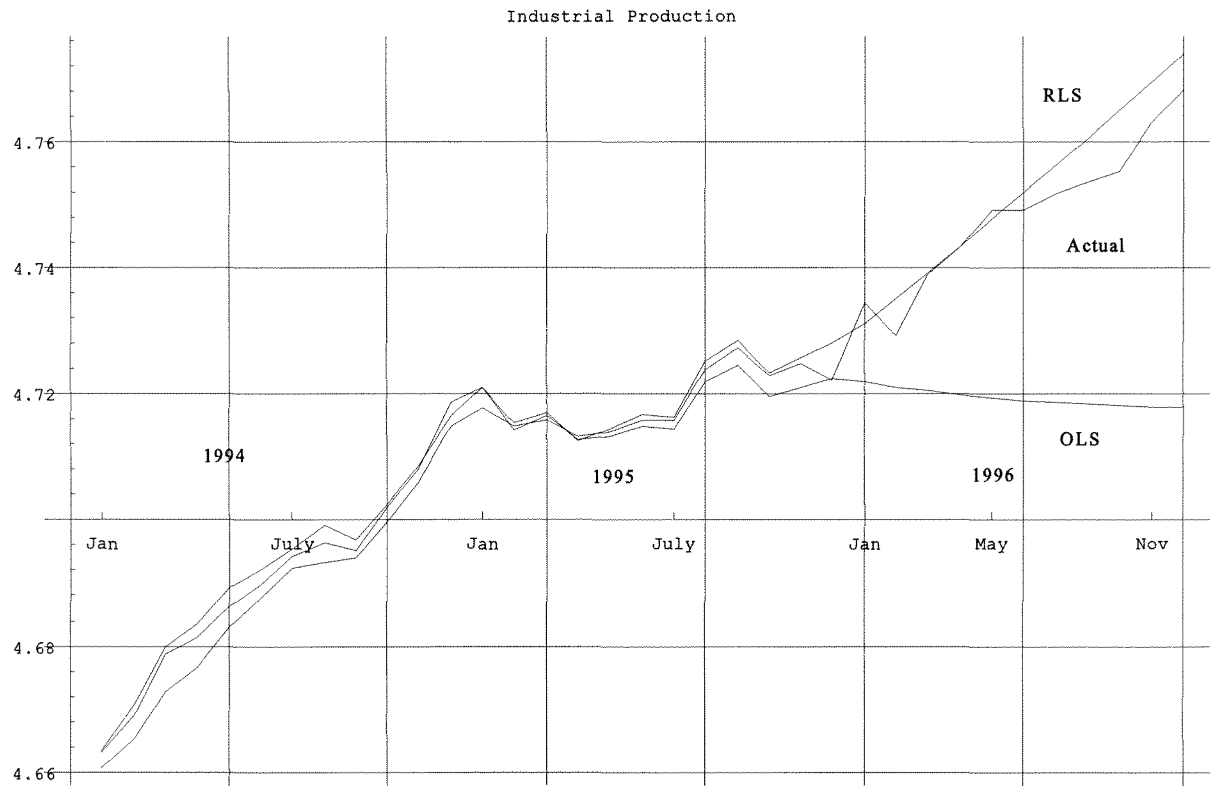

Figure 3 Industrial Production: In-Sample and Out-of-Sample Forecast Using Fed Funds Rate as Control Variable

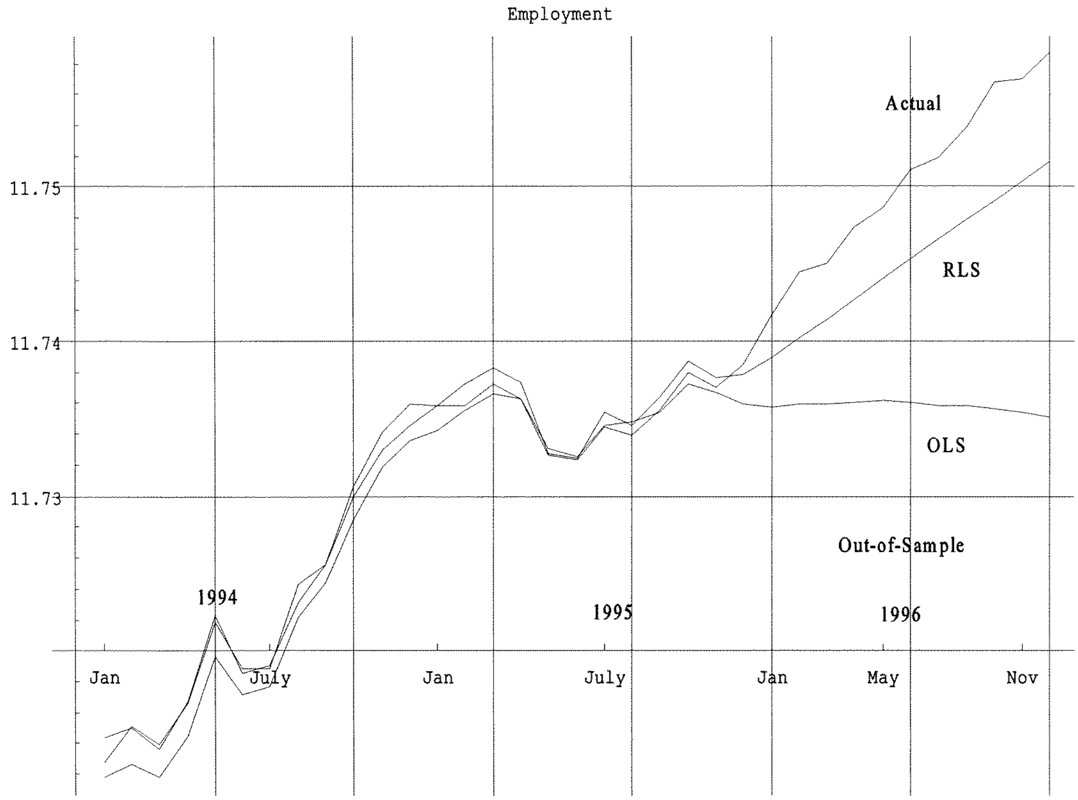

Figure 4 Employment: In-Sample and Out-of Sample Forecast Using Fed Funds and Federal Deficit as Control Variables

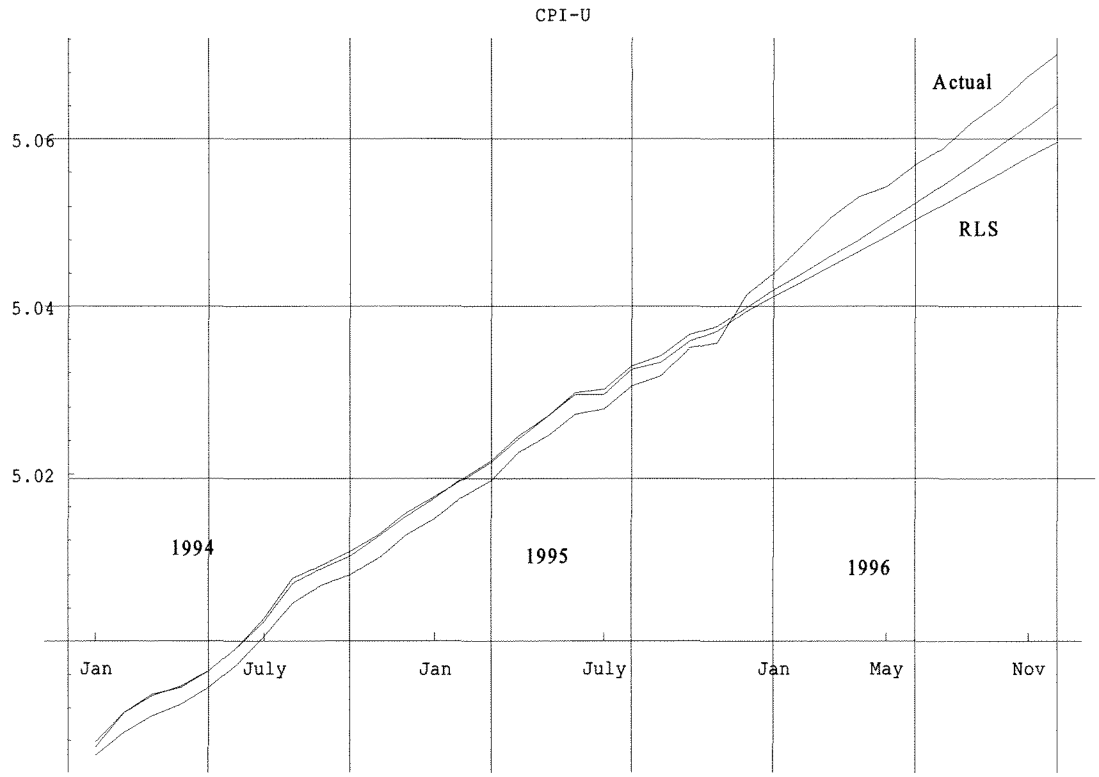

Figure 5 Consumer Price Index - Urban: In-Sample and Out-of-Sample Forecast Using Federal Deficit as Control Variable

Figure 6 M2: In-Sample and Out-of-Sample Forecast Using the Federal Deficit as a Control Variable

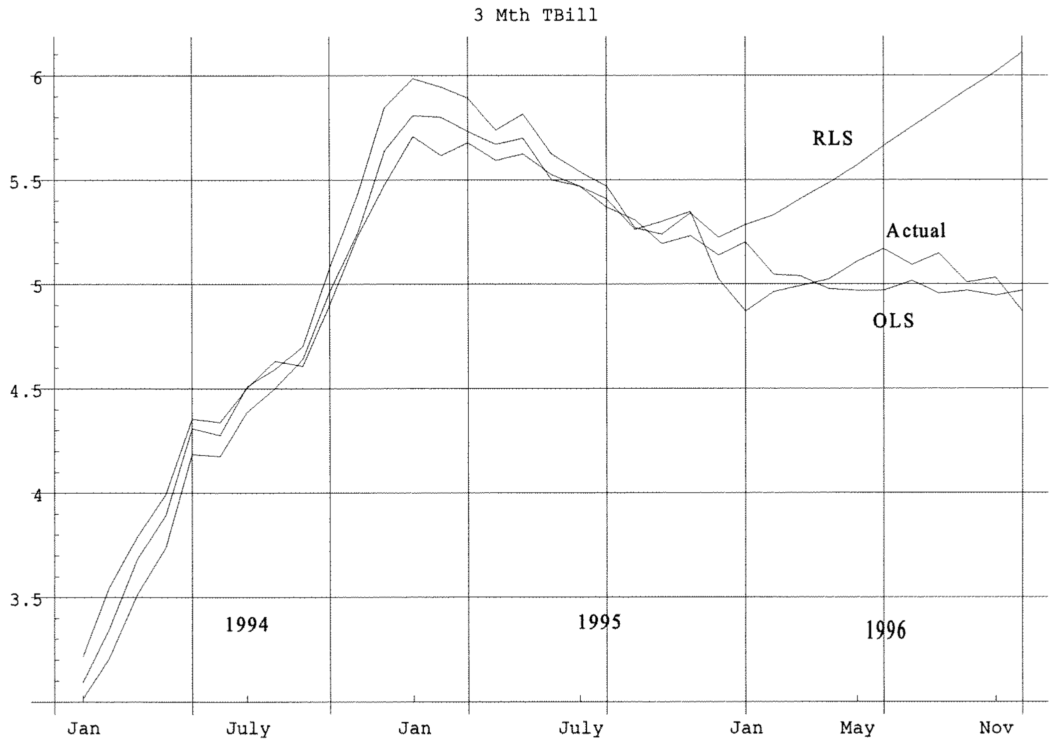

Figure 7 3 Month Treasury Bill Rate: In-Sample and Out-of Sample Forecast Using Fed Funds and Federal Deficit as Control Variables

Table 1: RMSE of Out of Sample Forecast of State Variables

Variable	Fed Funds as Control		Deficit as Control		Both as Control	
	RLS	OLS	RLS	OLS	RLS	OLS
Consumption	0.135	0.050	0.043	0.043	0.137	0.041
Investment	0.305	0.781	0.261	0.945	0.314	0.845
Industrial Production	0.110	0.642	0.111	0.664	0.117	0.620
Mfg. Inventories (Change)	534.9	2198.5	725.1	2275.5	525.2	2330.7
Retail Inventories (Change)	119.0	85.6	111.9	87.1	118.2	85.3
Mfg. I/S Ratio	2.702	3.192	1.184	3.315	2.774	2.939
Retail I/S Ratio	1.376	2.411	1.871	2.509	1.360	2.330
Employment	0.045	0.127	0.058	0.143	0.045	0.129
CPI-Urban	0.095	0.105	0.136	0.089	0.096	0.110
M2	0.266	0.152	0.093	0.164	0.269	0.154
3 Mth T Bill	13.29	3.125	11.889	3.229	13.481	2.947
Fed Funds	--	--	13.890	5.259	--	--
Fed Deficit	0.051	1.002	--	--	--	--

Table 2: RMSE of Out of Sample GDP Forecast (1/96-12/96)

	Fed Funds	Deficit	Both
OLS A, b, c	101.4	103.1	93.0
RLS A, b, OLS c	39.8	60.0	53.4
RLS A,b, one step ahead, OLS c	26.3	64.8	64.8

Table 3: Forecast of Quarterly GDP with Fed Funds as Control

Fed Fundsas Control	1996:Q1		1996:Q2		1996:Q3		1996:Q4	
	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr
OLS	0.1	1.2	-0.1	1.0	-0.3	-0.0	-0.5	-0.2
RLS	1.3	1.5	1.8	1.8	1.8	1.3	1.8	1.7
RLS (One Step Ahead)	2.4	1.8	3.1	2.4	1.6	1.8	1.5	2.1
Actual	2.0	1.7	4.7	2.7	2.0	2.2	3.8	3.2
Blue Chip (Feb. 1996)	-	1.7		2.2		1.9		2.0
Fair Model	2.7	-	2.8		2.3		2.3	

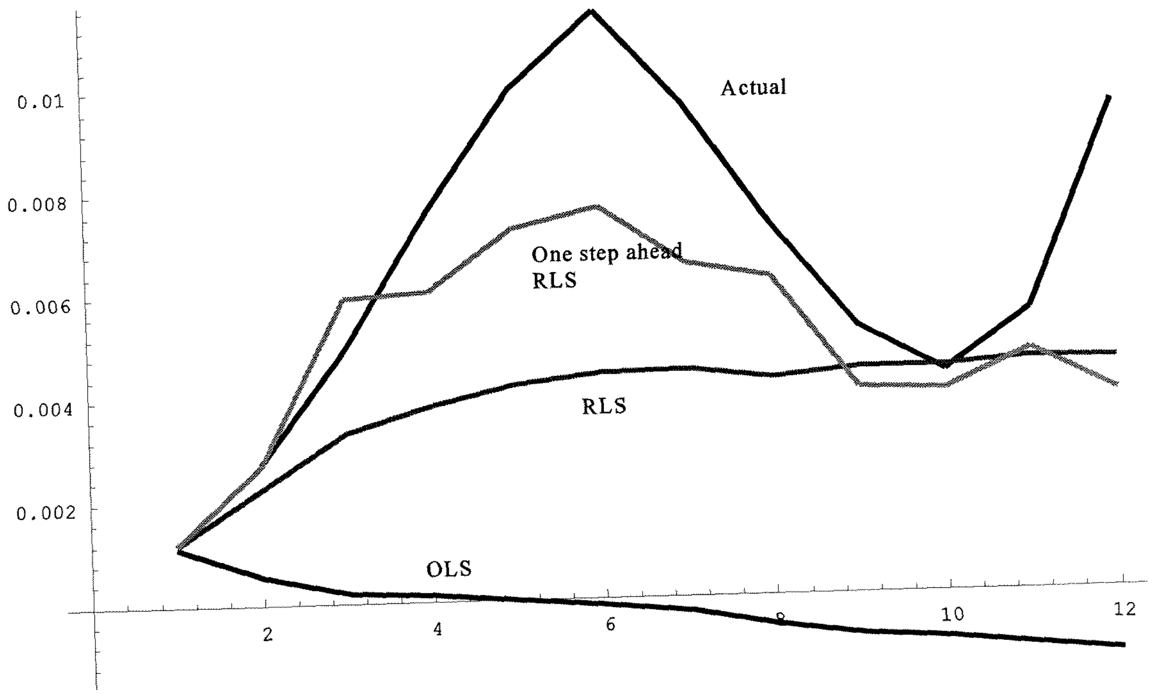

Figure 8 GDP Forecast: Fed Funds as Control

Table 4: Forecast of Quarterly GDP with Federal Deficit as Control

Federal Deficit as Control	Q1		Q2		Q3		Q4	
	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr
OLS	0.5	1.3	0.0	1.1	-0.2	0.1	-0.5	-0.1
RLS	1.8	1.6	1.7	1.9	1.5	1.3	1.3	1.6
RLS (One Step Ahead)	3.3	2.0	4.3	2.9	2.9	2.6	3.4	3.4
Actual	2.0	1.7	4.7	2.7	2.0	2.2	3.8	3.2
Blue Chip (Feb. 1996)	-	1.7		2.2		1.9		2.0
Fair Model	2.7	-	2.8		2.3		2.3	

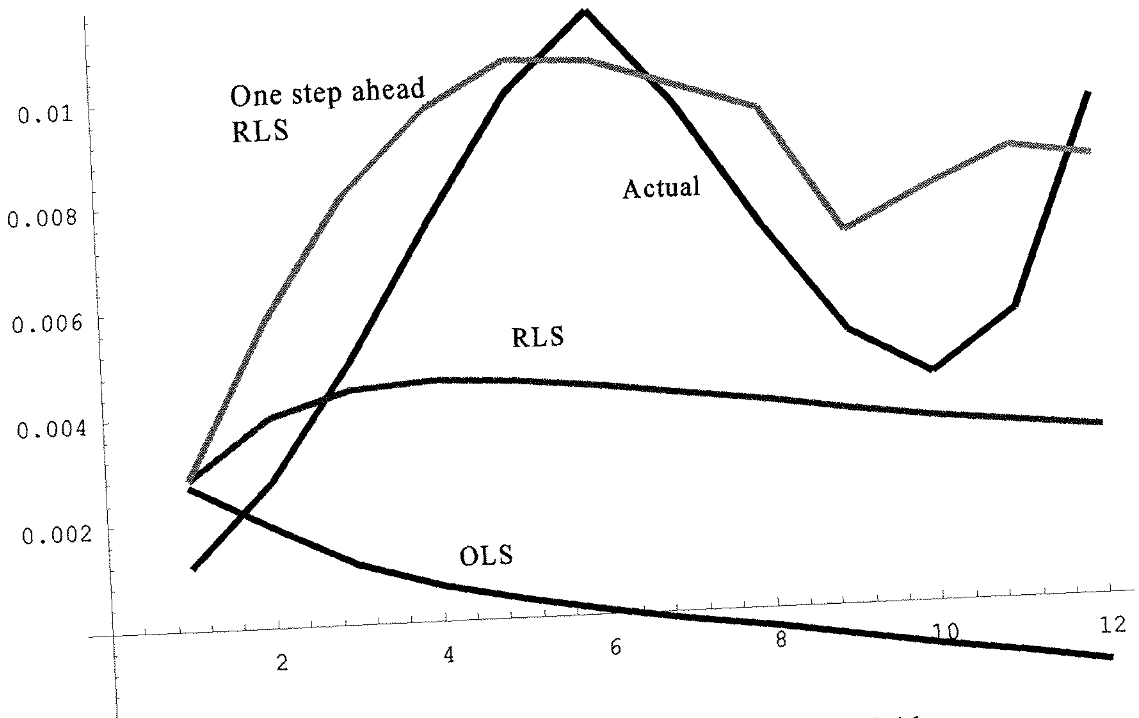

Figure 9 1996 GDP Forecast with Federal Deficit as Control Variable

Table 5: Forecast of Quarterly GDP with Fed Funds and Federal Deficit as Control								
Fed Funds and Federal Deficit	Q1		Q2		Q3		Q4	
	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr	Ann Rate	Yr/Yr
OLS	0.5	1.3	0.4	1.2	0.1	0.3	-0.0	0.2
RLS	2.0	1.7	2.2	2.1	2.0	1.6	4.1	2.5
RLS (One Step Ahead)	3.3	2.0	4.3	2.9	2.9	2.6	3.4	3.4
Actual	2.0	1.7	4.7	2.7	2.0	2.2	3.8	3.2
Blue Chip (Feb. 1996)	-	1.7		2.2		1.9		2.0
Fair Model	2.7	-	2.8		2.3		2.3	

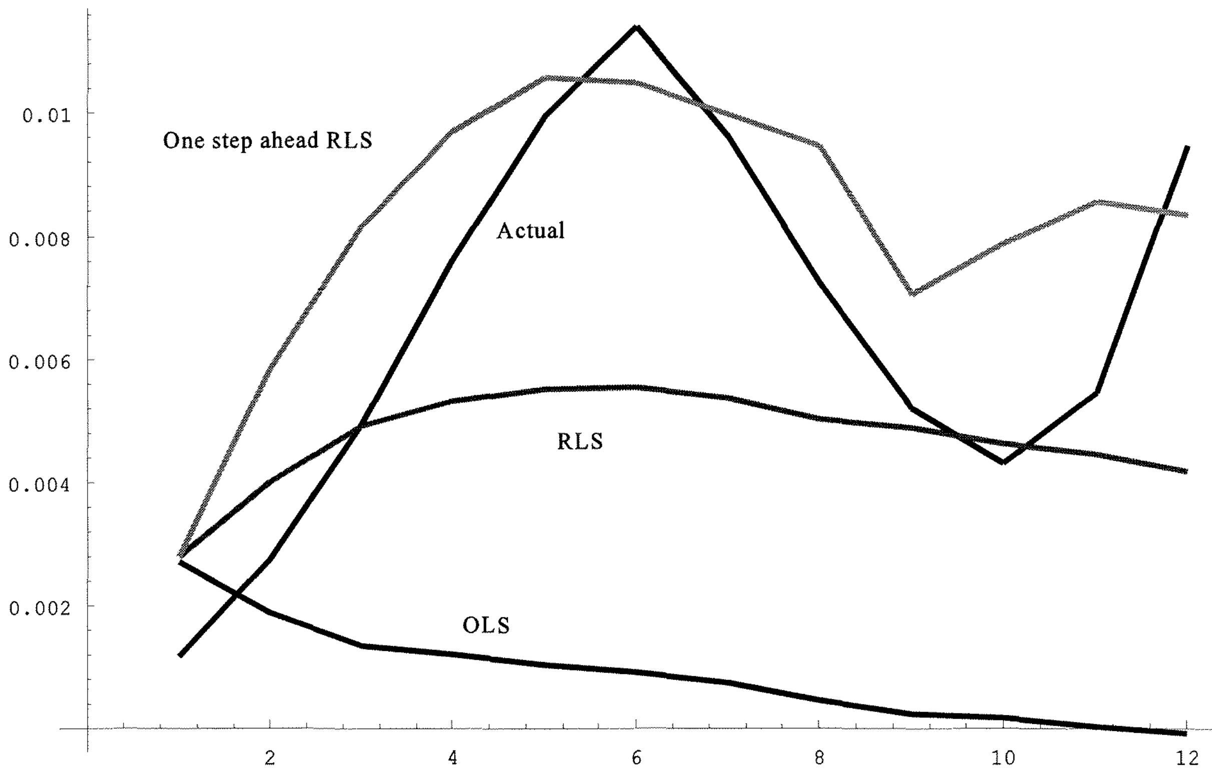

Figure 10 GDP Growth Forecast with both Fed Funds and Federal Deficit as Control Variables.