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Derek Laing
Victor E. Li
Ping Wang

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FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

INFLATION AND ECONOMIC ACTIVITY IN A MULTIPLE MATCHING MODEL OF MONEY

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Abstract

This paper investigates the relationship between money growth, inflation, and productive activity in a general equilibrium model where search frictions motivate the transactions role of money. The use of a multiple matching technique, where search frictions are captured by limited consumption variety, allows us to study price determination in a search-theoretic environment with divisible money and goods. We find that in such a setting, a positive feedback between work and shopping effort decisions create a channel by which inflation can positively influence real activity. This feature also creates the possibility of multiple steady state equilibria. We also analzye the impact of inflation on capital accumulation, the role search frictions play in determining the extent to which inflation distorts relative prices, and the effect of money growth on firm entry on trade frictions. In doing so, we demonstrate that a multiple matching model of money is amendable to study a wide range of traditional issues in monetary theory.

JEL Classifications: E31, E41

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Derek Laing
Pennsylvania State University

Victor E. Li
Federal Reserve Bank of St. Louis
and Pennsylvania State University

Ping Wang
Pennsylvania State University

I. Introduction

The relationship between money growth, inflation, and real activity is a classic and much debated issue in monetary economics. Contrary to the Phillips curve view of decades past, general equilibrium models of money tend to favor the conclusion that steady inflation is disruptive to economic activity. For example, money growth in cash-in-advance models with production [Stockman (1981) and Cooley and Hansen (1989)] generates a pure inflation tax effect which discourages market activities requiring cash. As a result, consumption, work effort, output, and the capital stock all decline with the inflation rate. Shopping time and money-in-the utility function models [e.g., McCallum and Goodfriend (1988)] also have a similar prediction. However, these approaches and their predictions have raised some concerns.

First, they approximate trade frictions which give rise to a transactions role for money in an otherwise Walrasian setting. Such a theoretical short-cut leads these approaches to overlook the impact of money growth and inflation on the very frictions which give rise to money as a medium of exchange. Secondly, evidence of a consistently negative relationship between inflation and economic activity is far from conclusive. While some cross-country studies and evidence from hyperinflation episodes [e.g., Fischer(1983), Cooley and Hansen (1989), and Aiyagari and Eckstein (1994)] find a negative correlation between inflation and output growth, these findings may be influenced by the observation that countries with sustained high inflation also experience highly variable inflation.[1] A recent study by Bullard and Keating (1995) finds that a negative money-output growth correlation is absent from stable price industrialized countries.

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[1] As argued by Jones and Manuelli (1995), it may be this variability of high inflation, rather than the level itself, which generates distortions and disrupts economic activity.

This paper evaluates the consequences of money growth and inflation on economic activity in the context of a search/matching model of money that highlights the decentralized and costly nature of the exchange process. Search theoretic approaches to monetary theory emphasize that the use of a medium of exchange minimizes the time or resource costs associated with searching for exchange opportunities, hence alleviating the "double coincidence of wants" problem with barter. The seminal work of Kiyotaki and Wright (1989,1991,1993) formalizes this aspect of monetary exchange in the search equilibrium paradigm of Diamond (1982,1984).

The particular search framework we adopt is based upon a "multiple" matching model of money developed by Laing, Li, Wang (1997). Such an approach, while embodying the "double coincidence of wants" frictions, utilizes an environment which allows us to relax restrictions on the divisibility and storability of goods and money often imposed in search-theoretic models of money.[2] The key features which allows us to accomplish this in a tractable manner are (i) abandoning a sequential search structure and having buyers contact *multiple* numbers of sellers in a given period and (ii) households having a preference for consumption variety and consuming baskets of goods. This ensures that there will always be a subset of goods among those contacted which the household

[2] In the prototypical search model of money, exchange is characterized by one-for-one swaps of goods and money, implying fixed prices. Extensions of the Kiyotaki-Wright model with divisible goods but indivisible money to include pricing include Trejos and Wright (1993,1995) and Shi (1994). Among the first to consider the implications of inflation in search-theoretic models of money is Li(1994,1995). However, because of these restrictions, inflation was modeled as a tax on money balances given fixed nominal prices.

finds desirable and hence keeps the steady state distribution of cash/goods trivial.[3] Search frictions and market incompleteness are captured by limitations in the number of sellers that buyers can contact in a given period and hence limited consumption variety. An analogy of this process is a consumer who shops in a marketplace and encounters many different products but not all desired products in the economy. The model is closed by specifying prices set by monopolistically competitive firms selling differentiated products and a circular flow of income between households and firms. Laing, Li, and Wang (1996) demonstrates that, given the double coincidence problem, monetary exchange improves trading opportunities relative to barter by increasing consumption variety.

Since the emphasis of this study is on inflation and monetary rather than barter exchange, the model simplifies and extends Laing, Li, and Wang (1997) to focus on a pure currency search economy. In our basic set-up there is a competitive labor market and a product market with random matching. Households allocate their time over work effort, shopping time, and leisure. They supply labor to firms and receive a cash wage payment. They then proceed to the goods market and are randomly matched with a subset of monopolistically competitive firms that set prices. It is the choice of shopping time which endogenizes the matching technology and influences the extent of trade frictions. Once cash is exchanged for desired goods, consumption occurs and firms use receipts to finance wage payments.

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[3] The main (technical) difficulty behind direct extensions of the Kiyotaki-Wright framework to include prices and divisible inventories is that it leads to an endogenous distribution of cash and goods which must be determined jointly with prices. Recent work attempting to characterize pricing behavior and the distribution of cash include Green and Zhou (1995), Corbae and Camera (1996), Zhou (1996), and Molico (1996). Shi (1997) circumvents the distributional issues with a structure where large households consist of a continuum of traders.

This framework is then used to study the effects of trade frictions, money growth and steady inflation on exchange activity, labor allocation, and production decisions. With a given time allocated to shopping, an exogenous reduction in trade frictions increases labor supply, overall work effort, and economic activity. On the other hand, money growth creates an inflation tax which a reallocation away from work effort to leisure. Similar to conventional models, inflation discourages market activity and real output.

However, by allowing shopping time and the matching technology to vary in response to the money growth rate, the results can be very different. In particular, not only can money growth and steady inflation encourage both work effort and shopping effort, but there also exists the possibility of multiple steady states. Intuitively, a greater matching rate encourages work effort and the higher labor income generated from work effort encourages shopping time. It is precisely this positive feedback between work and shopping efforts which why inflation can encourage market activity and the possibility of multiple equilibria.[4]

We then consider several variants of the basic model. First, the introduction of productive capital allows us to characterize equilibrium capital accumulation. We find that a positive relationship between inflation and capital can exist in equilibria where inflation positively affects work and shopping effort. Second, we analyze how search frictions and inflation distort relative prices in a model where households engage in the "home production" of a perfectly competitive homogenous good. Finally, we consider an alternative way of endogenizing the matching technology

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[4] It should also be noted that our notion of "shopping time" is very different from shopping time models of money. In these models money is valued because it directly increases the value of leisure. However, while possessing fiat currency in a world where it is generally accepted reduces exchange costs, it is not immediate why the quantity of money itself saves on these costs. Our model captures the notion that shopping time is a costly activity required for exchange.

by introducing firm entry. We find that not only that can money growth encourage firm participation, but if a preference for variety is sufficiently large, this entry effect may dominate the inflation tax effect, leading to a positive optimal rate of inflation.

The paper proceeds as follows. Section II will outline the basic model and characterize equilibrium conditions. Section III then analyzes the steady state of the model with both a fixed matching rate and endogenous shopping time. Section IV looks at extensions to the basic model to include productive capital, relative prices, and firm entry. Finally, Section V will conclude with a summary.

II. A Multiple Matching Model of Money

Goods, Preferences, and Production

Time is discrete and the economy is populated by a continuum of infinitely lived households (indexed by $h \in H$) and firms, with each of their masses normalized to unity. There is a large number of differentiated commodities of mass one, indexed by $\omega \in \Omega$. Each firm can only produce a particular good using labor as the sole input so that firms can also be indexed by ω. A household of type h desires a variety of goods over a subset $\Omega(h) \subset \Omega$. The commodity space is ordered in so that a worker of type h, employed by a particular firm, produces a good outside of his/her preference domain, $\Omega(h)$, so that there is no double coincidence of wants between them.[5] In this way, we rule

[5] This model can support the possibility of a double coincidence of wants and barter between households and firms by specifying carefully households' and owners' preferences over a *random subset* of goods. Laing, Li and Wang (1997) does precisely this, proves the existence of both barter and pure monetary equilibria, and shows that under some conditions, the pure monetary equilibrium is welfare-enhancing compared to barter. Since the present study focuses strictly on the pure monetary equilibrium, the detailed structure to support fiat currency will not be elaborated.

out the uninteresting case of autarky as well as any possible matches/exchanges between a worker and his/her employer. We also adopt the Diamond-Yellen (1990) convention in that associated with each firm ω is an infinitely lived owner who desires good ω and acts as the residual claimant of the firm's output.[6] All exchanges occur between households and firms as only workers/shoppers are mobile. Both goods and money are perfectly divisible and agents can store money and their own production goods in any amount without cost.

We make the following assumptions regarding household and firm owner preferences and the production technology.

Assumption 1: (*Household Preferences*). The lifetime utility for household h ∈ H is given by,

$$V = \sum_{t=0}^{\infty} \beta^t U(D_t(\omega), L_t) \tag{1}$$

where U[•] is strictly increasing and quasi-concave in its arguments, and D_t is a composite consumption good given by

$$D_t = \left\{ \int_{\omega \subset \Omega(h)} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega \right\}^{\frac{\gamma}{\gamma-1}} \tag{2}$$

where $\beta \in (0,1)$ is the subjective time-discount factor, L_t is leisure at time t, $c(\omega)$ is household consumption of good ω, and the composite consumption good captures the preference for consumption variety and has the constant elasticity form with $\gamma > 1$ denoting the elasticity of

[6] This feature of the model is without loss of generality since it is, as we shall see below, consistent with profit maximization. Alternatively, we can also consider a more complex environment where households are themselves the owners of firms and receive dividend payments via a stock market.

substitution across varieties.[7]

Assumption 2: (*Firm Owner Preferences*). For the owner of firm $\omega \in \Omega$ desires good ω, his lifetime utility is

$$\hat{V} = \sum_{t=0}^{\infty} \beta^t \hat{c}_t(\omega) \tag{3}$$

where $\hat{c}$ is ownership consumption of his own production good.

Assumption 3: (*Production Technology*). The production technology of firm ω is given by

$$y_t(\omega) = f[l_t(\omega)] \tag{4}$$

where $l(\omega)$ is the employment (density) and f satisfies $f' > 0, f'' < 0, f(0) = 0$ and the Inada conditions, $lim_{l \to 0} f'(l) = \infty$ and $lim_{l \to \infty} f'(l) = 0$.

Labor and Product Markets

At the beginning of each period households allocate their time to either work effort, l_t, shopping time (or "effort"), s_t, and leisure, $L_t = 1 - l_t - s_t$. Household's possess the ability to produce many types of goods but can only be productive at a single firm per period. Firm $\omega \in \Omega$ offers a competitive labor contract to households $h \in H$ which pays a nominal cash wage $W_t(\omega)$ in exchange for the household's labor services l_t.[8] With this, the firm produces output $y(\omega)$ according to the production technology given by (3).

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[7] For large values of γ, varieties are closer substitutes. This type of preferences is standard in the monopolistic competition literature, e.g., Dixit and Stiglitz (1977).

[8] In the generalized version of the model with barter and monetary exchange, this contract can also consist of wage payments in the firm's output. The composition of this optimal contract between goods and cash then determines equilibrium trading regimes. A pure monetary economy is one where this contract pays only cash wages.

Once household $h \in H$ receive wages from the competitive labor market, they travel to the goods market in which they are randomly matched with a set of $\chi_t \subset \Omega(h)$ firms with measure α_t. We make the following assumption regarding this matching technology:

Assumption 4: (*Matching Technology*). The measure of firms contacted by a particular household h is given by $\alpha(s_t)$, where $\alpha'(s_t) \geq 0$ and $\alpha(0) \geq 0$.

Thus, α_t can be thought of as a "matching" rate which measures the severity of search frictions in the goods market. It is endogenized by the household investment decision in shopping time.[9].

After matching, trades occur at monetary prices $P_t(\omega)$ set by the relevant monopolistically competitive firms, households consume $c_t(\omega)$ for each $\omega \in \chi$, and firms owners consume their residual output $\hat{c}(\omega)$.

The Money Supply Process

Lump-sum transfers of cash from the monetary authority occur to both households and firms after the labor market closes but before the goods market opens. Thus, firms must finance wage payments with cash receipts accumulated from the previous period's sales.[10] Let X_t denote this cash transfer, where a portion $T_t = \theta X_t$ is given to households and $\hat{T}_t = (1-\theta)X_t$ is given to firms, with θ

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[9] Technically, the set of firms contacted by households contain a countable number of firms. However, given a sufficiently dense product space, we approximate the consumer's aggregation over his desirable commodities as a continuum as defined in (1). An immediate consequence of this approximation is that the pricing behavior of firms will involve pure substitution effects and eliminate wealth effects.

[10] This timing of events should not be thought of as a cash-in-advance constraint on firms. It is the ex-post outcome of the richer environment where firms have the option of accumulating goods for the payment of wages.

$\in [0.1]$.[11] With this, we can write the (per capita) money supply process as $M^s_{t+1} = M^s_t + X_t = (1+\mu)M^s_t$ where μ is the money growth rate, and $X_t = T_t + \hat{T}_t$.

Optimization and Equilibrium

In each period, each household of type h is matched with a set of χ products with measure α in their desirable consumption set $\Omega(h)$. Included in this set are firms setting a common monetary price P and a set of positive measure of deviating firms (denoted by Ω'), with the representative deviating firm (indexed by ω') setting a monetary price of P'.

The representative household's problem is given by choosing $\{c_t(\omega), c_t(\omega'), l_t, s_t, M_{t+1}\}$ to maximize (1), subject to

$$M_t + W_t l_t - \int_{\chi(h)} P_t(\omega) c_t(\omega) d\omega + T_t - M_{t+1} \geq 0 \tag{6}$$

where

M_t

$$D_t = \left\{ \int_{\chi(h)} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega \right\}^{\frac{\gamma}{\gamma-1}}, \tag{5}$$

and M_{t+1} is the *beginning-of-period* household money holdings. With λ denoting the multiplier associated with (6) the first-order conditions, evaluated at the limiting case where the measure of Ω' vanishes, are given by

$$U_D(D,L) \{ \int_{\chi} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega \}^{\frac{1}{\gamma-1}} c_t(\omega)^{-\frac{1}{\gamma}} = \lambda_t P_t \tag{6}$$

[11]The liquidity effect literature [Lucas(1990), Fuerst(1992)] motivates a special case of this cash transfer process where $\theta \rightarrow 0$ and firms use the additional transfers to finance their wage bill.

$$U_D(D,L)\{\int_{\chi} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega\}^{\frac{1}{\gamma-1}} c_t(\omega')^{-\frac{1}{\gamma}} = \lambda_t P_t' \tag{7}$$

$$U_L(D,L) = \lambda_t W_t \tag{8}$$

$$U_L(D,L) = U_D(D,L)\frac{\partial D}{\partial s_t} \tag{9}$$

$$M_{t+1}\{\lambda_t - \beta\lambda_{t+1}\} = 0 \tag{10}$$

Equations (1) and (2) imply a relationship between $c(\omega)$ and $c(\omega')$ given by

$$\left[\frac{c_t(\omega)}{c_t(\omega')}\right]^{-1/\gamma} = \frac{P_t}{P_t'}$$

Substituting this into (6) yields the household's consumption demands:

$$c_t(\omega) = \frac{W_t l_t + T_t}{\alpha(s_t)P_t} = c \ ; \quad c_t(\omega') = \frac{W_t l_t + T_t}{\alpha(s_t)P_t^{1-\gamma}(P_t')^{\gamma}} \tag{11}$$

Equation (11) implies each consumer's demand, $c(\omega')$, decreases with its price P' and at a rate that depends upon the elasticity of substitution γ. An increase in total cash receipts, given by $W_t l_t + T_t$ raises the demand for all goods proportionately. The consumer's preference for variety implies that the share of their income apportioned to each good declines with the number of trading partners α contacted.

Noting that as the set of deviating firms are arbitrarily small, $\partial D/\partial s_t = \gamma/(\gamma-1)\alpha^{1/(\gamma-1)}\alpha'(s_t)$ c. Using this, (6), (8), and (9), the efficiency conditions for work effort and shopping time are

$$U_L(D,L) = U_D(D,L)\frac{W_t}{P_t}\alpha(s_t)^{\frac{1}{\gamma-1}} \tag{12}$$

$$\frac{W_t}{P_t} = \frac{\gamma}{\gamma-1}\alpha'(s_t)c_t \tag{13}$$

Equation (12) simply equates the marginal disutility of work effort with the marginal utility of consumption that can be supported by the additional wage income. Equation (13) says that while work effort raises the overall level of consumption by the additional real wage, the marginal benefits of shopping time is the additional variety which can be purchased with a given level of income. The latter is strictly increasing in the preference for variety (i.e. decreasing in γ).

Finally, from (10) note that a necessary condition for $M_{t+1} > 0$ is given by $\lambda_t = \beta\lambda_{t+1}$ or $\{U_{dt}D_{ct}/\beta U_{Dt+1}D_{ct+1}\}(P_{t+1}/P_t) = 1$. This condition implies that the opportunity cost of holding cash (or implicit nominal interest rate) is zero. We will impose that this cost be strictly positive, which in the steady state corresponds to the restriction that $\mu > \beta$-1. Consequently, since the cash transfer occurs before the goods market opens, the household ends each period with zero money holdings or $M_{t+1} = 0 \; \forall$ t. Thus, the household chooses an optimal sequence $\{c_t, c_t', l_t, s_t\}$ solving (11), (12), and (13) given prices and wage $\{P_t, P_t', W_t\}$.

We now consider the optimal price setting behavior of a deviating firm ω' which takes the price set by all other firms as given and sets the best response Nash equilibrium price P$'$. In a pure monetary equilibrium each owner consumes the residual output of it's firm so that the firm's problem is consistent with profit maximization. The representative deviating firm takes the consumption demands of each household contacted in (6) as given and chooses $\{\hat{c}_t(\omega'), l_t(\omega'), P_t(\omega'), \hat{M}_{t+1}\}$ which solves,

$$\max_{\hat{c}(\omega'),l(\omega'),P'} \sum_{t=0}^{\infty} \beta^t \hat{c}_t(\omega')$$

subject to

$$f[l_t(\omega')] - \alpha c_t(\omega') - \hat{c}_t(\omega') \geq 0 \tag{14}$$

$$\hat{M}_t + \alpha P_t(\omega')c_t(\omega') + \hat{T}_t - W_t l_t(\omega') - \hat{M}_{t+1} \geq 0 \tag{15}$$

$$\hat{M}_t \geq W_t l_t(\omega') \tag{16}$$

Inequality (14) is the firm's resource constraint, and says that output is either consumed or else sold to other households. Inequality (15) is the firm's flow budget constraint requiring that total cash balances at the beginning of next period cannot exceed the sum of current period money balances, receipts from sales, and the monetary transfer less cash wage payments. Finally, (16) is due to the absence of capital markets, and indicates that the firm cannot hire more labor than is warranted by its current cash balances.

It will be convenient to characterize a stationary equilibrium by scaling all nominal variables by the beginning-of-period money stock. With $\hat{m}_t \equiv \hat{M}_t/M_t^s$, $w_t = W_t/M_t^s$, and $p_t = P_t/M_t^s$, we can write (15) and (16) as

$$\hat{m}_{t+1} = \frac{\hat{m}_t + \alpha p_t(\omega')c_t(\omega') - w_t l_t(\omega') + (1-\theta)\mu}{1+\mu} \tag{15'}$$

$$\hat{m}_t \geq w_t l_t(\omega') \tag{16'}$$

and express the firm's value function as

$$V(\hat{m}_t) = \max_{l(\omega'), p'} f[l_t(\omega')] - \alpha c_t(\omega') + \beta V(\hat{m}_{t+1})$$

With (15') and (16') strictly binding, the first order conditions, given in the Appendix, yields a Nash equilibrium in the price-setting game where

$$p_t' = \left(\frac{\gamma}{\gamma-1}\right)\frac{w_{t+1}(1+\mu)}{\beta f'(l_{t+1})} \tag{17}$$

Intuitively, the monopolistic markup of price over next period wages depends negatively on γ and next period's marginal productivity. Since firms must finance wage payments with cash receipts carried over from last period, the marginal cost of hiring labor, and hence the markup, is increasing with the inflation rate μ. As $\mu \rightarrow \beta$-1 and $\gamma \rightarrow \infty$, the inverse markup approaches the marginal productivity of labor.

The firm chooses an optimal sequence $\{p_t', \hat{c}_t, l_t\}$ solving (14), (16'), and (17) given prices and wages $(p_t, w_t\}$. Labor and money market clearing implies that l_t chosen by firms and households are identical and that $\hat{m}_t = \hat{m}_{t+1} = 1$. We now characterize the steady state of the economy's equilibrium.

Definition 1: A symmetric steady-state monetary equilibrium is given by quantities $\{c^*, (c^*)', l^*, s^*\}$ and prices $\{p^*, (p^*)', w^*\}$ satisfying

$$p^* = \left(\frac{\gamma}{\gamma-1}\right)\frac{w^*(1+\mu)}{\beta f'(l^*)} \tag{18}$$

$$c^* = \frac{1+\theta\mu}{\alpha(s^*)p^*} \tag{19}$$

$$\frac{w^*}{p^*}\alpha(s^*)^{\frac{1}{\gamma-1}} = \frac{U_L(D^*,L^*)}{U_D(D^*,L^*)} \tag{20}$$

$$\frac{w^*}{p^*} = (\frac{\gamma}{\gamma-1})\alpha'(s^*)c^* \tag{21}$$

where p* = (p*)′, c* = (c*)′, w* = $1/l^*$, L* = 1 - l^* - s^*, and from (18), (19), and (2),

$$D^* = \alpha(s^*)^{\frac{\gamma}{\gamma-1}}c^* = \alpha^{\frac{1}{\gamma-1}}(\frac{1+\theta\mu}{1+\mu})(\frac{\gamma-1}{\gamma})\beta f'(l^*)l^* \tag{22}$$

Notice that a convenient way of expressing condition (20) is by substituting in (18), (19) and (22) and writing it in terms of the ratio of the elasticity of substitution of leisure to composite consumption

$$\Gamma(D^*,L^*)\frac{l^*(1+\theta\mu)}{1 - l^* - s^*} = 1 \tag{23}$$

where $\Gamma \equiv \xi_L/\xi_D$ and $\xi_L = U_L L$, $\xi_D = D_D D$.

III. Trade Frictions, Inflation, and Real Activity

This section analyzes the existence of steady state equilibria and investigates the model's steady state implications for money growth, inflation, and real activity. First, we will consider equilibria with a fixed shopping effort and matching rate. Then we consider the general model which allows shopping effort to vary optimally.

For convenience, and to make our analysis more concrete, we will adopt some specific functional forms for preferences and technology. In particular, let $f(l) = l^{\phi}$, consider a linear matching technology $\alpha(s) = \alpha_0 + \alpha_1 s$, $\alpha_0, \alpha_1 \geq 0$, and let preferences be given by $U(D,L) = [\eta D^{\rho} + (1-\eta)L^{\rho}]^{1/\rho}$, where $\eta \in (0,1)$ and $\rho \in [0,1]$. This CES specification embodies both the linear case where $\rho \to 1$ and the Cobb Douglas case where $\rho \to 0$. It implies that the elasticity of substitution ratio is given by $\Gamma = \{(1-\eta)/\eta\}(L/D)^{\rho}$. With this, condition (23) is given by

$$\frac{1-\eta}{\eta}\frac{l^*(1+\theta\mu)}{(D^*)^{\rho}(1-l^*-s^*)^{1-\rho}} = 1 \tag{24}$$

Equilibria with a Fixed Matching Technology

Consider the case where the matching rate is fixed at $\alpha = \alpha_0$ so that $s^* = 0$.

Proposition 1. Given $\alpha = \alpha_0$, there exists a unique steady state equilibrium $\{c^*, p^*, l^*, D^*\}$ solving (18), (19), (20) and (22).

Proof: From (24) a sufficient condition for this is that (l/D^{ρ}) is strictly increasing in l. Substituting in (22) gives

$$(l/D) = \frac{l^{1-\rho}(1+\mu)}{\alpha_0^{\gamma/(\gamma-1)}(1+\theta\mu)\beta f'(l)}$$

Thus, $\partial(l/D^{\rho})/\partial l > 0$ and there exists a unique l^* satisfying (24). With this, (22), gives D^* and (18) and (19) gives $p^* = [\gamma/(\gamma-1)]\,[(1+\mu)/\beta f'(l^*)]$ and $c^* = [(\gamma-1)/\gamma]\,[(1+\theta\mu)/(1+\mu)]\,[\beta f'(l^*)l^*/\alpha_0]$. ■

Consider now the impact of an exogenous increase in the matching rate α_0 and money growth rate μ:

Proposition 2. *(Impact of Trade Frictions)* In a pure monetary equilibrium with $\alpha = \alpha_0$, $\partial l^*/ \alpha_0 > 0$, $\partial D^*/ \alpha_0 > 0$, $\partial (w/p)^*/ \alpha_0 > 0$, and $\partial p^*/ \alpha_0 < 0$.

Proof: Substituting (22) into (24) gives the equilibrium locus determining l^*:

$$\frac{1-\eta}{\eta}(1+\theta\mu)^{1-\rho} = \left\{\alpha_0^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\frac{\beta f'(l)}{1+\mu}\right\}^{\rho}\left\{\frac{1-l}{l}\right\}^{1-\rho} \tag{25}$$

since the right hand side of (25) is strictly decreasing in l and increasing in α_0, $\partial l^*/ \alpha_0 > 0$. From (22), D is increasing in both α_0 and l so that $\partial D^*/\alpha_0 > 0$. From (18) $w^*/p^* = [\gamma/(\gamma-1)]\ \beta f'(l^*)/(1+\mu)$ and thus $\partial (w/p)^*/\alpha_0 < 0$. Finally, since $f'(l)l$ is increasing in l, $\partial p^*/\alpha_0 < 0$. ■

Intuitively, an increase in the matching rate increases the marginal benefit of wage income, as it is able to purchase more consumption variety. This shifts labor supply out and lowers the equilibrium real wage. The resultant increase in equilibrium work effort and matching rate increases real incomes and composite consumption.

Proposition 3. *(Impact of Money Growth and Inflation)* In a pure monetary equilibrium with $\alpha = \alpha_0$, $\partial l^*/ \mu < 0$, $\partial D^*/ \mu < 0$, $\partial (w/p)^*/ \mu < 0$, and $\partial p^*/ \mu > 0$.

Proof: From (25) it is immediate that a higher money growth rate increases the left hand side while reducing the right hand side. As the right hand side of (25) is strictly decreasing in l, it must be that $\partial l^*/\partial\mu < 0$. Since D is increasing in l from (22), $\partial D^*/ \mu < 0$. From (18) a decreasing l^* implies a higher nominal wage and lower marginal product and this gives $\partial p^*/ \mu > 0$. From (20) and CES preferences, note that $(w/p)^* = [(1-\eta)/\eta][D^*/(1-l)]^{1-\rho}$; as μ discourages D* and work effort, $\partial (w/p)^*/ \mu < 0$.

These results are not too surprising. Money growth creates an inflation tax effect which, for a given matching rate, decreases both labor demand and supply and equilibrium work effort. Real money balances used to finance labor declines and lower real incomes reduces composite consumption. This negative wealth effect of inflation is consistent with many standard general equilibrium models which predict a negative relationship between inflation and market activity. However, as we shall see below, the ability of traders in the economy to affect the "frequency" of exchange opportunities and the extent of search frictions can drastically change the characterization of steady state equilibria and even the impact of inflation on real activity.

Equilibria with Endogenous Shopping Effort

We now return to the general model outlined in Section II, where $\alpha = \alpha(s) = \alpha_0 + \alpha_1 s$. For a given shopping time allocation s, equation (24) corresponds to an efficiency condition for optimal work effort. Substituting (22) into (24) gives the LL locus:

$$\frac{1-\eta}{\eta}(1+\theta\mu)^{1-\rho}(1+\mu)^{\rho} = \left\{\alpha(s)^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\beta f'(l)\right\}^{\rho}\left\{\frac{1-l-s}{l}\right\}^{1-\rho} \tag{26}$$

For a given work effort allocation l, equation (21) corresponds to an efficiency condition for optimal shopping effort. Substituting (19) into (21) gives the SS locus:

$$l = \frac{\alpha(s)}{\alpha'(s)}\frac{\gamma-1}{\gamma(1+\theta\mu)} \tag{27}$$

A steady state can be characterized by $\{l^*, s^*\}$ satisfying (26) and (27). These conditions lead to the follow propositions:

Proposition 4. (*Characterization of LL and SS Loci*)

(i) For $\rho \geq 0$ sufficiently small, $\mathrm{d}l/\mathrm{d}s|_{LL} < 0$, for $\rho \leq 1$ sufficiently large $\mathrm{d}l/\mathrm{d}s|_{LL} > 0$, and there exists $0 < \rho < 1$ such that $\mathrm{d}l/\mathrm{d}s|_{LL} > 0$ for $s < \bar{s} < 1-l$ and $\mathrm{d}l/\mathrm{d}s|_{LL} < 0$ for $\bar{s} < s < 1$.

(ii) The SS locus is strictly increasing in the (s,l) space: $\mathrm{d}l/\mathrm{d}s|_{SS} > 0$.

Proof: See Appendix

The LL locus denotes the optimal response of work effort to a change in shopping effort. For ρ sufficiently large, a greater substitutability between composite consumption and leisure implies that an exogenous increase in s raises the marginal benefits of work effort and causes a substitution towards composite consumption. For ρ sufficiently small, less substitutability between composite consumption and leisure implies that an exogenous increase in s will actually reduce incentives for work effort as households substitutes towards leisure. The SS locus denotes the optimal response of shopping effort to a change in work effort. An exogenous increase in work effort lowers the marginal benefit of labor supply and, at the optimum, this must be equated with the marginal benefits of shopping effort. Since consumption per type, c^*, is strictly decreasing in s, an increase in shopping effort is necessary.

In light of these properties, we can divide the characterization of equilibria into several cases and analyze the effects of search frictions and inflation for each.

Proposition 5. Given ρ sufficiently small, there exists a unique steady state equilibrium $\{l^*, s^*\}$ such that $\partial s^*/\partial\alpha_0 < 0$, $\partial l^*/\partial\alpha_0 > 0$, $\partial s^*/\partial\mu > 0$, and $\partial l^*/\partial\mu < 0$.

Proof: See Appendix.

Intuitively, a reduction in trade frictions, as captured by an increase in α_0, generates a positive wealth effect which causes households to lower shopping effort and enjoy greater leisure and a substitution effect towards work effort. The SS locus shifts upwards in the (s,l) plane as shown in Figure 1. Consequently, there is an increase in composite consumption and real balances, and real wages decline from the increase in labor supply.

An increase in the inflation rate induces household's to substitute away from work effort and towards shopping time. Figure 1 illustrates this unique steady state and shows that an increase in μ shifts both the LL and SS loci downward. Note that in the case where $\rho = 0$ and $\theta = 0$, money is superneutral. However, the Cobb-Douglas specification is a "knife-edge" case where the elasticity of substitution ratio Γ is completely independent of the inflation tax effects. With ρ small, these results primarily stems from a wealth effect created by cash transfers to households.

Proposition 6. For ρ sufficiently large, there exists a unique steady state $\{l^*, s^*\}$ where

(i) $\partial s^*/\partial\alpha_0 < 0$ while the effect on l^* is generally ambiguous (with $\partial s^*/\partial\alpha_0 = 0$ for $\rho = 1$).

(ii) For $\gamma\text{-}1 > 1/(1\text{-}\phi)$, $\partial s^*/\partial\mu < 0$ and $\partial l^*/\partial\mu < 0$,

(iii) For $\gamma\text{-}1 < 1/(1\text{-}\phi)$, $\partial s^*/\partial\mu > 0$, and for $1 < (\gamma\text{-}1) < 1/(1\text{-}\phi)$, or θ sufficiently small, $\partial l^*/\partial\mu > 0$.

Proof: See Appendix.

Recall that with ρ sufficiently large, both the SS and LL locus are upward sloping. An increase in α_0 tends to reduce the optimal choice of s for a given l, shifting the SS locus upward in the (s,l) plane. This is the pure wealth effect of the improved matching technology . However, it also increases the optimal choice of l given s, shifting the LL locus upward. While both effects lead to a reduction in shopping time, the impact on work effort depends upon whether or not the substitution effect of α_0 outweighs the wealth effect. In the linear example where $\rho = 1$, these effects exactly cancel and there is no overall change in either the matching rate or equilibrium work effort (see Figure 2).

To obtain some intuition for these results, consider the case where $\theta = 0$. A greater money growth rate lowers work effort for a given shopping effort, shifting the LL locus downward in the (s,l) plane. This is the negative wealth effect of the inflation tax. For γ sufficiently large, the SS locus will be steeper LL and the decline in work effort lowers the marginal incentives to invest in shopping effort (see Figure 3, Case II). Notice that from (22) composite consumption D* falls, and, since U_L/U_D is a constant, (20) implies an increase in real wages as the marginal product of labor rises. *Intuitively, if the preference for variety is small, and hence search frictions are not important, inflation decreases investment in shopping time, employment, and economic activity.*

However, for γ sufficiently small the LL locus will be steeper than SS and the decline in work effort creates a substitution towards shopping effort. Consequently, the resulting increase in the matching rate increases the marginal benefits of wage income and the incentive to increase labor supply. It is precisely this positive feedback which can lead to an overall increase in work effort and employment (see Figure 3, Case I). *Intuitively, if the preference for variety is large, and hence search frictions are important, then inflation can increase shopping time, employment, and economic activity.* In this case an increase in the money growth rate increases composite consumption, D*, and lowers the real wage rate. This result is in stark contrast to the model which simply assumes a fixed matching rate.

Proposition 7. *(Multiple Equilibria)* For $0 < \rho < 1$, there exists the possibility of multiple (non-degenerate) steady states. If so,

(i) Equilibria can be ranked by a monotone increasing relationship between l^* and s^*,

and for θ sufficiently small,

(ii) $\text{Sign}\{\partial l^*/\partial\mu\} = \text{Sign}\{\partial s^*/\partial\mu\}$,

(iii) there will be at least one equilibria where $\partial l^*/\partial\mu > 0$ and $\partial s^*/\partial\mu > 0$.

This proposition can be verified graphically. Consider the case where the LL locus is upward sloping for s small and downward sloping for s large. Since the SS locus is upward sloping and all equilibria must occur along it, (i) is immediate. Since SS is linear, if LL is steeper than SS at the origin, then there is either a unique equilibria [described by Proposition 5 or Proposition 6(ii)] or an odd number of steady states (Figure 4, Case I). Since a higher money growth rate shifts the entire LL locus downward, movement of equilibria along the SS locus implies l^* and s^* must move in the same direction and for $2n$-1, $n \geq 2$, steady states implies that n - 1 of those equilibria will be characterize by $\partial l^*/\partial\mu > 0$ and $\partial s^*/\partial\mu > 0$. If SS is steeper than LL close to the origin, there is at least two steady states or, in general, an even number (Figure 4, Case II). Again, a higher money growth rate shifts SS downward, implying that l^* and s^* must move together. Furthermore, for every equilibria where $\partial l^*/\partial\mu < 0$ and $\partial s^*/\partial\mu < 0$, there exists one where $\partial l^*/\partial\mu > 0$ and $\partial s^*/\partial\mu > 0$.

The possibility of multiple equilibria again arises from the positive feedback effects between the optimal choices of work effort and investment in exchange activity. It is this interaction between employment and shopping time which not only creates a channel by which inflation can lead to increased economic activity but can also generate a multiplicity of steady states.

As an illustration of the existence of multiple equilibria, consider $\gamma = 2$, $\eta = 0.4$, $\rho = 0.8$, $\phi = 0.8$, $\alpha_0 = 0$, $\alpha_1 = 8$, $\beta = 0.99$. Figure 6 plots the roots of (26) as a function of l, where s has been substituted out from (27). For $\mu = 0$, it indicates that there is a low output equilibria, where $l = 0.104$ and $s = 0.208$ and a high output equilibria, where $l = 0.313$ and $s = 0.626$. Raising the inflation rate to $\mu = 0.10$ increases work and shopping effort in the low output equilibria to 0.125 and 0.256 and reduces work and shopping effort in the high output equilibria to 0.298 and 0.609.

IV. Some Extensions and Appliations

This section considers several extensions to our multiple matching model which enables it to address some other important issues relating to inflation and real activity. First, we look at the relationship between inflation and the capital stock, second, we analyze relative price determination, and third, endogenous firm entry.

Inflation and the Capital Stock

One of the central issues in monetary economics, dating back to Mundell (1963) and Tobin (1965), is how productive capital accumulation is related to inflation. Money-in-the-utility function models [Sidrauski (1967) and Brock (1975)] tends to support a superneutrality result, cash-in-advance models with either endogenous labor [Cooley and Hansen (1989)] or a finance constraint on capital goods [Stockman (1981)] predicts that inflation depresses the capital stock, and overlapping generations approaches [e.g., Drazen (1981)] support the presence of a "Mundel-Tobin" effect where inflation encourages capital investment.

We can incorporate productive capital quite easily into our matching framework by allowing firms to store unsold output which is then used in the following period's production process. With this, consider a standard Cobb-Douglas technology given by $f(l_t,k_t) = l_t^{\phi}k_t^{1-\phi}$, where k_t is the capital stock, $0<\phi<1$, and assume full capital depreciation. With this, the firm's value function can be expressed as

$$V(\hat{m}_t,k_t) = \max_{l_t,p_t',k_{t+1}} \; f(l_t,k_t) - \alpha c_t(\omega') - k_{t+1} + \beta V(\hat{m}_{t+1},k_{t+1})$$

The firm's first order conditions, evaluated at steady state, will imply

$$p^* = \left(\frac{\gamma}{\gamma-1}\right)\frac{w^*(1+\mu)}{\beta f_l(l^*,k^*)} \tag{28}$$

$$\beta f_k(l^*,k^*) = 1 \tag{29}$$

Thus, a steady state equilibrium is given by $\{p^*,c^*,l^*,s^*,k^*\}$ solving (19), (20), (21), (28), and (29). The modified versions of the SS and LL loci are given by (27) and

$$\frac{1-\eta}{\eta}(1+\theta\mu)^{1-\rho}(1+\mu)^{\rho} = \left\{\alpha(s)^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\beta J\right\}^{\rho}\left\{\frac{1-l-s}{l}\right\}^{1-\rho} \tag{30}$$

here $J \equiv \phi[\beta(1-\phi)]^{(1-\phi)/\phi}$. Since the steady state capital-labor ratio is constant, so is the marginal product of labor, given by J. Thus, (30) and (27) solve exclusively for $\{l^*,s^*\}$, with the steady state capital stock k* determined by (29). Consequently, since the marginal product of capital is increasing in labor, any equilibrium increase in l^* will lead to an increase in k*.

The characterization of this LL locus in (30) remains largely unchanged from that given in Proposition 4 for ρ bounded away from unity. Thus, we can conclude the following:

(i) for ρ sufficiently small, there exists a unique steady state where $\partial s^*/\partial\mu > 0$, $\partial l^*/\partial\mu < 0$, and $\partial k^*/\partial\mu < 0$.

(ii) for ρ sufficiently large, there exists a unique steady state where $\partial s^*/\partial\mu > 0$, $\partial l^*/\partial\mu > 0$, and $\partial k^*/\partial\mu > 0$.

(iii) for $0 < \rho < 1$, there exists the possibility of multiple equilibria. Among those where $\partial l^*/\partial\mu > 0$, we have $\partial k^*/\partial\mu > 0$.

Result (i) is immediate in that at the limiting case of $\rho = 0$, s^* and l^* will correspond to exactly Proposition 5. Thus, there will be a negative impact of inflation on the capital stock. To see (ii) note that for $\rho = 1$, the constancy of the marginal product of labor implies that (30) will determine a unique s^* while (27) pins down l^*. The reasoning behind (iii) is entirely analogous to the multiple equilibria discussion in the previous section. Thus, our model does contain equilibria where inflation can positively impact capital accumulation.

Relative Prices

The previous sections emphasized how money growth and inflation influences real activity through it's impact on household investment in exchange activity and hence search frictions. In this section we consider a different channel and analyze how the extent of trading frictions and inflation influence relative prices.

In particular, we simplify and extend the model by exogenously fixing the matching rate, $\alpha = \alpha_0$, and leisure and introduce a second sector specializing in the production of a homogenous good. Households have preferences over both the composite consumption good, given by (2) and sold on the search market as before, and homogenous good Q. They possess a home production technology with which to produce and sell Q on a frictionless and competitive market at price Z_t. Households allocate their unit of time between supplying labor to the market, l, and home production, h.

Assumption 4. *(Preferences and Technology)*

(i) Household lifetime utility is given by,

$$V = \sum_{t=0}^{\infty} \beta^t U(D_t(\omega), Q_t) \tag{31}$$

where D_t is a composite consumption good given by (2) and Q_t is a homogenous good.

(ii) The market production technology is given by (4) and each household's home-production technology is given by $g(h)$, where g satisfies $g' > 0$, $g'' < 0$, $g(0) = 0$ and the Inada conditions, $lim_{l \to 0} g'(h) = \infty$ and $lim_{l \to \infty} g'(h) = 0$.

Household's thus choose $\{c_t, c_t', Q_t, l_t\}$ to maximize (1) subject to

$$M_t + W_t l_t - \int_{\chi(h)} P_t(\omega) c_t(\omega) d\omega + Z_t g(1-l_t) - Z_t Q_t + T_t - M_{t+1} \geq 0$$

The firm's problem is identical to before and the market-clearing condition for the competitive good is $Q_t = g(1-l_t)$. This leads to the following steady state equilibrium conditions:

$$\frac{U_Q(D^*, Q^*)}{U_D(D^*, Q^*)} = \alpha_0^{\frac{1}{\gamma-1}} \frac{z^*}{p^*} \tag{32}$$

$$\frac{w^*}{z^*} = g'(1-l^*) \tag{33}$$

$$\frac{w^*}{p^*} = \left(\frac{\gamma-1}{\gamma}\right) \frac{\beta f'(l^*)}{1+\mu} \tag{34}$$

$$c^* = \frac{1+\theta\mu}{\alpha(s^*) p^*} \tag{35}$$

where $z^* = (Z_t/M_t^s)^*$ and $w^* l^* = 1$. Equation (32) equates the marginal benefits of consuming from the matching market sector with that of consuming from the competitive home production market, and (33) states that the implicit real wage earned from home production is equated with it's marginal product.

Rearranging these conditions, we arrive at a single condition determining the steady state l^*,

$$\Gamma[D^*, g(1-l^*)](1+\theta\mu) = \frac{\gamma-1}{\gamma} \frac{g(1-l^*)}{l^* g'(1-l^*)} \tag{36}$$

where $\Gamma \equiv \xi_L/\xi_D$ and $\xi_L = U_L L$, $\xi_D = D_D D$, and a condition determining the relative price of home to matching market goods,

$$\frac{z^*}{p^*} = \frac{\gamma-1}{\gamma}\frac{\beta}{1+\mu}\frac{f'(l^*)}{g'(1-l^*)} = \Gamma(D^*,Q^*)(\frac{1+\theta\mu}{1+\mu})\frac{\beta f'(l^*)l^*}{g(1-l^*)} \tag{37}$$

Notice that from (23), as the preference for variety diminishes and $\mu \to \beta$-1, this relative price converges to the competitive value of the ratio of marginal productivity across the home and market sectors. With this we make the following observations:

Proposition 8. (*Trade Frictions, Inflation, and Relative Prices*)

(i) Given $\Gamma_D < 0$ and $\Gamma_L > 0$, $\partial l^*/\partial\alpha_0 > 0$, $\partial(z^*/p^*)/\partial\alpha_0 < 0$, $\partial l^*/\partial\mu < 0$, $\partial(z^*/p^*)/\partial\mu < 0$

(ii) Given $\Gamma_D = 0 = \Gamma_L$, $\partial l^*/\partial\alpha_0 = 0$, $\partial(z^*/p^*)/\partial\alpha_0 = 0$; for $\theta = 0$, $\partial l^*/\partial\mu = 0$, $\partial(z^*/p^*)/\partial\mu < 0$, and for $\theta > 0$, $l^*/\partial\mu < 0$, $\partial(z^*/p^*)/\partial\mu < 0$.

Proof: See Appendix

Intuitively, case (i) indicates that as the severity of market frictions diminish, and α_0 rises, there is a shift in demand away from homogenous goods to differentiated products, reducing the relative price of the home production good. As a result, there is an increase in the allocation of work effort to the matching market sector, leading to a reduction in the real wage. An increase in the inflation rate directly lowers real income and the real wage earned in matching market production. As a result, demand is shifted away from matching market to home production. While the increase in demand tends to increase the relative price of home-produced goods, the inflation tax effect on real wages in the market sector dominates and the relative price of matching market goods rises.

The Cobb-Douglas case (ii) is once again a knife-edge case where the elasticity ratio is independent of consumption levels. An increase in α_0 leads to a greater matching rate and demand for matching market goods. However, this effect is exactly off-set by an equi-proportional decrease in demand for each consumption type. Also, if $\theta = 0$, money will be superneutral in affecting the equilibrium employment allocation, but the inflation tax still increases the relative price of matching market goods. For $\theta > 0$, households receive cash transfers prior to shopping, and this creates an offsetting effect that reduces work effort in the matching market while still increasing the relative price of matching market goods.

Several empirical studies [e.g., Garber (1982) and Rogers and Wang (1993)] offer evidence in support of our finding that inflation discourages market activity and increases the relative price of market goods.

Firm Entry, the Matching Technology, and Welfare

In this final extension, we consider an alternative method to endogenize the transactions technology. Previously, we have always normalized the measure of firms to be unity; this section considers the issue of optimal firm entry and how such decisions are influenced by the money growth rate. To isolate this effect, we simplify the model with an inelastic labor supply and shopping effort and set $\theta = 0$. Letting N denote the measure of firms, it will be convenient to denote R(N) as the ratio of the mass of firms contacted by each household to the total measure of firms, $R(N) \equiv \alpha(N)/N$.

Assumption 5: (*Matching Technology*) The ratio of matches to the measure of firms, R(N), satisfies $R'(N) \geq 0$ and $R(0) = 0$.

We impose a fixed per-period firm entry cost of $\kappa > 0$ and allow the measure of firms to vary subject to an ex-post zero profit condition given by $\hat{c}(\omega') = \kappa$ or

$$f[l_t(\omega')] - \frac{\alpha}{N} c_t(\omega') = \kappa \tag{38}$$

Since both households and firms take this matching technology as given, their optimization problems will be identical to before, with shopping time and leisure normalized to zero. With U(D) = D, a *steady state with firm entry* is thus characterize by {p*,l*,D*,N*} satisfying $l^* = 1/N$, (18), (19), (22), and $\hat{c}^* = \kappa$. Substituting (18) into (19) and both into (38) gives

$$c^* = \frac{w^*}{R(N)Np^*} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{\beta f'(1/N)}{R(N)N}\frac{1}{1+\mu} \tag{39}$$

$$f\left(\frac{1}{N}\right)\left\{1 - \frac{1}{N}\frac{f'(1/N)}{f(1/N)}\frac{\beta}{1+\mu}\left(\frac{\gamma-1}{\gamma}\right)\right\} = \kappa \tag{40}$$

roposition 9. The measure of firms and hence product variety is strictly increasing in the money growth rate μ, i.e., $dN/d\mu > 0$.

Proof: See Appendix

The intuition behind Proposition 4 is straightforward. An increase in the inflation rate increases the monopolistic markup and, for a given N, firm profits and owner consumption. This encourages the entry of new firms and product variety. Of course, this may be an oversimplified result as generalizing to allow households a work effort choice may put a limit on the this entry effect of inflation.

Given the absence of leisure and zero profits on the part of firms, it is quite straight forward to check whether or not this inflation effect on entry improves the matching technology sufficiently to improve steady state household and aggregate welfare. This involves computing the equilibrium effect on $V^* = [1/(1-\beta)]D^*$ and leads to the follow conclusion:

Proposition 10.

With endogenous entry, the optimum rate of money growth is determined as follows:

(i) Given $R'(N) = 0$, the unique optimal money growth rate which maximizes steady-state welfare is $\mu^* = \beta - 1$.

(ii) Given that $R'(N) > 0$ and $NR'(N)/R(N)$ is non-increasing in N, there is a unique positive optimal money growth rate $\mu^* > 0$ for sufficiently low time-discounting and a sufficiently large curvature for the preference over variety.

Proof: See Appendix.

By Proposition 9, a higher rate of money growth encourages firm entry. Since both the number and fraction of products contacted is increasing in N, firm entry increases the density of the product space and reduces trading frictions. If the matching technology is linear, then the optimal inflation rate exactly corresponds to what can be interpreted as "Friedman's rule" of $\mu^* = \beta - 1$ and this is a unique interior optimum. If the matching technology is such that the fraction of firms located is increasing in the number of entrants, the optimal inflation rate can indeed be positive. Any inflation rate lower than this worsens search frictions and impedes consumption variety. As a result a positive inflation rate permits households to enjoy greater variety and, as long as the subjective discounting factor is not too small and product variety is sufficiently important, this effect can dominate the inflation tax effect on households discussed in the last section.

VI. Conclusion

This paper has investigated implications of a multiple matching model of money for the effects of monetary growth and inflation on economic activity. The use of a multiple matching technique, where search frictions are captured by limited consumption variety, allows the model generalizes various aspects of the traditional money-search literature, including price determination and the divisibility and storability of goods and money. A variety of issues linking inflation to real activity are analyzed. The common conclusion shared by all of them is that the frictions which motivate monetary exchange are not simply a veil, they have real consequences on the economic activity, and are themselves generally not invariant to monetary and inflation policy.

The basic model we analyze is a production variant of a multiple matching model of money where both optimal work and shopping effort are determined jointly. We find that a positive feedback between these two decisions not only creates a channel by which inflation positively influences productive market activity, but also generate the possibility of multiple steady state equilibria. This latter finding suggests that the lack of strong empirical evidence supporting a positive or negative impact of steady inflation in industrialized countries may be the result of an economy in transition across multiple equilibria. However, our steady state analysis cannot address the stability of these steady states and the transitional dynamic response to changes in the money growth rate. Given our results, this undertaking appears to be a fruitful avenue for future work.

We then considered several related issues. First, incorporating capital accumulation into the model preserves many of these features, with the additional result that a Mundel-Tobin effect, where steady state inflation and the capital stock are positively related, can exist in equilibria where inflation increases work and shopping effort. Second, in our home production model, we find that the extent of search frictions can influence relative prices and that inflation can increase and distort and the relative price of goods across markets with varying degrees of trading frictions. Third, we endogenizing the matching technology with the optimal entry of firms. In this example, inflation, which raises the monopolistic mark-up, encourages firm entry and product variety. The resultant reduction of search frictions can lead to a positive optimal rate of inflation.

The results of this paper also complementary to some earlier work by Li (1994,1995) evaluating the consequences of inflation in search-theoretic models of money. In a fixed price indivisible search model of money, these papers concluded that a tax on money balances can indeed positively influence search activity, stimulate the accumulation of inventories, and increase welfare. Our paper suggests that these conclusions may not have been just an artifact of the indivisible nature of fiat money and inventory restrictions assumed by these models and are robust to generalizations to the search environment.

Finally, this paper has demonstrated that it is possible to construct search theoretic models of money which can be applied to a wide variety of issues in monetary economics. For example, once could analyze if such a model can capture the liquidity effects of monetary shocks and their implications for the cyclical behavior of real variables. For future work, it may also interesting to allow firms to borrow from credit markets such that they are not completely subject to the *ex post* cash constraint for wage bills. In so doing, one may analyze the consequences of inflation on the credit market, especially, the financial intermediation ratio and the loan-deposit interest rate spread.

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APPENDIX

Firm's First Order Conditions

Letting $\hat{\lambda}$ be the multiplier associated with (16') and assuming the constraints are strictly binding, the firm's first order conditions for l_t and p_t' can be written as:

$$\frac{\beta V_m(\hat{m}_{t+1})}{1+\mu}[c_t(\omega') + \frac{\partial c_t(\omega')}{\partial p'}p'] = \frac{\partial c_t(\omega')}{\partial p'}$$

$$f'[l_t(\omega')] = [\frac{\beta V_m(\hat{m}_{t+1})}{1+\mu} + \hat{\lambda}_t]w_t$$

and where, from (11), $\partial c_t'/\partial p_t' = -\gamma c_t'/p_t'$. The envelope condition is given by

$$V_m(\hat{m}_t) = \frac{\beta V_m(\hat{m}_{t+1})}{1+\mu} + \hat{\lambda}_t.$$

Proofs to Propositions

Proposition 4

(i) Without loss of generality, let $\alpha_0 = 0$. Notice that the left hand side of (26) is independent of l and s. The right hand side of (26) can be written as

$$RHS_{LL} = \left\{\frac{\gamma-1}{\gamma}\beta\phi\alpha_1^{\frac{1}{\gamma-1}}\right\}^{\rho}\frac{(1-l-s)^{1-\rho}}{l^{1-\phi\rho}}s^{\frac{\rho}{\gamma-1}}$$

Thus, the right had is strictly decreasing in l [$d(RHS_{LL})/dl<0$]. Differentiating this expression with respect to s gives

$$\frac{dRHS_{LL}}{ds} = \left\{\frac{\gamma-1}{\gamma}\beta\phi\alpha_1^{\frac{1}{\gamma-1}}\right\}^{\rho}s^{\frac{\rho}{\gamma-1}}\frac{(1-l-s)^{-\rho}}{l^{1-\phi\rho}}\left[\frac{\rho}{\gamma-1}\frac{1-l-s}{s} - (1-\rho)\right] \tag{A1}$$

For $\rho = 0$, this expression is strictly negative, implying $dl/ds|_{LL} < 0$, and for $\rho = 1$, this expression is strictly positive, implying $dl/ds|_{LL} > 0$. We also see that for s sufficiently small, (A1) is positive and for $s \to (1-l)$, (A1) is negative. Thus, for a given l there exists

$\bar{s}=(\sigma/1+\sigma)(1-l)$, where $\sigma = \rho/[(1-\rho)(1-\gamma)]$, such that (A1) is positive for $s < \bar{s}$, implying $dl/ds|_{LL} > 0$ and negative for $s > \bar{s}$, implying $dl/ds|_{LL} < 0$.

(ii) From (27), a sufficient condition for $dl/ds|_{SS} > 0$ is given by $\alpha''(s) \leq 0$. This is certainly satisfied with our linear matching technology, which implies an increasing and linear SS locus. ■

Proposition 5.

Consider the limiting case where $\rho = 0$, the Cobb-Douglas case. Equations (26) and (27) can be written as

$$\frac{1-\eta}{\eta}(1+\theta\mu) = \frac{1-l-s}{l} \tag{A2}$$

$$l = \frac{(\alpha_0+\alpha_1 s)}{\alpha_1}\frac{\gamma-1}{\gamma(1+\theta\mu)} \tag{A3}$$

Since the (LL) locus in (A2) is downward sloping while the (SS) locus in (A3) in upward sloping in the (s,l) plane, there exists a unique steady state $\{l^*,s^*\}$. An increase in α_0 increases l and shifts SS upwards. As a result,$\partial s^*/\partial\alpha_0 < 0$, $\partial l^*/\partial\alpha_0 > 0$. To analyze the effect of an increase in money growth μ, substitute (A3) into (A2) to get

$$\frac{1-\eta}{\eta} = \frac{(1\ -\ s)\alpha_1}{\alpha_0\ +\ \alpha_1 s}\left(\frac{\gamma}{\gamma-1}\right) - \frac{1}{1+\theta\mu} \tag{A4}$$

since the right hand side of (A4) is increasing in μ and decreasing is s, $\partial s^*/\partial\mu > 0$. From (A2), $\partial l^*/\partial\mu < 0$. From (A3), the overall matching rate α increases.

Proposition 6. Consider the limiting case where $\rho = 1$, the linear specification. Equations (26) can be written as

$$\frac{1-\eta}{\eta}(1+\mu) = \alpha(s)^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\beta f'(l) \tag{A5}$$

d

and (27) is given by (A3). Since the right hand side of (A5) is strictly increasing in s and decreasing in l, both the SS and LL locus are upward sloping in the (s,l) plane. By substituting (A3) into (A5) we can verify a unique steady state given by

$$s^* = \{E(1+\mu)(1+\theta\mu)^{\phi-1}\}^{\psi}/\alpha_1 - \alpha_0/\alpha_1 \tag{A6}$$

$$l^* = \{E(1+\mu)(1+\theta\mu)^{\phi-1}\}^{\psi}(\frac{\gamma-1}{\gamma})\frac{1}{\alpha_1(1+\theta\mu)} \tag{A7}$$

where $E \equiv [(1-\eta)/\eta][\gamma/(\gamma-1)]^{\phi}/[\beta\phi\alpha_1^{1-\phi}]$ and $\psi \equiv (\gamma-1)/[1-(\gamma-1)(1-\phi)]$.

(i) It is immediate that $\partial s^*/\partial\alpha_0 = -1/\alpha_1 < 0$ and $\partial l^*/\partial\alpha_0 = 0$.

(ii) For the effects of inflation, we set $\alpha_0 = 0$ for convenience. Consider the case where $\psi < 0$, which is guaranteed for a sufficiently large $(\gamma-1) > 1/(1-\phi)$. From (A6) it is clear that $\partial s^*/\partial\mu < 0$. To analyze the impact on work effort, notice from (A7) that $(\phi-1)\psi - 1 > 0$. This implies that $\theta>0$ supports a positive impact of μ on l^*. Take the limiting case where $\theta = 1$, the exponential on $(1+\mu)$ becomes $\psi\phi - 1 < 0$, implying $\partial l^*/\partial\mu < 0$.

(iii) Next, consider the case where $\psi > 0$, which is guaranteed for a sufficiently small $(\gamma-1) < 1/(1-\phi)$. From (A6) it is clear that $\partial s^*/\partial\mu > 0$. To analyze the impact on work effort, notice from (A7) that for $\theta = 0$, $\partial l^*/\partial\mu > 0$. As $\theta > 0$ supports a negative impact of μ on l^*, take the limiting case where $\theta = 1$, the exponential on $(1+\mu)$ becomes $\psi\phi - 1$, which is positive for $(\gamma-1) > 1$, implying $\partial l^*/\partial\mu > 0$, and negative for $(\gamma-1) < 1$, implying $\partial l^*/\partial\mu < 0$.■

Proposition 8

(i) Suppose $\Gamma_D < 0$ and $\Gamma_L > 0$. From (22) we have $D = [(\gamma-1)/\gamma]\alpha_0^{1/(\gamma-1)}[(1+\theta\mu)/(1+\mu)]\beta f'(l)l$, increases with l and α_0 and decreases with μ. An increase in α_0 decreases the left hand side of (36), while the right hand side is decreasing in l. Thus, $\partial l^*/\partial\alpha_0 > 0$. From (37) it is immediate that $\partial(z^*/p^*)/\partial\alpha_0 < 0$. An increase in μ decreases D and increases the left hand side of (36), implying $\partial l^*/\partial\mu < 0$. From the second equality in (37), $\partial(z^*/p^*)/\partial\mu < 0$.

(ii) Given $\Gamma_D = 0 = \Gamma_L$, it is clear that α_0 will not have any overall effects. For $\theta = 0$, changes in μ will have no effects on l while, from (37), $\partial(z^*/p^*)/\partial\mu < 0$. For $\theta > 0$, an increase in μ raises the left hand side of (36), leading to $l^*/\partial\mu < 0$ and $\partial(z^*/p^*)/\partial\mu < 0$. ■

Proposition 9

With our functional form for the production technology, equation (37) may be written as

$$f(\frac{1}{N})(1 - \frac{\phi\beta}{1+\mu}\frac{\gamma-1}{\gamma}) = \kappa$$

which implies

$$N^{\phi} = \frac{1}{\kappa}[1 - \frac{\phi\beta}{1+\mu}(\frac{\gamma-1}{\gamma})] \tag{A8}$$

Differentiating with respect to μ and rearranging gives

$$\frac{dN}{d\mu} = \frac{N^{1-\phi}}{\kappa}\frac{\beta}{(1+\mu)^2}(\frac{\gamma-1}{\gamma}) > 0 \qquad \text{(A9)}$$

which verifies the proposition. ■

Proposition 10

(i) From (22) V is proportional to $\Psi \equiv [N^{\gamma/(\gamma-1)-\phi}R(N)^{1/(\gamma-1)}]/(1+\mu)$. Differentiating with respect to μ gives

$$\frac{d\Psi}{d\mu} = (1+\mu)N^{\frac{1}{\gamma-1}-\phi}R^{\frac{1}{\gamma-1}}[(\frac{\gamma}{\gamma-1}-\phi) + (\frac{1}{1-\gamma})\frac{NR'(N)}{R}]\frac{dN}{d\mu} - N^{\frac{\gamma}{\gamma-1}-\phi}R^{\frac{1}{\gamma-1}}$$

Equating to zero, substituting in dN/dμ from (A9) and simplifying this expression yields the optimality condition

$$\left[\frac{\gamma}{\gamma-1} - \phi + N(\frac{1}{\gamma-1})\frac{R'(N)}{R}\right]\frac{1}{N^{\phi}\kappa}\frac{\beta}{1+\mu}(\frac{\gamma-1}{\gamma}) = 1 \qquad \text{(A10)}$$

Suppose $\alpha(N) = \alpha_0$, which is independent of N. Since the left-hand side of (A10) is strictly decreasing in μ there exists a unique optimal μ which satisfies this condition. Substituting (A8) into (A10) gives

$$[1 - \frac{\phi(\gamma-1)}{\gamma}]\,[1 - \frac{\phi\beta}{1+\mu}\frac{\gamma-1}{\gamma}]^{-1}\frac{\beta}{1+\mu} = 1,$$

which implies $\mu^* = \beta - 1$, thus verifying (i).

(ii) Suppose instead that R′(N) > 0. Provided that NR′(N)/R(N) is non-increasing in N, (A10) again implies a unique μ satisfying this condition as the left hand side is strictly decreasing in μ. Substituting (A8) into the left hand side of (A10) gives

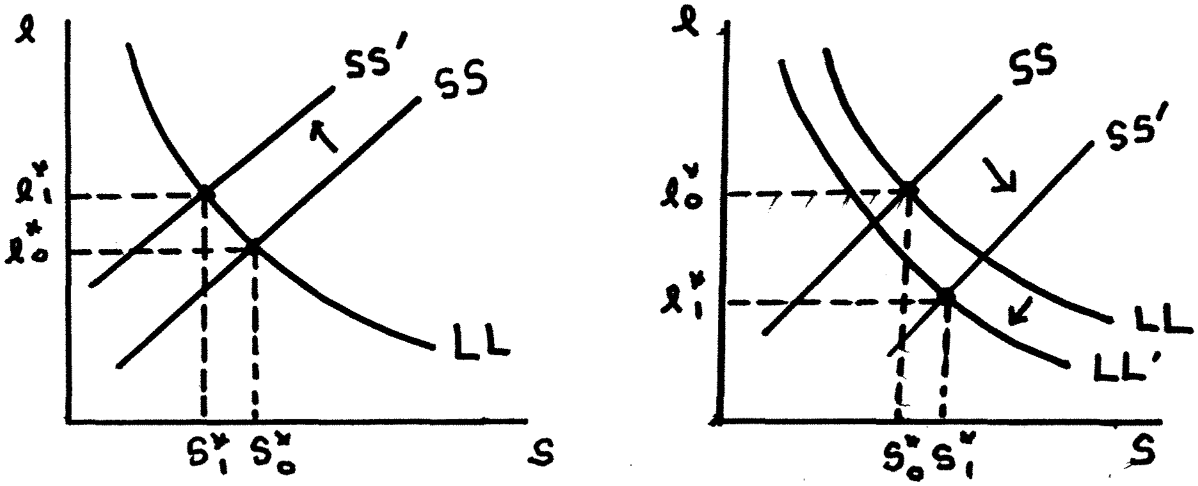
$$\Phi(\mu) \equiv \left[1 - \phi(\frac{\gamma-1}{\gamma}) + \frac{1}{\gamma}\frac{NR'(N)}{R}\right]\left[\frac{1+\mu}{\beta} - \phi(\frac{\gamma-1}{\gamma})\right]^{-1} = 1$$

where $d\Phi/d\mu < 0$. Taking, for example, $\beta = 1$, we can easily show that

$$\Phi(0) \equiv 1 + \frac{1}{\gamma}\frac{NR'(N)}{R(N)}\left[1 - \phi(\frac{\gamma-1}{\gamma})\right]^{-1} > 1$$

which implies a unique $\mu^* > 0$. Similar result can be obtained for sufficiently high values of β when γ is sufficiently small, thus verifying (ii). ■

FIGURE 1 - Equilibria with $\rho = 0$



Effect of Increase in α_0 | Effect of Increase in μ

FIGURE 2 - Equilibria with $\rho = 1$,
Effect of Increase in α_0

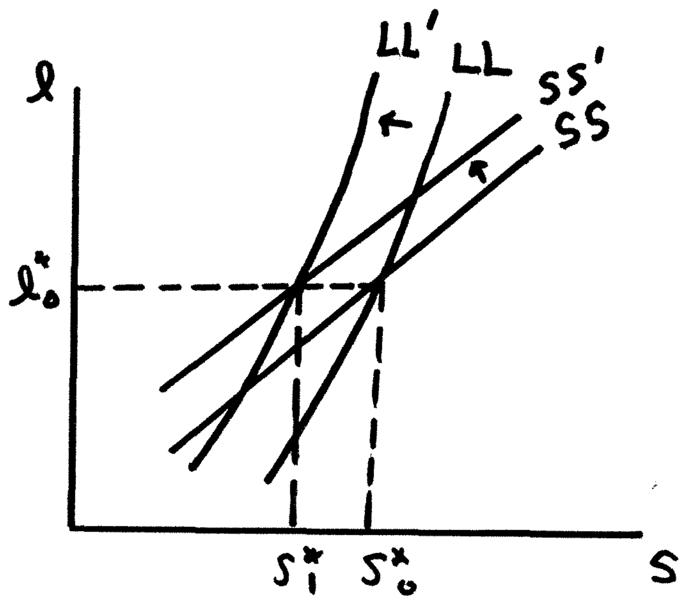
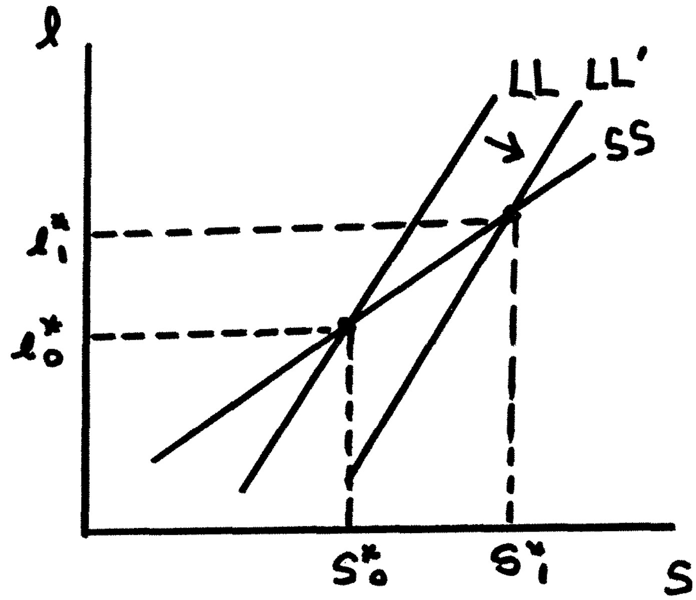


FIGURE 3 - Equilibria with $\rho = 1$,
Effect of Increase in μ, $\theta = 0$

Case 1: γ Sufficiently Small



Case 2: γ Sufficiently Large

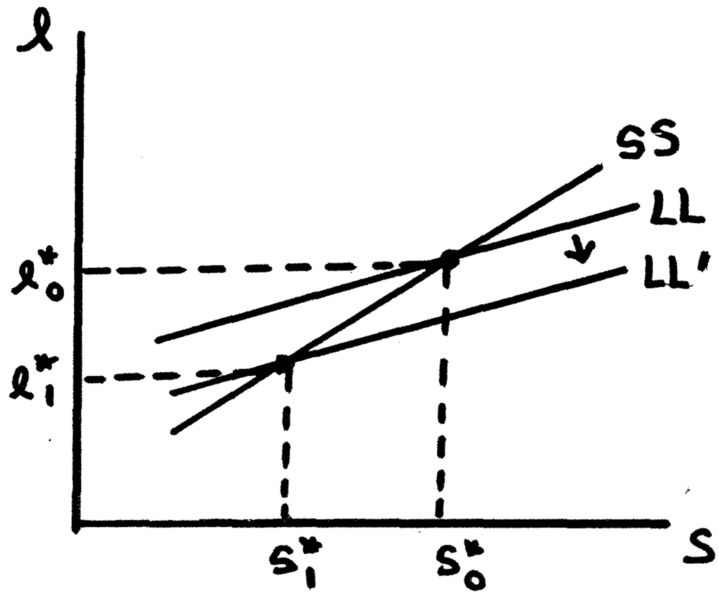
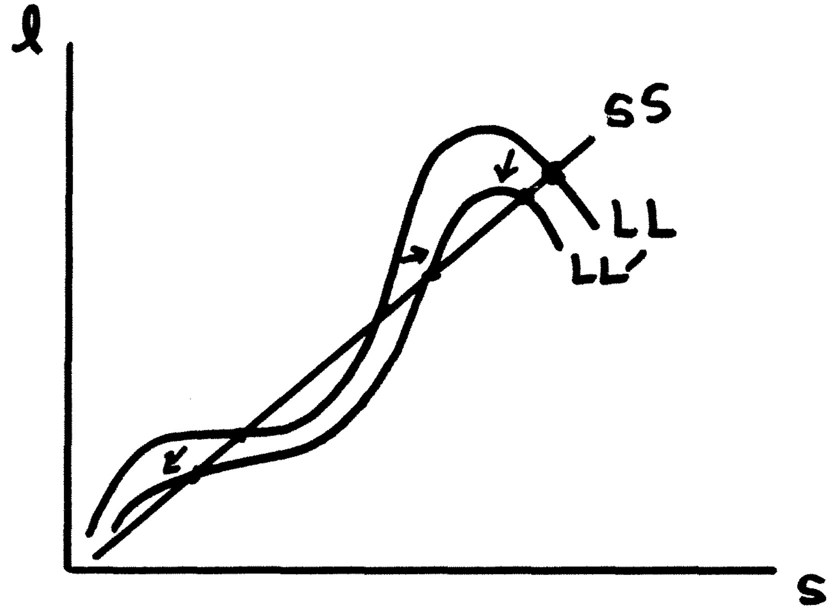


FIGURE 4 - Multiple Equilibria,
Effect of Increase in μ ($\theta = 0$)

Case 1: LL steeper than SS at Origin



Case 2: SS steeper than LL at Origin

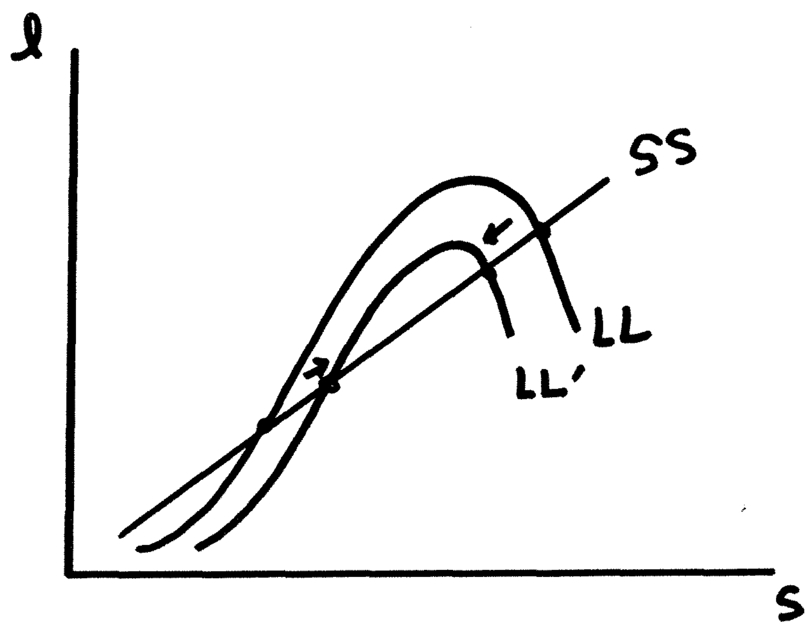


FIGURE 5 - Example of Multiple Equilibria

Roots of the Equilibrium Condition

($\gamma = 2$, $\eta = 0.4$, $\rho = 0.8$, $\phi = 0.8$, $\alpha_0 = 0$, $\alpha_1 = 8$, $\beta = 0.99$ and $\mu = 0,0.1$)