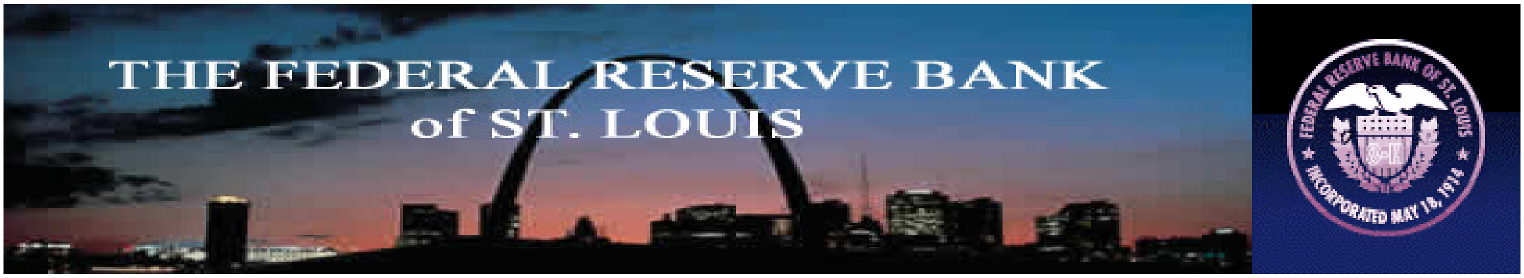

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ON THE SENSITIVITY OF VAR FORECASTS TO ALTERNATIVE LAG STRUCTURES

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1. INTRODUCTION

The forecasting literature has a long-standing tradition of comparing the accuracy of alternative models. Early papers (e.g., Cooper (1972) and Nelson (1972)) found that relatively simple time-series models yielded short-run forecasts as accurate as those produced by large-scale econometric models. Some builders of large econometric models, however, eschew the usefulness of time series methods in forecasting for other than short time horizons (e.g., Klein (1984)).

Recent comparisons of alternative forecasts (e.g., McNees (1986)) have examined the relative accuracy of an increasingly popular class of time-series models known as vector autoregressive (VAR) models.[1] Because VAR models need not impose theoretical priors, the forecaster's concern lies solely with the variables to include and the lag structure to be used. A substantial body of research indicates that the inclusion or exclusion of variables in a VAR model can have dramatic effects on the estimation and forecasting results.[2] There exists, however, only a nascent literature concerning the effects of alternative lag structures on forecasts derived from VAR models.

Lag length selection for VAR models has been based on a variety of criterion. Many researchers have constrained the lags on each right-hand-side variable to be equal, based either on ad hoc choice or by using statistical procedures that test the constraint that lags beyond somepoint jointly equal zero. (See Hakkio and Morris (1984), esp. pp. 33-34.) Imposition of equal lag lengths can be questioned, however, since under- or over-parameterization of a VAR model can produce estimates that are biased, inefficient or both.

In this study we investigate the sensitivity of macroeconomic forecasts obtained from VAR models that differ <u>only</u> in the lag structure. Holding constant the variables included and the time period examined, we focus on the effects of changes in the models' dynamic structure. Several lag selection approaches are used. One is to a priori constrain the lags to be one-quarter on each variable or to be four quarters. A simple AR(4) model also is estimated. In addition, five statistical procedures are used. Two are based on a mean-square-error criterion, two are derived from Bayesian criteria and one is the commonly used F-test. Our results demonstrate that forecasting accuracy differs dramatically across alternative dynamic structures. While there does not appear to be any one lag length selection criterion that consistently yields the most accurate forecasting model, the evidence suggests that relatively short-lagged models generally are more accurate than models with relatively long lags.

The format of the paper is as follows. Section 2 presents the model and data used. Section 3 briefly describes the lag length selection criteria and the results obtained when applying these criteria. Alternative forecasts are compared in section 4, followed by a summary and our conclusions in section 5.

2. THE MODEL AND DATA

The popularity of VAR models stems in part from their alleged atheoretical construction. In contrast to large-scale macroeconomic models that require imposing numerous identifying restrictions, VAR models use"theory" only to determine the included variables and the lag structure.[3] Thus, VAR models need not be derived from competing economic theories, but rather are predicated on statistical regularities. A VAR model is described by a system of equations

$$X = C + \beta(L)X + \epsilon \qquad (1)$$

where X is an (Nx1) vector of endogenous variables, C is an (Nx1) vector of constant terms, $\beta(L)$ represents an (NXN) polynomial in the lag operator (L), and ϵ is an (Nx1) vector of error terms. This system consists of N estimated equations, each consisting of lags of all N variables. If the lag operators (L) are identical for all equations, the system may be estimated using ordinary least squares (OLS) without loss of efficiency. If the lag structures are not equal across equations, however, a generalized least squares (GLS) estimation procedure is necessary.

The VAR model used here consists of four variables: the narrow money stock (M1), real GNP ($1982), the implicit GNP deflator (1982=100) and the three-month Treasury bill rate. While other variables could be added, we opt for a smaller model for two reasons. First, the forecasting literature generally focuses on these four variables. Since our purpose is not to find the "best" VAR model, focusing on the impact of alternative lag structures using these four commonly used variables seems adequate. Second, because each additional variable greatly increases the computer time used in estimating the lag selection algorithms, the number of variables is restricted to keep our task manageable.

VAR models assume that the relevant stochastic processes are stationary, a requirement generally achieved by detrending the data. Detrending can be done by assuming trend stationarity (TS), regressing a variable on time and using the estimated residuals as stationary series. The alternative is to assume first-difference stationarity (DS), where the time series can be modeled as first-differences (usually in logarithms), a procedure used widely in most time series studies. Given recent results of Nelson and Kang (1984) and Stulz and Wasserfallen (1985), we use the DS procedure.[4] All variables are quarterly observations, seasonally adjusted at the source. Although the estimation period runs from I/1960 through IV/1985, the existence of lagged estimators necessitates the use of data since IV/1956. While complete time series are available for output, prices and interest rates, the current definition of M1 is available only since I/1959. Consequently, values for M1 prior to 1959 were generated by splicing the "old" and "new" M1 series.

3. LAG LENGTH SELECTION

A popular approach used to estimate VAR models is to constrain the lag length on each right-hand-side variable in the VAR system to be equal. For example, Sims (1980) uses lags from 1 to 12 months on each variable; Fischer (1981) uses three quarterly lags on each variable; Friedman (1983) and Porter and Offenbacher uses eight lags; and Litterman's (1986) 7-variable model uses sixlags. Although fixing the lag lengths is a popular procedure, truncating some lags prematurely will yield biased estimates, and including too many lags will reduce the efficiency of the estimates. Thus, arbitrarily fixing the N^2 lag structures in an N-variable VAR system may increase the variance of the forecast error relative to an approach that uses some statistical lag selection procedure.[5]

3.1 Statistical Selection Procedures[6]

We have chosen five procedures from the set of available lag length selection criteria. While these criteria do not exhaust the available set, they are well suited for our purpose since they represent a broad range of alternatives in the bias-efficiency trade-off.

A commonly used criterion is the F-test, implemented by calculating the statistic

$$F_{L-j} = \frac{(SSR_{L-j-1} - SSR_L) \quad (T-L-1)}{SSR_L \quad (j+1)} \qquad j=0,\ldots,L \tag{2}$$

where SSR is the sum of square residuals either from the model estimated with the maximum lag length L or from the model with shorter lag lengths based on j+1 of the maximum lags constrained to equal zero, and T is the sample size. Calculated sequentially, the "best" lag length is determined at the point where adding one more lag does not improve the overall fit of the model at some predetermined statistical level. The F-test in practice has been found to choose relatively long lags.

Two mean squared error criterion also are used. One is Mallow's (1973) Cp statistic, which is defined as

$$Cp_{L-j} = \frac{SSR_{L-j} \quad (T-L-1)}{SSR_L} - T+2(L+1-j) \qquad j=0,\ldots,L \tag{3}$$

The optimal lag is chosen by minimizing Cp for the regression with j lag restrictions imposed. Akaike's (1970) final prediction error (FPE) criterion also is based on the mean squared error criterion.[7] The FPE criterion chooses the lag structure that minimizes

$$(4)\quad FPE_{L-j} = \frac{SSR_{L-j}}{T} \cdot \frac{T+(L+1-j)}{T-(L+1-j)} \qquad j=0,\ldots,L$$

Imposing j lag restrictions, both the Cp and FPE criterion trade off efficiency for reduced bias in the coefficient estimates. Geweke and Meese (1981) note that these criteria "have the advantage that asymptotically the chosen model is never too small, and the disadvantage that the probability of choosing the correct model is bounded away from unity asymptotically and the resulting estimates are therefore in general asymptotically inefficient."[8]

In contrast to the Cp and FPE criteria,two Bayesian criterion are used that place relatively more weight on efficiency. Schwarz's (1978) Bayesian information criterion (BIC) is based on minimizing

$$(5)\quad BIC_{L-j} = \ln\left[\frac{SSR_{L-j}}{(T-L-1+j}\right] + \frac{(L+1-j)\ln T}{T} \qquad j=0,\ldots,L$$

for j lag restrictions imposed. Geweke and Meese (1981) develop the Bayesian estimation criterion (BEC) which minimizes

$$(6)\quad BEC_{L-j} = \frac{SSR_{L-j}}{T-L-1+j} + \frac{SSR_{L}\;(L+1-j)}{T-L-1} \cdot \frac{\ln T}{T-L-1} \qquad j=0,\ldots,L$$

for j lag restrictions. Geweke and Meese use Monte Carlo tests and show that the BIC and BEC criteria choose the correct lag length asymptotically. They also note, however, that in small samples (T < 100) these two criteria tend to underfit, an outcome also reported by Batten and Thornton (1984) and Thornton and Batten (1985).

3.2 Lag Selection Results

Because our purpose is to compare forecast accuracy, we use I/1960-IV/1979initially to estimate the models, and then forecast over the

I/1980-IV/1985 period. Optimally, the lag structure would be updated each quarter, the VAR model re-estimated, and a new set of forecasts generated. To keep the task managable and still provide a reasonable application of the procedures discussed, we have chosen to use two sets of lag structures. One lag structure is determined by applying the selection procedures to data for the period I/1960-IV/1979. Recognizing that the lag structure may have changed from IV/1979 to IV/1985, we estimate a second set of lag structures using data for I/1960 through IV/1985. Using these two data sets to allows us to investigate the relative accuracy of the VAR models when the informational content is constrained to the pre-1980 period versus incorporating some information through 1985.

The results from applying the lag selection criteria to the I/1960 -IV/1979 data are presented in Table 1. The maximum lag length(L) always is limited to 12 quarters. The F-test generally selects the longest lag, often choosing the maximum lag. Inspection of the individual coefficients (not reported) revealed that in such instances the F-test usually is influenced by a significant lag at high order, even though most lower order lags are statistically insignificant. This result conforms with previous findings that the F-test may select lags that are "too long," thus reducing the efficiency of the coefficient estimates.

The other criteria generally select shorter lag lengths than the F-test. Consistent with Geweke and Meese's (1981) and Lutkepohl's (1985) simulation studies, Cp and FPE criteria often select the same dynamic structure. The Cp and FPE lag selections in Table 1, for example, differ only for the inflation rate variable in the interest rate equation. The BIC and BEC criteria also select similar, very short lag structures although the specific lag selections reveal more variation.

Using the data from I/1960-IV/1985 yields the lag structures reported in Table 2. These lag structures again show that the F-test yields the longest lags with the BIC and BEC tests selecting the shortest. Comparing the two sets of lag structures in Table 1 and 2, the Bayesian criteria appear to be the least sensitive to changes in the data set. For example, there are four changes in the lag structure using the BIC criterion and only three change using the BEC procedure (out of a possible 16). In contrast, extending the sample 1979 through 1985 produces 8 changes in the lags estimated using the F-test and 12 changes using the Cp and FPE criteria. Moreover, the lag length changes in the BIC and BEC are relatively small. The BIC based lags on M1 in the interest rate equation changes from two when the sample endpoint is IV/1979 to four using the IV/1985 endpoint. In contrast, the F-test indicates a change from zero to 10. Thus, the evidence in Tables 1 and 2 indicates that the lags derived from the non-Bayesian selection procedures are more sensitive to the data sample used. Our main concern, however, is the sensitivity of forecasts to changes in the lag structure, to which we now turn.

4. FORECAST RESULTS

VAR models were estimated using each of the lag structures reported in Table 1. . In addition, the two fixed-lag VAR models were estimated--one with lags on variable constrained to be one quarter (Fix-1) and the other with each lag constrained to equal four quarters (Fix-4)--along with the simple AR(4) model. Dynamic forecasts from each model are calculated for one, four and eight quarters ahead. The models initially are estimated through IV/1979 and forecasts are made for I/1980-IV/1981. Each model then is re-estimated (using the same lag structure) through I/1980 to generate forecasts for II/1980-I/1982. This process continues until the final one-quarter forecast is made for IV/1985. Note that while this procedure uses a fixed lag structure, it allows the parameter estimates to vary as new information is added.

Each model's root-mean-square error (RMSE) statistic for the j-period ahead forecast is calculated using the formula:

$$\text{RMSE}_j = \left[\sum_{t=\text{I}/1980}^{\text{IV}/1985} (X_{t+j} - \hat{X}_{t+j})^2 / T \right]^{1/2} \tag{7}$$

where X_{t+j} represents the actual value at t+j and $\hat{X}_{t+j}$ is the forecasted value for t+j made at time t. T is the number of j-period ahead forecasts.[9] The models' relative forecasting accuracy is compared based on their RMSE for each horizon.[10]

4.1 The 1960-1979 Lag Structure Forecasts

To illustrate the effects on forecast accuracy from changes solely in the dynamic structure, the VAR forecasts are presented separately for the one-quarter, four-quarter and eight-quarter horizons. The RMSEs generated using the I/1960-IV/1979 lag structures (from Table 1) are presented in Table 3.

4.1a One-quarter Horizon

The one-quarter ahead forecast results indicate that dramatic improvements in forecasting accuracy often are achieved using low order lag structures. For example, the ranking of lag structures (lowest RMSE first) in forecasting inflation are the BIC (1.42 percent), the simple AR model (1.43 percent), the BEC criterion (1.45 percent) and the *ad hoc* Fix-1 model (1.59 percent) up to an RMSE of 3.20 percent using the lengthy lag structure found using the F-test. To further illustrate the differences, the inflation forecast RMSE from the BIC-based model is 56 percent lower than that of the F-test lag structure and 30 percent lower than the Cp and FPE based models.

The BIC and BEC criteria also yield models with the lowest RMSE when forecasting real GNP growth and changes in the interest rate. As with inflation, the relative gain in the accuracy of forecasting real GNP growth is striking: a 21 percent reduction in accuracy relative to the CP and FPE lag structures and a 15 percent reduction compared with the F-test model. In contrast, the results are much closer across lag structures for the change in the interest rate. Even though the BEC model yields the lowest RMSE (1.72 percent), this figure is only 6 percent lower than the highest RMSE, that obtained from the Fix-4 and F-test models.

The lowest RMSE for one-quarter ahead M1 growth forecasts comes from the Cp and FPE lag structures. Their RMSE of 4.28 percent is 20 percent below that generated by the AR model (5.37 percent) and 18 percent below the Fix-1 and BIC models. In fact, the results show that the relatively more accurate forecasts of M1 growth come from the longer-lagged models.

The evidence indicates that except for M1 growth relatively more accurate one-quarter ahead forecasts are obtained from VAR models that severely constrain the lag structure. Short-lag models (Fix-1, BIC and BEC) generally yield the most accurate forecasts. Simply fixing the lag on each variable to one-quarter produces forecasts that are more accurate than the models based on the relatively sophisticated F, Cp and FPE lag selection procedures.

4.1b Four-Quarter Horizon

Extending the forecast horizon to a year significantly alters the choice for the most accurate model. Longer-lagged models are relatively more accurate, with the model based on the F-test criterion yields the lowest RMSE in forecasting inflation and M1 growth, and real GNP growth is forecast most accurately by the Fix-4 model. The inflation forecasting results show that while the F-test model has the lowest RMSE, the BIC and BEC determined lag structures produce the second and third most accurate forecasts. In fact, the Cp, FPE and Fix-4 lag structures all generate RMSEs that are over 20 percent larger than the models with constrained lag structures. The M1 growth forecasts, however, indicate that the most accurate forecasts come from (in order) the F-test, Cp and FPE models. In this instance, use of the BIC and BEC lag structures increases the RMSE by 27 percent and 22 percent, compared with the F-test model.

The short-lag models, in particular the BEC and Fix-1 lag structures, produce the lowest RMSEs only for the change in Treasury-bill rate variable. (The range of RMSE values for this variable is much wider than for the one-quarter ahead forecasts.) For inflation and real GNP, however, the BIC model still yields the second lowest RMSE.

4.1C Eight-Quarter Horizon

The RMSEs for a two-year forecast horizon are presented in the lowest tier of Table 3. The results indicate that the shorter-lag models again do quite well relative to their longer-lag alternatives. The model using the Fix-1 lag structure, for example, produces the second best forecast result for all of the variables. Moreover, the AR(4) model has the lowest RMSE for both M1 growth and the Treasury-bill rate change variables; relative to the F-test models, the AR(4) model lowers the RMSE by 35 percent for the M1 growth forecasts and 55 percent for the Treasury-bill change forecasts.

The eight-quarter forecasts indicate that short-lagged models generally outperform the longer lag versions. The results suggest, perhaps surprisingly, that VAR models with relatively short lag structures are not only easier to construct and manage, but also are relatively more accurate than models with long lag structures even over long horizons.

4.2 The 1960-1985 Lag Structure Forecasts

Using information through 1985 to generate the lag structures (but not the parameter values) of the VAR models, forecasts for the 1980-85 period were generated as before. The results are reported in Table 4. In lieu of a detailed analysis of Table 4, note that no lag selection technique consistently produces the best forecast. As a group, however, the low-order lag structure models, in particular the Fix-1 model, again do quite well overall at each forecasting horizon.

Adding the information from 1980-85 to re-estimate the lag structures often produces noticable reductions in the RMSEs. For example, the one-quarter ahead inflation forecast generated by the models based on the Cp and FPE criterion using the 1960-85 lag structures reduce their RMSEs by about 25-30 percent. The RMSE from the model using the F-test lag structures is reduced by a whopping 56 percent when the lag structure is updated. A summary of the relative impacts of the forecast accuracy for the statistical lag length selection criteria from changing the lags is presented in Table 5. The results in Table 5 not only show that changing the lag structure often produces dramatic changes in a given model's forecast record, but also demonstrates that the updated lag structure does not automatically yield more accurate forecasts. In 25 out of the 60 instances reported in Table 5, changes in the lag structure yield RMSEs that were worse (17) or no different (8) from those using the 1960-79 lag structure.

5. CONCLUSION

There exists no overriding *ex ante* rule for selecting lag lengths in a time series model. Although alternative statistical criterion exist, each forces the user to choose the lag structure on the grounds of a bias-efficiency trade off and the ease of implementation.

In this study we have examined the effect on forecast accuracy of VAR models that differ only in their dynamic structure. In addition to simple, *ad hoc* selection procedures, we used five statistical criteria that employ differing bias and efficiency tradeoffs to construct VAR models. Based on quarterly forecasts of inflation, money growth, output growth and changes in the three-month Treasury-bill rate for 1980-85, we found that VAR forecasts are very sensitive to changes in the lag structure. The relative accuracy of the forecasts varied substantially across forecast horizons (from one-quarter to two years) and variables. Our results suggest that models with relatively short lag structures (as a group) tend to outperform longer lag models.

Footnotes

1. VAR forecasts have been used by themselves (for example, see Litterman (1986) and Lupoletti and Webb (1986)) and in combination with econometric model forecasts (see McNees (1986)).

2. Recent policy-oriented analyses (e.g., Sims (1980), Gordon and King (1983), King (1983) and Braun and Mittnik (1985)), have indicated that small-scale VAR models are sensitive to the choice of included variables. In addition, Cooley and LeRoy (1985) note that structural interpretations of estimated VAR models require identifying restrictions as stringent as those used in structural models. They contend that non-structural VAR models may be a useful forecasting device, a view shared by Porter and Offenbacher (1983).

3. Statements like this have been vigorously debated. See Cooley and LeRoy (1985) for a position very different than that of Sims (1980).

4. The Treasury-bill rate is made stationary by simple first-differencing. Recent work by Eichenbaum and Singleton (1987) raises some questions about differencing. Their VAR results indicate that policy conclusions are sensitive to the degree of differencing. This issue is not pursued in our research.

5. The crucial role of lag selection in testing for Granger-causality in a bivariate system has been illustrated by Thornton and Batten (1985). Webb (1985) and Fackler and Krieger (1986) estimate multivariate VAR models and demonstrate that statistical lag length selection procedures that reduce model parameterization improve forecasts. In contrast to these arguments, Litterman (1986, p.27) contends that statistical formulae for selecting lag lengths are ill-advised, since "what such formulas ignore is that the reason one wants to choose a lag length in the first place is because one has prior information that more recent values of the variable in question have more information than now distant values."We use several lag length selection procedures that vary the bias-efficiency trade-off to examine the sensitivity of VAR forecast accuracy to changes in lag structure.

6. The following draws heavily from Batten and Thornton (1984) and Thornton and Batten (1985). We would like to thank them for making available the lag-length selection algorithm used here.

7. Akaike's FPE has been used by McMillin and Fackler (1984), Fackler and Kreiger (1985) and Webb (1986) in the context of multivariate VAR models.

8. Geweke and Meese (1981), p. 56.

9. The number of forecasts varies among the horizons. There are 24 one-quarter ahead forecasts, 21 four-quarter ahead forecasts and 17 eight-quarter ahead forecasts used to calculate the RMSEs.

10. Our comparisons are based only on the RMSE. See McNees (1986) for alternative ranking methods.

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Table 1

Alternative Lag Structures
Period: I/1960 - IV/1979

Selection Procedure	Dependent Variable	Independent Variable DP	DM1	DRGNP	DTB
F	DP	0	1	2	12
	DM1	5	4	11	12
	DRGNP	12	12	12	12
	DTB	11	0	12	10
Cp	DP	4	2	0	1
	DM1	0	9	0	6
	DRGNP	4	7	0	0
	DTB	1	4	0	4
FPE	DP	4	2	0	1
	DM1	0	9	0	6
	DRGNP	4	7	0	0
	DTB	2	4	0	4
BIC	DP	2	2	0	0
	DM1	0	1	0	1
	DRGNP	1	1	0	0
	DTB	0	2	1	3
BEC	DP	2	0	0	1
	DM1	0	1	0	0
	DRGNP	0	0	0	0
	DTB	0	1	1	0

Notes: The mennomics used for the variables are:
P represents the implicit GNP deflator (1982=100), M1 is the narrow definition of money, RNGP is real GNP ($82), and TB is the three-month Treasury-bill rate. All variables except the T-bill are measured as changes in the logarithm. DTB is measured as the change in the level of the Treasury-bill rate.

Table 2

Alternative Lag Structures
Period: I/1960 - IV/1985

Selection Procedure	Dependent Variable	Independent Variable DP	DM1	DRGNP	DTB
F	DP	0	5	5	12
	DM1	7	1	12	12
	DRGNP	12	12	12	12
	DTB	6	10	12	0
Cp	DP	3	0	0	1
	DM1	1	9	3	1
	DRGNP	3	2	0	7
	DTB	0	7	10	3
FPE	DP	3	1	0	1
	DM1	1	9	3	1
	DRGNP	3	2	0	7
	DTB	0	12	10	3
BIC	DP	2	0	0	1
	DM1	0	1	0	1
	DRGNP	1	1	0	7
	DTB	0	4	0	3
BEC	DP	2	0	0	1
	DM1	0	1	0	1
	DRGNP	1	0	0	0
	DTB	0	1	0	0

See notes accompanying Table 1.

Table 3

Forecast Summary Statistics (RMSE)
Lag Structure: I/1960-IV/1979
Forecast Period: I/1980-IV/1985

Alternative Lag Structures/RMSEs

1-Quarter Horizon

Variable	AR	Fix-1	Fix-4	F	Cp	FPE	BIC	BEC
DP	1.43	1.59	2.07	3.20	2.03	2.03	1.42*	1.45
DM1	5.37	5.22	4.69	4.75	4.28*	4.28*	5.23	5.08
DRGNP	4.73	4.50	4.86	5.11	5.51	5.51	4.33*	4.99
DTB	1.81	1.76	1.84	1.83	1.78	1.79	1.76	1.72*

4-Quarter Horizon

Variable	AR	Fix-1	Fix-4	F	Cp	FPE	BIC	BEC
DP	2.24	2.31	2.46	1.68*	2.41	2.41	1.99	2.03
DM1	4.99	4.66	4.41*	4.03	4.27	4.28	5.13	4.92
DRGNP	5.24	4.95	4.61	6.91	5.32	5.31	4.69	5.27
DTB	2.00	1.54*	2.10	3.16	1.93	1.94	1.82	1.54*

8-Quarter Horizon

Variable	AR	Fix-1	Fix-4	F	Cp	FPE	BIC	BEC
DP	3.35	2.56	3.17	2.27*	3.04	3.02	2.75	2.76
DM1	4.14*	4.15	4.39	6.34	4.97	4.99	4.26	4.29
DRGNP	4.80	4.75	4.85	6.50	5.31	5.31	4.61*	4.81
DTB	1.05*	1.32	1.76	2.32	1.67	1.68	1.98	1.33

*denotes lowest RMSE

Table 4

Forecast Summary Statistics (RMSE)
Lag Structure: I/1960-IV/1985
Forecast Period: I/1980-IV/1985

Alternative Lag Structures/RMSEs
1-Quarter Horizon

Variable	AR	Fix-1	Fix-4	F	Cp	FPE	BIC	BEC
DP	1.43	1.59	2.07	1.40*	1.40*	1.50	1.45	1.45
DM1	5.37	5.22	4.69	4.62*	5.12	5.12	5.23	5.23
DRGNP	4.73	4.50	4.86	5.11	4.66	4.66	4.33*	4.58
DTB	1.81	1.76	1.84	1.75	1.70*	1.76	1.73	1.73

4-Quarter Horizon

Variable	AR	Fix-1	Fix-4	F	Cp	FPE	BIC	BEC
DP	2.24	2.31	2.46	2.09	2.10	2.19	2.04	2.02*
DM1	4.99	4.66	4.41	3.72*	4.85	4.76	5.07	4.79
DRGNP	5.24	4.95	4.61	5.86	4.73	4.75	4.65	4.81
DTB	2.00	1.54*	2.10	1.79	1.92	1.97	1.83	1.55

8-Quarter Horizon

Variable	AR	Fix-1	Fix-4	F	Cp	FPE	BIC	BEC
DP	3.35	2.56*	3.17	2.97	3.00	2.85	2.75	2.75
DM1	4.14	4.15	4.39	4.96	4.12	4.06*	4.20	4.30
DRGNP	4.80	4.75	4.85	6.97	4.92	4.73	4.66*	4.69
DTB	1.65	1.32*	1.76	1.78	1.53	1.49	1.49	1.33

*denotes lowest RMSE.

Table 5

Percentage Change in RMSE
Table 4 relative to Table 3

Alternative Lag Structures/Percentage Change in RMSE

1-Quarter Horizon

Variable	F	Cp	FPE	BIC	BEC
DP	-56	-31	-25	2	0
DM1	- 3	20	20	0	3
DRGNP	0	-15	-15	0	-8
DTB	- 4	- 4	- 2	2	1

4-Quarter Horizon

Variable	F	Cp	FPE	BIC	BEC
DP	24	- 9	- 9	- 3	1
DM1	- 8	14	11	- 1	-3
DRGNP	-15	-11	-11	- 1	-9
DTB	-43	- 1	2	1	1

8-Quarter Horizon

Variable	F	Cp	FPE	BIC	BEC
DP	31	- 1	- 6	0	0
DM1	-22	-17	-19	- 1	0
DRGNP	7	- 7	-11	1	-2
DTB	-23	- 8	-11	-25	0