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Social Learning and Monetary Policy Rules

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Abstract

We analyze the effects of social learning in a widely-studied monetary policy context. Social learning might be viewed as more descriptive of actual learning behavior in complex market economies. Ideas about how best to forecast the economy's state vector are initially heterogeneous. Agents can copy better forecasting techniques and discard those techniques which are less successful. We seek to understand whether the economy will converge to a rational expectations equilibrium under this more realistic learning dynamic. A key result from the literature in the version of the model we study is that the Taylor Principle governs both the uniqueness and the expectational stability of the rational expectations equilibrium when all agents learn homogeneously using recursive algorithms. We find that the Taylor Principle is not necessary for convergence in a social learning context. We also contribute to the use of genetic algorithm learning in stochastic environments.

Keywords: New Keynesian macroeconomics, genetic algorithm learning.

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1 Introduction

1.1 Overview

Recent research has emphasized how policy choices may influence the stability properties of rational expectations equilibrium. In a typical analysis, a policymaker may commit to a particular policy rule, stating how adjustments to a control variable will be made in response to disturbances to the economy. The policy rule, together with optimizing private sector behavior, may imply that there is a unique rational expectations equilibrium associated with the policy rule, and that the equilibrium has desirable welfare properties. However, the equilibrium may or may not be robust to small expectational errors. If the expectations of the players in the economy are initially not rational, but deviate from rational expectations by a small amount, behavior of the players in the economy will be changed. This will then have effects on the price and quantity outcomes in the economy, feeding back into the learning process. Such a dynamic may or may not converge to the rational expectations equilibrium which is the policymaker's target. When the process does converge, it is called an *expectationally stable*, or *learnable* equilibrium.

We study learnability in a standard context, the model of monetary policy of Woodford (2003). A standard result, discussed in Woodford (2003) and Bullard and Mitra (2002), is that in a simple version of the model, the rational expectations equilibrium will be learnable provided the policymaker follows the *Taylor Principle*.[1] This means that the policymaker must react sufficiently aggressively to economic developments, such as deviations of inflation from target or the deviation of output from the flexible price, or potential, level of output. Failure to do so will create a rational expectations equilibrium which is unstable in the recursive learning dynamic. Such an equilibrium is unlikely to be successfully implemented in actual policymaking. Even small expectational errors would drive the economy away from the intended equilibrium.

[1] For a discussion of the Taylor Principle, see Woodford (2001).

The standard results are derived under the assumption of homogeneous expectations which are updated via recursive algorithms. This is the approach discussed extensively in Evans and Honkapohja (2001). By assuming homogeneous expectations and recursive algorithms, analytical results can be obtained concerning the expectational stability properties of equilibria across a wide variety of models. In this paper we study an alternative approach to learning, one that can be viewed as more realistic in terms of actual learning in complicated market economies. In it, agents are initially heterogeneous with respect to the models they use to forecast the future. Forecast rules are updated via genetic operators, meant to simulate the process of learning from neighbors and others in the economy. Results are not analytic but are based on computational experiments. We will call this alternative approach *social learning*.

Social learning has been studied in a wide variety of contexts in economics, but not in the standard New Keynesian model where many of the other findings concerning learnability have been presented. One reason is that the New Keynesian model is inherently stochastic, and the genetic algorithm applications which are drawn from the artificial intelligence literature are deterministic.[2] The genetic algorithm is meant to find "good" solutions to complicated problems with no known best solution. One purpose of this paper is to understand how insights from the genetic algorithm learning literature may be applied in a stochastic context.

1.2 Main findings

We conduct a series of computational experiments with social learning in the setting studied by Bullard and Mitra (2002). Our main finding is that the Taylor Principle does not have to be met in order for agents to coordinate on a rational expectations equilibrium of the model via the social learning dynamic. This stands in marked contrast to the findings in the recent learning literature.

[2] That is, the set of problems which have been considered are deterministic, although the algorithm itself is necessarily stochastic.

1.3 Recent related literature

Woodford (2003) contains the definitive statement of the nature of the New Keynesian model of monetary policy. Bullard (2006) surveys some of the literature on monetary policy and expectational stability, along with related issues. Genetic algorithm learning in economic contexts has been surveyed by Arifovic (2000).

Our paper is related to the recent literature on heterogenous learning. For example, Giannitsarou (2003) as well as Honkapohja and Mitra (2005, 2006) distinguish the following forms of heterogeneity in learning: different initial perceptions, different learning rules, and different degrees of inertia in updating in the same learning rule. Giannitsarou (2003) finds that when agents use least squares learning, E-stability implies learnability in the case of different initial perceptions. But for the other types of heterogeneity, the stability under homogenous learning does not necessarily imply stability under heterogenous learning. Our social learning approach encompasses a greater degree of heterogeneity than previous studies in this area, as a finite number of agents each have a different model within a given class of models.

Honkapohja and Mitra (2006) add structural heterogeneity to their analysis (agents respond differently to their forecasts), and study how transient and persistent heterogeneity in learning affects the learnability of the fundamental (MSV) solution. They find that transient heterogeneity in learning does not change the convergence conditions even in the presence of structural heterogeneity. But in case of persistent heterogeneity in learning, E-stability conditions do not in general imply learnability in structurally homogeneous and heterogeneous economies. Honkapohja and Mitra (2005) study the performance of interest rate rules in the presence of heterogeneous forecasts by the private sector and the central bank in New Keynesian model. They find that E-stability conditions are necessary but not sufficient for learnability with heterogeneity in learning.

Negroni (2003) studies heterogeneity in adaptive expectations. He considers two sources: heterogeneity of expectations (different gains) and heterogeneity of fundamentals. He finds that in the presence of heterogeneity, the conditions for convergence of heterogeneous adaptive beliefs to the stationary REE are not the same as for homogeneous beliefs.

Our agents have the same form of the learning rule but different initial beliefs about the values of the coefficients in the perceived law of motion. They update their beliefs at the same rate, so the economy is structurally homogeneous. Our agents are able to learn from the other agents (social learning), whereas in all the models with heterogenous learning mentioned above agents proceed to update their beliefs without knowing what and how well the rest of the agents are doing. Our results suggest that the social aspect is important for learning the rational expectations equilibrium.

Branch and Evans (2004) show that heterogeneity can arise under certain conditions as an endogenous outcome when agents choose between misspecified models. In our study, agents have the correct specification of the REE model, although they start with different beliefs about the coefficients in the correct specification. Our question is whether agents are able to learn the fundamental (MSV) values of the coefficients.

1.4 Organization

In the next section we discuss the New Keynesian model that we wish to study in this paper. Much has been written about this model, but here we only provide the reader with a minimal outline of the key equations, as the model itself is not the focus of this analysis. We then turn to a discussion of the social learning dynamic as we have implemented it in the New Keynesian model. Our main findings are the results of computational experiments, which we compare to standard results from the literature. The concluding section summarizes our findings and suggests a few directions for future research.

2 Environment

2.1 Overview

We study the simple version of the New Keynesian model employed by Bullard and Mitra (2002) and Woodford (2003). The economy is populated by a continuum of infinitely-lived household-firms that maximize utility and profits. Household-firms consume all goods but produce only one good on the continuum. Firms are monopolistically competitive and face a Calvo-style sticky price friction when determining their price. The model consists of three equations along with an exogenously specified stochastic process. The first equation is the linearized version of the first order condition for household utility maximization. The second equation is the linearized version of the first order condition for firm maximization of profits. The third equation is a Taylor-type interest rate feedback rule that describes the behavior of the monetary authority.[3] The system is given by

$$z_t = z^e_{t+1} - \sigma^{-1}[r_t - \pi^e_{t+1}] + \sigma^{-1} r^n_t \quad (1)$$

$$\pi_t = \kappa z_t + \beta \pi^e_{t+1} \quad (2)$$

where z_t is the output gap, π_t is the deviation of the inflation rate from a prespecified target, r_t is the deviation of the short-term nominal interest rate from the value that would hold in a steady state with the level of inflation at target and output at the level consistent with fully flexible prices. A superscript e denotes a subjective expectation that can initially be different from a rational expectation. All variables are expressed in percentage point terms and the steady state is represented by the point $(z_t, \pi_t, r_t) = (0, 0, 0)$. The parameter $\beta \in (0, 1)$ is the discount factor of the representative household, $\sigma > 0$ controls the intertemporal elasticity of substitution of the household, and $\kappa > 0$ relates to the degree of price stickiness in the economy. A standard calibration suggested by Woodford (2003) and widely used in the literature sets $(\beta, \sigma, \kappa) = (0.99, 0.157, 0.024)$. The natural rate of interest,

[3] Optimal policy and learnability can also be studied—see Evans and Honkapohja (2003).

r_t^n, is a stochastic term which follows the process

$$r_t^n = \rho r_{t-1}^n + \epsilon_t, \tag{3}$$

where ϵ_t is *i.i.d.* noise with variance σ_ϵ^2, and $0 \leq \rho < 1$ is a serial correlation parameter. The interest rate feedback rule of the monetary authority is given by

$$r_t = \varphi_\pi \pi_t + \varphi_z z_t, \tag{4}$$

where φ_π and φ_z are policy parameters taken to be strictly positive. The policymaker is committed to this rule and does not deviate from it. Substituting (4) into (1), we obtain

$$z_t = z_{t+1}^e - \sigma^{-1}[\varphi_\pi \pi_t + \varphi_z z_t - \pi_{t+1}^e] + \sigma^{-1} r_t^n. \tag{5}$$

2.2 Determinacy and learnability

Equations (2),(3), and (5) can be written as:

$$y_t = \alpha + B y_{t+1}^e + \chi r_t^n \tag{6}$$

where $\alpha = 0$, $y_t = [z_t, \pi_t]'$,

$$B = \frac{1}{\sigma + \varphi_z + \kappa\varphi_\pi} \begin{bmatrix} \sigma & 1 - \beta\varphi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \varphi_z) \end{bmatrix}, \tag{7}$$

and

$$\chi = \frac{1}{\sigma + \varphi_z + \kappa\varphi_\pi} \begin{bmatrix} 1 \\ \kappa \end{bmatrix}. \tag{8}$$

In order to analyze the effects of homogeneous recursive learning in this environment, Bullard and Mitra (2002) proceeded as follows. Assume that all agents have the following perceived law of motion (PLM)[4]

$$y_t = a + c r_t^n, \tag{9}$$

[4]The assignment of the PLM is not arbitrary but corresponds to the equilibrium law of motion of the economy.

which describes their belief concerning the equilibrium law of motion of the economy. With this perceived law of motion, they form expectations as

$$E_t y_{t+1} = a + c\rho r_t^n = \begin{bmatrix} z_{t+1}^e \\ \pi_{t+1}^e \end{bmatrix}. \tag{10}$$

The actual law of motion (ALM) is then found by substituting (10) into (6)

$$y_t = Ba + (Bc\rho + \chi) r_t^n. \tag{11}$$

The minimal state variable (MSV) solution is

$$y_t = \bar{a} + \bar{c} r_t^n \tag{12}$$

where $\bar{a} = 0$ and $\bar{c} = [I - \rho B]^{-1}\chi$. At $(\bar{a}, \bar{c})$, the actual law of motion coincides with the perceived law of motion and rational expectations equilibrium has been attained. If the actual law of motion has dynamics which tend to this fixed point, we say that the equilibrium is learnable.

Bullard and Mitra (2002) determine the necessary and sufficient condition for a rational expectations equilibrium to be determinate in the sense of Blanchard and Kahn (1980) as

$$\kappa(\varphi_\pi - 1) + (1 - \beta)\varphi_z > 0. \tag{13}$$

Bullard and Mitra (2002) also show that this same condition is necessary and sufficient for the expectational stability of rational expectations equilibrium. Inequality (13) is a statement of the Taylor Principle. In particular, consider the simplified condition $\varphi_z = 0$, so that the central bank does not respond to deviations of output from potential when setting its nominal interest rate target. Since $\kappa > 0$, the condition requires $\varphi_\pi > 1$, which is to say that the nominal interest rate must be adjusted more than one-for-one in response to deviations of inflation from target.

Bullard and Mitra (2002) concluded that condition (13) governs both uniqueness of rational expectations equilibrium as well as expectational stability of that equilibrium in this simple model.[5] Expectational stability is a

[5] The relationship between determinacy and learnability is less clear in more complicated settings.

notional time concept, but Evans and Honkapohja (2001) show that it governs the stability of the real time system formed when agents estimate the coefficients in (9) using recursive algorithms such as least squares. We now turn to examine the robustness of this finding when homogenous recursive learning is replaced with social learning.

3 Social learning

3.1 Overview

We study the behavior of evolutionary learning agents. Agents are initially heterogeneous with respect to their perceived law of motion (9), in the sense that each agent has a separate and possibly different set of coefficients. Thus each agent initially has a different forecasting model. The coefficients are updated using social evolutionary learning instead of least squares learning. Our objective is to see whether MSV solutions are learnable by evolutionary learning agents.

3.2 Initialization

We introduce heterogeneity as follows. There are N agents in the private sector. Each agent, $i = 1, ..N$ has a perceived law of motion (PLM)

$$z_t = a_{1,i,t} + c_{1,i,t} r_t^n \tag{14}$$

$$\pi_t = a_{2,i,t} + c_{2,i,t} r_t^n \tag{15}$$

We stress that r^n is a stochastic term, and that finding equilibrium values of a and c will depend on evaluating how well each forecast rule works even though there is noise in the system. This is not a typical feature of evolutionary learning environments. It is true that the genetic operators we discuss below are inherently stochastic, but the fitness calculation does not normally have to contend with exogenous stochastic terms.

The initial values for the coefficients are each randomly generated from a normal distribution with mean equal to the respective MSV value. The standard deviation for coefficients c_1 and c_2 is equal the largest of the absolute values of the MSV values of these coefficients. We used a smaller initial standard deviation for the coefficients a_1 and a_2. The MSV values for these coefficients are 0 and are smaller than MSV values for the coefficients c_1 and c_2. Therefore, we used initial standard deviation for coefficients a half as large as for the coefficients c. When setting the values of initial standard deviations we have pursued several objectives—namely, starting with a diverse population of rules, injecting diverse new rules through mutation, and keeping the diversity of new rules commensurate with the MSV values.

3.3 Expectations and the actual law of motion

Agents form their expectations of the output gap and deviation of inflation from target using (3), (14), (15) as

$$\begin{aligned} z^e_{i,t+1} &= a_{1,i,t} + c_{1,i,t}\rho r^n_t, && (16) \\ \pi^e_{i,t+1} &= a_{2,i,t} + c_{2,i,t}\rho r^n_t. && (17) \end{aligned}$$

The average expectations of the output gap and the deviation of inflation from target are computed as

$$\begin{aligned} z^e_{t+1} &= \frac{1}{N}\sum_{i=1}^{N} z^e_{i,t+1}, && (18) \\ \pi^e_{t+1} &= \frac{1}{N}\sum_{i=1}^{N} \pi^e_{i,t+1}. && (19) \end{aligned}$$

The actual values of the output gap and deviation of inflation from target are obtained from

$$y_t = \alpha + B \begin{bmatrix} z^e_{t+1} \\ \pi^e_{t+1} \end{bmatrix} + \chi r^n_t. \tag{20}$$

3.4 Forecast rule performance

Agents assess the performance, or *fitness*, of their forecasting model using mean squared forecast error as a criterion. Agents compute the mean squared forecast error for the output gap and the deviation of inflation over all periods following an initial history. We stress that it is important not to base the performance on only the most recent forecast error because the environment is stochastic.[6]

The fitness is computed as

$$F_{i,t} = -\frac{1}{t}\sum_{k=1}^{t}(z_k - z_{i,k}^f)^2 - w\frac{1}{t}\sum_{k=1}^{t}(\pi_k - \pi_{i,k}^f)^2 \tag{21}$$

where z_k^f is the forecast value of z for period k, and π_k^f is the forecast value of π for period k, and w is the relative weight on the MSE for inflation. An agent is characterized by a set of coefficients $(a_{1,i,t},\ a_{2,i,t}\ ,\ c_{1,i,t},\ c_{2,i,t})$ at each date t. The terms z_k^f and π_k^f are the forecasts of the output gap and the deviation of inflation from target that agent i could have computed in period k, if he had used the current, date t, set of coefficients $(a_{1,i,t},\ a_{2,i,t},\ c_{1,i,t},\ c_{2,i,t})$. The forecasts z_k^f, π_k^f are computed by agent i as

$$z_{i,k}^f = a_{1,i,t} + c_{1,i,t}\rho r_{k-1}^n \tag{22}$$

$$\pi_{i,k}^f = a_{2,i,t} + c_{2,i,t}\rho r_{k-1}^n \tag{23}$$

The weight w is used to give equal importance to the prediction error for the output gap and the deviation of inflation from target as the values of the MSE for these two variables can differ in order of magnitude. Without reasonable weighting, the fitness measure puts insufficient emphasis on the first or the second term in (21), leading to drift in coefficients away from MSV values.

First, we considered simulations with weight $w = 1$, implying output forecast error volatility and inflation forecast error volatility have the same weight in the assessment of forecast rules.[7] From these simulations, we collected the data on fitness and its composition: the first and the second

[6] Branch and Evans (2004, p. 3) assume that "... agents make their choices based on unconditional mean payoffs rather than on the most recent period's realized payoff. This is more appropriate in our stochastic environment since otherwise agents would frequently be misled by single period anomalies."

[7] The genetic operators used in these simulations are described below. Here, we simply wanted to discuss the fitness criterion.

summation terms in (21). This data indicated that the MSE for z was several orders of magnitude larger than the MSE for π, and therefore, agents effectively did not care very much about the accuracy of their prediction for π when assessing their forecast rule. As a result, the coefficients diverged away from MSV values (see quantitative details in section (5.4)).

The source of the difference in magnitudes of the MSE for the output gap and the MSE for the inflation deviation can be explained. From the time series of z and π, we observed that the output gap assumes larger values than inflation deviations. This comes from the values of the coefficients in equation (20) for the computation of the actual output gap, z, and inflation deviation, π. At the standard calibration we use, the coefficients for the computation of z are several times larger than the coefficients for the computation of π. This makes values of z larger than values of π, and so the squared prediction error for z larger than for π. In turn, this implies that in the fitness calculation, the first summation term in (21) is considerably larger than the second summation term (most frequently by a factor of 100). We used the weight w to adjust for this asymmetry. In particular, we set w such that the first and second summation terms in (21) are of the same order of magnitude. We use weight equal 100 for the simulations reported in this paper.

The criterion (21) with z_k^f set to zero is a version of the objective function for the central bank that is often employed in models of optimal monetary policy. In studies of this type, w would represent the central bank's relative preference for inflation versus output volatility. This objective is also often rationalized as an approximation to the utility of the representative household in this economy, as suggested by Woodford (2003). In the optimal policy literature, w takes on a relatively large value. There the weight on inflation stabilization is typically set to one, and the weight on output stabilization is close to zero, so that the relative weight on inflation stabilization is quite large. In the present paper, the agents are concerned with the forecasting performance of their forecasting model, and so forecast performance matters and z_k^f as well as π_k^f are non-zero. However, the relatively large value of w that delivers the best performance of the social learning model is similar.

3.5 Genetic operators

A hallmark of the evolutionary learning literature is that agents update their current state using genetic operators. These operators are meant to simulate the exchange of information in a large, complex economy, and are based on the principles of population genetics. Agents can meet other agents, exchange information concerning their current forecast rule, and possibly copy the partner's forecast rule, either in whole or in part. This process is implemented as described below.

We follow the literature in this area and use three genetic operators, namely crossover, mutation, and tournament selection. Our genetic system is real-valued. Crossover is implemented first. Two agents in the set of N agents are randomly matched without replacement. With probability of crossover $mcross$, their sets of coefficients can be subjected to crossover: If a random draw from a uniform distribution is less than or equal $mcross$, the agents exchange each type of coefficient with probability 0.5.

Mutation is implemented following crossover. An agent changes each coefficient with probability of mutation $mprob$ in the following way

$$new = old + random * mutdeviation, \tag{24}$$

where $random$ is a random number drawn from a standard normal distribution, old is the current value of the coefficient, and $mutdeviation$ is the standard deviation used for mutation. We set $mutdeviation$ to be decreasing over time according to

$$mutdeviation = deviation * (1 - decrease * t/T) \tag{25}$$

where $deviation$ is the standard deviation used to generate initial set of rules, t is current date, T is the total number of periods in the simulation, and $decrease$ is a coefficient. We set $decrease$ equal 0.95, it is intended to allow non-zero mutation standard deviation even in the last period of the simulation. Mutation can be very destructive late in a simulation when the N forecast rules may be very close to optimal, REE forecast rules, because a random choice of a new coefficient will cause a new round of genetic variation. The term (25) is meant to control this effect.

After mutation, agents compute the fitness of their coefficients according to (21).

The final genetic operator is tournament selection. Agents are randomly selected in pairs with replacement N times. For each pair of agents, the fitness values of the forecast rules are compared. The agent with the higher fitness value is copied into the next generation of agents. This creates a new generation of N agents. After this update is finished, agents go to the next period of the simulation. Tournament selection will provide most of the selection pressure in this evolutionary learning environment, as weaker forecasting rules are systematically discarded during this process.

4 Computational experiments

4.1 Overview

We conduct a set of computational experiments in order to understand the behavior of the economy under social learning. We begin our simulations by generating an initial history for the system at the rational expectations equilibrium, that is, using the MSV values for the coefficients a and c. We then conduct simulations that last for 3000 periods, and we set the length of the initial history to 100 periods. We use the parameter values from Woodford (2003), namely, $\sigma = 0.157$, $\kappa = 0.024$, $\beta = 0.99$, and $\rho = 0.35$. The standard deviation of r^n is 3.72. We consider a range of values for the parameters in the Taylor-type monetary policy rule. For values of the coefficient on the output gap, we use $\varphi_z \in [0.2, 1.1]$. For the coefficient on inflation, we use $\varphi_\pi \in [0.5, 2]$. At these parameter values, condition (13) is met for some policy parameter pairs but not for others, and is governed primarily by the value of φ_π.

We use the following parameter values in the genetic algorithm. The probability of mutation is 0.1 and the probability of crossover is 0.5. The number of agents is 30. The value of w in the fitness criterion is 100.

4.2 Main findings

We found that agents are able to learn MSV values of coefficients for most of the policy parameter pairs (φ_z, φ_π), both in the determinate and E-stable region as well as in the indeterminate and E-unstable region. A series of four figures shows our main results.

A typical simulation result for the policy rule characterized by $\varphi_\pi = 2.0$ and $\varphi_z = 0.2$ is given in Figure 1, and for the policy rule $\varphi_\pi = 1.5$ and $\varphi_z = 0.5$ in Figure 2. These policy rules are associated with a determinate rational expectations equilibrium and expectational stability. The figures show the time series of the deviation of each of the four coefficients from their MSV values averaged across all agents. The figure also shows ± 1 standard deviation for each coefficient's deviation from MSV values, showing the extent of the dispersion in coefficients in use at date t in the population of agents. Figures 1 and 2, along with other simulations using policy rules consistent with determinacy and learnability, suggest that long-run predictions from analyses using recursive learning and analyses using evolutionary learning are similar. In particular, both approaches predict convergence to the rational expectations equilibrium. This result breaks down when we consider other policy rules, however.

Figures 3 and 4 show typical simulation results for $\varphi_\pi = 0.5$ and $\varphi_z = 0.5$ or $\varphi_z = 0.3$, respectively. These policy rules are associated with indeterminacy and expectational instability. The figures again show the time series of the deviation of each of the four coefficients from their MSV values averaged across all agents, and ± 1 standard deviation. Here, the evolutionary learning dynamic converges to the MSV solution once again, even though an analysis based on least squares learning would predict instability in the learning dynamics. These findings suggest that, provided one is willing to take an evolutionary learning perspective, the less aggressive policy rules are not as disturbing as they may have appeared to be.

In order to provide more details concerning these results, we performed 1000 simulations for different policy rules (φ_π, φ_z) and collected data for the deviations of coefficients from their MSV values for each simulation. During each simulation, for each coefficient, we computed the average value of deviation from the MSV value for each period. Then we computed average value of the average deviations during the last 100 periods of simulation.[8] We also compute average of absolute values of deviations from MSV values during last 100 periods. In addition we collected data on percentage deviations from MSV values for coefficients c_1 and c_2 and computed average of (absolute) percentage deviations during last 100 periods of the simulations. (We cannot compute percentage deviations for coefficients a_1 and a_2 as their MSV values are zero). For each policy rule (φ_π, φ_z), we perform 1000 simulations, collect the above described statistics for each simulation, and report means and standard deviations for each statistic over 1000 simulations.

Table 1 reports means and standard deviations for average deviations from MSV values for a variety of fixed policy rules. The policy rules presented in this table include some that induce a determinate and E-stable rational expectations equilibrium, as well as others that induce indeterminacy and expectational instability. The policy rules that induce determinacy and learnability according to condition (13) will have larger values of φ_π and φ_z, which tend to be located toward the northeast part of the table. Relatively small values for φ_π and φ_z are associated with indeterminacy and expectational instability, and tend to be located in the southwest portion of the table.

We can make the following observations from Table 1. Perhaps most importantly, for the policy rules considered, regardless of whether they are consistent with determinacy and learnability or not, the population coefficients are quite close to their MSV values. The genetic algorithm we have implemented allows mutation up to date T in the simulation and so does not attempt to eliminate variation entirely, yet the table indicates that the

[8] The results are not qualitatively different for data computed for last 100-period, last 10-period and last 1-period; therefore, we only report results for last 100-period data.

population is quite close to the one that would use MSV values exclusively (all values in the table are very close to zero). To the extent there are differences from MSV values, the deviations for the constant coefficients a_1 and a_2 can be somewhat higher than those for the slope coefficients c_1 and c_2. Standard deviations indicate that there is some variety in the population even during the last 100 periods of the simulation, but the extent of the variety is not very large.

Table 2 presents means and standard deviations for absolute values of the deviations from the MSV values. This table also presents the percentage absolute deviation for the slope coefficients c_1 and c_2. These percentages for the absolute deviations range from about 3.0 to 11.0, and do not seem to vary systematically with the policy rule.

The previous tables illustrate convergence of each individual coefficient. We would also like to present a measure of convergence for a complete set of coefficients—how close all coefficients are to MSV values at the same time. Table 3 reports the number of simulations out of 1000 that satisfy specific convergence criteria based on averages of absolute deviations over last 100 periods of simulation. As different coefficients deviate from MSV values by different amounts, we present results of the application of two criteria for convergence. Criterion 1 requires that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 requires that the absolute deviation from the MSV value for a_1 is less than or equal 0.5, and that the absolute deviations from the MSV values for a_2, c_1, and c_2 are less than or equal to 0.3.[9]

Table 3 perhaps indicates a result more in conformity with previous findings in the learning literature: The number of simulations out of $1,000$ satisfying either convergence criterion clearly tends to decline as one moves toward the southwest in Table 3, that is, as one moves toward the region of the parameter space that is associated with indeterminacy and expectational

[9] The number of simulations satisfying criterion 2 is very close to the number of simulations satisfying a criterion which requires that the absolute value of the deviation of coefficient c_2 from its MSV value is less than 0.03, and the rest of the coefficients satisfy criterion 2.

instability. This is perhaps clearest when comparing the most northeasterly cell in the table with the cell in the southwest corner. The former is associated with determinacy and expectational stability, while the latter is not. In the northeast corner we observe values of 963 and 995, respectively, for the two convergence criteria, while in the southwest corner we observe values of 145 and 403. This would seem to be a clear indication that it is somehow "more difficult" for the social learning system to converge upon the MSV solution when expectational stability and determinacy conditions fail. However, we do not wish to press this point too hard. The cell associated with $\varphi_\pi = 1.0$ and $\varphi_z = 0.2$ has values of 208 and 578 for the two convergence criteria, respectively, not very different from the results for the cell in the southwest corner. Yet these parameter values satisfy condition (13); rational expectations equilibrium here is unique and expectationally stable. Furthermore, Tables 1 and 2 indicated that whatever failure to converge may exist, actual values are not very different from MSV values, and would probably not be meaningful in economic terms.

In some simulations, we can observe deviations of average values of coefficients a_1 and a_2 from their MSV counterparts, even though agents are always able to learn MSV values of c_1 and c_2 quite closely. Again considering Table 2, to the extent that agents are inaccurate in learning MSV values, it is due to the coefficients a_1 and a_2, as the deviation of these coefficients from MSV is the largest among all coefficients. In the least squares learning model of Bullard and Mitra (2002), as pointed by Woodford (2003, pp. 271-272), "... it is in fact the possible instability of the dynamics of estimates of the constant terms Γ_0 in the forecasting model that is the relevant threat; and whether this occurs or not is determined by whether or not the Taylor principle is adhered to ...." In our notation, Γ_0 corresponds to the coefficients a_1 and a_2. Similarly, Honkapohja and Mitra (2004) point out that "In Bullard and Mitra (2002, p. 1757), the constant term was the key to E-stability of the MSV solution .... " However, our simulations show that the system under evolutionary learning behaves somewhat differently. While the values of a_1 and a_2 may not be as close to their MSV values as the values of c_1 and

c_2, this effect occurs whether or not the Taylor principle holds.

5 Modifications and robustness

5.1 Overview

We performed several modifications of the simulations described above. These included using different fitness criterion and not using crossover. We now turn to a description of these modifications and their effects on the results.

5.2 Alternative performance evaluation

As we stressed earlier, the weighting of the two dimensions in the fitness criterion is essential to convergence of the social learning systems we study. Without reasonable weighting, the fitness measure puts insufficient emphasis on one dimension or the other, leading to drift in coefficients away from MSV values. The modification considered in this section has each agent compute the mean squared error for forecasting the deviation of inflation from target and the output gap separately, and simply consider them separately without combining them into one fitness measure. In particular, agent i computes mean squared errors for the output gap and inflation as

$$F_{i,t}^{z} = -\frac{1}{t}\sum_{k=1}^{t}(z_k - z_{i,k}^{f})^2, \tag{26}$$

$$F_{i,t}^{\pi} = -\frac{1}{t}\sum_{k=1}^{t}(\pi_k - \pi_{i,k}^{f})^2, \tag{27}$$

where $z_{i,k}^{f}$, $\pi_{i,k}^{f}$ are computed as in (22) and (23).

The change in performance criterion also has affects on the tournament selection operator. We modified the operator as follows. Again, N pairs of agents are randomly selected from the current generation with replacement, and fitness is compared for each pair. A new member of the next generation adopts the coefficients for forecasting output gap from the agent with higher

$F^z_{i,t}$ (lower mean squared error for forecasting the output gap) and the coefficients for forecasting the deviation of inflation from the target from the agent with higher $F^\pi_{i,t}$ (lower mean squared error for forecasting inflation). In this way the next generation of agents is created, and more fit forecasting rules are systematically selected while weaker rules are systematically discarded.

The results of these simulations are reported in Table 4. This table reports the same data as Table 2 for the baseline simulations. The results are qualitatively the same as for the baseline simulations. Table 5 reports the number of simulations that satisfy convergence criteria. We find similar effects when moving from northeast to southwest in this table as we did in Table 3.

5.3 Simulations without crossover

Crossover is considered a powerful operator in the genetic algorithm literature. One is taking "building blocks of good solutions" and combining them to create new possible solutions. This is thought to be a much faster way to find a good solution to a difficult problem than to merely rely on a mutation process. Especially for our real-valued, multidimensional problem, it can take a long time for mutation alone to find the best solution. In this subsection, we show that crossover is essential to our findings. To do this, we consider systems in which crossover has been discarded completely from the genetic algorithm. These simulations are done in the same way as the baseline simulations described above, with the sole modification that there is no crossover. Table 6 reports the some of the same data for simulations without crossover as Table 2 for the baseline simulations. The simulations without crossover have the following results. The constant coefficients, a_1 and a_2, approach the MSV values of zero. However, the slope coefficients, c_1 and c_2, deviate from the MSV values by 96 to 99 percent. We conclude that crossover is an important GA operator for learning the MSV solution in this model.

5.4 Relative weight equal to unity

For completeness, we also report results of the simulations in which the weight w was simply set equal to one. As we have indicated, the convergence properties are not as good for this parameterization. The results are shown in Table 7 where the coefficients tend to be farther from MSV values at the end of the simulation as compared to the baseline simulation. Table 8 shows that the convergence criteria are met less often as well.

6 Conclusion

A key finding in the literature on learning in New Keynesian models of monetary policy is that nominal interest rate feedback policies which are too close to an interest rate peg tend to be associated with indeterminacy and instability in the recursive learning dynamics. The policymaker must react sufficiently aggressively to economic developments in order to assure determinacy of rational expectations equilibrium and expectational stability of that equilibrium. This has been promoted as an important reason to discard policy rules which are insufficiently aggressive,[10] and this idea has gained widespread acceptance in monetary policy discussions.

We have investigated whether this result is robust to the substitution of an evolutionary learning dynamic for the recursive learning dynamic. Our main finding is that the evolutionary learning dynamic does not put a premium on policy rules which obey the Taylor Principle. Instead, evolutionary learning converges to a small neighborhood of the MSV solution whether or not the policymaker obeys that principle.

When the Taylor Principle is violated, equilibrium is indeterminate. It is well-known that sunspot equilibria exist in a neighborhood of an indeterminate rational expectations equilibrium. This is another important reason why insufficiently aggressive policy rules may be considered poor policy. In the recursive learning literature, it has generally been difficult to obtain ex-

[10] See, for instance, Woodford (2001, 2003).

pectational stability of sunspot equilibria.[11] An interesting extension of our analysis would be to analyze the stability of sunspot equilibria under the evolutionary learning dynamic.

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Table 1.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	-0.001	0.078	0.000	0.063	0.001	0.066	0.001	0.078
	a2	-0.002	0.081	-0.001	0.076	-0.002	0.071	-0.002	0.066
	c1	-0.002	0.044	-0.001	0.043	0.000	0.042	0.001	0.042
	c2	-0.002	0.003	-0.002	0.003	-0.002	0.003	-0.002	0.003
1.0	a1	-0.001	0.089	0.000	0.070	0.001	0.073	0.002	0.089
	a2	-0.002	0.089	-0.001	0.082	-0.002	0.077	-0.002	0.071
	c1	-0.002	0.048	-0.001	0.047	0.000	0.046	0.001	0.045
	c2	-0.002	0.003	-0.002	0.003	-0.002	0.003	-0.002	0.003
0.9	a1	-0.001	0.102	0.000	0.078	0.001	0.082	0.002	0.103
	a2	-0.002	0.098	-0.002	0.090	-0.002	0.084	-0.002	0.078
	c1	-0.002	0.053	-0.001	0.052	0.000	0.051	0.001	0.050
	c2	-0.002	0.004	-0.002	0.004	-0.002	0.004	-0.002	0.003
0.8	a1	-0.001	0.120	0.000	0.088	0.001	0.094	0.003	0.121
	a2	-0.002	0.110	-0.002	0.100	-0.003	0.092	-0.002	0.084
	c1	-0.003	0.060	-0.001	0.058	0.000	0.056	0.001	0.055
	c2	-0.003	0.004	-0.003	0.004	-0.003	0.004	-0.002	0.004
0.7	a1	-0.001	0.146	0.000	0.101	0.002	0.111	0.003	0.145
	a2	-0.002	0.125	-0.002	0.113	-0.003	0.102	-0.002	0.092
	c1	-0.004	0.068	-0.002	0.066	0.000	0.064	0.002	0.062
	c2	-0.003	0.005	-0.003	0.005	-0.003	0.004	-0.003	0.004

Table 1: Means and standard deviations of coefficients for last 100 periods for a variety of policy rules.

TABLE 1 CONTINUED.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
0.6	a1	-0.002	0.183	0.000	0.120	0.003	0.134	0.003	0.182
	a2	-0.002	0.145	-0.003	0.129	-0.003	0.115	-0.002	0.103
	c1	-0.005	0.078	-0.002	0.075	0.000	0.073	0.002	0.070
	c2	-0.004	0.005	-0.003	0.005	-0.003	0.005	-0.003	0.005
0.5	a1	-0.002	0.240	0.000	0.146	0.003	0.168	0.004	0.234
	a2	-0.003	0.170	-0.003	0.150	-0.003	0.131	-0.002	0.115
	c1	-0.007	0.092	-0.003	0.088	0.000	0.085	0.003	0.082
	c2	-0.004	0.006	-0.004	0.006	-0.004	0.006	-0.003	0.005
0.4	a1	-0.003	0.343	0.001	0.186	0.005	0.222	0.005	0.318
	a2	-0.003	0.212	-0.004	0.178	-0.003	0.152	-0.002	0.131
	c1	-0.010	0.112	-0.004	0.107	0.000	0.102	0.004	0.097
	c2	-0.005	0.008	-0.005	0.007	-0.004	0.007	-0.004	0.007
0.3	a1	-0.002	0.548	0.003	0.256	0.008	0.320	0.010	0.463
	a2	-0.003	0.280	-0.004	0.221	-0.004	0.182	-0.002	0.151
	c1	-0.016	0.144	-0.007	0.135	0.001	0.127	0.007	0.121
	c2	-0.007	0.010	-0.006	0.009	-0.006	0.008	-0.005	0.008
0.2	a1	-0.004	1.051	0.007	0.405	0.017	0.523	0.016	0.735
	a2	-0.005	0.399	-0.004	0.291	-0.005	0.226	-0.002	0.173
	c1	-0.030	0.200	-0.013	0.183	0.001	0.170	0.011	0.158
	c2	-0.010	0.014	-0.008	0.012	-0.007	0.011	-0.006	0.010

TABLE 2.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.061	0.049	0.050	0.039	0.052	0.040	0.059	0.051
	a2	0.046	0.067	0.043	0.062	0.041	0.059	0.038	0.054
	c1	0.035	0.027	0.035	0.026	0.034	0.025	0.033	0.025
	c2	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	perc c1	4.265	3.226	4.236	3.204	4.205	3.172	4.178	3.165
	perc c2	10.129	6.779	10.015	6.833	9.992	6.717	9.819	6.478
1.0	a1	0.069	0.056	0.055	0.043	0.058	0.045	0.066	0.059
	a2	0.050	0.073	0.047	0.067	0.044	0.063	0.041	0.058
	c1	0.039	0.029	0.038	0.028	0.037	0.028	0.036	0.027
	c2	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	perc c1	4.280	3.238	4.253	3.206	4.213	3.196	4.183	3.177
	perc c2	10.145	6.843	9.937	6.763	9.967	6.699	9.873	6.533
0.9	a1	0.079	0.065	0.062	0.047	0.065	0.051	0.076	0.070
	a2	0.056	0.081	0.052	0.074	0.048	0.069	0.045	0.063
	c1	0.043	0.032	0.042	0.031	0.040	0.031	0.040	0.030
	c2	0.004	0.002	0.004	0.002	0.004	0.002	0.003	0.002
	perc c1	4.294	3.254	4.272	3.227	4.224	3.208	4.210	3.190
	perc c2	10.231	6.818	10.059	6.785	10.048	6.703	9.814	6.532
0.8	a1	0.093	0.077	0.070	0.054	0.074	0.059	0.088	0.083
	a2	0.063	0.090	0.058	0.082	0.053	0.076	0.048	0.068
	c1	0.048	0.036	0.046	0.035	0.045	0.034	0.044	0.033
	c2	0.004	0.003	0.004	0.003	0.004	0.003	0.004	0.002
	perc c1	4.339	3.275	4.285	3.235	4.247	3.222	4.226	3.198
	perc c2	10.284	6.834	10.048	6.883	9.981	6.842	9.700	6.530
0.7	a1	0.111	0.094	0.080	0.062	0.086	0.070	0.104	0.101
	a2	0.072	0.103	0.065	0.092	0.059	0.084	0.053	0.075
	c1	0.054	0.041	0.052	0.040	0.051	0.038	0.049	0.037
	c2	0.005	0.003	0.005	0.003	0.004	0.003	0.004	0.003
	perc c1	4.371	3.307	4.320	3.269	4.278	3.235	4.250	3.217
	perc c2	10.408	7.025	10.180	6.934	10.085	6.770	9.690	6.552

Table 2: Means and standard deviations for absolute values of 100-period data for a variety of policy rules. The values of the a1 and a2 coefficients are sometimes farther from the MSV solution, which is zero.

Table 2 continued.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
0.6	a1	0.138	0.120	0.095	0.073	0.103	0.085	0.128	0.130
	a2	0.083	0.119	0.074	0.105	0.067	0.094	0.059	0.084
	c1	0.062	0.047	0.060	0.046	0.058	0.044	0.056	0.042
	c2	0.005	0.004	0.005	0.003	0.005	0.003	0.005	0.003
	perc c1	4.411	3.336	4.362	3.308	4.307	3.254	4.284	3.231
	perc c2	10.443	7.044	10.286	6.908	10.093	6.750	9.645	6.479
0.5	a1	0.180	0.159	0.115	0.089	0.128	0.109	0.160	0.171
	a2	0.098	0.139	0.087	0.122	0.076	0.107	0.067	0.094
	c1	0.074	0.056	0.071	0.053	0.068	0.051	0.065	0.049
	c2	0.006	0.004	0.006	0.004	0.006	0.004	0.005	0.004
	perc c1	4.484	3.375	4.418	3.342	4.356	3.303	4.316	3.278
	perc c2	10.522	7.044	10.312	6.918	10.172	6.816	9.701	6.569
0.4	a1	0.254	0.230	0.147	0.113	0.166	0.147	0.212	0.237
	a2	0.123	0.173	0.104	0.145	0.090	0.123	0.077	0.106
	c1	0.090	0.068	0.085	0.065	0.081	0.062	0.077	0.059
	c2	0.008	0.005	0.007	0.005	0.007	0.005	0.006	0.004
	perc c1	4.567	3.453	4.487	3.398	4.409	3.350	4.353	3.309
	perc c2	10.941	7.189	10.346	6.994	10.188	6.845	9.816	6.710
0.3	a1	0.395	0.379	0.204	0.154	0.233	0.219	0.300	0.352
	a2	0.162	0.228	0.130	0.179	0.107	0.147	0.090	0.121
	c1	0.116	0.087	0.108	0.082	0.101	0.077	0.096	0.073
	c2	0.010	0.007	0.009	0.006	0.008	0.006	0.008	0.005
	perc c1	4.716	3.552	4.584	3.473	4.504	3.410	4.463	3.360
	perc c2	11.160	7.344	10.716	7.166	10.165	6.796	9.596	6.470
0.2	a1	0.739	0.747	0.324	0.244	0.369	0.371	0.462	0.571
	a2	0.234	0.323	0.174	0.233	0.137	0.180	0.107	0.136
	c1	0.162	0.122	0.146	0.111	0.135	0.103	0.126	0.095
	c2	0.014	0.009	0.012	0.008	0.011	0.007	0.010	0.007
	perc c1	4.972	3.752	4.774	3.609	4.649	3.544	4.570	3.462
	perc c2	11.647	7.625	10.918	7.340	10.204	6.906	9.753	6.623

TABLE 3.

Parameter	φ_π	0.5	1.0	1.5	2.0
φ_z	Criterion				
1.1	1	950	960	964	963
	2	990	992	994	995
1.0	1	933	947	955	957
	2	982	990	989	991
0.9	1	901	931	945	935
	2	973	981	989	991
0.8	1	866	911	921	902
	2	967	972	979	987
0.7	1	810	863	880	860
	2	952	962	971	974
0.6	1	725	801	822	807
	2	932	945	961	966
0.5	1	604	694	739	717
	2	901	925	944	938
0.4	1	453	554	607	619
	2	830	888	919	889
0.3	1	280	379	467	490
	2	668	800	847	815
0.2	1	145	208	275	306
	2	403	578	677	690

Table 3: The number of simulations for which each of the criteria is satisfied, the total number of simulations is 1000. Criterion 1 means that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 means that absolute deviations from MSV value for $a1$ is less than or equal 0.5 and that absolute deviations from MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.

Table 4.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.064	0.053	0.050	0.039	0.054	0.044	0.066	0.062
	a2	0.061	0.083	0.057	0.078	0.053	0.072	0.050	0.066
	c1	0.034	0.027	0.033	0.026	0.033	0.026	0.032	0.025
	c2	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	perc c1	4.117	3.230	4.091	3.199	4.059	3.187	4.036	3.168
	perc c2	9.102	6.030	8.946	6.035	8.812	5.899	8.687	5.867
0.8	a1	0.099	0.086	0.069	0.055	0.079	0.068	0.100	0.102
	a2	0.084	0.114	0.077	0.103	0.070	0.093	0.063	0.084
	c1	0.046	0.036	0.045	0.035	0.044	0.034	0.042	0.033
	c2	0.004	0.002	0.004	0.002	0.003	0.002	0.003	0.002
	perc c1	4.189	3.280	4.149	3.250	4.112	3.224	4.081	3.204
	perc c2	9.208	6.131	9.037	6.042	8.893	5.929	8.678	5.868
0.5	a1	0.202	0.190	0.115	0.091	0.141	0.130	0.188	0.211
	a2	0.133	0.179	0.116	0.154	0.100	0.130	0.087	0.113
	c1	0.071	0.056	0.068	0.054	0.065	0.051	0.063	0.049
	c2	0.006	0.004	0.005	0.004	0.005	0.003	0.005	0.003
	perc c1	4.330	3.400	4.262	3.349	4.208	3.303	4.151	3.263
	perc c2	9.469	6.365	9.200	6.211	8.946	5.981	8.759	5.936
0.2	a1	0.886	0.938	0.325	0.258	0.417	0.460	0.567	0.715
	a2	0.314	0.410	0.236	0.301	0.178	0.218	0.138	0.164
	c1	0.156	0.122	0.141	0.111	0.130	0.102	0.121	0.095
	c2	0.012	0.008	0.011	0.007	0.010	0.007	0.009	0.006
	perc c1	4.794	3.756	4.608	3.627	4.477	3.520	4.380	3.442
	perc c2	10.346	6.848	9.801	6.478	9.243	6.309	8.841	6.074

Table 4: Means and standard deviations for absolute values of 100-period data for a variety of policy rules for simulations with separate mean squared error for output gap and deviation of inflation from target.

Table 5.

Parameter	φ_π	0.5	1.0	1.5	2.0
φ_z	Criterion				
1.1	1	921	941	952	949
	2	975	980	984	987
0.8	1	828	874	893	869
	2	946	959	968	974
0.5	1	553	669	710	682
	2	861	898	924	920
0.2	1	127	193	267	275
	2	371	549	620	619

Table 5: The number of simulations for which each of the criteria is satisfied for simulations with separate fitness, the total number of simulations is 1000. Criterion 1 means that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 means that absolute deviations from MSV value for $a1$ is less than or equal 0.5 and that absolute deviations from MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.

Table 6.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	0.825	0.001	0.812	0.001	0.800	0.001	0.789	0.001
	c2	0.030	0.000	0.029	0.000	0.029	0.000	0.028	0.000
	perc c1	99.570	0.063	99.570	0.063	99.569	0.063	99.569	0.063
	perc c2	96.999	0.632	96.972	0.637	96.947	0.639	96.919	0.641
0.8	a1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	1.097	0.001	1.075	0.001	1.054	0.001	1.034	0.001
	c2	0.039	0.000	0.038	0.000	0.038	0.000	0.037	0.000
	perc c1	99.570	0.063	99.570	0.063	99.570	0.063	99.569	0.063
	perc c2	97.075	0.620	97.037	0.623	97.001	0.631	96.967	0.637
0.5	a1	0.002	0.001	0.002	0.001	0.002	0.001	0.002	0.001
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	1.639	0.001	1.591	0.001	1.545	0.001	1.503	0.001
	c2	0.059	0.000	0.057	0.000	0.055	0.000	0.054	0.000
	perc c1	99.571	0.063	99.571	0.063	99.570	0.063	99.570	0.063
	perc c2	97.214	0.597	97.160	0.603	97.108	0.614	97.055	0.623
0.2	a1	0.004	0.002	0.003	0.002	0.003	0.001	0.003	0.001
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	3.238	0.002	3.055	0.002	2.892	0.002	2.746	0.002
	c2	0.117	0.001	0.110	0.001	0.104	0.001	0.099	0.001
	perc c1	99.574	0.063	99.573	0.063	99.572	0.063	99.572	0.063
	perc c2	97.566	0.553	97.460	0.569	97.366	0.580	97.271	0.589

Table 6: Means and standard deviations for absolute values of 100-period data for a variety of policy rules for simulations with no crossover.

Table 7.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.072	0.060	0.050	0.039	0.062	0.050	0.087	0.082
	a2	0.093	0.107	0.088	0.102	0.084	0.096	0.081	0.092
	c1	0.035	0.027	0.035	0.026	0.034	0.026	0.033	0.025
	c2	0.004	0.003	0.004	0.003	0.004	0.003	0.004	0.003
	perc c1	4.266	3.229	4.236	3.189	4.212	3.181	4.177	3.165
	perc c2	13.579	10.732	13.243	10.246	13.147	10.178	13.113	10.429
1.0	a1	0.084	0.070	0.055	0.043	0.071	0.058	0.102	0.095
	a2	0.103	0.118	0.095	0.111	0.092	0.105	0.088	0.099
	c1	0.039	0.029	0.038	0.028	0.037	0.028	0.036	0.027
	c2	0.005	0.004	0.004	0.003	0.004	0.003	0.004	0.003
	perc c1	4.278	3.238	4.254	3.199	4.221	3.188	4.190	3.173
	perc c2	13.574	10.894	13.488	10.378	13.030	9.919	13.064	9.981
0.9	a1	0.098	0.082	0.062	0.048	0.081	0.068	0.119	0.112
	a2	0.113	0.127	0.106	0.123	0.099	0.114	0.094	0.105
	c1	0.043	0.032	0.042	0.031	0.041	0.031	0.040	0.030
	c2	0.005	0.004	0.005	0.004	0.005	0.004	0.004	0.003
	perc c1	4.320	3.255	4.275	3.219	4.234	3.205	4.199	3.190
	perc c2	13.529	10.498	13.678	10.678	13.196	10.409	12.900	10.094
0.8	a1	0.117	0.101	0.070	0.054	0.094	0.080	0.142	0.137
	a2	0.126	0.144	0.116	0.131	0.109	0.125	0.103	0.116
	c1	0.048	0.036	0.046	0.035	0.045	0.034	0.044	0.033
	c2	0.006	0.004	0.005	0.004	0.005	0.004	0.005	0.004
	perc c1	4.345	3.288	4.292	3.239	4.270	3.210	4.231	3.191
	perc c2	13.735	11.008	13.519	10.442	13.160	10.672	12.865	9.970
0.7	a1	0.145	0.126	0.081	0.062	0.114	0.100	0.175	0.168
	a2	0.146	0.161	0.131	0.149	0.122	0.141	0.114	0.125
	c1	0.054	0.041	0.052	0.039	0.051	0.038	0.049	0.037
	c2	0.006	0.005	0.006	0.005	0.006	0.005	0.005	0.004
	perc c1	4.375	3.317	4.329	3.259	4.296	3.241	4.253	3.217
	perc c2	13.886	11.064	13.414	10.421	13.236	10.616	12.421	9.706

Table 7: Means and standard deviations for absolute values of 100-period data for a variety of policy rules for weight=1. The values of the a1 and a2 coefficients are sometimes farther from the MSV solution, which is zero.

Table 7 continued.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
0.6	a1	0.188	0.166	0.095	0.073	0.143	0.128	0.219	0.220
	a2	0.170	0.188	0.148	0.168	0.139	0.159	0.125	0.139
	c1	0.062	0.047	0.060	0.045	0.058	0.044	0.056	0.043
	c2	0.007	0.006	0.007	0.005	0.006	0.005	0.006	0.005
	perc c1	4.420	3.348	4.361	3.294	4.315	3.263	4.288	3.244
	perc c2	13.889	10.786	13.384	10.591	13.003	10.128	12.418	9.586
0.5	a1	0.250	0.224	0.116	0.089	0.184	0.166	0.278	0.286
	a2	0.201	0.216	0.177	0.198	0.158	0.176	0.137	0.152
	c1	0.074	0.056	0.070	0.053	0.068	0.051	0.066	0.049
	c2	0.008	0.007	0.008	0.006	0.007	0.006	0.007	0.005
	perc c1	4.496	3.398	4.409	3.338	4.354	3.299	4.341	3.278
	perc c2	13.886	11.099	13.256	10.314	12.960	10.521	12.636	9.875
0.4	a1	0.362	0.333	0.148	0.114	0.255	0.234	0.378	0.387
	a2	0.249	0.268	0.214	0.243	0.184	0.206	0.153	0.169
	c1	0.090	0.068	0.085	0.065	0.081	0.061	0.078	0.060
	c2	0.010	0.008	0.009	0.007	0.009	0.007	0.008	0.006
	perc c1	4.593	3.436	4.491	3.392	4.422	3.341	4.365	3.355
	perc c2	13.874	10.723	13.386	10.320	12.975	9.905	12.438	9.436
0.3	a1	0.577	0.548	0.207	0.157	0.368	0.352	0.568	0.602
	a2	0.315	0.345	0.265	0.292	0.212	0.239	0.182	0.198
	c1	0.116	0.087	0.108	0.081	0.101	0.077	0.097	0.075
	c2	0.013	0.010	0.012	0.009	0.011	0.008	0.010	0.007
	perc c1	4.740	3.564	4.590	3.465	4.505	3.437	4.503	3.450
	perc c2	14.270	10.902	13.578	10.776	13.438	9.905	12.359	9.337
0.4	a1	1.130	1.111	0.333	0.248	0.621	0.627	0.931	1.002
	a2	0.453	0.498	0.353	0.396	0.268	0.291	0.210	0.225
	c1	0.162	0.124	0.147	0.111	0.135	0.103	0.127	0.097
	c2	0.018	0.014	0.016	0.012	0.014	0.010	0.013	0.009
	perc c1	4.996	3.801	4.779	3.609	4.641	3.536	4.597	3.524
	perc c2	15.276	11.323	13.768	10.315	13.254	9.774	12.555	9.319

Table 8.

Parameter	φ_π	0.5	1.0	1.5	2.0
φ_z	Criterion				
1.1	1	850	881	881	850
	2	934	953	953	955
1.0	1	819	858	861	845
	2	921	941	940	946
0.9	1	781	826	840	802
	2	908	921	929	936
0.8	1	741	793	802	739
	2	878	907	915	928
0.7	1	662	734	749	682
	2	853	874	889	905
0.6	1	569	676	674	596
	2	806	847	867	876
0.5	1	447	556	579	514
	2	759	805	838	822
0.4	1	299	418	422	399
	2	663	751	791	733
0.3	1	201	261	308	269
	2	507	633	677	608
0.2	1	78	137	157	163
	2	276	418	484	452

Table 8: The number of simulations for which each of the criteria is satisfied, the total number of simulations is 1000, weight=1. Criterion 1 means that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 means that absolute deviations from MSV value for $a1$ is less than or equal 0.5 and that absolute deviations from MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.

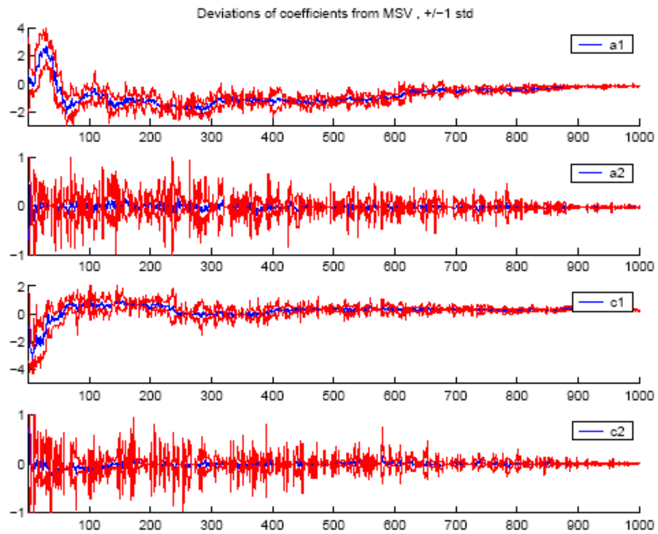

Figure 1: Simulation for determinate and E-stable region: $\phi_\pi = 2$, $\phi_z = 0.2$.

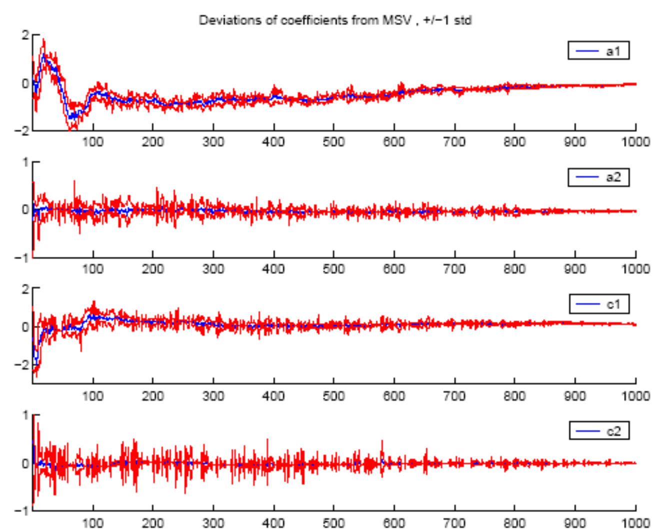

Figure 2: Simulation for determinate and E-stable region: $\phi_\pi = 1.5$, $\phi_z = 0.5$.

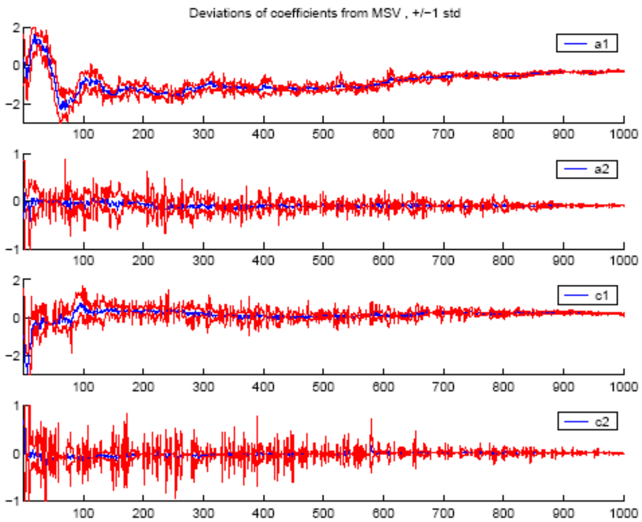

Figure 3: Simulation for indeterminate and E-unstable region: $\phi_\pi = 0.5$, $\phi_z = 0.3$.

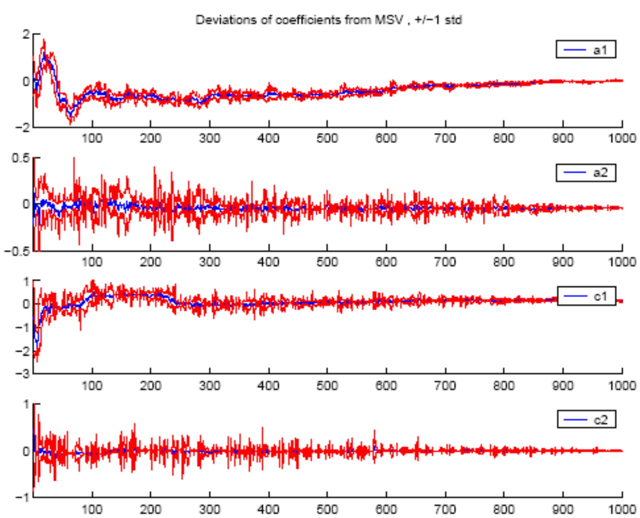

Figure 4: Simulation for indeterminate and E-unstable region: $\phi_\pi = 0.5$, $\phi_z = 0.5$.