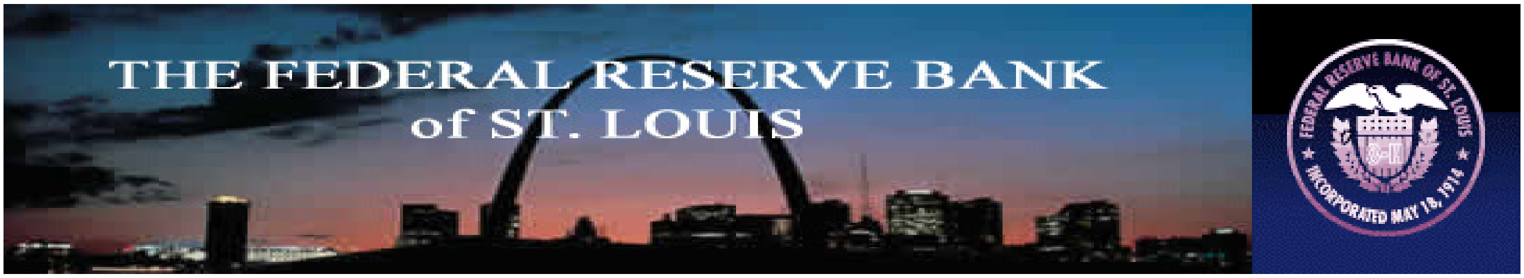

WORKING PAPER SERIES

Do Fundamentals, Bubbles or Neither Determine Stock Prices? Some International Evidence

Gerald P. Dwyer, Jr.
and
R. W. Hafer

Working Paper 1989-003A
http://research.stlouisfed.org/wp/1989/1989-003.pdf

1989

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

DO FUNDAMENTALS, BUBBLES OR NEITHER DETERMINE STOCK PRICES? SOME INTERNATIONAL EVIDENCE

Gerald P. Dwyer, Jr.* and R. W. Hafer**

Federal Reserve Bank of St. Louis

89-003

*Clemson University and Federal Reserve Bank of St. Louis

**Federal Reserve Bank of St. Louis

 This paper was prepared for presentation at the Federal Reserve Bank of St. Louis' Thirteenth Annual Economic Policy Conference, October 21-22, 1988.

I. INTRODUCTION

It has been one year since the global decline in stock prices and the principal cause remains a mystery. For some observers, the decline was a natural conclusion to an unfounded runup in prices that occurred especially during 1987, both in the United States and abroad. For example, the Brady Commission noted that "[A]s in the U.S., stock valuation in these [foreign] markets in 1987 began to rise above levels apparently justified by historical precedent or economic factors." [Brady Commission (1988), p. 9] Indeed, in its study of stock price behavior in other markets, the section exploring the events of October 1987 is titled "Bursting the Bubble."[1/]

The notion that stock price behavior can be explained by bubbles suggests that stock prices deviate from the level implied by their underlying fundamental value. The idea that stock prices may deviate from their fundamentals is not unique to commissions appointed after a market crash, however. For example, a recent study by Hardouvelis (1988) concludes that, based on an analysis of stock prices for Japan, the United Kingdom and the United States, the hypothesis of a bubble is not inconsistent with stock price behavior leading up to October 1987. Previous investigations of the role of "fundamentals" has been given content by associating the fundamentals with dividends. There is a relatively large literature that finds much of the variance in stock prices is not explained by dividends [Shiller (1981) and Campbell and Shiller (1987)].

At the same time, other studies present evidence that bubbles do not play an important role in explaining the movement in stock prices. Diba and Grossman (1988) recently have provided evidence that supports the fundamentals hypothesis. Using U.S. annual stock price data from

1871 to 1986, their evidence does not support the bubble hypothesis. Using daily data for the periods before the 1929 and 1987 crashes, Dwyer and Santoni (1988) also find no support for the hypothesis that rational bubbles are consistent with the time-series characterization of stock prices during these two episodes.

Most previous tests of the fundamentals versus bubble models of stock prices focus on the behavior of U.S. stock prices, use data of long periodicity, or both. Because the crash of 1987 offers an almost-laboratory setting in which to test these alternative hypotheses, we set out first to see whether the behavior of stock prices during the final stages of the recent runup is consistent with the implications of a rational bubble model. We do this using daily stock price data for Australia, Canada, France, Germany, Japan, the United Kingdom and the United States.

We also investigate the importance of fundamentals as determinants of stock prices. Because daily data are not well suited to this task, we use monthly stock price data for the past 15 years. Our analysis extends beyond the United States and includes stock markets in Canada, Germany, Japan and the United Kingdom. The analysis using monthly data allows us to examine the generality of the fundamentals model across different markets.

The format of our study is as follows. To provide some perspective, the second section presents a descriptive analysis of daily stock prices for the year preceding the October 1987 crash. In the third section we present a theoretical analysis of a model of stock prices with a rational bubble. We also present empirical tests of this model's implications for the behavior of daily stock prices for our sample of seven countries. We examine the fundamentals model in the fourth section, deriving some implications of this model under the alternative assumptions of constant and varying expected real interest rates. Empirical tests derived from the theoretical discussion also are presented. Conclusions close the paper.

II. DESCRIPTIVE ANALYSIS OF DAILY DATA

The daily stock price data used in this study are taken from Morgan Stanley's Capital International Perspective.[2/] We use the index values for Australia, Canada, France, Germany, Japan, the United Kingdom and the United States. The daily stock price indexes used are available since July 29, 1986. Our analysis uses the data from this point through December 31, 1987, a period that encompasses the worldwide decline in equity values in October 1987.

To summarize the relative behavior of the seven indexes, percentage changes in the indexes over various periods are presented in table 1. These periods are: (1) from the beginning of the data sample to each index's peak; (2) from the peak to October 12, 1987; and (3) from October 26, 1987 through December 31, 1987. We end the second sample period on the Monday prior to the crash of the U.S. market because the dating of the crash is not the same in all markets, thus making uniform timing impossible.[3/] Also, the notion that stock prices around the world ratcheted up in some kind of global bull market is best examined using data prior to the declines that occurred around October 19. These periods allow us to compare the relative magnitudes of the price increases, to assess the movements of the indexes prior to the crash and to compare their recovery. The percentage changes for these periods are presented in the upper half of table 1.

The statistics reported in the first column indicate that the increase in equity values was quite disparate. The best performer in own currency terms was Australia, with an increase of 114.46 percent. In contrast, the German index only gained 10.68 percent from July 1986 to its peak, and France 19.32 percent. The average gain during this period was 51.32 percent.

The second column shows that most stock price indexes had lost ground even before the crash.4/ From the respective peak dates to October 12, the decline in stock prices ranged from 4.28 percent for the United Kingdom to 14.46 percent for France. Indeed, in Germany and France, the declines from their respective peaks to October 12 essentially wiped out the gains realized since July 1986. All of the entries indicate that market values fell prior to the crash and, in some instances, had been declining for some time.

The period after the crash, here measured from October 26 through December 31, 1987, also reveals disparate behavior in the different stock indexes. After the crash, stock prices continued to fall in Australia, France, Germany and Japan, with the German index registering an 18.52 percent decline. In Canada, the United Kingdom, and the United States, stock prices increased from 1.92 percent in the United Kingdom to 10.88 percent in Canada.

It also is instructive to compare the behavior of the different stock price indexes when measured in some common currency. This is done in the lower half of table 1, where the indexes are denominated in dollars. The largest percentage increase again is for Australia, with a 159.84 percent increase. The smallest is Germany, showing a 27.37 percent increase. The average increase in dollar terms is 67.92 percent, compared with 51.31 percent when the indexes are denominated in local currencies.

The most striking difference between the dollar-denominated and own currency measures of the stock indexes comes after the crash. Again using October 26 as the beginning of the post-crash period, the data presented in the lower panel of table 1 show that only the German index continues to decline after the crash, falling 8.57 percent further by the end of 1987. In contrast to the own-currency measures, the dollar-denominated indexes for Australia, France and Japan now show increases during the post-crash period.

III. TESTS FOR RATIONAL BUBBLES IN STOCK PRICES

Depending on the time-series properties of the fundamentals, any time-series process can be used to characterize stock prices and can be consistent with the fundamentals. That said, we show in the first part of this section that a particular class of rational bubbles adds certain patterns to the time-series behavior of stock prices. We then examine daily stock price indexes for evidence of these patterns and test the notion suggested in the Brady Report (1988) that stock markets around the world ratcheted up together.

A. Implications of Bubbles and Fundamentals

Assume that investors equate the expected holding period return on a stock to a constant expected real interest rate, r. With the price measured at the start of the period just after the dividend is paid, the expected holding period return in period t then is

$$(1) \quad r = \frac{E_t p_{t+1} + E_t d_{t+1} - p_t}{p_t} ,$$

where $E_t(\cdot)_{t+1}$ is the expected value of $(\cdot)$ in period t+1 conditional on all information available in period t including the price in period t,

p_t, and the dividend in period t, d_t. Equation (1) can be rewritten as

(2) $p_t = \beta(E_t p_{t+1} + E_t d_{t+1})$,

where $\beta = (1+r)^{-1}$. Equation (2) is the basis for both the model of the fundamentals and the model for bubbles.

The Fundamentals

The fundamental price in period t, p_t^f, is the discounted present value of the expected future stream of dividends,[5/]

(3) $p_t^f = \sum_{i=1}^{\infty} \beta^i E_t d_{t+i}$

given the transversality conditions that

$$\lim_{i\to\infty} \beta^i E_t d_{t+i} = \lim_{i\to\infty} \beta^i E_t p_{t+i} = 0.$$

The implications of the fundamentals model can be shown using a particularly simple time-series process for expected dividends. If the expected growth rate of dividends is constant, the fundamental model implies that proportional changes in the stock price are unpredictable except for a constant growth rate. If dividends grow at a constant rate g^d and deviations of dividends from this constant rate are unpredictable, then

(4) $d_{t+1} = (1+g^d)d_t + \eta_{t+1}$,

where η_{t+1} is the unexpected part of dividend growth in period t+1, and $E_t \eta_{t+1} = 0$. Substitution of equation (4) into (3) and calculation of the proportional change in the fundamental price yields:

$$(5)\quad \frac{\Delta p^f_{t+1}}{p^f_t} = g^d + \frac{\eta_{t+1}}{p^f_t} \sum_{i=1}^{\infty} \beta^i (1+g^d)^i .$$

Letting g^f denote the growth rate of the fundamental price, equations (4) and (5) then imply that

$$(6)\quad E_t g^f_t = g^d.$$

An immediate result of equation (6) is that the proportional changes in the price of the stock are unpredictable except for a constant.

Rational Bubbles

Suppose that a bubble is superimposed on this simple structure. Assume that the actual stock price in period t deviates from the fundamental price by an amount b_t with a positive probability $\pi > 0$ of continuing each period. This specification of a bubble follows Blanchard and Watson (1982), although a deterministic specification yields similar conclusions [Diba and Grossman (1988)]. A solution for the price that includes the bubble, p^b_t, and is consistent with equation (2) is

$$(7a)\quad p^b_t = p^f_t + b_t ,$$

where

$$(7b)\quad b_t = \begin{cases} (1+r)\pi^{-1} b_{t-1} & \text{with probability } \pi \\ 0 & \text{with probability } 1-\pi. \end{cases}$$

The term π^{-1} appears in the bubble part of the price because purchasers of the stock market must be compensated for the possibility that the bubble will burst: a lower probability of the bubble continuing is associated with a higher rate of increase of the bubble part of the stock price if the bubble continues. As long as the bubble continues, the bubble part of the price grows at the rate

$$(8)\quad g^b = r + (1+r)\,\frac{(1-\pi)}{\pi}\ .$$

Because $1-\pi$ and π are greater than zero, the bubble part of the price grows at a rate greater than the real interest rate. When the bubble bursts, the expected growth rate of this part of the price is zero, and the stock price is determined by the fundamentals modeled above.

If there is a bubble, however, proportional changes in the stock price are predictable for any finite period. If the expected dividend grows at a constant rate, from equation (7a) the proportional change in price including the bubble is

$$(9)\quad \frac{\Delta p^b_{t+1}}{p^b_t} = g^f_t\,\frac{p^f_t}{p^b_t} + \frac{\Delta b_{t+1}}{p^b_t}\ .$$

From this definition and using equation (7a) to rearrange it,

$$(10)\quad \frac{\Delta p^b_{t+1}}{p^b_t} = g^f_t - g^f_t\,\frac{b_t}{p^b_t} + \frac{\Delta b_{t+1}}{p^b_t}\ .$$

The result of using equations (7b) and (8) to replace Δb_{t+1} and rearrangement is

$$(11)\quad \frac{\Delta p^b_{t+1}}{p^b_t} = g^f_t + (g^b - g^f_t)\,\frac{b_t}{p^b_t}\ .$$

Using equation (7a), we find that

$$(12) \quad \frac{\Delta p^b_{t+1}}{p^b_t} = g^f_t + (g^b - g^f_t) \frac{1}{1 + p^f_t / b_t} .$$

The proportional change in the stock price equals the proportional change in the fundamental price plus a term due to the bubble. The first part of the term due to the bubble is $g^b - g^f_t$, which has a strictly positive expected value because $g^b > r$, and the expected value of g^f_t is $g^d < r$. The second part of the term due to the bubble is not stationary in finite time. To see this, let period 0 be any period when the bubble is on. The ratio of the fundamental price to the bubble part of the price in later periods when the bubble is on is

$$(13) \quad \frac{p^f_t}{b_t} = \frac{(1+g^f_t)^t}{(1+g^b)^t} \frac{p^f_0}{b_0} ,$$

where p^f_0/b_0 is constant for all $t > 0$. Because the expected value of g^f_t is less than g^b, this ratio is a decreasing function of time. This means that, for any finite period when a bubble is on, the proportional change in price is an increasing function of time. Consequently, the proportional change in observed stock prices is predictable from its own past value. As time goes to infinity, the ratio in equation (13) goes to zero, because the bubble part of the price eventually dominates the fundamental component in the stock price.[6/]

B. Tests for Unit Roots

Before we actually test for the presence of bubbles in the different stock indexes, it is useful to determine whether the data can be characterized in a manner that is consistent with the simple model presented above. An implication of the model is that stock prices should be difference stationary. This property of the data can easily be tested using the procedures of Dickey and Fuller (1979) to determine whether there is a unit root in the levels and first difference of the stock prices for the countries in our sample.

To test for unit roots in the levels of the stock price indexes, changes in stock prices are regressed on a constant, the lagged level of the index and one lagged value of the change. The test statistic is the reported t-ratio. If the calculated t-ratio is greater than the critical value, one cannot reject the hypothesis that the series contains a unit root. If the t-ratio is smaller than or equal to the critical value, however, then the notion of a unit root is rejected. For example, with a sample size of 250 observations, the critical value is -2.88 at the 5 percent level of significance.[7/] Calculated t-ratios smaller than this are inconsistent with the hypothesis of a unit root in the series at this significance level.

The Dickey-Fuller test is conducted on the levels and the first-differences of the stock prices for each country. Moreover, two periods are used for each country: one period uses data from the beginning of the data set in July 1986; the other uses data from the beginning of 1987. In both instances, the endpoint is the peak in each country's index. In this way, we can determine whether stock prices behave differently during the final stages of the bull market.[8/] The results of the Dickey-Fuller test for unit roots in the stock prices are presented in table 2.[9/]

The test statistics reported for the levels of stock prices, regardless of country or sample period, indicate that one cannot reject the hypothesis of a unit root in the series. Not only are the reported t-ratios far below the relevant critical values, but some even are incorrectly signed. The evidence for the first-differences, however, uniformly rejects the hypothesis of a second unit root. The calculated test statistics are much less than the critical value (approximately -2.88).

C. Time Trend Regressions

Equation (13) indicates that if a bubble is on, the proportional increase in the stock price is an increasing function of time. This is because a portion of the observed price rise is due to the fact that the bubble component increasingly dominates the stock price. In addition to being a positive function of time, the observed proportional change of the stock price should increase at a decreasing rate. This characterization of the data can be tested by regressing the proportional change in stock prices on time.

Table 3 provides the results of regressions of the proportional changes in stock prices on time for the period from the beginning of the sample to each country's peak. Two regression results are reported: one in which time enters only as a linear term and another that adds a quadratic term. In every instance, the regressions with time entered only linearly indicate that the proportional change in stock prices has no statistically significant trend (5 percent level). Moreover, adding the quadratic term does not alter this finding except in the case of Germany. For Germany, both the linear and quadratic terms are highly significant. Even so, the signs of the coefficients are counter to the theory: they indicate negative and then increasing rates of increase of stock prices up to the peak.

To determine if the data are consistent with a bubble at work in the latter stages of the bull market, the time regressions are re-estimated for the period from January 1987 to the respective peak of each stock market. These results, reported in table 4, yield many more significant estimated coefficients than those from the longer sample. This is true for Canada and the United States. Even so, the coefficients are incorrectly signed to support the bubble hypothesis. Only for Germany is there evidence that supports the bubble hypothesis: These results indicate a positive and significant trend term for 1987.[10/]

D. Autoregressions

We also can test for the presence of bubbles by examining autoregressions with the proportional change in stock prices. If bubbles are present, observed changes in stock prices should exhibit significant serial correlation. To test this, the proportional change in the stock price index from each country is regressed on a constant and 25 lagged values of the change. The bubble hypothesis is rejected if the lag terms are not positive and significant.

The relevant statistics for this test are reported in table 5. Again results are presented for two periods as in previous tests. To test the hypothesis that the autoregressive parameters are significant as a group, χ^2-statistics are calculated. For the longer period, the estimated χ^2 is significant for Canada and Germany. Only for Canada, however, do we also find that the sum of the lagged coefficients is significant at some reasonable level. This evidence is therefore consistent with the hypothesis that a bubble was present in Canadian stock prices during 1986-87.

At first glance, the results for 1987 only seem to provide even stronger evidence of bubbles in stock prices. During this period of the bull market, the χ^2 statistics are significant for Canada, France, Germany, Japan and the United States, thus supporting the bubble hypothesis. Note, however, that for France, Japan and the United States we find that the coefficients sum is negative, contrary to the bubble hypothesis being tested. Moreover, the summed terms for Canada and Germany are not significantly different from zero. Thus, except for some evidence of bubbles at work in Canadian stock prices, the data are largely inconsistent with the presence of bubbles in the different stock price indexes.

E. Contagious Bubbles

The rise of stock prices since the early 1980s in markets around the world and their subsequent crash in October 1987 have led some observers to argue that this coincident behavior cannot be explained by chance alone. The Brady Commission, for example, argues that "investors made comparisons of valuations in different countries often using higher valuations in other countries as justification for investing in lower valued markets. Consequently, a process of ratcheting up among worldwide stock markets began to develop." [Brady Report (1988), p. 10] This view suggests that higher prices in one market beget higher prices in another, a hypothesis which might be called one of a "contagious bubble."

We can investigate this claim by examining the behavior of relative stock prices during 1986 and 1987. To do this, consider the following. Assume that transaction costs to buy and sell stock anywhere in the world are zero. If investors are risk-neutral, the expected return from holding stock in any common currency must be the same anywhere. This implies that

$$(14)\quad Eh_t^i = Eh_t^j + E\Delta e_t ,$$

where Eh_t^i is the expected rate of return from holding stock in country i in terms of i's currency in period t, Eh_t^j is the expected return from holding stock in country j in terms of j's currency and $E\Delta e_t$ is the expected rate of change in the price of country j's currency in terms of country i's.11/ If the condition in equation (14) holds, the expected return from holding stock in country i is the same as the expected return from holding a foreign stock, a condition which might be called "stock-return parity" by analogy with "interest-rate parity."

Equation (14) can be written in ex post terms as

$$(15) \quad h_t^i = h_t^j + \Delta e_t + \varepsilon_t^i - \varepsilon_t^j - \varepsilon_t^e ,$$

where ε^i and ε^j terms are the unexpected part of the holding period returns for stocks i and j, and ε^e is the unexpected part of the rate of change of the exchange rate. If expectations are rational, the ε's are independent of the expected portions of the holding period returns and the unexpected part of the change in the exchange rate. If dividends are zero, equation (15) can be written in terms of the levels of stock prices as

$$(16) \quad p_t^i - p_{t-1}^i = p_t^j - p_{t-1}^j + e_t - e_{t-1} + \varepsilon_t^i - \varepsilon_t^j - \varepsilon_t^e ,$$

where p is the logarithm of the stock's price, and e is the logarithm of the exchange rate. If we define a stock price relative as

$$(17) \quad x = p^i - p^j - e ,$$

equation (16) can be rewritten as

$$(18) \quad x_t = x_{t-1} + \varepsilon_t^i - \varepsilon_t^j + \varepsilon_t^e.$$

Equation (18) is useful because it shows that relative stock prices this period equal their value last period plus the difference between the unexpected parts of the holding period returns (the ε^i's and ε^j's) and the unexpected part of the change in the exchange rate (ε^e). What equation (18) says is: even if expected rates of return among different stocks are the same, relative stock prices are a random walk. This conclusion is specific to the assumption of stock-return parity but, in general, relative stock prices will be characterized as having a unit root.[12/]

To test whether stock prices ratcheted up worldwide in a contagious bubble, we examine the levels of the relative stock prices for unit roots over the period from the beginning of the sample through October 12, 1987 and from January 1987 through October 12. The results of the Dickey-Fuller tests on the levels of the stock price relatives are presented in table 6.[13/] The top half of the table reports the results for the longer period. There we find no test statistics that are smaller than the relevant 5 percent critical value of about -2.88. These results are inconsistent with the notion that stock prices ratchet up or down with each other. When the sample period is shortened to only data from 1987, this outcome does not always hold. As shown in the bottom half of table 6, the hypothesis of a single unit root in the level of the stock price relatives is rejected for Canada and Germany, Germany and the United Kingdom, and Germany and the United States. The statistical evidence using data only from 1987 suggests that German stock prices increased in unison with those in other markets.

The test statistics for Germany are the most consistent with the Brady Commissions' notion of prices rising only because others did. How much weight should be attached to this result for Germany? It is interesting that the result exists only if one deletes data from the second half of 1986. This suggests that finding or rejecting unit root processes in these stock price relatives is sensitive to the sample period tested.

IV. STOCK PRICES, DIVIDENDS AND BOND YIELDS: TESTS FOR FUNDAMENTALS

We now develop, in this section, implications for the joint time-series properties of stock prices, dividends and interest rates, focusing on the relationships between the levels of the series. Because the results in the second part of this section provide little support to the fundamentals model, we then estimate some simple regressions to examine how much of the variance of stock prices is explained by some readily-available factors suggested by the fundamentals model.

A. Stock Prices and the Fundamentals

We develop the implications for the joint behavior of stock prices and dividends first with the maintained hypothesis that the expected real interest rate is constant. We then develop the implications for stock price behavior when the expected real interest rate is allowed to vary.

A Constant Expected Real Interest Rate

If stock prices are governed by the fundamentals as defined in equation (3) with a constant expected real interest rate, there are testable implications concerning the relationship between stock prices, dividends and bond yields. Equation (3) for the fundamental price (p^f) can be written as

$$(19) \quad p_t^f = d_t \sum_{i=1}^{\infty} \beta^i + \sum_{i=1}^{\infty} \beta^i \, (E_t d_{t+i} - d_t) \; .$$

Suppose that the actual price p_t equals the fundamental price. If the price and the dividends each have one unit root (as turns is consistent with the data), then equation (19) implies that the price and dividends are cointegrated.[14/] That is, a regression of stock prices on dividends of the form

$$(20) \quad p_t = \gamma_0 + \gamma_1 d_t + \varepsilon_t$$

will yield a residual that does not have a unit root. To see this, note that

$$(21) \quad E_t d_{t+i} - d_t = \sum_{j=1}^{i} (E_t d_{t+j} - E_t d_{t+j-1}) ,$$

which implies that, after simplification, equation (14) can be rewritten as

$$(22) \quad p_t^f = r^{-1} d_t + r^{-1} \sum_{i=1}^{\infty} \beta^i (E_t d_{t+i+1} - E_t d_{t+1}) .$$

By the hypothesis that the dividend has one unit root, $E_t d_{t+i+1} - E_t d_{t+i}$ does not have a unit root. In addition, the discount factors form a set of geometrically declining coefficients on these stationary values. This also implies that the second term in equation (22) does not have a unit root.[15/]

In addition to assuming that changes in dividends do not have a unit root, assume that expected changes in dividends are strictly stationary. If the expected real interest rate is constant and the fundamentals determine stock prices, the cointegrating regression is the projection of equation (22) on d_t, which will have stationary residuals if d_t has one unit root.[16/]

This implication for a projection of the stock price on dividends does not hold when a rational bubble is added to the stock price, however. If bubbles are an important component of the observed stock price, then the stock price will not be cointegrated with dividends.[17/] This is because, as discussed above, when a bubble is on, the bubble part of the price increases with time. Since the bubble part of the price is independent of the dividend by assumption, it therefore appears in the residual of equation (22). Hence, the estimated residuals in equation (22) increase with time, reflecting the bubble part of the price, and have a root greater than one.[18/]

A Variable Expected Real Interest Rate

What are the implications for the relationship between stock prices and dividends if the expected real interest rate is not constant? The basis of the relationship is the arbitrage condition, equation (2), which can be written as:

$$(23)\quad p_t^f = \beta_t[E_t d_{t+1} + E_t p_{t+1}] ,$$

where $\beta_t = (1+r_t)^{-1}$. The discount factor now has a time subscript to reflect its nonconstancy. To simplify the notation, we suppress the superscript "f" for the fundamental price. The solution for the fundamental price is based on iteration and successive substitution using equation (23). Iterating equation (23), we find that:

$$(24)\quad E_t p_{t+1} = E_t[\beta_{t+1}\ [E_{t+1} d_{t+2} + E_{t+1} p_{t+2}]] ,$$

and so forth. By repeated substitution into equation (23) with transversality conditions imposed,

$$(25)\quad p_t = \beta_t E_t d_{t+1} + \beta_t E_t[\beta_{t+1} E_{t+1} d_{t+2}]$$

$$+ \beta_t E_t[\beta_{t+1} E_{t+1}[\beta_{t+2} E_{t+2} d_{t+3}]]$$

$$+ \beta_t E_t[\beta_{t+1} E_{t+1}[\beta_{t+2} E_{t+1}[\beta_{t+3} E_{t+3} d_{t+4}]]$$

$$+ \ldots .$$

This equation is the analogue with a variable interest rate of the fundamental equation (3).

This rather complicated fundamental equation for the stock price can be rewritten in terms of contemporaneous conditional expectations and covariance terms. Because β_t is known, the first term on the right in equation (25) is simply the known discount factor times the expected dividend next period. The remaining terms are the conditional expectations of products of random variables.[19/] These terms are more complicated but actually have a simple structure.

It is useful to have a general characterization of the joint distribution of the interest rate and dividend series. While it may be somewhat stronger than necessary, we assume that the joint distribution of changes in dividends and changes in the discount factors is stationary. By Wold's decomposition theorem (Hannan, 1970, pp. 136-37; Sargent, 1979, pp. 256-60), there exists a joint moving-average representation of the form

$$(26)\quad \Delta\beta_t = \alpha^{\beta} + \sum_{j=0}^{\infty} w_j^1 \varepsilon_{t-j}^{\beta} + \sum_{j=0}^{\infty} w_j^2 \varepsilon_{t-j}^{d}$$

$$\Delta d_t = \alpha^{d} + \sum_{j=1}^{\infty} w_j^3 \varepsilon_{t-j}^{\beta} + \sum_{j=0}^{\infty} w_j^4 \varepsilon_{t-j}^{d} ,$$

where α^{β} and α^{d} are deterministic,

$$w_0^1 = w_0^4 = 1,$$

$$E\varepsilon_t^\beta = \varepsilon_t^d = 0,$$

$$E\varepsilon_t^\beta \varepsilon_t^\beta = \sigma_\beta^2,\ E\varepsilon_t^d \varepsilon_t^d = \sigma_d^2,\ \text{and}\ E\varepsilon_t^\beta \varepsilon_s^d = 0 \text{ for all s, t.}$$ [20]

For notational simplicity, we suppress the deterministic part of this moving-average representation.

Consider the second term on the right-hand side of equation (25). With the multiplier β_t suppressed, rearrangement using the law of iterative expectations reduces this term to

$$(27)\quad E_t[\beta_{t+1}E_{t+1}d_{t+2}] = E_t\beta_{t+1}E_t d_{t+2}$$
$$+ E_t[\beta_{t+1}-E_t\beta_{t+1}][E_{t+1}d_{t+2}-E_t d_{t+2}]$$

where $E_t[x][y]$ denotes the conditional covariance of x and y.[21] By definition and from equation (26),

$$(28)\quad \beta_{t+1} - E_t\beta_{t+1} = \beta_{t+1} - \beta_t - E_t\Delta\beta_{t+1}$$
$$= \varepsilon_{t+1}^\beta + w_0^2\varepsilon_{t+1}^d$$

and similarly

$$(29)\quad E_{t+1}d_{t+2} - E_t d_{t+2} = \varepsilon_{t+1}^d ,$$

Because the conditional and unconditional expectation of the covariance of these innovations is the same, the second term in equation (25) reduces to

$$(30)\quad \beta_t E_t[\beta_{t+1}E_{t+1}d_{t+2}] = \beta_t\ E_t\beta_{t+1}E_t d_{t+2}$$
$$+ \beta_t\ (w_0^2)^2\ \sigma_d^2.$$

The remaining terms are more complex, but the end result is the same, with the terms reducing to expectations conditional on information available at t and variance terms.22/

For example, consider the third term in equation (25), namely

(31) $\beta_t \; E_t[\beta_{t+1}E_{t+1}[\beta_{t+2}E_{t+2}d_{t+3}]]$.

For compactness of representation, define

(32) $x_{t+3} = \beta_{t+2}E_{t+2}d_{t+3}$.

Then

(33) $E_t[\beta_{t+1}E_{t+1}x_{t+3}] = E_t\beta_{t+1}E_tx_{t+3}$

$$+ E_t[\beta_{t+1}-E_t\beta_{t+1}][E_{t+1}x_{t+3}-E_tx_{t+3}] \; .$$

The first term in this expression is the product of conditional expectations at t, and the second term is a conditional covariance term. Evaluating the covariance term is tedious but reasonably straightforward. Equation (28) provides $\beta_{t+1} - E_t\beta_{t+1}$. By the definition of first differences,

(34) $\beta_{t+2} = \beta_t + \Delta\beta_{t+1} + \Delta\beta_{t+2}$

and

$$E_{t+2}d_{t+3} = d_t + \Delta d_{t+1} + \Delta d_{t+2} + E_{t+2}\Delta d_{t+3} \; .$$

This implies that

(35) $x_{t+3} = \beta_t d_t + \beta_t(\Delta d_{t+1}+\Delta d_{t+2}+E_{t+2}\Delta d_{t+3})$

$$+ d_t(\Delta\beta_{t+1}+\Delta\beta_{t+2}) + \Delta\beta_{t+1}(\Delta d_{t+1}+\Delta d_{t+2}+E_{t+2}\Delta d_{t+3})$$

$$+ \Delta\beta_{t+2}(\Delta d_{t+1}+\Delta d_{t+2}+E_{t+2}\Delta d_{t+3}).$$

Despite the messiness of this equation, $E_{t+1}x_{t+3} - E_t x_{t+3}$ in equation (33) reduces to a manageable function of the parameters in the moving-average representation, as does the expression $E_t[\beta_{t+1} - E_t\beta_{t+1}]\ [E_{t+1}x_{t+3} - E_t x_{t+3}]$.

From this analysis, the third term on the right-hand side of equation (25) can be written

$$(36)\quad \beta_t E_t[\beta_{t+1}E_{t+1}[\beta_{t+2}E_{t+2}d_{t+3}]]$$

$$= \beta_t E_t \beta_{t+1} E_t \beta_{t+2} E_t d_{t+3} + X_3 ,$$

where X_3 is a linear function of the variances. By induction, it is not hard to show that similar algebra goes through for further terms in equation (25).

The final result of all this is that

$$(37)\quad p_t = \sum_{j=1}^{\infty} \delta_{t+j} E_t d_{t+j} + X ,$$

where

$$w_{t+j} = \prod_{i=1}^{j} E_t \ddot{o}_{t-i+j} ,$$

where $E_t\beta_t = \beta_t$ and X is a linear function of the variances.

If we suppress the constant term due to the covariance terms, equation (37) can be rewritten as

$$(38)\quad p_t = d_t \sum_{j=1}^{\infty} \delta_{t+j} + \sum_{j=1}^{\infty} \delta_{t+j}(E_t d_{t+j} - d_t) .$$

To make this equation manageable, we add one further assumption, namely, that the expected real interest rate is constant over the relevant horizon. If the term structure is flat, then

$$\delta_{t+j} = \beta_t^j ,$$

where $\delta_t = (1+r_t)^{-1}$. This simplifies equation (38) to

$$(39) \quad p_t = \frac{d_t}{r_t} + \sum_{j=1}^{\infty} \beta_t^j (E_t d_{t+j} - d_t) .$$

After further simplification, equation (39) can be written in the form

$$(40) \quad p_t = \frac{d_t}{r_t} + \frac{1}{r_t} \sum_{i=1}^{\infty} \beta_t^i \, (E_t d_{t+i+1} - E_t d_{t+1}) .$$

where β_t is defined analogously to above.

At least under some assumptions, equation (40) is the basis for a cointegrating regression. Suppose that the expected one-period change in dividends is a constant, c, over the horizon. Then equation (40) becomes

$$(41) \quad p_t = \frac{d_t}{r_t} + \frac{c}{r_t^2} .$$

This particular assumption suggests regressing the stock price on the dividend divided by the interest rate and on the inverse of the square of the interest rate. Different assumptions about the dividend process would yield different cointegrating regressions.[23/]

This analysis suggests possible cointegrating regressions. Perhaps the simplest is the one suggested by equation (41). In the next section, we implement a first-order approximation of this equation by regressing the stock price on the dividend relative to the interest rate and on the inverse of the interest rate.[24/]

Interpretation in Terms of Nominal Interest Rates

Measuring an expected long-term real interest rate on a monthly basis is a daunting task. It is fortunate that all of the analysis of a variable real interest rate can be interpreted in terms of the nominal interest rate instead. If equation (23) is interpreted as an equation in nominal terms for all variables and equation (26) is assumed to hold in nominal terms, then all of the analysis goes through in terms of nominal variables and the nominal interest rate. In the final estimating equation, the price of stock and other variables measured in terms of current currency values can be placed in real terms by dividing by the price level. The major assumption that is different than in the previous section is an assumption of the existence of a stationary representation of the joint process of changes in nominal dividends and changes in the nominal interest rate. This is not the same as assuming that there is a stationary representation of changes in real dividends and changes in the real interest rate. Given the tenuousness of any estimate of the expected real interest rate on a monthly basis, we use nominal interest rates in the empirical analysis.

B. Tests for Cointegration

The analyses in the last section leading to cointegrating equations differ in terms of assumptions about the interest rate. In both analyses, it is assumed that real stock prices and real dividends have unit roots. This sets up the testable implication that stock prices and dividends are cointegrated. In the theoretical analysis, two different assumptions about the interest rate are used. The first is that the expected real interest rate is constant. This is the assumption used by Campbell and Shiller (1987), Diba and Grossman (1988) and Santoni and

Dwyer (1988). If the expected real interest rate is constant, a regression of the real stock price on the real dividend per share is a cointegrating regression relating the two: the residuals of this regression do not have a unit root. The second assumption about the interest rate loosens the restriction by allowing the real interest rate to vary over time, but the expected interest rate over the horizon is constant at any point in time. This suggests that a regression of the real stock price on the ratio of the real dividend to the interest rate and on the inverse of the interest rate yields residuals which do not have a unit root. These implications can be tested.

Empirical Results with International Data

We examine these hypotheses with monthly data for Canada, Germany, Japan, the United Kingdom and the United States for April 1973 through December 1987.[25/] The underlying data on stock prices are indexes from Capital International Perspectives with and without dividends reinvested.[26/] These indexes are measured at the end of the period. We calculate dividends from the relationship between the indexes with and without dividends reinvested.[27/]

The first issue is to test for the existence of unit roots in the stock price indexes and dividends per share. The test statistics for unit roots are presented in table 7. The table contains unit root tests for real stock prices, real dividends per share, the current yield (dividends per share over the stock price) and the long-term interest rate. The results for the current yield are presented for later reference. The table reports the estimated deviation of the first-order coefficient from 1 and the estimated t-ratio of this deviation from 1.[28/] Given the number of observations in the regressions, the t-ratios for rejecting the hypothesis of a unit root and their marginal significance levels are about -2.58 at the 10 percent significance level and -3.5 at the 1 percent marginal significance level [Fuller (1976), p. 373]. At the 5 percent marginal significance level, one reported test statistic is inconsistent with the null hypothesis of a unit root. Under the null hypothesis of a unit root, the test statistic for real dividends per share for Japan are smaller than would be expected. The finding of a unit root for stock prices and not for dividends per share in Japan is inconsistent with the hypothesis of cointegration of real stock prices and real dividends. If the real stock price has a unit root and real dividend per share does not, then the real dividend per share cannot possibly be related to the unit-root component of real stock prices.[29/]

The results of estimating cointegrating regressions for stock prices and dividends as well as the unit root tests on the residuals of these regressions are presented in table 8. For each country we present the slope coefficient from a regression of real stock prices on real dividends per share, this coefficient's ordinary least squares "t-statistic" and the regressions' R^2 and Durbin-Watson test statistics. Because there is no reason for the regressions to have serially uncorrelated residuals, the "t-statistics" produced by the regression program are not distributed as student's t. With some assistance from the Durbin-Watson though, these t-ratios provide some guidance as to whether there is any relationship between real stock prices and real dividends per share. If the t-ratio is small and substantial positive serial correlation is present in the residuals, then the evidence is clear that there is little relationship between real stock prices and real dividends per share.

The test statistic in the last column of table 8 is the estimated t-ratio from the Dickey-Fuller test for a unit root in the regression residuals. The maintained hypothesis in this test is that real stock prices and real dividends per share have unit roots. The null hypothesis tested is that real stock prices and real dividends per share are not cointegrated. Under the maintained hypotheses and this null hypothesis, the t-ratio in the unit root test of the residuals is zero. With the number of observations in the regressions in table 8, the null hypothesis that a t-ratio is zero is inconsistent with the data at the 5 percent marginal significance level if the t-ratio is less than or equal to -3.37 [Engle and Yoo (1987)]. If the t-ratio is zero, then the joint hypothesis that the fundamentals model is correct and the expected real interest rate is constant is inconsistent with the data. A small t-ratio (negative and large in magnitude) is consistent with this joint hypothesis.

There is hardly any evidence in table 8 that supports the hypothesis that real stock prices and real dividends per share are related. The relationships between real stock prices and real dividends per share in the cointegrating regressions are weak at best for Japan and the United States. From the theoretical analysis above, the estimated coefficients in the cointegrating regression should be positive and equal to one over the real interest rate.[30] A negative coefficient estimate implies a negative expected real interest rate, which implies that real stock prices or dividends should be negative. These are unappealing implications of the estimated negative coefficient for Japan. Even ignoring the serial correlation of the residuals, it is clear that the hypothesis that real stock prices are unrelated to real dividends per share cannot be rejected for Japan and the United States. The evidence from the Dickey-Fuller tests for unit roots also is unambiguous. Not a single t-ratio in the last column of table 8 approaches -3.37. There is no evidence in this table which lends any support to the fundamentals theory of stock prices.

Perhaps this result should not be surprising however. The tests for unit roots in table 7 indicate that the current yields of stocks have unit roots. The algebra which leads to the cointegrating regressions in table 8, however, suggests that current yields do not have a unit root. Indeed, this can be a different way of testing the same hypothesis. Note that equation (22) can be rewritten as

$$(42) \quad \frac{d_t}{p_t} = r - \sum_{i=1}^{\infty} \beta^i \, (E_t d_{t+i+1} - E_t d_{t+1})/p_t \; .$$

In this equation, if expected changes in future dividends relative to the current price are a stationary process, then the current yield does not have a unit root. Such a specification goes through for certain types of processes on the dividends and prices. For example, if the growth rate of dividends is a stationary process and the expected real interest rate is constant, then the right-hand side of equation (42) is stationary. It is clear in table 7 that current yields have a unit root. A unit root in current yields basically is inconsistent with the hypothesis that the dividend and the price are cointegrated.

Allowing for variable discount rates provides little help. Table 9 presents the results of the cointegrating regressions for real stock prices, real dividends per share and the long-term nominal interest rates. In general, these regressions appear to be more satisfactory than those in table 8. The R^2 of the regressions generally are nontrivial, a marked improvement over table 8. Even so, none of the coefficients indicates that the variables are cointegrated at the 5 percent significance level. The number of observations in these equations requires a t-ratio of about -3.85 to reject the hypothesis of a unit root in the residuals and, therefore, the hypothesis that stock prices are not cointegrated with dividends and bond yields. The tests for unit roots in the residuals provide no support for the hypothesis of cointegration.

The cointegration tests provide no support for the hypothesis that stock prices are determined by the fundamentals.[31/] These results may reflect many things, including the length of the sample or the loose relationship between the yields on long-term government bonds and the discount rate on stock. In this regard, it is worth noting that the test results with interest rates will be negative if the discount rate on stock is not cointegrated with the yield on government bonds. Put in other terms, risk premia on stocks with unit roots are consistent with the fundamentals models and with our results. It also could be that stock prices are just determined by whim and fancy. Before abandoning ourselves to this conclusion, however, we examine some simple regression suggested by the fundamentals.

C. Atheoretic Estimates of the Fundamentals Relationship

Tests for cointegration are one approach to analyzing the fundamentals model. Another way of examining the importance of the fundamentals is in terms of the importance of likely fundamental factors affecting changes in stock prices. To do this, we estimate straightforward regressions of changes in the logarithm of real stock prices on changes in the logarithm of the long-term interest rate and on a proxy for unexpected changes in real dividends. We assume that changes in the logarithms for real stock prices and changes in the logarithm of the long-term interest rates are approximately the same thing as the unexpected changes in each. We estimate the unexpected part of the change in dividends from a 12th-order autoregression for the change in the logarithm of real dividends.

The results of these regressions, reported in table 10, suggest some generalizations. Unexpected changes in the long-term interest rate do appear to be negatively related to changes in stock prices. This provides some modicum of support for the fundamental model. Dividends, on the other hand, appear to be quite unimportant in the regressions. This may reflect problems with the way dividends are measured. The most obvious problem is our complete ignorance of the timing of dividend announcements. There is evidence which suggests that announcements of dividends do affect share prices.[32/] The nonexistence of a statistically significant relationship between dividends and stock prices may reflect the poor measurement of unexpected dividends. In the last analysis, though, we are inclined to think that dividends are relatively unimportant in explaining monthly changes in stock prices.[33/]

The R^2 of these regressions also are not overwhelming. The lowest R^2 is .068 for Canada, and the highest is only .127 for the United Kingdom. While each is statistically significant, there is little doubt that much of the variance of stock prices is not directly related to our measures of long-term interest rates and dividends.

As a further examination of the importance of the fundamentals, changes in the logarithm of real stock prices are regressed on a set of macroeconomic variables that are available monthly and are likely to be important for stock prices if the fundamentals model is correct. In addition to the long-term interest rate in each country, proxies for the unexpected change in the logarithm of industrial production are included. For the non-U.S. estimates, possible influences of foreign developments are incorporated by including a proxy for the unexpected change in the logarithm of industrial production in the United States, the change in logarithm of the long-term interest rate in the United States and the change in the real exchange rate relative to the United States. The unexpected part of the change of industrial production is estimated by the residual from a 12th-order autoregression. We do not explore possible international links beyond those between the United States and each of these countries, nor do we explore links between the United States and the other countries. Such an exploration is beyond the limited scope of our analysis.

The results of these regressions are reported in table 11. These additional variables do not explain much of the variation of real stock prices. Because industrial production in the United States does little, the regression for the United States looks little different from its counterpart in table 10. The regressions for Germany, Japan and the United Kingdom also show little change from their counterparts in table 10. The regression for Canada shows the largest increase in its R^2, increasing from .068 to .186. This increase appears to be mostly related to the inclusion of changes in the real exchange rate, although industrial production does contribute some to the explanation of changes in real Canadian stock prices. That said, there appears to be some relationship between changes in stock prices and interest rates, but the fraction of variation in these indexes explained is hardly overwhelming.

V. CONCLUSION

The results of our analysis can be stated succinctly. The implications of the hypothesis of rational bubbles in stock prices find little support in the data. Our tests of this model are based on daily data taken for the year preceding the October 1987 crash across several countries. Although there are some individual results consistent with the notion of bubbles in stock prices, the weight of the evidence is against this theory. We also find little support for a hypothesis of a contagious bubble driving up stock prices worldwide. An alternative model, a version of the fundamentals, also is examined in a variety of ways using monthly observations that span the past 15 years for several countries. This model also has sparse support in the data.

Our finding that neither rational-bubble nor the fundamentals models adequately characterizes the behavior of stock prices in several countries raises some interesting questions, all of which form a direction for further research efforts. Have we adequately modeled both processes? While arguing that a model is found wanting because it is not properly specified can lead to vacuous circularity, the proper specification in testing the fundamentals model is crucial. After all, the econometrician is forced to use some measure of interest rates that may not be that used by the market in forming its valuation. Similarly, our test procedures may be unable to disentangle a rational bubble in the variance found in daily stock prices.

Our finding of no cointegration of stock prices and dividends is at odds with previous work by Campbell and Shiller (1987), Diba and Grossman (1988) and others. A natural question that arises concerns the robustness of test results to the time period. Their examinations rely on annual data spanning the past 100 years of U.S. financial history. Our analysis is based on only the past 15 years. Moreover, the periodicity of our data is monthly, not annual. It may be that stock prices and dividends are closely related, but only across a broad enough span of time. If this is so, an important issue is what this implies about the joint process of stock prices, dividends and interest rates.

FOOTNOTES

1/ The conclusions of the Brady Commission are the subject of several studies, all of which find little support for the conclusion or recommendations of the Commission. See Roll (1988) and Santoni (1988).

2/ See the data appendix for more detail on these data.

3/ Roll (1988) discusses the timing of the crashes.

4/ The peak dates used are Australia--9/21/87; Canada--8/13/87; France--3/26/87; Germany--8/17/87; Japan--6/11/87; United Kingdom--7/16/87; and United States--8/25/87.

5/ If the limit of the present value of expected dividends sufficiently far into the future were not zero, there would be no forward solution to equation (2) such as (3).

6/ An idea of the order of magnitude of a bubble's effect on the serial correlation of the price can be gained from Diba and Grossman's simulations (1988, pp. 527-28). Their simulations suggest that a rational bubble adds a component to the stock price which is highly positively serially correlated with a first order serial correlation coefficient that is typically 0.8 or above and which has a slowly-damped autocorrelation function.

7/ The critical values are taken from Fuller (1976, p. 373).

8/ Note that the models suggest that passing the peak value, not the date of a large downward adjustment some time later, is the correct date for the end of a rational bubble.

9/ All regression results reported are based on White's (1980) heteroskedasticity-consistent estimates. For the most part, ordinary least squares regressions yielded the same qualitative conclusions.

10/ It should be noted that the true peak of the German index in 1987 was in January. That is, from January 6 to the peak used in the regression analysis (August 17), the market actually fell and recovered slightly, although never regaining the highest 1987 value.

11/ We ignore the second-order term $h\Delta e$ in this discussion.

12/ Another interpretation of the test for unit roots in the relative stock prices is in terms of the cointegration of the stock price indexes with an imposed coefficient of unity in the cointegrating equation [Granger (1986); Engle and Granger (1987)].

13/ Tests also were run for changes in the relatives. The existence of second unit roots is inconsistent with the data.

14/ A variable does not have a unit root if none of the roots of the stochastic difference equation characterizing the variable equals one. Two variables are cointegrated (of order one) if each variable has a unit root and the residuals of the simple linear equation characterizing the relationship of the two variables does not have a unit root. Engle and Granger (1987) provide a more general definition and some explanatory discussion.

This implication for stock prices and dividends has been derived in somewhat different fashions by Campbell and Shiller (1987) and Diba and Grossman (1988).

15/ Under certain assumptions, the two terms on the right-hand-side of equation (22) are not independent. Nonetheless, the part orthogonal to d_t does not have a unit root. For example, suppose that d_t has a constant expected growth rate. Then the constant expected growth rate is impounded into the first term in the cointegrating regression.

16/ The projection of $r^{-1}d_t$ on d_t yields a coefficient of r^{-1}. Suppress any deterministic component of expected changes in dividends. Then the projection on d_t of the remainder of the equation, which does not have a unit root, yields a coefficient of zero. Because d_t has a unit root and therefore is nonstationary and since $E_t d_{t+i+1} - E_t d_{t+1}$ is stationary, the projection of $E_t d_{t+i+1} - E_t d_{t+1}$ on d_t must be zero. Any constant coefficient in the projection other than zero would imply that either both variables are stationary or both are nonstationary.

17/ Diba and Grossman (1988) present a related analysis.

18/ The root is not a unit root, because the root is $1 + r\pi^{-1}$. As a practical matter, the ability to discern this root in a finite sample depends on the proportion of periods with bubbles on, without bubbles and with crashes.

19/ The conditional expectations at t + j, j > 0, are random variables from the standpoint of a conditional expectation at t.

20/ The contemporaneous orthogonality of the ε's is a result of including ε_t^d in the equation for $\Delta\beta_t$. This representation imposes no restrictions on the data not imposed in the assumptions above. This simply is a representation, not the representation of the relationships. This particular one is convenient.

21/ The law of iterative expectations says that $E_t[E_{t+1}[d_{t+2}]] = E_t[d_{t+2}]$, because the information set at t is a subset of the information set at t + 1. By the assumptions we have made, namely that β_{t+1} and d_{t+1} are known at t + 1 but not t, the expectation at t + 1 is conditional on a proper superset of the information available at t.

22/ On another level, the variance term is a covariance term because it reflects the covariance of Δβ and Δd.

23/ For example, suppose that the expected growth rate of dividends is constant. This assumption is inconsistent with the hypothesis that changes in the level of dividends are strictly stationary. This assumption implies that the price of stock is a nonlinear function of the real interest rate and the dividend of the form:

$$p_t = \frac{1+g^d}{r_t - g^d} d_t .$$

This equation is nonlinear in the interest rate and the dividend and cannot be written as a linear function of a simple term such as the ratio of the two.

24/ We included the dividend in addition to these two variables in a set of regressions. This has no effect on the conclusion.

25/ In addition to these countries, we also collected data for France. In the process of constructing the data on dividends, we found substantial oddities in the behavior of dividends leading us to drop that country. Data availability also forced us to remove Australia from the sample.

26/ Capital International Perspectives has two series with dividends reinvested. The one which we use has gross dividends reinvested. The other, which we do not use, has dividends net of tax reinvested.

27/ Define x1 as the ratio of the index with dividends reinvested to its value last period and x2 as the ratio of the index without dividends reinvested to its value last period. The yield is x1/x2 - 1. We recover the dividend per share from this yield by multiplying by the level of the index without dividends reinvested.

In a few periods, the calculated yield and dividend are negative or are relatively large followed by a relatively small value (or vice versa) compared to surrounding values. Negative values make no sense, and large inverse changes in consecutive periods also suggest errors. As a result of these likely problems, a few observations are not used. We do not use yields for: West Germany, January and February 1974, May through August 1946; Japan, November and December 1985; and United Kingdom, November and December 1982. Capital International Perspectives is investigating these apparent problems with the data.

28/ For each variable for each country, each equation is estimated with from 0 to 2 lagged changes in the variable. If a t-ratio exceeds 1.95 in absolute value, the lag distribution includes that lag and any lesser lags. For real stock prices, all of the estimated equations include no lag except for the world index which includes one lag. For real dividends per share, all of the estimated equations include two lags except for the United Kingdom which includes one lag. For the yield on stocks, all of the equations include no lag except for Germany and the United Kingdom which include one lag. For the long-term interest rates, the equation for the Canadian rate includes no lag, the equations for Germany and Japan include one lag, and the equations for the United Kingdom and the United States include two lags.

29/ The estimates indicate that the real stock price index for Japan has a unit root with an estimated first-order autoregressive coefficient of 1.009, and the coefficient for real dividends per share is .918. Arguably, these are best interpreted as a preliminary indicator of a lack of cointegration of real stock prices and real dividends per share.

30/ As the analysis in Santoni and Dwyer (1988) indicates, even though this coefficient can be affected by the expected real growth rate of dividends per share, it still should be positive.

31/ It is obvious that a "souped-up" version of the model of the fundamentals above always can explain any observation. The same holds true for the rational bubbles model in the last section. This is the same as saying that, in the same sense that utility maximization is vacuous, both the fundamental theory and the bubble theory are vacuous. With an emphasis on the impossibility of refuting the fundamentals model, Hamilton and Whiteman (1985) discuss these points.

32/ Among others, see Litzenberger and Ranaswamy (1982).

33/ Indeed, recall that support for the fundamentals model often comes from tests based on long time series of annual data. See, for example, Diba and Grossman (1988).

REFERENCES

Blanchard, Olivier, and Mark Watson. "Bubbles, Rational Expectations, and Financial Markets," in P. Wachtel, ed., Crises in the Economic and Financial Structure (Lexington: Lexington Books, 1982).

Brady Commission. Report of the Presidential Task Force on Market Mechanisms, 1988.

Campbell, John, and Robert Shiller. "Cointegration and Tests of Present Value Models," Journal of Political Economy, 95 (October 1987), pp. 1062-88.

Diba, Behzad T., and Herschel I. Grossman. "Explosive Rational Bubbles in Stock Prices?" American Economic Review, 78 (June 1988), pp. 520-29.

Dickey, David A., and Wayne A. Fuller. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," Journal of the American Statistical Association (June 1979), pp. 427-31.

Dwyer, Gerald P., Jr., and R. W. Hafer. "Are National Stock Markets Linked?" Federal Reserve Bank of St. Louis Review (November/ December 1988), pp. 3-14.

______, and G. J. Santoni. "Tests for Rational Bubbles in the 1920s and 1980s Bull Markets," unpublished paper, Federal Reserve Bank of St. Louis, September 1988.

Engle, Robert F., and C. W. J. Granger. "Co-integration and Error Correction: Representation, Estimation, and Testing," Econometrica 55 (March 1987), pp. 251-76.

Flood, Robert P., and Peter M. Garber. "Market Fundamentals versus Price-Level Bubbles: The First Tests," Journal of Political Economy 88 (August 1980), pp. 745-70.

Fuller, Wayne A. Introduction to Statistical Time Series (New York: John Wiley & Sons, Inc., 1976).

Granger, C. W. J. "Developments in the Study of Cointegrated Economic Variables," <u>Oxford Bulletin of Economics and Statistics</u>, 48 (August 1986), pp. 213-28.

Hamilton, James D., and Charles H. Whiteman. "The Observable Implications of Self-Fulfilling Expectations," <u>Journal of Monetary Economics</u>, 16 (November 1985), pp. 353-73.

Hannan, E. J. <u>Multiple Time Series</u> (New York: John Wiley & Sons, Inc., 1970).

Hardouvelis, Gikas A. "Stock Market Bubbles Before the Crash of 1987: Japan, United States and Great Britain," unpublished paper, Federal Reserve Bank of New York, April 1988.

Litzenberger, Robert H., and Krishna Ramaswamy. "The Effects of Dividends on Common Stock: Tax Effects or Information Effects?" <u>Journal of Finance</u>, 37 (May 1982), pp. 429-43.

Roll, Richard W. "The International Crash of October 1987," in Robert Kamphuis, Roger Kormendi and J. W. Henry Watson, eds., <u>Black Monday and the Future of Financial Markets</u> (Chicago: Mid America Institute, 1988).

Santoni, G. J., and Gerald P. Dwyer, Jr. "Bubbles or Fundamentals: Some Evidence from the Great Bull Markets of 1924-29 and 1982-87," paper presented at Conference on Crashes and Panics in Historical Perspective, Salomon Brothers Center for the Study of Financial Institutions, New York University, October 19, 1988.

Sargent, Thomas. <u>Macroeconomic Theory</u> (New York: Academic Press, 1979).

White, Halbert. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," <u>Econometrica</u>, 48 (May 1980), pp. 817-38.

Table 1
Summary Statistics
Percentage Changes in Indexes

Country	7/29/86-Peak 1/	Periods Peak-10/12/87	10/26/87-12/31/87
		Own currency	
Australia	114.46%	-5.66%	-0.91%
Canada	47.99	-8.66	10.88
France	19.32	-14.46	-9.58
Germany	10.68	-7.92	-18.52
Japan	65.46	-5.95	-4.35
United Kingdom	59.73	-4.28	1.92
United States	41.59	-8.09	7.55
		U.S. dollars	
Australia	159.84%	-7.01%	1.22%
Canada	54.50	-6.97	11.87
France	34.87	-14.09	0.37
Germany	27.37	-6.21	-8.57
Japan	81.31	-6.40	11.76
United Kingdom	75.93	-3.02	12.73
United States	41.59	-8.09	7.55

1/ The dates of the peaks are: Australia--9/21/87; Canada--8/13/87; France--3/26/87; Germany--8/17/87; Japan--6/11/87; United Kingdom--7/16/87; and United States--8/25/87.

Table 2
Unit Root Tests on Stock Price Indexes 1/
Daily Data (in own currency)

Country	Sample 2/	T-ratio on lag variable: Levels	T-ratio on lag variable: Changes	Durbin-Watson: Levels	Durbin-Watson: Changes
Australia	8/4/86-9/21/87	1.162	-9.190	2.00	2.01
	1/2/87-9/21/87	0.887	-7.899	2.00	2.00
Canada	8/4/86-8/13/87	-0.094	-8.275	1.97	2.00
	1/2/87-8/13/87	-2.081	-6.473	1.95	2.00
France	8/4/86-3/26/87	-0.350	-7.264	2.02	1.99
	1/2/87-3/26/87	-0.439	-5.569	1.89	1.90
Germany	8/4/86-8/17/87	-1.486	-8.725	1.98	2.00
	1/2/87-8/17/87	-2.032	-6.561	1.97	1.99
Japan	8/4/86-6/11/87	0.208	-7.952	2.02	1.99
	1/2/87-6/11/87	-0.637	-6.258	2.01	1.98
United Kingdom	8/4/86-7/16/87	1.706	-8.843	1.98	1.99
	1/2/87-7/16/87	-0.091	-7.540	1.98	1.98
United States	8/4/86-8/25/87	0.010	-10.122	1.99	1.99
	1/2/87-8/25/87	-1.656	-7.337	1.97	1.97

1/ All regressions include a constant term and lagged dependent variable.

2/ Sample period endpoints represent peaks in stock price indexes.

Table 3
Time Regression Results
(own currency, percentage changes)
Sample Period: July 30, 1986-Peak 1/

Country	Constant	Time	(Time)	R^2	DW	X^2(MSL)
Australia	0.002	$2.696x10^{-6}$		0.001	1.79	0.217
	(2.32)	(0.46)				(0.642)
	0.004	$-2.353x10^{-5}$	$8.861x10^{-8}$	0.004	1.80	1.710
	(2.66)	(-1.09)	(1.24)			(0.425)
Canada	0.001	$3.223x10^{-6}$		0.001	1.42	0.373
	(1.20)	(0.61)				(0.541)
	0.001	$5.729x10^{-6}$	$-9.285x10^{-9}$	0.001	1.42	0.382
	(0.66)	(0.25)	(-0.12)			(0.826)
France	-0.0001	$1.448x10^{-5}$		0.006	1.82	1.106
	(-0.11)	(1.05)				(0.293)
	0.002	$-4.351x10^{-5}$	$3.391x10^{-7}$	0.012	1.83	2.754
	(0.80)	(-0.91)	(1.27)			(0.252)
Germany	$9.222x10^{-5}$	$2.642x10^{-6}$		0.0003	1.78	0.097
	(0.07)	(0.31)				(0.755)
	0.005	$-9.936x10^{-5}$	$3.750x10^{-7}$	0.028	1.84	8.337
	(2.16)	(-2.50)*	(2.74)*			(0.015)
Japan	0.001	$1.176x10^{-5}$		0.003	1.82	0.635
	(0.52)	(0.80)				(0.426)
	0.001	$9.648x10^{-6}$	$9.406x10^{-9}$	0.003	1.82	0.636
	(0.40)	(0.19)	(0.04)			(0.728)
United Kingdom	0.0003	$1.260x10^{-3}$		0.011	2.07	2.873
	(0.33)	(1.70)				(0.090)
	0.0004	$1.007x10^{-5}$	$1.010x10^{-8}$	0.011	2.07	2.984
	(0.28)	(0.35)	(0.10)			(0.225)
United States	0.001	$5.205x10^{-6}$		0.002	1.84	0.591
	(0.51)	(0.77)				(0.442)
	0.001	$-2.357x10^{-6}$	$2.720x10^{-8}$	0.002	1.84	0.795
	(0.53)	(-0.08)	(0.29)			(0.672)

1/ The dates of the peaks are: Australia--9/21/87; Canada--8/13/87; France--3/26/87; Germany--8/17/87; Japan--6/11/87; United Kingdom--7/16/87; and United States--8/25/87.

Table 4
Time Regression Results
(own currency, percentage changes)
Sample Period: January 2, 1987-Peak 1/

Country	Constant	Time	(Time)	R^2	DW	χ^2(MSL)
Australia	0.001	8.939×10^{-6}		0.003	1.83	0.582
	(1.47)	(0.76)				(0.446)
	0.012	-4.268×10^{-5}	2.775×10^{-7}	0.008	1.84	1.697
	(1.76)	(-0.81)	(1.01)			(0.428)
Canada	0.003	-1.722×10^{-5}		0.012	1.17	2.350
	(2.85)	(-1.53)				(0.125)
	0.007	-0.0002	8.337×10^{-7}	0.057	1.23	11.194
	(4.39)	(-3.30)*	(3.09)*			(0.004)
France	0.001	3.385×10^{-5}		0.004	1.77	0.206
	(0.44)	(0.45)				(0.650)
	0.004	-2.789×10^{-4}	5.126×10^{-6}	0.027	1.82	1.920
	(0.87)	(-0.79)	(0.99)			(0.383)
Germany	-0.005	5.635×10^{-5}		0.037	1.86	5.644
	(-1.76)	(2.38)*				(0.018)
	-0.007	1.291×10^{-4}	-4.491×10^{-7}	0.041	1.86	5.731
	(-1.63)	(1.34)	(-0.87)			(0.057)
Japan	0.004	-7.800×10^{-6}		0.0004	1.89	0.051
	(1.80)	(-0.22)				(0.822)
	0.004	-5.014×10^{-6}	-2.423×10^{-8}	0.0004	1.89	0.052
	(1.14)	(-0.04)	(-0.02)			(0.974)
United Kingdom	0.003	-2.074×10^{-6}		0.000	2.13	0.014
	(2.02)	(-0.12)				(0.905)
	0.030	-1.226×10^{-4}	8.612×10^{-7}	0.019	2.16	3.577
	(2.84)	(-1.80)	(1.89)			(0.167)
United States	0.003	-1.038×10^{-5}		0.003	1.88	0.578
	(1.94)	(-0.76)				(0.447)
	0.007	-1.584×10^{-4}	8.809×10^{-7}	0.040	1.96	6.433
	(3.38)	(-2.52)*	(-2.40)*			(0.040)

1/ The dates of the peaks are: Australia--9/21/87; Canada--8/13/87; France--3/26/87; Germany--8/17/87; Japan--6/11/87; United Kingdom--7/16/87; and United States--8/25/87.

Table 5
Autoregression Test Results 1/

Country	9/3/86–Peak		1/2/87–Peak	
	χ^2(MSL)	$\Sigma\beta_i$(MSL)	χ^2(MSL)	$\Sigma\beta_i$(MSL)
Australia	28.961	-0.098	31.874	0.069
	(0.27)	(0.75)	(0.16)	(0.85)
Canada	81.098	0.419	67.471	0.451
	(0.00)	(0.09)	(0.00)	(0.12)
France	28.182	-0.181	90.337	-1.187
	(0.30)	(0.64)	(0.00)	(0.20)
Germany	44.686	0.112	50.891	0.301
	(0.01)	(0.75)	(0.00)	(0.43)
Japan	24.001	-0.140	41.819	-1.108
	(0.52)	(0.72)	(0.02)	(0.08)
United Kingdom	30.946	0.112	29.848	-0.104
	(0.19)	(1.00)	(0.23)	(1.00)
United States	32.645	-0.190	39.070	-0.348
	(0.14)	(0.52)	(0.04)	(0.43)

1/ The dates of the peaks are: Australia--9/21/87; Canada--8/13/87; France--3/26/87; Germany--8/17/87; Japan--6/11/87; United Kingdom--7/16/87; and United States--8/25/87. Marginal significance levels in parentheses.

Table 6
Unit Root Tests on Relative Stock Prices: Levels
Daily Data (in U.S. dollars)

Period: 8/4/86-10/12/87 (N = 307)

Country	Australia	Canada	France	Germany	Japan	United Kingdom
Canada	-1.205					
France	-0.392	-0.869				
Germany	-0.724	-1.011	-1.260			
Japan	-1.620	-1.906	-0.693	-0.802		
United Kingdom	-2.216	-0.918	0.334	-0.411	-1.917	
United States	-1.252	-1.546	-1.173	-1.096	-1.494	-0.678

Period: 1/2/87-10/12/87 (N = 200)

Country	Australia	Canada	France	Germany	Japan	United Kingdom
Canada	-0.284					
France	-0.010	-1.395				
Germany	-1.801	-3.508*	-2.033			
Japan	-1.099	-1.618	-1.264	-2.845		
United Kingdom	-1.368	-1.208	-0.709	-2.937*	-1.406	
United States	-0.609	-1.916	-1.187	-4.163*	-1.499	-1.515

* Denotes significance at 5 percent level

Table 7
Unit Root Tests: Levels 1/
April 1973 to December 1987

Country	Real stock price	Real dividend per share	Current yield	Long-term interest rate
Canada	-.056	-.025	-.063	-.030
	(-2.51)	(-1.09)	(-2.40)	(-1.86)
Germany	-.020	-.097	-.039	-.019
	(-1.34)	(-2.28)	(-1.41)	(-1.75)
Japan	.009	-.082	-.002	-.016
	(0.98)	(-3.76)	(-0.15)	(-1.01)
United Kingdom	-.026	-.066	-0.082	-.036
	(-1.55)	(-2.81)	(-2.85)	(-1.99)
United States	-.049	-.071	-.043	-.022
	(-2.69)	(-1.80)	(-2.15)	(-1.62)

1/ Estimated t-ratios are in parentheses.

Table 8
Tests of Cointegration
Real Stock Prices and Real Dividends Per Share
April 1973 to December 1987

Country	Cointegrating regression: Estimated coefficient 1/	R^2	D-W	Dickey-Fuller tests: Estimated coefficient
Canada	158.02 (4.35)	.098	.114	-.063 (-2.46)
Germany	172.15 (2.88)	.047	.054	-.014 (-0.76)
Japan	-348.20 (-1.39)	.011	.016	.008 (0.87)
United Kingdom	363.43 (7.20)	.230	.100	-.044 (-1.81)
United States	45.49 (0.61)	.002	.061	-.045 (-2.43)

1/ Estimated t-ratios are in parentheses.

Table 9
Tests of Cointegration
Real Stock Prices, Real Dividends Per Share and Interest Rates
April 1973 to December 1987

	Cointegrating regression				Dickey-Fuller tests
	Estimated coefficients 1/				
Country	d/R	1/R	R^2	D-W	Estimated coefficient
Canada	17.03 (4.48)	-9.83 (-3.46)	.105	.116	-.062 (-2.43)
Germany	6.80 (1.88)	6.26 (4.23)	.408	.076	-.032 (-1.47)
Japan	10.98 (1.26)	40.40 (10.12)	.542	.114	-.017 (-0.64)
United Kingdom	10.46 (2.89)	34.61 (7.17)	.731	.246	-.115 (-3.07)
United States	-5.81 (-1.20)	11.32 (4.30)	.457	.119	-.065 (-2.53)

1/ Estimated t-ratios are in parentheses.

Table 10
Regressions of Changes in Real Stock Prices on Real Dividends Per Share and Interest Rates
April 1973 to December 1987

Country	Estimated coefficients 1/ Constant	Unexpected change in real dividends	Change in interest rate	R^2 / s_e	F	D-W
Canada	-.0001	-.0270	-.3864	.068	5.985	1.864
	(-0.030)	(-0.202)	(-3.441)	.0574		
Germany	.0014	.0942	-.3499	.071	5.057	1.822
	(0.353)	(1.401)	(-2.848)	.0475		
Japan	.0042	.1406	-.2545	.071	5.453	2.057
	(1.280)	(0.901)	(-3.183)	.0394		
United Kingdom	-.0012	-.2168	-.6568	.127	10.740	1.895
	(-0.204)	(-1.219)	(-4.473)	.0728		
United States	.0002	.0553	-.3561	.087	7.715	1.987
	(0.004)	(0.335)	(-3.862)	.0491		

Note: All variables are changes in the logarithms of the variables.

1/ Estimated t-ratios are in parentheses.

Table 11
Regressions of Changes in Real Stock Prices on
Interest Rates, Industrial Production and Exchange Rates
April 1973 to December 1987

	Estimated coefficients [1/]								
		Change in interest rate		Unexpected change in industrial production					
Country	Constant	Domestic	United States	Domestic	United States	Change in real exchange rate	R^2 se	F	D-W
Canada	.0016	-.0247	-.0068	.2975	-.0068	-1.0801	.186	7.590	1.874
	(0.414)	(-2.165)	(-0.550)	(1.010)	(-0.550)	(-3.427)	.051		
Germany	-.0005	-.0451	.1647	.0613	-.3285	.1647	.089	3.348	1.852
	(-0.130)	(-2.742)	(1.480)	(1.480)	(-0.655)	(1.480)	.051		
Japan	.0042	-.0457	.0032	.2355	-.0553	-.0651	.096	3.518	1.926
	(1.204)	(-3.589)	(0.369)	(0.806)	(-0.125)	(-0.575)	.045		
United Kingdom	-.0008	-.0586	.0002	-.0852	0.232	-0.1127	.158	6.416	2.031
	(-0.162)	(-5.471)	(0.019)	(-0.258)	(0.336)	(-0.678)	.068		
United States	-.0012	-.0326		0.1301			.078	7.378	2.003
	(-0.332)	(-3.827)		(0.273)			.049		

1/ Estimated t-ratios are in parentheses.

DATA APPENDIX

Stock Prices

The stock price indexes used are generated from "Morgan Stanley Capital International Perspective," edited by Capital International Perspective S. A. in Geneva, Switzerland, and published by Morgan Stanley & Co., Inc. in New York. For each index, the sample of companies included represents an attempt to match the industry profile of the country. Each index is a market weighted concept. Indexes measured in terms of U.S. dollars adjust for fluctuations in the local currency relative to the dollar. The indexes with dividends, available only on a monthly basis, consist of the price index with dividends reinvested. Daily index values are available beginning July 29, 1986; monthly values begin December 1969.

Prices

The price data are series 64 in International Financial Statistics.

Canada: Data for all cities with a population of over 30,000. Weights correspond to family expenditure patterns of 1982, base 1981.

Germany: Cost of living index for all households, base 1980. Index covers 753 items in 118 municipalities with 1980 weights.

Japan: Index covers whole country excluding one-person households and those engaged mainly in agriculture, forestry and fishing. Base is 1985.

United Kingdom: General index of retail prices, all items, base January 1987. The weights are revised annually on the basis of a continuing family expenditure survey.

United States: Index covers all urban consumers representing about 80 percent of the non-institutional population. Beginning January 1983, cost of shelter to homeowners is measured by a rental equivalence. From January 1987, an enhanced housing survey represents more adequately both owners and renters in estimating cost of shelter. Weights based on Consumer Expenditure Survey, carried out from 1982 through 1984.

Long-term Interest Rates

These data are series 61 in International Financial Statistics.

Canada: Average yield to maturity of issues with original maturity of 10 years or more.

Germany: Calculated as the weighted average of all bonds with an average remaining life to maturity of more than three years, or four years for bonds included before January 1977. Monthly figures are calculated as averages of four bank week return dates including the end-of-month yield of the preceding month.

Japan: Arithmetic average yield to maturity of all government bonds with seven years to maturity. Monthly series are compiled from end-of-month prices quoted on the Tokyo stock exchange.

United Kingdom: Issue at par with 20 years to maturity. Data for 1972 onwards are averages of Wednesday yields.

United States: Data based on 10-year constant maturities.

Industrial Production

These data are series 66 in International Financial Statistics.

Canada: Beginning 1984, data refer to industrial production in constant prices, base year 1981. Industrial production includes services related to mineral extraction as part of the mining sector.

Germany: Index including construction, base 1980.

Japan: Series covers 9 mining and 523 manufacturing industries weighted by 1980 value-added data.

United Kingdom: Base 1980, series covers manufacturing industries, energy and water.

United States: Data are an index produced with value-added weights.

Exchange Rates

Series used is ag (end-of-period) in International Financial Statistics.

Canada: Midpoint rate quoted by the Bank of Canada at noon in the Montreal-Toronto interbank exchange market.

Germany: Midpoint rate determined officially during the official session of the Frankfurt foreign exchange market.

Japan: Midpoint rate in the interbank foreign exchange market in Tokyo.

United Kingdom: Midpoint rate at noon in the London market.