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Abstract: Shleifer and Vishny (1997) pointed out some of the practical and theoretical problems associated with assuming that rational risk-arbitrage would quickly drive asset prices back to long-run equilibrium. In particular, they showed that the possibility that asset price disequilibrium would worsen, before being corrected, tends to limit rational speculators. Uniquely, Shleifer and Vishny (1997) showed that "performance-based asset management" would tend to reduce risk-arbitrage when it is needed most, when asset prices are furthest from equilibrium. We analyze a generalized Shleifer and Vishny (1997) model for central bank intervention. We show that increasing availability of arbitrage capital has a pronounced effect on the dynamic intervention strategy of the central bank. Intervention is reduced during periods of moderate misalignment and amplified at times of extreme misalignment. This pattern is consistent with empirical observation.

Keywords: Intervention, foreign exchange, limits to arbitrage, arbitrage, noise trader

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The behavior of foreign exchange rates has puzzled economists since the breakdown of Bretton Woods in 1973. First, exchange rates do not covary with interest rate differentials in any explicable way in the short- and medium-term (Hodrick (1987), Engel (1996) and Meredith and Chinn (1998)). Second, a large literature has documented successful trend-following technical trading rules in foreign exchange markets (e.g., Sweeney (1986), Neely, Weller and Dittmar (1997)). Third, foreign exchange rates are only weakly connected to fundamentals over long horizons. (Meese and Rogoff (1983), Kilian (1999), Engel (2000), Mark and Sul (2001) and Rapach and Wohar (2002), Neely and Sarno (2002). One might interpret the evidence to indicate that exchange rates are connected to fundamentals in the long-run and/or under extreme conditions, but that exchange rates can deviate substantially from their fundamental values for significant periods. This misalignment of exchange rates presents a puzzle: Why is there apparently insufficient risk-arbitrage to keep foreign exchange rates in line with fundamentals?

Some researchers have sought to create general equilibrium models in which exchange rates seem to be disconnected from fundamentals, e.g. Duarte and Stockman (2002). Such models are not yet wholly convincing; they cannot explain the behavior of risk premia or variation in exchange rates. Models containing features such as noise trading and/or limits to arbitrage are also widely used (e.g., Devereux and Engel (2002), Duarte and Stockman (2001)).

Other researchers have turned to bounded rationality or behaviorally based departures from rationality to generate the apparently volatile expectations of exchange rates. For example, Frankel (1996) argues that exchange rates are detached from fundamentals by swings in expectations about future values of the exchange rate. Four pieces of evidence suggest that overly volatile expectations are to blame for such behavior: 1) Survey measures of exchange rate expectations are very poor forecasts and are often not internally consistent (Frankel and Froot, 1987, Sarno and Taylor 2001); 2) the failure of uncovered interest parity (UIP) seems to hinge on irrational expectations (Engel, 1996); 3) Trend-following trading rules make risk-adjusted excess returns (Neely, 1997; Neely, Weller, and Dittmar, 1997); 4) Switching from a fixed to a floating exchange rate changes the volatility of real exchange rates and the ability of UIP to explain exchange rate changes (Mussa, 1986).

The volatile expectations of an apparently economically significant group of agents have created misalignments—predictable long-term returns—that are potentially exploitable. Monetary authorities have invested in foreign exchange in anticipation of such long-run reversion to fundamentals. Neely (2005) summarizes evidence that major central banks, those of the United States, Germany, Japan, Switzerland and Australia, have made excess returns–returns on a zero investment strategy–on their foreign exchange intervention by "buying-low and selling-high" (Leahy (1995), Neely (1998), Sweeney (1997), Sjöö and Sweeney (2001)). These predictable long-term returns are associated with deviations from PPP fundamentals.

There are doubtless private agents with realistic expectations who similarly profit from fundamental-based investments. But these agents—coupled with central banks—do not seem to have enough market power to prevent persistent and large departures from fundamentals. Why is there not more private risk arbitrage?

A growing literature argues that various limits to arbitrage reduce the speed with which rational speculators can push rates back to fundamentals. Noise trading in the presence of fundamental risk makes arbitrage risky (De Long et al., 1990). Shleifer and Vishny (1997) (hereafter SV) explored how traders are constrained by risk and principal-agent problems. In particular, SV argue that arbitrage is limited in asset markets because the marginal investor in asset markets is a highly specialized agent who loses resources precisely when asset prices diverge far from fundamental values. Specifically, the SV argument depends on the existence of performance-based arbitrage (PBA). PBA means that the capital available to arbitrageurs depends on the recent returns to their portfolios. When foreign exchange rates diverge further from fundamentals, fundamental-based traders (FBTs) lose money. As the divergence worsens, so does the performance of FBTs, both directly because of their losses and because principals provide less capital. The loss of capital means that risk-arbitrage investment in anticipation of a return to fundamentals fails precisely as divergence from fundamentals worsens, leading to greater divergence.

Thus, although risky arbitrage will certainly exert a force in the long run to correct deviations from fundamentals, this force is attenuated in the short run. The weakness of risky arbitrage will lead to potentially significant misallocation of resources as trade and investment decisions are made on the basis of distorted price signals.

The limits-to-arbitrage argument is especially relevant for foreign exchange, where there is a great deal of uncertainty about fundamentals and very long horizons for mean reversion. While such uncertainty surely exacerbates the PBA that stems from principal-agent problems, monetary authorities are likely to be less affected by the forces that drive PBA. Central banks have proved that they are willing to take long-term positions on reversion to fundamentals and they have a vested interest in reducing misalignments with sterilized intervention.[1]

This paper seeks to answer the following: How does the presence of capital dedicated to risk arbitrage influence the intervention strategy of a central bank facing fundamental uncertainty? To this end, we extend the SV model to include a central bank and fundamental uncertainty in addition to rational arbitrageurs.

[1] Krugman and Miller (1993) make a related argument in showing how central bank intervention can stabilize exchange rates if speculators are subject to rules that require them to limit their maximum "drawdown".

Much of the theoretical debate about the impact of intervention has focused on informational issues. Our analysis complements the existing literature by looking at the strategic interaction between arbitrageurs and the central bank in an environment in which the bank has a longer time horizon and is not subject to the same short-term pressures that limit the actions of professional arbitrageurs.

We examine optimal intervention policy in a model which extends that of SV by introducing a central bank who plays the role of a Stackelberg leader in an intervention game. The bank's objective function values both trading profitability and stabilization of the exchange rate around its fundamental value. We are particularly interested in characteristics of intervention strategy in the situation where the effects of arbitrage are weakened. The presence of arbitrageurs in the market makes a difference to the optimal intervention policy. If the central bank intervenes aggressively during periods of moderate misalignment, but when there is a possibility that the misalignment will get worse, it risks weakening the synergistic effect of risk arbitrage. This happens because the intervention increases the short run losses of arbitrageurs and reduces the funds they have available to bear against misalignment in the future. Using the same logic, we identify a "high-powered" intervention effect when deviations from fundamentals are unusually large. We show that the combination of these two effects has a substantial impact on the dynamic intervention strategy of the central bank.

2. The Limits to Arbitrage Literature

The limits-to-arbitrage concept has significantly influenced thinking on asset pricing. Indeed, Barberis and Thaler (2002) define limits to arbitrage as one of the building blocks of behavioral finance.

While SV made a generic argument about the limits to arbitrage in general asset markets others have used the concept to explain or rationalize behavior in specific contexts. Collins, Gong and Hribar (2003) argue that limits to arbitrage prevent institutional investors from exploiting the apparently abnormal returns enjoyed by firms with low institutional ownership. Brunnermeier and Nagel (2004) find that rational investors (hedge funds) captured much of the upswing in technology stocks but were able to reduce their exposure before the crash. They argue that limits to arbitrage might contribute to the preference for rational investors to ride bubbles and destabilize prices. Brav, Heaton and Rosenberg (2004) criticize both rational and behavioral finance for a lack of testable predictions. But they argue that ex post explanations support the limits of arbitrage arguments on which behavioral finance relies. Gabaix, Krishnamurthy, and Vigneron (2005) examine the mortgage-backed securities (MBS) market through the lens of limits to arbitrage theory. They find that the pricing of homeowner prepayment risk is consistent with the specialized arbitrageur hypothesis. Massa, Peyer, and Tong (2005) examine the equity performance and investment in the 2 years after a firm is added to an index. They find behavior that supports a limits to arbitrage theory. McMillan (2005) compares nonlinear dynamics in asset returns from European to Asian markets, concluding that limits to arbitrage are greater in Asian markets. Stein (2005) uses the limits-to-arbitrage concept to explain the existence of open-end funds and the coexistence of large mispricings with rational, competitive arbitrageurs. Greenwood (2005) develops testable predictions from a limits-to-arbitrage framework with multiple risky assets; Nikkei 225 data support these predictions. Jiang, Lee, and Zhang (2005) examine the relationship between uncertainty in estimates of firm value and equilibrium returns. Their findings are consistent with models that incorporate limits to rational arbitrage.

Not all papers find support for limits-to-arbitrage models. Gallagher and Taylor (2001), for example, evaluate a risky arbitrage hypothesis versus a limits-to-arbitrage hypothesis in U.S. equity markets. They find that the speed of mean reversion favors the risky arbitrage hypothesis.

3. The Model

We extend the model developed by Shleifer and Vishny (1997) in the following ways. We apply the model specifically to the foreign exchange market, introduce a central bank in addition to arbitrageurs and also permit uncertainty about fundamentals. For concreteness we assume that the problem involves U.S. based arbitrageurs, noise traders, a foreign central bank, and that the foreign currency is the euro. There are three time periods. The expected fundamental value of the euro (the exchange rate measured as the dollar price of the euro) at time t is V_t . It becomes known either at time 2 or at time 3. It can take on two values, V_H and V_L with equal probability. Its expected value is $\overline{V}$. The exchange rate at time t, t =1, 2 is p_t . In each of the first two periods noise traders experience a pessimism shock. At time 1, the first period shock S_1 is known to arbitrageurs and to the central bank, but the second period shock, S_2, is not. The shocks affect noise trader demand for the euro at time t, Q_t^N . (Note that the cumulative second period shock is the sum of the first period shock and the increment, S_2, to the first shock.)

$$Q_1^N = \frac{V_1 - S_1}{p_1}; \qquad Q_2^N = \frac{V_2 - S_1 - S_2}{p_2}. \tag{1}$$

At time 2 the exchange rate either returns to fundamental value and all uncertainty is resolved or there is a further pessimism shock which occurs with probability q. Thus, $V_1 = \overline{V}$ and

$$V_2 = \begin{cases} \overline{V} \text{ with probability } q \\ V_H \text{ with probability } 0.5(1\text{-}q) \\ V_L \text{ with probability } 0.5(1\text{-}q) \end{cases} \qquad S_2 = \begin{cases} S > 0 \text{ with probability } q \\ -S_1 \text{ with probability } (1\text{-}q) \end{cases}$$

Each period risk neutral arbitrageurs have a maximum amount of dollars F_t that they can invest. F_1 is assumed to be given and F_2 is determined endogenously, through performance in the first period. If the exchange rate does not return to fundamental value in period 2, arbitrageurs want to invest all their resources in the euro since $p_2 < \overline{V}$ in equilibrium. So total cumulative demand for the euro by arbitrageurs in this case is F_2 / p_2. The central bank may intervene at time 1 or time 2. Intervention at time *t* is denoted I_t. Assuming unit supply of the euro, the exchange rate at times $t = 1,2$ is given by:

$$p_1 = V_1 - S_1 + D_1 + I_1 \tag{2}$$

$$p_2 = V_2 - S_1 - S_2 + D_1 + D_2 + I_1 + I_2,$$

where D_i is incremental arbitrageur demand in period *i*, measured in dollars. Again, note that I_2 is the incremental intervention in period 2. We allow intervention at time 2 to depend on the state, so the level of intervention will depend on the realization of the noise shock. In period 1 arbitrageurs may choose not to invest all their resources in the euro, but rather to hold some funds back in case the currency becomes even more undervalued in the future. Thus $D_1 \le F_1$.

The model captures two sources of uncertainty in a simple way: First, there is uncertainty about the speed with which the exchange rate will revert to fundamental value. It may take either two or three periods. Second, there is uncertainty about what the true fundamental value is.

We examine first the decisions of arbitrageurs. We assume that they maximize their expected terminal wealth, which is equivalent to maximizing the expected dollar value of the currency portfolio under suitable conditions. Performance-based arbitrage dictates that the supply of funds at time 2 depends on results from period 1. Investors place funds with arbitrageurs, but lack a detailed understanding of the strategy followed by the arbitrageurs. They evaluate the skill of the arbitrageur simply by observing results. Profitable portfolio managers attract additional funds while loss-making managers experience an outflow of funds. This outflow comes about as a consequence of the fact that some investors infer that the loss-making managers are of lower ability than they had previously thought.

We follow SV in assuming a linear specification for the function relating gross return and fund inflows or outflows. The expression for such incremental fund flows in period 2 is given by the sum of the performance based measure and the funds that were not invested at all in period 1 ($F_1 - D_1$).

$$D_2 = -aD_1(1 - p_2/p_1) + F_1 - D_1 \qquad a \geq 1. \tag{3}$$

If $a = 1$ then the arbitrageurs' gains and losses simply reflect changes in portfolio value. But if $a > 1$ then there is a multiplicative effect of dollar gains and losses. A given dollar loss causes investors to withdraw funds and conversely a dollar gain generates a cash inflow. Incremental demand from arbitrageurs at time 2 is then composed of any funds not invested at time 1, $F_1 - D_1$, together with the fund flows described in (3). So the arbitrageurs' total dollar demand for the euro at time 2 is:

$$F_2 = D_1 + D_2 = D_1 + F_1 - D_1 - aD_1(1 - p_2/p_1) = F_1 - aD_1(1 - p_2/p_1). \tag{4}$$

This specification is unaffected by the presence of the central bank because it describes the allocation of capital based on performance, although in equilibrium exchange rates and so arbitrageur resources will be affected by intervention.

If $S_2 = -S_1$, the exchange rate returns to its fundamental value, arbitrageurs close out their positions and at time 1 their expected wealth $E[W]$ is

$$E[W] = F_1 - aD_1(1 - \overline{V}/p_1). \tag{5}$$

If $S_2 = S > 0$, then $E[W] = (F_2/p_2)\overline{V}$ or (using the expression in (4) for F_2)

$$E[W] = (\bar{V}/p_2)(F_1 - aD_1(1 - p_2/p_1)). \tag{6}$$

Since we have assumed that the probability that $S_2 = -S_1$ is $1-q$, the arbitrageur's problem in period 1 is:

$$\max_{D_1} (1-q)(F_1 - aD_1(1 - \bar{V}/p_1)) + q(\bar{V}/p_2)(F_1 - aD_1(1 - p_2/p_1)) \text{ subject to } D_1 \le F_1$$

and the first order condition is given by:

$$(1-q)(\bar{V} - p_1) + q\bar{V}(1 - p_1/p_2) \ge 0, \tag{7}$$

where the inequality $D_1 \le F_1$ holds with complementary slackness.

Next we consider the objective of the central bank. We assume that it wishes to stabilize the exchange rate subject to an expected (shadow) loss constraint. The bank is itself uncertain about the true fundamental value of the currency but is aware of the possible impact of noise traders and arbitrageurs. Like the arbitrageurs, it observes the noise shock S_1 but does not know whether in period 2 the exchange rate will be driven further from its expected fundamental value by a deepening surge of pessimism. In addition it does not know whether the true fundamental value revealed in periods 2 or 3 will be V_H or V_L.

In contrast to another strand of the literature on intervention which focuses on the effect of targeting a value for the exchange rate different from fundamental value, we suppose that the bank's objective is to bring about full allocative efficiency. In other words, its target for the exchange rate corresponds to its best estimate of true fundamental value. However, we introduce a cost of intervention that is related to uncertainty about fundamentals and the possible losses the bank may incur. Since the shadow cost of foreign exchange is $\bar{V}$ the expected cost of intervention is

$$(0.5(V_H - \bar{V}) + 0.5(V_L - \bar{V}))(I_1 + qI_2), \tag{8}$$

which is zero. The probability q multiplying the level of intervention in period 2 reflects the fact that intervention is state contingent. In the event that the noise shock deepens, the level of intervention is I_2. If on the other hand the shock disappears (with probability $(1 - q)$), there is no intervention. Thus a risk neutral central bank facing no other costs of intervention would choose perfect stabilization at expected fundamental value. Since there is good reason to believe that banks worry about losses on intervention, we capture this by assuming that the bank attaches a higher weight to losses (π_L) than to gains (π_H). The expected cost of intervention is as follows:

$$\pi_H\left(V_H - \overline{V}\right) + \pi_L\left(V_L - \overline{V}\right)\left(I_1 + qI_2\right) \qquad \pi_H = 1 - \pi_L < 0.5\,. \tag{9}$$

One can interpret $\pi_L - \pi_H \geq 0$ as an index of the risk aversion of the central bank. It is a measure of the degree to which losses are given greater weight than gains. Introducing the notation $\Delta V \equiv V_H - \overline{V} = \overline{V} - V_L > 0$,the central bank then faces the optimization problem:

$$\min_{I_1, I_2} \frac{1}{2}\left[\left(\overline{V} - p_1\right)^2 + q\left(\overline{V} - p_2\right)^2\right] - k\left[\left(\pi_H - \pi_L\right)\left(\left(I_1 + qI_2\right)\Delta V\right)\right] \ , \tag{10}$$

where $k > 0$ denotes the relative weight that the central bank places on profitability versus stabilization. The first term in the objective function represents the expected benefit from stabilization. Since p_1 is not a random variable, although it will depend on the level of intervention at time 1, it occurs without a probability weight. The value of p_2 is to be interpreted as that which occurs conditional on $S_2 = S > 0$, i.e. pessimism increases. In the event that the exchange rate returns to fundamental value at time 2, at time 1 its expectation is $\overline{V}$ and the term disappears from the objective function. So although the central bank does not know what value p_2 will take at the time that it formulates its optimal intervention strategy, it can compute its value conditional on an additional noise shock as a function of the level of intervention at time 2. This allows it to formulate an optimal state-contingent intervention strategy.

The second term in the objective function captures the cost of intervention. It depends both on the central bank's subjective attitude to losses as measured by $\pi_H - \pi_L$ and on its objective uncertainty about fundamentals, measured by ΔV. An alternative approach to modeling the costs of intervention would be to use realized rather than shadow profits and losses. However, in the present framework this has the rather unappealing implication that the central bank—even if it were risk neutral—would refrain from perfect stabilization at expected fundamental value in order to generate an expected profit from the market. In other words it would choose to exploit its market power to destabilize the market to generate profits.

4. Equilibrium and Comparative Statics

We analyze an equilibrium in which arbitrageurs are price takers and the central bank follows an optimal dynamic intervention strategy that takes into account the impact of its intervention on the trading activity of arbitrageurs. We first present two special cases that are helpful in developing an intuitive understanding of the solution to the model. We assume that arbitrageurs have invested up to the limit of the funds available, i.e. $D_1 = F_1$, a situation termed "extreme circumstances" by Shleifer and Vishny.

In the first case we look at the marginal effect of intervention at time 1 on the exchange rate at time 2, holding intervention at time 2 fixed. Then we can show that:

$$\frac{dp_2}{dI_1} = -\frac{aF_1 p_2}{p_1\left(p_1 - aF_1\right)} < 0, \tag{11}$$

given the stability condition $p_1 > aF_1$.[2] So increased intervention at time 1 actually worsens things—i.e., causes prices to deviate further from fundamentals—at time 2. The reason is that reducing the deviation of exchange rate from fundamental at time 1 increases the arbitrageurs'

[2] If this condition does not hold then arbitrageurs are better off not trading. See Shleifer and Vishny (1997, p. 46).

losses at time 2. The resulting fund outflows mean that the arbitrageurs are less able to correct the second period shock. We call this situation *destabilizing intervention.*

In the second case we consider the effects of intervention at time 2, conditional on a deepening of the noise shock, and holding intervention at time 1 fixed. Then we find that:

$$\frac{dp_2}{dI_2} = \frac{p_1}{p_1 - aF_1} > 1, \tag{12}$$

We get a "multiplier" effect on the exchange rate since there is both the direct effect and an indirect effect coming from the fact that now arbitrageurs lose less in the second period. We call this situation *high-powered intervention.*

These effects illustrate the fact that there is an important interplay between the strategy of the central bank and the trades of the arbitrageurs. At the margin, intervention is more effective the further the exchange rate has been driven from its fundamental value. These observations lead us to pose the following question: If the central bank pursues an optimal dynamic intervention strategy, what is the effect on this strategy of an increase in the availability of arbitrage capital? This leads us to state the following:

Proposition Let I_1 be intervention at time 1, and I_2 be intervention at time 2 conditional on a deepening of the noise shock ($S_2 = S > 0$). Assume that arbitrageurs are fully invested at $t = 1$. Then optimal intervention adjusts to an increase in arbitrage capital (at $F_1 = 0$) as follows:

$$\frac{dI_1}{dF_1} = -1 - q\left(\frac{ap_2}{p_1^2}\left(\overline{V} - p_2\right)\right) < -1$$

$$\frac{dI_2}{dF_1} = a\left(\frac{\left(\overline{V} - p_2\right) - \left(p_2 - p_1\right)}{p_1}\right) + q\left(\frac{ap_2}{p_1^2}\left(\overline{V} - p_2\right)\right) > 0. \tag{13}$$

Proof: See Appendix

The interpretation of this result is consistent with the intuition developed in the two special cases (11) and (12). The effect of introducing a "small" amount of arbitrage capital leads the central bank to reduce its intervention more than dollar for dollar in the first period, and to increase its intervention in the second period contingent on an increase in noise trader pessimism. Thus the presence of arbitrage capital leads to a significant asymmetry in the optimal intervention strategy of the bank. It intervenes less aggressively than before when the deviation of the exchange rate from fundamental value is moderate, and more aggressively when the deviation is extreme. The explanation lies in the effect that the presence of performance-based arbitrage has on efficient pricing. Larger noise shocks force arbitrageurs to partially liquidate their positions. Their ability to bear against mispricing is reduced precisely when the mispricing is most extreme. This comes about because arbitrageurs report interim losses which cause investors to withdraw funds. Central bank intervention in period 1 can actually increase period 2 losses by keeping the period 1 exchange rate relatively close to fundamentals, leading to greater changes in the exchange rate from period 1 to period 2, which result in larger losses for the arbitrageurs. The central bank finds it advantageous to reduce these losses by scaling back intervention in period 1, thus increasing the amount of arbitrage capital available to stabilize the exchange rate in period 2.

5. Numerical Results

Next we show that this qualitative property of equilibrium, which we have shown to hold when the quantity of arbitrage capital is "small", is more generally true. We present a numerical example using the following parameter values: $\bar{V} = 1$, $S_1 = 0.3$, $S_2 = 0.1$, $q = 0.2$,

$\pi_L - \pi_H = 0.2$, $\Delta V = 0.25$, $k = 0.1$. Arbitrageurs always choose to be fully invested at $t = 1$ with these parameter values. In Figure 1 we fix *a* = 1.2, the parameter that governs the rate at which funds are withdrawn when arbitrageurs make losses. For small values of F_1 the central bank intervenes more aggressively in period 1 than in period 2. But as F_1 increases the relationship eventually reverses, despite the fact that the incremental noise shock in period 2 is considerably smaller than that in period 1.

In Figure 2 we fix $F_1 = 0.2$ and allow *a* to vary. Increasing *a* shifts intervention from period 1 to period 2, as our earlier discussion would lead us to expect. That is, as PBA becomes more important, the central bank intervenes less aggressively in period 1 – to reduce period 2 losses to arbitrageurs – and more aggressively in period 2. Note that the level of intervention in period 2 shown in both figures is contingent on an increase in the pessimism of noise traders. If the exchange rate reverts to fundamental value at time 2 as a result of the early dissipation of market pessimism, the central bank is able to close out its intervention position, so that $I_2 = -I_1$.

This tendency of the central bank to intervene less when the importance of PBA increases (*a* rises) explains why it is not optimal for the central bank to simply fully offset the noise shock in the first period. As explained in the previous section, intervention in period 1 will increase period 2 losses by keeping the period 1 exchange rate relatively close to fundamentals, leading to greater changes in the exchange rate from period 1 to period 2, which result in larger losses for the arbitrageurs.

Figure 3 illustrates the effect of simultaneous variation in F_1 and *a*. Not surprisingly we see that the effect of varying *a* becomes more pronounced as F_1 increases. That is, larger resources for arbitrageurs magnify the PBA effect on intervention.

Our results provide a potential explanation for the fact that the frequency of intervention has tended to decline over time, but the magnitude of interventions has increased.[3] As the quantity of arbitrage capital devoted to currency trading has risen, so we would predict that intervention when exchange rates deviate only to a moderate degree from fundamentals would decline, whereas periods of extreme divergence would attract an increased volume of intervention.

Finally, we can examine how the results would change if the central bank's risk aversion ($\pi_L - \pi_H$) or the uncertainty about fundamentals (ΔV) increases. The value of k, the weight in the central bank's optimization function graphs subsumes both variables: An increase in k is equivalent to an increase in risk aversion or an increase in fundamental uncertainty. So, we can examine how intervention in both periods changes as a function of k. Figure 4 duplicates Figure 1, but for 3 different values of k, k = 0.1, 0.7, and 1.3. As k increases, intervention in both periods tends to decline, but intervention in period 1 declines faster. At the same time, the sensitivity of intervention in period 2 to F_1 increases with the value of k.

6. Discussion and Conclusion

The behavior of the foreign exchange market has puzzled economists since the inception of floating exchange rates in 1973. In particular, exchange rates appear to have shown persistent misalignments with fundamentals. While volatile expectations seem to be a plausible theory as to why exchange rates should deviate from long-run fundamentals, it is less clear why there is not more risk-arbitrage on a return to long-term fundamentals. Shleifer and Vishny (1997)

[3] For example, the U.S. authorities intervened 281 times from January 1, 1986 to January 1, 1996, but only twice since January 1, 1996. At the same time, the magnitude of a given intervention in the latter period was almost 5 times larger than an intervention in the former period. Similarly, the German authorities did not intervene from 1995 to the transition to the European Central Bank (ECB) in 1999. And the ECB has reportedly intervened sparingly since its inception.

provide at least a partial answer by pointing out some problems associated with assuming that rational risk-arbitrage will quickly drive asset prices back to long-run equilibrium. In particular, they show that "performance-based asset management" may generate situations in which asset price disequilibrium will worsen, before being corrected. This in turn limits the effect of rational risk-arbitrage.

This paper extends the Shleifer and Vishny (1997) model to include central bank intervention and fundamental uncertainty. Our results indicate that increasing availability of arbitrage capital has a pronounced effect on the dynamic intervention strategy of the central bank. Intervention is reduced during periods of moderate misalignment and amplified at times of extreme misalignment. This pattern is consistent with empirical observation. We show also that the central bank's uncertainty about the true fundamental value of the exchange rate and its aversion to the risk of making losses are factors that will limit its intervention activity.

It is worth noting that the logic of our argument is not confined to the foreign exchange market. But it is certainly true that episodes in which monetary authorities have intervened to influence the stock market are much less common. One such occurrence was in August 1998, when the Hong Kong Monetary Authority purchased HK$118 billion in shares in the 33 large capitalization stocks of the Hang-Seng Index over a period of ten days (Bhanot and Kadapakkam, 2004). This was a massive intervention, representing roughly twenty times the average daily trading volume on the Hong Kong market. To the extent that extreme deviations from fundamentals are less common in a stock market index than in the market for foreign exchange, and that more private capital is devoted to arbitrage in the stock market, our model provides a possible explanation for the difference in observed frequency of intervention, and for the fact that when such interventions do take place in the stock market, they are particularly large.

A primary longer-term goal of any research program that seeks to understand the intervention behavior of central banks will need to explain the diminishing appetite for such intervention. The increasing importance of private sector risk-arbitrage, the diminishing ratio of intervention resources to trading volume, and greater reliance on verbal interventions may be important factors in modeling this trend.

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Appendix

Proof of Proposition

We assume that parameters generate an equilibrium in which arbitrageurs are constrained in the first period, i.e. $D_1 = F_1$. It is convenient to introduce the variable $I_C \equiv I_1 + I_2$ to denote cumulative intervention at time 2. Prices in periods 1 and 2 are given by:

$$p_1 = \overline{V} - S_1 + F_1 + I_1 \tag{A.1}$$

$$\begin{aligned} p_2 &= \frac{p_1}{p_1 - aF_1}\left(\overline{V} - S_1 - S_2 + F_1(1-a) + I_C\right) \\ &= \frac{\overline{V} - S_1 + F_1 + I_1}{\overline{V} - S_1 + (1-a)F_1 + I_1}\left(\overline{V} - S_1 - S_2 + F_1(1-a) + I_C\right) \end{aligned} . \tag{A.2}$$

The expression for p_2 has incorporated the impact of performance-based arbitrage on demand at time 2 as captured in equation (3) in the paper. The central bank's problem is:

$$\min_{I_1, I_C} \frac{1}{2}\left[\left(\overline{V} - p_1\right)^2 + q\left(\overline{V} - p_2\right)^2\right] - k\left[(\pi_H - \pi_L)((1-q)I_1 + qI_c)\Delta V\right].$$

We substitute in the expressions for p_1 and p_2 above and derive the first-order necessary conditions for an optimum by setting the partial derivatives with respect to I_1 and I_C equal to zero. We write these conditions as implicit functions of the variables of interest.

$$\begin{aligned} f^1(I_1, I_C, F_1) &= 0 \\ f^C(I_1, I_C, F_1) &= 0 \end{aligned} . \tag{A.3}$$

This generates the following system of equations:

$$\begin{bmatrix} \partial f^1/\partial I_1 & \partial f^1/\partial I_C \\ \partial f^C/\partial I_1 & \partial f^C/\partial I_C \end{bmatrix}\begin{bmatrix} dI_1/dF_1 \\ dI_C/dF_1 \end{bmatrix} = -\begin{bmatrix} \partial f^1/\partial F_1 \\ \partial f^C/\partial F_1 \end{bmatrix}. \tag{A.4}$$

If we denote the matrix of partial derivatives by *J*, then we find that:

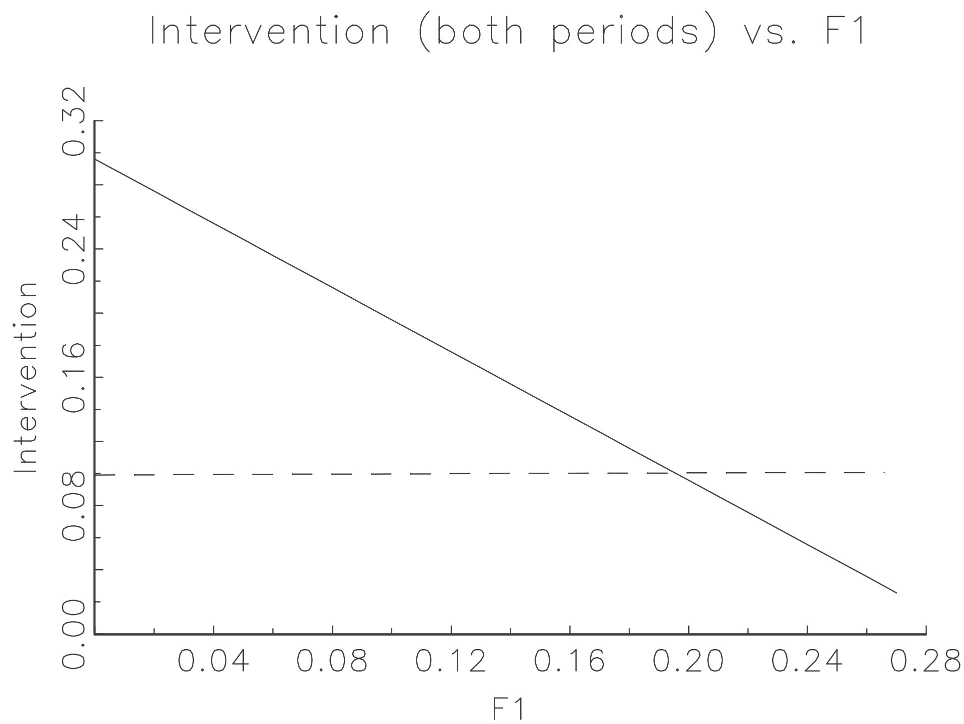
$$\begin{bmatrix} dI_1/dF_1 \\ dI_C/dF_1 \end{bmatrix} = -J^{-1} \begin{bmatrix} \partial f^1/\partial F_1 \\ \partial f^C/\partial F_1 \end{bmatrix}. \tag{A.5}$$

Evaluating the elements of J at $F_1 = 0$ we find that it is equal to the identity matrix. Therefore

$$\begin{bmatrix} dI_1/dF_1 \\ dI_C/dF_1 \end{bmatrix} = -\begin{bmatrix} \partial f^1/\partial F_1 \\ \partial f^C/\partial F_1 \end{bmatrix}. \tag{A.6}$$

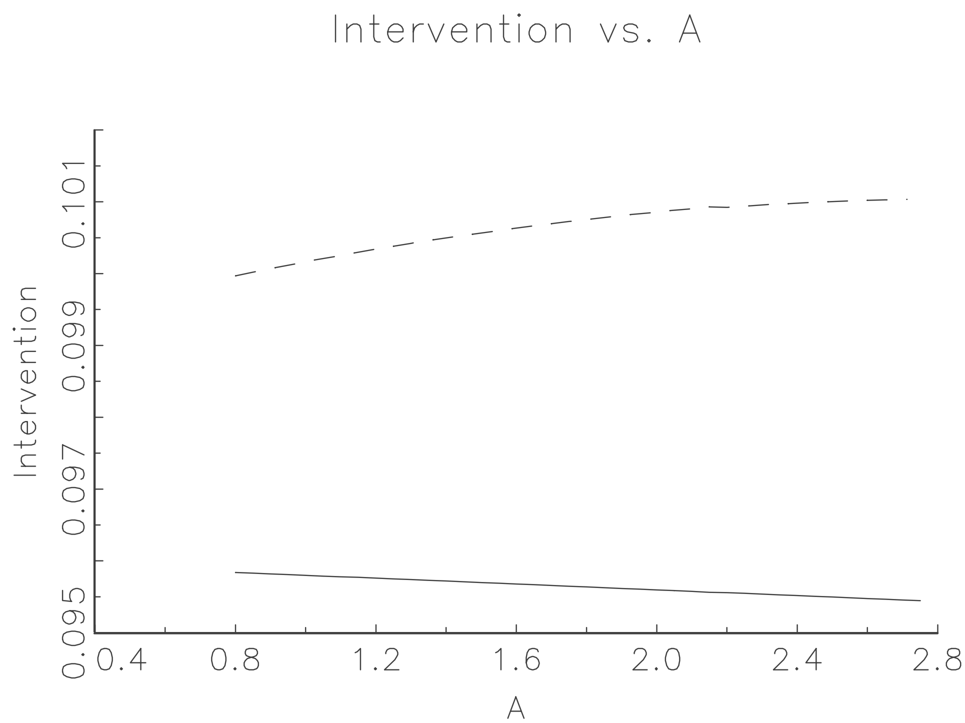
We obtain the results in the text by evaluating the partial derivatives on the right-hand-side of (A.6) at $F_1 = 0$, and noting that $dI_2/dF_1 = dI_C/dF_1 - dI_1/dF_1$.

Figure 1: Intervention as a function of the level of arbitrage capital, *F1*.



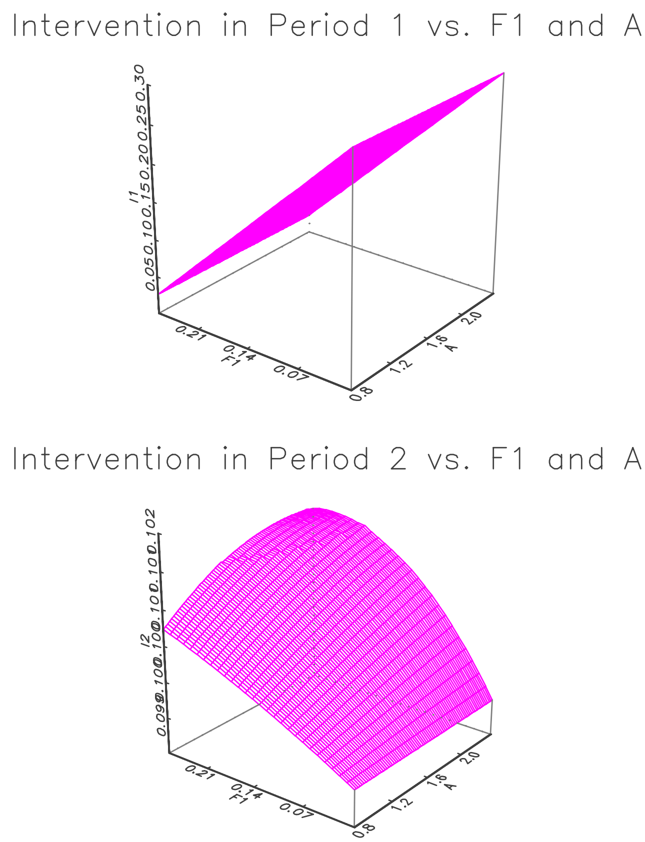
Notes: The solid line denotes intervention in period 1 (I_1) and the dashed line is intervention in period 2 contingent on a further pessimistic shock (I_2).

Figure 2: Intervention as a function of *a*.



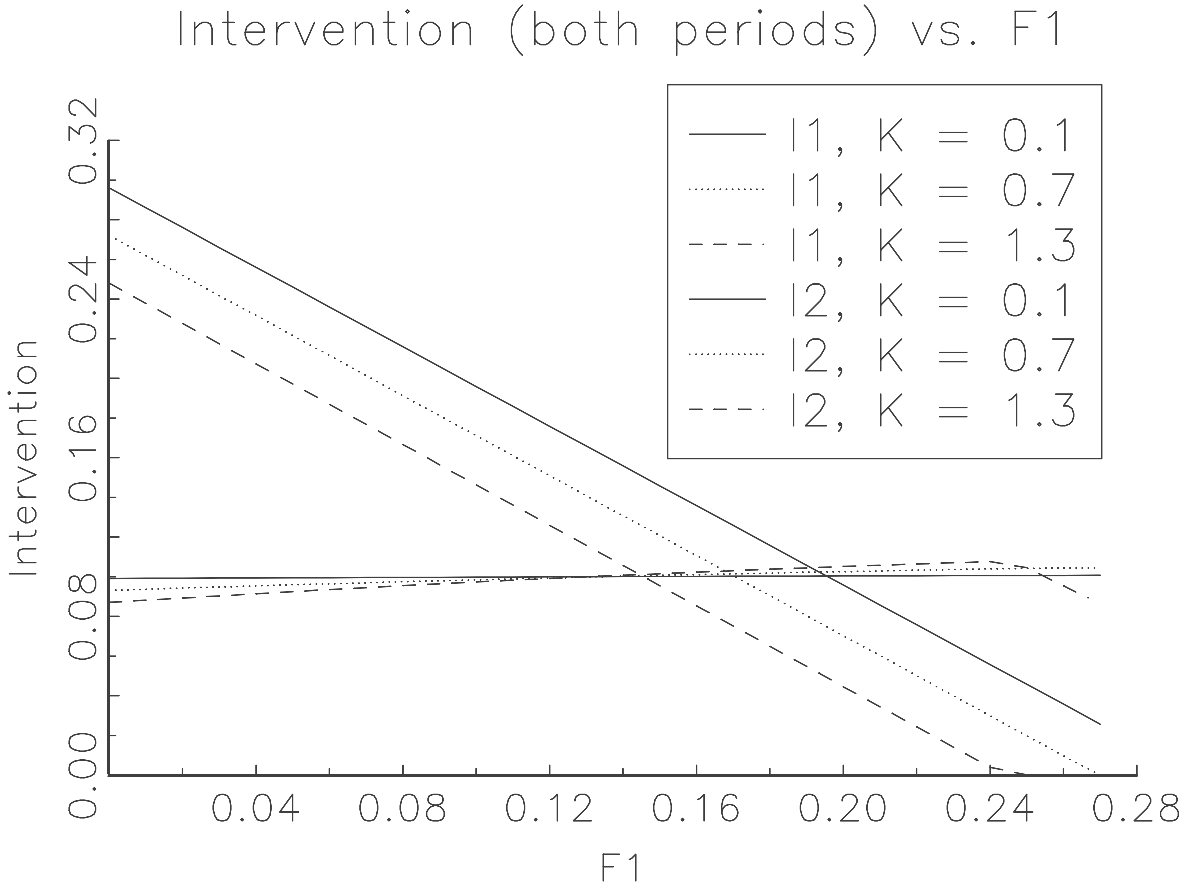
Notes: The solid line denotes intervention in period 1 (I_1) and the dashed line is intervention in period 2 contingent on a further pessimistic shock (I_2).

Figure 3: Intervention as a function of *F1* and *a*.



Notes: The top (bottom) panel shows intervention in period 1 (2) as a function of F1 and A.

Figure 4: The effect of changing the central bank's risk aversion or uncertainty about fundamentals



Notes: The figure shows the effect of changing the central bank's risk aversion or uncertainty about fundamentals, which are both mediated through k.