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Working Paper 2002-013C

<http://research.stlouisfed.org/wp/2002/2002-013.pdf>

July 2002

Revised May 2005

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# Time-Varying Risk Premia and the Cross Section of Stock Returns

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**Abstract:** This paper develops and estimates a heteroskedastic variant of Campbell's (1993) ICAPM, in which risk factors include a stock market return and variables forecasting stock market returns or variance. Our main innovation is the use of a new set of predictive variables, which not only have superior forecasting abilities for stock returns and variance, but also are theoretically motivated. In contrast with the early authors, we find that Campbell's ICAPM performs significantly better than the CAPM. That is, the additional factors account for a substantial portion of the two CAPM-related anomalies, namely, the value premium and the momentum profit.

JEL classification: G10, G12

Keywords: Stock return predictability, Time-varying investment opportunities, Value premium, Momentum profit, and Cross section of stock returns

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<sup>a</sup> Acknowledgements: I appreciate helpful suggestions from Bill Emmons, two anonymous referees, and participants at the 2002 Washington Area Finance Association Meeting, the 2002 Kansas-Missouri Joint Seminar on Stochastic Theory and Applications, the 2003 Midwest Finance Association Meeting, the 2003 Eastern Finance Association Meeting, and the 2003 FMA European Meeting. I also thank Narasimhan Jegadeesh and Kenneth French for providing data. Jason Higbee provided excellent research assistance. The views expressed in this paper are those of the author and do not necessarily reflect the official positions of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

## 1. Introduction

In the past two decades, financial economists have challenged the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965). In particular, there are three well-established CAPM-related anomalies: (1) the size premium (e.g., Basu, 1977 and Banz, 1981); (2) the value premium (e.g., Fama and French, 1992); and (3) the momentum profit (e.g., Jegadeesh and Titman, 1993). Some authors, e.g., Fama and French (1996) and Carhart (1997), argue that these anomalies reflect systematic risk and include them as additional risk factors in the empirical asset pricing models; others, however, attribute them to data mining or irrational pricing.

This paper attempts to provide some insight on this debate by investigating whether, as first pointed out by Merton (1973), the CAPM-related anomalies reflect a hedge demand for changes in investment opportunities. We first develop a discrete-time heteroskedastic intertemporal CAPM (ICAPM), which is a simple extension of Campbell's (1993) model. In our model, risk factors include a stock market return and variables forecasting stock market returns or variance. Another innovation of the paper is the use of a new set of forecasting variables—the consumption-wealth ratio (e.g., Lettau and Ludvigson, 2001), realized stock market variance, and the stochastically detrended risk-free rate—as proxy for time-varying investment opportunities. These variables have important advantages. First, they have significant out-of-sample predictive power for stock market returns and subsume the information content of the variables commonly used by the early authors (e.g., Guo, 2006).<sup>1</sup> Second, these variables are also strong predictors of stock market volatility—an important measure of investment opportunities in our ICAPM (e.g., Lettau and Ludvigson, 2002). Third, they are theoretically motivated (e.g., Guo, 2004 and Bernanke and Gertler, 1989).

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<sup>1</sup> Bossaerts and Hillion (1999) and Goyal and Welch (2003) show that the variables used by the early authors, e.g., the dividend yield, the term premium, and the default premium, have negligible out-of-sample predictive power.

We estimate Campbell's ICAPM using portfolios formed according to (1) the size of market capitalization, (2) the book-to-market value ratio, and (3) the past returns, respectively. For example, at the beginning of each period, we sort stocks into ten portfolios by each of these criteria and rebalance the portfolios in the next period and so forth. The size premium is the difference between the return on the decile with smallest capitalization and the return on the decile with largest capitalization, and the value premium and the momentum profit are defined in a similar manner. Our results indicate that the heteroskedastic ICAPM is a statistically significant improvement over the CAPM, which fails to explain the value premium and the momentum profit.<sup>2</sup> In particular, unlike the CAPM, the heteroskedastic ICAPM is not rejected by data at the conventional significance level in either conditional or unconditional specifications. More importantly, the difference between the two models is economically important. For example, loadings on stock market risk account for a momentum profit of only 0.08 percent per quarter, while the heteroskedastic ICAPM implies an expected momentum profit of 2.54 percent, which is close to the sample average of 3.49 percent. Moreover, the momentum strategy is found to be closely related to the dynamic of stock market volatility. These results, to our best knowledge, are innovative.

Similarly, while loadings on stock market risk imply a *negative* value premium of  $-0.75$  percent per quarter, the contribution from loadings on the consumption-wealth ratio is 0.95 percent. Overall, Campbell's ICAPM implies a value premium of 0.18 percent per quarter, which is a dramatic improvement over the CAPM; nevertheless, it is noticeably smaller than the sample average of 1.06 percent. This discrepancy should not be too surprising because many authors, e.g., Lakonishok et al. (1994) and Conrad et al. (2003), suggest that, for various reasons, we cannot fully

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<sup>2</sup> The size premium seems to have disappeared in our post-World War II sample from 1952 to 2000.

attribute the value premium to rational pricing. That said, we want to emphasize that a significant portion of it reflects loadings on the hedging factors of the ICAPM proposed in this paper.

Our results are consistent with the concurrent papers by Campbell and Vuolteenaho (2004) and Brennan et al. (2004); however, they are in contrast with the early authors, e.g., Campbell (1996), Li (1997), and Chen (2002). The conflicting results reflect the fact that Campbell's ICAPM is not a general equilibrium model and thus its empirical performance is sensitive to poor instrumental variables used by the early authors.

The remainder of the paper is organized as follows. We discuss a variant of Campbell's ICAPM in Section 2 and explain data in Section 3. The empirical results are presented in Section 4, and some concluding remarks are provided in Section 5.

## 2. The Heteroskedastic Campbell ICAPM

As in Campbell (1993), an agent maximizes his Epstein and Zin (1989) objective function,

$$(1) \quad U_t = \left\{ (1 - \beta) C_t^{1-(1/\sigma)} + \beta (E_t U_{t+1}^{1-\gamma})^{[1-(1/\sigma)]/(1-\gamma)} \right\}^{1/[1-(1/\sigma)]}$$

$$= [(1 - \beta) C_t^{(1-\gamma)/\theta} + \beta (E_t U_{t+1}^{1-\gamma})^{1/\theta}]^{\theta/(1-\gamma)},$$

subject to the intertemporal budget constraint,

$$(2) \quad W_{t+1} = R_{m,t+1} (W_t - C_t),$$

where  $C_t$  is consumption,  $W_t$  is aggregate wealth,  $R_{m,t+1}$  is the return on aggregate wealth,  $\beta$  is the time discount factor,  $\gamma$  is the relative risk aversion coefficient,  $\sigma$  is the elasticity of intertemporal substitution, and  $\theta = (1 - \gamma) / [1 - (1 / \sigma)]$ .

Assuming a joint log-normal distribution or using a second-order Taylor approximation, we can write the Euler equations in the log-linear form:

$$(3) \quad E_t \Delta c_{t+1} = \mu_t + \sigma E_t r_{m,t+1},$$

$$(4) \quad E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii,t}}{2} = \theta \frac{V_{ic,t}}{\sigma} + (1-\theta)V_{im,t},$$

where  $\mu_t = \sigma \log(\beta) + \frac{1}{2} \frac{\theta}{\sigma} V_t[\Delta c_{t+1} - \sigma r_{m,t+1}]$ ;  $r_{i,t+1}$  is the return on asset  $i$ ;  $r_{f,t+1}$  is the risk-free rate;

and  $V$  is variance or covariance, e.g.,  $V_{im,t} = E_t[(r_{i,t+1} - E_t r_{i,t+1})(r_{m,t+1} - E_t r_{m,t+1})]$ . Throughout the paper, we use lower case letters to denote logs. We log-linearize equation (2), the intertemporal budget constraint, around the mean log consumption-wealth ratio,  $c - w$ , and obtain

$$(5) \quad \Delta w_{t+1} \approx r_{m,t+1} + k_w + (1 - \frac{1}{\rho})(c_t - w_t),$$

where  $\rho = 1 - \exp(c - w)$  and  $k_w$  are constants. If the consumption-wealth ratio,  $c_t - w_t$ , is stationary, equation (5) implies

$$(6) \quad c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{m,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}.$$

After substituting equation (3) into equation (6), we obtain

$$(7) \quad c_{t+1} - E_t c_{t+1} = r_{m,t+1} - E_t r_{m,t+1} + (1 - \sigma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \mu_{t+j}.$$

We assume that there are  $(K-1)$  state variables,  $x_{t+1} = [x_{1,t+1}, \dots, x_{K-1,t+1}]$ , lags of which forecast the return on aggregate wealth or its volatility. As in Campbell (1996), we also assume that  $r_{m,t+1}$  and  $x_{t+1}$  follow a first-order vector autoregressive (VAR) process:

$$(8) \quad s_{t+1} = A_0 + A s_t + \varepsilon_{t+1},$$

where  $s_{t+1} = [r_{m,t+1}, x_{1,t+1}, \dots, x_{K-1,t+1}]$ ,  $A_0$  is a  $K$ -by-1 vector of constants,  $A$  is a  $K$ -by- $K$  matrix, and  $\varepsilon_{t+1} = [\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, \dots, \varepsilon_{K,t+1}]$  is a  $K$ -by-1 vector of error terms with a variance-covariance matrix  $\Omega$ .

The revision to expected returns is then equal to

$$(9) \quad r_{h,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} = e_1' \rho A (I - \rho A)^{-1} \varepsilon_{t+1} = \lambda_h' \varepsilon_{t+1},$$

where  $e1' = [1, 0, \dots, 0]'$  is a 1-by- $K$  vector with the first cohort equal to one and the other cohorts equal to zero;  $I$  is a  $K$ -by- $K$  identity matrix; and  $\lambda_h' = e1' \rho A (I - \rho A)^{-1} = [\lambda_{h1}, \lambda_{h2}, \dots, \lambda_{hK}]'$  is a 1-by- $K$  vector.

**Proposition 1.** *If conditional variance and covariance terms of  $\varepsilon_{t+1}$  in equation (8) are a linear function of lagged state variables*

$$(10) \quad V_{ij,t} = \text{cov}_t(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) = \omega_{ij,0} + \omega'_{ij} s_t, \quad i, j = 1, \dots, K,$$

where  $\omega_{ij,0}$  is a scalar and  $\omega'_{ij} = [\omega_{ij,1}, \omega_{ij,2}, \dots, \omega_{ij,K}]'$  is a 1-by- $K$  vector, then  $\mu_t$  is a linear function of conditional state variables:

$$(11) \quad \mu_t = \mu_0 + \psi_1 E_t r_{m,t+1} + \psi_2 E_t s_{2,t+1} + \dots + \psi_K E_t s_{K,t+1} = \mu_0 + \psi' E_t s_{t+1},$$

where  $\psi' = [\psi_1 \psi_2 \dots \psi_K]'$  is a 1-by- $K$  vector.

**Proof.** Available upon request.

Equation (10) can be motivated from Merton's ICAPM, in which the expected stock market return is determined by its own variance and its covariances with other risk factors. We also assume that restrictions have been imposed on parameters  $\omega_{ij,0}$  and  $\omega'_{ij}$  in equation (10) so that the variance-covariance matrix is well defined.

**Proposition 2.** *Equilibrium return on asset  $i$  is determined by its covariance with the state variables:*

$$(12) \quad E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma V_{im,t} + \left( \sum_{j=1}^K ((\gamma - 1) \lambda_{hj} + \frac{\theta}{\sigma} \lambda_{vj}) V_{ij,t} \right),$$

where  $\lambda_v' = [\lambda_{v1}, \lambda_{v2}, \dots, \lambda_{vK}]' = \rho \psi' A (I - \rho A)^{-1}$  is a 1-by- $K$  vector;  $V_{ij,t}$ ,  $j=1, \dots, K$ , is the conditional covariance between  $r_{i,t+1}$  and the  $k^{\text{th}}$  cohort of vector  $[r_{m,t+1}, x_{t+1}]'$ ; and  $V_{im,t} = V_{i1,t}$ .

**Proof.** Available upon request.

Equation (12) nests two interesting specifications in Campbell (1993), who imposes some restrictions on parameters in equation (11). In the first case,  $\mu_t$  is a linear function of only the expected stock market return ( $\mu_t = \mu_0 + \psi_1 E_t r_{m,t+1}$ ) and the associated asset pricing equation is

$$(13) \quad E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma V_{im,t} + \sum_{j=1}^K [(\gamma - 1) - \frac{\theta \psi_1}{\sigma}] \lambda_{ij} V_{ij,t}.$$

In the second case,  $\mu_t$  is a constant ( $\mu_t = \mu_0$ ) and the associated asset pricing equation is

$$(14) \quad E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma V_{im,t} + \left( \sum_{j=1}^K (\gamma - 1) \lambda_{ij} V_{ij,t} \right).$$

Moreover, if the hedging factors have zero prices in equation (12), we obtain the familiar CAPM:

$$(15) \quad E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma V_{im,t}.$$

We estimate variants of Campbell's ICAPM using the generalized method of moments (GMM) by Hansen (1982). In particular, to mitigate the small sample problem, we follow the advice of Ferson and Forester (1994) and use the iterative GMM.<sup>3</sup>

Suppose that there are  $N$  portfolio returns,  $r_{i,t+1}$ ,  $i=1, \dots, N$ . Our identifying system includes three blocks, as in Campbell (1996). First, there are  $K(K+1)$  orthogonality conditions to identify  $K(K+1)$  parameters in the VAR system of equation (8):

$$(16) \quad \begin{bmatrix} r_{m,t+1} \\ x_{t+1} \end{bmatrix} - A_0 - A \begin{bmatrix} r_{m,t} \\ x_t \end{bmatrix} = \varepsilon_{t+1} \perp \begin{bmatrix} 1 \\ r_{m,t} \\ x_t \end{bmatrix},$$

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<sup>3</sup> Some authors have suggested that the identity matrix is more reliable than the optimal weighting matrix when the number of time-series observations is small relative to the number of orthogonality conditions. However, as argued by Hodrick and Zhang (2001), the increase in the standard errors associated with the identity matrix severely affects the inference about the validity of asset pricing models. Interestingly, they find that results obtained from using the optimal weighting matrix are similar to those using the weighting matrix advocated by Hansen and Jagannathan (1997).



where  $x_t \perp y_t$  denotes  $\sum_{t=1}^{T \rightarrow \infty} x_t y_t = 0$ . Second, there are  $N(K+1)$  orthogonality conditions to identify

$N(K+1)$  parameters in conditional asset return equations:

$$(17) \quad r_{i,t+1} - r_{f,t+1} - B_{i0} - B_i \begin{bmatrix} r_{m,t} \\ x_t \end{bmatrix} = \eta_{i,t+1} \perp \begin{bmatrix} 1 \\ r_{m,t} \\ x_t \end{bmatrix}, \quad i = 1, \dots, N.$$

The last block is the asset pricing equation, and we consider the four specifications mentioned above, respectively. First is the general heteroskedastic ICAPM in equation (12). Under the null hypothesis of the test, the pricing error is orthogonal to a constant and to lagged state variables. Because risk prices are complicated functions of the underlying structural parameters, we focus only on its unrestricted implication, i.e., risk prices are parameters to be estimated. For this specification, the system is over-identified with  $N(K+1)-K$  degrees of freedom. The second specification is the simplified heteroskedastic ICAPM in equation (13). There are  $N(K+1)$  orthogonality conditions to identify two structural parameters,  $\gamma$  and  $\frac{\theta\psi_1}{\sigma}$ .<sup>4</sup> The system is over-identified with  $N(K+1)-2$  degrees of freedom. Equation (13) has some restrictions on asset prices:

$$(18) \quad \begin{aligned} p_1 &= \gamma + [(\gamma - 1 - \frac{\theta\psi_1}{\sigma})]\lambda_{h1} \\ p_j &= [(\gamma - 1 - \frac{\theta\psi_1}{\sigma})]\lambda_{hj}, \quad j = 2 \dots K \end{aligned} .$$

The third specification is the homoskedastic ICAPM in equation (14), in which  $\gamma$  is the only parameter to be estimated. For this specification, the system is over-identified with  $N(K+1)-1$  degrees of freedom. The restrictions on asset prices imposed by equation (14) are

$$(19) \quad \begin{aligned} p_1 &= \gamma + [(\gamma - 1)]\lambda_{h1} \\ p_j &= [(\gamma - 1)]\lambda_{hj}, \quad j = 2 \dots K \end{aligned} .$$

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<sup>4</sup> As in Campbell (1996), we treat  $\rho$  as a constant: It is set to be equal to 0.98 in our quarterly data.

The last specification is the CAPM in equation (15), in which  $\gamma$  is the only parameter to be estimated. The conditional CAPM has the same number of orthogonality conditions and of the over-identified restrictions as the homoskedastic ICAPM.

For the unconditional specification, we use only a constant as the instrumental variable for equation (17) and equations (12), (13), (14), and (15). Given the orthogonality conditions, we obtain the parameter estimates by minimizing the quadratic form  $J = g' \omega g$ , where  $g$  is the sample average of orthogonality conditions and  $\omega$  is the optimal weighting matrix. Under the null hypothesis that the pricing model is correctly specified, the minimized value of the quadratic form  $J$  has a  $\chi^2$  distribution with degrees of freedom equal to the number of over-identifying restrictions; it provides a goodness-of-fit test to the pricing model. Since the specifications of asset pricing equations (12), (13), (14), and (15) are nested, we also use the D-test proposed by Newey and West (1987) to test the restrictions across these specifications:

$$(20) \quad g_r' \omega_u g_r - g_u' \omega_u g_u \sim \chi^2,$$

where  $g_r$  is the sample average orthogonality conditions of the restricted model,  $g_u$  is the sample average orthogonality conditions of the unrestricted model, and  $\omega_u$  is the optimal weighting matrix usually estimated using the unrestricted model. The D-test has degrees of freedom equal to the number of restrictions.

### 3. Data

We use the consumption-wealth ratio, *cay*, realized stock market variance,  $\sigma_m^2$ , and the stochastically detrended risk-free rate, *rrel*, as forecasting variables for stock returns and variance. It is worth noting that the cointegrating vector used in computing *cay* is estimated over the full sample. This methodology has been questioned because it might introduce a look-ahead bias,

especially in the context of out-of-sample predictability. However, we see no apparent reason why it should spuriously affect our results. If *cay* has no economic content, it follows immediately that investors do not care about shocks to *cay* and thus the shocks should not help explain the cross section of stock returns. Therefore, our analysis provides additional insight on this debate.

Because *cay* is available on a quarterly basis, we analyze a quarterly sample spanning from 1952:Q4 to 2000:Q4, with a total of 193 observations. Following Merton (1980) and many others, realized stock market variance is the sum of the squared deviation of the daily excess stock return from its quarterly average in a given quarter.<sup>5</sup> The stochastically detrended risk-free rate is the difference between the risk-free rate and its average over the previous four quarters: The quarterly risk-free rate is approximated by the sum of the monthly risk-free rate in a given quarter. We obtain *cay* from Martin Lettau at New York University. We use the daily stock market return data constructed by Schwert (1990) before July 1962 and use the value-weighted daily stock market return data from the Center of Research for Security Prices (CRSP) at the University of Chicago thereafter. The daily risk-free rate is not directly available, but we assume that it is constant within a given month. The monthly risk-free rate is also obtained from CRSP.

We assume that the return on aggregate wealth is equal to the value-weighted stock market return from CRSP. As stipulated by Campbell's ICAPM, we use real stock market returns instead of excess returns as in the CAPM. Given that the two variables have a correlation coefficient of 0.997 in our sample, our results are not sensitive to the particular choice of stock market returns.

We focus on only three sets of stock portfolios formed according to size, book-to-market value ratio, and past returns, although we find very similar results using portfolios formed according

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<sup>5</sup> Because of the October 1987 stock market crash, realized stock market variance in that quarter is much higher than the sample average. Following Guo (2006) and many others, we replace it with the next highest observation.

to many other criteria. We obtain the momentum portfolio data, which span the period 1965:Q1 to 1998:Q4, from Narasimhan Jegadeesh at the University of Illinois and obtain all the other portfolio data spanning the period 1952:Q4 to 2000:Q4 from Kenneth French at Dartmouth College. See Jegadeesh and Titman (2001) and Fama and French (1992) for details about the portfolio data.

We estimate the unconditional specification using decile portfolios of each characteristic, respectively, which yield a total of 40 orthogonality conditions given that  $K$  is equal to 4 in this paper, compared with a total of 193 time-series observations (136 for momentum portfolios). For the conditional Campbell ICAPM, we use three portfolios—the bottom 30 percentile, the next 40 percentile, and the top 30 percentile—for each characteristic, respectively, which yield a total of 50 orthogonality conditions.

[Insert Table 1 here]

Table 1 provides summary statistics for the four state variables and a size premium,  $r_{smb}$ , a value premium,  $r_{hml}$ , and a momentum profit,  $r_{wml}$ . The size premium is the return on a portfolio that is short in the decile with largest market capitalization and is long in the decile with smallest market capitalization, and the value premium and the momentum profit are defined in a similar manner. As shown in panel A, all the forecasting variables are moderately correlated with each other and with the portfolio returns. They are also correlated with a business cycle indicator,  $BCI$ , which is equal to 1 during economic recessions and equal to zero during expansions. Panel B shows that the size premium appears to have disappeared in our sample, with an average of only 0.2 percent per quarter. In contrast, there is a substantial value premium of 1.1 percent and a striking momentum profit of 3.7 percent. Given that the value premium and the momentum profit are negatively related to stock market returns (panel A), their positive average returns cannot be explained by the CAPM.

Table C of Table 1 reports the regression results of forecasting one-quarter-ahead returns and variance, with the White (1980) corrected t-statistics in parentheses. We find negligible predictability in the size premium and the value premium; in contrast, our forecasting variables explain over 15 percent of variations in the momentum profit.<sup>6</sup> To our best knowledge, this result is innovative. Consistent with Lettau and Ludvigson (2001) and Guo (2006),  $\sigma_m^2$ , *cay*, and *rrel* are all significant predictors and jointly account for 20 percent of variations of stock market returns. We also replicate the results by Lettau and Ludvigson (2002) that  $\sigma_m^2$  and *cay* are strong predictors of stock market variance. Although the latter specification does not guarantee a positive expected volatility, the fitted value is always positive in our sample. For robustness, we also assume that stock market variance is a linear function of only its own lag in equation (16) and find qualitatively the same results, which are available upon request. Lastly, we want to emphasize that our forecasting variables subsume the information content of those used by Campbell (1996), Li (1997), and Chen (2002). Therefore, the strong support for Campbell's ICAPM documented in this paper is mainly due to our superior forecasting variables.<sup>7</sup>

## 4. Empirical Results

### 4.1. The Conditional Campbell ICAPM

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<sup>6</sup> The relation between stock market volatility and the momentum profit is not sample-specific: We find very similar results over various subsamples from 1926 to 2000, which are available upon request.

<sup>7</sup> The early authors estimate the Campbell ICAPM using monthly data. However, data frequencies do not explain the difference between their results and ours since we confirm their results using quarterly data. Cochrane (1996) and many others also test asset pricing models using quarterly data over a similar sample period.

Table 2 reports four nested specifications of Campbell's ICAPM for each set of portfolios. Model I is the homoskedastic ICAPM in equation (14); model II is the simplified heteroskedastic ICAPM in equation (13); model III is the general heteroskedastic ICAPM in equation (12); and model IV is the CAPM in equation (15). In models I and II, we estimate the structural parameters and then use equations (19) and (18), respectively, to calculate the price of risk for each factor and obtain the standard deviation using the delta method outlined by Campbell, et al. (1997). In contrast, we estimate the price of risk directly for models III and IV.

[Insert Table 2 here]

Following Campbell (1996), we orthogonalize and normalize the shocks to state variables so that they have the same unconditional variance as that of stock market returns, with Sims' (1980) ordering  $r_m$ ,  $cay$ ,  $\sigma_m^2$ , and  $rrel$ . We assume that the stock market return is the most important risk factor so that our results can be directly compared with the CAPM. The ordering is somewhat ad hoc; however, it is important to note that our main result, that the ICAPM outperforms the CAPM, does not depend on any particular choice of Sims' ordering. For example, in Tables 2 and 3, Sims' ordering affects only the magnitude of the price of risk but not the inference about the statistical significance and the specification tests. Similarly, in Table 4, it affects the relative contribution of each risk factor but not the pricing error.

We find strong support for the heteroskedastic ICAPM (models II and III) relative to the CAPM (model IV) and the homoskedastic ICAPM (model I) using three size portfolios, as shown in panel A of Table 2. First, the CAPM is overwhelmingly rejected by Hansen's J-test. We also strongly reject the CAPM relative to the simplified heteroskedastic ICAPM (model II) using the Newey and West (1987) D-test. Second, while the J-test does not reject the homoskedastic ICAPM at the conventional significance level, it is rejected relative to the simplified heteroskedastic ICAPM

at the 5 percent significance level. Third, the J-test fails to reject both heteroskedastic specifications at the conventional significance level. Moreover, the parameter for heteroskedasticity in model II,  $\frac{\theta_{11}}{\sigma}$ , is statistically significant, indicating that time-varying volatility has an important effect on asset prices. Lastly, we cannot reject model II relative to model III—the general heteroskedastic specification—at almost the 20 percent significance level. Therefore, despite its parsimonious specification, the simplified heteroskedastic ICAPM advocated by Campbell (1993) provides a good description for the effect of time-varying volatility on asset prices.

The point estimate of the structural parameter is plausible in panel A of Table 2. The relative risk aversion coefficient,  $\gamma$ , is found to be significantly positive in all specifications and its point estimate is, for example, 14.1 in model II, the preferred specification. We note that  $\gamma$  is much larger than the price of stock market risk, which is only 3.7 in model II. This pattern is consistent with Campbell (1996), who suggests that the mean reversion in stock prices reduces the price associated with stock market risk. While our results provide support for a positive risk-return tradeoff in the stock market, it is important to note that the prices of the other factors are all statistically significant and their absolute values are as big as that of stock market risk.

We find very similar results from the book-to-market portfolios and the momentum portfolios, as shown in panels B and C of Table 2, respectively. First, the J-test does not reject model II at the 9 percent significance level for the book-to-market portfolios and at the 15 percent level for the momentum portfolios. Also, we cannot reject model II relative to model III at the conventional significance level. Second, in contrast, the J-test overwhelmingly rejects the conditional CAPM and the D-test overwhelmingly rejects the conditional CAPM relative to model II, the preferred specification. Third, the D-test shows that the heteroskedastic specification (model II) also performs significantly better than the homoskedastic specification (model I). Similarly, the

parameter for the heteroskedasticity,  $\frac{\theta\sqrt{1}}{\sigma}$ , is significantly negative in both panels. Lastly, the structural parameter  $\gamma$  as well as the risk prices are almost always statistically significant, and their point estimates are strikingly similar to those reported in panel A of Table 2.

To summarize, the heteroskedastic ICAPM provides a statistically significant improvement over the CAPM in explaining the cross section of stock returns, indicating that time-varying stock market return and variance both have important effects on asset prices.

#### *4.2. The Unconditional Campbell ICAPM*

We report the estimation results of the unconditional Campbell ICAPM in Table 3. In addition to the three sets of decile portfolios, we also analyze a set of nine mixed portfolios, including the bottom 30 percentile, the next 40 percentile, and the top 30 percentile of momentum, book-to-market, and size, respectively. Since we find no statistical difference between models II and III using the D-test, to conserve space, we report only the results from models I, II, and IV.

[Insert Table 3 here]

Again, Table 3 shows that Campbell's ICAPM fits data well and provides a statistically significant improvement over the CAPM in many cases. First, the J-test indicates that we cannot reject Campbell's ICAPM at the conventional significance level for all sets of portfolios. Second, in contrast, we overwhelmingly reject the CAPM using the J-test in all cases except for the size portfolios. Third, we reject the CAPM in favor of the heteroskedastic ICAPM (model II) using the D-test at the 1 percent significance level for the momentum portfolios and at the 10 percent significance level for the book-to-market portfolios. Lastly, the point estimates of the structural parameters and the risk prices are very similar to those reported in Table 2.

However, there are two noticeable differences between Tables 2 and 3. First, the D-test indicates that we cannot reject model I relative to model II at the 20 percent significance level in all



panels of Table 3. Similarly,  $\frac{\theta_{M1}}{\sigma}$  is always insignificant, although it is negative in three of four panels. Second, while we cannot reject the model at a significance level much higher than that in Table 2, the price of risk is imprecisely estimated in some cases of Table 3. One possible explanation for the difference is that, as explained by Cochrane (1996), in the conditional model, we also implicitly include a set of managed portfolios that exploit the predictability of stock returns. The managed portfolios usually have a large dispersion in average returns and, therefore, pose a more stringent test to the asset pricing model than portfolios formed simply according to size, industry, or beta.<sup>8</sup> Therefore, we usually find stronger support for an asset pricing model when using the unconditional model than when using the conditional model (also see Hodrick and Zhang, 2001). However, because of a large dispersion in loadings on the risk factors, the managed portfolios allow us to precisely identify the underlying risk prices.

#### 4.3. *The Cross Section of Stock Returns*

As shown in Tables 2 and 3, Campbell's ICAPM appears to provide a reasonably good explanation for data. However, as pointed out by Cochrane (1996) and others, we might fail to reject an asset pricing model simply because it has large pricing errors. In this section, we show that this is not the case in our estimation.

[Insert Table 4 here]

Table 4 provides a decomposition of volatility-adjusted average return,  $\overline{er}_i + (V_{ii} / 2)$ , into loadings on the four risk factors, based on the corresponding estimation results of model II reported in Table 3. Panel A presents the decomposition for the size deciles. Consistent with Campbell (1996), almost all the variations of the cross-sectional returns are explained by loadings on stock

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<sup>8</sup> The book-to-market and the momentum portfolios are also the managed portfolios.

market risk. This result should not be a surprise because we have shown in panel A of Table 3 that the CAPM (model IV) provides a good explanation for the returns on the size portfolios. Of course, our evidence reflects the fact that the dispersion of loadings on the hedging factors is small among the size portfolios rather than that the hedging demand in the ICAPM is economically unimportant. This result highlights that it is important to test the asset pricing model using portfolios with a large dispersion in conditional returns such as the book-to-market and momentum portfolios, which we discuss below. As shown in the upper left panel of Figure 1, realized and expected volatility-adjusted returns lineup along the 45-degree line, indicating that pricing errors are very small.

[Insert Figure 1 here]

Panel B of Table 4 presents the decomposition for the ten book-to-market portfolios. Again, loadings on stock market risk are the most important determinant of the return on each portfolio. However, the compensation for stock market risk implies a value premium of  $-0.75$  percent per quarter, compared with the sample average of  $1.06$  percent. That is, consistent with the early literature, the CAPM leaves a substantial value premium of  $1.81$  percent per quarter unexplained. This result explains why the J-test rejects the CAPM overwhelmingly in panel B of Table 3. In contrast, the value premium is not so puzzling for Campbell's ICAPM because loadings on the other risk factors make significant contributions to it. Especially, loadings on the consumption-wealth ratio account for a value premium of  $0.95$  percent per quarter.<sup>9</sup> Overall, Campbell's ICAPM implies a value premium of  $0.18$  percent per quarter, a dramatic increase from  $-0.75$  percent implied by the CAPM. Therefore, a substantial portion of the value premium reflects intertemporal pricing. This result also confirms the specification tests in panel B of Table 3 that we cannot reject

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<sup>9</sup> This result is not sensitive to Sims' ordering. For example, loadings on the consumption-wealth ratio account for a value premium of  $0.91$  percent if we use the ordering  $r_m$ ,  $rrel$ ,  $\sigma_m^2$ , and  $cay$ .

Campbell's ICAPM at the conventional significance level and that the heteroskedastic ICAPM performs significantly better than the CAPM at the 10 percent level.

However, the explained value premium is still somewhat smaller than the sample average of 1.06 percent. This discrepancy should not be too surprising because the value premium cannot be fully explained by rational pricing for at least two reasons. First, Lakonishok et al. (1994) argue that the value premium reflects irrational pricing because investors tend to be more risk averse toward value stocks than growth or glamour stocks.<sup>10</sup> Second, Conrad et al. (2003) attribute half of the observed value premium to data snooping. Moreover, we have not taken into account transaction costs associated with the value strategy, which could substantially reduce its profitability and prevent investors from exploiting the value premium. These rationales are consistent with recent evidence by Schwert (2003) that the value premium has substantially attenuated in the past decade.

The upper right panel of Figure 1 provides some clue about the source of pricing errors for the book-to-market portfolios. The four bottom book-to-market deciles are consistently overpriced relative to the six top deciles, possibly indicating that investors might have been more risk averse toward value stocks than growth stocks, as argued by Lakonishok et al. (1994). However, while the irrational pricing explanation is potentially interesting, it is important to stress again that we cannot fully attribute the value premium to pricing errors either. That is, as discussed above, our results indicate that a significant portion of the value premium cannot be explained by the CAPM because it reflects loadings on the hedging factors in the ICAPM proposed in this paper.

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<sup>10</sup> In an early version of this paper, we allow  $\gamma$  to vary across the book-to-market portfolios. We find that value stocks have significantly higher  $\gamma$  than growth stocks and find similar results using portfolios formed according to various characteristics such as the dividend-price ratio, the earning-price ratio, and the cash flow-market capitalization ratio.

Results in panel C of Table 4 for the momentum portfolios are qualitatively similar to those in panel B. Stock market risk is again the most important determinant of the return on each portfolio. However, the other factors, especially realized stock market variance, explain most variations of the cross section of stock returns. In particular, loadings on stock market risk contribute only 0.08 percent to the average momentum profit of 3.49 percent, compared with 2.50 percent from  $\sigma_m^2$ , 0.45 percent from *rrel*, and -0.49 percent from *cay*.<sup>11</sup> Again, these results confirm the specification test in panel C of Table 3 that Campbell's ICAPM performs significantly better than the CAPM. It is interesting to note that stock market volatility is important to explaining the momentum profit.<sup>12</sup> This result should not be very surprising since, as shown in Table 1, realized stock market variance is a strong predictor of the momentum profit.

Similar to the value premium, Campbell's ICAPM does not fully account for the momentum profit either. Especially, the first decile (past losers) is severely overpriced, with a pricing error of -0.57 percent per quarter. This result is consistent with enormous evidence that the momentum profit might have been exaggerated if we take into account factors such as transactional costs and tax-motivated trading strategies (e.g., Grinblatt and Moskowitz, 2002). Nevertheless, our estimation shows that Campbell's ICAPM accounts for a substantial momentum profit of 2.54 percent, suggesting an important role for rational pricing. The lower left panel of Figure 1 confirms that

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<sup>11</sup> Again, this result is not sensitive to alternative Sims' orderings. For example, we find 1.80 percent from  $\sigma_m^2$ , 1.15 percent from *rrel*, and -0.49 percent from *cay* if we use the ordering  $r_m$ , *cay*, *rrel*, and  $\sigma_m^2$ .

<sup>12</sup> This result appears to be consistent with some recent authors, who find that momentum is related to some measures closely related to stock market volatility. For example, Harvey and Siddique (2000) find that momentum is related to co-skewness; Pastor and Stambaugh (2003) find that momentum is related to some measure of liquidity; Lee and Swaminathan (2000) document a link between momentum and trading volume.

Campbell's ICAPM provides a good explanation for the momentum portfolios: The realized and expected returns lineup around the 45-degree line nicely.

Lastly, panel D reports the decomposition of the returns on nine mixed portfolios, which are consistent with those discussed above. In particular, loadings on *cay* decrease from past losers to past winners, from value to growth, and from small to big market capitalization. Also, loadings on  $\sigma_m^2$  increase from past losers to past winners, from value to growth, and from small to big market capitalization; loadings on *rrel* increase from past losers to past winners, from growth to value, and from big to small capitalization. In general, the lower right panel of Figure 1 shows that the realized and expected returns lineup well around the 45-degree line, except that past losers (M1) and value stocks (B3) exhibit some sizable pricing errors (also see Table 4).

Overall, the decomposition indicates that, consistent with the specification tests reported in Table 3, the heteroskedastic ICAPM provides a better explanation for the cross section of stock return, i.e., has substantially smaller pricing errors, than the CAPM does.

## 5. Conclusions

In this paper, we evaluate the empirical performance of a heteroskedastic variant of Campbell's ICAPM using a new set of conditioning variables. The heteroskedastic ICAPM explains the cross section of stock returns significantly better than the CAPM does. In particular, it accounts for a substantial portion of two CAPM-related anomalies, namely, the value premium and the momentum profit.

Our results also shed light on the on-going debate about the risk-return relation by showing that there is a distinction between a positive risk aversion coefficient and a positive risk-return relation. In this paper, we find that both the relative risk aversion coefficient and the price of stock

market risk are significantly positive. Given that a hedge for time-varying investment opportunities is a significant determinant of stock market returns, it is possible to find a negative risk-return relation if the hedge and risk components are negatively related, even though the relative risk aversion coefficient is positive (also see Guo and Whitelaw, 2005).

Campbell's ICAPM is not a general equilibrium model: Campbell (1993) takes stock return predictability as given and derives a set of non-arbitrage restrictions across asset returns based on shareholders' optimization. Therefore, any test of Campbell's ICAPM is related to a specific asset pricing model through the choices of the forecasting variables. In this sense, our results provide direct support for the limited stock market participation model by Guo (2004), who explains why the consumption-wealth ratio and realized stock market variance forecast stock returns. Limited stock market participation is a relatively new literature, and our results highlight its promising role in explaining the asset price movement, which warrants attention in future research.

Lastly, our paper does not provide an explicitly explanation for the mechanism of the momentum profit. Given that our state variables forecast the momentum profit, we suspect that, as argued by Chordia and Shivakumar (2002), the momentum profit reflects the cross-sectional dispersion of expected stock returns. That is, past winners (losers) continue to perform well (poorly) because their expected returns are persistent. A further investigation along this line should provide a direct explanation to the momentum profit and we leave it for future research.

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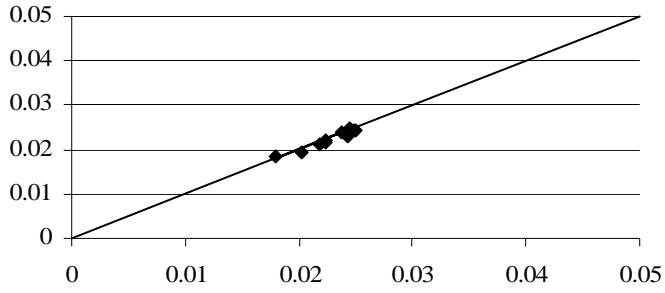
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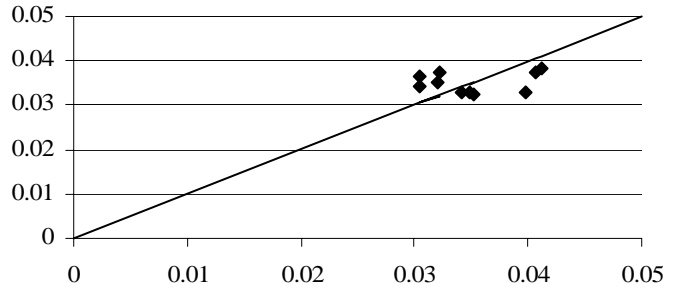
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Figure 1: Realized (Horizontal Axis) vs. Expected (Vertical Axis) Returns

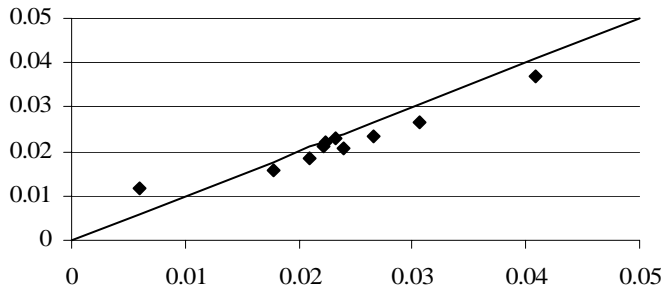
10 Size Portfolios



10 Book-to-Market Portfolios



10 Momentum Portfolios



9 Mixed Portfolios

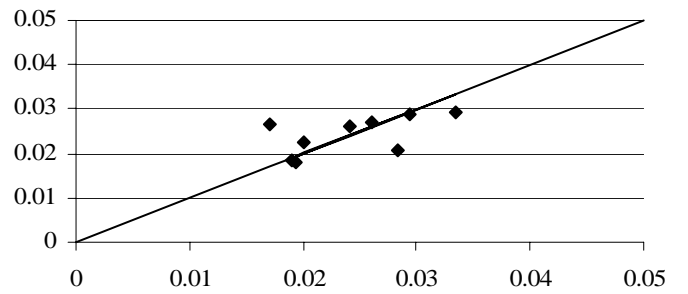


Table 1 Summary Statistics of Risk Factors and Portfolio Returns

	$r_{smb}$	$r_{hml}$	$r_{wml}$	$r_m$	$\sigma_m^2$	$cay$	$rrel$
Panel A: Correlation Matrix							
$r_{smb}$	1.00	0.38	-0.25	0.30	-0.27	0.12	-0.12
$r_{hml}$		1.00	-0.18	-0.17	0.02	0.01	-0.04
$r_{wml}$			1.00	-0.16	0.05	-0.23	0.02
$r_m$				1.00	-0.40	0.30	-0.30
$\sigma_m^2$					1.00	-0.34	-0.02
$cay$						1.00	-0.16
$rrel$							1.00
$BCI$	-0.08	-0.04	-0.01	-0.08	0.19	0.12	-0.28
Panel B: Univariate Summary Statistics							
Mean	0.002	0.011	0.037	0.019	0.004	0.613	0.000
Standard error	0.089	0.085	0.074	0.083	0.004	0.012	0.003
Autocorrelation	-0.03	0.04	-0.15	0.07	0.43	0.83	0.71
Panel C: Forecasting Quarterly Stock Returns							
$r_{smb}$				0.186 (1.811)	3.408 (1.564)	0.655 (1.029)	-2.166 (-0.969)
			$\bar{R}^2=0.03$				
$r_{hml}$				0.168 (1.884)	1.177 (0.592)	0.114 (0.214)	3.534 (1.594)
			$\bar{R}^2=0.00$				
$r_{wml}$				0.177 (1.617)	-6.907 (-2.307)	-1.292 (-1.827)	1.390 (0.556)
			$\bar{R}^2=0.15$				
$r_m$				0.031 (0.466)	7.312 (4.877)	2.458 (5.199)	-5.852 (-3.020)
			$\bar{R}^2=0.20$				
$\sigma_m^2$				0.005 (1.290)	0.393 (4.162)	-0.085 (-3.361)	0.070 (0.953)
			$\bar{R}^2=0.24$				

Table 2 Conditional Campbell ICAPM

Model	$\gamma$	$\frac{\theta\psi_1}{\sigma}$	Risk Prices For				OIR
			$r_m$	$cay$	$\sigma_m^2$	$rrel$	
Panel A: Three Size Portfolios							
I	11.507 (2.608)		6.441 (4.284)	9.424 (2.040)	9.633 (3.573)	-20.506 (-3.990)	$\chi^2(14)=17.710$ (0.220)
II	14.149 (2.862)	-9.723 (-2.041)	3.677 (2.352)	12.406 (2.241)	12.500 (3.646)	-10.456 (-3.159)	$\chi^2(13)=15.830$ (0.258)
III			3.418 (2.448)	5.198 (0.726)	14.915 (3.933)	-6.847 (-1.332)	$\chi^2(11)=14.001$ (0.233)
IV			4.918 (4.015)				$\chi^2(14)=34.667$ (0.002)
I vs. II: $\chi^2(1)=6.170$ (0.013) II vs. III: $\chi^2(2)=3.259$ (0.196) IV vs. II: $\chi^2(1)=16.586$ (0.000)							
Panel B: Three Book-to-Market Portfolios							
I	11.984 (3.138)		4.216 (3.306)	10.315 (2.361)	7.479 (3.358)	-11.904 (-3.262)	$\chi^2(14)=28.817$ (0.011)
II	13.852 (3.205)	-8.430 (-2.043)	3.638 (2.514)	11.934 (2.485)	10.650 (3.494)	-9.018 (-2.737)	$\chi^2(13)=19.879$ (0.098)
III			3.781 (2.436)	13.563 (1.945)	12.206 (3.081)	-5.849 (-1.078)	$\chi^2(11)=18.034$ (0.081)
IV			5.339 (4.380)				$\chi^2(14)=45.376$ (0.000)
I vs. II: $\chi^2(1)=5.244$ (0.022) II vs. III: $\chi^2(2)=0.356$ (0.837) IV vs. II: $\chi^2(1)=24.097$ (0.000)							
Panel C: Three Momentum Portfolios							
I*	8.978 (2.614)		3.020 (2.323)	4.917 (1.584)	8.577 (3.233)	-18.770 (-5.512)	$\chi^2(14)=40.143$ (0.000)
II	11.578 (2.039)	-15.375 (-2.307)	1.664 (0.667)	11.588 (1.701)	18.516 (3.178)	-18.898 (-3.598)	$\chi^2(13)=18.180$ (0.151)
III**			3.828 (1.132)	11.364 (1.276)	39.179 (5.529)	-3.308 (-0.549)	$\chi^2(11)=21.468$ (0.029)
IV			3.568 (2.659)				$\chi^2(14)=60.562$ (0.000)
I vs. II: $\chi^2(1)=6.684$ (0.010) II vs. III: $\chi^2(2)=0.924$ (0.630)*** IV vs. II: $\chi^2(1)=14.364$ (0.000)							

Table 3 Unconditional Campbell ICAPM

Model	$\gamma$	$\frac{\theta\psi_1}{\sigma}$	Risk Prices For				OIR
			$r_m$	$cay$	$\sigma_m^2$	$rrel$	
Panel A: Ten Size Portfolios							
I	6.529 (2.416)		3.341 (3.106)	3.693 (1.685)	2.273 (1.791)	-1.869 (-1.388)	$\chi^2(9)=3.719$ (0.929)
II	3.382 (1.011)	2.994 (1.175)	3.755 (2.685)	-0.427 (-0.118)	-0.273 (-0.118)	0.238 (0.118)	$\chi^2(8)=2.646$ (0.955)
IV			3.750 (2.736)				$\chi^2(9)=2.684$ (0.976)
I vs. II: $\chi^2(1)=1.195$ (0.274)							
IV vs. II: $\chi^2(1)=0.014$ (0.906)							
Panel B: Ten Book-to-Market Portfolios							
I	20.462 (3.314)		6.931 (4.191)	16.772 (2.601)	5.453 (1.870)	-6.179 (-1.777)	$\chi^2(9)=7.837$ (0.551)
II	22.273 (2.634)	-2.907 (0.420)	6.568 (3.628)	19.234 (1.947)	7.174 (1.347)	-7.720 (-1.435)	$\chi^2(8)=6.401$ (0.602)
IV			6.463 (3.868)				$\chi^2(9)=25.500$ (0.002)
I vs. II: $\chi^2(1)=0.251$ (0.616)							
IV vs. II: $\chi^2(1)=3.109$ (0.078)							
Panel C: Ten Momentum Portfolios							
I	20.148 (3.127)		6.667 (2.974)	8.794 (1.381)	19.525 (4.007)	-13.305 (-2.612)	$\chi^2(9)=10.012$ (0.350)
II	18.981 (2.909)	-1.763 (-0.412)	6.250 (2.619)	7.759 (1.221)	19.229 (3.925)	-13.212 (-2.554)	$\chi^2(8)=10.307$ (0.244)
IV			6.459 (3.520)				$\chi^2(9)=49.822$ (0.000)
I vs. II: $\chi^2(1)=0.173$ (0.677)							
IV vs. II: $\chi^2(1)=6.885$ (0.009)							
Panel D: Nine Mixed Portfolios							
I	19.412 (1.895)		5.804 (1.947)	14.092 (1.453)	15.358 (2.030)	-20.665 (-2.049)	$\chi^2(8)=(9.154)$ (0.329)
II	16.008 (1.881)	-3.949 (0.781)	4.135 (1.737)	11.807 (1.446)	12.857 (1.925)	-16.631 (-1.907)	$\chi^2(7)=10.132$ (0.181)
IV			4.417 (2.693)				$\chi^2(8)=27.170$ (0.001)
I vs. II: $\chi^2(1)=0.677$ (0.411)							
IV vs. II: $\chi^2(1)=1.398$ (0.237)							

Table 4 Factor Contributions to Expected Returns

Portfolios	$\overline{er}_i$ (1)	$\overline{er}_i + (V_{ii} / 2)$ (2)	$r_m$ (3)	$cap$ (4)	$\sigma_m^2$ (5)	$rrel$ (6)	Error (7)
Panel A: Ten Size Portfolios							
1(smallest)	1.71	2.50	2.45	-0.03	0.04	0.00	0.05
2	1.75	2.44	2.47	-0.02	0.02	-0.01	-0.02
3	1.86	2.48	2.41	-0.02	0.02	0.00	0.07
4	1.79	2.38	2.39	-0.02	0.02	0.00	-0.01
5	1.91	2.44	2.29	-0.01	0.01	0.00	0.16
6	1.77	2.24	2.19	-0.01	0.01	0.00	0.06
7	1.78	2.24	2.21	-0.01	0.00	0.00	0.04
8	1.77	2.19	2.11	0.00	0.00	0.00	0.07
9	1.68	2.03	1.95	0.00	0.00	0.00	0.08
10(largest)	1.51	1.80	1.83	0.01	0.00	0.00	-0.04
SMB	0.20	0.70	0.62	-0.04	0.04	0.00	0.09
Panel B: Ten Book-Market Portfolios							
1(lowest)	2.56	3.05	4.06	-0.43	0.15	-0.14	-0.59
2	2.84	3.21	3.55	-0.19	0.09	0.03	-0.28
3	2.86	3.22	3.37	0.20	0.13	0.02	-0.50
4	2.71	3.05	3.16	0.52	-0.04	-0.20	-0.38
5	3.14	3.42	2.76	0.72	-0.10	-0.08	0.12
6	3.23	3.51	2.89	0.62	-0.13	-0.14	0.27
7	3.19	3.49	2.79	0.32	0.11	0.06	0.21
8	3.64	3.97	2.95	0.54	-0.20	-0.01	0.70
9	3.71	4.06	3.01	0.77	-0.09	0.03	0.34
10(highest)	3.62	4.11	3.31	0.52	-0.15	0.13	0.29
HML	1.06	1.06	-0.75	0.95	-0.30	-0.27	0.88
Panel C: Ten Momentum Portfolios							
1(loser)	-0.38	0.60	4.50	0.32	-3.34	-0.30	-0.57
2	1.07	1.77	3.97	0.34	-2.48	-0.25	0.19
3	1.52	2.10	3.76	0.24	-2.01	-0.15	0.26
4	1.72	2.23	3.60	0.23	-1.72	0.01	0.10
5	1.78	2.23	3.48	0.18	-1.54	0.10	0.02
6	1.89	2.33	3.49	0.17	-1.53	0.15	0.04
7	1.97	2.40	3.51	0.08	-1.69	0.19	0.31
8	2.21	2.66	3.64	0.06	-1.58	0.20	0.34
9	2.54	3.06	3.87	-0.03	-1.39	0.22	0.39
10(winner)	3.33	4.09	4.58	-0.17	-0.84	0.15	0.38
WML	3.71	3.49	0.08	-0.49	2.50	0.45	0.95
Panel D: Nine Mixed Portfolios							
M1(loser)	0.22	0.96	2.82	0.86	-1.46	-0.33	-0.94
M2	1.48	1.95	2.42	0.51	-0.90	0.10	-0.18
M3(winner)	2.18	2.77	2.77	0.16	-0.84	0.25	0.42
B1(lowest)	1.09	1.55	2.54	-0.37	0.15	-0.52	-0.26
B2	1.32	1.63	1.98	0.22	-0.31	-0.40	0.15
B3(highest)	2.12	2.48	2.10	0.22	-0.76	0.13	0.78
S1(smallest)	1.34	2.14	2.95	0.92	-2.01	0.22	0.05
S3	1.47	2.04	2.69	0.41	-0.92	-0.04	-0.10
S3(largest)	1.24	1.57	2.17	-0.23	0.06	-0.47	0.05

## Note for Tables

### Table 1

The table reports summary statistics of portfolio returns and the risk factors in Campbell's ICAPM.  $r_{smb}$  is the return on a portfolio short in stocks from the top capitalization decile and long in stocks from the bottom capitalization decile.  $r_{hml}$  is the return on a portfolio short in stocks from the bottom book-to-market decile and long in stocks from the top book-to-market decile.  $r_{wml}$  is the return on a portfolio short in stocks from the decile of lowest past returns and long in stocks from the decile of highest past returns. Also,  $r_m$  is the real stock market return;  $\sigma_m^2$  is realized stock market variance;  $cay$  is the consumption-wealth ratio; and  $rrel$  is the stochastically detrended risk-free rate.  $BCI$  is a business cycle indicator: It is equal to 1 for economic recessions and equal to zero for expansions. We use a quarterly sample from 1952:Q4 to 2000:Q4 for all the variables except  $r_{wml}$ , which is available over the period 1965:Q1 to 1998:Q4. In the forecasting regression reported in panel C, the White (1980) heteroskedastic-consistent t-statistics are reported in parentheses.

### Table 2

The table reports the iterative GMM estimation results of four nested specifications of Campbell's ICAPM using three sets of portfolios formed according to (i) size, (ii) book-to-market, and (iii) past returns. Each set has three portfolios: the top 30 percentile, the next 40 percentile, and the bottom 30 percentile of the corresponding characteristic. Equations (16) and (17) are the common blocks for all specifications. Model III uses equation (12), a general case of Campbell's ICAPM with heteroskedastic stock returns. Model II is equation (13), a simplified heteroskedastic ICAPM. Model I is equation (14), the homoskedastic ICAPM. Model IV is equation (15) or the CAPM, in which we restrict the price of risk to zero for factors other than stock market risk. These specifications are nested and we show in the lower part of each panel the Newey and West (1987) D-test, as specified in equation (20). The White (1980) corrected t-statistics are reported in parentheses. The price of risk is directly estimated for models III and IV; it is calculated using equations (19) and (18) for models I and II, respectively, with the t-statistics from the delta method outlined by Campbell, et al. (1997). The OIR column reports the J-test by Hansen (1982). The instrument variables include a constant; the real stock market return,  $r_m$ ; realized stock market variance,  $\sigma_m^2$ ; the consumption-wealth ratio,  $cay$ ; and the stochastically detrended risk-free rate,  $rrel$ . We use a quarterly sample from 1952:Q4 to 2000:Q4 for the size and book-to-market portfolios, and from 1965:Q1 to 1998:Q4 for the momentum portfolios. \* Iterative GMM is not converged after 1,000 iterations. We use the point estimates from model II as the initial parameters and use 5 iterations. \*\* Iterative GMM is not converged after 1,000 iterations. We use the point estimates and the implied risk price from model II as the initial parameters, and we use 5 iterations. \*\*\* Given that iterative GMM is not converged for model III, we use the optimal weighting matrix from model II to calculate the D-test.

### Table 3

The table reports the estimation results of the unconditional Campbell ICAPM. That is, we use only a constant as the instrumental variable for equation (17), equation (14) for model I, equation (13) for model II, and equation (15) for model IV. Panel A through panel C use decile portfolios formed according to the corresponding characteristic, and panel D use three portfolios from each characteristic as discussed in the note on Table 2. Momentum data span the period 1965:Q1 to 1998:Q4, and the other portfolio data span the period 1952:Q4 to 2000:Q4. See the note on Table 2 for other details.

### Table 4

The table decomposes the average realized return according to their loadings on the four risk factors. The decomposition is based on the estimation results of model II reported in Table 3. Momentum data span the period 1965:Q1 to 1998:Q4, and the other portfolio data span the period 1952:Q4 to 2000:Q4.