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Worldwide Macroeconomic Stability and Monetary Policy Rules

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Abstract

We study the interaction of multiple large economies in dynamic stochastic general equilibrium. Each economy has a monetary policymaker that attempts to control the economy through the use of a linear nominal interest rate feedback rule. We show how the determinacy of worldwide equilibrium depends on the joint behavior of policymakers worldwide. We also show how indeterminacy exposes all economies to endogenous volatility, even ones where monetary policy may be judged appropriate from a closed economy perspective. We construct and discuss two quantitative cases. In the 1970s, worldwide equilibrium was characterized by a two-dimensional indeterminacy, despite U.S. adherence to a version of the Taylor principle. In the last 15 years, worldwide equilibrium was still characterized by a one-dimensional indeterminacy, leaving all economies exposed to endogenous volatility. Our analysis provides a rationale for a type of international policy coordination, and the gains to coordination in the sense of avoiding indeterminacy may be large.

Keywords: Indeterminacy, sunspot equilibrium, Taylor principle, great inflation.

JEL codes: E52, F41, F42.

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1 Introduction

1.1 Policy-induced endogenous volatility

It has been widely documented that the 1970s and early 1980s were characterized by substantially more macroeconomic volatility than the later 1980s or the 1990s in the major industrialized economies.[1] In an influential paper, Clarida, Gali, and Gertler (2000) explored the possibility that the earlier era might be viewed as a sunspot equilibrium induced by poor monetary policy. Their empirical results suggested that U.S. policymakers did not obey the *Taylor principle*[2] during this era, and their theoretical findings suggested that failure to obey the Taylor principle can be associated with indeterminacy of rational expectations equilibrium and the possibility of sunspot equilibria. Under this interpretation, the volatility observed during the 1970s was facilitated by poor policy.[3]

A natural question is how the Clarida, Gali, and Gertler (2000) analysis may be altered in an open economy setting. With several large economies interacting, determinacy of the resulting world equilibrium hinges on the joint actions of world policymakers. It is not very clear *a priori* how the determinacy conditions might be influenced by the nature of policy in each of the countries, the nature of the economic interactions between the economies, or the relative size of the economies involved. Our principle goal in this paper is to explore an international version of the Clarida, Gali, and Gertler (2000) argument. We use one of the recent extensions of the standard New Keynesian model to multiple, large industrialized economies. We seek to understand how the monetary policies in the various economies impinge on the determinacy of worldwide equilibrium.

In the closed economy literature, indeterminacy of rational expectations equilibrium has been viewed as an outcome to be avoided if at all possible.

[1] See, for instance, McConnell and Perez-Quiros (2000), van Dijk and Sensier (2004), Doyle and Faust (2005), Kim and Nelson (1999), and Stock and Watson (2003).

[2] For a discussion, see Woodford (2001, 2003).

[3] A related view, but one we do not explore in this paper, is the expectations trap hypothesis of Chari, Christiano, and Eichenbaum (1998).

This is because indeterminacy is associated with the existence of, at least potentially, quite volatile rational expectations equilibria in which the volatility is unrelated to the fundamental disturbances buffeting the economy.[4] We take the same view of the worldwide equilibrium studied in this paper. But because there are multiple policymakers in the international setting, we are also interested in the implications that might be drawn for international monetary policy coordination. The coordination can be designed, in our setting, primarily to avoid indeterminacy of worldwide equilibrium.

1.2 Main findings

We begin by showing how determinacy of worldwide equilibrium depends on the joint behavior of the world's policymakers in the model. We are able to relate the conditions for determinacy that apply in the open economy setting to certain conditions that are available from related closed economy analyses. We find that the open economy setting puts a relatively sharp upper bound on how aggressive each policymaker can be in its policy rule with respect to inflation deviations in order to remain consistent with determinacy. This finding is consistent with some of the related small open economy literature and suggests that analyses of major industrialized economies in closed economy settings—surely the bulk of the analysis to date in the large and rapidly growing New Keynesian literature—may be misleading from the perspective of the discussion of which types of monetary policy rules are consistent with equilibrium determinacy.

We are interested in the idea that policymakers in a large economy may be able to take a simple unilateral action to guarantee determinacy of worldwide equilibrium. For example, the monetary authority in a large economy might be able to adopt a policy rule of a specific sort that effectively coordinates expectations worldwide and renders worldwide equilibrium determinate, even in a situation where monetary policy in partner economies would be, by itself, inappropriate for generating a determinate worldwide

[4] See Woodford (1999, pp. 67-69) for a statement of this problem.

equilibrium. However, we find that in the model we study, the scope for one country to take a simple unilateral action to guarantee determinacy of world equilibrium is limited. It can be done, to be sure, in certain situations, but generally speaking if a foreign economy is pursuing a policy sufficiently inconsistent with determinacy, domestic policymakers would have no simple options that would render worldwide equilibrium determinate.[5] Instead, they would have to suffer with indeterminacy and the potential for endogenous volatility, or try to persuade the policymakers in the foreign economy to change their approach to policymaking. One may have the intuition, as we did, that policymakers in a large economy could adopt policies and influence macroeconomic adjustment to shocks in such a way as to avoid the worst types of exposure to endogenous volatility, but such is not the case in the economy we study. We discuss this and related results further in the main text.

When worldwide equilibrium is indeterminate, all countries are exposed to endogenous volatility. We are interested in understanding how this volatility plays out across the world economy. We simulate sunspot equilibria for several calibrated, three-country cases, and verify the extent to which endogenous volatility originating in one country can be transmitted across borders in each case. The dimension of indeterminacy can be as large as nine in a three country model, a clear change from the closed economy analysis. This means that multiple sunspot processes can be operating simultaneously, and in this sense the world economy can be exposed to endogenous volatility originating from many sources. We find that even economies in which policymakers pursuing what may be viewed as an appropriate policy—a policy rule consistent with determinacy when viewed from a closed economy perspective—are exposed to additional volatility in the sunspot equilibrium. Those pursuing inappropriate policies fare even worse. This finding suggests that policymakers from large economies running what appears from a

[5] In the class of models we study, leading examples of policies "sufficiently inconsistent with determinacy" include a policy rule which is too close to an interest rate peg, or a policy rule which is too aggressive with respect to inflation deviations from target or the output gap.

closed economy perspective to be very reasonable monetary policies may still have much to fear from the potential for endogenous volatility worldwide. This concern would be especially pronounced in cases where a large partner economy was pursuing a monetary policy inconsistent with worldwide equilibrium determinacy.

The model we analyze is not rich enough to match international data in a completely convincing way,[6] but in keeping with the provocative analysis of Clarida, Gali, and Gertler (2000), we end the paper with a consideration of some empirical estimates of monetary policy rules for the three largest economies in the 1970s, an era sometimes associated with indeterminacy, and in the 1990s, an era often described approvingly as being associated with better monetary policy worldwide. Using these estimates, our global perspective suggests that the earlier era was characterized by a two-dimensional indeterminacy in the worldwide equilibrium. The U.S. can actually be viewed as following a rule conducive to equilibrium determinacy, but still, because the partner countries were not, the world equilibrium would still have been indeterminate, leaving the U.S. as well as all other countries exposed to endogenous volatility. For the more recent era, worldwide equilibrium is characterized by a one-dimensional indeterminacy, so that the world economy is still exposed to endogenous volatility. Our point is to emphasize that calculations like these would clearly depend on joint policymaker behavior in the large economies, and may not be evaluated effectively in a closed economy model.

Finally, we stress the implications of our findings for concepts of international monetary policy coordination. The theoretical, fully optimal cooperative worldwide monetary policy has been worked out for the model we use.[7] But our paper is written from a positive perspective, and our empirical findings suggest that not every industrialized country has employed at each moment in time a monetary policy rule consistent with determi-

[6] For open economy estimates based on a richer model, see Lubik and Schorfiede (2005). Our model has the virtue of collapsing to the standard, simple version of Woodford (2003) when one of the economies is large and closed.

[7] See Clarida, Gali, and Gertler (2002).

nacy of worldwide equilibrium. Some have, to be sure, but others have not, and according to our estimates the major economies we look at have not simultaneously pursued policy sufficient to induce determinacy of worldwide equilibrium. One implication may be that the world's policymakers should not be content to let each large economy pursue monetary policy unilaterally. As we show, coordination in our model most likely means direct discussions with foreign policymakers in an attempt to convince them to follow a policy rule which will, in joint operation with other monetary policies, generate a unique rational expectations equilibrium worldwide. Viewed from this perspective, the gains from international monetary policy coordination may be large.

1.3 Recent related literature

We are not the first authors to study indeterminacy in an open economy setting. A number of papers have addressed determinacy issues and the connection to monetary policy for small open economies. De Fiore and Liu (2005) study a small open economy version of Cooley and Hansen (1989). They find that monetary policy rules associated with determinacy in the closed economy may not be associated with determinacy in the open economy. We have similar results for our model with large economies interacting. The De Fiore and Liu (2005) findings contrast with earlier work by Carlstrom and Fuerst (1999) for a small open economy, which suggested that determinacy conditions were largely unaffected by trade openness. But Carlstrom and Fuerst (1999) did not have the terms of trade effects that play an important role in De Fiore and Liu (2005) and in the present paper. Zanna (2003) also works in a small open economy setting, in continuous time and under alternative assumptions relative to the present paper. He finds that the degree of openness and the degree of exchange rate pass-through are key factors for generating a determinate equilibrium under a given monetary policy rule. Linnemann and Schabert (2004) work in continuous time as well and with a somewhat different small open economy model. They also find that equilibrium determinacy generally depends on the degree of trade openness.

Batini, Levine, and Pearlman (2004) discuss indeterminacy issues in a symmetric two-bloc model related to ours. The model used by Batini, *et al.*, (2004) is a more elaborate, two-country version of the n-country model we use. The focus of Batini, *et al.* (2004) is to use root-locus methods to analyze how equilibrium determinacy is related to the forecast horizon of policymakers that react to expected inflation alone with policy inertia. They find, as we do, that there is an upper bound on how aggressive policymakers can be if the goal is to generate determinacy of worldwide equilibrium, and they also argue that longer forecast horizons tend to be associated with indeterminacy. The forecast horizon is fixed at one in our model to maintain comparability to Clarida, Gali, and Gertler (2000). The related paper by Batini, Justiniano, Levine, and Pearlman (2006) contains an analysis of optimally robust monetary policy rules in a closely related environment. Bullard and Schaling (2005) discuss both determinacy and learnability in a two country model similar to the one used here, but from a purely theoretical perspective, and considering a wide variety of monetary policy rules, including targeting rules and situations of asymmetric policy. Also, we do not examine the learning issue in this paper. For a closed economy analysis of that question, see Honkapohja and Mitra (2004).

There is a large literature on international monetary policy cooperation. Benigno and Benigno (2006a), for instance, work in a context similar to the one used here, and show that in general there are theoretical gains from international policy cooperation. This type of result also occurs in our model, as discussed by Clarida, Gali, and Gertler (2002). Benigno and Benigno (2006b) discuss strategic aspects of monetary policy cooperation, showing how the optimal cooperative allocations can be implemented when each country follows an inflation targeting regime. Similar themes are contained in Obstfeld and Rogoff (2002). They study a symmetric two-country model related to the one in this paper, and find plausible conditions under which the Nash equilibrium of a game in which each participant is choosing a monetary policy rule approximates the fully optimal cooperative equilibrium. Corsetti and Pesenti (2005) also work in a model related to the one in this paper, and find that the degree of exchange rate exposure is a key determinant of whether purely inward-looking policies or ones that involve international cooperation are to be preferred.

There is also a large literature on the estimation of Taylor-type monetary policy rules which we cannot effectively summarize here. One related line of research stems from Orphanides (2001), who has argued that monetary policy rules should be estimated using only data available to policymakers at the time that policy actions are being decided. Orphanides (2005) re-estimates the Clarida, Gali, and Gertler (2000) policy rules using real-time data. He argues that the real-time estimates are consistent with the Federal Reserve abiding by the Taylor principle, so that there was no risk of indeterminacy. We do not use real time data in this paper to obtain our estimates across three countries and two time periods. However, our baseline estimates are consistent with Orphanides (2005) for the U.S. in the 1970s, in that we obtain a sufficiently strong reaction to deviations of inflation from target that we would conclude that the policy rule in use was consistent with equilibrium determinacy, if the economy was closed. The two-dimensional indeterminacy we find for this period is due to monetary policy in the partner economies, not the U.S.

Lubik and Schorfheide (2004) estimate closed economy, one-dimensional sunspot equilibria directly based on subsamples of U.S. postwar data. We think it would be interesting to apply this methodology to our multi-economy, multi-dimensional setting. Belaygorod, Chib, and Dueker (2006) extend the Lubik and Schorfheide closed economy analysis to a fully dynamic, full sample approach in which the economy can switch between determinacy and indeterminacy. Beyer and Farmer (2004) discuss identification in the Lubik and Schorfheide (2004) context.

1.4 Organization

In the next section we present the model we use in this paper. We then turn to a discussion of determinacy conditions for worldwide equilibrium, and we relate these conditions to analogous findings in closed economy settings. In the subsequent section we explore how all variables worldwide can be volatile as part of a sunspot equilibrium, even in those countries which, from a closed economy perspective, are pursuing policies consistent with determinacy of rational expectations equilibrium. In the final section we turn to estimates of policy rules in the spirit of Clarida, Gali, and Gertler (2000). We combine these estimates with a simple calibration, which suggests that worldwide equilibrium was indeterminate in the 1970s and in fact remains indeterminate today.

2 Environment

2.1 An open economy new Keynesian model

2.1.1 Preliminaries

The new Keynesian model has been extended to the international context in a variety of ways, and any of these extensions could be used for the purpose at hand. We choose to use an n country version of the model of Clarida, Gali, and Gertler (2002). This particular extension has several advantages from our point of view. The chief among these is that the model collapses to the simplest New Keynesian closed economy as a special case. We will use this feature extensively to understand how determinacy conditions are influenced by open economy considerations and to calibrate the model in a way that can be related to existing, closed economy literature.

The model itself is not the focus of this paper. Our goal in this section is to keep the paper self-contained by giving the reader sufficient information to understand all of the main assumptions.

The world economy consists of n countries. We sometimes refer to country one as the domestic or home country, with the remaining $n-1$ countries designated as "foreign." Each country is populated by a continuum of households indexed by $h \in [0,1]$. Households in all countries consume differentiated goods produced by firms of all n countries. World population is normalized to unity, and each country has a mass γ_j, $j = 1, ..., n$, where

$0 \leq \gamma_j \leq 1$, and $\sum_{j=1}^{n} \gamma_j = 1$. There is no population growth. Apart from the potential difference in size (if $\gamma_j \neq 1/n$) all the countries are the same in terms of preferences and technologies. Each country has a monopolistically competitive intermediate goods producing sector which uses labor as an input. Nominal prices are sticky in the intermediate goods sector in the sense of Calvo (1983). The number of intermediate goods producers in each country is normalized to unity. The number of final goods producing firms in each country is equal to the number of households. Final goods producers are perfectly competitive and take intermediate goods as inputs. We assume the law of one price holds.

2.1.2 Households

Preferences Households live forever and maximize utility defined over consumption and leisure. A household in country j maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t^j) - V\left[N_t^j(h) \right] \right\}, \tag{1}$$

where

$$U\left(C_t^j\right) - V\left[N_t^j(h)\right] = \frac{(C_t^j)^{(1-\sigma)}}{1-\sigma} - \frac{N_t^j(h)^{1+\phi}}{1+\phi},$$

with $\sigma > 0$, $\phi > 0$, and where $\beta \in (0,1)$ is the discount factor. In this expression, C_t^j is an index of country j consumption and $N_t^j(h)$ represents the labor supply of household h in country j. The consumption index is given by

$$C_t^j = \prod_{k=1}^{n} C_{j,k,t}^{\gamma_k}, \tag{2}$$

for $j = 1, ..., n$, where $C_{j,k,t}$ is the consumption of the final good from country k consumed by households in country j at time t. The parameter γ_k, $k = 1, 2, ..., n$ controls not only the size of a country but also its degree of openness. If $\gamma_k \to 0$ for all $k \geq 2$, the home country large and closed, all other countries are vanishingly small and open, and all consumers worldwide consume the goods produced by the home country. If $\gamma_k \to 1$ for some k, then that foreign country is large and closed and all other economies are vanishingly small and open. The consumption index C_t^j is associated with a consumption price index in country j given by

$$P_{j,t}^C = \mathcal{K}^{-1} \prod_{k=1}^{n} P_{j,k,t}^{\gamma_k}, \tag{3}$$

where $P_{j,k,t}$ is the producer price of foreign good k in country j at time t, and where the constant $\mathcal{K}$ is given by $\mathcal{K} \equiv \prod_{k=1}^{n} \gamma_k^{\gamma_k}$.

Budget constraints The sequence of household budget constraints can be expressed in nominal terms. Let $W_t^j(h)$ represent the nominal wage associated with the labor supply $N_t^j(h)$ of household h in country j. The asset structure consists of complete, contingent, one period nominal bonds denominated in country one currency. Let D_{t+1}^j denote the country j consumer's holding of such a bond purchased at date t which yields a payoff at date $t+1$. Let $\hat{Q}_{t,t+1}^j$ be the stochastic discount factor in country j. Finally, let T_t^j denote lump sum taxes and Γ_t^j denote lump sum profits accruing from ownership of intermediate goods firms in country j. Then households maximize their expected lifetime utility (1) subject to a sequence of budget constraints

$$P_{j,t}^C C_t^j + E_t\{\hat{Q}_{t,t+1}^j D_{t+1}^j\} = W_t^j(h) N_t^j(h) + D_t^j - T_t^j + \Gamma_t^j$$

for $t = 0, ..., \infty$.

Labor supply In each country, each household faces a constant elasticity demand function for its labor services

$$N_t^j(h) = \left(\frac{W_t^j(h)}{W_t^j} \right)^{-\eta_t} N_t^j, \tag{4}$$

where N_t^j is per capita employment in country j, and

$$W_t^j \equiv \left(\int_0^1 W_t^j(h)^{1-\eta_t} dh \right)^{1/(1-\eta_t)}$$

is the associated aggregate wage index in country j. The elasticity of labor demand, η_t, is the same across workers, but may vary over time. The first-order condition for labor supply implies

$$\frac{W_t^j(h)}{P_{j,t}^C} = (1+\mu_t^w)\left[N_t^j(h)\right]^{\phi}\left(C_t^j\right)^{\sigma}, \tag{5}$$

where $\mu_t^w = 1/\eta_t - 1$ is the optimal wage markup. Wages are perfectly flexible. We follow Clarida, Gali, and Gerter (2002) and allow for exogenous variation in the wage markup arising from shifts in η_t, interpretable as exogenous variation in workers' market power.[8] Because wages are flexible, all workers will charge the same wage and work the same level of hours. Thus we can write

$$\begin{aligned} W_t^j(h) &= W_t^j \\ N_t^j(h) &= N_t^j \end{aligned}$$

for all h and all t.

Consumption The first-order necessary conditions for consumption for country j are

$$P_{j,i,t}C_{j,i,t} = \gamma_i P_{j,t}^C C_t^j, \tag{6}$$

where $i = 1, 2, ..., n$, and

$$\beta\left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\sigma}\left(\frac{P_{j,t}^C}{P_{j,t+1}^C}\right) = \hat{Q}_{t,t+1}^j. \tag{7}$$

Taking expectations we obtain

$$\beta R_t^j E_t\left\{\left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\sigma}\left(\frac{P_{j,t}^C}{P_{j,t+1}^C}\right)\right\} = 1,$$

where R_t^j denotes the gross nominal yield on a one-period discount bond in country j, the inverse of the expected value of the stochastic discount factor.

[8] We have kept this feature for comparability but we do not use it in the analysis that follows.

2.1.3 Final goods producers

Each final goods firm uses a continuum of intermediate goods to produce output, according to the CES technology

$$Y_t^j = \left(\int_0^1 Y_t^j(f)^{(\xi-1)/\xi} df \right)^{\xi/(\xi-1)} \tag{8}$$

where Y_t^j denotes aggregate output in country j, while $Y_t^j(f)$ is the input produced by intermediate goods firm f. Both variables are expressed in per capita terms. Profit maximization, taking the price of the final good as given, implies the following set of demand equations for intermediate goods,

$$Y_t^j(f) = \left(\frac{P_{j,j,t}(f)}{P_{j,j,t}} \right)^{-\xi} Y_t^j, \tag{9}$$

and associated producer price index in country j

$$P_{j,j,t} = \left(\int_0^1 P_{j,j,t}(f)^{1-\xi} df \right)^{1/(1-\xi)} .$$

2.1.4 Intermediate goods producers

Each intermediate goods firm produces output using a technology that is linear in labor input

$$Y_t^j(f) = A_t N_t^j(f),$$

where A_t is an exogenous technology parameter which is the same across all countries and $N_t^j(f)$ (normalized by population size), is the labor used by each firm. It is a CES composite of individual household labor, where each household is a monopolistically competitive supplier of labor

$$N_t^j(f) = \left(\int_0^1 N_t^j(h)^{(\eta_t-1)/\eta_t} dh \right)^{\eta_t/(\eta_t-1)} .$$

Intermediate goods firms set prices on a staggered basis as in Calvo (1983), where θ is the probability a firm keeps its price fixed in a given period and

$(1-\theta)$ is the probability it changes it, where probability draws are *i.i.d.* over time. The cost minimization problem of the firm yields

$$MC_t^j = \frac{(1-\tau)(W_t^j/P_{j,j,t})}{A_t}, \tag{10}$$

where MC_t^j denotes real marginal cost and τ percent of the wage bill is the subsidy that each firm receives. Since in equilibrium each household charges the same wage and supplies the same number of hours, we can treat the firm's decision problem over total labor demand as just involving the aggregates $N_t(f)$ and W_t.

The pricing problem in period t involves choosing the reset price $P_{j,j,t}^0$ to maximize

$$E_t \sum_{s=0}^{\infty} \theta^s \hat{Q}_{t,t+s}^j Y_{t+s}^j(f)(P_{j,j,t}^0 + P_{j,j,t+s} MC_{t+s}^j)$$

subject to the demand curve (9). The solution to this problem implies that firms set their price equal to a discounted stream of expected future nominal marginal cost

$$E_t \sum_{s=0}^{\infty} \theta^s \hat{Q}_{t,t+s}^j Y_{t+s}^j(f)(P_{j,j,t}^0 - (1+\mu^p) P_{j,j,t+s} MC_{t+s}^j) = 0 \tag{11}$$

where $\mu^p = 1/(\theta - 1)$. The law of large numbers implies that the country j price index evolves according to

$$P_{j,j,t} = [\theta(P_{j,j,t-1})^{1-\xi} + (1-\theta)(P_{j,j,t}^0)^{1-\xi}]^{1/(1-\xi)}. \tag{12}$$

2.2 Equilibrium, monetary policy, and dynamics

2.2.1 Linearization

We follow Clarida, Gali, and Gertler (2002) and represent the equilibrium of this economy in terms of logarithmic deviations from steady state values. We let $x = \ln X$, and write these equations as

$$\tilde{y}_t^j = E_t \tilde{y}_{t+1}^j - \sigma_{j,o}^{-1} \left[r_t^j - E_t \pi_{t+1}^j - \overline{rr}_t^j \right], \tag{13}$$

$$\pi_t^j = \beta E_t \pi_{t+1}^j + \lambda_{j,o} \tilde{y}_t^j + u_t^j. \tag{14}$$

for each economy $j = 1, ..., n$, where u_t^j is the marginal cost shock in country j. We supplement these equations with a policy rule determining the deviation of the nominal interest rate from the level consistent with inflation at target and output at potential, r_t^j, in the next subsection. In these equations, $\tilde{y}_t^j$ is the output gap, π_t^j is the deviation of the producer price inflation rate from a target set by the monetary authority. The parameters $\sigma_{j,o}$ and $\lambda_{j,o}$ are conglomerates of underlying parameters, defined as follows:

$$\sigma_{j,o} = \sigma - \kappa_{j,o} \tag{15}$$

where $\kappa_{j,0} \equiv (1-\gamma_j)(\sigma-1)$. If $\gamma_j \to 1$, which is to say that country j is large and all other countries are vanishingly small, then $\kappa_{j,o} \to 0$ and $\sigma_{j,o} = \sigma$, which is as it would be for a closed economy. Also,

$$\lambda_{j,o} = \delta \kappa_{j,j} \tag{16}$$

where

$$\kappa_{j,j} = \sigma + \phi - \kappa_{j,o},$$

and

$$\delta = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

Again, if $\gamma_j \to 1$ so that $\kappa_{j,o} \to 0$, the value of $\lambda_{j,o} \to \delta(\sigma + \phi)$ which is the same as one would infer for the closed economy model.

The term $\overline{rr}_t^j$ is the country j natural real interest rate (conditional on foreign output), given by

$$\overline{rr}_t^j = \sigma_{j,o} E_t \Delta \bar{y}_{t+1}^j + \sum_{s=1,\ s \neq j}^{n} \kappa_{j,s} E_t \{\Delta y_{t+1}^s\},$$

where $\Delta \bar{y}^j$ is the growth rate of the flexible price level of output in country j, Δy^s is the growth rate of the level of output in foreign economy s, and

$$\kappa_{j,s} = \gamma_s (\sigma - 1).$$

2.2.2 Monetary policy

We specify the policy rule to be forward looking with interest rate smoothing. We assume that each country pursues a Taylor-type rule featuring consumer price index, or CPI, inflation. This is intuitively plausible as in an open economy CPI inflation, not domestic producer price inflation, is often the variable of interest for the monetary authority. We show that targeting CPI inflation is equivalent to having a conventional Taylor-type rule augmented by a third term which is the *terms of trade*. The monetary policy rule in country j is given by

$$r_t^j = \varphi_\pi^j E_t \pi_{j,t+1}^C + \varphi_y^j E_t \tilde{y}_{t+1}^j + \varphi_r^j r_{t-1}^j, \tag{17}$$

where the deviation of consumer price inflation from target in country j is

$$\pi_{j,t}^C = \pi_t^j + \sum_{i=1, i \neq j}^{n} \gamma_i \Delta s_{j,i,t}, \tag{18}$$

$\Delta s_{j,i,t}$ the rate of change of the terms of trade between countries j and i, and where φ_π^j, φ_y^j, and φ_r^j are the policymaker coefficients on expected consumer price inflation deviations, the output gap, and the lagged interest rate, respectively. Using (18) in (17) implies that country j policymakers effectively respond to both the expected domestic inflation rate and to the expected rate of change of the terms of trade

$$r_t^j = \varphi_\pi^j E_t \pi_{t+1}^j + \varphi_y^j E_t \tilde{y}_{t+1}^j + \varphi_\pi^j \sum_{i=1, i \neq j}^{n} \gamma_i E_t \Delta s_{j,i,t+1} + \varphi_r^j r_{t-1}^j. \tag{19}$$

In the special case where the country j economy is closed, CPI and domestic inflation are equivalent and the domestic central bank simply responds to domestic inflation.

The rule (19) can be further simplified. The terms of trade between countries j and i can be written as

$$s_{j,i,t+1} = y_{t+1}^j - y_{t+1}^i,$$

so that

$$\Delta s_{j,i,t+1} = s_{j,i,t+1} - s_{j,i,t} = (y^j_{t+1} - y^j_t) - \left(y^i_{t+1} - y^i_t\right).$$

Since $\tilde{y}^j_{t+1} = y^j_{t+1} - \bar{y}^j_{t+1}$, we can re-write the above equation in terms of differences in the output gap between countries

$$\Delta s_{j,i,t+1} = \left[(\tilde{y}^j_{t+1} + \bar{y}^j_{t+1}) - (\tilde{y}^j_t + \bar{y}^j_t)\right] - \left[(\tilde{y}^i_{t+1} + \bar{y}^i_{t+1}) - \left(\tilde{y}^i_t + \bar{y}^i_t\right)\right].$$

Rearranging the terms we obtain

$$\Delta s_{j,i,t+1} = (\tilde{y}^j_{t+1} - \tilde{y}^i_{t+1}) - (\tilde{y}^j_t - \tilde{y}^i_t) + (\bar{y}^j_{t+1} - \bar{y}^i_{t+1}) - (\bar{y}^j_t - \bar{y}^i_t),$$

and letting $\Delta \bar{s}_{j,i,t+1} = (\bar{y}^j_{t+1} - \bar{y}^i_{t+1}) - (\bar{y}^j_t - \bar{y}^i_t)$ be the rate of change of the natural level of the terms of trade between country j and i,

$$\Delta s_{j,i,t+1} = (\tilde{y}^j_{t+1} - \tilde{y}^i_{t+1}) - (\tilde{y}^j_t - \tilde{y}^i_t) + \Delta \bar{s}_{j,i,t+1}.$$

We conclude that the monetary policy rule is given by

$$\begin{aligned} r^j_t = {} & \varphi^j_\pi E_t \pi^j_{t+1} + \varphi^j_y E_t \tilde{y}^j_{t+1} + \varphi^j_r r^j_{t-1} \\ & + \varphi^j_\pi \sum_{i=1, i\neq j}^{n} \gamma_i E_t \left[(\tilde{y}^j_{t+1} - \tilde{y}^i_{t+1}) - (\tilde{y}^j_t - \tilde{y}^i_t) + \Delta \bar{s}_{j,i,t+1}\right]. \end{aligned}$$

This last term is the expected rate of change of output gap differentials between country j and all other economies, appropriately weighted by country size. For the dynamic system, it will be convenient to rearrange and simplify term to write the rule as

$$\begin{aligned} r^j_t = {} & \varphi^j_\pi E_t \pi^j_{t+1} + \varphi^j_y E_t \tilde{y}^j_{t+1} + \varphi^j_r r^j_{t-1} \\ & + \sum_{i=1, i\neq j}^{n} \varphi^j_{s,i} (E_t \tilde{y}^j_{t+1} - \tilde{y}^j_t) - \sum_{i=1, i\neq j}^{n} \varphi^j_{s,i} (E_t \tilde{y}^i_{t+1} - \tilde{y}^i_t) \\ & + \sum_{i=1, i\neq j}^{n} \varphi^j_{s,i} E_t \Delta \bar{s}_{j,i,t+1}, \end{aligned}$$

where $\varphi^j_{s,i} = \varphi^j_\pi \gamma_i$.

2.2.3 Dynamic system

Three countries We wish to apply the model to a three country world setting. With the specified monetary policies, the general formulation of the dynamic system has the following reduced form

$$B_{11}\mathcal{X}_t^1 = B_{12}E_t\mathcal{X}_{t+1}^1 + B_{13}\mathcal{X}_t^2$$

$$\mathcal{X}_t^2 = R\mathcal{X}_{t-1}^1 + S\mathcal{X}_{t-1}^2 + U_t$$

where $\mathcal{X}_t^1$ is a 9×1 vector of free variables, $\mathcal{X}_t^2$ is a 12×1 vector of predetermined variables,[9] U_t is a 12×1 vector of fundamental disturbances, and R and S are conformable matrices. The vectors are given by

$$\mathcal{X}_t^1 = \left[\tilde{y}_t^1, \pi_t^1, r_t^1, \tilde{y}_t^2, \pi_t^2, r_t^2, \tilde{y}_t^3, \pi_t^3, r_t^3\right]',$$

and

$$\mathcal{X}_t^2 = \left[r_{t-1}^1, g_t^1, u_t^1, w_t^1, r_{t-1}^2, g_t^2, u_t^2, w_t^2, r_{t-1}^3, g_t^3, u_t^3, w_t^3\right]',$$

where

$$\begin{aligned} g_t^j &= \sigma_{j,o}^{-1}\overline{rr}_t^j, \\ w_t^j &= \sum_{i=1, i\neq j}^{3} \varphi_{s,i}^j E_t \Delta\bar{s}_{j,i,t+1}, \end{aligned}$$

and where fundamental shocks are defined by

$$\begin{aligned} g_t^j &= \rho_g g_{t-1}^j + \epsilon_{g,t}^j, \\ u_t^j &= \rho_u u_{t-1}^j + \epsilon_{u,t}^j, \\ w_t^j &= \rho_w w_{t-1}^j + \epsilon_{w,t}^j, \end{aligned}$$

with $\epsilon_g^j, \epsilon_u^j$, and ϵ_w^j *i.i.d.*, and

$$U_t = \left[0, \epsilon_g^1, \epsilon_u^1, \epsilon_w^1, 0, \epsilon_g^2, \epsilon_u^2, \epsilon_w^2, 0, \epsilon_g^3, \epsilon_u^3, \epsilon_w^3\right].$$

[9] We follow Blanchard and Kahn (1980) in using this terminology.

The matrix B_{11} is

$$B_{11} = \begin{bmatrix} B_{1,1}(1,1) & B_{1,1}(1,2) & B_{1,1}(1,3) \\ B_{1,1}(2,1) & B_{1,1}(2,2) & B_{1,1}(2,3) \\ B_{1,1}(3,1) & B_{1,1}(3,2) & B_{1,1}(3,3) \end{bmatrix}$$

where the diagonal matrices are given by

$$B_{11}(j,j) = \begin{bmatrix} 1 & 0 & \sigma_{j,0}^{-1} \\ -\lambda_{j,0} & 1 & 0 \\ \sum_{i=1,i\neq j}^{3} \varphi_{s,i}^{j} & 0 & 1 \end{bmatrix}$$

and all the off diagonal matrices for all i and j are given by

$$B_{11}(j,i) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\varphi_{s,i}^{j} & 0 & 0 \end{bmatrix}.$$

The matrix B_{12} is

$$B_{12} = \begin{bmatrix} B_{12}(1,1) & B_{12}(1,2) & B_{12}(1,3) \\ B_{12}(2,1) & B_{12}(2,2) & B_{12}(2,3) \\ B_{12}(3,1) & B_{12}(3,2) & B_{12}(3,3) \end{bmatrix}$$

where the diagonal matrices are given by

$$B_{12}(j,j) = \begin{bmatrix} 1 & \sigma_{j,0}^{-1} & 0 \\ 0 & \beta & 0 \\ \varphi_{y}^{j} + \sum_{i=1,i\neq j}^{3} \varphi_{s,i}^{j} & \varphi_{\pi}^{j} & 0 \end{bmatrix}$$

and all the off diagonal matrices for all i and j are given by

$$B_{12}(j,i) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\varphi_{s,i}^{j} & 0 & 0 \end{bmatrix}.$$

The matrix B_{13} is a 9×12 matrix given by

$$B_{13} = \begin{bmatrix} B_{13}(1,1) & 0 & 0 \\ 0 & B_{13}(2,2) & 0 \\ 0 & 0 & B_{13}(3,3) \end{bmatrix}$$

where

$$B_{13}(j,j) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \varphi_{r}^{j} & 0 & 0 & 1 \end{bmatrix}.$$

Redefining the matrices such that $B_1 = (B_{11})^{-1}B_{12}$ and $C = (B_{11})^{-1}B_{13}$ we obtain

$$\mathcal{X}_t^1 = B_1 E_t \mathcal{X}_{t+1}^1 + C\mathcal{X}_t^2$$

and

$$\mathcal{X}_t^2 = R\mathcal{X}_{t-1}^1 + S\mathcal{X}_{t-1}^2 + U_t.$$

An autoregressive representation We follow Evans and Honkapohja (2001) and Honkapohja and Mitra (2004). Let $\eta_{t+1} = \mathcal{X}_{t+1}^1 - E_t\mathcal{X}_{t+1}^1$, and write the dynamic system as

$$\mathcal{X}_t^1 = B_1\mathcal{X}_{t+1}^1 + C\mathcal{X}_t^2 - B_1\eta_{t+1} \tag{20}$$

$$\mathcal{X}_{t+1}^2 = R\mathcal{X}_t^1 + S\mathcal{X}_t^2 + U_{t+1} \tag{21}$$

Thus the dynamic system can be written as a vector autoregressive process

$$\begin{bmatrix} \mathcal{X}_t^1 \\ \mathcal{X}_t^2 \end{bmatrix} = J \begin{bmatrix} \mathcal{X}_{t+1}^1 \\ \mathcal{X}_{t+1}^2 \end{bmatrix} + L \begin{bmatrix} U_{t+1} \\ \eta_{t+1} \end{bmatrix} \tag{22}$$

where

$$J = \begin{bmatrix} I & -C \\ R & S \end{bmatrix}^{-1} \begin{bmatrix} B_1 & 0 \\ 0 & I \end{bmatrix} \tag{23}$$

and

$$L = \begin{bmatrix} I & -C \\ R & S \end{bmatrix}^{-1} \begin{bmatrix} 0 & -B_1 \\ -I & 0 \end{bmatrix} \tag{24}$$

The rational expectations equilibrium is determinate in the sense of Blanchard and Kahn (1980) if the number of eigenvalues of J inside the unit circle equals the number of free variables (the dimension of $\mathcal{X}^1$). If instead the number of eigenvalues inside the unit circle is less than the number of free variables in the model, equilibrium is indeterminate.

Stationary non-fundamental equilibria We now write the system in a form that characterizes non-fundamental equilibria. We will use this form to simulate some sunspot equilibria. Let matrix J be diagonalizable. Then we have that $Q^{-1}JQ = \Lambda$ and we partition Q^{-1} and $(\mathcal{X}_t^{1'}, \mathcal{X}_t^{2\prime})'$ such that

$$Q^{-1} = \begin{bmatrix} Q^{11}(1,1) & Q^{11}(1,2) & Q^{12}(1) \\ Q^{11}(2,1) & Q^{11}(2,2) & Q^{12}(2) \\ Q^{21}(1) & Q^{21}(2) & Q^{22} \end{bmatrix}$$

and

$$\begin{bmatrix} \mathcal{X}_t^1 \\ \mathcal{X}_t^2 \end{bmatrix} = \begin{bmatrix} \mathcal{X}_t^{1,*} \\ \mathcal{X}_t^{1,\#} \\ \mathcal{X}_t^2 \end{bmatrix}.$$

The system then becomes

$$Q^{-1} \begin{bmatrix} \mathcal{X}_t^{1,*} \\ \mathcal{X}_t^{1,\#} \\ \mathcal{X}_t^2 \end{bmatrix} = \begin{bmatrix} \Lambda_1^* & 0 & 0 \\ 0 & \Lambda_1^\# & 0 \\ 0 & 0 & \Lambda_2 \end{bmatrix} Q^{-1} \begin{bmatrix} \mathcal{X}_{t+1}^{1,*} \\ \mathcal{X}_{t+1}^{1,\#} \\ \mathcal{X}_{t+1}^2 \end{bmatrix} + \begin{bmatrix} \bar{L}_1^* \\ \bar{L}_1^\# \\ \bar{L}_2 \end{bmatrix} \nu_{t+1}$$

where $\nu_{t+1}' = (U_{t+1}', \eta_{t+1}')$ and $\bar{L} = Q^{-1}L$. The matrix Λ_1^* contains eigenvalues with modulus less than one whereas $\Lambda_1^\#$ and Λ_2 contain eigenvalues with modulus greater than one. We exclude cases where the values lie on the unit circle. The allocation of roots between $\Lambda_1^\#$ and Λ_2 is not unique but here we follow the standard procedure and arrange eigenvalues in a nondecreasing order of modulus.

Consideration of the first and second blocks of equations gives us conditions that rule out explosive solutions. Define

$$\begin{bmatrix} p_t^* \\ p_t^\# \end{bmatrix} = \begin{bmatrix} Q^{11}(1,1) & Q^{11}(1,2) \\ Q^{11}(2,1) & Q^{11}(2,2) \end{bmatrix} \begin{bmatrix} \mathcal{X}_t^{1,*} \\ \mathcal{X}_t^{1,\#} \end{bmatrix} + \begin{bmatrix} Q^{12}(1) \\ Q^{12}(2) \end{bmatrix} \mathcal{X}_t^2.$$

Then

$$\begin{bmatrix} p_{t+1}^* \\ p_{t+1}^\# \end{bmatrix} = \begin{bmatrix} \Lambda_1^* & 0 \\ 0 & \Lambda_1^\# \end{bmatrix}^{-1} \begin{bmatrix} p_t^* \\ p_t^\# \end{bmatrix} - \begin{bmatrix} \Lambda_1^* & 0 \\ 0 & \Lambda_1^\# \end{bmatrix}^{-1} \begin{bmatrix} \bar{L}_1^* \\ \bar{L}_1^\# \end{bmatrix} \nu_{t+1}.$$

For the first block, that is p_t^*, the i^{th} component is

$$p_{t+1}^{*,i} = \left(\lambda_1^{*,i}\right)^{-1} p_t^{*,i} - \left(\lambda_1^{*,i}\right)^{-1} \bar{L}_1^{*,i} \nu_{t+1},$$

provided $\lambda_1^{*,i} \neq 0$. Taking expectations, $E_t p_{t+1}^{*,i} = \left(\lambda_1^{*,i}\right)^{-1} p_t^{*,i}$ and $E_t p_{t+s}^{*,i} = \left(\lambda_1^{*,i}\right)^{1-s} E_t p_{t+1}^{*,i}$ for all $s > 1$. Since $\left|\lambda_1^{*,i}\right| < 1$, $E_t p_{t+s}^{*,i} \to \infty$ as $s \to \infty$ unless $E_t p_{t+s}^{*,i} = 0$. Hence the class of stationary solutions satisfies $p_t^{*,i} = 0$ for all i, which implies that $p_t^* = 0$. This translates into the following restriction for stationary equilibria

$$Q^{11}(1,1)\mathcal{X}_t^{1,*} + Q^{11}(1,2)\mathcal{X}_t^{1,\#} + Q^{12}(1)\mathcal{X}_t^2 = 0$$

Since $\left|\lambda_1^{\#}\right| > 1$, $E_t p_{t+s}^{*,i} \to 0$ as $s \to \infty$ and therefore we do not obtain any restrictions from the second block. Then

$$p_t^{\#} = \left(\Lambda_1^{\#}\right)^{-1} p_{t-1}^{\#} - \left(\Lambda_1^{\#}\right)^{-1} \bar{L}_1^{\#}\nu_t.$$

Combining the two we can write the class of stationary autoregressive sunspot equilibria as

$$\begin{bmatrix} \mathcal{X}_t^{1,*} \\ \mathcal{X}_t^{1,\#} \end{bmatrix} = \left(Q^{11}\right)^{-1} \begin{bmatrix} -Q^{12}(1)\mathcal{X}_t^2 \\ z_t \end{bmatrix},$$

where

$$\begin{aligned} z_t = {} & -Q^{12}(2)\mathcal{X}_t^2 - \left(\Lambda_1^{\#}\right)^{-1} Q^{11}(2,1)\mathcal{X}_{t-1}^{1,*} \\ & + \left(\Lambda_1^{\#}\right)^{-1} Q^{12}(2)\mathcal{X}_{t-1}^2 + \left(\Lambda_1^{\#}\right)^{-1} Q^{11}(2,2)\mathcal{X}_{t-1}^{1,\#} - \left(\Lambda_1^{\#}\right)^{-1} \bar{L}_1^{\#}\nu_t. \end{aligned}$$

3 Determinacy of worldwide equilibrium

3.1 Closed economy benchmarks

We begin by analyzing determinacy of worldwide equilibrium, focusing primarily on how the determinacy of worldwide equilibrium is affected by the monetary policies of the various countries. This depends on the matrix J. An analogous closed economy case with monetary policy inertia has been analyzed by Bullard and Mitra (2006). They show that the necessary conditions for determinacy with the forward looking policy rule[10] are, in our

[10] In the monetary policy rule studied by Bullard and Mitra (2006), the central bank targets domestic inflation, which would correspond to producer price index inflation in the present model. But, in a closed economy there is no distinction between PPI and CPI.

notation,

$$\lambda(\varphi_\pi + \varphi_r - 1) + (1-\beta)\varphi_y > 0 \tag{25}$$

$$\left[\lambda\sigma^{-1} + 2(1+\beta)\right]\varphi_r + 2(1+\beta) > \sigma^{-1}\left[\lambda\left(\varphi_\pi - 1\right) + (1+\beta)\varphi_y\right] \tag{26}$$

if $\varphi_y < 2\sigma^{-1}$ (see proposition 3 in Bullard and Mitra (2006)). In addition, if $\varphi_r \geq 1$, then the necessary and sufficient conditions are given by (26).

Some intuition for these conditions is available if we consider the case of $\varphi_y = 0$, that is, the policymaker does not react to the output gap in the policy rule. In this situation, given that $\lambda > 0$, condition (25) becomes $\varphi_\pi + \varphi_r > 1$, a modification of the oft-stated *Taylor principle* condition that policymakers must react more than one-for-one to deviations of inflation from target in order to generate a determinate rational expectations equilibrium.

However, because condition (26) must also be met, it is not sufficient to think only in terms of this modified Taylor principle. Condition (26) puts meaningful quantitative restrictions on what rules the policymaker can adopt and remain consistent with determinacy of rational expectations equilibrium. At baseline parameter values, including standard estimated values for φ_r, Bullard and Mitra (2006) found that φ_y had to be relatively low, less than about 0.5, and that there would be an upper bound on the value of φ_π, but that this upper bound was large enough that it would not be relevant for policy discussions. If we use our baseline parameter values discussed below and assume the home country is closed, this upper bound on φ_π is approximately 44 when $\varphi_y = 0$. The upper bound actually increases somewhat as the home country is allowed to be more open.

However, the Bullard and Mitra (2006) analysis was for a closed economy, and the determinacy condition by definition then refers to what, in our model, would be producer price inflation. We have, more realistically, included consumer price inflation in the monetary policy rule. This changes the determinacy conditions, but in complicated ways, and only in the case where an economy is somewhat open, since the closed economy case corresponds exactly to Bullard and Mitra (2006). We use this fact to now turn to a numerical analysis of the determinacy conditions for the multi-country model here. It will turn out that the upper bound on φ_π with consumer price inflation in the policy rule and open economies becomes much smaller and reaches levels that are relevant for policy discussions.

3.2 Calibrated values

We choose the calibration so that each economy collapses to the one studied by Woodford (2003) when the mass of the rest of the world tends to zero. Woodford's (2003) values have been widely used and provide a simple benchmark. These values are $\beta = 0.99$, $\sigma = 0.157$, $\phi = 0.11$, and $\lambda = 0.024$. To obtain this last value when the rest of the world has zero mass, we note that in this case $\lambda = \delta\left(\sigma + \phi\right)$, with $\delta = \left[\left(1-\theta\right)\left(1-\beta\theta\right)\right]/\theta$. Given other parameters, this means that $\theta = 0.745$ to obtain $\lambda = 0.024$. In a quarterly model this means that intermediate goods firms can optimally reset their prices about once per year on average with this value of the Calvo parameter.

3.3 Determinacy conditions

Figure 1 shows how the determinacy condition is altered relative to the closed economy case with monetary policy inertia and a forward looking rule as analyzed by Bullard and Mitra (2006). We use a three country model, varying the country weights or openness parameters γ_1, γ_2, and γ_3. In these calculations, we fix the monetary policy rule in two countries and allow the coefficients on inflation deviations and the output gap to vary in one country, while holding the coefficient on lagged interest rate fixed at 0.65. In the other two countries, the coefficients on inflation deviations, the output gap, and the lagged interest rate are 1.0, 0.2, and 0.65, respectively.

The first panel has $\gamma_1 = 0.99$, so that the economy is nearly completely closed, and the determinacy regions compare favorably to the Bullard and Mitra (2006) analysis.[11] The other five panels in Figure 1 show how the region consistent with determinacy is altered as γ_1 decreases (with γ_2 and

[11] See their Figure 2, middle panel.

γ_3 equal to each other and increasing) making the home country more open. The most interesting finding is that there is an upper limit on the reaction to inflation which becomes more severe as γ_1 is decreased, even if $\varphi_y = 0$. The final panel shows how the condition applies in the case where all three countries are of equal size and hence equally open.

The upper limit on φ_π comes from the interaction of open economy considerations with the fact that policymakers are targeting consumer price index inflation in this setting. The upper bound is sharp enough that it might impinge on actual policymaker considerations. The advice coming from the first, closed economy panel is that monetary policymakers can be as aggressive as they wish (there is an upper bound, but it is far from actual practice) with respect to consumer price index inflation, and still remain consistent with determinacy of equilibrium. A roughly realistic open economy case might be the fifth panel in the figure, in which the home country produces half of world output, and two partner countries produce one quarter each. But in this case, the degree to which policymakers can be aggressive with respect to inflation is very limited, the upper bound for the home country being on the order of 1.1 when $\varphi_y = 0$. This is a sense in which open economy considerations may dramatically change one's perceptions of appropriate policy choices.

The calculations in Figure 1 primarily involve variations in the openness parameters γ_i. If we fix these country weights and set both $\varphi_y = 0.2$ and $\varphi_r = 0.65$ for each of the three countries, we can calculate how coefficients with respect to inflation deviations interact internationally. We now turn to this task.

We wish to interpret the γ_i as representing the share of world gross domestic product. We want to consider two different periods as discussed by Clarida, Gali, and Gertler (2000), namely 1969-1979 and the more recent era, 1990-2004. We use data that measures these shares using data from the Penn World Tables[12] based on purchasing power parity adjusted prices worldwide. For the earlier era, we interpret the three countries as, in order,

[12] Summers and Heston (1991).

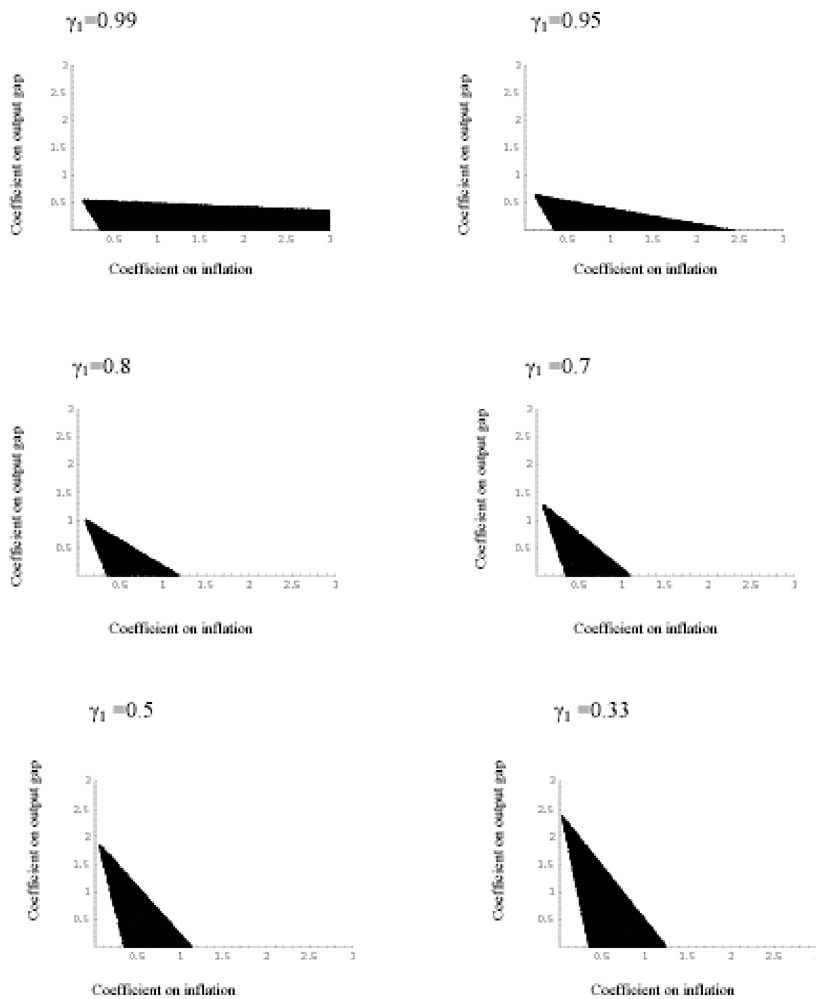

Figure 1: Determinacy from the perspective of the home country. The shaded region shows the policy rule parameter choices consistent with determinacy, taking other countries policy rules as given. When the economy is nearly closed (upper left), conditions correspond to those in Bullard and Mitra (2006). As the economy becomes more open (lower right), policy rules which are too aggressive with respect to inflation are associated with indeterminacy.

the U.S., Germany, Japan. These measures imply the U.S. share as 0.61, Germany's share as $\gamma_2 = 0.16$, and Japan's share as $\gamma_3 = 0.23$. For the later era, we interpret the three countries as, in order, the U.S., a unified Europe under a monetary policy conducted by the European Central Bank, and Japan. These measures imply the U.S. share as 0.46., the Euro area share as $\gamma_2 = 0.36$, and Japan's share as $\gamma_3 = 0.18$. There are certainly other methods of computing these shares but we do not think it is critical for the analysis we provide below.

Figure 2 shows the trade-off between monetary policy in the U.S. as against the Euro area (Germany in the earlier era) and Japan. This figure is somewhat different from Figure 1 in that each axis represents a policy parameter from a different country, whereas in Figure 1 both policy parameters were from the same country. The shaded region represents the combinations of policy parameters representing aggressiveness toward inflation deviations in two countries that are consistent with determinacy of worldwide equilibrium.[13] Perhaps the most striking finding from this picture, hinted at in Figure 1, is that there are sharp lower *and* upper limits on the policy parameters of each country. This finding holds in both eras and with respect to both (really, all three) countries. In particular, if the either the U.S., Japan, or Germany/Euro area is either too passive *or* too aggressive with respect to inflation deviations from target, world equilibrium will be indeterminate.

The idea that one country can unilaterally induce indeterminacy is not borne out by Figures 1 and 2. There are policy rules which, if chosen by one or another of the foreign policymakers, leave the domestic policymaker unable to induce determinacy of worldwide equilibrium, at least within the class of policy rules we consider. The relationship with respect to determinacy between φ_π in the home country vis-a-vis the two foreign economies is not continuous. According to Figure 2, for example, adoption of a value like $\varphi_\pi = 2.0$ by just one of the countries will leave world equilibrium indeterminate, and the other two policymakers will be unable to correct this

[13] We sampled 10,000 points randomly in this space to produce these figures.

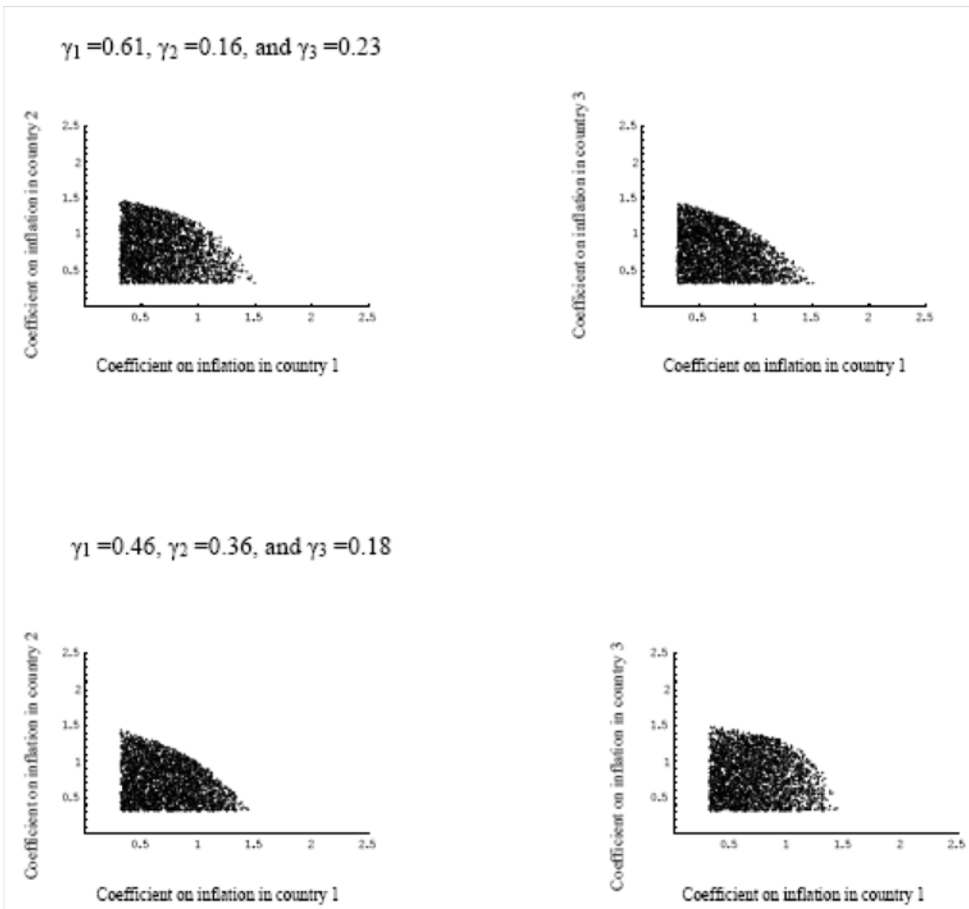

Figure 2: The trade-off between inflation aggressiveness in the home economy versus economies two and three, taking the remaining economy's policy rule as given, with openness weights from the 1970s (top) and the 1990s (bottom). The shaded region is associated with determinacy of world equilibrium. If the foreign policymaker is relatively aggressive, the home policymaker may be able to achieve determinacy by being less aggressive. But for foreign policy choices $\varphi_\pi \lessapprox 0.25$ or $\varphi_\pi \gtrapprox 1.5$, the home policymaker cannot make a simple adjustment to maintain worldwide determinacy.

situation with unilateral action.[14] We next turn to an investigation of the implications of this finding for each country.

3.4 Transmission of sunspot shocks across borders

One may have the intuition that, even if world equilibrium is indeterminate, a country will not suffer if it follows a "good" policy, suitably defined. In this section we show that this intuition is generally not correct, especially if the sunspot shock originates in a large country which is following a "poor" policy from the point of view of determinacy of worldwide equilibrium.

To address this issue, we simulate sunspot equilibria with no fundamental shocks to show the extent of transmission of sunspot shocks across borders. We draw the shocks from a standard normal distribution. Each simulation is for 60 periods, corresponding to approximately 15 years, and the volatilities reported are for averages of $1,000$ simulations. The results are reported in Table 1. In the first panel (panel A) all the countries are of equal size. For column 1 of panels B and C we use the 1969-1979 weights for the three countries, and in column 2 of panels B and C we use 1990-2004 weights.

We stress that any volatility for any variable in Table 1 below is coming only from the sunspot shock, as there are no fundamental shocks. Because the shocks follow a standard normal, the volatilities reported can be interpreted relative to a unit standard deviation for the driving sunspot process. A number smaller than unity means that the variable is less volatile than the driving sunspot process, while a number larger than unity means that variable is more volatile.

We call the monetary policy rule of a given country *determinacy-consistent* if it adheres to the modified Taylor principle $\varphi_\pi + \varphi_r > 1$. Of course, given the above discussion, it is not actually appropriate to think in terms of individual country policies inducing determinacy of worldwide equilibrium, and one could easily mis-characterize the situation by relying too heavily on this

[14] We think this lack of ability to influence worldwide determinacy conditions comes from the relatively weak international interactions in this economy. When we say the other policymakers are unable to correct the situation, we mean within the class of policy rules we examine.

simple principle. But for the calibration we are using and for the estimated policy rules we discuss later in the paper, this characterization turns out to be very useful. In particular, the dimension of indeterminacy of worldwide equilibrium will always equal the number of countries following determinacy-inconsistent policies for all of the quantitative cases we consider.

In all the three panels of Table 1 we assume that the coefficients on inflation deviations, the output gap, and the lagged interest rate in the countries following a determinacy-consistent policy are 0.9, 0.2, and 0.65, respectively. In the country following a determinacy-inconsistent policy these coefficients are 0.2, 0.2, and 0.65. This "poor" policy rule is not aggressive enough to induce determinacy of worldwide equilibrium when used in conjunction with the other two policies. The size of the country following the determinacy-inconsistent policy differs across these panels. In all of the panels, A, B, and C, the sunspot shock originates on inflation in the country following the determinacy-inconsistent policy rule. In our simulations we found that if the sunspot shock on inflation originates in countries following a determinacy-consistent policy rule, then the sunspot volatility generated by these shocks is not substantial.[15]

Panels A, B, and C, show that sunspot volatility is always transmitted across borders in this model—fundamental shocks are null in these simulations. Panel A suggests that the effects are rather large for the output gap in equally-sized countries. In this panel, there are no shocks of any type in the U.S. or Japan, only a unit standard deviation sunspot shock in originating on German inflation.[16]

The extent of this effect, however, depends on the size of the country following the determinacy-inconsistent policy. When relatively small

[15] The sunspot shock could originate on any of the three variables in any of the three countries. Multiple sunspot processes could be operating simultaneously as well. Thus there are many possibilities, and we do not explore them all in this paper.

[16] The fact that all countries are affected by sunspot shocks originating in a single country helps address a criticism of the Clarida, Gali, and Gertler (2000) paper, namely that the volatility of the 1970s was a global phenomenon, and that the likelihood of all countries simultaneously experiencing a sunspot would be low (see, e.g., Nelson (2005)). Simultaneous sunspot shocks are not necessary in this paper.

Table 1. Transmission of Sunspot Shocks

Panel A

	Equal γ_i		
	U.S.	Germany	Japan
Output gap	1.40	3.12	1.40
Inflation	0.31	2.24	0.31
Interest rate	0.12	1.70	0.12

Panel B

	1970s γ_i			*1990s* γ_i		
	U.S.	Germany	Japan	U.S	Euro area	Japan
Output gap	0.77	2.70	0.06	0.67	0.49	2.71
Inflation	0.12	2.13	0.02	0.13	0.09	2.13
Interest rate	0.04	1.54	0.04	0.05	0.05	1.55

Panel C

	1970s γ_i			*1990s* γ_i		
	U.S.	Germany	Japan	U.S.	Euro area	Japan
Output gap	3.88	3.91	4.47	3.49	2.90	1.78
Inflation	2.38	1.02	1.14	2.32	0.64	0.43
Interest rate	1.94	0.38	0.37	1.81	0.20	0.21

Table 1: Transmission of sunspot shocks across borders. In Panel A, Germany has the determinacy-inconsistent policy rule. In Panel B, the smallest economy has a determinacy-inconsistent policy rule (Germany in the 1970s, Japan in the 1990s). In Panel C the largest country (the U.S.) has the determinacy-inconsistent monetary policy rule. The sunspot shock is on inflation in the country with determinacy-inconsistent policy and follows a standard normal process. There are no fundamental shocks. Entries in the table are percent standard deviations, averaged across 1,000 simulations. Volatility is transmitted worldwide, often with amplitude greater than that of the driving sunspot process.

countries—Germany in the 1969-1979 era and Japan in the 1990-2004 era—are pursuing determinacy-inconsistent policies which allow the world economy to follow a sunspot equilibrium, the effect on the other two economies is mitigated, in the sense that the other two economies experience additional volatility (above zero, which is the volatility of fundamentals), but less than unity, the standard deviation of the driving sunspot process. This is shown in panel B. This result is very compelling, as it is in accord with much intuition that suggests effects originating in relatively small economies should not have undo influence on larger partner economies.

Panel C tells a different story. If the largest country—U.S. in both the eras—is pursuing the determinacy inconsistent policy and the world subsequently coordinates on a sunspot equilibrium, then there can be large spillover effects on the relatively small countries. This is also in accord with a great deal of intuition about the effects of poor policy choices by large countries.

We conclude that sunspot volatility is transmitted across borders in the equilibria we examine, and that this effect can be particularly acute when the sunspot shock originates in a large country pursuing a determinacy-inconsistent policy.

4 Evidence of postwar sunspot equilibria

4.1 Estimates of actual policy

4.1.1 Overview

We now follow Clarida, Gali, and Gertler (2000) and turn to estimation of monetary policy rules for the three largest economies during the 1969-1979 era and during the 1990-2004 era. We wish to provide some evidence as to whether the policymakers jointly satisfied worldwide determinacy conditions in either the earlier era or the later era. We use quarterly data.[17]

[17] Estimates using similar methodology for an intermediate period, 1979-1994, across Germany, Japan, and the U.S. are provided in Clarida, Gali, and Gertler (1998). We do not consider the 1979-1990 era in this paper, as discussed further below.

We use the estimation procedure of Clarida, Gali, and Gertler (2000) to maintain comparability to the analysis there, except for two differences which are important in our context. Clarida, *et al.*, (2000) use a second-order partial adjustment process of the actual interest rate and the central bank reacts to the contemporaneous output gap. This policy rule is somewhat altered from the one they use to calculate determinacy conditions. We follow a different specification in order to keep our estimated policy feedback rule exactly compatible with the policy feedback rule in our dynamic system. Otherwise, we cannot be sure if estimated coefficients are consistent with worldwide equilibrium determinacy or not.[18] To obtain compatibility, we estimate policy rules in which the actual interest rate follows a first-order partial adjustment process and the central back reacts to the one-period-ahead output gap. For estimation, the policy rule (17) can be rewritten as, omitting the country superscripts,

$$r_t = (1-\varphi_r)\left[rr^* - \left(\frac{\varphi_\pi}{(1-\varphi_r)} - 1\right)\pi^*\right] + \varphi_\pi \pi_{t,k} + \varphi_y \tilde{y}_{t,q} + \varphi_r r_{t-1} + \varepsilon_t \quad (27)$$

where r_t is the actual nominal interest rate, $\pi_{t,k}$ is the rate of inflation between periods t and $t+k$, $\tilde{y}_{t,k}$ is the measure of the average output gap between period t and period $t+k$, $\varphi_r \in [0,1]$ is the interest rate smoothing parameter, rr^* is the real rate of interest, and π^* is the inflation target. The error term is then a linear combination of forecast errors and hence it is orthogonal to any variable in the information set Ω_t. The error is given by

$$\varepsilon_t = -(1-\varphi_r)\left[\frac{\varphi_\pi}{(1-\varphi_r)}\left(\pi_{t,k} - E_t\left[\pi_{t,k}|\Omega_t\right]\right) + \frac{\varphi_y}{(1-\varphi_r)}\left(\tilde{y}_{t,q} - E_t\left[\tilde{y}_{t,q}|\Omega_t\right]\right)\right]. \quad (28)$$

We let z_t be variables within the central bank's information set, and we assume these variables are known when the central bank sets its interest rate r_t. We estimate a constant and the parameter vector $(\varphi_\pi, \varphi_y, \varphi_r)$ using

[18] Later in this section we present estimates of policy rules using the Clarida, Gali, and Gertler (2000) specification, and check determinacy via the suitably altered dynamic system.

GMM since our set of orthogonality conditions given by

$$E_t[\{r_t - ((1-\varphi_r)\left[rr^* - \left(\frac{\varphi_\pi}{(1-\varphi_r)} - 1\right)\pi^*\right] + \varphi_\pi \pi_{t,k} + \varphi_y \tilde{y}_{t,q} + \varphi_r r_{t-1})\} z_t\ ] = 0$$

are the same as Clarida, Gali, and Gertler (2000). We estimate four parameters but the number of instruments in the set z_t exceeds four which implies some overidentifying restrictions that we can test.

4.1.2 Data

1969-1979 period The starting date of our sample is mainly driven by data considerations and the end date is driven by the findings of Clarida, Gali, and Gertler (2000) that indicate a significant shift in the U.S. monetary policy around 1979. For the U.S.the interest rate is the average federal funds rate in the first month of each quarter. The annualized rate of change of the consumer price index between two subsequent quarters is our measure of inflation. The output gap is obtained by taking the deviation of log real GDP from a fitted quadratic trend. The instruments used in the estimation are four lags of the federal funds rate, inflation, the output gap, the spread between the long-term bond rate and the 3-month treasury bill rate, the log difference of the spot commodity price index, and a constant.

The variables in z_t for (West) Germany and Japan are somewhat different from the U.S. as detailed in the Appendix. For countries and time periods with a non-seasonally adjusted CPI, we use percent change from a year ago as a measure of inflation. For both Germany and Japan, we use the world commodity price index instead of the U.S. commodity price index. We also include the log difference of the real exchange rate, dm/dollar for Germany and yen/dollar for Japan.

1990-2004 period In Clarida, Gali, and Gertler (2000), the second era is 1979-1996. We chose the 1990-2004 era in order to explore the seemingly "passive" monetary policy rule in Japan and its implications for worldwide equilibrium. In addition, instead of Germany, we want to estimate the monetary policy rule for the Euro-area in this period. The U.S. data sources

Table 2. Estimates for the 1969-1979 Period.

Country	φ_π	φ_y	φ_r	Interpretation
U.S.	0.27	0.22	0.75	*Determinacy-consistent*
	(0.03)	(0.04)	(0.06)	
Germany	0.30	0.46	0.58	*Determinacy-inconsistent*
	(0.10)	(0.03)	(0.03)	
Japan	0.14	0.04	0.80	*Determinacy-inconsistent*
	(0.01)	(0.02)	(0.02)	

Table 2: Baseline estimates of monetary policy rules for the 1970s era suggest a two-dimensional indeterminacy of worldwide equilibrium, even though the U.S. policy may be viewed as reasonable taken in isolation. Standard errors are reported in parentheses.

are the same as for the earlier era, but the data for Japan is slightly different. In particular, the CPI is seasonally adjusted, real GDP is chain-weighted, and the interest rate and spread measures are based on better interest rate data (see the Data Appendix for details). We use the IMF's measure of world commodity prices for the Euro-area[19] and Japan. But since this series starts in 1992, it reduces the sample size for both the Euro-area and Japan. The Euro-area sample is even shorter since the interbank overnight rate begins in 1994. Thus we have sample data beginning in 1990 for the U.S., 1992 for Japan, and 1994 for the Euro-area. We think this is a good compromise in trying to include as much data as possible but still restricting attention to the period between 1990 and 2004.

4.2 Estimation results

4.2.1 Baseline estimates

The estimates reported in Tables 2 and 3 are based on the specification of the interest rate rule summarized by equation (27) with one lag of the interest rate. The target horizon for inflation and output gap is one quarter, $k = q = 1$.[20] We estimate a constant and the parameter vector $(\varphi_\pi, \varphi_y, \varphi_r)$ using GMM. In our estimation, the number of parameters we estimate is less than the number of instruments we use, and so we check the validity of the overidentifying restrictions by computing the J-statistic. Under the null hypothesis that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is asymptotically χ^2 with degrees of freedom equal to the number of overidentifying restrictions. The $p-$values associated with this test for each economy in each era was always large, 0.80 or greater, and so we do not comment further on these tests here.

We begin by discussing the estimated policy rule coefficients and their implications for determinacy of worldwide equilibrium. We then turn to a discussion of the estimated constant term and the implications for implied inflation targets in the three countries.

4.2.2 Interpretations for the 1970s and 1990s

To evaluate implications for worldwide equilibrium determinacy, we take the estimates we have obtained by following the approach of Clarida, Gali, and Gertler (2000) at face value and combine them with Woodford's (2003) calibrated values in the dynamic system described earlier. We reach the following conclusions.

[19] The Euro-area consists of 12 countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

[20] In Clarida, Gali, and Gertler (2000), they also have a one period ahead target horizon on the output gap. But according to their footnote 5, "...$x_{t,q}$.includes GDP generated between the beginning of period t and the beginning of period $t+q$ (it includes periods t, $t+1$,...and $t+q-1$). Having $q = 1$, then implies that the regression has the contemporaneous output gap as a regressor in Clarida, Gali, and Gertler (2000). In this paper, we report the estimation result for the case where one-period-ahead output gap is the regressor. This matches our dynamic system.

Table 3. Estimates for the 1990-2004 period.

Country	φ_π	φ_y	φ_r	Interpretation
U.S.	0.08	0.07	0.94	*Determinacy-consistent*
	(0.10)	(0.03)	(0.03)	
Euro-area	0.21	0.11	0.91	*Determinacy-consistent*
	(0.05)	(0.02)	(0.01)	
Japan	−0.04	0.01	0.90	*Determinacy-inconsistent*
	(0.02)	(0.01)	(0.01)	

Table 3: Baseline estimates of monetary policy rules for the 1990s era suggest a one-dimensional indeterminacy of worldwide equilibrium, even though the U.S. and European policies may be viewed as reasonable. Standard errors are reported in parentheses.

First, in the earlier era, there would have been a two dimensional indeterminacy of worldwide equilibrium. The U.S. policy was determinacy-consistent, according to the estimates in Table 2, but the monetary policies in Japan and Germany were not. Thus, indeterminacy characterized the 1970s, but not in the same way suggested by Clarida, Gali, and Gertler (2000). Combined with the findings summarized in Table 1, this suggests that a sunspot equilibrium, were it to have been followed, would have affected the U.S., regardless of the origin of the sunspot shock. U.S. policymakers would have been buffeted by endogenous volatility, even though their own policy seemed reasonable and the actual sunspot events were originating elsewhere. In addition, the indeterminacy is multi-dimensional, meaning more than one process could be operating in equilibrium. The total quantitative effect would of course depend on the variance of the sunspot shock (which in principle could be arbitrarily large), and its correlation with fundamental shocks. Our Table 1 simply scales all variability to the scale of the sunspot standard deviation, and considers a shock which has zero correlation with fundamentals.[21]

The worst-case scenario in Table 1 is one where the largest country is following a determinacy-inconsistent policy, and then transmitting sunspot volatility to its partner countries. According to these baseline empirical estimates, this was not the case in the 1970s era.[22] However, the alternative estimates described below actually point to this worst case scenario.

In the later era, which is normally viewed approvingly as one where monetary policy has been better, we still find a one dimensional indeterminacy. The "poor" policy from a worldwide determinacy perspective is coming from Japan. Thus according to these estimates there is scope for endogenous volatility even in the current economic environment. However, Table 1 suggests the effect on the U.S. would be mitigated because the size of the Japanese economy relative to the U.S. is small.

These results suggest that a global perspective is critical in evaluating prospects for equilibrium determinacy.

4.2.3 Estimated inflation targets

Clarida, Gali, and Gertler (2000) showed how to identify a separate estimate of the inflation target. We think these estimates are unreliable in our samples, as we now discuss. The nominal federal funds rate is given by (27), which is reproduced here for convenience

$$r_t = (1-\varphi_r)\left[rr^* - \left(\frac{\varphi_\pi}{(1-\varphi_r)} - 1\right)\pi^*\right] + \varphi_\pi \pi_{t,k} + \varphi_y \tilde{y}_{t,q} + \varphi_r r_{t-1} + \varepsilon_t, \quad (29)$$

where the error term is given by (28). Clarida, Gali, and Gertler (2000) wished to separately identify an estimate of the inflation target π^*. They

[21] We use the 1969-1979 time period following Clarida, Gali, and Gertler (2000), but it might be viewed as inappropriate for Japan and Germany on the grounds that the breakdown of Bretton Woods is included and may have had a larger impact on these economies. We re-estimated policy rules for these economies using just the short sample period 1973-1979 and found results were still consistent with a one-dimensional indeterminacy (the German policy now being determinacy-consistent). The short sample makes these estimates less interesting, in our view.

[22] Countries like Japan and Germany certainly have smaller, tightly integrated trading partners that could be affected by their determinacy-inconsistent policy. We think this is an interesting area for future research.

Table 4. Estimates of π^* given rr^*.

	rr^*		π^*	
Country	1969-1979	1990-2004	1969-1979	1990-2004
U.S.	−0.03	1.55	25.88	6.35
			(35.79)	(15.79)
Germany/Euro-area	1.14	1.11	−0.45	2.47
			(2.64)	(0.19)
Japan	−1.59	0.76	8.33	−0.57
			(0.72)	(0.08)

Table 4: Sample average ex-post real interest rates, expressed in percent, along with inflation target estimates using the Clarida, Gali, and Gertler (2000) methodology. If the modified Taylor principle holds exactly, the inflation target estimate is undefined. This nearly occurs for the U.S. in both eras, producing unreliable estimates. Standard errors are reported in parentheses.

used the following procedure, which according to our results does not change the estimated coefficients φ_π, φ_y, and φ_r. They viewed the sample average ex-post real interest rate as a good proxy for rr^*. It is calculated by taking the relevant sample mean of the federal funds rate and subtracting from it the sample average of the annualized CPI inflation rate. Given rr^*, $\pi^\star$ can be estimated using the restriction implied by (29). The values for rr^* as well as the estimates of π^* are reported in Table 4.

The inflation target estimates in Table 4 are sometimes far from what seems sensible, such as the sample mean for the era, and in addition are sometimes imprecisely estimated. There is a clear connection between these instances and the determinacy conditions we have outlined above, which we now illustrate using the U.S. case for the 1969-1979 era. When we estimate (29) with a constant term, the value of the constant is -0.4259. The value of rr^* for this period is -0.03. The coefficient estimates from Table 2 indicate that the modified Taylor principle narrowly holds, that is, that $\varphi_\pi + \varphi_r = 0.2673 + 0.7489 = 1.0162 > 1$.[23] Rearranging the constant term in (29) and equating it to -0.4259 indicates that

$$\pi^* = \frac{-0.43 - (1 - \varphi_r) rr^*}{(1 - \varphi_\pi - \varphi_r)} = 25.88,$$

which is exactly the estimated value. In fact, if the modified Taylor principle holds exactly, so that $\varphi_\pi + \varphi_r = 1$, then the value of π^* would be undefined in this calculation. If the estimates of φ_π and φ_r should sum to slightly less than one—which is certainly within the realm of sampling variability according to these estimates—then the estimate of π^* would switch sign, becoming large and negative. For this reason we expect unreliable estimates of π^* using the Clarida, Gali, and Gertler (2000) methodology when the modified Taylor principle is close to holding exactly, as is the case for some of the estimates given in Table 2 and 3 above.

4.2.4 Alternative estimates

Our econometric estimates have so far indicated that a two-dimensional indeterminacy characterized the worldwide equilibrium in the 1970s, and that a one-dimensional indeterminacy characterized the world equilibrium in the 1990s. These results are based on a particular policy rule being used by each country in each era. We can consider other policy rules, at some expense since alternative policy rules change the nature of our dynamic system. In this section we estimate the policy rules considered by Clarida, Gali, and Gertler (2000), assuming that each of the three largest economies employed rules of this form. The Clarida, Gali, and Gertler (2000) rule has the monetary authority reacting to the current output gap instead of our expected output gap, and allows for a second order partial adjustment process in the nominal interest rate, but otherwise is the same. We keep our specification of consumer price index inflation, as opposed to domestic producer price inflation, as the relevant measure of inflation in the policy

[23] For this calculation we use point estimates with four significant digits to avoid significant rounding error.

TABLE 5. ESTIMATES FOR THE 1969-1979 PERIOD,
CLARIDA, GALI, AND GERTLER (2000) SPECIFICATION.

Country	φ_π	φ_y	$\varphi_{r,1}$	$\varphi_{r,2}$	Interpretation
U.S.	0.24	0.24	0.79	−0.08	*Determinacy-inconsistent*
	(0.03)	(0.03)	(0.06)	(0.04)	
Germany	0.24	0.38	0.56	−0.02	*Determinacy-inconsistent*
	(0.11)	(0.04)	(0.04)	(0.03)	
Japan	0.10	0.02	1.17	−0.36	*Determinacy-inconsistent*
	(0.01)	(0.01)	(0.03)	(0.03)	

Table 5: Estimates of monetary policy rules for the 1970s era using the Clarida, Gali, and Gertler (2000) policy rule specification suggest a three-dimensional indeterminacy of worldwide equilibrium. Here the U.S. policy may be viewed as inconsistent with deteminacy, which is a version of the reported results of Clarida, Gali, and Gertler (2000). This specification allows for a second order partial adjustment process in the nominal interest rate, so that two coefficients are reported. Standard errors are reported in parentheses.

rule. This issue did not arise in Clarida, Gali, and Gertler (2000), as that paper was a closed economy study.

The estimates reported in Table 5, which apply to the 1969-1979 era, now suggest a three-dimensional indeterminacy of worldwide equilibrium. The finding for the U.S., that a version of the modified Taylor principle, namely[24] $\varphi_\pi + \varphi_{r,1} + \varphi_{r,2} > 1$, fails to hold, is consistent with the original findings reported by Clarida, Gali, and Gertler (2000). However, in worldwide equilibrium, all three countries are failing to meet this principle, and when we check determinacy conditions, we indeed obtain a three dimensional indeterminacy. Again, sunspot equilibria could arise from many

[24] Here $\varphi_{r,1}$ and $\varphi_{r,2}$ refer to the coefficients on the lagged interest rates in the policy rule for $t-1$ and $t-2$ respectively.

Table 6. Estimates for the 1990-2004 period, Clarida, Gali, and Gertler (2000) specification.

Country	φ_π	φ_y	$\varphi_{r,1}$	$\varphi_{r,2}$	Interpretation
U.S.	0.12	0.01	1.58	−0.65	*Determinacy-consistent*
	(0.03)	(0.01)	(0.05)	(0.04)	
Euro-area	0.15	0.07	1.12	−0.22	*Determinacy-consistent*
	(0.05)	(0.01)	(0.04)	(0.04)	
Japan	−0.04	0.00	0.98	−0.08	*Determinacy-inconsistent*
	(0.02)	(0.01)	(0.07)	0.06	

Table 6: Alternative estimates of monetary policy rules for the 1990s era suggest a one-dimensional indeterminacy of worldwide equilibrium, as in the baseline specification. Standard errors are reported in parentheses.

sources, and indeed multiple sunspots could be influencing worldwide economic dynamics simultaneously. These estimates and the baseline estimates for this era contained in Table 2 suggest that the dimension of indeterminacy in the 1970s may well have been larger than one and that the era could have been as volatile as it was due in part to self-fulfilling fluctuations.

The alternative estimates reported in Table 6, which apply to the 1990-2004 period, are similar to the baseline estimates contained in Table 3. They suggest, again, that while monetary policy was generally better during the more recent era, there was still some risk of endogenous volatility due to a one-dimensional indeterminacy in the worldwide equilibrium.[25]

[25] These alternative estimates can, using the Clarida, Gali, and Gertler (2000) methodology, be obtained along with an estimate of the implied inflation target. This would again be subject to the caveat discussed above. For the sake of completeness, we report this estimates here, with standard errors in parentheses. For the 70s, U.S. −0.54 (3.63), Germany 3.08 (0.60), Japan 8.78 (0.24); for the 90s, U.S. 2.81 (0.50), Europe 2.51 (0.40), Japan −0.59 (0.12).

5 Conclusion

We have shown how international monetary policy considerations impinge on determinacy conditions for worldwide rational expectations equilibrium, an international version of a famous result due to Clarida, Gali, and Gertler (2000). Failure to achieve determinacy has been considered a severe shortcoming in the closed economy new Keynesian literature, and we view such a failure the same way in the international context. The difference is that in the international context, determinacy is influenced jointly, most importantly by the large policymakers on the world scene. We have considered a simple, n-economy version of the new Keynesian model discussed in Clarida, Gali, and Gertler (2002). This particular international version is certainly not the only one available, but has the distinct advantage for us that it collapses to the standard, simple new Keynesian model widely studied following Woodford (2003) when any of the economies becomes completely closed.

Our analysis indicates that the degree of openness of each economy has clear effects on determinacy conditions, a finding that is consistent with previous studies. We have shown how the closed economy case is consistent with known findings from the literature, and also how these conditions are altered as an economy becomes more open. We found clear upper limits on how aggressive a policymaker can be with respect to inflation in the monetary policy rule. This limit would exist in the closed economy case but only at extreme values which would seem to be unlikely to impinge on actual policy. One finding is then that policymakers for large economies in this model can neither be too passive nor too aggressive with respect to deviations of inflation from target if they wish to remain consistent with determinacy of worldwide rational expectations equilibrium. We also explore some of the tradeoffs that exist between the policy rule adopted by one nation versus that adopted by another. We find that there is little or no scope for a large country to render the worldwide equilibrium determinate via simple, unilateral action in cases where a partner country is pursuing a policy sufficiently inconsistent with determinacy.

We have also investigated the extent to which sunspot shocks originating in a single country may be transmitted across borders in a worldwide sunspot equilibrium. The general finding is that all variables in all countries are more volatile in such an equilibrium. The strength of this effect depends on the source of the sunspot shock, the degree of openness of each of the economies, and the monetary policy rules in effect in each economy. One generality is that sunspot volatility originating in a relatively small economy following a policy which is inconsistent with determinacy of worldwide equilibrium has muted effects on large economy volatility. Sunspot shocks originating in a large country following a policy which is inconsistent with worldwide determinacy, on the other hand, have large effects on the partner economies. Both of these results are in accord with a great deal of intuition about poor policies being followed in large versus small countries.

In keeping with the original closed economy analysis of Clarida, Gali, and Gertler (2000), we estimate monetary policy rules for three large economies, and assess determinacy conditions for worldwide equilibrium using these estimates. We find in our baseline estimates that the 1970s was associated with a two-dimensional indeterminacy of worldwide rational expectations equilibrium, and thus that endogenous volatility was a distinct possibility during that era. Interestingly, the U.S. policy can be viewed as reasonable according to these estimates, as a version of the Taylor principle is met. But in conjunction with partner country policies in Germany and Japan, worldwide equilibrium was indeterminate. For the more recent 1990-2004 era, we find a one-dimensional indeterminacy of worldwide equilibrium. Thus, while policy has generally been better in the more recent era, the world economy has still been exposed to the threat of endogenous volatility.

A long-standing debate in monetary policy circles concerns possible welfare gains from international monetary policy coordination. At issue is the extent to which policymakers in large economies such as the United States, the Euro area, and Japan should adjust policy in reaction to economic events outside their own borders. In the multiple country version of the new Keynesian model we study, each large policymaker can contribute, but only contribute, importantly to the determinacy of worldwide equilibrium. Failure to ensure determinacy worldwide would open the door to unnecessary equilibrium fluctuations in the world rational expectations equilibrium. These unnecessary fluctuations, we have argued, would affect all economies. The poor macroeconomic performance potentially associated with indeterminacy could reduce the welfare of households worldwide dramatically as the magnitude of the unnecessary equilibrium fluctuations could, in principle, be quite large. Policymakers in major economies may want to coordinate to avoid such an outcome. This is a very different reason for international policy coordination than those that are normally advanced, but one that we think may warrant further study.

References

[1] Batini, N., P. Levine, J. Pearlman. 2004. "Indeterminacy with Inflation-Forecast-Based Rules in a Two-Bloc Model." International Finance Discussion Paper #797, Board of Governors of the Federal Reserve System, April.

[2] Batini, N., A. Justiniano, P. Levine, and J. Pearlman. 2006. "Robust Inflation-Forecast-Based Rules to Shield Against Indeterminacy." *Journal of Economic Dynamics and Control*, forthcoming.

[3] Belaygorod, A., S. Chib, and M. Dueker. 2006. "Indeterminacy, Adverse Reactions to Interest Rate Shocks and the Great Inflation of the 1970s." Manuscript, Olin School of Business, Washington University, and Federal Reserve Bank of St. Louis.

[4] Benhabib, J., and R. Farmer. 1999. "Indeterminacy and Sunspots in Macroeconomics." In Taylor, J.B., and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1A. Elsevier. Chapter 6, pp. 387-448.

[5] Benigno, G., and P. Benigno. 2006*a*. "Designing Targeting Rules for International Monetary Policy Cooperation." *Journal of Monetary Economics*, forthcoming.

[6] Benigno, G., and P. Benigno. 2006*b*. "Implementing International Monetary Cooperation Through Inflation Targeting." *Macroeconomic Dynamics*, forthcoming.

[7] Beyer, A., and R. Farmer. 2004. "On the Indeterminacy of New Keynesian Macroeconomics." European Central Bank Working Paper #323.

[8] Blanchard, O., and C. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48(5): 1305-1312.

[9] Bullard, J., and K. Mitra. 2006. "Determinacy, Learnability, and Monetary Policy Inertia." *Journal of Money, Credit, and Banking*, forthcoming.

[10] Bullard, J., and E. Schaling. 2006. "Monetary Policy, Determinacy, and Learnability in the Open Economy." Manuscript, Federal Reserve Bank of St. Louis.

[11] Carlstrom, C., and T. Fuerst. 1999. "Optimal Monetary Policy in a Small, Open Economy: A General Equilibrium Analysis." Working Paper #99-11, Federal Reserve Bank of Cleveland.

[12] Chari, V., L. Christiano, and M. Eichenbaum. 1998. "Expectations Traps and Discretion." *Journal of Economic Theory* 81(2): 462-492.

[13] Clarida, R., J. Gali, and M. Gertler. 1998. "Monetary Policy Rules in Practice: Some International Evidence." *European Economic Review* 42: 1033-1067.

[14] Clarida, R., J. Gali, and M. Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*, February, pp. 147-180.

[15] Clarida, R., J. Gali, and M. Gertler. 2002. "A Simple Framework for International Monetary Policy Analysis." *Journal of Monetary Economics* 49: 879-904.

[16] Corsetti, G., and P. Pesenti. 2005. "International Dimensions of Optimal Monetary Policy." *Journal of Monetary Economics* 52: 281-305.

[17] Cooley, T., and G. Hansen. 1989. "The Inflation Tax in a Real Business Cycle Model." *American Economic Review* 79: 733-748.

[18] De Fiore, F., and Z. Liu. 2005. "Does Trade Openness Matter for Aggregate Instability?" *Journal of Economic Dynamics and Control* 29(7): 1165-1192.

[19] Doyle, B., and J. Faust. 2005. "Breaks in the Variability and Co-Movement of G-7 Economic Growth." *Review of Economics and Statistics* 87(4): 721-740.

[20] Evans , G., and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton University Press.

[21] Honkapohja, S., and K. Mitra. 2004. "Are Non-fundamental Equilibria Learnable in Models of Monetary Policy?" *Journal of Monetary Economics* 51(8): 1743-70.

[22] Kim, C-J, and C. Nelson. 1999. "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle." *Review of Economics and Statistics* 81(4): 608-16.

[23] Linnemann, L., and A. Schabert. 2004. "Net Foreign Assets, Interest Rate Policy, and Macroeconomic Stability." Working paper, Money Macro and Finance Research Group.

[24] Lubik, T., and F. Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review* 94(1): 190-217.

[25] Lubik, T., and F. Schorfheide. 2005. "A Bayesian Look at New Open Economy Macroeconomics." *NBER Macroeconomics Annual.*

[26] McConnell, M., and G. Perez Quiros. 2000. "Output Fluctuations in the United States: What has Changed Since the Early 1980s?" *American Economic Review* 90(5): 1464-1476.

[27] Nelson, E. 2005. "The Great Inflation of the Seventies: What Really Happened?" *Advances in Macroeconomics* 5(1), Article 3.

[28] Obstfeld, M., and K. Rogoff. 2002. "Global Implications of Self-Oriented National Monetary Rules." *Quarterly Journal of Economics* 117(2): 503-535.

[29] Orphanides, A. 2001. "Monetary Policy Rules Based on Real-Time Data." *American Economic Review* 91: 964-985.

[30] Orphanides, A. 2005. "Monetary Policy Rules, Macroeconomic Stability, and Inflation: A View from the Trenches." *Journal of Money, Credit, and Banking* 36(2): 151-175.

[31] Stock, J., and M. Watson. 2003. "Has the Business Cycle Changed and Why." *NBER Macroeconomics Annual.*

[32] Summers, R., and A. Heston. 1991. "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988." *Quarterly Journal of Economics* 106(2): 327-68.

[33] van Dijk, D., and M. Sensier. 2004. "Testing for Volatility Changes in U.S. Macroeconomic Time Series." *Review of Economics and Statistics* 86(3): 833-39.

[34] Woodford, M. 1999. "Optimal Monetary Policy Inertia." NBER Working Paper #7261, July.

[35] Woodford, M. 2001. "The Taylor Rule and Optimal Monetary Policy." *American Economic Review Papers and Proceedings* 91(2): 232-237.

[36] Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.

[37] Zanna, L-F. 2003. "Interest Rate Rules and Multiple Equilibria in the Small Open Economy." International Finance Discussion Paper #785, Board of Governors of the Federal Reserve System.

A Data source appendix[26]

Appendix Table 1: Data for the U.S.

Series	ID	Source	Description
Effective fed funds rate	FEDFUNDS	FRED	H:15 Selected interest rates
CPI	PCU@USECON	Haver/BLS	CPI-U, all items
GDP	GDPH@USECON	Haver/BEA	Real GDP (SAAR, Bil.Chn.2000$)
Short term rate	FTBS3@USECON	Haver/FRB	3-Month Treasury, Secondary Market (% p.a.)
Long term bond rate	FCM10@USECON	Haver/FRB	10-Year Treasury Yield at Constant Maturity (% p.a.), nominal
Commodity price index	PZALL@USECON	Haver/CRB	KR-CRB Spot Commodity Price Index: All Commodities (1967=100)

[26] Data on international liquidity, money and banking and international transactions cover the former FRG and GDR beginning mid-1990. Data on prices, production, employment and national accounts refer only to the former Federal Republic of Germany.

Appendix Table 2: Data for Germany

Series	ID	Source	Description
Interbank overnight rate	C134IM@IFS	Haver/IMF	Germany: MMkt Rate: Interbank: Overnight (%)
CPI	CW34CZ @OECDMEI	Haver/OECD	West Germany: CPI All Items (SA, 1990=100)
GDP	CW34GPCN @OECDMEI	Haver/OECD	West Germany: GDP (SAAR, Bil.90.Euros)
Short term rate	C134FRIO @OECDMEI	Haver/OECD	Germany: 3-Month FIBOR: Frankfurt Interbank Offer Rate (%)
Long term rate	C134IB@IFS	Haver/IMF	Germany: 3 Years & Over Government & Agency Bond Yield, Wtd Avg (%)
Nominal exchange rate	C134ECME@IFS	Haver/IMF	Germany: Exchange Rate: Market or Par (EOP, DM/US$)
World commodity price	C001CXAP	IMF (discontinued)	World: Commodity Price Index: All Commodities (1995=100)

Appendix Table 3: Data for Japan

Series	ID	Source	Description
Interbank rate overnight	C158IM@IFS	Haver /IMF	Japan: MMkt Rate: Tokyo Overnight Call Money (%)
CPI	C158PC@IFS (1969-1979)	Haver /IMF	Japan: Consumer Prices (2000=100, NSA)
	C158CZ@OECDMEI (1990-2004)	Haver /OECD	Japan: CPI: All ltems Incl Imputed Rent (SA, 2000=100)
GDP	GDP_95.Q.JA (69-79)	Fame	JAPAN – Real GDP
	C158GDPC @OECDMEI (90-04)	Haver /OECD	Japan: GDP (SAAR, Bil.Chn.2000.Yen)
Short term rate	C158IDEP@IFS (1969-1979)	Haver /IMF	Japan: 3-Month Unregulated Time Deposit Rate (%)
	C158FRCD @OECDMEI (90-04)	Haver /OECD	Japan: 3-Month Certificates of Deposit {Gensaki Rate} (% per annum)
Long term rate	C158IB@IFS (1969-1979)	Haver /IMF	Japan: Yield to maturity of all ordinary Government bonds (EOM, %)
	C158FYGL @OECDMEI (90-04)	Haver /OECD	Japan: 10-Year Central Government Bond Yield (% per annum)
Nominal exchange rate	C158ECME@IFS	Haver /IMF	Japan: Exchange Rate: Market or Par (EOP, Yen/US$)
World commodity price	C001CXAP (69-79)	IMF (disc.)	World: Commodity Price Index: All Commodities (1995=100)
	C001CXAP@IFS (1990-2004)	Haver /IMF	World: Commodity Price Index: All Commodities (1995=100)

Appendix Table 4: Data for Euro-area

Series	ID	Source	Description
Interbank overnight rate	C163ID@IFS	Haver /IMF	Euro Area+GR in 2001: Overnight Interbank Rate (%)
CPI	C025CZN @OECDMEI	Haver /OECD	Euro Zone 12 {incl Greece}: CPI: Total {OECD computation} (NSA, 2000=100)
GDP	C025GDPC @OECDMEI	Haver /OECD	Euro Zone 12 {incl. GR}: GDP (SA, Bil.2001.Euro)
Short term rate	C163IM@IFS	Haver /IMF	Euro Area+GR in 2001: Interbank Rate: 3-Month (% per annum)
Long term bond rate	C163IB@IFS	Haver /IMF	Euro Area+GR in 2001: Government Bond Yield (%)
Nominal exchange rate	X111EXR @EUROSTAT	Haver /Eurostat	Euro Exchange Rate: U.S. Dollar (Ave, USD/EUR)
World commodity price	C001CXAP @IFS	Haver /IMF	World: Commodity Price Index: All Commodities (1995=100)