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# PRICES, WAGES, AND EMPLOYMENT IN THE U.S. ECONOMY: A TRADITIONAL MODEL AND TESTS OF SOME ALTERNATIVES

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## ABSTRACT

In this paper, we outline the cost minimizing behavior of oligopoly firms and the price adjustment process in the labor market which underlie the traditional formulation of aggregate wage-price behavior in the U.S., and show that resulting equations applied to U.S. data remain stable before and after the significant change in the monetary policy rule that had taken place in 1979. This result contradicts the prediction of the Lucas critique applied to this context that, in response to a major change of the monetary policy rule, the Phillips curve and the price setting equation of firms would have undergone significant changes. We test several competing hypotheses for the price level determination, including the possibility that more direct effect of the money supply should be relevant, and show that our formulation dominates alternatives in non-nested tests. Finally, we present evidence that the nature of capital is putty-clay rather than fully malleable, together with a demand function for labor based on this recognition.

In the process of these inquiries, we contrast our formulation with that proposed by Layard and Nickell in England.

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# I INTRODUCTION

In our earlier paper we reported on the performance of the price-wage sector of the MPS model (one of the models maintained at the Board of Governors of the Federal Reserve System) between the late 1960s and 1990 and supplemented the analysis with a series of tests designed to check if the specification of this part of the model was missing any direct effects of the money supply on the movements of prices and wages. (Ando, Brayton & Kennickell, 1991)

We have two purposes in writing the present paper. First, listening to the reaction from readers of our earlier paper, it is evident that we have failed to communicate our basic message, especially the theoretical framework underlying our empirical results.[1] The closest parallel work to ours is that of Layard and Nickell (Layard and Nickell, 1985 and 1986) (hereafter L-N), which has received a great deal of attention, especially in the United Kingdom. Despite their parallel appearance, there are sufficient differences between our own and the L-N formulations that a clarification of the exact differences helps us to understand better the nature of both formulations. In part II of this paper, we show what these two formulations say about the behavior of wages, prices and employment on the steady state growth path. In part III, we proceed to review the dynamic adjustment processes and also the ability of our formulation to account for the pattern of actual data in the U.S. In part IV, we pick up some ideas from L-N and test their relevance against U.S. data, as well as some other hypotheses incorporating

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[1]See especially Wallis (1992 ). In addition to the lack of communication about our underlying framework, Wallis appears to object to our use of dynamic simulation as a measure of the performance of our system of equations. While acknowledging the technical result of Pagan(1989) on this question, we do not believe that his result makes all uses of dynamic simulations unacceptable for all purposes. We will, however, limit the use of dynamic simulations to a minimum in this paper in order to avoid a methodological controversy and to concentrate our attention on the analysis of the mechanism determining prices, wages and employment.

alternative views of how prices and wages are determined, especially those associated with a more direct role for the money supply.

## II THEORETICAL BACKGROUND

### A. Optimal Capital-Output Ratio and Price-Wage Frontier.

#### 1. Simplified MPS Formulation

We must take note of two basic assumptions at the outset. First, we share with L-N a view that firms face oligopolistic markets for their products and that they can set the price of output, although the ability of individual firms to set output price is narrowly limited.

For many firms, the output market can probably be characterized as monopolistically competitive, and therefore the output price must be very close to minimized average cost. For other firms in a genuinely oligopolistic market, insofar as they produce goods having common features and share a similar production technology, the minimized average cost must be similar. They also share the knowledge that, if they set their product price much higher than the minimized average cost in an attempt to extract exceptionally large oligopoly rents, then they face the threat that a new firm may enter the market and undercut their price. If this is the environment in which most firms operate, then a reasonable approximation to their pricing policy is to start with the description of their minimized average cost, and then to enrich it by considering factors that may affect the size of the mark-up. Given the assumption that the production function is homogenous of degree one in labor and capital and that it is common to all firms competing in the market, we may identify the minimized average cost for some arbitrarily given output and assume that the cost minimizing factor proportions remain the same for any other given level of output. We can then consider the price setting behavior of firms in two distinct stages. We can first model their determination of the minimized total average cost and associated factor proportions for a given output and factor prices. We can then analyze determination of the mark-up and the dynamic adjustments of the price to changes in environment in a separate analysis.

Our second critical assumption is about the nature of capital. Based on persistent evidence in our estimation of the investment function, starting with the work of Bischoff (Bischoff, 1971) and continuing to the one reported below, we believe that capital goods, at least producer's equipment, are "putty-clay" in nature. That is, before installation, production possibilities faced by a firm can be described by a flexible production function and any factor proportion is possible to accommodate relative factor prices. However, once equipment is put in place, it is no longer possible to change factor proportion insofar as this particular equipment is concerned. Nor is it possible to take advantage of new technology developed after the equipment is put in place by upgrading the equipment to improve its productivity. This has a number of implications both on the form and interpretation of several equations.

Let us begin by considering a "stripped down" case of the firm's optimization problem given by the following:

$$\underset{E,I}{Min}\ \beta(E,I;XCA) = WE + RI \tag{1}$$

$$Subject\ to\text{: } XCA = f(Ee^{\gamma t}, I) \tag{2}$$

$f$ is assumed to be homogeneous of degree one in $Ee^{\gamma t}$ and I. (2) may be specialized to :

$$XCA = f = B\left[a(Ee^{\gamma t})^{\frac{\sigma-1}{\sigma}} + (1-a)I^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2'}$$

The definition of symbols are collected at the end of the paper.

Most of the discussions below use the CES production function (2') rather than (2) both because it is a particularly easy case to analyze, and because we have reasonable empirical support for the hypothesis that aggregate production possibilities before equipment is put in place can be approximated by the CES production function. Indeed, the Cobb-Douglas function is acceptable for most purposes. We will, however, have an occasion to make use of the CES formulation rather than the Cobb Douglas to clarify some technical points.

For the problem at hand, we assume that the decision maker who is responsible for capital investment and for output price takes factor prices, $W$ and $R$, as given. It is important to be clear about the nature of the production function $f$. It is an ex-ante description of the production possibilities associated with new investment currently undertaken, not a description of the relationship among total output, total capital stock and total employment for the firm or for the industry. Such a description of the total production process must recognize that there exists a whole series of capital with different vintages which require different labor inputs for a given output. We will discuss the implications of this structure for the labor demand equation in Section III.B.2 below. It should be noted here, however, that the non-malleable nature of capital makes it necessary to look forward at least as long as the currently acquired capital is expected to remain, and that the length of life of equipment is determined by the timing of when the labor cost of producing one unit of output with the equipment in question becomes greater than the total cost of producing the same output using new equipment incorporating the then newest technology and the then prevailing prices.[2]

[2]While the formulation of the problem incorporating, in addition to the putty-clay assumption, an anticipation of relevant future events and replacement requirements can become quite complex, under a certain set of assumptions, the qualitative nature of the solution can be shown to remain unchanged from

The Lagrange function of this problem is given by

$$L = \beta + \lambda(XCA - f) \tag{3}$$

where $\lambda$, the Lagrange multiplier, can be interpreted as the cost of increasing the requirement $XCA$ by one unit at the point of the minimum; in other words, $\lambda$ is the minimized total unit cost. If the market for output is perfect, then the price of output, $P$, must be equal to $\lambda$. We assume, however, that the market for output is oligopolistic with some restrictive features as discussed above, so that the price of output is expected to be characterized by a mark-up on the minimized unit cost, that is,

$$P = \mu\lambda \tag{4}$$

where $\mu$ is the mark-up factor. We will discuss factors affecting the size and the movement of $\mu$ over time later, but for the moment let us suppose that $\mu$ is constant and exogenous. Then the first order conditions for the optimum in the CES case are:

$$\frac{I}{XCA} = B^{\sigma-1}(1-\alpha)^{\sigma}\left(\frac{P}{\mu R}\right)^{\sigma} \tag{5a}$$

$$\frac{Ee^{\gamma t}}{XCA} = B^{\sigma-1}\alpha^{\sigma}\left(\frac{P}{\mu We^{-\gamma t}}\right)^{\sigma} \tag{5b'}$$

or

$$\frac{\mu We^{-\gamma t}}{P} = B^{\frac{\sigma-1}{\sigma}}\alpha\left(\frac{XCA}{Ee^{\gamma t}}\right)^{\frac{1}{\sigma}} \tag{5b}$$

and

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the ones presented here. Such an analysis was worked out in some detail in Ando, Modigliani, Rasche and Turnovsky (1974 ).

$$\frac{Ee^{\gamma t}}{I} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma} \left(\frac{R}{We^{-\gamma t}}\right)^{\sigma} \tag{5c}$$

The Cobb-Douglas case can be obtained by equating $\sigma$ to unity in (5a), (5b), and (5c). (5a) serves as the starting point for the definition of the optimal capital output ratio, and hence of the investment equation. It may be noted that the ratio $R/P$ involves the price of output, $P$, and hence the argument appears circular. But this is not really so, because $R/P = (\rho+\delta)\frac{P^I}{P}$, and hence it involves the relative price $P^I/P$, which can be taken as given for our purposes here.

(5b), on the other hand, can be rewritten in the form

$$P = \mu We^{-\gamma t} \frac{B^{\frac{\sigma-1}{\sigma}}}{\alpha} \left(\frac{Ee^{\gamma t}}{XCA}\right)^{\frac{1}{\sigma}} \tag{5bb}$$

and it can be thought of as the pricing equation for output by firms. Note that the ratio $\left(Ee^{\gamma t}/XCA\right)$ is uniquely determined once the ratio $(I/XCA)$ is determined by equation (5a). Hence, (5bb) makes the real wage rate a function of the mark-up factor, the productivity trend, a parameter of the production function and the user cost of capital $(\rho+\delta)$ $(P^I/P)$.[3] (5bb) with $(\rho+\delta)(P^I/P)$ substituted for $Ee^{\gamma t}/XCA$ is sometimes referred to as " the price-wage frontier".

[3]In the Cobb-Douglas Case, (5bb) can be reduced to

$$\alpha \frac{P\,XCA}{\mu} = WE$$

$P\,XCA/\mu$ is the total value added, so that this is nothing but the familiar propositions that with the Cobb-Douglas, the share of labor in value added is equal to the exponent of labor.

## 2. Comparison with the Layard-Nickell Formulation

While we realize that L-N have restated their formulation several times in the literature, we find it most convenient to base our interpretation of their formulation on their two relatively early papers dating 1985 and 1986.

L-N take the capital stock as exogenously given, and under the assumption of fully malleable capital, they jointly derive the employment equation and the pricing rule for output. Indeed, their employment equation and the pricing rule for output are the same equation in the steady state, differing only in dynamics. In this section we focus our attention on their pricing rule.

While we find the reasoning leading to their pricing rule given as equation (6c) in (L-N, 1986) and equation (19) in (L-N, 1985) somewhat difficult to follow, these equations are identical to our equation (5bb) above.[4] While (5bb) is specialized to the CES production

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[4]The basic issue here is how the capital stock was determined in the first place. Although L-N say that capital stock is given in their verbal discussion, based on their algebraic presentation we are under the impression that the rental rate is taken as given and the capital stock was implicitly determined consistently with this rental rate and the demand for output. If this is so, then the price-wage frontier referred to immediately after equation (5bb) and explicitly derived as equation (6*) in footnote 7 below is well defined. This means that, corresponding to a given rental rate for capital and the production function, the real wage rate is uniquely determined up to the mark-up factor, as in our formulation.

On the other hand, L-N may have in mind a different formulation, in which the real wage rate is determined in the labor market prior to other decisions by firms. They also take the stock of capital to be given. Since the cost of capital is presumably the "sunk" cost in this case, the firm will accept whatever the real wage rate imposed upon it, and hire workers up to the point where the marginal value product of labor is equal to the wage rate. Since the marginal product of labor is declining (given that the capital stock is given), after paying all workers the uniform real wage, there remains residual income and this determines the quasi-rent for capital. Presumably, this is accomplished by letting the market price of capital float so that the rate of return on capital is consistent with the rate of interest, but there is nothing to insure that the market value of capital is consistent with the replacement cost for it.

If this is what L-N have in mind as the model of the firm behavior, then the optimization problem must be formulated in a different way from the algebraic exposition sketched in their 1986 paper. Note also that the formulation outlined in the preceding paragraph is a very short-run solution, and in order to explain the pattern of prices, wages, and employment over time, a description of how the capital stock in the following period is determined is needed. But if the process of capital accumulation is to be made endogenous, the relationship between the replacement cost and the market value of capital cannot be ignored. We are thus forced back to the standard formulation we have presented in the text.

function, it is nothing but the proposition that the wage rate should be equal to the marginal value product of labor adjusted for the mark-up, and so is equation (6c) in (L-N, 1986).[5] In their 1985 paper, L-N argued that this rule was the result of profit maximization with respect to the output price, but in their 1986 paper they deemphasized the profit maximization story, in our opinion correctly.

A problem arises in the empirical implementation of this equation by L-N. Abstracting from short-run dynamics, their estimated equation for price is given by

$$\ln \frac{P}{W} = \frac{1}{.456}\left[-4.18+.0381\,G-.486\,\ln\left(\frac{I}{L}\right)\right] \tag{6}$$

where G is a set of demand variables.(L-N, 1986, Table 5) $L$ is the labor force rather than employment, but for the purposes of looking at the equilibrium properties of the system, the difference between $E$ and $L$ could be compounded into the constant. We have

$$\frac{I}{E} = \frac{I}{XCA}\;\frac{XCA}{E}$$

and the optimal capital - output ratio, $\left(\frac{I}{XCA}\right)^{*}$ is a function of $\frac{\mu R}{P}$. Let us consider the case in which the price of $I$ is proportional to the price of $XCA$, and the real rate of interest

[5]When factor proportions are at the point where the total unit cost is minimized, the marginal cost of producing an extra unit of output using additional labor or additional capital is the same, and they are both equal to the minimized average cost. This is not true at any other point on the production function, so that it is somewhat misleading to say that the pricing rule is a mark-up on marginal cost without specifying exactly what is meant by marginal cost.

and all other factors entering $R$ are constant. In that case, $\left(\frac{I}{XCA}\right)^{*}$ is constant and hence, we have

$$\ln\frac{P}{W} = const. + 1.067\ \ln\frac{E}{XCA} \tag{6a}$$

In other words, at a fixed cost of capital, any increase in labor productivity results in an increase in the real wage rate with an elasticity of 1.067. In such a world, if the productivity of labor continues to rise as we hope it would, then the share of labor income in total value added will continually rise, because the real wage rate is rising faster than the rate of increase of productivity per manhour. Thus, L-N's price equation is inconsistent with the existence of a steady state growth path. Since they restrict the steady state elasticity of $I/E$ with respect to $P/W$ to be the same in the price equation and in the labor demand equation, their labor demand equation is also inconsistent with the existence of a steady state growth path.

Since L-N's estimated elasticity of 1.067 is barely significantly different from 1.0, we assume that they could have restricted it to unity without changing their result significantly. On the other hand, if they wanted to give their data the possibility of generating the elasticity of the optimal labor-output ratio with respect to the real wage rate different from unity, they could have easily based their estimates more explicitly on the CES production function with labor augmenting technical progress.

The basis for such a reformulation is given by equation (5b) or (5bb) above. Recasting it in the form that L-N use in their empirical work, we have, for a given cost of capital,

$$\ln\frac{W}{P} = const. + \frac{1}{\sigma}\ln\frac{XCA}{E} + \left(1-\frac{1}{\sigma}\right)\gamma t \tag{6b}$$

(6b) is different from (6a) only by the presence of the last term, $\left(1-\frac{1}{\sigma}\right)\gamma t$. But this is the term that insures the proper homogeneity, making it possible for the system to have productivity growth while maintaining constant shares of total value added between capital and labor. When $\sigma$ goes to one - the Cobb-Douglas Case -- the coefficient of $\ln(XCA/E)$ becomes unity while at the same time the last term vanishes. One cannot let the coefficient of $\ln(XCA/E)$ deviate from unity while ignoring the emergence of the last term. This is also an easy modification to implement empirically, since the constraint is linear. L-N report that they attempted to introduce a productivity trend term in their estimation equation but its estimated coefficient was always insignificant.[6] There should be no question of significance, since only one independent coefficient needs to be estimated between two independent variables, $\ln(X/E)$ and $\gamma t$, if $\gamma$ has been estimated elsewhere. Alternatively, $\sigma$ might be estimated, for example, in the investment equation, then introduced here so that $\gamma$ can be estimated here. Estimation of these two equations together, on the other hand, is fairly difficult because of critical non-linearity in such a system.[7]

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[6]In other words, L-N recognize that the rate of change of the productivity term belongs here as we have shown in (6b) above, but they did not seem to have realized the importance of the constraint imposed on the coefficient of $\ln(X/E)$ and that of $\gamma t$ by the homogeneity of the production function. If this constraint is not taken into account in estimation, and $\sigma$ is reasonably close to unity as in their case, it is natural that the coefficient of $\gamma t$ turns out to be statistically insignificant. But even a very small, statistically insignificant coefficient here has a critically important role to play on the long run properties of the model. It is not the exact numerical value but the constraint relating two coefficients that is important.

[7]L-N may respond that they have in mind a more general production function than a CES. Consider then a general homogeneous production function of degree one in inputs,

$$XCA = f(Ee^{\gamma t}, I)$$

and define

$$f_1 = \frac{\partial t}{\partial (Ee^{\gamma t})}; \qquad f_E = e^{\gamma t} f_1$$

## B. PHILLIPS CURVE

### 1. Traditional MPS Formulation

Since Phillips (1958) observed the relationship between the rate of change of the nominal wage rate and the rate of unemployment, a number of different interpretations of this relationship have been offered, but we prefer to view it as a description of a particular equilibrating process in the labor market. Let us begin our discussion by observing that, when Phillips worked on this topic, the most popular formulation of price dynamics was the one proposed by Samuelson (1947) which posited that the rate of change of a price is an increasing function of the excess demand for the corresponding good. We may assume

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(5b) then becomes

$$\frac{uWe^{-\gamma t}}{P} = f_1(Ee^{\gamma t}, I)$$

Since f is homogenous of degree one in $Ee^{\gamma t}$ and $I$, $f_1$ is homogeneous of degree zero and, hence,

$$\frac{\mu We^{-\gamma t}}{P} = f_1(Ee^{\gamma t} / I) \qquad \text{(6bb)}$$

(6bb) is the exactly the same form as (5bb) and makes $\frac{\mu W}{P}$ proportional to $e^{\gamma t}$. Note that

$$f_l(Ee^{\gamma t}, I) = f_l(Ee^{\gamma t} / I) = \frac{\mu R}{P}.$$

Hence, provided that the inverse function $f_l^{-1}$ exists, $Ee^{\gamma t}/I$ is uniquely determined for a given $\frac{\mu R}{P}$. By substituting the last expression into (6bb), we obtain

$$\frac{\mu We^{-\gamma t}}{P} = f_1\left[f_l^{-1}\left(\frac{\mu R}{P}\right)\right] \qquad \text{(6*)}$$

(6*) expresses $\frac{W}{P}$ as a function of the real interest rate. As noted in comments immediately following eq. (5bb), the expression (6*) is sometimes referred to as the "price-wage frontier".

that what Samuelson meant by price was the price of the good in question relative to the price index of some basic basket of goods.

In the context of the aggregate labor market, taking the relative price of labor as the ratio of the nominal wage rate to some general price index and the unemployment rate as an indicator of excess demand (supply) in the labor market, the Samuelson hypothesis can be written as,

$$\Delta(W / P^c) / (W / P^c) = \phi(u) \tag{7}$$

or

$$\Delta W / W = \phi(u) + \Delta P^c / P^c \tag{7a}$$

If the price term in (7a) is interpreted as the expected rate of inflation, then it can be immediately viewed as the expectations-augmented Phillips curve. In what follows, in order to keep our exposition as simple as possible, we replace the term $(\Delta P^C/P^C)$ by $(\Delta P^C/P^C)_{-1}$[8]. The separation of the wage and price terms in (7a) recognizes that, in the context of the structure of the labor market in the U.S., actions of employers and employees determine the nominal wage rate, not the real wage. Neither employers nor employees in a particular market has much control over an aggregate price measure such as $P^C$. This point is important in the comparison of our formulation with that of L-N.

In the original Samuelson formulation, which referred to a specific commodity such as corn, the rate of change of the price becomes zero at the point where the excess demand is zero. When the formulation is applied to the wage rate and the labor market, the situation

[8]The basic characteristics of the system do not change significantly if this simplification is replaced by a lengthy distributed lag, or by the assumption of perfect forsight.

is more complex. Equilibrium in the labor market, defined as the level of u that is consistent with a "stationary" relationship between $W$ and $P^C$, may depend on factors such as the relative bargaining power of employers and employees, the generosity of unemployment insurance benefits compared with potential earnings, and shifts in the demographic composition of the labor force.

In further elaborating the specification of the Phillips curve, two "wedges" between price variables that are of interest to workers and firms need to be considered. One is the difference between the total cost to the firm of employing a worker relative to worker's take-home pay. The wage rate $W$ is defined as total compensation inclusive of employer contributions to social insurance representing the total hourly cost of employing a worker. The wage rate that matters to employees, on the other hand, is likely to exclude certain items, especially employment taxes on employers. In the American context, the latter include employer contributions to the social security system (including Medicare) and to the unemployment insurance program. We assume, therefore, that when contribution rates on these items are increased, some fraction of such increases will show up as an increase in $W$ with only a partial reduction in take-home pay.

The second wedge represents any divergence of consumption prices from product prices. An explicit relationship between these prices is needed because eq.(7a) involves $P^C$ while we have only explained $P$ in equation (5bb). We will make use of the definition:

$$P^c C = PX + P^{IM}\ IM \tag{8}$$

and the approximation

$$\Delta P^c / P^c = (\Delta P / P) + (P^{IM} IM / P^c C)[(\Delta P^{IM} / P^{IM}) - (\Delta P / P)] \tag{9}$$

where $P^{IM}$ is the price of intermediate inputs, including domestic and imported raw materials such as oil and agricultural products, and *C*, *X*, and *IM* are consumption expenditure, value added and intermediate inputs in base year prices. The relationship is exact if the ratio *IM*/*C* is constant over time[9].

Finally, suppose that, for whatever reason, the difference between the actual value of the real wage rate, *W*/*P*, and its equilibrium value, (*W*/*P*)*, defined by equation (5b), turns out to be unusually large. It seems reasonable, under such conditions, that the adjustment process for *P* can be seen as too slow and that the participants in the labor market would also attempt to adjust *W* to correct for this situation. This may be especially true in economies in which institutional indexation of wages is prominent.

All these considerations imply that the simple Phillips curve (7a) should be modified to include several additional variables, resulting in:

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[9]From (8), we have

$$\Delta P^{c} \equiv \frac{X}{C}\Delta P + \frac{IM}{C}\Delta P^{IM} \;\; ; \frac{X}{C} = 1 - \frac{IM}{C}$$

so long as $\frac{\text{IM}}{\text{C}}$ is nearly constant. We then have

$$\frac{\Delta P^{c}}{P^{c}} = \frac{XP}{P^{c}C}\frac{\Delta P}{P} + \frac{P^{IM}IM}{P^{c}C}\frac{\Delta P^{IM}}{P^{IM}}$$

$$\frac{\Delta P^{c}}{P} = \left(1 - \frac{P^{IM}IM}{P^{c}C}\right)\frac{\Delta P}{P} + \frac{P^{IM}IM}{P^{c}C}\frac{\Delta P^{IM}}{P^{IM}}$$

Equation (9a) above follows immediately.

$$\Delta W / W = \phi(u^w) + (\Delta P / P)_{-1} + \theta_1(\Delta T / W) + \theta_2(B^u / W)_{-1}$$

$$+\theta_3[(\Delta P^{IM} / P^{IM}) - (\Delta P / P)]_{-1} + \theta_4[(W / P) - (W / P)^*]_{-1} \qquad (7b)$$

where θ's are parameters to be estimated, $u^W$ is a demographically weighted unemloyment rate[10], $T$ is the dollar amount of employment taxes collected per hour, and $B^u$ is unemployment insurance benefits. Other variables have been previously defined.

Along the steady-state growth path, the wage equation simplifies to

$$\Delta W / W = \phi(u^W) + (\Delta P / P)_{-1} + \theta_2(B^u / W) \qquad (7b')$$

A second structural relationship between $P$ and $W$ is given by the price equation (5bb). Since we are focusing our attention here on the steady-state growth path, let us replace $E_t/XCA_t$ in (5bb) by $(E_0/XCA_0)e^{\gamma t}$ and then take the time derivative of (5bb), holding μ constant along the growth path:

$$\Delta P / P = (\Delta W / W) - \gamma \qquad (5bb')$$

Substitution of (5bb') into (7b') yields the necessary condition for constant wage inflation:

$$\phi(u^W) - \gamma + \theta_2(B^u / W) = 0 \qquad (10)$$

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[10]That is, $u^w$ is the unemployment rate computed keeping the demographic structure of the labor force constant as it was in the base year.

The unemployment rate that satisfies (10) is defined as the "natural" rate of unemployment and denoted by u* [11].

We complete the discussion of the Phillips curve by noting that, because the response of both the demand and supply of labor to the level of the real wage rate is quite weak in the short run, the level of the unemployment rate may be taken as given for the purposes of analyzing short-run wage-price behavior at the macroeconomic level. With regard to labor supply, accumulated evidence in the U.S. indicates that primary workers -- those who are between 25 and 60 years old and the main income earners for their families -- are not very responsive to variations in the real wage rate. That is, they tend to remain in the labor force under most conditions. On the other hand, secondary workers tend to be quite responsive positively to their own wage rate, but they are also responsive negatively to the wage rate of primary workers. Since the aggregated wage rate is an average of the wage rates for both groups, we find that the participation rate of workers does not respond much to the aggregate wage rate.

With regard to the effect of the real wage on labor demand, we must remember that the stock of capital available to firms is a collection of vintage capital. The real wage rate is equated to the marginal value product of labor associated with the newest vintage, and the same nominal wage must be paid to labor working with all vintages of capital. The price of output is also determined on the basis of the cost associated with the newest vintage, as described in the preceding section. The cost of using older equipments, other than maintenance expenses, is a "sunk cost" and not relevant for current decisions. Thus, so long as the gross labor costs of using older vintage machines is less than the price of

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[11](10) yields $u^{W*}$, that is, the natural rate in terms of $u^{W}$, and the unemployment rate for various demographic groups in the labor force. We can then compute the standard definition of the unemployment rate corresponding to this solution, and designate it as u*. See footnote 22 below.

output and sufficient demand for output exists, production with the older vintages will be undertaken. The price of output less gross labor cost defines the residual rent for older vintage equipment, and the present value of current and future rents determines the market value of this capital. The only effect of the real wage rate on the demand for labor is associated with the decision to utilize or not to utilize the least efficient vintage of capital. After utilizing various vintages of capital in order of efficiency, if the demand for output is such that firms can sell more output, they must compare the potential revenue associated with the next vintage of machines, and the cost of labor for operating them. If the real wage rate is such that the net revenue after the wage payment is still positive, firms will utilize these marginal machines, otherwise they will not utilize them. It is generally expected that firms do not face this type of decisions because their investment policy should have insured that there should be enough efficient machines to meet the demand without utilizing really inefficient machines. The short run effect of real wage rate on the demand for labor, therefore, is expected to be quite small. We present some evidence for this proposition in Section III. C. below.

## 2. Layard-Nickell Formulation of Wage Rate Determination

As in the case of the price setting behavior of firms, the basic framework that L-N use to describe wage setting behavior in the labor market contains some parallel considerations to the one we have used in formulating the Phillips curve in the MPS model. They both describe responses of the wage rate to excess demand (supply) in the labor market, that is, the interaction between the behavior of firms and that of workers. We believe, on the other hand, that the differences between the two formulations are much more basic in this case than in the case of the price setting equation. Let us begin by introducing the L-N formulation explicitly. The basic formulation remains the same in their 1985 and 1986 papers, and it is given by

$$(1 - g\psi)(\ln W - \ln P) = (\ln P^e - \ln P) + g_0 + g_1(\ln L - \ln N)$$
$$+ g_2(\ln K - \ln L) + g_3 Z + g_4 \gamma t + g_5 G \qquad (11)$$

where $\psi$ is the lag operator.

In both 1985 and 1986 papers, empirical estimates of parameters in (11) were based on the above equation with one modification ($\ln L$-$\ln N$ is replaced by $\ln u$, where $u$ is the unemployment rate) and some restrictions on parameters ($g$=0, $g_4$=0, and $g_2$ is set to be equal to the coefficient of $I/L$ in (6) in equilibrium). Given that g is restricted to be zero, this equation determines the level of the real wage rate, and it plays an entirely different role from the Phillips curve which determines the rate of change of the nominal wage rate.[12]

Before we attempt to understand the nature of this equation, we wish to note one mechanical point, the same one we raised with regard to L-N's pricing equation, recast in the form of equation (6b) above. In the pricing rule, we pointed out that if the coefficient ($1/\sigma$) of $\ln(K/L)$ is not equal to unity, then the equation must contain an additional term $\left(1-\frac{1}{\sigma}\right)\gamma t$ in order to maintain the homogeneity implied by the production function and to

[12]In their 1985 paper, L-N offer a transformation of (11) which appears to be more like a conventional Phillips curve. It is given as (11a) below.

$$\ln W - \ln W_{-1} = (\ln P^e - \ln P_{-1}) + g_0 + (g-1)(\ln W_{-1} - \ln P_{-1}) + g_1 u + g_2(\ln K - \ln L) + \text{etc} \qquad (11a)$$

We believe that (11a) has a virtue of making clear exactly what $P^e$ has to be. It must be the expected value for P in the current period, and $\ln P^e$- $\ln P_{-1}$ must be the first difference from the pricing equation under the assumption of rational expectation. It then becomes critical that the homogeneity restriction discussed throughout in our review of L-N be observed both in (11) and (6b), because otherwise these equations imply that wage inflation increases as the level of productivity in the economy increases, other things equal.

insure that it is possible for the labor share of total value added to remain constant. Exactly the same argument holds here, and since L-N have imposed the restriction that the coefficient of $\ln(K/L)$ must be the same with the opposite sign here as in the pricing equation, their estimates of the wage equation are not consistent with the implications of the homogeneous production function.[13]

In spite of a fairly lengthy discussion by L-N, we find this equation rather difficult to interpret. One possible interpretation is that it is the supply function of labor, but then the variable like $K$, which belongs strictly to firms who employ workers, does not belong in this equation. We offer here one possible interpretation, although it will leaves some puzzles in its economic motivation. Suppose that we write the supply equation of labor as :

$$\ln L = l_0 + l_1 \ln(W / P) + \ln\ LT + ZS \qquad (12)$$

where $LT$ is the size of all potential workers in the economy, for instance, civilian, non-institutional population of working age, $ZS$ is the set of all other variables that may affect the participation rate, and $l_i$'s are numerical parameters. That is, (12) represents the participation behavior of workers.

Let us take L-N's labor demand equation ((L-N, 1986), eq.17 and Table 4) and write its steady state implication as

$$lnN = n_0 + n_1 (W/P) + lnK + ZD \qquad (13)$$

[13]Of course, in the U.K., it may be that the share of labor in domestic business value added steadily increased during the sample period and L-N's estimates account for the increase. But even in such a situation, it is more desirable to impose the homogeneity restriction and then to search for causes of such shifts in factor shares of income.

where $ZD$ represents all variables in L-N's demand for labor equation (in its steady state form) other than the one explicitly shown in (13) above, and $n_i$'s are parameters.

By subtracting (13) from (12), we have

$$\ln L - \ln N = (l_0 - n_0) + (l_1 - n_1)\ln(W/P) + \ln LT - \ln K + ZS - ZD \tag{14}$$

Solving (14) for ln($W/P$), we finally have

$$\ln(W/P) = \frac{1}{(l_1 - n_1)}[-(l_0 - n_0) + \ln(K/LT) + (\ln L - \ln N) - ZS + ZD] \tag{14a}$$

(14a) has almost the same form as L-N's wage equation ((L-N, 1986), eq.(23) and Table 6), except that L-N replaced ln$L$-ln$N$, or its approximation $u$, with ln$u$, thus making it impossible to match coefficients, especially the requirement that the coefficients of $ln(K/LT)$ and (ln$L$-ln$N$) should both be $[1/(l_1-n_1)]$. If (14a) provides the proper interpretation, then L-N's requirement that the coefficient of ln$K/L$ in their wage equation equal the reciprocal of the (steady-state) coefficient of ln$W/P$ in their labor demand equation is equivalent to saying that the supply of labor is not responsive to the real wage rate. In the case of the U.S., we have argued that this is not a bad assumption, but we wonder if L-N meant to impose such a restriction.

Equation (14) may be thought of as a reduced form equation which gives the maintainable level of the unemployment rate for a hypothetical value of the real wage rate, given other exogenous variables. If, in addition, we take the real wage rate to be given by the price-wage frontier defined in Section II.A.1. and equation (6*) in footnote 7, then equation (14) is consistent with the system that we have outlined as representing the structure for prices and wages in the MPS model, except that the relationship between $P$ and $P^C$ must be made explicit.

In writing this relationship in the form of (14a) rather than as in (14), L-N appear to assert that the real wage may be set at a value different from that given by (6*) in footnote 7. L-N may mean to suggest that workers and employers bargain between them to determine the sharing of the mark-up, while accepting the basic real wage rate before the mark-up given by the price-wage frontier (6*). If this is their intention, there is little difficulty in integrating those features of labor market with which L-N are concerned with the optimizing behavior of firms[14]. On the other hand, if L-N's formulation calls for participants in the labor market to impose the real wage rate different from that given by (6*) which cannot be accommodated by a redistribution of the mark-up, then such a model must face all the difficulties outlined in footnote 4 above. We do not believe that the short-run optimization scheme of the type outlined in footnote 4 can be a basis for explaining dynamic pattern of the real wage rate over a long period of time, and for generating an estimate of basic equilibrium concepts such as the natural rate of unemployment.

## III. Empirical Implementation of Producer Behavior in the MPS Model and Tests of Some Popular Propositions

[14]This formulation is especially plausible for economies in which a centralized bargaining between the workers and employers is effective. This is especially true if workers and employers recognize implications of their activities, and agree to index the basic wage rate to the value added price index rather than to the final good price index. Then the arrangement implies that income shares between labor and capital are set by the bargaining insofar as such sharing can be accommodated by distributing the mark-up. This appears to be the scheme followed by Japan since 1980.

## A. Some General Comments

In this part, we report the empirical implementation of four equations discussed in Part II, namely, the investment equation for producer equipment, the manhours equation (demand for labor), the pricing rule for the output by firms, and the Phillips curve. These four equation are the collection of equations minimally needed to describe the mechanism generating the price-wage behavior and the level of employment in the MPS model. They must be supplemented by a few semi-definitional equations, approximately in the form of (8) and (9), in order to relate specific prices needed elsewhere to the value added price index determined by our main pricing equation[15].

These equations have been a part of the MPS model for more than 20 years, and they have gone through a series of reformulations, although their basic form has not been changed significantly. Most of the changes have to do with forms of dynamic adjustments, but in our experience respecifications of this sort do not affect the behavior of the system much, provided that they are so formulated to preserve the steady state properties of each equation.

To give ourselves an opportunity to check on this proposition and for purposes of exposition, we have reported in this paper formulations of the equations in question that are somewhat different in their adjustment processes from those used in the model itself. We simply report here that in no case does the change created significant difference in the behavior of the system as a whole both in terms of its steady state properties and also in its

[15]We will refer to these equations as "producer behavior" equations, although it is missing two critical sets of equations for producer behavior, namely, inventory equations and/or scheduling of output production, as well as the equation for producers' structure, while the Phillips curve is not exclusively the behavior of producers. We cannot quite call them the supply block because the supply of saving is excluded. We find it least misleading to call them the producer behavior equations.

dynamic behavior. We now proceed to summarize empirical findings for each of four basic equations.

## B. ESTIMATION OF THE BASIC EQUATIONS

### 1. Investment in Producer Equipment

The starting point of the investment equation is (5a) shown in Section II.A.1, which gives the optimal capital-output ratio as a function of the ratio of minimized average cost (which is equal to the output price divided by the mark-up factor) to the gross rental price of capital. We observe two simplifications. First, in repeated estimations of the investment equation over many years, we have never found $\sigma$ to be significantly different from unity. We therefore conclude that the relevant production function here is Cobb-Douglas, and assume that $\sigma$ is in fact unity. Second, gross rent, $R$, is simply equal to $P^I(\rho+\delta)$ in the absence of various fiscal interventions, but it can become a fairly complex expression once taxes and subsidies are introduced. Fortunately, however, almost all of these fiscal interventions contribute to the definition of the cost of capital through a complex multiplicative factor to $R$. We will therefore rewrite (5a) as

$$\frac{I}{XCA} = A(1-\alpha)\left[\frac{P}{\mu P^I(\rho+\delta)}F(\tau)\right] = Av \qquad (5a')$$

where $F(\tau)$ is the summary measure of fiscal intervention on the cost of capital, and we denote the optimal capital output ratio by $v$. Because some variables, especially prices, are measured in arbitrary units, we need the scale factor, $A$.

For estimation, $XCA$ is moved to the right hand side and lags are introduced to reflect the time it takes for firms to recognize needs for new investment, to design the appropriate equipments corresponding to the best available technology and relative prices, to order such equipment and then to wait for its delivery. The second reason for lags is that the relevant addition to capacity on which investment should be based is the needed addition in a future period, while we must approximate it in terms of currently available information. Traditionally, a double distributed lags originally proposed by Bischoff have been used.

$$\sum_{i=0}^{T} b_i \, XCA_{t-i} \, v_{t-i-1} + \sum_{i=1}^{T} c_i \, XCA_{t-i} \, v_{t-i}$$

We will work, however, with a somewhat different form here. We may rewrite (5a') by dividing gross investment into a net addition to capacity, $\Delta XC$, and replacement of depreciating capacity, $\delta XC_{-1}$, and express the steady state gross investment, $I^*$, as

$$I^* = Av\,\Delta XC + Av\,\delta\, XC_{-1} \tag{5aa}$$

Note that the constant $A$ is added to take account of the arbitrary scale of price indices. The empirical investment equation expresses investment in terms of current and lagged values of the variables on the right hand side of (5aa), with capacity output being approximated by moving averages of actual output:

$$I = \sum_{i=0}^{17} A1_i \ \Delta XB_{-i} \ v_{-i-1} + \sum_{i=1}^{18} A2_i \ \delta_{-i} XB_{-i} \ v_{-i}$$

$$\sum A1_i = .35\ (6.0); \quad \sum A2_i = .13\ (33.5) \qquad (15)$$

*Sample period*: 1967.1–1991.4; $\overline{R}^2 = .995$; $se = 4.81$

Detailed estimation results are given in Appendix I[16].

The putty-clay assumption is imposed on (5aa) and (15) by the absence of the term $XC_{-1}\Delta v$ which would reflect investment undertaken to change the capital intensity on existing vintages. In (5aa) and (15), changes in relative factor prices only affect investment to the extent that firms are undertaking gross additions to capacity. We have undertaken a test of whether or not the term $XC_{-1}\Delta v$, contributes significantly to the explanation of I. If it is added to (15) as another distributed lag, it is clearly insignificant as reported in Table 1.

## 2. Demand for Employment

Given that the nature of capital is putty-clay, labor demand in the short run is determined by an approximation to the labor requirements associated with the sum of the fixed coefficient production technologies for the existing vintages of capital, used in order of efficiency, to produce the output required. With the assumption that the ex-ante production function is Cobb-Douglas, in time t, the manhours required to produce output associated with capital of vintage s is given by

[16]For the simplicity of exposition all estimated equations reported in the text are abbreviated. The full results are given in Appendix 1.

$$\ln E(t,s) = Const. + \ln X(t,s) - [(1-\alpha)/\alpha] \ln \hat{v}(t,s) - \gamma s \qquad (16)$$

The first argument in the parenthesis refers to the time of production, while the second argument refers to the vintage of capital used. $E(t,s)$ and $X(t,s)$, respectively, are the manhours required to utilize equipment of vintage $s$ fully in period $t$, and output that can be produced using equipment of vintage $s$ fully in period $t$. $\hat{v}(t,s)$ represents a weighted average of $v(t\text{-s-}\tau)$, $\tau = 1, 2, \ldots T$ where $T$ is the same as the limit in the summation in (15)[17].

Let us construct the optimal capital-output ratio averaged across all vintages of capital in place at time $t$, $\bar{v}$(t), recursively by

$$\bar{v}(t)XC(t) = (1-\delta_t)\ \bar{v}(t-1)\ \ XC(t-1) + .25\hat{v}(t,o)\ XCA(t) \qquad (17)$$

.25 is necessary because $XCA$, gross addition to capacity, is measured at an annual rate, and time is measured in quarters.

We can then show, through rather tedious algebra, that the aggregate demand for manhours in the steady state growth path can be described as

$$\ln E^*_t = const^* + \ln X - [(1\text{-}\alpha\ \ _t)/\alpha\ \ ] \ln \bar{v}_t - \gamma t \ ^{18} \qquad (18)$$

---

[17]In other words, (16) may be thought of as a specific realization of the production function (2) in period t-s, except that a fraction of equipments then produced has disappeared through the normal depreciation process, and that the investment equatio (15) made an error in period t - s.

[18]Because ln(E(v)) is not equal to E(ln(v)), equation (18) must be regarded as an approximation. However, if, for example, v is distributed by the log-nromal distribution, and its variance is constant over time, then the equation is exact except for an adjustment needed to the constant.

where $E^*_t$ is the path of $E_t$ corresponding to a stationary growth path of $X_t$. The basic assumption is that $\hat{v}_t$ is stationary over time, and that $X_t$ is also stationary around $X_0( e^{(\gamma+\eta)t} )$, where $\eta$ is the rate of growth of the potential labor force. The same $\gamma$ as in (16) holds here, but *const** is not the same as *const** in (16). The numerical value of $[(1-\alpha)/\alpha]$ in the U.S. is estimated to be .285.

The estimated labor demand equation is specified in an error correction format, except that adjustment to $\bar{v}$ is assumed to be instantaneous as this term already is very long weighted average of past relative prices.[19]

$$\underset{}{\Delta (\ln E + .258 \ln\bar{v})} = \underset{(3.5)}{-.15} + \underset{(5.8)}{.31}\, \Delta (\ln E + .258 \ln\bar{v})_{-1}$$

$$+ \underset{(13.0)}{.46}\, \Delta \ln X - .00015t - \underset{(3.2)}{.06}(\ln E + \underset{(3.5)}{.258}\ln\bar{v} - \ln X)_{-1} \qquad (19)$$

Sample period : 1061.1 - 1991.4, $\bar{R}^2 = .734$, se = .0044

(19) is again shown in an abbreviated from, and detailed estimation results are given in Appendix I.

[19]The formulation that is actually in the MPS model differs from (19) in its dynamics and takes account of the fact that the structure of the vintage capital makes labor less and less productive as actual output increases relative to capacity, while the presence of overhead workers has the opposite effects, leading to some non-linearity in the relationship between employment and output in the short run. The dynamic behavior of (19) and that of the one in the model, however, appears to be very similar.

### 3. Price of the Value-Added Measure of Output

The price equation is based on (5b), the first order condition of the cost minimization for labor input, rearranged to express the price level as a markup on unit labor costs. Invoking once again the Cobb-Douglas assumption, we have

$$\ln P^* = const + \ln W - \ln(E/X)^* + \mu \qquad (20)$$

where (E/X)* is the labor-output ratio excluding the effects of short-run, cyclical fluctuations, and it is constructed by taking the predicted value of $E$ excluding short-run, cyclical terms in equation (19) and then dividing the result by $X$. Note that, by construction, $(E/X)^*$ has the long-run trend of $\gamma$.[20] The mark-up factor, $\mu$, is assumed to vary with cyclical conditions and the degree of price competition from imports, represented by real terms of trade, $\varepsilon$.

$$\mu = \mu(u^W, \varepsilon)$$

The actual price equation is estimated in error correction form:

$$\Delta \ln P = \underset{(5.3)}{.11} + \underset{(5.6)}{.12}\,(\ln W - \ln(E/X)^* - \ln P)_{-1}$$

$$- \underset{(2.7)}{.001} u^W_{-1} + \underset{(4.2)}{.015}\varepsilon_{-1} + \sum_{i=1}^{4} B4_i\, \Delta \ln P_{-i} \qquad (21)$$

$$\sum_i B4_i = .26\ (2.3)$$

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[20]We may note that, in (5b), E/X referred to the employment and output associated with the currently installed, latest vintage of capital, while in (20), we are using total employment and the value added of the private, domestic, non-farm business sector. The justification for this switch is the argument leading to the proposition that $\gamma$ in equation (18) is the same as $\gamma$ in equation (16).

Sample period : 1963.1- 1991.4; $\overline{R}^2 = .661$; se = .0043

As before, the detailed estimation results are given in Appendix I. Although not shown in (21) above, the estimated equation contains dynamic terms reflecting the speed with which energy and agricultural prices are passed through to final product prices.

## 4. Phillips Curve

The final equation needed in this block is the Phillips curve, which determines the nominal wage rate (total compensation per manhour). Our estimated equation is a straightforward dynamic implementation of equation (7b), except that we did not explicitly split $\frac{\Delta P^c}{P^c}$ into its components as in (9), nor did we introduce the term in the real wage rate $\left[\left(\frac{W}{P}\right)-\left(\frac{W}{P}\right)^*\right]$. The summary of the result is shown below as (22).

$$\Delta \ln W = \underset{(3.2)}{.007} - \underset{(5.2)}{.002}u^W + \underset{(3.0)}{.25}\ \Delta \ln W_{-1}$$

$$+ \sum_{i=1}^{6} B3_i\ \Delta \ln P^c_{-i} + \sum_{i=0}^{3} B7_i (B^u / W)_{-i} \qquad (22)$$

$$\sum_i B3_i = .75\ (9.0); \qquad \sum_i B7_i = .03\ (2.1)$$

Sample period 1963.1 to 1991.4; $\overline{R}^2 = .684$, se = .0037

Here again, (22) is an abbreviated report of the result, and full estimation results are given in Appendix I. They show that the additional terms representing employer contributions to social insurance and the labor force participation of the female population are significant in explaining the inflation rate of the wage rate. The wage-price control measure, represented by a dummy variable corresponding to its introduction and removal, is marginally significant.

## C. Tests of Alternative Dynamics

We have already discussed the test of the putty-clay assumption in the context of the investment equation and the result is reported in Table 1. We see no evidence that this assumption is inconsistent with the data. Analogous tests have been performed in the course of reestimating the investment function for producer's equipment in the MPS model whenever major revisions of data took place, and the results have always been the same. We believe, therefore, that we are on a reasonably sound basis in accepting the putty-clay hypothesis.[21]

In comparing our formulation with that of L-N, a critical issue related to the question of the malleability of capital is the response of labor demand to variations in the real wage rate. We have introduced the term ($\ln W$ - $\ln P$ - $\ln (X/E)^*$) into the manhours equation (19) to check whether or not the demand for labor responds to the current level of the real product wage, adjusted for average productivity, over and above the impact of the real wage on labor demand incorporated into our formulation through $\bar{v}$. As reported in Table 1, this term is totally insignificant. This, of course, does not mean that the demand for labor does not depend on the real wage rate. It does mean that the channel through which the real wage rate affects the demand for labor goes through $\bar{v}$, and it takes a long time.

Finally, L-N stressed the point that bargaining in the labor market between employers and employees determine the level of the real wage rate, in contrast to the traditional Phillips curve which assigns the role of determining movements of the nominal wage rate to the

[21]A similar result was obtained in the context of the econometric model of the Bank of Italy using Italian data.

labor market. We indicated above that we do not find their argument fully convincing, but we are prepared to entertain the possibility that labor market participants pay close attention to the level of the real wage rate in bargaining about the nominal wage. To test this possibility, we have introduced into equation (22) the term ($\ln W$ - $\ln P$ - $\ln(X/E)^*$). The result is again reported in Table 1, and again this terms proves to be totally insignificant.

We conclude, therefore, that, as far as U.S. data are concerned, there is little evidence that the MPS formulation of the price-wage sector misspecified the role of the real wage and its determination, nor is there any evidence that its demand function for labor is missing some additional channels through which the real wage rate affects the demand for labor.

**D. The Natural Rate of Unemployment**

Implicit in the estimated structure of the wage and price equations (21) and (22) is the equilibrium, or natural, rate of unemployment (u*) -- the unemployment rate consistent with a constant rate of inflation. Although u* could be defined in terms of all factors affecting wages and prices, including highly transitory ones, it is more illuminating to calculate it as a medium-run concept which excludes transitory influences on wages and prices. Based on the estimated wage and price equations, the medium-term natural rate varies over time with the demographic structure of the labor force, the rate of growth of "permanent" productivity, and the unemployment insurance replacement ratio. Because the rate of growth of wages is specified to depend on the rate of growth of consumption prices, rather than the price of value added, the spread between the growth rates of these two prices can also be viewed as influencing the natural rate, especially if the growth rates diverge for a substantial period of time. On the other hand, short term fluctuations of this spread, due for instance to effects of weather variations on agricultural prices, should not be included in the definition of the natural rate. Unfortunately, it is usually quite difficult to distinguish between the long term movements of the spread from short term fluctuations. In order to provide the maximum information, we report two measures of the natural rate, one $(u_1^*)$ excluding the effects of the inflation spread and the second $(u_2^*)$ including them[22].

Both measures of the natural rate display considerably variation over time. Chart 1 plots $u_1^*$ in the upper panel and the contributions to the variation in $u_1^*$ over time of demographics, productivity growth and unemployment insurance. Chart 2 plots $u_2^*$ and the additional contribution of the inflation spread. The general slow-down in productivity growth has raised the natural rate over time: changing demographic structure gradually raised the natural rate from the early 1960s through the mid 1970s and slowly lowered it since. The effect of the replacement ratio is lagged, with considerable volatility at high frequencies (reflecting factors such as the legislation of temporary benefits) around a small secular downtrend. The effect of the spread between consumption and product price inflation also has tended to be erratic, although in recent years this spread has been persistently positive, tending to raise the natural rate.

We should perhaps observe here that the magnitude of the task of explaining the movement of the natural rate from the 1960s to the present is very different in the U.S. from the parallel task in most European countries. In the U.K., for instance, it seems to

[22]The formula for the first natural rate measure is

$$u_1^* = 5.66 + (u - u^w) - 522.7\,\Delta(X/E)^* + 15.3\,(B^u/W)$$

Each explanatory variable is expressed as a deviation from its mean so that the constant is the sample average of the natural rate. The second measure of the natural rate is defined as

$$u_2^* = u_1^* + 522.7(1/8)\sum_{i=0}^{7}\left[\left(\frac{\Delta P^c}{P^c}\right)_{-i} - \left(\frac{\Delta P}{P}\right)_{-i}\right]$$

In order to include only persistent effects of the inflation spread on the natural rate, the spread is measured as in eight quarter moving average.

have risen 6 to 8 percentage points during the past 30 years or so depending on how it is defined and measured. In the U.S., on the other hand, including the inflation spread in the definition, it rose from the low of less than 5% in 1960s to the high of close to 8% in the late 1970s, and fell to roughly 6% to 6.5% in early 1990s. The maximum movement recorded is therefore some 3.5 percentage point. While we have found a number of problems in the analytical framework used by L-N, their work contains some suggestions which may be helpful in explaining differences between the U.S. and U.K. economies in this regard.

## IV. A Closer Examination of the Wage and Price Equations

### A. The Motivation

In discussions of macroeconomic theory and policy since the mid-1970s, it is often taken for granted that conventional macroeconomic models have failed in some way to account for the course of economic history from the 1970s to 1990s, and that one of the prime culprits is the wage-price block of these models. As Visco(1991) has shown in some detail, even among better known models, there is sufficient variation in both the specification and the estimates of these equations that their performance probably has not been uniform, and we do not know the performance of models other than the MPS model. For those of us who have followed the performance of the modified Phillips curve and associated equations in the MPS model since the 1970s, however, the perception of the failure of these equations to explain movements in wages and prices has been a puzzle, since these equations have been more stable and reliable than most other empirical macroeconomic relationships in the MPS model during the period in question.

This section provides evidence for this view. We first examine the stability of these empirical relationships and then test the hypothesis that they are misspecified by failing to include a direct effect of the money supply on inflation. This latter question is important because, if the hypothesis is true, then even in a serious recession, the monetary authority may hesitate to follow an aggressively stimulative monetary policy if it resulted in a higher rate of growth of the money supply. On the other hand, if the money supply affects the rate of inflation only through excess demand or supply in the labor and goods and services markets, then the monetary authority may feel more inclined to follow active policies to keep the economy near the growth path consistent with the maintenance of the natural rate of unemployment even if this causes the supply of money to grow at a rate different from some predetermined target.

**B. Stability Tests**

We focus our stability tests on the beginning of the 1980s when U.S. monetary policy shifted to a strongly anti-inflationary stance. In particular, if our formulation of adaptive inflation expectations is wrong and economic agents shifted their view of future inflation around the time of this major policy change, this should show up as evidence of instability in the MPS wage and price equations. In the case of the price equation, the hypothesis of structural stability cannot be rejected at standard significance levels (p-value of .12), while stability of the wage equation can be rejected (p-value of .04). While the evidence regarding the wage equation appears to be at odds with the claim made above, the instability has to do with very short-run dynamics, not with wage movements over periods of a year or longer. Chart 3 plots the actual change in compensation per hour from 1980 to 1991 along with the predictions of two versions of the wage equation, one estimated from 1963 to 1979 and the other over the period graphed.[23] The chart thus compares an out-of-sample estimate of wage growth since 1980 with an in-sample estimate. The version of the wage equation estimated through the end of 1979 actually does quite a

[23]The predicted values are static estimates based on the actual values of all explanatory variables.

good job of tracking the behavior of wage inflation since 1980. Not surprisingly, the out-or-sample predictions are poorer than the in-sample ones: the root mean square error of the former is about one-third larger than the latter. But if the errors are averaged over four quarter intervals, the out-of-sample measure is only five percent larger than the in-sample measure. This suggests to us that, despite the formal statistical result, there really is no evidence of any fundamental shift in wage behavior.

## C. An Out-of-Sample Simulation of Inflation Since 1980

The second approach to testing the robustness of the wage and price equations is through out-of-sample simulation. Chart 4a reports the results of such an exercise using versions of the wage and price equations estimated through the end of 1979 in a dynamic simulation starting in the first quarter of 1980 and ending in the second quarter of 1993.[24] The equations accurately track the speed of disinflation through 1982, but then under-predict inflation somewhat through the mid-1980s, and over-predict it somewhat since the late 1980s. The root mean squared forecast error of the rate of price inflation (GDP deflator) is 2.16 percentage points while the mean forecast error is only -.06 percentage point as the periods of under-prediction and over-prediction largely cancel out. For an out-of-sample simulation that is 13 years long, this performance seems rather good.

A closer examination of results shows that the prediction errors are associated to a large extent with the estimated response of the price markup to the real terms of trade. The version of the price equation estimated through 1979 has a coefficient on the real terms of trade that is twice the value estimated over the full sample. As a consequence, in the dynamic simulation the appreciation of the dollar in the first half of the 1980s and its

[24]The set of equations simulated consists of price and wage equations, (21) and (22), augmented with simple equations for the personal consumption deflator and the GDP deflator. In the simulations, the ratios of energy and farm prices to the GDP deflator are held at their historical values, as are other variables such as the real terms of trade and the unemployment rate.

subsequent depreciation lead to movements in the price markup--first falling and then rising -- that are too large. Nonetheless, because the value of the real terms of trade is about the same at the beginning and end of the simulation period, the movements in the markup are offsetting.

Pagan (1989), Hendry(1986) and others have questioned the appropriateness of using dynamic simulations in evaluating the performance of a system of equations. We have already reported in Section IV.B. above the stability of individual equations in the system over a period of major policy changes by a more standard procedure and found that our equations do not show much instability. We supplement these sets of information by providing in Chart 4b the static simulation of the same set of equations as in Chart 4a. That is, the only difference between Chart 4a and Chart 4b is that, in generating Chart 4b, we have utilized actual values of lagged endogenous variables on the right hand side of equations, while in Chart 4a, we used lagged endogenous variables internally generated by the system. In both cases, equations were estimated using data through the 4th quarter, 1979, and predicted values are the simultaneous solution of equations in question.

Without getting into the technical issues of interpreting dynamic simulation errors, looking at Charts 4a and 4b, at least intuitively, it should be clear that it is helpful to look at them together rather than one or the other alone. Chart 4b suggests that the system performs well in extrapolation, and there is little evidence of a clear break in the structure. Chart 4a, on the other hand, highlights the problem, hinting at its cause, namely, overestimation of the effects of the terms of trade.

### D. Tests of a Direct Influence of Money on Prices

In the MPS wage and price equations, the role of monetary quantities is indirect: a change in the supply of money affects prices over time in the full model, but the causal chain extends from the supply of money to short-term nominal interest rates, to real interest rates, to the demand for output relative to supply, to the unemployment rate, and finally to wages and prices. Although this approach in which effects of money on prices and wages are indirect is similar to much of the empirical literature on U.S. inflation, over the years a variety of inflation models have been proposed that contain a direct influence of the supply of money. Examples include the PSTAR model of Hallman, Porter and Small (1991) and the Rotemberg's (1982) adjustment cost model of price. This section provides evidence that the MPS wage and price equations are not misspecified by omitting a direct role of money on wage and price setting.

The PSTAR model is the monetary model of inflation that has received the most attention in recent years, and it is the focus of our misspecification tests. The monetary variable in the PSTAR model is a measure of money market disequilibrium, $gap^m$, equal to the log difference between *M2* per unit of potential output and the price level:

$$gap^m = \ln M2 - (\ln GDPC + \ln PGDP) = (\ln VEL + gap^x) \qquad (23)$$

where *M2* and *GDP* are standard official definitions, *GDPC* is the capacity *GDP*, P*GDP* is the deflator of *GDP*, and *VEL* is the *GDP* velocity of *M2*. $gap^x$ is the difference between ln *GDP* and ln *GDPC*. The second equality of (23) indicates that the money gap can be expressed in terms of velocity and the output gap. $gap^m$ is similar to what one might expect to find in a money demand equation as a measure of short-run money market disequilibrium except that potential output is used in place of actual output, and the effects of interest rates are excluded. But the differences between $gap^m$ and short-run disequilibrium are transitory. The output gap is stationary by construction and the relevant interest rate -- the difference between market interest rates and rates on *M2* deposits -- has little low frequency movement. Thus, it seems reasonable to characterize $gap^m$ as a form of money market disequilibrium.

The PSTAR model specifies that the rate of change of inflation ($\Delta^2 p$) depends on the lagged money gap and lagged changes in inflation:

$$\Delta^2(\ln PGDP) = a_0 + a_1\ gap^m_{-1} + \sum_{i=1}^{4} a_{2i}\,\Delta^2(\ln PDGP)_{-i} \tag{24}$$

We also consider a second monetary model of inflation in which the rate of inflation depends on lagged inflation and lagged money growth:

$$\Delta\ln(PGDP) = b_0 + \sum_{i=1}^{4} b_{1i}\,\Delta\ln(PGDP)_{-i} + \sum_{i=1}^{12} b_{2i}\,(\ln M2)_{-i} \tag{25}$$

Estimates of equation (24) and (25) are shown in the upper part of Table 2.

The first set of tests comparing the MPS wage-price equations to these two monetary price specifications are based on the "C test" of Davidson and MacKinnon (1981). This non-nested test estimates the regression

$$\pi = c_0 + c_1\pi^{mps} + c_2\pi^m \tag{26}$$

where $\pi$ is the actual rate of inflation (=ΔlnPGDP), $\pi^{mps}$ the predicted rate of inflation from the MPS model, and $\pi^m$ the predicted rate of inflation from one of the monetary models.[25] Davidson and MacKinnon describe why the C test is inferior to other non-nested tests they present, but this seems to be the only test that can be implemented without serious difficulty given that the MPS specification is a multi-equation system. The non-nested test results are presented in the lower part of Table 2. In each case, the estimated coefficient on the MPS inflation prediction is insignificantly different from unity, and the coefficient on the inflation prediction from the monetary model is insignificantly different from zero. Thus, the C tests show no evidence that the MPS equations are misspecified by excluding a direct role of money on prices.

To buttress the non-nested test results, a set of regressions was estimated in which the income velocity of money -- the monetary component of the money gap used in the PSTAR model -- is included in the MPS wage and price equations.[26] Velocity is not significant in either equation.[27]

## V. Some Concluding Remarks

In this paper, we have attempted to present a reasonably comprehensive review of the performance of the price-wage sector of the MPS model, with the emphasis on the period

[25]The MPS inflation predictions are the one-step-ahead (i.e static) forecasts from the wage-price block. This set of equations consists of price and wage equations, (21) and (22), augmented with simple equations for the personal consumption deflator (which appears on the right hand side of the wage equation) and the GDP deflator (the price series explained by the monetary models). For each of these auxiliary price equations, the rate of change of the price series depends on its own lagged values and contemporaneous and lagged growth rates of other prices in the system. To put the information structure of forecasts of the wage-price block on a comparable basis with that of the monetary models, which require information only through the prior quarter, the version of the wage- price block used in the tests also contains autoregressive equations for the energy and farm price indexes and the unemployment rate.

[26]As shown above, the money gap can be decomposed into velocity and the output gap. Only velocity is included in the regressions because we are interested in whether there is a direct role of money, not if the output gap is a better cyclical measure than the corresponding variables in the wage and pric equations.

[27] The regressions are estimated from 1963.1 to 1991.4 with the first lag of the logarithm of M2 velocity added to the specifications reported in (21) and (22). Estimated velocity coefficients (t- statistics) are .018 (1.4) and -.22 (0.1) in the wage and price equations, respectively.

starting in the late 1970s when it is presumed that a major change in the monetary and fiscal policy in the U.S. took place. As a part of this review, we have also tried to lay out the underlying theoretical structure for factor demands of firms and their pricing behavior for output, and the hypothesis concerning the workings of labor market leading to the wage setting behavior embodied in the Phillips curve. We found it useful to contrast our formulation with the more recent work of Layard and Nickell on the same subject, which has become well known, especially in the United Kingdom.

At the empirical level, we have extended the results reported in our earlier paper (Ando *et.al* 1991) and tried to show that the relevant equations, taken individually or as a system, do not show any indication of break or a shift in their structure at the time of the important policy rule change at the beginning of the 1980s. Equations estimated through the end of the 1970s did a reasonable job of predicting the development of the inflation pattern up to the early 1990s, except that these equations tended to overestimate the effects of the real terms of trade on the domestic prices. While this is somewhat surprising, we are observing the same phenomenon for other countries, in some cases in a rather dramatic manner.[28] We need to understand the mechanism generating this pattern better, especially because, for almost every country, trade is becoming more important over time and the interaction between domestic prices and the real terms of trade is bound to play a more significant role in determine the inflation pattern everywhere.

On the theoretical level, we found what we believe to be a slip in Layard and Nickell's formulation which makes their equations violate the homogeneity constraints. More importantly, we believe that the nature of capital is putty-clay, and we have presented some evidence for our belief. If we are even partly correct, Layard and Nickell's

[28]For example, while the lira was devalued significantly between September, 1992, and January, 1993, domestic inflation in Italy measured in terms of the CPI actually declined.

formulation needs reconsideration, especially the demand for labor equation. We also find their wage equation difficult to understand. Taken literally, it is in conflict with the optimization process of firms with homogeneous production function and its implication on the relationship between the rental rate for capital and the real product wage for labor. Layard and Nickell may have meant to describe the process in which management and labor bargain to share the oligopoly mark-up while respecting the cost structure of firms. If this was their intention, their wage equation must be reformulated substantially.

On the other hand, we have found their list of additional factors affecting the dynamic adjustment process of prices, wages and employment serves as a reasonable check list, and it has led us in particular to check if there are direct effects of the deviation of the real wage rate from its equilibrium value on movements of wages and employment. In the case of the United States, the results were negative, but it may be significant in other countries depending on the specific structure of the labor market in the country. Our tests of whether or not our system is misspecified because of the absence of the direct effect of money supply on prices and wages also came out uniformly negative.

In an empirical investigation in social sciences, no evidence is ever conclusive. For the description of the price-wage mechanism for the U.S. economy and for many of OECD countries, however, evidence presented here together with results of many preceding studies suggest that it would be more fruitful to work on revisions and improvements of the existing framework rather than a radical overhaul.

## List of Variables

| | |
|---|---|
| $A$ | a scale factor in the investment function |
| $a$ | parameter in the production function |
| $B$ | a scale factor in the production function |
| $B^u$ | unemployment insurance benefits per hour |
| $\beta$ | total factor costs of production |
| $C$ | total consumption |
| $DPW$ | dummy variable for price-wage control |
| $\delta$ | the rate of depreciation of producer equipment |
| $E$ | employment in manhours |
| $(E/X)^*$ | labor-output ratio excluding cyclical fluctuations; see section III.B.3 for further discussion |
| $\varepsilon$ | real terms of trade |
| $F(\tau)$ | tax and subsidy effects on the cost of capital |
| $G$ | demand factors as defined by Layard and Nickell |
| $GDP$ | gross domestic production in base year price |
| $GDPC$ | capacity $GDP$ |
| $\gamma$ | the rate of labor augmenting technical progress per period |
| $I$ | gross investment in producer equipment in constant dollars |
| $IM$ | intermediate inputs, including imports and agricultural goods |
| $K$ | total capital stock, used by Layard and Nickell |
| $L$ | labor force, in manhours |
| $LF$ | female labor force, number of persons |
| $LT$ | total non-institutional, civilian population |
| $\lambda$ | Lagrange multiplier in the cost minimization by firms, minimized total average cost. |
| $M$ | money stock outstanding, $M2$ |
| $M^s$ | the supply of money |
| $\mu$ | the mark-up factor (a number somewhat greater than units) |
| $P$ | price of value added measure of output for private, domestic nonfarm business sector |
| $p^b$ | price of gross output that is, $XB$ |
| $p^c$ | price of consumption goods |
| $p^E$ | price of energy |
| $p^f$ | price of farm products |

| | |
|---|---|
| $P^I$ | price of investment goods |
| $P^{IM}$ | price of *IM* |
| *PGDP* | implicit deflator of *GDP* |
| $\pi$ | actual rate of inflation in terms of *PGDP* |
| $\pi^{mps}$ | prediction of the price-wage sector of MPS model for $\pi$ |
| $\pi^m$ | prediction of the monetary models for $\pi$ |
| *R* | gross rent per unit of capital per period |
| $\rho$ | the real rate of interest |
| $\sigma$ | elasticity of substitution in the production function |
| *T* | employment tax in dollars per manhour |
| $\Delta T$ | changes in employment tax per manhour |
| u | unemployment rate (standard definition) |
| $u^w$ | unemployment rate with fixed weights for components of labor force |
| $u_1^*$ | the natural rate of unemployment excluding effects of the spread between inflation rates of $P^c$ and *P* |
| $u_2^*$ | the natural rate of unemployment including effects of the spread between inflation rate of $P^c$ and *P* |
| *v* | optimal capital-output ratio |
| *VEL* | *GDP* velocity of *M*2 |
| *W* | rate of compensation per manhour |
| $W^l$ | wage rate per manhour net of employment taxes and employee contribution to social insurance |
| $w^e$ | the ratio of the value of energy input to total value added |
| $w^f$ | the ratio of the value of farm products to total value added |
| *X* | value added output of the private, domestic non-farm, business output, in constant dollars |
| *XB* | gross output corresponding to *X* |
| *XC* | capacity output corresponding to *X* |
| *XCA* | gross addition to *XC* |

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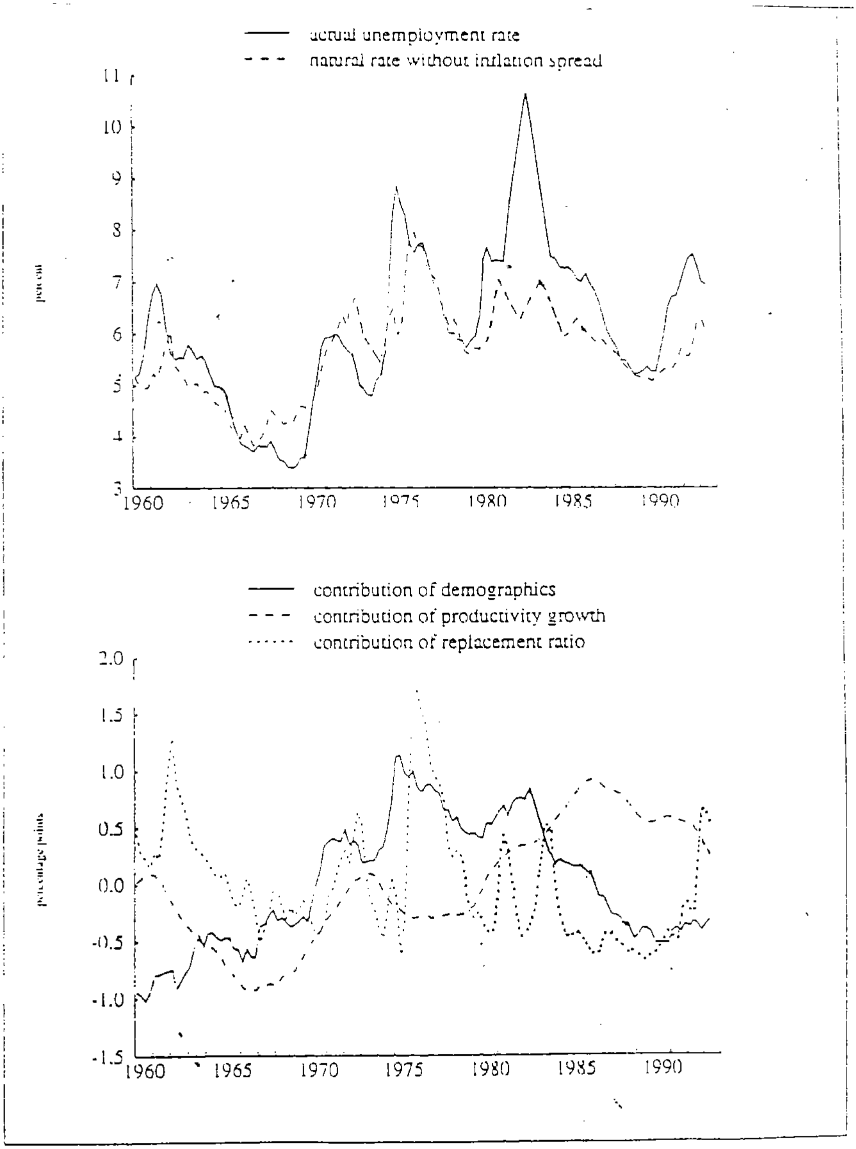
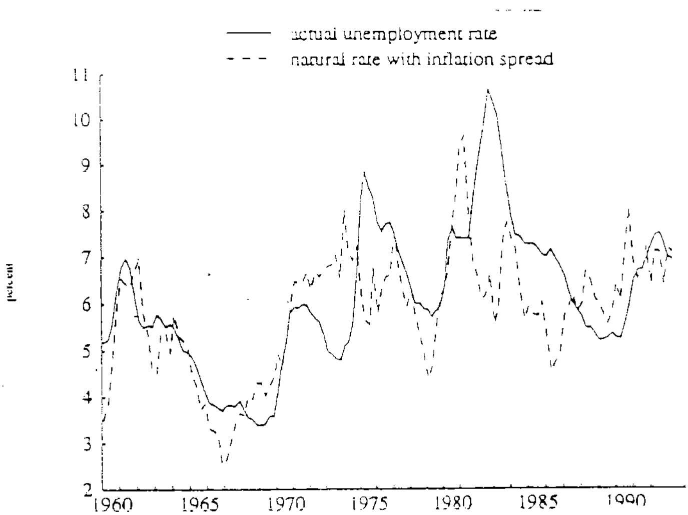
Chart 1

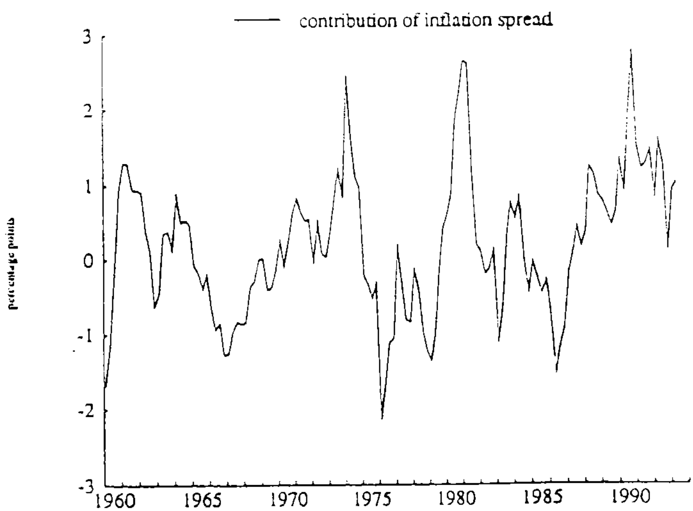
**The Natural Rate of Unemployment $u_1^*$**



## Chart 2

### The Natural Rate of Unemployment $u_2^*$



**Prediction Errors of Wage Equation since 1980 (Measured in Logarithmic First Difference of Compensation per manhour)**

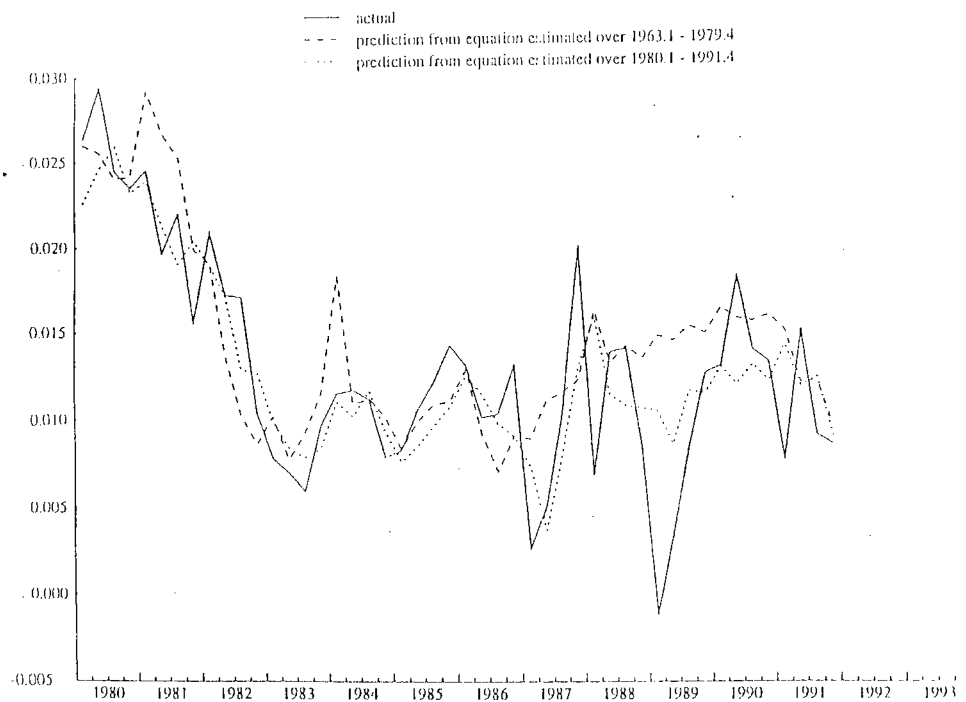


Chart 4a
Out-of-Sample Dynamic Simulation of Wage-Price subsystem
(GDP Implicit Deflator, annual rate of change)

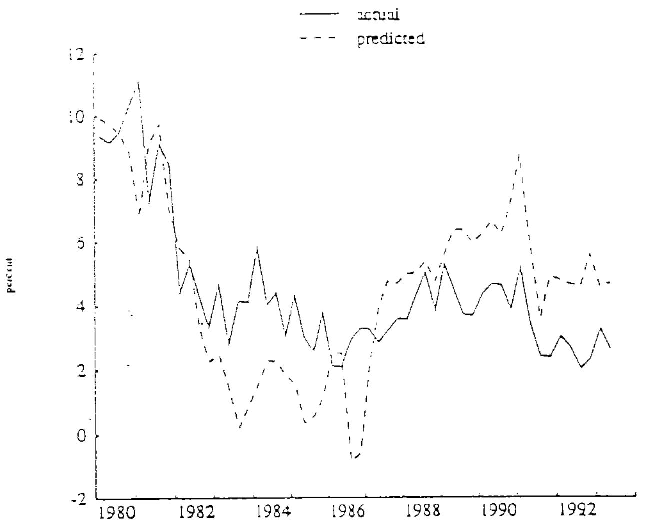



**Chart 4b**
**Out-of-Sample Static Simulation of Wage-Price Subsystem**
**(GDP Implicit Deflator, annual rate of change)**

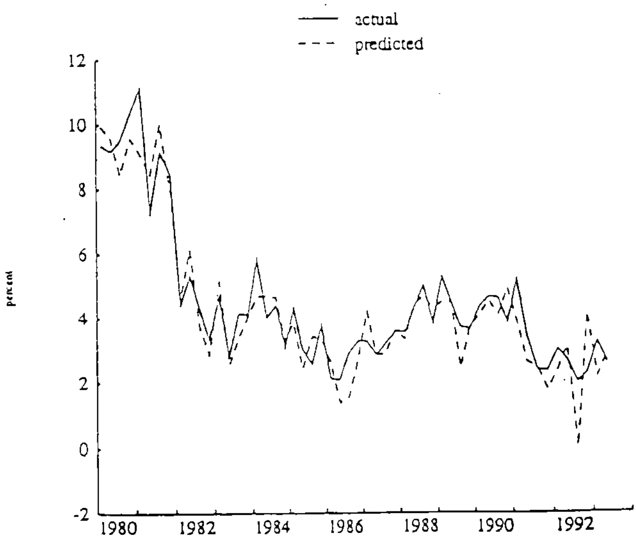


Table 1

Tests of Alternative Dynamics in Some Key Equations

| Equation | Additional variable | Coefficient[1] |
|---|---|---|
| Employment[2] | $[\ln W - \ln P - \ln(x/E)^*]_{-1}$ | -.006<br>(.33) |
| Wage[3] | $[\ln W - \ln P - \ln(x/E)^*]_{-1}$ | -.002<br>(.11) |
| Investment[4] | $\sum_{i=0}^{11} a_i \, XB_{-i-1} \, \Delta v_{-i}$ | -.006[5]<br>(.14) |

[1] $t$-statistics are reported in parentheses.

[2] See Eq.(19) in Section III.B.2. and discussion in Section III.C.

[3] See Eq.(22) in Section III.B.4. and discussion in Section III.C.

[4] See Eq.(15) in Section III.B.1 and discussion immediately following it.

[5] Coefficient sum. An F test of the joint significance of the set of lags has a p value of .78.

Table 2[1]

Tests for a Direct Effect of Money on Prices
(1963.1 - 1991.4)

A. Monetary Models of Inflation

**1. P* Model [Eq.(24), Section IV.D.]**

$$\Delta^2(\ln PGDP) = .203 + .040\, gap^{m}_{-1} + \sum_{i=1}^{4} a_{21}\, \Delta^2(\ln PGDP)_{-i}$$

(4.2) (4.2)

$$\sum_i a_{2i} = -1.252$$

(4.2)

$\overline{R}^2 = .333$; se = .00357; Durbin -H = -.11

**2. Traditional Money Model [Eq.(25), Section IV. D.]**

$$\Delta(\ln PGDP) = -.001 + \sum_{i=1}^{4} b_{li}\, \Delta(\ln PGDP)_{-i} + \sum_{i=1}^{12} b_{2i}\, \Delta(\ln M2)_{-i}$$

(0.8)

$$\sum_i b_{1i} = .833; \sum_i b_{2i} = .178$$

(12.3) (2.0)

$\overline{R}^2 = .649$; se = .00371; dw = 1.99

[1] $t$ - statistics are shown in parentheses.

B. Non-nested tests[2]

$$\pi = c_0 + c_1 \pi^{mps} + c_2 \pi^{m}$$

| Monetary model | $c_0$ | $c_1$ | $c_2$ |
|---|---|---|---|
| 1. P* Model | .0002 | .86 | .13 |
| | (.2) | (5.4) | (.7) |
| 2. Traditional Model | .0001 | .84 | .16 |
| | (.1) | (6.2) | (1.1) |

[2]The design of this set of tests and discussion of the result is given in Section IV.D. of the text, following equation (26), and in footnote 25.

Appendix 1

Detailed Estimation Results of Key Equations

1. INVESTMENT EQUATION

$$I = \sum A1_i \Delta XB_i V_{-i-1} + \sum A2_i \delta_{-i} XB_{-i} v_{-i}$$

Restrictions :

$A1$ : third degree polynomial
$A2$ : third degree polynomial

```
ESTIMATION TECHNIQUE:  AUTOREGRESSION OF ORDER  1
SAMPLE PERIOD:  1967 1  TO  1991 4  ( 100  OBSERVATIONS)

RHO              STD ERROR      T-STAT

.91983           .036822        24.98


SUMS OF DISTRIBUTED LAG COEFFICIENTS
COEFFICIENT           SUM        T-STATISTIC
A1                  .34835          6.0404
A2                  .12518          33.473

R-SQUARE                      =   .99537
R-SQUARE (CORRECTED)          =   .99496
DURBIN WATSON STATISTIC       =   2.5071
SUM OF SQUARED RESIDUALS =   2106
F-STATISTIC(  9 ,  91 )    =   2172.6
DEGREES OF FREEDOM (ADJUSTED) =   91
F-PROBABILITY                 =  -8.4466E-7
STD ERROR OF REGRESSION  =   4.8106
```

INVESTMENT EQUATION (Continued)

| NAME | LAG | COEFF | STD ERROR | T-STATISTIC |
|---|---|---|---|---|
| A1 | 0 | .021372 | .0032121 | 6.6535 |
| A1 | 1 | .028551 | .003046 | 9.3732 |
| A1 | 2 | .033281 | .0034075 | 9.7672 |
| A1 | 3 | .035846 | .003828 | 9.3641 |
| A1 | 4 | .036525 | .00414 | 8.8225 |
| A1 | 5 | .035603 | .0043268 | 8.2285 |
| A1 | 6 | .033361 | .0044235 | 7.5418 |
| A1 | 7 | .030081 | .0044776 | 6.718 |
| A1 | 8 | .026044 | .0045287 | 5.751 |
| A1 | 9 | .021534 | .0045939 | 4.6876 |
| A1 | 10 | .016833 | .0046628 | 3.61 |
| A1 | 11 | .012221 | .0046995 | 2.6006 |
| A1 | 12 | .0079826 | .0046538 | 1.7153 |
| A1 | 13 | .0043984 | .0044731 | .9833 |
| A1 | 14 | .0017509 | .0041205 | .42491 |
| A1 | 15 | 3.2211E-4 | .0036109 | .089204 |
| A1 | 16 | 3.9422E-4 | .0031164 | .1265 |
| A1 | 17 | .0022493 | .0031748 | .70848 |
| A2 | 1 | .0072908 | .0064091 | 1.1376 |
| A2 | 2 | .0071372 | .0041562 | 1.7173 |
| A2 | 3 | .0073301 | .0030997 | 2.3648 |
| A2 | 4 | .0077828 | .0029021 | 2.6818 |
| A2 | 5 | .0084086 | .0029411 | 2.859 |
| A2 | 6 | .0091209 | .0028805 | 3.1664 |
| A2 | 7 | .0098328 | .0026815 | 3.667 |
| A2 | 8 | .010458 | .0024463 | 4.2749 |
| A2 | 9 | .010909 | .0023314 | 4.6792 |
| A2 | 10 | .0111 | .002436 | 4.5567 |
| A2 | 11 | .010944 | .0027022 | 4.0499 |
| A2 | 12 | .010354 | .0029754 | 3.4797 |
| A2 | 13 | .0092432 | .0031167 | 2.9656 |
| A2 | 14 | .0075254 | .0030675 | 2.4533 |
| A2 | 15 | .0051137 | .0029511 | 1.7328 |
| A2 | 16 | .0019214 | .0032768 | .58639 |
| A2 | 17 | -.0021382 | .0047272 | -.45231 |
| A2 | 18 | -.0071518 | .0074667 | -.95782 |

Appendix 1

Detailed Estimation Results of Key Equations

2. EMPLOYMENT EQUATION

---

$$\Delta(\ln e \; -.258 \ln \bar{v}) \; = \; AO + A1\Delta(\ln E - .258 \ln \bar{v})_{-1} + A2\Delta \ln X$$

$$+A3 \; t \; + \; A4(\ln E \; - \; .258 \ln \bar{v} - \; \ln X)_{-1}$$

---

ESTIMATION TECHNIQUE: OLS
SAMPLE PERIOD: 1961 1 TO 1991 4 ( 124 OBSERVATIONS)

---

| NAME | LAG | COEFF | STD ERROR | T-STATISTIC |
|---|---|---|---|---|
| A0 | 0 | -.14686 | .041705 | -3.5214 |
| A1 | 1 | .31096 | .053911 | 5.7679 |
| A2 | 0 | .45629 | .035163 | 12.977 |
| A3 | 0 | -1.521E-4 | 4.6895E-5 | -3.2434 |
| A4 | 0 | -.064247 | .0184 | -3.4917 |

R-SQUARE = .7432
R-SQUARE (CORRECTED) = .73457
DURBIN-H STATISTIC = -.8312
SUM OF SQUARED RESIDUALS = .0023374
STD ERROR OF REGRESSION = .0044319

Appendix 1

Detailed Estimation Results of Key Equations

3. PRICE EQUATION

$$\Delta \ln P = B0 + B1(\ln W - \ln(E/X)*\ln P)_{-1} + \sum B2_{-i} w_i^e \Delta \ln(P^E / P_{-1})_{-i}$$
$$+ B3 w^f \Delta \ln(P^f / P_{-1}) + \sum B4_i \Delta \ln P_{-i} + B5\, u_{-1}^w$$
$$+ B6\, \varepsilon_{-i}$$

Restrictions :

$B2$ : second degree polynomial
$B4$ : second degree polynomial

ESTIMATION TECHNIQUE: OLS
SAMPLE PERIOD: 1963 1 TO 1991 4 ( 116 OBSERVATIONS)

| NAME | LAG | COEFF | STD ERROR | T-STATISTIC |
|---|---|---|---|---|
| B0 | 0 | .1078 | .020238 | 5.3266 |
| B1 | 0 | .11771 | .021158 | 5.5633 |
| B2 | 0 | -.44349 | .075722 | -5.8568 |
| B2 | 1 | .074673 | .058848 | 1.2689 |
| B2 | 2 | .24027 | .055104 | 4.3603 |
| B2 | 3 | .053312 | .084605 | .63013 |
| B3 | 0 | -.54916 | .11474 | -4.7862 |
| B4 | 1 | -.0027032 | .081441 | -.033192 |
| B4 | 2 | .15198 | .052045 | 2.9201 |
| B4 | 3 | .14132 | .0526 | 2.6866 |
| B4 | 4 | -.034694 | .071649 | -.48422 |
| B5 | 0 | -.0010108 | 3.7781E-4 | -2.6754 |
| B6 | 0 | .014671 | .0035089 | 4.1811 |

SUMS OF DISTRIBUTED LAG COEFFICIENTS

| COEFFICIENT | SUM | T-STATISTIC |
|---|---|---|
| B2 | -.075229 | -.50445 |
| B4 | .2559 | 2.3554 |

R-SQUARE = .69086
R-SQUARE (CORRECTED) = .66142
DURBIN-H STATISTIC = .64866
SUM OF SQUARED RESIDUALS = .0019749
STD ERROR OF REGRESSION = .0043369

INVESTMENT EQUATION

Appendix 1

Detailed Estimation Results of Key Equations

4. WAGE EQUATION (SECTION III.B.4 IN THE TEST)

$$\Delta \ln W = BO + B1u^{w} + B2\Delta \ln W_{-1} + \sum B3_i \Delta \ln P^c_{-i} + B4\,\Delta \ln(T/W)$$

$$+B5DPW + B6\Delta(LF/LT) + \sum B7_i (B^{w}/W)_{-i}$$

Restrictions:

$B2 + \Sigma B3 = 1$
$B3$ : third degree polynomial
$B7$ : first degree polynomial

ESTIMATION TECHNIQUE: OLS
SAMPLE PERIOD: 1963 1 TO 1991 4 ( 116 OBSERVATIONS)

| NAME | LAG | COEFF | STD ERROR | T-STATISTIC |
|---|---|---|---|---|
| B0 | 0 | .0068876 | .002139 | 3.2201 |
| B1 | 0 | -.0019133 | 3.4554E-4 | -5.5373 |
| B2 | 0 | .25158 | .083392 | 3.0169 |
| B3 | 1 | .19433 | .10836 | 1.7935 |
| B3 | 2 | .15511 | .080615 | 1.9241 |
| B3 | 3 | .13876 | .072049 | 1.9259 |
| B3 | 4 | .12617 | .0746 | 1.6913 |
| B3 | 5 | .098229 | .08182 | 1.2005 |
| B3 | 6 | .035809 | .10849 | .33005 |
| B4 | 0 | .76334 | .21378 | 3.5707 |
| B5 | 0 | -.0092389 | .0032985 | -2.801 |
| B6 | 0 | -.0049128 | .0030209 | -1.6263 |
| B7 | 0 | -.014766 | .01572 | -.93929 |
| B7 | 1 | -3.9734E-5 | .0062436 | -.0063639 |
| B7 | 2 | .014686 | .0061014 | 2.407 |
| B7 | 3 | .029412 | .015551 | 1.8913 |

SUMS OF DISTRIBUTED LAG COEFFICIENTS

| COEFFICIENT | SUM | T-STATISTIC |
|---|---|---|
| B3 | .74842 | 8.9747 |
| B7 | .029293 | 2.0874 |

R-SQUARE = .71105
R-SQUARE (CORRECTED) = .68353
DURBIN-H STATISTIC = -.38265
SUM OF SQUARED RESIDUALS = .0014188

## APPENDIX II

### A Note on the Money Market and the Price Level

This note reminds the reader of a well known property of the standard demand function for money, and explains why this property forms an important background for the effort invested in the study of the Phillips curve and its alternatives.

In the tradition of macroeconomic analysis leading up to the book by Patinkin (1965), the economy is viewed as divided into the real sector and the monetary sector, and conditions are defined for the so-called dichotomy of these two sectors under which all quantities and relative prices are determined in the real sector, and the price level is determined by the demand and supply of money. Even Milton Friedman (1971) expressed his adherence to this mental framework.

Unfortunately, this set-up, which is useful in comparative static analysis up to a point, is unlikely to work when it is applied to the dynamic process in the real economy. To see this, consider the standard demand function for money and equate it with the supply of money, and write

$$\ln M^s = m_0 + m_x \ln X + \ln P + m_r \ln \left[ \rho + \left( \frac{\dot{P}}{P} \right)^e \right]$$

where $M^s$ is the supply of money, $X$ is the measure of real income and $P$ its price, and $\rho$ is the real rate of interest. $m_i$'s are numerical coefficients, and we know, from the standard theory of the demand for money, that $.5 < m_x < 1.0$, and $-.5 < m_r < 0$.

Suppose now that, for the expected rate of inflation, $\left(\frac{\dot{P}}{P}\right)^{e}$, we adapt the perfect foresight assumption and let $\left(\frac{\dot{P}}{P}\right)^{e} = \frac{\dot{P}}{P}$. This is not unreasonable because in this equation the nominal rate of interest should be a relative short term rate, say for one week to three months. In the framework just outlined, this equation then is a first order differential equation in $P$, because $M^S$ is exogenously given, and $X$ and $\rho$ are determined in the real sector of the economy and exogenously given for this equation.

With a suitable transformation of the variable, $\rho$, this equation can be shown to be equivalent to a linear differential equation in $P$, and it is then easy to show that, for any reasonable volumes of parameters, it is unstable when it is solved forward, with the initial condition defined in $t$. In other words, if the system is started in period $t$ with a given path of $M^s$, $X$, and $\rho$ and an initial value of $P$, $P$ will increase at an increasing rate or decline to zero, depending on the initial conditions. Thus, this set-up is incapable of determining the path of $P$ that is reasonable.

Of course, if the first order differential equation is unstable forward, it is stable backward. Therefore, if we have the terminal condition in a distant future and then solve the equation backward, we will have a stable path and the initial price level is well determined. This leads some analysts with rational expectations models to conclude that the stability problem does not exist here. Note, however, that, if the system is solved backward, a slight change in the path of forcing variables or the terminal condition must cause a significant, though finite, shift in the price level at $t$. Hence, we should observe frequent and large jumps in the price level over time if this is the mechanism by which the price level is determined. This seems inconsistent with the nature of observed data on prices.

It is true that we can avoid this problem by some strong assumptions. For example, if $m_r$ is exactly zero, so that the demand for money is independent of the rate of interest, then this problem does not arise. We believe, however, that the case we have discussed here is the reference case and points to the basic problem of why any model in which the basic price level is directly determined by the money supply is unlikely to be successful for economics with a reasonable price pattern. That is, our argument would not necessarily apply to hyperinflationary economies.

Of course, this does not mean that the money supply does not have an impact on the price level. It merely means that the causal chain is indirect and complex, and the Phillips curve and its variations are the most promising hypothesis of how such an indirect causal chain works so far available.