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FLUCTUATIONS, INSTABILITY, AND AGGLOMERATION

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ABSTRACT

Recent models in economic geography suggest that there may be very large numbers of equilibrium spatial structures. Simulations suggest, however. that the structures that emerge are surprisingly orderly, and often seem approximately to follow simple rules about the spacing of urban sites. This paper offers an explanation in terms of the process by which a spatial economy diverges away from an even distribution of activity across the landscape. It shows that a small divergence of activity away from spatial uniformity. even if it is highly irregular, can be regarded as the sum of a number of simple periodic fluctuations at different spatial "wavelengths"; these fluctuations grow at different rates. There is a particular "preferred wavelength" that grows fastest; provided that the initial distribution of activity across space is flat enough, this preferred wavelength eventually dominates the spatial pattern and becomes the typical distance between cities. The approach suggests that surprisingly simple principles of self-organization may lie beneath the surface of models that appear at first to yield hopelessly complex possibilities.

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Over the last few years there has been a substantial revival of interest in regional and urban economics. Much of this revival is due to the rediscovered usefulness of regions and metropolitan areas as empirical laboratories, whose evolution Can shed light on such questions as the nature of the macroeconomic adjustment process or the character of external economies. There has also, however, been a resurgence of theoretical work on spatial economics, thanks in large part to the application of modeling techniques previously developed in industrial organization, international trade, and growth theory.

In several recent papers (Krugman 1991, 1992, 1993a, 1993b) I have explored one particular approach to spatial modeling that, while admittedly capturing only some of the reasons why spatial structure emerges in real economies, has the virtue of being particularly easy to work with. In this approach, an economy with two or more locations is assumed to consist of two sectors: a constant-returns, geographically immobile sector ("agriculture"), and an increasing-returns, monopolistically competitive, geographically mobile sector ("manufacturing"). When one adds transportation costs in the manufacturing sector, and also adds some simple dynamics, models of this type exhibit spontaneous spatial self-organization: even if all locations are identical in resources and technology, manufacturing firms have an incentive to concentrate production close to the markets and supplies that other manufacturing firms provide, thus producing a "centripetal" tendency toward agglomeration. Working against this centripetal tendency, however, is the "centrifugal" pull of the immobile agricultural sector.

In a two-location model, the tension between centripetal and centrifugal forces can be treated analytically; one can derive a criterion, depending in an economically meaningful way on the parameters, which determines whether or not manufacturing concentrates in one location. Beyond this case, however, it becomes very difficult to derive analytical results. Simulations show that there may be equilibria with multiple manufacturing concentrations: they also indicate that as the number of locations grows, there typically start to be a very large number of equilibria.

And yet there seems to be some underlying order under this complexity. When one starts from a random distribution of manufacturing on a linear, landscape, for example, one typically finds that a roughly regular spacing of manufacturing concentrations emerges. Furthermore, the distance between these concentrations is relatively insensitive to the starting position, and appears to depend in a sensible way on the model's parameters. It is easy to offer some intuition about why this should happen: one may argue, following Arthur (1990), that successful manufacturing concentrations tend to cast an "agglomeration shadow" over nearby rivals, leading to a roughly equal spacing. Yet we would like a more specific explanation. And one would like a model of the model -- something that helps us to understand what our computer is telling us.

In this paper I offer a somewhat novel approach that helps explain the behavior of these particular models, and that may turn out to have application in a variety of spatial models. Instead of focussing on the long-run equilibrium that the economy eventually attains, this approach focusses on the process of divergence away from the unstable equilibrium in which manufacturing is evenly distributed across space.

In what sense is an even distribution of manufacturing unstable? Suppose that the actual distribution is slightly perturbed away from perfect flatness. Such a perturbation, even if it is highly irregular, can be thought of as a Fourier series (in space, of course, rather than time) -- the sum of a number of periodic fluctuations, with different wavelengths. And some of these periodic fluctuations will tend to be self-reinforcing, growing over time. In particular, we can show that there is one wavelength that is the most unstable in the sense that a fluctuation at that wavelength tends to grow more rapidly than fluctuations at any other wavelength. Given enough time, this spatial wavelength Will dominate the divergence from even distribution -- and the peaks and valleys of the divergence will dictate the locations of the eventual agglomerations. Thus the spacing of manufacturing concentrations will be approximately equal to the preferred wavelength of the dynamic process of divergence, a preferred wavelength that is determined by the parameters of the model. In particular, the preferred wavelength is inversely proportionalto transportation costs and positively related both to the degree of scale economies and to the share of manufacturing in the economy.

The paper begins with a review of the general approach to spatial dynamics used here, and describes the suggestive results of some simulation exercises. It then turns to a specific model of a linear economy, and shows how the evolution of this model near an even distribution of manufacturing can be viewed in terms of the growth rates of fluctuations of different frequencies. Finally, the paper shows why the economy has a preferred wavelength, and how this wavelength depends on the parameters.

1. A basic spatial model

Consider an economy in which there are a number of locations, indexed by $j = 1,...,J$. Let D_{jk} be the distance between any pair of locations j and k.

In this economy there are two factors of production: immobile "farmers" and mobile "workers". It will be convenient to choose units so that there are a total of $1-\mu$ farmers and μ workers. Also, in this paper I will restrict attention to economies in which spatial structure is completely endogenous, so the farmers will be assumed to be equally divided among the locations.

Everyone in this economy shares the same tastes, which may be represented by a two-level structure. At the upper level, there are Cobb-Douglas preferences between agricultural goods and a manufacturing aggregate:

$$U = C_M^{\mu} C_A^{1-\mu} \tag{1}$$

At the lower level, manufacturing is a CES composite of a large number of symmetric differentiated products:

$$C_M = \left[\sum_i c_i^\rho\right]^{1/\rho} \tag{2}$$

where $\sigma = 1/(1-\rho)$ is the elasticity of substitution.

Each factor is specific to the production of one sector. Farmers produce agricultural output with constant returns to scale. Workers produce manufactured goods. There are economies of scale in this production, specific both to the firm and to the particular variety produced; these are represented as a linear cost function,

$$L_{Mi} = \alpha + \beta Q_{Mi} \tag{3}$$

We also introduce transport costs. For the sake of tractability, there are assumed to be zero transport costs for agricultural goods. Transport costs on manufactured goods are of Samuelson's "iceberg" form. If one unit of a manufactured good is shipped from location j to location k, only $\exp(-YD_{jk})$ units arrive, with λ the transportation cost per unit distance.

It is a familiar proposition that if we take the spatial distribution of workers as given, a model of the form just described yields a monopolistically competitive equilibrium in which all profits are competed away. This equilibrium includes an equilibrium level of the real wage at each location: differences in these real wage rates are what drive the economy's dynamics.

Workers are assumed to move gradually toward locations that offer them above-average real wages. Let λ_j be the fraction of workers currently in location j. Then the average real wage rate can be defined as a weighted average of real wage rates at each location,

$$\bar{\omega} = \sum_j \lambda_j \omega_j \tag{4}$$

and the assumed dynamics take the form[1]

$$\frac{d\lambda_j}{dt} = \gamma(\omega_j - \bar{\omega})\lambda_j \tag{5}$$

The dynamic behavior of this model can be thought of as a sequence of general-equilibrium problems. For any given distribution of manufacturing across locations, the economy reaches an equilibrium that determines the real wage at each location. This vector of real wages then determines, via (4) and (5), the distribution of workers a short time later, and the calculation can be repeated until the model economy converges on some long-run equilibrium geographical pattern.

In Krugman (1992) I show that the equilibrium of this model at any point in time can usefully be described as the simultaneous solution of four sets of equations. First, the income of any location is the sum of the earnings of its immobile farmers and the workers who are currently located there:

$$Y_j = \frac{1-\mu}{J} + \mu\lambda_j w_j \tag{6}$$

where w_j is the wage rate measured in terms of the agricultural

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[1]In all of my models to date, I have ignored two important aspects of real-world spatial economics -- forward-looking behavior by agents who try to anticipate future spatial patterns, and large agents, such as shopping mall developers, who try to influence these patterns. The excuse for these omissions is, of course, tractability.

good.

Second, the true price index of manufactures at any given location depends on the distribution of manufacturing, transportation costs, and wage rates:

$$T_j = \left[\sum_k \lambda_k w_k^{1-\sigma} e^{-\tau(\sigma-1)D_{jk}}\right]^{1/(1-\sigma)} \tag{7}$$

Third, the equilibrium wage rate at any location depends on incomes, true price indices, and transportation costs to all other locations:

$$w_j = \left[\sum_k Y_k T_k^{\sigma-1} e^{-\tau(\sigma-1)D_{jk}}\right]^{1/\sigma} \tag{8}$$

Finally, the real wage rate at location j depends on the nominal wage rate in terms of agricultural goods and the local true price index of manufactured goods:

$$\omega_j = w_j T_j^{-\mu} \tag{9}$$

These equations are fairly simple, and are very easy to solve numerically -- one simply starts with guesses at the wage and true price vectors, and iterates over (6), (I), and (8) until convergence. Analytical results, however, in anything larger than a two-region model, are another matter. Thus to date explorations of multi-location settings have relied on numerical examples. In the next section I briefly describe one set of examples, as a motivation for the subsequent discussion.

2. Evidence from numerical examples

In an effort to understand the formation of systems of cities, I carried out a series of simulations on a particular version of the model described in part 1; these results are reported both in Krugman (1992) and Krugman 1993a. In these simulations the economy was assumed to consist of 12 locations symmetrically placed around a circle, like a clock face. (The number 12 was chosen because it is a relatively small number with a large number of divisors). Each run began with a random allocation of manufacturing across locations, and the model was then allowed to evolve until convergence.

For the most interesting range of parameters, the result of these experiments was that 'the model economy organized itself into a spatial structure with all manufacturing in 2 or 3 locations, more or less symmetrically located around the circle. Figure 1 illustrates the results of a typical run, with the first set of bars representing the initial shares of manufacturing, the second set the final shares. In this case all manufacturing ended up in locations 6 and 11, almost but not quite opposite each other on the circle. For the parameters used for this run, about 60 percent of the runs led to two cities 5 apart, almost all other runs to two cities 6 apart, and a few runs to three symmetrically placed cities.

At one level these examples demonstrate the complexity of the possible outcomes, even in such a relatively small model. After all, there are 12 ways to locate concentrations 5 apart on a 12-location circle, 6 ways to locate them 6 apart, and 4 ways to locate 3 concentrations 4 apart. Thus even this example seems to have 12+6+4=22 stable locational equilibria.

And yet in some sense the model's results are not as arbitrary as one might suppose. For these parameters, one always gets 2 or on rare occasions 3 concentrations, never more or less. And the concentrations are always at least roughly evenly spaced. This suggests that there is a sort of natural distance between manufacturing concentrations that the model is "trying" to produce, within the limits of what the initial conditions allow.

It would certainly be desirable to understand vhy the model has a tendency to produce some particular spacing between concentrations. Not only would it help us understand this model, but it would raise hopes that economic geography will yield more definite results than we might otherwise fear. Models with agglomeration economies typically have many equilibria, and one therefore worries whether all that theory will tell us is that lots of things could happen -- a result that would make the theory untestable as well as useless. But the numerical examples suggest that there may be a tendency to some kind of approximate regularity, which will be a testable and useful prediction even if we do not know precisely which equilibrium will emerge.

But isn't this a lot to be resting on a small Set Of numerical examples? Indeed it is, and we might want to try a much broader set of examples before being sure of our generalizations. or, alternatively, we might look again at the theory and see whether there is an analytical basis for the observed near-regularities. What we will do now is see that there is such a basis. Indeed, linear spatial models along the lines we have been discussing (and probably many other spatial models as well) will always tend to produce a regular spacing of agglomerations if the initial distribution is sufficiently smooth.

3. Fluctuations and aaalomeration: some intuition

Before proceeding to the formal analysis, it will be useful to try to get some intuition about the story we are about to tell.

Imagine, then, a "long, narrow" spatial economy, sufficiently long that we may treat its length as infinite (that is, ignore edge effects) and sufficiently narrow that we may treat it as one-dimensional. And imagine that initially manufacturing is distributed almost evenly along this line -- almost, but not quite.

We first ask the following question: how does an increase in the amount of manufacturing at one location, say z, affect the real wage of workers at another location x?

This is not an easy question to answer rigorously, because of the general equilibrium effects: a geographical redistribution of manufacturing will in general change wage rates at all locations. But we can think loosely in terms of partial effects. At given wage rates, an increase in the concentration of manufacturing at z will have three effects on workers at x. First, it will enlarge their market, since they Can sell to z; second, it will improve the supply of goods, since workers at x will buy goods from z; but finally, it will increase the competition that workers at x face in other markets.

Do the positive effects (which we can think of as backward and forward linkages, respectively) prevail over the negative? The answer depends on how far z is from x. Roughly, we can think of the typical market in which z competes with x as being halfway between the two locations. If z and x are very close, then the market that z provides is essentially as close to, and therefore as important to x as the market in which z competes with x. In this case the linkages outweigh the competitive effect. But if z is very far away, it is also much further away than the typical market in which x and z compete. Both the linkages and the competitive effect will be weak, but the linkages will be weaker (since the relevant distance is twice as large). As a result, a thickening of the distribution at a distant z is likely to reduce real wages at x.

We can think, then, of some critical distance that defines the range of positive agglomeration economies. An increase in manufacturing at any point raises the real wages of workers within that range, while depressing the real wages of workers beyond it.

Now let us take the crucial step. Let us suppose for a moment that the divergence of manufacturing from a completely flat distribution is not erratic, but instead takes the form of a periodic function -- indeed, let it be a sine curve. And let us ask what is likely to happen to this divergence over time.

Suppose that we look at a peak of this divergence. Will this peak be marked by real wages that are above average, roughly average, or below average? The answer depends on the distribution of manufacturing around this peak. High concentrations of manufacturing near the peak, within the range of positive agglomeration economies, raise the peak's real wage: below-average concentrations within that range lower it. Outside the range of positive agglomeration, things are reversed: low manufacturing concentrations raise the peak's real wage, high concentrations reduce it.

What this implies is that the real wage at the peak will depend on the _frequency_ (or wavelength) of the distribution. Suppose that the wavelength is very small relative to the range of positive agglomeration, as in Figure 2. Then within that range there will be about as many troughs as peaks, roughly cancelling each other out: the real wage at the central peak will be just about average.

On the other hand, suppose that the wavelength of the distribution of manufacturing is very long compared with the range of positive agglomeration, as in Figure 3. Then much of the "high ground" surrounding our peak will lie on the wrong side of the range of agglomeration, exerting a _negative_ effect on the real wage there. The real wage at the peak may well actually be below average.

The real wage at the peak is most likely to be high when the wavelength of the manufacturing distribution is approximately equal to the range of agglomeration, as in Figure 4. In this case all of the nearby deviations from flat manufacturing distribution work to reinforce the high real wage at the peak: the high ground lies inside the range of positive agglomeration, the low ground outside (where it also therefore makes a positive contribution).

Now let us consider what will happen to the the amplitude of these fluctuations over time. In the case illustrated in Figure 2, where the wavelength of the fluctuation is very short compared with the range of agglomeration, peaks and troughs in the manufacturing distribution will offer real wages that are little different from the average. Thus there will be no particular tendency for the fluctuations to change over time.

In the case illustrated in Figure 3, where the wavelength of the distribution is very long compared with the range of agglomeration, peaks will tend to have below-average real wages, troughs above-average. In this case, then, peaks and troughs will tend to shrink over time: the fluctuations will die out.

In the central case, however, peaks will offer clearly above-average real wages, troughs below-average; thus workers will migrate away from troughs and toward peaks, amplifying the fluctuation over time.

All of this reasoning depends, of course, on the assumption that manufacturing is distributed across space in a regular sine wave. What relevance can it have to a situation in which the distribution of manufacturing is not so regular? The answer is that irregul r adistribution of m anufacturing Can be decomposed into a sum Of sine waves of different frequencies and amplitudes. And of these regular fluctuations, those with very long wavelengths will die out over time, those with very short wavelengths will grow only slowly, while those with more or less the right wavelength will grow rapidly. In particular, there is some wavelength (corresponding to the range of positive agglomeration) that will grow most rapidly. Call this the "preferred wavelength". Over time, the divergence of manufacturing from a flat distribution will tend to become dominated by a fluctuation at the preferred wavelength.

This process cannot, of course, go on forever. For one thing, we will see in the next section that the reasoning here is only strictly valid as long as we are able to represent the economy by a linear approximation around a flat distribution of manufacturing. As the fluctuations grow, this linear approximation will break down. Above all, at some point there will be no manufacturing left in some locations; at that point the smooth curves of the fluctuations will start gathering themselves into the spikes of Figure 1. But if the initial position of the economy is a sufficiently flat distribution of manufacturing, the process of divergence Will firmly establish peak concentrations of manufacturing at intervals roughly equal to the preferred wavelength, and these peak concentrations will then gather themselves into cities.

A final point: notice that in this model, instability is the source of self-organization. The economy organizes itself into a spatial structure of cities and rural areas precisely because a flat, unorganized spatial structure is unstable; the intervals at which cities are located are determined by the particular wavelength of fluctuation for which the flat structure is <u>most</u> unstable.

This is about as far as we can go in an intuitive discussion. Let us now turn to a formal treatment.

4. <u>Dynamics near a flat spatial structure</u>

For the formal analysis, we will consider a version of the basic model presented in part 1 in which farmers are distributed evenly along a line of infinite extent. Workers will also, at any point in time, be distributed along that line; we let $\lambda(x)$ be the density of workers at position x, normalized so that with a flat distribution $\lambda=1$ everywhere.

For this economy, equations (6)-(9) may be rewritten in the following form (the constant terms are added so that when the distribution is flat, $Y(x) = W(x) = T(x) = 1$ for all x is a solution):

$$Y(x) = 1 - \mu + \mu\lambda(x)\, w(x) \tag{10}$$

$$T(x) = \left[\frac{\tau(\sigma-1)}{2}\int_{-\infty}^{\infty}\lambda(z)\, w(z)^{1-\sigma} e^{\tau(1-\sigma)|x-z|}dz\right]^{1/(1-\sigma)} \tag{11}$$

$$w(x) = \left[\frac{\tau(\sigma-1)}{2}\int_{-\infty}^{\infty} Y(z)\, T(z)^{\sigma-1} e^{-\tau(\sigma-1)|x-z|} dz\right]^{1/\sigma} \tag{12}$$

$$\omega(x) = w(x)\, T(x)^{-\mu} \tag{13}$$

These are a fairly nasty-looking set of nonlinear equations. Suppose, however, that we restrict our attention to situations in which $\lambda(X)$ is close to 1, that is, where the distribution of manufacturing is fairly flat. Then we can take linear approximations to the equations. Let a prima on a variable represent deviation from 1: then the approximate linearized model takes the form

$$Y'(X) = \mu\lambda'(x) + \mu w'(x) \tag{14}$$

$$T'(x) = \frac{1}{1-\sigma}\frac{\tau(\sigma-1)}{2}\left[\int_{-\infty}^{\infty}\lambda'(z)\, e^{-\tau(\sigma-1)|x-z|}dz + \int_{-\infty}^{\infty} w'(z)\, e^{-\tau(\sigma-1)|x-z|}dz\right] \tag{15}$$

$$w'(x) = \frac{1}{\sigma}\frac{\tau(\sigma-1)}{2}\left[\int_{-\infty}^{\infty} Y'(z)\, e^{-\tau(\sigma-1)|x-z|}dz + (\sigma-1)\int_{-\infty}^{\infty} T'(z)\, e^{-\tau(\sigma-1)|x-z|}dz\right] \tag{16}$$

$$\omega'(x) = w'(x) - \mu T'(x) \tag{17}$$

These equations do not, at first sight, appear any more tractable than the nonlinear version. But now let us, following the suggestion of part 3, assume for a moment that the distribution of manufacturing follows a simple periodic distribution, say

$$\lambda'(x) = \delta\cos(\phi x) \tag{18}$$

Now let us simply guess that if the divergence of $\lambda(x)$ from 1 follows this simple periodic form, the divergences of all of the other variables from 1 will be constant multiples of $\lambda(x)$. (This conclusion is actually obvious from the spatial symmetry and the linearity). That is, we guess that there is a solution of the form

$$Y'(x) = a_Y\lambda'(x) \tag{19}$$

$$T'(x) = a_T\lambda'(x) \tag{20}$$

$$w'(x) = a_w\lambda'(x) \tag{21}$$

$$\omega'(x) = a_\omega\lambda'(x) \tag{22}$$

If this is a valid solution, then we have managed to reduce a general equilibrium problem that is, strictly speaking, the solution of an infinite number of nonlinear equations to the solution of four linear equations.

Let us, then, substitute (18) into (19)-(22). When we do so, we will see repeatedly a term of the form

$$K(z) = \frac{\tau(\sigma-1)}{2}\int_{-\infty}^{\infty}\cos(\phi z)\, e^{-\tau(\sigma-1)|x-z|}dz \qquad (23)$$

With a little grinding, it is possible to show that

$$K(z) = H(\phi,\tau,\sigma)\cos(\phi z) \qquad (24)$$

where

$$H(\phi,\tau,\sigma) = \frac{\text{(o-11'}}{(\sigma-1)^2+(\phi/\tau)^2} \qquad (25)$$

H represents a sort of discount factor -- the ratio of the impact of a fluctuation to what would happen if there was a uniform increase in the same variable that raised the level at x by the same amount. Thus in the equation for the true price index we know that an equal increase in all wage rates would raise the price index at x by an amount equal to the increase in the wage rate at x; a fluctuation will raise the price index by H times the increase at x, with the ratio H depending on the frequency of the fluctuation. It is immediately obvious that for for very high frequencies, H approaches zero, while for low frequencies it approaches 1.

We can now write our equations as

$$a_Y = \mu a_w + \mu \tag{26}$$

$$a_T = -\frac{1}{\sigma-1}H + Ha_w \tag{27}$$

$$a_w = \frac{1}{\sigma}Ha_Y + \frac{\sigma-1}{\sigma}Ha_T \tag{28}$$

and

$$a_\omega = a_w - \mu a_T \tag{29}$$

These equations can be solved to yield the crucial result that

$$a_\omega = \frac{\mu}{\sigma-1}H + (1-\mu H)\frac{\mu H - H^2}{\sigma-(\sigma-1+\mu)H} \tag{30}$$

Why is this the crucial result? Because the linearized version of the dynamic equation (5) is

$$\frac{d\lambda'(x)}{dt} = \omega'(x)\gamma = a_\omega\gamma\lambda'(x) = g_\phi\lambda'(x) \tag{31}$$

where g is the rate of growth of a fluctuation at that frequency.

Now we note that a perturbation of the spatial distribution of manufacturing around $\lambda=1$ can be represented as the sum of a number of sine waves of different wavelengths:

$$\lambda'(x) = \lambda'_1(x) + \lambda'_2(x) + \ . \ . \ . \tag{32}$$

And the growth of the perturbation may be written

$$\frac{d\lambda'(x)}{dt} = g_1\lambda'_1(x) + g_2\lambda'_2(x) + \ . \tag{33}$$

so that we can think of each periodic fluctuation as growing at its own characteristic rate. The fluctuation that will grow fastest is the one with the largest (positive) response of the real wage rate to manufacturing concentration: and given sufficient time that fluctuation will dominate the spatial pattern.

So all we have to do to determine the preferred wavelength is find the maximum of (30). It is straightforward to determine three results. First,

$$a_1 = 0 \text{ when } H = 0, \text{ that is, when } \phi \to \infty \tag{34}$$

That is, as our intuitive discussion in part 3 suggested, fluctuations at very high frequencies = very short wavelengths will not tend to grow.

Second,

$$a_1 < 0 \text{ when } H=1, \text{ provided that } \mu < \frac{\sigma-1}{\sigma} \tag{35}$$

The condition here is a familiar one, appearing also in Krugman (1991). It says, in effect, that economies of scale are not so large that workers would prefer all to be concentrated in the same place no matter how high transportation costs are. Given this condition, we find that very low frequency fluctuations, those with very long wavelengths, tend to die out.

Finally, at H=0 we find

$$\frac{da_\omega}{dH} = \frac{\mu}{\sigma-1} + \frac{\mu}{\sigma} > 0 \tag{36}$$

Taken together, these observations imply that the relationship between the growth rate of a fluctuation and H has the shape indicated in Figure 5. Growth is slow at very short wavelengths, negative at high wavelengths, and most rapid at some intermediate wavelength.

The preferred wavelength, the wavelength of most rapid divergence, is a function of the three parameters τ, σ, and μ. The transport cost τ enters the solution in only one place, in the definition of H in (25). It is thus obvious that the preferred frequency is strictly proportional to τ, and thus that the preferred wavelength is inversely proportional. This is obvious with hindsight, since the wavelength and the transportation cost can both be changed in the same proportion by redefining the unit of distance, with no real change in the model.

It is more painful to derive the impact of changes in the elasticity of substitution and the share of manufacturing. It is, however, straightforward to calculate the preferred frequency numerically for given μ and σ. This is shown in Table 1; we see that higher elasticities of substitution, which imply lower equilibrium economies of scale, tend to reduce the preferred wavelength, while a higher manufacturing share tends to increase the preferred wavelength.

5. Conclusions and implications

Interesting models of the emergence of structure in a spatial economy generally involve a tug of war between centripetal forces that tend to produce agglomerations and centrifugal forces that tend to pull them apart. Such models typically have many equilibria. Yet both observation of the world and experiments with numerical models suggest that there is a surprising amount of order in the actual outcomes. What is the source of this orderliness?

In this paper I have suggested that the origins of order may lie in the dynamics of divergence away from an unstructured, roughly flat spatial distribution of economic activity. In this model, and probably in a number of others as well, it is possible to think of the divergence from that unstable "flat" equilibrium as the sum of a number of spatially periodic fluctuations, which grow at different rates. Out of instability emerges order, because the fluctuations vith the fastest growth rates tend over time to dominate the scene. If the initial distribution of activity is sufficiently close to flat, the eventual distribution will be closely determined by a single preferred wavelength, which is preferred precisely because it is the most unstable.

There are two obvious extensions to the present analysis. The first is to two dimensions. Here one would be looking for a shape as well as a size of fluctuations. It seems intuitively obvious that starting from a smooth distribution across a large plain, the economy will tend to arrange itself into a hexagonal pattern, but

I have not yet been able to **show** this.

The second extension is to multiple industries, with different transport costs and/or economies of scale. A hypothesis is that a Christaller-type hierarchy will emerge, with the distributions of the activities determined by sums **of** fluctuations at different frequencies -- and with each frequency an integer multiple of the previous one. Again, I have not yet been able to confirm this appealing notion.

still, even the results of the simple model in this paper are exciting, both for what they say about the real world and what they **say** about modeling. They suggest that surprisingly simple principles of organization may lurk, at least as approximate rules, beneath what appear to be hopelessly complex spatial systems. And they suggest that the theory of spatial economies, which has increasingly come to rely on numerical methods, may still yield some secrets to paper-and-pencil analysis as well.

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Table 1: Preferred values of ϕ/τ

μ	.2	.3	.4
	5.82	4.48	3.60
	7.76	6.11	5.00
	10.00	7.64	6.25

FIGURE 1

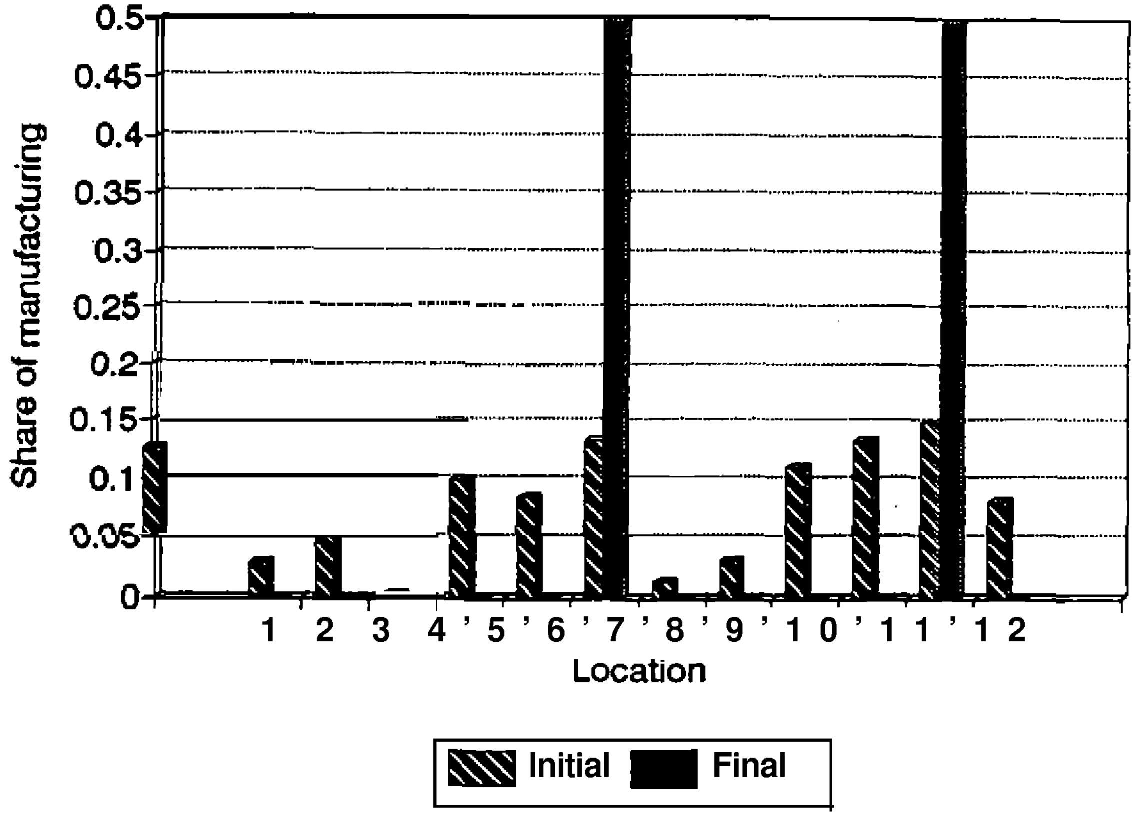

0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Share of manufacturing
1 2 3 4 ' 5 ' 6 ' 7 ' 8 ' 9 ' 1 0 ' 1 1 ' 1 2
Location
Initial
Final

FIGURE 2

Short wavelengths grow slowly

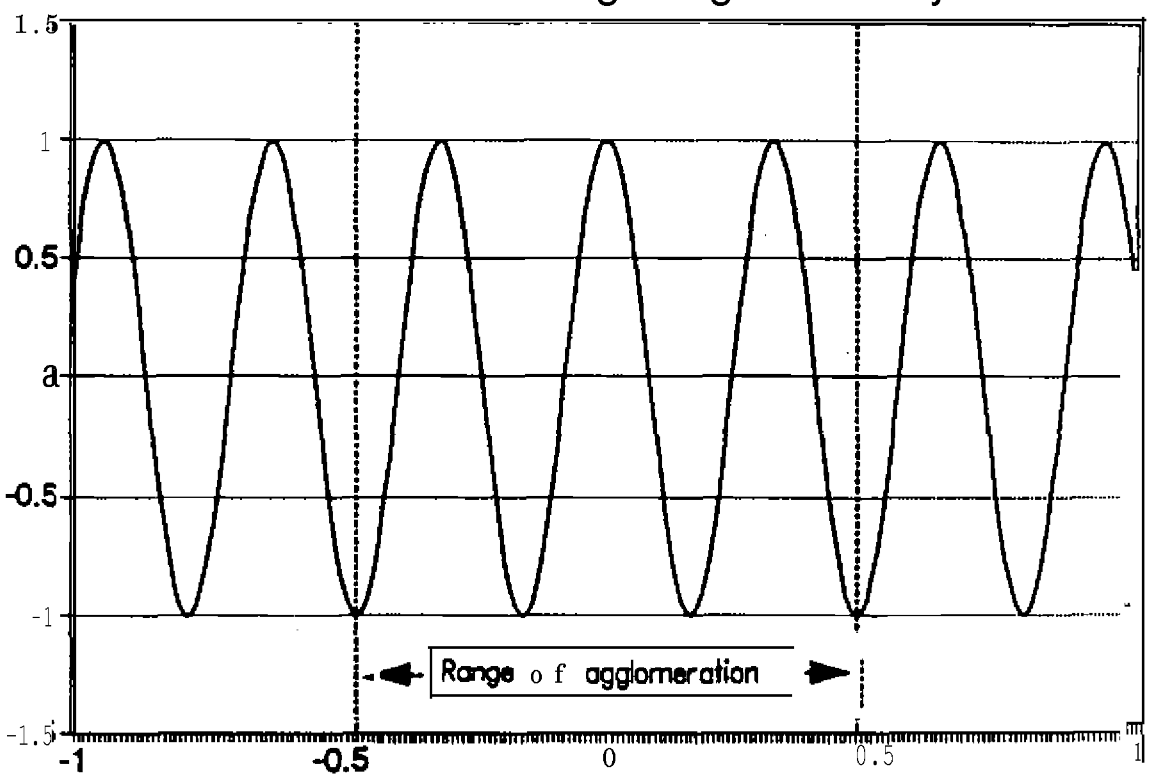


FIGURE 3

Long wavelengths also grow slowly

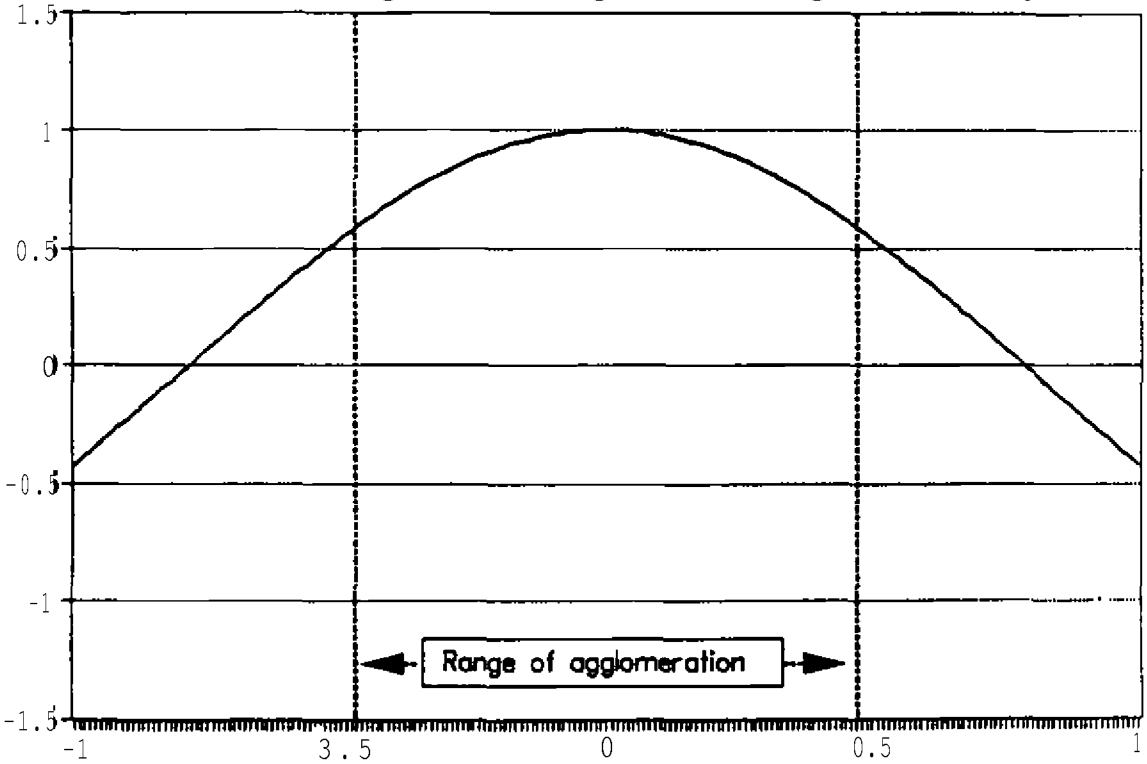


FIGURE 4
The preferred wavelength

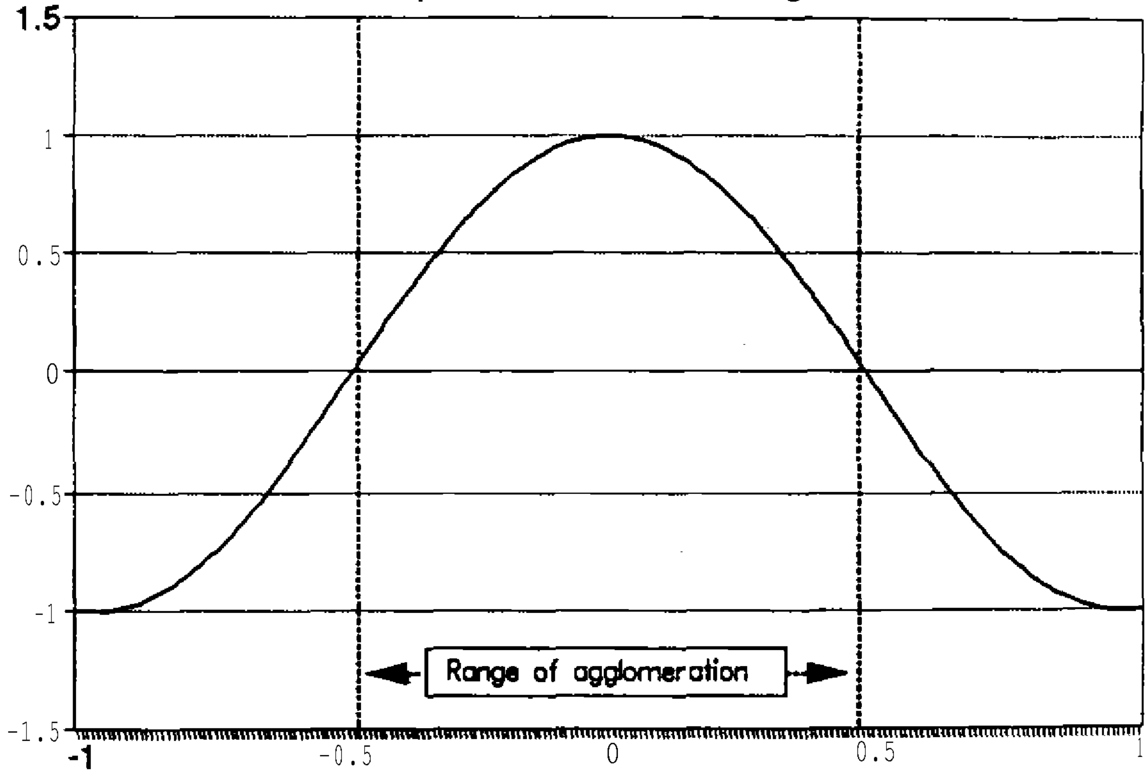
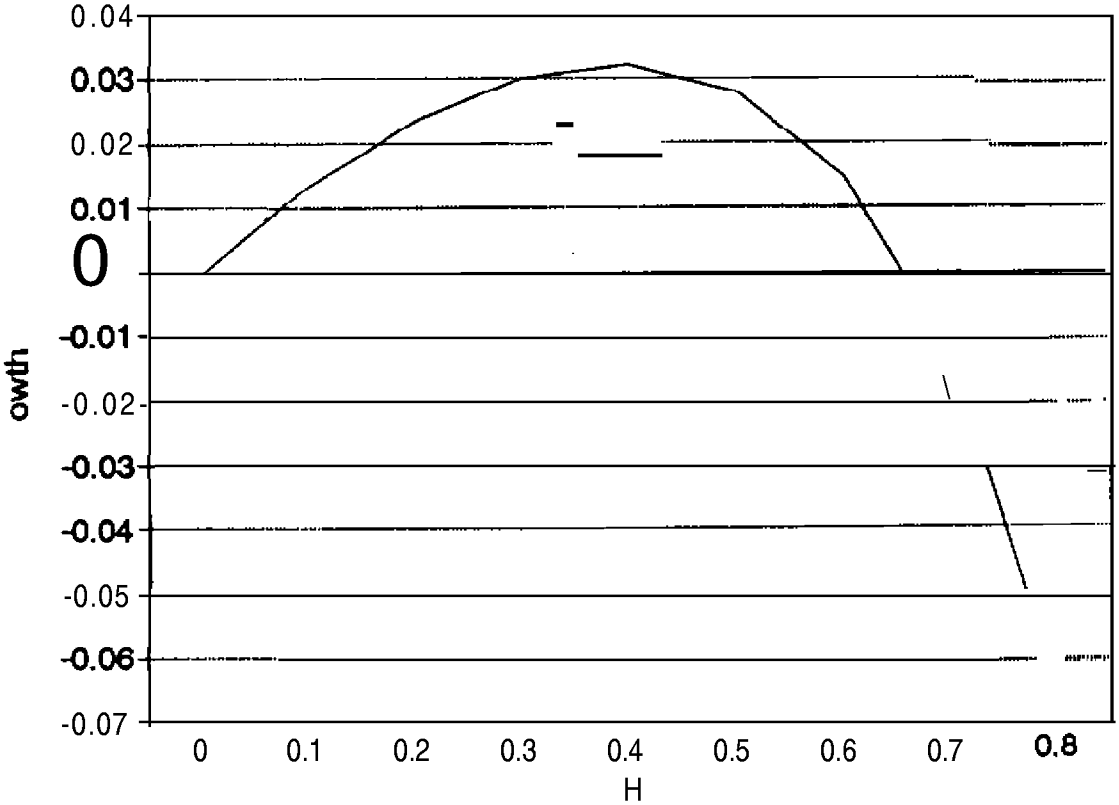


FIGURE 5


0.04
0.03
0.02
0.01
0
-0.01
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-0.04
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