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ABSTRACT

This paper studies the role of asset-market completeness for the properties of optimal policy. A suitable framework for this purpose is the small open economy with complete international asset markets. For in this environment changes in policy represent country-specific risk diversifiable in world markets. Our main finding is that the fundamental public finance principle whereby when taxes on all final goods are available, it is optimal to tax final goods uniformly fails to obtain. In general, uniform taxation is optimal because it amounts to a nondistorting tax on fixed factors of production. In the open economy this principle fails because when households can insure against the risk of a policy reform, initial private asset holdings are contingent on actual policy and thus no longer represent an inelastically supplied source of income. Two further differences between optimal policy in the closed and open economies with complete markets are: (a) In the open economy, optimal consumption and income tax rates are unchanged in response to government purchases shocks. By contrast, in the closed economy tax rates do respond to innovations in public spending. (b) In the open economy, the Friedman rule is optimal only if the Ramsey planner has access to consumption taxes. In the absence of consumption taxes, deviations from the Friedman rule are large. On the other hand, in the closed economy, the availability of either consumption or income taxes suffices to render the Friedman rule optimal.

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1 Introduction

This paper presents an investigation into the role of asset market completeness for the properties of optimal policy in dynamic general equilibrium models of the macroeconomy. We focus on environments in which it is possible for individual agents to insure themselves against policy uncertainty. An ideal framework for this purpose is the small open economy model with complete international asset markets. In this environment, policy changes represent idiosyncratic, country-specific shocks, against which domestic residents can insure by trading in international financial markets.

Since the seminal work of Lucas and Stokey (1983), a large body of research has been devoted to characterizing optimal fiscal and monetary policy in dynamic macroeconomic settings. Most of the existing work, however, has limited attention to closed economy environments. In the closed economy, policy changes represent aggregate uninsurable risk. We instead study Ramsey allocations in a small open economy with access to complete international asset markets. To facilitate comparison, all other aspects of the model economy are deliberately kept as in existing related closed-economy studies. In particular, money demand is motivated by a cash-in-advance constraint, and an interest-elastic velocity of circulation is rationalized by assuming the existence of cash and credit goods as in Lucas and Stokey (1983). The government must finance a stream of unproductive consumption by printing money, issuing one-period, state-noncontingent nominal debt, and levying proportional taxes on income and consumption. The government is assumed to be benevolent in the sense that it conducts policy so as to maximize the welfare of the representative agent. The government has access to a commitment technology, so that announced policies are perfectly credible. Output is produced using labor as the sole input. We further restrict attention to the case of fully flexible prices.

In the closed-economy version of our model, when the government has access to labor and consumption taxes, optimal policy features a uniform tax on all final goods. This finding is related to Coleman (2000) who shows that uniform taxation is optimal in a closed, real economy with capital accumulation. In fact, these results are applications to dynamic settings of a general principle of modern public finance. Namely, that in the presence of an inelastically supplied factor, taxing all final goods at the same proportional rate amounts to an undistorting capital levy on the inelastically supplied factor.

A central result of this paper is to show that the uniform taxation principle fails to obtain in an open economy with complete asset markets. The reason for the failure of this general result is that in the open economy under complete markets the initial wealth of private agents is no longer an inelastically supplied factor. Households can purchase assets whose payoff is contingent on the tax regime in place in future dates and states. As a consequence, the initial net foreign asset position of the country is a function of the tax regime and therefore represents an endogenous variable for the Ramsey planner.

The dependence of net foreign assets on tax policy introduces an additional margin for the Ramsey planner to exploit. In particular, the Ramsey government faces a tradeoff between a policy that eliminates wedges between marginal rates of substitution and marginal rates of transformation and one that maximizes the amount of foreign assets that households will bring into the state in which the Ramsey reform takes place. The first source of tension calls for taxing all final goods at the same rate. The second source of tension requires setting a low average tax rate on consumption, as foreign assets are fully allocated to consumption over the agent's lifetime. The result of this tradeoff is a tax structure that deviates from the principle of uniform taxation in favor of lower consumption tax rates and higher income taxes.

We find that for plausible parameterizations the deviations from the principle of uniform taxation are severe. In our economy there are three types of good, cash goods, credit goods, and leisure. The government has access to proportional consumption and income taxes. Consumption taxes apply to both consumption goods uniformly. Income taxes represent a subsidy on leisure. In addition, the government controls the nominal interest rate. The nominal interest amounts to an indirect tax on consumption of cash goods. Therefore, under uniform taxation the nominal interest rate should be zero and the consumption tax rate should be equal to the negative of the income tax rate. In our economy the Ramsey planner sets the nominal interest rate to zero thus taxing both types of consumption at the same rate. However, the Ramsey planner does not tax consumption and leisure uniformly. Indeed, for the baseline calibration, it is optimal for the government to tax consumption and to subsidize leisure. Specifically, the Ramsey policy features a consumption tax rate of 9 percent and an income tax rate (a leisure subsidy) of 55 percent.

One may wonder why the violation of the uniform taxation principle occurs across consumption and leisure but does not extend to the consumption of cash and credit goods. In other words, one may ask why the Friedman rule turns out to be Ramsey optimal. The intuition behind this result is that a positive nominal interest rate is a less efficient way of taxing consumption than is an explicit consumption tax. The reason is that positive nominal interest rates represent a tax on only a fraction of total consumption expenditures, namely, expenditures on cash goods. Therefore positive nominal interest rates introduce a wedge between the marginal rate of substitution and the marginal rate of transformation of cash for credit goods. A consumption tax, on the other hand, applies uniformly to both cash and credit goods, and thus does not distort this margin.

A further consequence of assuming that residents of the small open economy have access to complete international asset markets is that neither the Ramsey real allocation nor its associated consumption and income tax rates adjust in response to government spending shocks. Government purchases shocks introduce pure wealth effects, which domestic agents can fully insure against via international financial markets. Thus, given tax rates, the household has no incentive to alter either consumption nor labor supply. The government therefore finds it optimal to keep tax rates unchanged. It is clear from this argument that the neutrality of government spending shocks vanishes when agents cannot hedge against such shocks. This is the case either when the economy is closed, so that government purchases shocks become aggregate uninsurable risk, or when markets are incomplete in the open economy.

A natural question then is how the Ramsey government manages to finance government purchases shocks in the open economy with complete markets. The Ramsey planner collects the necessary revenue entirely through surprise changes in the price level. Unexpected inflation represents a capital levy on private holdings of nominal public debt. Thus, surprise changes in the price level are tantamount to a non-distorting lump-sum tax.

The fiscal consequences of productivity shocks, although not entirely, are also accommodated to a large extend through surprise changes in inflation. Thus, under the optimal policy inflation is highly volatile. Under the baseline calibration, our model implies that the optimal standard deviation of inflation is 11 percentage points per year. It follows that high volatility of the inflation rate is a necessary characteristic of optimal policy in the open economy with consumption and income taxes. This need not be the case in the closed economy. We show that when both consumption and income taxes are available, the Ramsey planner can support the Ramsey allocation in the closed economy with a constant path of prices.

A further differences between the Ramsey policy in the open and the closed economies is that in the open economy the availability of consumption taxes is a prerequisite for the optimality of the Friedman rule. Indeed, in the absence of consumption taxes, optimal deviations from the Friedman rule can be large. In the open economy with income taxes only, we find that for the baseline parameterization the average nominal interest rate is 30 percent per year.

The reason why the Ramsey planner resorts to the inflation tax in the open economy when consumption taxes are unavailable is that a tax on domestic output does not cover all sources of income. National income in the open economy consists of the sum of domestic value added and interest income on the country's net foreign asset holdings. An output tax thus captures only the non-interest sources of income. Because, as pointed out above, the nominal interest rate represents a tax on purchases of cash goods, and because eventually all income is spent on consumption, by setting a positive nominal interest rate, the Ramsey planner can collect indirectly tax revenue from all sources of national income. In the closed economy, the Friedman rule is optimal whether the planner has access to consumption taxes or not. The reason is that in the closed economy an output tax applies to all sources of income and thus there is no need for the Ramsey planner to use the consumption tax as an indirect income tax.

2 Anticipated Ramsey Reform and the Uniform Taxation Principle: a Simple Example

In this section, we show by means of a simple example that uniform taxation ceases to be Ramsey optimal when the Ramsey reform is anticipated and agents can insure against this eventuality in international asset markets. Consider an economy in which agents live for 2 periods, period -1 and period 0. In period -1, agents' only activity is to trade in financial markets. In period 0 agents consume and work. In period 0 the economy is either in state 1 or in state 2. In state 1, the government undergoes a Ramsey reform. The probability that state 1 occurs is exogenous and denoted by $\pi \in (0,1)$. The Ramsey planner values only the utility of the representative agent in state 1. We assume that the Ramsey government can perfectly commit to any preannounced policy. We will show that the Ramsey planner finds it in its own best interest to announce already in period -1 his tax policy for state 1. In this way, the planner can design tax policies so as to induce private agents to shift resources into the state in which it is in power, state 1, via international capital markets.

Agents are endowed with claims against the government promising to pay b units of consumption in period 0. In period 0, the government can levy proportional labor income taxes, τ, and consumption taxes, ϕ. The budget constraint of the representative household in period -1 is then given by:

$$E_{-1}r[b+(1-\tau)wh] \geq E_{-1}r(1+\phi)c, \tag{1}$$

where the random variable r denotes the price of one unit of consumption in a particular state of the world in period 0 divided by the probability of occurrence of that state, w denotes the wage rate in period 0, h denotes labor supply, and c denotes consumption in period 0. The left-hand side of the budget constraint represents the present discounted value of income. Income consists of asset returns and after tax wage receipts. The right-hand-side of the budget constraint shows the present discounted value of after-tax consumption expenditures. The representative household chooses contingent plans for consumption and hours worked, $\{c, h\}$, so as to maximize expected utility,

$$\max E_{-1}U(c,h)$$

subject to (1) taking as given b and the stochastic processes for $\{\tau, \phi, r\}$. Let $\bar{\theta} > 0$ denote the Lagrange multiplier associated with the household's intertemporal budget constraint (1). Then the household's first-order optimality conditions are (1) holding with equality and

$$U_c(c,h) = \bar{\theta}r(1+\phi) \tag{2}$$
$$-U_h(c_0,h_0) = \bar{\theta}r(1-\tau) \tag{3}$$

for every state.

Output is assumed to be produced with a linear technology

$$y = h.$$

For firms to produce a positive and finite quantity it must be the case that $w = 1$.

The government in each state of period 0 must satisfy its budget constraint:

$$\tau h + \phi c = g + b, \tag{4}$$

where g denotes government purchases, which are assumed to be exogenously given. Likewise government liabilities, b, are exogenous.

2.1 Ramsey Policy in a Closed Economy

We first characterize optimal anticipated Ramsey policy in a closed economy because in this case optimal policy conforms to the uniform taxation principle. The intuition why in this case uniform taxation is optimal is as follows. In our economy private agents are identical. As a consequence in equilibrium there is no borrowing or lending among them. Thus, in the closed economy the only assets in positive aggregate net supply are government bonds. However, by assumption, the amount of government bonds outstanding in each state is exogenous. It follows that private agents are not able to shift resources across states. In other words, initial private wealth in period 0 is inelastically given and, in particular, cannot be influenced by policy. Therefore, it is optimal (non-distorting) for the government to fully tax this inelastically supplied factor. Applying a uniform tax on consumption and leisure is an indirect way of levying a tax on initial wealth.

Let x_1 denote the value of a variable x in state 1. In state 1, the household's problem consists in maximizing $U(c_1, h_1)$ subject to $b_1 + (1 - \tau_1)h_1 \geq (1 + \phi_1)c_1$. The first-order conditions of this problem are the budget constraint holding with equality and

$$-\frac{U_h(c_1, h_1)}{U_c(c_1 h_1)} = \frac{1 - \tau_1}{1 + \phi_1}. \tag{5}$$

In a closed economy, goods markets must clear state by state, that is,

$$h_1 = g + c_1. \tag{6}$$

The Ramsey problem consists in maximizing $U(c_1, h_1)$ subject to (4) evaluated in state 1, (5), and (6).

The Pareto optimal allocation in state 1 is the solution to the problem of maximizing $U(c_1, h_1)$ subject to the feasibility constraint $h_1 = g + c_1$. Therefore, the Pareto optimal values of $\{c_1, h_1\}$ are given by the solution to the following two equations: $h_1 = c_1 + g$ and $-U_h(c_1, h_1)/U_c(c_1, h_1) = 1$. We now show that the Pareto optimal allocation can be supported as a competitive equilibrium by setting $\tau_1 = -\phi_1$. This implies that uniform taxation of all goods must be Ramsey optimal. Clearly, when $\tau_1 = -\phi_1$, the Pareto values of c_1 and h_1 solve (5) and (6). It remains to show that there exists a value ϕ_1 satisfying $\phi_1 = -\tau_1$ and (4) evaluated in state 1. It is straightforward from equations (4) and (6) that this value of ϕ_1 is given by $\phi_1 = -(g + b_1)/g$.[1]

It is worth pointing out that in this example uniform taxation emerges as the Ramsey optimal policy regardless of the fiscal regime of state 2. We next show that in the open economy, the Ramsey fiscal regime in one state departs from the uniform taxation principle regardless of the policy regime in place in the other state.

2.2 Ramsey Policy in the Open Economy

We now consider a small open economy with free capital mobility. The main insight is that in an open economy with access to complete international asset markets, uniform taxation fails to be Ramsey optimal. This result depends crucially on the fact that the Ramsey reform is anticipated. To keep the problem as close as possible to the closed economy case, assume that the household is endowed in period -1 with an asset which promises that the household will receive b units of goods in period 0. A competitive equilibrium in the open economy is then a set of stochastic processes $\{c, h\}$ and a positive constant $\bar{\theta}$ satisfying (1), (2), (3), and (4), given state contingent tax policies ϕ, τ, the exogenous pricing kernel r, the level of government spending g, and government liabilities b.

There are two formal differences between the open and the closed economies. First, in the open economy domestic absorption need not equal domestic production state by state.

[1] Of course, for there to exist a competitive equilibrium consistent with the Pareto values for c_1 and h_1, it must be the case that $\phi_1 > -1$, that is, that $b < 0$. Alternatively, if the representative household was assumed to be endowed with a sufficiently large amount of goods, then the Pareto allocation could be decentralized even with $b > 0$.

That is, in the open economy equation (6) does not apply. Second, in the open economy the contingent claim prices $\{\pi r_1, (1-\pi)r_2\}$ are no longer an endogenous variable but rather determined in international capital markets whose prices the small open economy has to take as given.

An important determinant of optimal policy is the assumed strategic behavior of governments. One possibility is to focus on Nash equilibria where each government i, $i = 1, 2$, takes the fiscal policy of the other government, τ_j, ϕ_j, $j \neq i$, as given. We believe that this specification of government behavior is not the most appealing one. For, in t;his case, players have no choice about their fiscal policy. Specifically, given τ_j and ϕ_j, there exists a unique pair τ_i, ϕ_i, $i \neq j$ that can be supported as a competitive equilibrium outcome. To see this, consider, for example, the reaction function of government 1. Since government 1 takes as given τ_2 and ϕ_2, equations (2), (3), and (4) evaluated in state 2 uniquely determine c_2, h_2, and $\bar{\theta}$ as functions of τ_2 and ϕ_2. It then follows from the intertemporal budget constraint (1) that $d(\tau_2, \phi_2) + b_1 = (1+\phi_1)c_1 - (1-\tau_1)h_1$, where $d(\tau_2, \phi_2) \equiv (1-\pi)r_2[b_2 + (1-\tau_2)h_2 - (1+\phi_2)c_2]/(\pi r_1)$. This expression together with (2), (3), and (4) evaluated in state 1 then implies unique values for ϕ_1 and τ_1. That is, when government 1 takes both τ_2 and ϕ_2 as given, then it in fact ends up with no choice over its own fiscal policy. Therefore, the resulting Nash equilibrium cannot be regarded as one in which any government follows a Ramsey-optimal policy. This result is driven by two features of the assumed behavior of governments: (a) Each government takes as given a tax policy for the other government that in general (off equilibrium) does not satisfies the other government's budget constraint. (b) In choosing a tax regime, each government is assumed to ensure that it is consistent with a competitive equilibrium, given the policy of the other government.

Alternatively, one could conceive a situation in which each government assumes that the other government will choose fiscal policies that ensure fiscal solvency under all possible circumstances (on or off equilibrium). Specifically, this can be achieved by postulating that each government chooses its fiscal policy assuming that the other government can choose only one tax instrument independently. For example, , suppose that government 1 chooses takes τ_2 as given.

To derive the reaction function of government 1, it is convenient to present a simpler characterization of the competitive equilibrium: Values for $\{c_1, h_1, \bar{\theta}\}$ satisfying

$$\pi r_1(h_1 - g - c_1) + (1-\pi)r_2[h(\bar{\theta}) - g - c(\bar{\theta})] = 0 \tag{7}$$

$$U_c(c_1, h_1)c_1 + U_h(c_1, h_1)h_1 - \bar{\theta} r_1[g + b + c_1 - h_1] = 0, \tag{8}$$

where $h(\bar{\theta})$ and $c(\bar{\theta})$ are the solution to (2), (3), and (4) evaluated in state 2, are the same as those satisfying (1)-(4).

To establish this claim, we first show that any $\{c_1, h_1, \bar{\theta}\}$ that satisfy (1)-(4) also satisfy (7) and (8). Use (4) to write (1) as (7). Then use (2) and (3) to eliminate ϕ_1 and τ_1 from (4) evaluated in state 1 to get (8). Next we show that any $\{c_1, h_1, \bar{\theta}\}$ that satisfy (7) and (8) also satisfy (1)-(4). From the definition of the functions $c(\bar{\theta})$ and $h(\bar{\theta})$, it follows that equations (2), (3), and (4) hold in state 2. Pick ϕ_1 and τ_1 so that (2) and (3) evaluated in state 1 hold. Now use these expression to eliminate the marginal utility terms from (8) to get that (4) holds in state 1. So all that is left to show is that (1) holds. Use (7) and replace $h - g - c$ in each state with (4). It follows that (1) is satisfied.

Recall that the Ramsey planner can announce in advance which policy he will follow should he be elected, that is, should state 1 materialize. Indeed, it is in the Ramsey planner's best interest to do so, for in this way he can attract resources into state 1. The Ramsey problem then consists in choosing $\{c_1, h_1, \bar{\theta}\}$ at time -1 so as to maximize (**??**) subject to (7) and (8), given b, g, contingent claim prices $\{r_0\}$, and the functions $h(\bar{\theta})$ and $c(\bar{\theta})$.

The question is whether optimal anticipated Ramsey policy displays uniform taxation, that is, is $\phi_1+\tau_1 = 0$, or in terms of the marginal rate of substitution, will $-U_h(c_1, h_1)/U_c(c_1, h_1) = 1$. The answer is in general not. To see this let λ and μ be the Lagrange multiplier on (7) and (8),respectively. The Lagrangian of the Ramsey problem can be written as

$$\begin{aligned} \mathcal{L} &= U(c_1, h_1) \\ &+\lambda \left\{\pi r_1(h_1 - g - c_1) + (1-\pi) r_2[h(\bar{\theta}) - g - c(\bar{\theta})]\right\} \\ &+\mu \left\{U_c(c_1, h_1)c_1 + U_h(c_1, h_1)h_1 - \bar{\theta} r_1[g + b + c_1 - h_1]\right\} \end{aligned}$$

The associated first-order conditions with respect to c_1 and h_1 are

$$\begin{aligned} U_c(1) - \lambda\pi r_1 + \mu \left[\frac{\partial(U_c(c_1, h_1)c_1 + U_h(c_1, h_1)h_1)}{\partial c_1} - \bar{\theta} r_1\right] &= 0 \\ -U_h(1) - \lambda\pi r_1 + \mu \left[-\frac{\partial(U_c(c_1, h_1)c_1 + U_h(c_1, h_1)h_1)}{\partial h_1} - \bar{\theta} r_1\right] &= 0 \end{aligned}$$

In the general case, in which μ, the multiplier on the government's budget constraint is non-zero, uniform taxation holds only if for the Ramsey real allocation

$$\frac{\partial(U_c(c_1, h_1)c_1 + U_h(c_1, h_1)h_1)}{\partial c_1} = \frac{\partial(U_c(c_1, h_1)c_1 + U_h(c_1, h_1)h_1)}{\partial h_1}$$

This is clearly not the case for many commonly used specifications of the period utility function. For example, in the business cycle literature attention is typically restricted to preference specifications that are consistent with the facts that per capita consumption is growing over time whereas per capita hours are not. King, Plosser, and Rebelo (1988) show that for period utility function of the form $U(c, \bar{h} - h)$, the only period utility function compatible with growth in consumption but not in hours is $U(C, \bar{h}-h) = c^{1-\sigma}/(1-\sigma)v(\bar{h}-h)$ for $\sigma \geq 0$ and $log(c) + v(\bar{h} - h)$ for $\sigma = 1$. For this class of utility functions, we have that $U_c c + U_h h = U(1 - \sigma + v'/vh)$. It follows that unless v'/vh is independent of h, uniform taxation fails. Therefore we conclude that for preference specifications that are standard in business cycle analyzes, anticipated Ramsey reforms do not abide to the principle of uniform taxation.

The intuition for this result is as follows. Uniform taxation is optimal because it acts as a non-distorting tax scheme on the initial wealth. Initial wealth is typically regarded as an inelastically supplied factor. As such it is optimal to tax it fully. However, when Ramsey reform are anticipated, then the amount of initial wealth that agents bring into the period is no longer an inelastically supplied factor. Indeed it is a function of the tax policies that will be implemented in that period. So the Ramsey planner has an incentive to design the tax structure in such a way that agents shift a lot of wealth out of state 2 into state 1. This can be achieved by taxing consumption relatively less. (Our numerical work in section 4 shows this for an infinite horizon monetary economy). In the argument above we assumed that the Ramsey planner can commit in period -1 to the policy actions he will take in period 0. Typically, in the study of Ramsey policy commitment is interpreted as the government binding its hands to do in later periods (in which it is in power) what it promised to do in the first period (it came into power). We extend the meaning of commitment by assuming that the Ramsey government can commit ex ante, that is, even before it takes over the government, to its future actions. If we were to assume the contrary, that is, the commitment technology becomes available only once the government is in power, then uniform taxation re-emerges as the central characteristic of optimal taxation. In that case the Ramsey government will take initial wealth as exogenously given and will choose to tax it fully. However, the latter assumption about the availability of the commitment technology is less appealing for one may wonder what exactly it is that allows the government to commit to its actions for any future date and any future contingency but deprives it from doing so one period before the government takes power. Clearly, if the commitment technology is in the control of the government it will want to use it even before it actually gets elected because as we have just shown it yields higher utility to its constituents.

3 The Model

In this section, we develop a simple infinite-horizon model of a small open economy. A demand for money is motivated by a cash-in-advance constraint. Sales are assumed to have to be carried out in the buyer's currency[2]. Prices are assumed to be flexible. Asset markets are complete. The government finances an exogenous stream of unproductive consumption by issuing non-state-contingent nominal debt, levying distortionary income and consumption taxes, and printing money.

3.1 Households

Consider an economy populated by a large number of identical households. Each household has preferences defined over processes of consumption of cash goods, c_t^m, credit goods, c_t^c, and labor effort, h_t. Preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^m, c_t^c, h_t), \tag{9}$$

where $\beta \in (0,1)$ denotes the subjective discount factor, and E_0 denotes the mathematical expectation operator conditional on information available in period 0. The single-period utility function U is assumed to be increasing in both consumption goods, decreasing in effort, strictly concave, and twice continuously differentiable.

Each period $t \geq 0$, households can purchase two types of financial assets: fiat money, M_t, and one-period, state-contingent, nominal assets, D_{t+1}, which pay one unit of currency

[2]Alternatively, one could assume that the seller's currency is used in transactions. For a comparison of the consequences of these two alternative transaction technologies for savings, investment, and the exchange rate, see Helpman and Razin (1984).

in a particular state of period $t+1$. Money holdings are motivated by a cash-in-advance constraint on purchases of cash goods of the form

$$M_t \geq P_t(1+\phi_t)c_t^m, \tag{10}$$

where P_t denotes the domestic currency price of consumption goods and ϕ_t stands for a proportional consumption tax rate. At the beginning of each period t, after all shocks are realized, the household chooses its desired holdings of money and contingent claims. Letting W_t denote the representative household's nominal financial wealth at the beginning of period t, we have that

$$W_t \geq M_t + E_t r_{t+1} D_{t+1}.$$

The variable r_{t+1} denotes the period-t price of a claim to one unit of currency in a particular state of period $t+1$ divided by the probability of occurrence of that state conditional on information available in period t. Thus, the second term on the right-hand side of the above expression denotes the cost of all contingent claims purchased at the beginning of period t. Note that $E_t r_{t+1}$ is the period-t price of an asset that pays one unit of currency in every state in period $t+1$. Thus $E_t r_{t+1}$ represents the inverse of the risk-free gross nominal interest rate. Formally, letting R_t denote the gross risk-free nominal interest rate, we have

$$R_t = \frac{1}{E_t r_{t+1}}. \tag{11}$$

After the financial market is closed, the household supplies hours in the labor market and shops for consumption goods. The household's wealth at the beginning of period $t+1$ is then given by

$$W_{t+1} = M_t + D_{t+1} + P_t(1-\tau_t)(w_t h_t + \Pi_t) - P_t(1+\phi_t)(c_t^m + c_t^c),$$

where τ_t denotes the income tax rate, w_t denotes the real wage rate, and Π_t denotes profits from the ownership of firms.[3] Note that because D_{t+1} is a random variable measurable with respect to the information set of period $t+1$, W_{t+1} is also a random variable measurable with respect to the information set available in period $t+1$. The right-hand side of this expression displays the difference between the sources of wealth at the beginning of period $t+1$ and consumption expenditures: money carried over from the previous period, M_t, plus the payoff of state-contingent claims, D_{t+1}, plus wage and profit income net of taxes, $P_t(1-\tau_t)(w_t h_t + \Pi_t)$, minus consumption expenditures, $P_t(1+\phi_t)(c_t^m + c_t^c)$. Combining the above three expressions yields the following period-by-period budget constraint

$$E_t r_{t+1} W_{t+1} \leq W_t - \frac{R_t - 1}{R_t} M_t + R_t^{-1}[P_t(1-\tau_t)(w_t h_t + \Pi_t) - P_t(1+\phi_t)(c_t^m + c_t^c)]. \tag{12}$$

In addition to this budget constraint, the household is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes:

$$\lim_{j\to\infty} E_t q_{t+j} W_{t+j} \geq 0, \tag{13}$$

[3] Note that we are imposing the same tax on labor income as on profits. Taxing profits at a rate less than one hundred per cent and different from the tax rate applied to labor income, may induce the Ramsey planner to deviate from the Friedman rule (see Schmitt-Grohé and Uribe, 2001). In this case the Ramsey planner taxes pure profits indirectly by taxing consumption of cash goods.

at all dates and under all contingencies. The variable q_t denotes the period-zero price of one unit of currency to be delivered in a particular state of period t divided by the probability of occurrence of that state given information available at time 0 and is given by

$$q_t = r_1 r_2 \dots r_t, \tag{14}$$

with $q_0 \equiv 1$. The assumption that preferences display no satiation implies that utility-maximizing households will always choose allocations such that constraints (12) and (13) both hold with equality. Sequences for $\{M_t, c_t^m, c_t^c, h_t\}$ satisfying these two constraints with equality are the same as those satisfying the following single intertemporal constraint:[4]

$$W_0 + E_0 \sum_{t=0}^{\infty} q_{t+1} P_t (1-\tau_t)(w_t h_t + \Pi_t) = E_0 \sum_{t=0}^{\infty} q_{t+1} \left[P_t (1+\phi_t)(c_t^m + c_t^c) + (R_t - 1) M_t \right]. \tag{15}$$

This expression states that total wealth in period zero, which consists of the sum of initial financial wealth and the present discounted value of after-tax labor and profit income, must equal the present discounted value of consumption expenditures including inflation-tax payments. The household chooses the set of processes $\{c_t^m, c_t^c, h_t, M_t\}_{t=0}^{\infty}$, so as to maximize (9) subject to (10) and (15), taking as given the set of processes $\{P_t, w_t, r_{t+1}, \tau_t, \phi_t, \Pi_t\}_{t=0}^{\infty}$ and the initial condition W_0. Let the multiplier on the intertemporal budget constraint be denoted by λ. Define

$$\lambda_t \equiv \lambda \frac{q_t P_t}{\beta^t}. \tag{16}$$

Then the first-order conditions associated with the household's maximization problem are (10), (15) holding with equality, and

$$U_1(c_t^m, c_t^c, h_t) = (1+\phi_t)\lambda_t \tag{17}$$

$$\frac{U_1(c_t^m, c_t^c, h_t)}{U_2(c_t^m, c_t^c, h_t)} = R_t \tag{18}$$

$$-\frac{U_3(c_t^m, c_t^c, h_t)}{U_1(c_t^m, c_t^c, h_t)} = \frac{(1-\tau_t) w_t}{(1+\phi_t) R_t} \tag{19}$$

$$(R_t - 1)[M_t - P_t(1+\phi_t) c_t^m] = 0. \tag{20}$$

The interpretation of these optimality conditions is straightforward. First-order condition (17) states that consumption taxes introduce a wedge between the marginal utility of consumption of cash goods and the marginal utility of real wealth. Optimality condition (18) shows that the nominal interest rate, R_t, breaks the equality between the marginal rate of substitution of cash and credit goods and their marginal rate of transformation (unity). As the opportunity cost of holding money, R_t, increases, households consume relatively more credit goods and less cash goods. Equation (19) shows that the income tax rate, the consumption tax rate, and the nominal interest rate distort the consumption/leisure margin. Given the wage rate, households will tend to work less and consume less the higher are τ_t, ϕ_t, or R_t. According to first-order condition (20), when the opportunity cost of holding money

[4] See Woodford (1994).

is strictly positive, $R_t > 1$, the cash-in-advance constraint is binding. In this case, money is dominated in rate of return by interest bearing assets, so households choose not to hold nominal balances beyond the amount strictly necessary to buy the desired quantity of cash goods.

3.2 Firms

Output, denoted by y_t, is produced with a concave technology that takes labor services, h_t, as the only factor input,

$$y_t = z_t h_t^{\eta}, \tag{21}$$

where z_t is an exogenous and stochastic productivity shock and $\eta \in (0, 1]$ is a constant parameter. In closed-economy studies of optimal monetary and fiscal policy it is customary to assume that the production function is linear in labor ($\eta = 1$). In the open economy this assumption is more problematic because it allows for the possibility of corner solutions. In particular, when $\eta = 1$, the equilibrium labor supply, and thus output, may be nil in certain states. We avoid this problem by assuming that the marginal product of labor goes to infinity as effort approaches zero. This problem does not arise in closed economies because in such environments zero output implies zero consumption (or negative consumption if public consumption is positive).[5]

Firms operate in a perfectly competitive environment. Firms can sell either in domestic or foreign markets. Sales must be carried out in the buyer's currency. Therefore, in each period $t \geq 0$, firms can sell goods in domestic markets at the price P_t or in foreign markets at the price P_t^*. Firms receive payments for goods sold in the goods market of period t and must pay for the factor inputs used in that market after the financial market of period t has closed. Like households firms have access to complete assets markets. Due to the presence of exchange rate risk, in the financial market of period $t + 1$, the real value of profits generated in the goods market of period t is random (measurable only with respect to the information set of period $t + 1$). Firms are assumed to fully insure against this risk. Thus, profit distributions in the financial market of period $t+1$ are measurable with respect to the information set of period t and are given by

$$P_t \Pi_t = z_t h_t^{\eta} [\alpha_t P_t + (1 - \alpha_t) P_t^* R_t E_t r_{t+1} e_{t+1}] - P_t w_t h_t. \tag{22}$$

Here, $\alpha_t \in [0, 1]$ denotes the fraction of production sold domestically. Note that profits generated in period t are available to households only at the beginning of period $t + 1$. This is why the exchange rate prevailing in period $t + 1$ is used to convert foreign sales revenues into domestic currency. The firm chooses α_t and h_t so as to maximize expected profits. It follows from the first-order conditions of the profit-maximization problem that for firms to be indifferent between selling domestically and abroad, it must be the case that

$$P_t = R_t P_t^* E_t r_{t+1} e_{t+1}. \tag{23}$$

Given this condition, firms will choose to demand labor so as to equate the marginal product of labor to the real wage rate

$$w_t = z_t \eta h_t^{\eta - 1}. \tag{24}$$

[5] The main results of the paper still obtain when η is assumed to be unity.

3.3 The government

The government faces a stream of public consumption, denoted by g_t, that is exogenous, stochastic, and unproductive. Government expenditures are financed by levying income taxes at the rate τ_t and consumption taxes at the rate ϕ_t, by printing money, and by issuing one-period, risk-free, nominal obligations, which we denote by B_t. We assume that the government cannot issue or hold state-contingent assets. We believe that, as a first approximation, this assumption describes well actual financing practices by national governments. Let A_t denote nominal government liabilities at the beginning of period t. In the financial market of period t the government issues money and bonds to finance these liabilities. That is,

$$A_t = M_t + B_t.$$

At the beginning of period $t+1$, total government liabilities are given by

$$A_{t+1} = M_t + R_t B_t + P_t g_t - P_t \tau_t (w_t h_t + \Pi_t) - P_t \phi_t (c_t^m + c_t^c).$$

Note that A_t belongs to the information set of period $t-1$ for all $t \geq 0$. Combining the above two expressions, we obtain the government's sequential budget constraint

$$A_{t+1} = R_t A_t + M_t(1 - R_t) + P_t[g_t - \tau_t(w_t h_t + \Pi_t) - \phi_t(c_t^m + c_t^c)], \tag{25}$$

for $t \geq 0$. The government is subject to a no-Ponzi-game constraint of the form

$$\lim_{j \to \infty} E_t q_{t+j} A_{t+j} \leq 0, \tag{26}$$

at all dates and under all contingencies. This constraint is a requirement for the existence of well defined Ramsey equilibria. In the closed economy, the no-Ponzi-game constraint on private households implies that in equilibrium $\lim_{j\to\infty} E_t q_{t+j} A_{t+j} \leq 0$. So there is no need to impose constraint (26). This is because in the closed economy the private sector's asset holdings must necessarily match the government's total liabilities. In the open economy this is not the case. For both the government and private agents have access to international capital markets. Thus, the no-Ponzi-game restriction on private households no longer guarantees that the government is not running a Ponzi scheme against the rest of the world. We assume that a prerequisite for the government to have access to international financial markets is the satisfaction of a borrowing limit like the one given in equation (26).

A benevolent government seeking to maximize the welfare of private agents will always choose asset processes such that (26) holds with strict equality. Otherwise, it could implement a lump-sum transfer to private agents, thereby making them better off, without violating its no-Ponzi-game constraint. The monetary/fiscal regime consists in the announcement of state-contingent plans for the nominal interest rate and the tax rates, $\{R_t, \tau_t, \phi_t\}$.

3.4 Equilibrium

We assume free capital mobility. This means that the following no-arbitrage condition must hold

$$r_{t+1}^* = r_{t+1} \frac{e_{t+1}}{e_t}; \quad t \geq 0, \tag{27}$$

where r^*_{t+1} denotes the period-t foreign-currency price of a claim to one unit of foreign currency delivered in a particular state of period $t+1$ divided by the probability of occurrence of that state conditional on information available in period t.

Let $R^*_t \equiv 1/E_t r^*_{t+1}$ denote the foreign risk-free nominal interest rate. Then combining (23) with the no-arbitrage condition (27) yields

$$P_t = e_t P^*_t \frac{R_t}{R^*_t}. \tag{28}$$

Note that this law-of-one-price condition differs from the usual one, namely $P_t = e_t P^*_t$, by the interest rate differential, R_t/R^*_t. This modification of the law-of-one-price condition is a consequence of two features of our model. First, goods must be purchased with the buyers' currency and second, in each period goods markets open only after financial markets have closed.[6]

We assume that (in period -1) households are able buy contingent claims whose pay-off is contingent upon the realizations of the exogenous shocks and any potential policy regime changes occuring in any period $t \geq 0$. As a result, we one can show that the individual household's marginal utility of wealth, λ_t, can be written as

$$\lambda_t = \theta \frac{q^*_t P^*_t}{\beta^t} \frac{R_t}{R^*_t}, \tag{29}$$

where θ is an exogenous parameter. To obtain this expression, consider the maximization problem of the household in period -1. Let $\tilde{\lambda}$ be the Lagrange multiplier on the household's intertemporal budget constraint in period -1 (i.e., the multiplier on the $t = -1$ version of equation (15)). Then, the first-order condition with respect to c^m_0 is $\beta U_1(c^m_0, c^c_0, h_0) = (1 + \phi_0)\tilde{\lambda}\tilde{q}_0 P_0$. Here $\tilde{q}_0$ denotes the price in period -1 of one unit of domestic currency in a particular state of period zero normalized by the probability of occurrence of that state conditional upon information available at date -1. At the same time, evaluating equations (16) and (17) at $t = 0$ yields $U_1(c^m_0, c^c_0, h_0) = (1 + \phi_0)\lambda P_0$. It follows that $\lambda = \tilde{\lambda}\tilde{q}_0/\beta$. Also note that by (27) and (28) $q_t P_t = e_0 P^*_t q^*_t R_t/R^*_t$ and that $\tilde{q}_0 = \tilde{q}^*_0 e_{-1}/e_0$. Combining the last three expressions and defining $\theta = \tilde{q}^*_0 e_{-1}\tilde{\lambda}/\beta$, yields $\lambda = \theta P^*_t q^*_t R_t/(P_t q_t R^*_t)$. Combining this expression with (16) we obtain equation (29). Note that θ depends on variables measurable with respect to information sets dated before period 0. The Ramsey planner in general will internalize the dependence of $\tilde{\lambda}$—and thus of θ—on his actions. However, it should be clear from the example presented in section 2 that as the probability of reform becomes small, this dependence vanishes. We assume that the probability of reform is indeed small, so that the Ramsey planner can treat θ as given. We note that the failure of uniform taxation does not depend on this assumption, as demostrated by the example of section 2.

Combining (22) with (23), equilibrium profits are given by

$$\Pi_t = z_t h^\eta_t - w_t h_t.$$

Using this expression in the government budget constraint (25) to eliminate profits, we obtain

$$A_{t+1} = R_t A_t + M_t(1 - R_t) + P_t[g_t - \tau_t z_t h^\eta_t - \phi_t(c^m_t + c^c_t)]. \tag{30}$$

[6]The usual law-of-one-price condition would obtain if one were to assume that transactions must be carried out in the sellers' currency, as is maintained, for example, in Stockman (1980) and Helpman (1981), or if one were to assume that goods markets open before that period's financial markets (Svensson timing).

For households to have well defined demand functions it must be the case that the nominal interest be non-negative, that is,

$$R_t \geq 1.$$

We are now ready to formally define an equilibrium.

Definition 1 (Competitive Equilibrium With Consumption and Income Taxes) *A competitive equilibrium is a set of stochastic processes* $\{P_t, e_t, r_{t+1}, M_t,\ A_{t+1}, c_t^c,\ c_t^m, h_t, q_t, w_t, y_t, \lambda_t\}$ *satisfying (10), (11), (14), (17), (18), (19), (20), (21), (24), (26) holding with equality, (27), (28), (29), and (30) given the parameter* θ*, exogenous stochastic processes* $\{g_t, z_t, P_t^*, r_{t+1}^*, q_t^*\}$*, policy sequences* τ_t*,* ϕ_t*, and* $R_t \geq 1$*, and the initial condition* A_0*.*

Note the definition of a competitive equilibrium involves neither the variable W_0 nor the household's intertemporal budget constraint, equation (15). The reason is that in equilibrium W_0 adjusts endogenously across different states of the world in period 0 to guarantee that (29) holds for a given value of θ. That is, given equilibrium values for contingent plans for $\{P_t, e_t, r_{t+1}, M_t,\ A_{t+1}, c_t^c,\ c_t^m, h_t, q_t,\ w_t, y_t, \lambda_t\}$ one can find the value of W_0 that is associated with the competitive equilibrium from equation (15).

3.4.1 Primal Form of the Competitive Equilibrium

The following proposition presents the primal form of the competitive equilibrium.

Proposition 1 (Competitive Equilibrium in Primal Form) *Given initial conditions* A_0 *and* e_0*, the positive parameter* θ*, and exogenous stochastic processes for* $\{z_t, g_t, P_t^*, r_{t+1}^*\}$*, contingent plans* $\{c_t^m, c_t^c, h_t\}$ *satisfying* $U_1(t)/U_2(t) \geq 1$ *and*

$$\theta \frac{A_0}{e_0} = E_0 \sum_{t=0}^{\infty} \beta^t \left[U_1(t) c_t^m + U_2(t) c_t^c + \eta^{-1} U_3(t) h_t + \theta x_t^* (z_t h_t^\eta - g_t - c_t^m - c_t^c)\right] \tag{31}$$

are the same as those satisfying $U_1(t)/U_2(t) \geq 1$*, (10) (11), (14), (17), (18), (19), (20), (21), (24), (26) holding with equality, (27), (28), (29), and (30), where* $x_t^* \equiv P_t^* q_t^* / (\beta^t R_t^*)$*,* $q_t^* = r_1^* \ldots r_t^*$*, and* $R_t^* = 1/E_t r_{t+1}^*$*.*

Proof: See appendix A. ■

The proposition states that any real allocation satisfying the single intertemporal implementability constraint given by equation (31) can be supported as a competitive equilibrium. By comparison, in the closed economy the primal form of the competitive equilibrium consists of the implementability constraint (31) coupled with a period-by period feasibility constraint requiring that domestic absorption equals domestic production, that is $c_t^m + c_t^c + g_t = z_t h_t^\eta$. It follows immediately that under the Ramsey allocation welfare must be at least as high in the open economy as in the closed economy.

4 Ramsey Allocations

As is well known, the Ramsey planner would like to inflate away the initial level of nominal liabilities by setting an infinite price level in period zero. This is because in this model initial inflation represents a lump-sum capital levy. We avoid this unrealistic feature of the unrestricted Ramsey plan by assuming that the initial nominal exchange rate, e_0, is arbitrarily fixed. The Ramsey optimization problem then consists in maximizing (9) subject to (31), and $U_1(t)/U_2(t) \geq 1$, given initial government liabilities measured in units of foreign currency, A_0/e_0.

4.1 Analytical Results

The following proposition shows that under weak restrictions on preferences the Friedman rule is optimal in the open economy when consumption and income taxes are available. The restrictions on preferences we impose are the same as those assumed in the related closed-economy literature.

Proposition 2 (Optimality of the Friedman Rule in the Open Economy When Consumption and Income Taxes are Available) *Assume that the single-period utility function is strictly increasing in c^m and c^c, strictly decreasing in h, and concave. Assume further that $U(c^m, c^c, h) = f(c^m, c^c)v(h)$, where f is homogeneous of degree k. Then, the Friedman rule is Ramsey optimal.*

Proof: The strategy of the proof is to solve a modified Ramsey problem in which we do not impose the constraint $U_1(t)/U_2(t) \geq 1$ and then to show that the solution does not violate it. In this case the Lagrangian takes the form

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(t) + \xi \left[U_1(t)c_t^m + U_2(t)c_t^c + \eta^{-1}U_3(t)h_t + \theta x_t^*(z_t h_t^\eta - g_t - c_t^m - c_t^c) \right] \right\} - \xi\theta \frac{A_0}{e_0},$$

where ξ is the Lagrange multiplier associated with the implementability constraint (31). The first-order optimality conditions with respect to $\{c_t^m, c_t^c, h_t\}_{t=0}^{\infty}$ and ξ are:

$$U_1(t) + \xi[U_{11}(t)c_t^m + U_1(t) + U_{21}(t)c_t^c + \eta^{-1}U_{31}(t)h_t] = \xi\theta x_t^* \tag{32}$$

$$U_2(t) + \xi[U_{12}(t)c_t^m + U_{22}(t)c_t^c + U_2(t) + \eta^{-1}U_{32}(t)h_t] = \xi\theta x_t^* \tag{33}$$

$$U_3(t) + \xi[U_{13}(t)c_t^m + U_{23}(t)c_t^c + \eta^{-1}U_{33}(t)h_t + \eta^{-1}U_3(t)] = -\xi\theta\eta x_t^* z_t h_t^{\eta-1} \tag{34}$$

$$\theta \frac{A_0}{e_0} = E_0 \sum_{t=0}^{\infty} \beta^t \left[U_1(t)c_t^m + U_2(t)c_t^c + \eta^{-1}U_3(t)h_t + \theta x_t^*(z_t h_t^\eta - g_t - c_t^m - c_t^c) \right] \tag{35}$$

Using the particular functional form of the period utility function assumed in the statement of the proposition, we can write the optimality conditions (32) and (33) as

$$U_1(t)[1 + \xi(k + \eta^{-1}v'(h_t)h_t/v(h_t))] = \xi\theta x_t^*$$

$$U_2(t)[1 + \xi(k + \eta^{-1}v'(h_t)h_t/v(h_t))] = \xi\theta x_t^*.$$

Taking the ratio of these expressions yields $U_1(t)/U_2(t) = 1$. Therefore, the constraint $U_1(t)/U_2(t) \geq 1$ is not violated. It follows from equation (18) that $R_t = 1$, that is, it follows that the Friedman rule is optimal. ■

The reason why the Friedman rule turns out to be optimal is quite intuitive. In this model, the nominal interest rate, R_t, represents an indirect tax on a subset of consumption goods, namely cash goods. This is because the nominal interest rate is the opportunity cost of holding real balances, which in turn are required to carry out purchases of cash goods. Thus, a positive interest rate introduces a wedge between the marginal rate of substitution between cash and credit goods and the marginal rate of transformation between these goods (unity). On the other hand, the proportional consumption tax ϕ_t represents a less distorting way to tax private spending as it applies to all types of goods uniformly. Consequently, the Ramsey planner chooses to give up completely the use of the nominal interest rate as a tax on consumption and to fully rely instead on regular consumption taxes for this purpose.

Having shown that the Friedman rule is optimal, it is straightforward to establish that uniform taxation fails to be Ramsey optimal.

Proposition 3 (Uniform Taxation is Not Ramsey Optimal) *Assume that the single-period utility function is strictly increasing in c^m and c^c, strictly decreasing in h, and concave. Assume further that $U(c^m, c^c, h) = f(c^m, c^c)v(h)$, where f is homogeneous of degree k. Then, uniform taxation, that is, $\tau_t = -\phi_t$, fails to be Ramsey optimal.*

Proof: Under the assumptions of the proposition equation (34) can be written as

$$U_3(t)[1 + \xi[k + \eta^{-1}v''(t)/v'(t)h_t + 1/\eta] = -\xi\theta\eta x_t^* z_t h_t^{\eta-1} \tag{36}$$

Combining this equation with (32) yields

$$-\frac{U_3(t)}{U_1(t)} = \eta z_t h_t^{\eta-1} \frac{[1 + \xi(k + \eta^{-1}v'(h_t)h_t/v(h_t))]}{[1 + \xi(k + \eta^{-1}v''(h_t)h_t/v'(h_t) + \eta^{-1})]}$$

Because for general preference specifications $\frac{[1+\xi(k+\eta^{-1}v'(h_t)h_t/v(h_t))]}{[1+\xi(k+\eta^{-1}v''(h_t)h_t/v'(h_t)+\eta^{-1})]}$ is different from unity, uniform taxation fails. ■

The intuition for the failure of uniform taxation is the same as the one developed in section 2 for the simple two-period economy. When the Ramsey reform is anticipated, then the Ramsey planner wants to follow a policy that induces private agents to bring a lot of wealth into the state of the world in which the Ramsey planner is in power. Uniform taxation, however, acts as a lump-sum tax on initial wealth and thus discourages agents to bring a large stock of wealth into the Ramsey state. As a result uniform taxation is no longer the optimal policy.

Finally, it is worth noting that except for the intertemporal implementability constraint, government purchases, g_t, do not enter in any of the Ramsey planner's optimality conditions. It follows immediately that the realization of the government spending shock in period t does not affect that period's real allocation. In other words, in the Ramsey solution, c_t^m, c_t^c, and h_t are uncorrelated with g_t. Moreover, it follows from the decentralized version of the primal form of the competitive equilibrium that if the real allocation is uncorrelated with government purchases, then so are the consumption and income tax rates, ϕ_t and τ_t (see equilibrium conditions (17), (19), and (29)). It is also clear from the first-order conditions of the Ramsey problem that in each period $t \geq 0$, the real allocation, c_t^m, c_t^c, and h_t, depends only on the realization of the productivity shock in that period, z_t, and on the Lagrange multiplier associated with the implementability constraint, which in turn depends only on the initial state of the economy (g_0,z_0,A_0/e_0). We highlight these results in the following proposition:

Proposition 4 (Neutrality of Government Purchases Shocks) *In the Ramsey solution, c_t^m, c_t^c, h_t, τ_t, and ϕ_t are uncorrelated with government purchases, g_t. The stochastic processes followed by all of these variables inherit the properties of the stochastic process of the productivity shock z_t.*

The intuition behind this result is straightforward. Innovations to government purchases have pure wealth effects on households. However, given our maintained assumption of complete international asset markets, households can fully insure this source of risk. It follows that given tax rates, households' demands for consumption and leisure are unaffected by government purchases shocks. This implies that the size and elasticity of the consumption and income tax bases are unchanged in response to government spending shocks. It is then not surprising that the Ramsey planner chooses to keep tax rates unchanged. One may then wonder how the planner finances the government purchases shock. The Ramsey government finances all innovations to government purchases with lump-sum (i.e., nondistorting) taxes on private holdings of nominal public debt via surprise changes in the price level. It follows also that innovations in government spending are associated with depreciations of the domestic currency. This is because by PPP the rate of depreciation of the domestic currency is proportional to domestic inflation.[7]

By contrast, shocks to the level of labor productivity alter the Ramsey allocation. The reason is that technology shocks affect the equilibrium wage rate and variations in the real wage rate cannot be insured against to the extend that only a domestic resident can earn the domestic wage rate. That is, to take advantage of, say, a higher wage rate, the domestic resident has to work more. No one else can do it for him. As a consequence, hours of work will depend on the level of technology.

4.2 Dynamic Properties of Ramsey Allocations

We now turn to the characterization of the dynamic properties of the Ramsey allocation. The optimality conditions of the Ramsey problem are too complex to be solved analytically. However, the exact numerical solution can be obtained. Before explaining our numerical solution algorithm, we present a baseline calibration of the model.

4.2.1 Calibration

The time unit is meant to be one year. We assume that prior to the implementation of the Ramsey policy, the economy is in a steady-state competitive equilibrium. To pin down the deep structural parameters of the model, we assign numerical values to a number of long-run relations. These values reflect key observed long-run features of industrialized open economies. In the pre-Ramsey competitive equilibrium, the average real interest rate is 4

[7]See equation (28) and recall that $R_t = 1$ under the Ramsey plan, that P_t^* is assumed to grow at a constant rate, and that R_t^* is assumed to be constant.

percent and the average inflation rate is also 4 percent. Thus, the nominal interest rate is 8 percent. The public-debt-to-GDP ratio is 40 percent and GDP money velocity is 5.8. In this steady state the consumption tax rate and the income tax rate are restricted to be the same. The share of government purchases to in GDP is assumed to be 20 percent. Households devote on average 20 percent of their time to work and the labor share in GDP is 80 percent. In the steady-state the country has a trade balance surplus of 2 percent of GDP. We assume that there no external shocks. That is, both r_t^* and P_t^* are assumed to be nonstochastic. In addition, we assume that the foreign inflation rate and nominal interest rate equal their domestic counterparts of 4 and 8 percent, respectively. This means that $r^* = 1/R^* = 1/1.08$ and that $\pi^* = P_t^*/P_{t-1}^* = 1.04$. Because in the domestic economy the discount factor must satisfy $\beta = \pi/R$, it follows that x^* is constant. We normalize x^* to unity.

We assume that the period utility function takes the form

$$U(c^m, c^c, h) = \frac{\left[(c^m)^{\alpha_m} (c^c)^{\alpha_c} (1-h)^{1-\alpha_m-\alpha_c}\right]^\gamma}{\gamma} \tag{37}$$

The parameter γ is assigned a value so that the intertemporal elasticity of substitution, $1/(1-\gamma)$, is equal to 1/3, a value that falls within the range typically used in equilibrium business cycle studies.

We assume that government purchases and productivity shocks follow independent two-state Markov processes. Specifically, z_t can take on the values $z^h = 1+\Delta^z$ or $z^l = 1-\Delta^z$ with $\Delta^z > 0$ and transition probabilities defined by the parameter $\phi^z \equiv \text{Prob}(z_{t+1} = z^i | z_t = z^i)$, $i = h, l$. We assume that z_t has a standard deviation of 0.04 and a first order serial correlation of 0.82. Similarly, g_t takes on the values $g^h = g + \Delta^g$ and $g^l = g - \Delta^g$, with $\Delta^g > 0$ and transition probabilities described by the parameter $\phi^g \equiv \text{Prob}(g_{t+1} = g^i | g_t = g^i)$, $i = h, l$. We assume that on average g_t is 20 percent of GDP and that g_t has a standard deviation of 0.00382 and a first-order serial correlation of 0.9.

The above restrictions imply the deep structural parameter values shown in table 1. A detailed derivation of how the structural parameters are identified can be found in appendix B.

4.2.2 Exact Numerical Solution Method

Using the particular functional form of the period utility function given in equation (37), optimality conditions (32) and (33) can be solved for c_t^m and c_t^c as functions of h_t, x_t^*, and ξ to obtain

$$c_t^m = c^m(h_t, x_t^*; \xi) \equiv \left\{ \left(\frac{\alpha_m}{\alpha_c}\right)^{\alpha_c \gamma} \frac{\xi \theta x_t^*}{\alpha_m v(h_t)[1 + \xi(k + \eta^{-1} v'(h_t) h_t / v(h_t))]} \right\}^{1/[(\alpha_m+\alpha_c)\gamma - 1]}$$

and

$$c_t^c = c^c(h_t, x_t^*; \xi) \equiv \frac{\alpha_c}{\alpha_m} c^m(h_t, x_t^*; \xi)$$

Using $c^m(h_t, x_t^*; \xi)$ and $c^c(h_t, x_t^*; \xi)$ to eliminate c_t^m and c_t^c from optimality condition (34) one obtains an expression of the form

$$H(h_t, z_t, x_t^*; \xi) = 0.$$

Table 1: Calibration

Parameter	Value	Description
β	0.9615	Subjective discount factor
α_m	0.0504	Preference parameter
α_c	0.1885	Preference parameter
γ	-2	Preference parameter
η	0.8	Technology parameter
θ	3.6560	Parameter governing domestic private wealth
A_0/e_0	0.1705	Initial public liabilities in foreign currency
R^*	1.08	Foreign gross nominal interest rate
x^*	1	External shock
g^h	0.0590	High realization of government consumption
g^l	0.0514	Low realization of government consumption
ϕ^g	0.95	Prob$(g_{t+1} = g^i \vert g_t = g^i)$, $i = h, l$
z^h	1.04	High realization of productivity shock
z^l	0.96	Low realization of productivity shock
ϕ^z	0.91	Prob$(z_{t+1} = z^i \vert z_t = z^i)$, $i = h, l$

Note: The time unit is one year.

Given values for z_t, x_t^*, and ξ, this expression can be solved numerically for h_t using any standard nonlinear equation solver. Recall the productivity shock z_t is assumed to take on two values, z^h and z^l, and that because we are assuming no external shocks, x_t^* is constant. Guess a value for ξ. Then solving the above equation for z^h and z^l one obtains two values for hours, $h^h(\xi)$ and $h^l(\xi)$. In turn, using the functions $c^m(h, x^*; \xi)$ and $c^c(h, x^*; \xi)$, yields two values for consumption of cash and credit goods, $c^{mh}(\xi)$, $c^{ml}(\xi)$, $c^{ch}(\xi)$, $c^{cl}(\xi)$. Now consider the optimality condition (35). Note that the expression $U_1(t)c_t^m + U_2(t)c_t^c + \eta^{-1}U_3(t)h_t + \theta x^*(z_t h_t^\eta - g_t - c_t^m - c_t^c)$ can be written as $x(z_t, g_t; \xi)$. Define the vector $X(\xi)$ as

$$X(\xi) = \begin{bmatrix} x(z^h, g^h; \xi) \\ x(z^l, g^h; \xi) \\ x(z^h, g^l; \xi) \\ x(z^l, g^l; \xi) \end{bmatrix}.$$

Now construct the vector

$$Y(\xi) = (I - \beta\Pi)^{-1}X(\xi),$$

where Π is the 4×4 transition probability matrix of the state vector $[z_t; g_t]'$. If the state of the world in period 0 is i ($i = 1, 2, 3, 4$), then ξ is the solution to optimality condition (35), which can be written as

$$\theta\frac{A_0}{e_0} = Y^i(\xi),$$

where Y^i denotes the i-th element of the vector Y. This is a nonlinear equation, which can be solved using standard numerical packages. In this way one obtains one value of ξ for each state in period zero.

4.2.3 Unconditional Moments and Impulse Responses

Table 2 displays a number of unconditional moments of key endogenous variables.[8] The

Table 2: Dynamic properties of the Ramsey allocation in the small open economy

Variable	Mean	Std. Dev.	Auto. corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
τ	55.1	0.373	0.82	1	0	1
ϕ	8.6	1.32	0.82	-1	0	-1
π	-2.6	11.1	0.0101	-0.394	0.323	-0.394
R	0	0	NaN	NaN	NaN	NaN
ϵ	-6.52	10.7	0.0101	-0.394	0.323	-0.394
y	0.0832	0.0113	0.82	1	0	1
h	0.0445	0.00537	0.82	1	0	1
c	0.185	0.00259	0.82	1	0	1
tb/y	-194	37.1	0.821	0.992	-0.126	0.992
a	0.17	0.035	0.873	0.586	-0.81	0.586
g	0.0552	0.00382	0.9	0	1	0
z	1	0.04	0.82	1	0	1

Note. τ, ϕ, π, R, ϵ, and tb/y are expressed in percentage points, and y, h, c, a, g, and z in levels.

Ramsey equilibrium looks strikingly different from the pre-Ramsey competitive equilibrium. The planner relies heavily on income taxes. The average income tax rate is 55 percent, whereas in the pre-Ramsey economy it is 12 percent. The Ramsey planner set the consumption tax rate at a much lower level than the income tax rate, 8.6 percent. Thus, the pre-Ramsey tax structure, with income and consumption tax rates restricted to be equal to each other is far from optimal. In addition, the Ramsey planner gives up completely the use of seignorage as a source of revenue, since, as shown above, the nominal interest rate is zero at all times. By contrast, in the pre-Ramsey economy, the interest rate is on average 8 percent.

The differences in tax structures have strong implications for the long-run levels of consumption and labor effort. In the pre-Ramsey economy workers dedicate 20 percent of their time to work. By contrast, the Ramsey economy is a leisure society. Workers allocate only 4.5 percent of their time to remunerated work. As a result, output is much lower in the Ramsey economy than in the pre-Ramsey steady state: 0.08 versus 0.28. At the same time, consumption falls slightly as the economy moves from the pre-Ramsey to the Ramsey policy

[8]The moments are unconditional with respect to the initial exogenous state, (z_0,g_0), but not with respect to the initial level of government liabilities A_0/e_0, which is fixed as described above.

from 0.215 to 0.185. The combination of a pronounced decline in output and a modest reduction in consumption results in a drastic trade balance deterioration under the Ramsey policy. The trade balance deficit is 200 percent of GDP in the Ramsey economy, while in the pre-Ramsey economy it displays a moderate surplus of 2 percent of GDP. The Ramsey economy is able to support a trade deficit of this magnitude because private agents are insured against the possibility of a Ramsey-type policy reform.

One may wonder whether the optimal tax structure in the open economy does resemble that of a closed economy with capital accumulation. After all, a country's net foreign asset position can be thought of as a capital stock. The answer to this question is that the tax structures are very different. In the closed economy with capital accumulation and consumption and labor income taxes, the Ramsey tax structure consists of taxing consumption and subsidizing labor effort at the same rate. Moreover the consumption tax/labor subsidy is typically well in excess of 100 percent (Coleman, 2000). Such a fiscal policy taxes all goods (consumption and leisure) at the same rate and is equivalent to a pure levy on the initial stock of physical capital. The tax structure we obtain in the open economy is quite different, for consumption is taxed but leisure is subsidized.[9] The reason behind this difference in results is that in the open economy, due to the existence of complete asset markets, the initial net foreign asset position of the economy is not an inelastically supplied factor. Rather, it is a function of the state of the world in period zero. One aspect of the state of the world in period zero is the particular tax policy in place. More specifically, before period zero, agents engage in asset transactions that ensure that their marginal utility of wealth is proportional to that of foreign agents, which is an exogenous variable, at all dates and under all possible contingencies. As a consequence, the country's asset position in period zero is state contingent as well.

Table 2 also shows that the Ramsey government 'front loads' taxes. That is, the benevolent government uses large unexpected changes in the price level as a lump-sum tax on nominal government bond holdings. The standard deviation of inflation is 11 percent. A two-standard error band around the mean of -2.6 percent encompasses inflation rates in the range -24.6 and +19.6 percent. Moreover, inflation is virtually uncorrelated over time, with a serial correlation of 0.01.[10] The fact that in the open economy the government chooses to use the price level to front load taxation when both income and consumption taxes are at the planners disposal stands in stark contrast to what happens in a closed economy. It can be shown that in the closed economy, the Ramsey planner can always choose consumption tax rates in such a way as to support full price stability.

The model also implies a high volatility of the depreciation rate of the domestic currency, $\epsilon_t \equiv e_t/e_{t-1}$. The depreciation rate is a constant fraction of domestic inflation. For reasonable calibrations the factor of proportionality is close to unity. To see this, use PPP

[9]With $\eta < 1$, τ_t is a tax on income rather than labor. However, we have verified that the qualitative features of the Ramsey tax structure are unchanged when η is set at unity, a value at which τ_t becomes a labor income tax rate.

[10]Indeed, $1/\pi_t$ is perfectly serially uncorrelated. To see this note that (17) implies that $\lambda_t = \beta R_t E_t \lambda_{t+1}/\pi_{t+1}$. Now use equation (29), the fact that $R_t = 1$, and the assumption of no external shocks to get $E_t[1/\pi_{t+1} - 1/\beta] = 0$. It follows that $Cov(1/\pi_t, 1/\pi_{t+1}) = E(1/\pi_t - 1/\beta)E_t(1/\pi_{t+1} - 1/\beta) = 0$.

(equation (28)) to write

$$\epsilon_t = \frac{\pi_t}{\pi_t^*} \frac{R_{t-1}}{R_t} \frac{R_t^*}{R_{t-1}^*}.$$

Because foreign inflation and the foreign interest rate are assumed to be constant, and the Friedman rule holds domestically, we have that

$$\epsilon_t = \frac{\pi_t}{\pi^*}.$$

We set π^* equal to 1.08, a number close to unity. As a result, the volatility of the domestic depreciation rate is approximately equal to that of the domestic inflation rate. A way to delink the behavior of the devaluation rate from that of domestic inflation is to introduce nontraded goods and either sectorial shocks or nominal frictions.

Movements in government spending do not affect the business-cycle properties of the Ramsey real allocation. Specifically, if the only source of uncertainty was government purchases shocks, then hours, consumption, and output would be constant over time. This is because households are fully insured against government purchases shocks (via international financial markets) and the economy is too small to affect world prices. The government is able to support constant tax rates in response to government purchases shocks because it can fully finance the resulting changes to its budget through unexpected inflation. Therefore, the price level is highly sensitive to government purchases shocks, as the capital levy embodied in surprised inflation serves as the sole absorber of such innovations to the fiscal budget.[11]

Figure 1 displays the impulse response of the Ramsey economy to a one-standard-deviation increase in government purchases. In response to a positive and persistent government purchases shock real government liabilities decline and converge to their long-run level gradually. The initial decline is the result of the surprise inflation engineered by the government, whose liabilities are all in nominal terms. Real government liabilities fall by much more than what is needed to finance the current increase in g_t. This is because the increase in g_t is persistent and therefore the government needs to collect revenues to finance not only the current increase in g_t but also the future expected increases.

A temporary increase in government purchases leads to a trade balance deterioration. In response to the increase in government purchases, the economy uses the trade balance to smooth out private consumption. This behavior illustrates the general principle in international finance that the trade balance is used to finance temporary endowment shocks.

Figure 2 depicts the response of the Ramsey economy to a one-standard-deviation increase in the productivity shock, z_t. Unlike in the case of government purchases shocks, productivity shocks induce movements in the real allocation and tax rates. It follows that all of the variation in the real allocation and tax rates is due to productivity shocks. Real variables and tax rates inherit the properties of the stochastic process of the technology shock. However, as in the closed economy with income taxes only, the income tax rate is remarkably smooth over the business cycle. The standard deviation of τ_t is only 0.4. A two-standard-deviation band around the mean tax rate of 55 percent lies within the interval 54 to 56 percent. On

[11]The fact that the correlation between π_t and g_t is low, 0.3, is not at odds with the intuition given above. For it is the correlation between π_t and innovations in g_t that should be close to one.

the other hand, the consumption tax rate is four times as volatile as the income tax rate, with a two-standard-deviation band spanning the range 6 to 11.2 percent.[12]

In response to a positive technology shock, the planner finds it optimal to increase the labor income tax rate and to decrease the consumption tax rate. The increase in the labor income tax rate is a consequence of the fact that periods of high productivity are particularly good times to tax income, as the tax base is larger at any level of effort. The planner therefore substitutes income tax revenue for consumption tax revenue. This substitution allows the government to keep the distortion of the leisure/consumption margin, which is given by $(1-\tau_t)/(1+\phi_t)$, relatively constant.

5 An Economy With Income Taxes Only

In closed economy environments, the ability of the government to levy income taxes suffices to make the Friedman rule part of the optimal policy mix. We show in this section that in the open economy environment laid out above this is not the case. Our analysis focuses on how the absence of a consumption tax instrument affects the quantitative features of the economy under optimal fiscal and monetary policy.

The competitive equilibrium of an economy with income taxes only can be defined directly by imposing $\phi_t = 0$ in the definition of the competitive equilibrium in the economy with both income and consumption taxes given in definition 1. Formally, we have

Definition 2 (Competitive Equilibrium With Income Taxes Only) *A competitive equilibrium with income taxes only is a set of stochastic processes* $\{P_t, e_t, r_{t+1}, M_t, A_{t+1}, c_t^c, c_t^m, h_t, q_t, w_t, y_t, \lambda_t\}$ *satisfying (11), (14), (18), (21), (24), (26) holding with equality, (27), (28), (29), and*

$$M_t \geq P_t c_t^m \tag{38}$$

$$U_1(c_t^m, c_t^c, h_t) = \lambda_t \tag{39}$$

$$-\frac{U_3(c_t^m, c_t^c, h_t)}{U_1(c_t^m, c_t^c, h_t)} = \frac{(1-\tau_t)w_t}{R_t} \tag{40}$$

$$(R_t - 1)(M_t - P_t c_t^m) = 0 \tag{41}$$

$$A_{t+1} = R_t A_t + M_t(1-R_t) + P_t(g_t - \tau_t z_t h_t^\eta) \tag{42}$$

given the exogenous stochastic processes $\{g_t, z_t, P_t^*, r_{t+1}^*, q_t^*\}$*, policy sequences* τ_t *and* $R_t \geq 1$*, and the initial condition* A_0*.*

The primal form of the competitive equilibrium is significantly different that of the economy with both consumption and income taxes. In particular, the primal form of the competitive equilibrium with income taxes only includes one additional restriction tieing the marginal utility of consumption of credit goods to an exogenous foreign variable at each date and state. The following proposition presents a formal statement of the primal form.

[12] The volatility of the consumption tax rate depends on the particular parameterization of preferences. For example, as γ approaches zero, preferences become log-linear in consumption and leisure, and the volatility of ϕ_t converges to zero.

Proposition 5 (Primal Form in an Economy With Income Taxes Only) *Given initial conditions A_0 and e_0, the positive parameter θ, and exogenous stochastic processes for $\{z_t, g_t, P_t^*, r_{t+1}^*\}$, contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (31), and*

$$U_2(c_t^m, c_t^c, h_t) = \theta x_t^* \tag{43}$$

are the same as those satisfying $U_1(t)/U_2(t) \geq 1$, (11), (14), (18), (21), (24), (26) holding with equality, (27), (28), (29), (38), (39), (40), (41), and (42), where $x_t^* \equiv P_t^* q_t^*/(\beta^t R_t^*)$, $q_t^* = r_1^* \ldots r_t^*$, and $R_t^* = 1/E_t r_{t+1}^*$.

Proof: See appendix A. ■

The Ramsey planner chooses stochastic processes c_t^m, c_t^c, and h_t so as to maximize (9) subject to $U_1(t)/U_2(t) \geq 1$, (31), and (43), taking as given the initial condition A_0/e_0, and the processes x_t^*, z_t, and g_t. It follows immediately from the first-order conditions of the Ramsey problem that, as in the economy with two tax instruments, the real allocation, c_t^m, c_t^c, and h_t, depends only on the current realization of the technology shock z_t and the Lagrange multiplier associated with the implementability constraint. This means, in particular, that the real allocation, the income tax rate, and the nominal interest rate are all uncorrelated with government purchases. It also implies that innovations in government purchases are completely financed by surprise changes in the price level. The following proposition summarizes this result.

Proposition 6 (Neutrality of Government Purchases Shocks in the Economy with Income Taxes Only) *In the economy with income taxes only, under the Ramsey solution, c_t^m, c_t^c, h_t, τ_t, and R_t are uncorrelated with government purchases, g_t. The stochastic processes followed by all of these variables inherit the properties of the stochastic process of the productivity shock z_t.*

Table 3 displays unconditional moments in the economy with income taxes only. In construction the table, the calibration of the structural parameters is identical to that used for the economy with both consumption and income taxes, which is summarized in table 1. The model admits an exact numerical solution method similar in nature to the one applied to the economy with two tax instruments described in section 4.2.2.

A striking feature of the economy with income taxes only is that the Friedman rule fails dramatically.[13] The average nominal interest rate is 30 percent per year. A positive nominal interest rate allows the Ramsey government to indirectly tax consumption. In particular, purchases of cash goods are subject to a cash-in-advance constraint. Thus, by rasing the opportunity cost of holding cash, the government is implicitly collecting revenue from consumption expenditures.

The planner does not substitute seignorage income one for one for consumption tax revenue. To see this note that in the economy with income and consumption taxes, the

[13] This result is related to independent work by Cunha (2002). Cunha studies Ramsey policies in a small, open, incomplete markets economy with traded and nontraded goods in which the Ramsey planner is assumed to have access only to a non-state-contingent labor income tax. Cunha shows that the Friedman rule is not optimal and conjectures that it would be optimal if the planner could tax consumption of tradables and nontradables at different rates.

Table 3: Dynamic Properties of the Ramsey Allocation in the Economy with Income Taxes Only ($\phi_t = 0$)

Variable	Mean	Std. Dev.	Auto. corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
τ	52.3	0.383	0.82	1	0	1
π	27.2	20.2	0.0495	-0.517	0.263	-0.517
R	29.3	1.48	0.82	-1	0	-1
ϵ	22	18.7	0.0606	-0.523	0.272	-0.523
y	0.0996	0.0121	0.82	1	0	1
h	0.0558	0.0057	0.82	1	0	1
c	0.192	0.00142	0.82	1	0	1
tb/y	-151	29.4	0.821	0.991	-0.132	0.991
a	0.131	0.0332	0.855	0.75	-0.661	0.75
g	0.0552	0.00382	0.9	0	1	0
z	1	0.04	0.82	1	0	1

Note. τ, ϕ, π, R, ϵ, and tb/y are expressed in percentage points, and y, h, c, a, g, and z in levels.

share of consumption of cash goods in total consumption is about 20 percent (and equal to $\alpha_m/(\alpha_m+\alpha_c)$. Thus, since the nominal interest rate is a tax only on cash goods, a one-for-one substitution of seignorage for consumption tax revenue would require setting the nominal interest rate roughly five times as high as the consumption tax rate in the economy with consumption taxes. This would require a nominal interest rate of about 45 percent. Instead, the planner settles for a lower nominal interest rate of 30 percent. The reason why the planner chooses not to make up the entire loss of consumption tax revenue with seignorage revenue is that a positive nominal interest rate is a less efficient way of taxing consumption than an explicit uniform consumption tax. This is because a positive nominal interest rate distorts not only the consumption leisure margin but also the cash-credit margin.

The non optimality of the Friedman rule depends on the particular parameterization of the model. In particular, when in the economy with income and consumption taxes the optimal consumption tax rate is positive, then in the economy with income taxes only the nominal interest rate is positive. On the other hand, for parameterizations for which the optimal consumption tax is negative in the economy where both tax instruments are available, the Friedman rule holds in the economy without consumption taxes. In this case the planner would like to set the nominal interest rate at a negative value so as to subsidize consumption. However this is not possible in a competitive equilibrium because it would create a pure arbitrage opportunity.[14]

The behavior of income tax rates is virtually unchanged by the disappearance of the consumption tax instrument. The mean income tax rate is 52 percent in the economy with income taxes only. This value is slightly less than the mean income tax rate of 55 percent

[14] For example, when η is lowered to 0.5, then in the economy with both tax instruments the optimal consumption tax is -17.2 percent, and in the economy with income taxes only the Friedman rule is optimal.

implied by the model with both income and consumption taxes. When the government loses the consumption tax instrument, income from regular taxation (i.e., income and consumption tax revenue) falls by about 17 percent. This shortfall in revenues is fully taken up by inflation taxes. Both forecast able and unforcastable changes in the price level become larger. The average inflation rate increases from -2.6 percent in the economy with two tax instruments to 27 percent in the economy with income taxes only, and the standard deviation of inflation increases from 11 to 20 percentage points.

Because the nominal interest rate varies over the business cycle, the model displays deviations of PPP from its mean. The coefficient of variation of PPP (i.e., std(PPP/mean(PPP)) is 3.3 percent.[15] It is worth noting that deviations from PPP occur in spite of the fact that there is a single traded good and no barriers to trade. The key factors behind the implied deviations from PPP are that trades are made in the buyer's currency and that the nominal interest rate is time varying. Absence of either one of these two factors implies the reemergence of PPP.[16]

6 An Economy With Consumption Taxes Only

In the economy with income and consumption taxes analyzed in section 3, we found that the Friedman rule holds under the Ramsey policy. On the other hand, deviations from the Friedman rule are large in the economy with income taxes only. A natural question is whether the availability of consumption taxes alone suffices to make the Friedman rule optimal, or whether it is the combination of both tax instruments that makes it optimal. Two other striking characteristics of optimal policy in both the economy with income and consumption taxes and the economy with income taxes only are the large trade balance deficits and the use of highly volatile and serially uncorrelated price changes as a way to tax nominal government debt in a lump-sum fashion. In this section, we investigate the robustness of these characteristics of the Ramsey policy in an open economy where the government has access only to consumption taxes.

The equilibrium conditions in the economy with consumption taxes only are the same as those associated with the economy where both consumption and income taxes can be levied with the exception that τ_t must be zero at all times and under all contingencies. Specifically, the labor supply schedule (19) and the government's sequential budget constraint (45) now become

$$-\frac{U_3(c_t^m, c_t^c, h_t)}{U_1(c_t^m, c_t^c, h_t)} = \frac{w_t}{R_t(1+\phi_t)} \tag{44}$$

$$A_{t+1} = R_t A_t + M_t(1-R_t) + P_t[g_t - \phi_t(c_t^m + c_t^c)] \tag{45}$$

We then define a competitive equilibrium as follows:

[15]This follows from the fact that std(PPP/mean(PPP))=std(R_t/mean(R)) $\times 100 = 0.0618/1.875 \times 100$, where R_t is the gross domestic nominal interest rate.

[16]It is immediate to see that if one assumes that transactions are carried out in the seller's currency, then, because shoppers acquire the desired amounts of domestic and foreign monies in the same period in which they plan to spend it, the condition $P_t = e_t P_t^*$ must hold. At the same time, when purchases are made in the buyer's currency but the domestic and foreign nominal interests are constant, then the condition $P_t = e_t P_t^* R_t / R_t^*$ collapses to $P_t = e_t P_t^* R/R^*$. Thus, the volatility of $P_t/(e_t P_t^*)$ is zero.

Definition 3 (Competitive Equilibrium With Consumption Taxes Only) *A competitive equilibrium in the economy with consumption taxes only is a set of stochastic processes* $\{P_t, e_t, r_{t+1}, M_t,\ A_{t+1}, c_t^c,\ c_t^m, h_t, q_t,\ w_t, y_t, \lambda_t\}$ *satisfying (10), (11), (14), (17), (18), (20), (21), (24), (26) holding with equality, (27), (28), (29), (44), and (45) given the exogenous stochastic processes* $\{g_t, z_t, P_t^*, r_{t+1}^*, q_t^*\}$, *policy sequences* ϕ_t *and* $R_t \geq 1$, *and the initial condition* A_0.

The following proposition provides the primal form of this competitive equilibrium.

Proposition 7 (Primal Form in an Economy with Consumption Taxes Only) *Given initial conditions* A_0 *and* e_0, *the positive parameter* θ, *and exogenous stochastic processes for* $\{z_t, g_t, P_t^*, r_{t+1}^*\}$, *contingent plans* $\{c_t^m, c_t^c, h_t\}$ *satisfying* $U_1(t)/U_2(t) \geq 1$, *(31), and*

$$-U_3(t) = \eta z_t h_t^{\eta-1} \theta x_t^* \tag{46}$$

are the same as those satisfying $U_1(t)/U_2(t) \geq 1$, *(10) (11), (14), (17), (18), (20), (21), (24), (26) holding with equality, (27), (28), (29), (44), and (45), where* $x_t^* \equiv P_t^* q_t^* /(\beta^t R_t^*)$, $q_t^* = r_1^* \dots r_t^*$, *and* $R_t^* = 1/(E_t r_{t+1}^*)$.

Proof: See appendix A. ■

The Ramsey problem consists in maximizing (9) subject to $U_1(t)/U_2(t) \geq 1$, (31), and (46), taking as given A_0/e_0, and the exogenous processes x_t^*, g_t, and z_t.

Consider a modified version of the Ramsey problem where the restriction $U_1(t)/U_2(t) \geq 1$ is dropped. Under the functional form for the period utility function assumed in proposition 2, Ramsey problem's first-order conditions are (31), (46), and

$$f_1[v(1+\xi k) + \xi v' h/\eta + \mu v'] = \xi \theta x^*$$

$$f_2[v(1+\xi k) + \xi v' h/\eta + \mu v'] = \xi \theta x^*$$

$$f[v' + v'\xi(\eta^{-1} + k) + \xi v'' h/\eta] + \xi \theta x^* \eta z h^{\eta-1} = -\mu[f v'' + \eta(\eta-1) z h^{\eta-2} \theta x^*],$$

where ξ and μ_t denote the Lagrange multipliers on (31), (46), respectively, and k is the degree of homogeneity of f. Dividing the first of these expressions by the second yields $f_1/f_2 = 1$. It follows immediately that $U_1(t)/U_2(t) = 1$. Thus, the unconstrained solution satisfies the restriction $U_1(t)/U_2(t) \geq 1$. Moreover, by equation (18) the nominal interest rate associated with the Ramsey allocation is zero at all dates and states, which is to say that the Friedman rule is optimal. We summarize this result in the following proposition.

Proposition 8 (Optimality of the Friedman Rule in the Open Economy When Only Consumption Taxes are Available) *Assume that the single-period utility function is strictly increasing in* c^m *and* c^c, *strictly decreasing in* h, *and concave. Assume further that* $U(c^m, c^c, h) = f(c^m, c^c)v(h)$, *where* f *is homogeneous of degree* k. *Then, the Friedman rule is Ramsey optimal in the economy with consumption taxes only.*

The intuition for the optimality of the Friedman rule in the economy with consumption taxes only is the following. As explained earlier, the nominal interest rate is an implicit tax on the consumption of cash goods. However, compared to a uniform tax on total consumption, a positive nominal interest rate is an inferior fiscal instrument. For it distorts an additional margin, namely, the cash-credit consumption margin. Thus, the availability of an explicit uniform tax on consumption of cash and credit goods makes the benevolent government give up completely the use of a positive nominal interest rate for fiscal purposes.

Simple inspection of the first-order conditions of the Ramsey problem reveals that the real allocation in period t is independent of the realization of government purchases in that period. As in the economies with consumption and income taxes and income taxes only, this is a consequence of the fact that private households can fully insure against government purchases shocks. It follows that : (1) Tax rates are uncorrelated with government purchases; (2) All innovations in government purchases are financed through unexpected changes in the price level; and (3) fluctuations in the real allocation are due exclusively to variations in the level of technology, z_t.[17] We summarize these findings in the next proposition.

Proposition 9 (Neutrality of Government Purchases Shocks in the Economy with Consumption Taxes Only) *In the Ramsey solution, c_t^m, c_t^c, h_t, and ϕ_t are uncorrelated with government purchases, g_t. The stochastic processes followed by all of these variables inherit the properties of the stochastic process of the productivity shock z_t.*

Table 4 presents unconditional moments of selected macroeconomic and policy variables

Table 4: Dynamic Properties of the Ramsey Allocation in the Economy with Consumption Taxes Only($\tau_t = 0$)

Variable	Mean	Std. Dev.	Auto. Corr.	Corr(x,y)	Corr(x,g)	Corr(x,z)
ϕ	30.6	1.52	0.82	1	0	1
π	-2.88	9.8	0.00742	-0.345	0.351	-0.345
R	0	0	NaN	NaN	NaN	NaN
ϵ	-6.79	9.41	0.00742	-0.345	0.351	-0.345
y	0.301	0.0232	0.82	1	0	1
h	0.223	0.0103	0.82	1	0	1
c	0.202	0.00118	0.82	1	0	1
tb/y	14.2	6.35	0.823	0.98	-0.201	0.98
a	0.17	0.0326	0.88	0.495	-0.869	0.495
g	0.0552	0.00382	0.9	0	1	0
z	1	0.04	0.82	1	0	1

Note. τ, ϕ, π, R, ϵ, and tb/y are expressed in percentage points, and y, h, c, a, g, and z in levels.

of the economy with consumption taxes only. The calibration and solution techniques are

[17]It can be shown that in the special case of a linearly homogeneous production technology, $\eta = 1$, the consumption tax rate is constant across dates and states. In this case, innovations in both government spending and productivity are financed via surprise inflation.

as explained in section 4. In the case with income taxes only, the lack of one regular tax instrument was in part made up by using the inflation tax. Here this does not happen. All lost income tax revenue is replace with consumption tax revenue. This is reflected in an increase in the consumption tax rate from 9 percent to 31 prevent when the Ramsey planner is forced to give up the income tax instrument.

Without the income tax (i.e., without subsidies to leisure), households increase their labor supply by almost a factor of 5 relative to the case in which both income and consumption taxes are levied optimally. Consequently, average domestic output increases by about 400 percent. Consumption also increases but by much less, roughly 10 percent. The trade balance therefore experiences a dramatic reversal from a deficit of 200 percent of GDP in the economy with two taxes to a surplus of 14 percent of GDP in the economy with consumption taxes only.

As in the economies with income and consumption taxes or income taxes only, the Ramsey planner uses the inflation rate as a lump-sum tax on private sector's holdings of nominal government liabilities. This reflected in the fact that price level changes are highly volatile and virtually serially uncorrelated. The standard deviation of inflation is 10 percentage points and its serial correlation is 0.007.

7 Incomplete Asset Markets

It is a well-known result that in the closed economy neoclassical model (i.e., in closed, real economies with capital accumulation), when the government has access to both consumption and income taxes, the Ramsey allocation is Pareto optimal. Moreover, the policy that implements the Ramsey plan features a constant consumption tax rate and a constant labor subsidy equal in size to the consumption tax (see, e.g., Coleman, 2000). One can show that a similar result obtains in monetary closed economies.

Proposition 10 (Ramsey Allocation in the Closed Economy With Income and Consumption Taxes) *Consider a closed-economy version of the monetary open-economy model developed in section 3. Assume that the Ramsey planner has access to consumption and income taxes, then the Ramsey allocation is Pareto optimal. The policy associated with the Ramsey plan satisfies $R_t = 1$ and $\phi_t = -\tau_t$ for all t. In addition, ϕ_t and τ_t can be taken to be constant.*

Proof: See Appendix C. ■

Proposition 10 states that it is optimal in the closed economy for the utilitarian government to tax all goods, that is, consumption and leisure, at a uniform rate.[18] This property of the Ramsey policy in closed economies stands in stark contrast to what is optimal in the open economy. In section 4 we found that not only is it not optimal for the Ramsey planner to tax all goods uniformly but instead under the optimal policy some goods are taxed while

[18]This result depends on the assumption that ϕ_0 is a choice variable of the Ramsey planner. If one assumes that the planner must take ϕ_0 as given, then the Ramsey real allocation ceases to be Pareto optimal, and the associated Ramsey policy features a path for consumption tax rates different from that of labor subsidies. For a formal treatment of this issue, see proposition 13 in appendix C and the discussion below it.

others are subsidized. In the calibrated open economy we study in section 4, consumption is taxed at an average rate of 9 percent whereas leisure is subsidized at a 55 percent rate. In addition, the constancy of optimal tax rates over time fails to obtain. For example, in our calibrated economy the standard deviation of the consumption tax rate is 1.3 percentage points.

To explain these different characteristics of optimal policy in the closed and open economies, it is crucial to understand the role played by international financial markets. In the closed economy, a uniform tax on consumption and a subsidy on labor of equal magnitude represents a tax on initial private wealth. Because initial private wealth is inelastically supplied, the optimal policy mix just described amounts to a non-distorting or lump-sum levy on initial wealth. This is the reason why the Ramsey plan can achieve the Pareto optimal allocation.

In the open economy, the Ramsey planner does not regard initial private wealth as inelastically supplied. This is because the existence of complete contingent claim markets allows residents of a small open economy to insure against potential policy reform, that is, domestic residents can buy assets whose pay-off is contingent on future tax policies. As a result, in any period, including the period in which the Ramsey reform is implemented, the amount of assets brought into the period by private agents is a function of the tax regime prevailing in that period. Residents of the small open economy are able to insure against policy reform because such reform represent a country specific risk that can be diversified through trade in international asset markets. The Ramsey planner internalizes the policy-dependence of initial wealth of private agents. Because initial wealth is no longer an inelastically supplied factor, it is also no longer optimal to devise a tax scheme aimed at exploiting that fact.

To illustrate how optimal policy in the open economy is affected by the assumed existence of markets for assets with payoffs contingent of the policy regime, in this section we consider a particular kind of market incompleteness by making this type of asset unavailable to domestic residents. This implies that households can no longer insure against potential policy changes.[19] Consequently, asset holdings of the private sector at the beginning of period 0 are independent of the realization of the tax regime to be put into place in that period. The central finding of this section is that in this case the results of the closed economy carry over to the open economy setting. The Ramsey allocation features no wedge between cash and credit goods consumption ($R_t = 1$) nor a wedge between consumption and leisure $\phi_t = -\tau_t$. That is, the classic public finance theorem of uniform taxation of final goods reemerges in the open economy when agents cannot insure against policy reforms. Furthermore, the optimal consumption and income tax rates are constant over time.

The main formal difference between a competitive equilibrium with complete markets and one with the particular type of market incompleteness considered in this section is that equilibrium condition (29) tying the domestic (after-tax) marginal utility of wealth to an exogenous variable is replaced by the intertemporal budget constraint of private households. Formally, using (22) to eliminate Π_t from the household's intertemporal budget constraint

[19] Insurance against domestic government purchases and technology shocks is still available.

(15), yields

$$W_0 + E_0 \sum_{t=0}^{\infty} q_{t+1} P_t (1-\tau_t) z_t h_t^{\eta} = E_0 \sum_{t=0}^{\infty} q_{t+1} \left[P_t (1+\phi_t)(c_t^m + c_t^c) + (R_t - 1) M_t \right], \qquad (47)$$

with W_0 exogenously given. The equations describing the household, firm, and government sectors given in sections 3.1, 3.2, and 3.3, respectively, continue to hold. We then can define a competitive equilibrium for the economy with incomplete markets as follows.

Definition 4 (Competitive Equilibrium Under Incomplete Asset Markets) *A competitive equilibrium is a set of stochastic processes $\{P_t, e_t, r_{t+1}, M_t, A_{t+1}, c_t^c, c_t^m, h_t, q_t, w_t, y_t, \lambda_t\}$ and a positive scalar λ satisfying (10), (11), (14), (16)-(21), (24), (26) holding with equality, (27), (28), (30), and (47), given sequences τ_t, ϕ_t, and $R_t \geq 1$, the exogenous stochastic processes $\{g_t, z_t, P_t^*, r_{t+1}^*\}$, and the initial conditions W_0 and A_0.*

The following proposition presents the primal form of the competitive equilibrium.

Proposition 11 (Primal Form Under Incomplete Asset Markets) *Given initial conditions A_0, W_0, and e_0, and exogenous stochastic processes for $\{z_t, g_t, P_t^*, r_{t+1}^*\}$, contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$ and*

$$\frac{W_0 - A_0}{e_0} = E_0 \sum_{t=0}^{\infty} \beta^t x_t^* [c_t^m + c_t^c + g_t - z_t h_t^{\eta}] \qquad (48)$$

are the same as those satisfying $U_1(t)/U_2(t) \geq 1$, (10), (11), (14), (16)-(21), (24), (26) holding with equality, and (27), (28), (30), and (47), where $x_t^* \equiv P_t^* q_t^* / (\beta^t R_t^*)$, $q_t^* = r_1^* \ldots r_t^*$, and $R_t^* = 1/E_t r_{t+1}^*$.

Proof: See appendix A. ∎

This proposition states that any real allocation satisfying the single intertemporal resource constraint given by equation (48) can be supported as a competitive equilibrium. Equation (48) states that the initial net foreign asset position of the economy, $(W_0 - A_0)/e_0$, must be equal to the present discounted value of current and future expected trade deficits. It is important to note that equation (48) is the only restriction of the unconstrained Pareto optimal problem. By unconstrained Pareto optimality we mean the best resource allocation that can be achieved in this economy in the presence of lump-sum taxes and the absence of the cash-in-advance constraint. Formally, the unconstrained Pareto problem consists in choosing stochastic processes $\{c_t^m, c_t^c, h_t\}$ so as to maximize the utility function (9) subject to (48), given $(W_0 - A_0)/e_0$ and exogenous processes $\{x_t^*, g_t, z_t\}$.

The Ramsey problem consists in choosing c_t^m, c_t^c, and h_t so as to maximize the utility function (9) subject to $U_1(t)/U_2(t) \geq 1$ and (48). Consider a modified Ramsey problem that does not impose the condition $U_1(t)/U_2(t) \geq 1$. That is, consider the problem of maximizing (9) subject to (48). This problem is identical to the unconstrained Pareto problem. Let $\xi > 0$ denote the Lagrange multiplier associated with the implementability constraint (48). Then, the first-order conditions associated with this problem are (48), $U_1(t) = \xi x_t^*$, $U_1(t)/U_2(t) = 1$, and $-U_3(t)/U_1(t) = \eta z_t h_t^{\eta-1}$. Note that the constraint $U_1(t)/U_2(t) \geq 1$ is satisfied. So the solution to the modified Ramsey problem is indeed the solution to the original Ramsey problem. Therefore, the Ramsey allocation coincides with the Pareto optimal allocation. Comparing the optimality conditions of the Ramsey problem with equilibrium conditions (18) and (19), it follows that the Ramsey policy associated with the Ramsey plan features $R_t = 1$ and $\tau_t = -\phi_t$. That is, the Ramsey plan imposes no wedge between cash and credit goods consumption or between consumption and leisure. Furthermore, combining the optimality condition $U_1(t) = \xi x_t^*$ with (17) yields $1 + \phi_t = \xi/(e_0\lambda)$. It follows that the consumption tax rate and the income subsidy are constant across dates and states.

8 Discussion and Conclusion

To a first approximation, all policy reforms are anticipated. In modern, democratic societies, policy-regime switches are preceded by lengthy public debate. A case in point is the process leading to tax harmonization and monetary union in Europe. Therefore, in this paper we argue that the scenario of greatest practical interest for the study of optimal policy is not one in which reform comes as a complete surprise to private agents, but one in which the probability of a policy switch, prior to its implementation, is positive and known to the public. The issue of the real effects of anticipated policy changes has been studied extensively (see, for instance, the seminal contribution of Calvo, 1986). The central focus of this paper is on the consequences of policy anticipation for optimal policy in environments in which agents can insure against policy reforms.

Most of the existing related literature on optimal monetary and fiscal policy has limited attention to closed-economy, representative-agent environments. In this setup it is not possible for agents to insure against policy reform. For policy changes represent aggregate risk, which, even in the presence of complete markets, is undiversifiable. It follows that the characteristics of anticipated Ramsey policy are independent of the degree of market completeness. A more suitable environment to study the effect of the degree of market completeness on anticipated Ramsey reforms is the open economy with complete international financial markets. In this economy, policy changes are country-specific shocks against which agents can hedge by trading in world asset markets.

In the open economy with complete asset markets, the amount of net foreign assets private agents will have in their possession in a particular date and state will depend on the policy regime households expected to be in place at that particular date and state. It follows that governments can affect the level of private wealth brought into the period in which the policy reform is implemented. All other things equal, the Ramsey government would like to announce a policy that induces agents to enter the state in which the Ramsey reform takes place with the largest possible stock of wealth. Because eventually all private wealth is allocated to consumption, the Ramsey planner has an incentive to minimize consumption tax rates. However, at the same time, the benevolent government would like to minimize distortions in relative prices. This objective calls for taxing all goods at the same rate. Because the government must raise revenue to finance its budget, there emerges a policy tradeoff between uniform taxation and low consumption taxation. The resolution of this conflict is a Ramsey policy that departs from the traditional ideal of uniform taxation of all final goods.

One fiscal instrument that is specific to the government of an open economy is the taxation of foreign trade in goods and services. In the above analysis, we have ignore taxation of foreign trade flows. However, it can be shown that all of the results of this paper can be interpreted as those pertaining to an economy where the government has access to income and trade balance taxes. Specifically, suppose that in the economy without taxation of foreign trade τ_t denotes the optimal income tax rate in period t and ϕ_t denotes the optimal consumption tax rate. Then the real allocation of such an economy is identical to an economy with an income tax rate equal to $(\tau_t + \phi_t)/(1 + \phi_t)$ and a tax on imports and a subsidy on exports at the uniform rate equal to ϕ_t.

We close by noting that the recent revival of interest in studying optimal policy in closed economy environments has been accompanied by the emergence of a rapidly growing literature that focuses on optimal monetary policy in small open economies (see, for instance, Galí and Monacelli, 2000, and the references cited therein). In this literature, fiscal policy considerations are typically ignored by assuming, either explicitly or implicitly, that the government has access to lump-sum taxation. An additional central element of this body of work is the assumption that prices are sticky. The basic insight of this line of investigation is that, as in the closed economy, optimal monetary policy should be geared toward fully stabilizing the price level. Specifically, the monetary authority must seek to smooth out prices in those sectors of the economy where price adjustment is costly. This result is in stark contrast with those we obtain in this paper. For we find that under the optimal policy the price level is highly volatile. Two features of our model account for this difference. First, we assume that all prices are flexible. So it is costless for the government to use the price level as a shock absorber by unexpectedly deflating or inflating the real value of its outstanding nominal liabilities in response to innovations in the fiscal budget. Second, we study the joint determination of optimal fiscal and monetary policy in environments where the government fiscal deficits cannot be financed via explicit lump-sum taxes. It is left for future research the analysis of how the presence of complete markets would affect the characteristics of anticipated Ramsey reforms in environments with sluggish price adjustment.

Appendix A

Proof of Proposition 1

We first show that plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (10) (11), (14), (17), (18), (19), (20), (21), (24), (26) holding with equality, (27), (28), (29), and (30), also satisfy $U_1(t)/U_2(t) \geq 1$ and (31). To this end divide (30) by R_t/q_t and use (11) to obtain,

$$E_t q_{t+1} A_{t+1} = q_t A_t + E_t q_{t+1} \left\{ M_t(1-R_t) + P_t[g_t - \tau_t z_t h_t^\eta - \phi_t(c_t^m + c_t^c)] \right\}.$$

Take expectations with respect to information available at time 0 and sum for $t = 0$ to $t = T$. This yields:

$$E_0 q_{T+1} A_{T+1} = q_0 A_0 + E_0 \sum_{t=0}^{T} q_{t+1} \left\{ M_t(1-R_t) + P_t[g_t - \tau_t z_t h_t^\eta - \phi_t(c_t^m + c_t^c)] \right\}.$$

Letting $T \to \infty$ and using (26) holding with equality, we have

$$A_0 = E_0 \sum_{t=0}^{\infty} q_{t+1} P_t \left[\frac{M_t}{P_t}(R_t - 1) + \tau_t z_t h_t^\eta + \phi_t(c_t^c + c_t^m) - g_t \right]$$

By (19) and (24), $(1-\tau_t) z_t h_t^\eta = -(1+\phi_t) R_t U_3(t)/U_1(t) h_t/\eta$; by (20) $(R_t - 1)M_t/P_t = (R_t - 1)(1+\phi_t)c_t^m$; and by (14) $q_{t+1} = q_t r_{t+1}$. Let $tb_t = z_t h_t^\eta - g_t - c_t^m - c_t^c$, then we have

$$\begin{aligned}
A_0 &= E_0 \sum_{t=0}^{\infty} q_t r_{t+1} P_t \left[(R_t - 1)(1+\phi_t)c_t^m + tb_t + (1+\phi_t)(c_t^m + c_t^c) + \frac{U_3(t) R_t (1+\phi_t)}{\eta U_1(t)} h_t \right] \\
&= E_0 \sum_{t=0}^{\infty} q_t r_{t+1} P_t (1+\phi_t) R_t \left[c_t^m + \frac{1}{R_t(1+\phi_t)} tb_t + \frac{1}{R_t} c_t^c + U_3(t)/U_1(t) h_t/\eta \right] \\
&= E_0 \sum_{t=0}^{\infty} \frac{q_t P_t (1+\phi_t)}{U_1(t)} \left[U_1(t) c_t^m + \frac{U_1(t)}{R_t(1+\phi_t)} tb_t + U_2(t) c_t^c + U_3(t) h_t/\eta \right] \\
&= E_0 \sum_{t=0}^{\infty} \frac{e_0}{\theta} \beta^t [U_1(t) c_t^m + U_2(t) c_t^c + U_3(t)/\eta h_t + \theta x_t^* tb_t].
\end{aligned}$$

In the third equality we used (11) and replaced $U_1(t)/R_t$ with (18). For the fourth equality, we combined (17), (27), (28), and (29) to express $p_t q_t (1+\phi_t)/U_1(t)$ as $\beta^t e_0/\theta$ and used (17) and (29) to replace $U_1(t)/(R_t(1+\phi_t))$ with θx_t^*. Note that the fourth equation is (31), which is what we wanted to show.

Next we show that if contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfy $U_1(t)/U_2(t) \geq 1$ and (31), then they also are consistent with (10), (11), (14), (17), (18), (19), (20), (21), (24), (26) holding with equality, (27), (28), (29), and (30) . Let R_t be given by (18), y_t by (21), w_t by (24), λ_t by (29), ϕ_t by (17), τ_t by (19), and P_0 by (28) evaluated at $t = 0$. Choose any process for M_t/P_t satisfying $M_t/P_t = c_t^m(1+\phi_t)$ if $R_t > 1$, and $M_t/P_t \geq c_t^m(1+\phi_t)$ if $R_t = 1$. Then, by construction, (10) and (20) are satisfied. Choose sequences for A_t and P_t sequentially as follows: Choose A_1 so that it satisfies (30) for $t = 0$. Then choose P_1 so that

$$\frac{A_t}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} R_{t+j}^{-1} \left[\frac{M_{t+j}}{P_{t+j}}(R_{t+j}-1) + \tau_{t+j} z_{t+j} h_{t+j}^{\eta} + \phi_{t+j}(c_{t+j}^m + c_{t+j}^c) - g_{t+j} \right], \quad (49)$$

for $t = 1$. Now use (30) to construct A_2. Continuing in this way, one obtains sequences for P_t and A_t. As will become clear shortly, it is useful to show that (49) also holds for $t = 0$. Manipulate the term under the sum in equation (31) as follows:

$$\begin{aligned}
& [U_1 c_t^m + U_2 c_t^c + U_3/\eta h_t + \theta x_t^* t b_t] &= \\
\frac{U_1(t)}{R_t(1+\phi_t)} \left[R_t(1+\phi_t)c_t^m + (1+\phi_t)c_t^c + (1+\phi_t)R_t U_3/U_1 h_t/\eta + \frac{R_t(1+\phi_t)}{U_1(t)}\theta x_t^* t b_t \right] &= \\
\lambda_t R_t^{-1} \left[(1+\phi_t)c_t^m R_t + (1+\phi_t)c_t^c - (1-\tau_t) z_t h_t^{\eta} + t b_t \right] &= \\
\lambda_t R_t^{-1} \left[(1+\phi_t)c_t^m (R_t - 1) + \phi_t(c_t^m + c_t^c) + \tau_t z_t h_t^{\eta} - g_t \right] &= \\
\lambda_t R_t^{-1} \left[M_t/P_t(R_t - 1) + \phi_t(c_t^m + c_t^c) + \tau_t z_t h_t^{\eta} - g_t \right] &
\end{aligned}$$

The first equality uses equations (18); the second uses equations (17), (19), (24), and (29); and the third uses equations (20) and (24). Divide (31) by λ_0 and note that $\lambda_0 = \theta P_0/e_0$. It follows that (49) also holds for $t = 0$. For $t \geq 1$, choose the nominal exchange rate e_t to satisfy (28), and let

$$r_{t+1} = \beta \frac{P_t \lambda_{t+1}}{P_{t+1}\lambda_t}. \quad (50)$$

This expression, together with (28) and (29) implies (27). Combining (49) evaluated at t and $t+1$ one can write

$$\frac{A_t}{P_t} = R_t^{-1} \left[\frac{M_t}{P_t}(R_t - 1) + \tau_t z_t h_t^{\eta} + \phi_t(c_t^m + c_t^c) - g_t \right] + \beta A_{t+1} E_t \frac{\lambda_{t+1}}{P_{t+1}\lambda_t}.$$

Using (30) to eliminate A_t yields

$$R_t^{-1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}.$$

Given our definition of r_{t+1}, this expression implies that (11) holds for all $t \geq 0$. Let q_t be given by (14). Finally, we need to show that (26) holds with equality at all dates and under all contingencies. Multiply (30) by q_{t+1}, apply the E_t operator, and use the fact that $E_t q_{t+1} = q_t E_t r_{t+1} = q_t/R_t$ to get

$$E_t q_{t+1} A_{t+1} = q_t A_t + q_t R_t^{-1} \{ M_t(1 - R_t) + P_t[g_t - \tau_t z_t h_t^{\eta} - \phi_t(c_t^m + c_t^c)] \}.$$

Summing for $j = 0$ to J yields

$$\begin{aligned}
& E_t q_{t+J+1} A_{t+J+1} = \\
q_t A_t + E_t \sum_{j=0}^{J} q_{t+j} R_{t+j}^{-1} P_{t+j} [M_{t+j}/P_{t+j}(1 - R_{t+j}) + (g_{t+j} - \tau_{t+j} z_{t+j} h_{t+j}^{\eta} - \phi_t(c_t^m + c_t^c))]. &
\end{aligned}$$

Now take the limit for $J \to \infty$ and use the fact that, by (14) and (50), $q_{t+j}P_{t+j} = q_t P_t \beta^j \lambda_{t+j}/\lambda_t$ to get

$$\lim_{J\to\infty} E_t q_{t+J+1} A_{t+J+1} =$$

$$P_t q_t E_t \left\{ \frac{A_t}{P_t} + \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} R_{t+j}^{-1} [M_{t+j}/P_{t+j}(1 - R_{t+j}) + (g_{t+j} - \tau_{t+j} z_{t+j} h_{t+j}^{\eta} - \phi_t(c_t^m + c_t^c))] \right\}.$$

But, since (49) holds for all $t \geq 0$, the right hand side equals zero for all $t \geq 0$. So condition (26) holds with equality for all $t \geq 0$. ■

Proof of Proposition 5

We first show that plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (11), (14), (18), (21), (24), (26) holding with equality, (27), (28), (29), (38), (39), (40), (41), and (42) also satisfy $U_1(t)/U_2(t) \geq 1$, (31), and (43). First, note that equations (18), (29), (39) imply condition (43). Next, divide (42) by R_t/q_t and use (11) to obtain,

$$E_t q_{t+1} A_{t+1} = q_t A_t + E_t q_{t+1} [M_t(1 - R_t) + P_t(g_t - \tau_t z_t h_t^{\eta})].$$

Take expectations with respect to information available at time 0 and sum for $t = 0$ to $t = T$. This yields:

$$E_0 q_{T+1} A_{T+1} = q_0 A_0 + E_0 \sum_{t=0}^{T} q_{t+1} [M_t(1 - R_t) + P_t(g_t - \tau_t z_t h_t^{\eta})]$$

Letting $T \to \infty$ and using (26) holding with equality, we have

$$A_0 = E_0 \sum_{t=0}^{\infty} q_{t+1} P_t \left[\frac{M_t}{P_t}(R_t - 1) + \tau_t z_t h_t^{\eta} - g_t \right]$$

By (24) and (40), $(1 - \tau_t) = -U_3(t)/U_1(t) R_t/(\eta z_t h_t^{\eta-1})$; by (41) $(R_t - 1)M_t/P_t = (R_t - 1)c_t^m$. We then have

$$\begin{aligned} A_0 &= E_0 \sum_{t=0}^{\infty} q_t E_t r_{t+1} P_t [(R_t - 1)c_t^m + z_t h_t^{\eta} - g_t + \eta^{-1} U_3(t)/U_1(t) R_t h_t] \\ &= E_0 \sum_{t=0}^{\infty} \frac{q_t P_t}{U_1(t)} [U_1(t) c_t^m + U_2(t) c_t^c + U_1(t) R_t^{-1} (z_t h_t^{\eta} - g_t - c_t^m - c_t^c) + \eta^{-1} U_3(t) h_t] \end{aligned}$$

The last equality uses equation (18). Multiply this expression by $U_1(0)/P_0$. Let $q_t^* = r_1^* \dots r_t^*$ and $q_0^* = 1$. Then by (27), (29), and (39), $P_t q_t U_1(0)/(P_0 U_1(t)) = \beta^t$ for any $t \geq 0$ and $U_1(0)/P_0 = \theta e_0$. This yields

$$\theta \frac{A_0}{e_0} = E_0 \sum_{t=0}^{\infty} \beta^t [U_1(t) c_t^m + U_2(t) c_t^c + \eta^{-1} U_3(t) h_t + \theta x_t^* (z_t h_t^{\eta} - g_t - c_t^m - c_t^c)],$$

which is (31). Now we show that if contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfy $U_1(t)/U_2(t) \geq 1$, (31), and (43), then they also are consistent with (11), (14), (18), (21), (24), (26) holding with equality, (27), (28), (29), (38), (39), (40), (41), and (42). Let λ_t be given by (39), R_t by (18), y_t by (21), w_t by (24), τ_t by (40), and P_0 by (28) evaluated at $t = 0$. Equations (18), (39), and (43) guarantee that (29) is also satisfied. Choose any process for M_t/P_t satisfying $M_t/P_t = c_t^m$ if $R_t > 1$, and $M_t/P_t \geq c_t^m$ if $R_t = 1$. Then, by construction, (38) and (41) are satisfied. Then choose processes for A_t and P_t sequentially as follows: Choose A_1 so that it satisfies (42) for $t = 0$. Then choose P_1 so that

$$\frac{A_t}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} R_{t+j}^{-1} \left[\frac{M_{t+j}}{P_{t+j}} (R_{t+j} - 1) + \tau_{t+j} z_{t+j} h_{t+j}^{\eta} - g_{t+j} \right] \tag{51}$$

for $t = 1$. Now use (42) to construct A_2. Continuing in this way, one obtains processes for P_t and A_t. As will become clear shortly, it is useful to show that (51) also holds for $t = 0$. Manipulate the term under the sum in equation (31) as follows:

$$\begin{aligned}
\left[U_1 c_t^m + U_2 c_t^c + \frac{U_3}{\eta} h_t + \theta x_t^* (z_t h_t^{\eta} - g_t - c_t^m - c_t^c) \right] &= U_1 \left[c_t^m + \frac{U_2}{U_1} c_t^c + \frac{U_3}{\eta U_1} h_t \right. \\
&\quad \left. + \frac{U_2}{U_1} (z_t h_t^{\eta} - g_t - c_t^m - c_t^c) \right] \\
&= U_1 \left[c_t^m + \frac{1}{R_t} (z_t h_t^{\eta} - g_t - c_t^m) \right. \\
&\quad \left. - \frac{1}{R_t} (1 - \tau_t) w_t \frac{h_t}{\eta} \right] \\
&= \frac{U_1}{R_t} \left[c_t^m (R_t - 1) - g_t + \tau_t z_t h_t^{\eta} \right] \\
&= \frac{\lambda_t}{R_t} \left[M_t/P_t (R_t - 1) - g_t + \tau_t z_t h_t^{\eta} \right]
\end{aligned}$$

The second equality uses equations (18) and (40); the third uses equations (24); and the fourth uses equations (41). Use the above expression to get rid of the expression in square brackets on the right of equation (31). Divide the resulting expression by λ_0 and note that by equations (28) evaluated at $t = 0$ and (29), $\lambda_0 = \theta P_0/e_0$. It follows that (51) also holds for $t = 0$. Pick the nominal exchange rate e_t to satisfy (28) and the pricing kernel r_{t+1} to satisfy (27). Combining (51) evaluated at dates t and $t+1$ one can write

$$\frac{A_t}{P_t} = R_t^{-1} \left[\frac{M_t}{P_t} (R_t - 1) + \tau_t z_t h_t^{\eta} - g_t \right] + \beta A_{t+1} E_t \frac{\lambda_{t+1}}{P_{t+1} \lambda_t}$$

Using (42) to eliminate A_t yields

$$R_t^{-1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}.$$

This expression together with (27), (28), and (29) implies that (11) holds for all $t \geq 0$. Let q_t be given by (14). Finally, we need to show that (26) holds with equality at all dates and under all contingencies. Multiply (42) by q_{t+1}, apply the E_t operator, and use the fact that $E_t q_{t+1} = q_t E_t r_{t+1} = q_t/R_t$ to get

$$E_t q_{t+1} A_{t+1} = q_t A_t + q_t R_t^{-1}[M_t(1 - R_t) + P_t(g_t - \tau_t z_t h_t^\eta)]$$

Summing for $j = 0$ to J yields

$$E_t q_{t+J+1} A_{t+J+1} = q_t A_t + E_t \sum_{j=0}^{J} q_{t+j} R_{t+j}^{-1} P_{t+j} \left[\frac{M_{t+j}}{P_{t+j}}(1 - R_{t+j}) + (g_{t+j} - \tau_{t+j} z_{t+j} h_{t+j}^\eta)\right]$$

Now take the limit for $J \to \infty$ and use the fact that, by (14), (27), and (28), $q_{t+j}P_{t+j} = q_t P_t \beta^j \lambda_{t+j}/\lambda_t$ to get

$$\lim_{J\to\infty} E_t q_{t+J+1} A_{t+J+1} = P_t q_t E_t \left\{\frac{A_t}{P_t} + \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t R_{t+j}} \left[\frac{M_{t+j}}{P_{t+j}}(1 - R_{t+j}) + g_{t+j} - \tau_{t+j} z_{t+j} h_{t+j}^\eta\right]\right\}$$

But, since (51) holds for all $t \geq 0$, the right hand side equals zero for all $t \geq 0$. So condition (26) holds with equality for all $t \geq 0$. ■

Proof of Proposition 7

We first show that plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (10), (11), (14), (17), (18), (20), (21), (24), (26) holding with equality, (27), (28), (29), (44), and (45) also satisfy $U_1(t)/U_2(t) \geq 1$, (31) and (46). The equilibrium conditions (17), (24), (29), and (44) taken together imply (46). To show that (31) holds divide (45) by R_t/q_t and use (11) to obtain,

$$E_t q_{t+1} A_{t+1} = q_t A_t + E_t q_{t+1} \{M_t(1 - R_t) + P_t[g_t - \phi_t(c_t^m + c_t^c)]\}.$$

Take expectations with respect to information available at time 0 and sum for $t = 0$ to $t = T$. This yields:

$$E_0 q_{T+1} A_{T+1} = q_0 A_0 + E_0 \sum_{t=0}^{T} q_{t+1} \{M_t(1 - R_t) + P_t[g_t - \phi_t(c_t^m + c_t^c)]\}.$$

Letting $T \to \infty$ and using (26) holding with equality, we have

$$\begin{aligned}
A_0 &= E_0 \sum_{t=0}^{\infty} q_{t+1} P_t \left[\frac{M_t}{P_t}(R_t - 1) + \phi_t(c_t^c + c_t^m) - g_t \right] \\
&= E_0 \sum_{t=0}^{\infty} q_{t+1} P_t \left[(1+\phi_t) c_t^m (R_t - 1) + (1+\phi_t)(c_t^c + c_t^m) - z_t h_t^\eta - g_t - c_t^m - c_t^c + z_t h_t^\eta \right] \\
&= E_0 \sum_{t=0}^{\infty} q_{t+1} P_t \frac{1+\phi_t}{U_1(t)} \left[U_1(t) c_t^m R_t + U_1(t) c_t^c - U_1(t)/(1+\phi_t) z_t h_t^\eta + U_1(t)/(1+\phi_t) tb_t \right] \\
&= E_0 \sum_{t=0}^{\infty} q_{t+1} P_t R_t \frac{1+\phi_t}{U_1(t)} \left[U_1(t) c_t^m + U_2(t) c_t^c + \eta^{-1} U_3(t) h_t + R_t^{-1} U_1(t)/(1+\phi_t) tb_t \right] \\
&= E_0 \sum_{t=0}^{\infty} q_t P_t \frac{1+\phi_t}{U_1(t)} \left[U_1(t) c_t^m + U_2(t) c_t^c + \eta^{-1} U_3(t) h_t + \theta x_t^* tb_t \right] \\
&= E_0 \sum_{t=0}^{\infty} \frac{e_0}{\theta} \beta^t [U_1(t) c_t^m + U_2(t) c_t^c + \eta^{-1} U_3(t) h_t + \theta x_t^* tb_t],
\end{aligned}$$

where we define $tb_t = z_t h_t^\eta - g_t - c_t^m - c_t^c$. In the last equality we combined (17), (27), (28), and (29) to express $p_t q_t (1+\phi_t)/U_1(t)$ as $\beta^t e_0/\theta$ and used (17). Note that the last equation is (31), which is what we wanted to show.

Next we show that if, given A_0 and e_0, contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfy $U_1(t)/U_2(t) \geq 1$ and equations (31) and (46), then they also are consistent with (10), (11), (14), (17), (18), (20), (21), (24), (26) holding with equality, (27), (28), (29), (44), and (45). Let R_t be given by (18), y_t by (21), w_t by (24), λ_t by (29), ϕ_t by (17). Combining (17), (24), (29), and (46), one obtains (44). Let P_0 be given by (28) evaluated at $t = 0$. Choose any process for M_t/P_t satisfying $M_t/P_t = c_t^m(1+\phi_t)$ if $R_t > 1$, and $M_t/P_t \geq c_t^m(1+\phi_t)$ if $R_t = 1$. Then, by construction, (10) and (20) are satisfied. Now choose sequences for A_t and P_t sequentially as follows: Choose A_1 so that it satisfies (45) for $t = 0$. Then choose P_1 so that

$$\frac{A_t}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} R_{t+j}^{-1} \left[\frac{M_{t+j}}{P_{t+j}}(R_{t+j} - 1) + \phi_{t+j}(c_{t+j}^m + c_{t+j}^c) - g_{t+j} \right], \tag{52}$$

for $t = 1$. Next use (45) to construct A_2. Continuing in this way, one obtains sequences for P_t and A_t. As will become clear shortly, it is useful to show that (52) also holds for $t = 0$. Manipulate the term under the sum in equation (31) as follows:

$$\begin{aligned}
\left[U_1 c_t^m + U_2 c_t^c + \eta^{-1} U_3 h_t + \theta x_t^* tb_t \right] &= \\
\frac{U_1(t)}{R_t(1+\phi_t)} \left[R_t(1+\phi_t) c_t^m + (1+\phi_t) c_t^c + \eta^{-1}(1+\phi_t) R_t U_3/U_1 h_t + \frac{R_t(1+\phi_t)}{U_1(t)} \theta x_t^* tb_t \right] &= \\
\lambda_t R_t^{-1} \left[(1+\phi_t) c_t^m R_t + (1+\phi_t) c_t^c - z_t h_t^\eta + tb_t \right] &= \\
\lambda_t R_t^{-1} \left[(1+\phi_t) c_t^m (R_t - 1) + \phi_t (c_t^m + c_t^c) - g_t \right] &= \\
\lambda_t R_t^{-1} \left[M_t/P_t (R_t - 1) + \phi_t (c_t^m + c_t^c) - g_t \right] &
\end{aligned}$$

The first equality uses equations (18); the second uses equations (17), (24), (29), and (44); and the third uses equations (20) and (24). Divide (31) by λ_0 and note that $\lambda_0 = \theta P_0/e_0$. It follows that (52) also holds for $t = 0$. For $t \geq 1$, choose the nominal exchange rate e_t to satisfy (28), and choose r_{t+1} to satisfy equation (27). Combining (52) evaluated at t and $t+1$ one can write

$$\frac{A_t}{P_t} = R_t^{-1}\left[\frac{M_t}{P_t}(R_t - 1) + \phi_t(c_t^m + c_t^c) - g_t\right] + \beta A_{t+1} E_t \frac{\lambda_{t+1}}{P_{t+1}\lambda_t}.$$

Using (45) to eliminate A_t yields

$$R_t^{-1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}.$$

This expression together with (27), (28), and (29) implies that (11) holds for all $t \geq 0$. Let q_t be given by (14). Finally, we need to show that (26) holds with equality at all dates and under all contingencies. Multiply (45) by q_{t+1}, apply the E_t operator, and use the fact that $E_t q_{t+1} = q_t E_t r_{t+1} = q_t/R_t$ to get

$$E_t q_{t+1} A_{t+1} = q_t A_t + q_t R_t^{-1}[M_t(1 - R_t) + P_t(g_t - \phi_t(c_t^m + c_t^c))]$$

Summing for $j = 0$ to J yields

$$E_t q_{t+J+1} A_{t+J+1} = q_t A_t + E_t \sum_{j=0}^{J} q_{t+j} R_{t+j}^{-1} P_{t+j}[M_{t+j}/P_{t+j}(1 - R_{t+j}) + (g_{t+j} - \phi_{t+j}(c_{t+j}^m + c_{t+j}^c))]$$

Now take the limit for $J \to \infty$ and use the fact that, by (14), (27), and (28), $q_{t+j}P_{t+j} = q_t P_t \beta^j \lambda_{t+j}/\lambda_t$ to get

$$\lim_{J\to\infty} E_t q_{t+J+1} A_{t+J+1} = P_t q_t E_t \left\{\frac{A_t}{P_t} + \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t R_{t+j}}\left[\frac{M_{t+j}}{P_{t+j}}(1 - R_{t+j}) + g_{t+j} - \phi_{t+j}(c_{t+j}^m + c_{t+j}^c)\right]\right\}$$

But, since (52) holds for all $t \geq 0$, the right hand side equals zero for all $t \geq 0$. So condition (26) holds with equality for all $t \geq 0$. ∎

Proof of Proposition 11

We first show that contingent plans $\{c_t^m, c_t^c, h_t\}$ that satisfy $U_1(t)/U_2(t) \geq 1$, (10), (11), (14), (16)-(21), (24), (26) holding with equality, and (27)-(30), for given A_0, W_0, and e_0 are consistent with (48). Divide (30), by R_t/q_t and use (11) to obtain,

$$E_t q_{t+1} A_{t+1} = q_t A_t + E_t q_{t+1}[M_t(1 - R_t) + P_t(g_t - \phi_t(c_t^m + c_t^c) - \tau_t z_t h_t^\eta)]$$

Take expectations with respect to information available at time 0 and sum for $t = 0$ to $t = T$. This yields:

$$E_0 q_{T+1} A_{T+1} = q_0 A_0 + E_0 \sum_{t=0}^{T} q_{t+1}[M_t(1 - R_t) + P_t(g_t - \phi_t(c_t^m + c_t^c) - \tau_t w_t h_t)]$$

Letting $T \to \infty$ and using (26) holding with equality we have

$$-A_0 = E_0 \sum_{t=0}^{\infty} q_{t+1}[M_t(1-R_t) + P_t(g_t - \phi_t(c_t^m + c_t^c) - \tau_t w_t h_t)]$$

Now add this expression to (47) to get

$$W_0 - A_0 = E_0 \sum_{t=0}^{\infty} q_t E_t r_{t+1} P_t [c_t^m + c_t^c - z_t h_t^{\eta} + g_t]$$

Let $q_t^* = r_1^* \ldots r_t^*$. Then by (11), (27) and (28) $P_t q_t E_t r_{t+1} = P_t^* q_t^* e_0 / R_t^*$. Using the definition $x_t^* = P_t^* q_t^* / (\beta^t R_t^*)$, equation (48) follows immediately. Next, we show that contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$ and (48) also satisfy $U_1(t)/U_2(t) \geq 1$, (10), (11), (14), (16)-(21), (24), (26) holding with equality, and (27)-(30). Given contingent plans $\{c_t^m, c_t^c, h_t\}$ construct R_t, y_t, and w_t, so that respectively, (18), (21), and (24) hold. Choose M_t/P_t so as to be consistent with (10) and (20). With e_0 given, construct P_0 from (28) evaluated at $t = 0$. Pick ϕ_0 so that the following expression holds for $t = 0$.

$$\frac{A_t}{P_t} \frac{U_1(t)}{1+\phi_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[U_1(t+j) c_{t+j}^m + U_2(t+j) c_{t+j}^c + \frac{U_3(t+j) h_{t+j}}{\eta} + \frac{U_1(0) e_0}{P_0(1+\phi_0)} x_{t+j}^* tb_{t+j} \right], \tag{53}$$

where $tb_t \equiv z_t h_t^{\eta} - g_t - c_t^m - c_t^c$. Note that this determines ϕ_0 uniquely. (It could be the case that $\phi_0 < -1$ or that $\phi_0 > 1$.) Find τ_0 from (19), then find A_1 from (30). Set $P_1(1 + \phi_1)$ so as to satisfy (53) evaluated at $t = 1$. Now use (17) to find λ_0 and λ_1/P_1. Let

$$r_{t+1} = \beta \frac{\lambda_{t+1}/P_{t+1}}{\lambda_t/P_t} \tag{54}$$

Find e_1 by imposing (27). Then find P_1 from (28). Thus ϕ_1 and λ_1 are also uniquely determined. Now repeat the above steps and in this way construct contingent plans for $\{A_{t+1}, r_{t+1}, \phi_t, P_t, \lambda_t, \tau_t, e_{t+1}\}$ that satisfy (17), (19), (27), (28), and (30) for all dates and under all contingencies. Let q_t be given by (14) and $q_0 = 1$. Set $\lambda = \lambda_0/(P_0 q_0)$. Therefore, (16) holds at $t = 0$. To see that (16) holds also for all $t > 0$, assume that it holds for some arbitrary $t' \geq 0$ and show that then it also holds for $t' + 1$. Consider (54) which can be written as $\lambda_{t'+1}/P_{t'+1} = \beta^{-1} r_{t'+1} \lambda_{t'}/P_{t'}$. By assumption, $\lambda_{t'}/P_{t'} = \lambda q_{t'}/\beta^{t'}$. This yields, $\lambda_{t'+1}/P_{t'+1} = \beta^{-1} r_{t'+1} \lambda q_{t'}/\beta^{t'} = \lambda q_{t'+1}/\beta^{t'+1}$, where the last equality uses (14). Hence, (16) holds for all $t \geq 0$. To show that equation (11) is satisfied, evaluate (53) for some arbitrary $t \geq 0$ and write it as:

$$\frac{A_t}{P_t} \frac{U_1(t)}{1+\phi_t} = U_1(t) c_t^m + U_2(t) c_t^c + \frac{U_3(t) h_t}{\eta} + \frac{U_1(0) e_0}{P_0(1+\phi_0)} x_t^* tb_t + \beta E_t \frac{A_{t+1}}{P_{t+1}} \frac{U_1(t+1)}{1+\phi_{t+1}}$$

Use (17) and (54) to rewrite this expression as

$$\frac{U_1(t)}{P_t(1+\phi_t)} A_t = U_1(t) c_t^m + U_2(t) c_t^c + \frac{U_3(t) h_t}{\eta} + \frac{U_1(0) e_0}{P_0(1+\phi_0)} x_t^* tb_t + \frac{U_1(t)}{P_t(1+\phi_t)} E_t A_{t+1} r_{t+1}$$

Now use (30) to get

$$\begin{aligned}\frac{U_1(t)}{P_t(1+\phi_t)}A_{t+1}(R_t^{-1}-E_t r_{t+1}) &= U_1(t)c_t^m+U_2(t)c_t^c+\frac{U_3(t)h_t}{\eta}+\frac{U_1(0)e_0}{P_0(1+\phi_0)}x_t^* tb_t\\ &\quad +\frac{U_1(t)R_t^{-1}}{P_t(1+\phi_t)}\left[M_t(1-R_t)+P_t[g_t-\tau_t z_t h_t^\eta-\phi_t(c_t^m+c_t^c)\right]\end{aligned}$$

Rearranging we obtain

$$\begin{aligned}(1-R_tE_tr_{t+1}) &= P_t(1+\phi_t)R_tc_t^m+P_t(1+\phi_t)c_t^c-P_t(1-\tau_t)z_th_t^\eta\\ &\quad+\frac{P_t(1+\phi_t)R_t}{U_1(t)}\frac{U_1(0)e_0}{P_0(1+\phi_0)}x_t^*tb_t\\ &\quad+M_t(1-R_t)+P_t[g_t-\tau_tz_th_t^\eta-\phi_t(c_t^m+c_t^c)\\ &= P_t(1+\phi_t)(R_t-1)c_t^m+P_t(1+\phi_t)(c_t^c+c_t^m)-P_t(1-\tau_t)z_th_t^\eta\\ &\quad+\frac{P_t(1+\phi_t)R_t}{U_1(t)}\frac{U_1(0)e_0}{P_0(1+\phi_0)}x_t^*tb_t\\ &\quad+M_t(1-R_t)+P_t[g_t-\tau_tz_th_t^\eta-\phi_t(c_t^m+c_t^c)]\\ &= -M_t(1-R_t)-P_t[g_t-\tau_tz_th_t^\eta-\phi_t(c_t^m+c_t^c)]-tb_t\\ &\quad+\frac{P_t(1+\phi_t)R_t}{U_1(t)}\frac{U_1(0)e_0}{P_0(1+\phi_0)}x_t^*tb_t\\ &\quad+M_t(1-R_t)+P_t[g_t-\tau_tz_th_t^\eta-\phi_t(c_t^m+c_t^c)]\\ &= \frac{P_t(1+\phi_t)R_t}{U_1(t)}\frac{U_1(0)e_0}{P_0(1+\phi_0)}x_t^*tb_t-P_ttb_t\\ &= 0,\end{aligned}$$

where the last equality follows from (16), (17), and (27). Thus, (11) holds. To show that (26) holds recall from above that by summing (30) one can obtain the following expression for any $t\geq 0$

$$\lim_{J\to\infty}E_tq_{t+J+1}A_{t+J+1}=\frac{P_tq_t}{\lambda_t}\left[\frac{\lambda_tA_t}{P_t}-E_t\sum_{j=0}^{\infty}\frac{P_{t+j}q_{t+j}\lambda_t}{P_tq_t\lambda_{t+j}}\left(U_1c^m+U_2c^c+\eta^{-1}U_3h+\frac{\lambda_{t+j}tb_{t+j}}{R_{t+j}}\right)\right]$$

Now note that by (16) $\frac{P_{t+j}q_{t+j}\lambda_t}{P_tq_t\lambda_{t+j}}=\beta^j$ and that by (16) and (27) $\lambda_{t+j}/R_{t+j}=U_1(0)/(P_0(1+\phi_0))e_0x_{t+j}^*$. Then equation (53) implies that the right side of the above expression equals zero. So $\lim_{J\to\infty}E_tq_{t+J+1}A_{t+J+1}=0$ for all $t\geq 0$, which is to say that (26) holds with equality. Finally to show that (47) holds, sum (30) and use (26) holding with equality to get

$$A_0=E_0\sum_{t=0}^{\infty}q_{t+1}P_t\left[(R_t-1)M_t-P_t(1-\tau_t)z_th_t^\eta+(1+\phi_t)P_t(c_t^m+c_t^c)+tb_t\right]$$

Note that by (27) $\beta^te_0x_t^*=P_tq_{t+1}$ and rearrange terms to write this expression as

$$\frac{A_0}{e_0}-\sum_{t=0}^{\infty}\beta^tx_t^*tb_t=E_0\sum_{t=0}^{\infty}q_{t+1}\frac{P_t}{e_0}\left[(R_t-1)M_t-P_t(1-\tau_t)z_th_t^\eta+(1+\phi_t)P_t(c_t^m+c_t^c)\right]$$

By (48) the left side of this expression equals W_0/e_0. Thus, (47) holds. ■

Appendix B

Calibration: Identification of Structural Parameters

We assign the following numerical values: $\eta = 0.8$, $\gamma = -2$, $\beta = \frac{1}{1.04}$, $\pi = 1.042$, $s_{tb} \equiv \frac{h^\eta - c^m - c^c - g}{y} = 0.02$, $h = 0.2$, $s_g \equiv \frac{g}{y} = 0.2$, $x^* = 1$, $R^* = 1.082$, $\frac{1}{s_m} = \frac{y}{m} = 5.8$, and $\frac{a}{y} = s_a \equiv \frac{m + \text{bonds}}{y} = 0.57$ Define total consumption as $c = c^m + c^c$. Then, we have $s_c \equiv c/y = (y - g - tb)/y = 1 - s_g - s_{tb} = 0.78$. Consumption velocity is then given by $c/m = c/y \times y/m = s_c/s_m = 4.52$. By (27), (28), and (29) we have that in steady state $R = \pi/\beta = 1.08$. In steady state, equation (30) becomes $\pi a = Ra + m(1-R) + g - \tau h^\eta - \phi c$ Divide by output, $y = h^\eta$ to get $(\pi - R)s_a = s_m(1-R) + s_g - \tau - \phi s_c$. Impose the assumption that in the pre-Ramsey competitive equilibrium $\phi = \tau$ to get $\tau = \phi = [(R-\pi)s_a + s_m(1-R) + s_g]/(1+s_c) = 0.1176$. By equation (18) we have that $\alpha_m/\alpha_c = Rc^m/c^c$. Because in the steady state of the pre-Ramsey competitive equilibrium the nominal interest rate is positive, we have that cash-in-advance constraint (10) holds with equality. Therefore, we can write $\alpha_m/\alpha_c = R/\left[(1+\phi)c/m - 1\right] = 0.2672$. By equation (19) we have that

$$\begin{aligned}
\frac{(1 - \alpha^m - \alpha^c)}{\alpha^m} &= \frac{1-h}{c^m}\frac{1-\tau}{(1+\phi)R}\frac{\eta y}{h} \quad \text{(we use } w = \eta y/h\text{)} \\
&= \left(\frac{1-h}{h}\right)\left(\frac{y}{c}\right)\left(\frac{c}{m}\right)\left(\frac{1-\tau}{R}\right)\eta \quad \text{(we use } m = (1+\phi)c^m\text{)}
\end{aligned}$$

Solving for α_c yields

$$\alpha^c = \left\{1 + \frac{R}{(1+\phi)c/m - 1}\left[1 + \left(\frac{1-h}{h}\right)\left(\frac{y}{c}\right)\left(\frac{c}{m}\right)\left(\frac{1-\tau}{R}\right)\eta\right]\right\}^{-1} = 0.1885.$$

It follows that $\alpha_m = 0.0504$. Solving equations (19) for c^m and (18) for c^c we obtain

$$c^m = \frac{\alpha^m}{1 - \alpha^m - \alpha^c}\frac{(1-\tau)(1-h)}{R(1+\phi)}\eta\frac{y}{h} = 0.0426,$$

and

$$c^c = \frac{\alpha^c}{\alpha^m}c^m R = 0.1727.$$

Then λ follows directly from (17), $\lambda = \frac{U_1}{1+\phi} = 3.9619$. Then θ is identified from equation (29), $\theta = \frac{\lambda}{Rx^*} = 3.6560$. The initial value of the nominal exchange rate determines the initial real debt of the government. By setting a high value of e_0 amounts to imposing a high capital levy on private agents' holdings of government liabilities. We choose e_0 so that the utility value of government liabilities in period zero is the same as in the steady-state of the pre-Ramsey equilibrium. By setting e_0 in this way, we ensure that the burden of public liabilities does not change in the period the Ramsey plan is implemented. This is just one of may possible strategies of assigning a value to e_0. Alternative values of e_0 will affect the level of tax rates and the real allocation, but will not alter their cyclical properties in fundamental ways. Specifically, note that $A_0/e_0 = (A_0/P_0)\lambda_0/\theta$. We then impose that $(A_0/P_0)\lambda_0 = s_a h^\eta \lambda$. This implies that $A_0/e_0 = 0.1705$.

Appendix C

Ramsey Plans in the Closed Economy With Consumption and Income Taxation

In this appendix, we develop the closed-economy version of the model presented in section 3. The description of households and the government is as in sections 3.1 and 3.3, respectively. Without loss of generality and to facilitate comparison with the related closed-economy literature, we focus on the case in which output is produced with a linear technology, $\eta = 1$. So we have,

$$y_t = z_t h_t. \tag{55}$$

Profits in period t are given by

$$z_t h_t P_t - P_t w_t h_t.$$

The firm chooses h_t so as to maximize expected profits. Firms will find it optimal to produce a finite and positive quantity of output as long as

$$w_t = z_t. \tag{56}$$

Equilibrium

In equilibrium, the goods market must clear,

$$z_t h_t = c_t^m + c_t^c + g_t. \tag{57}$$

This equilibrium condition differentiates the closed economy from the open economy. In the open economy, output need not equal domestic absorption at all times, since domestic residents have access to foreign goods markets. All households are assumed to be identical, so in equilibrium there is no borrowing and lending among them. The only assets in non-zero net supply are government bonds. Thus, for the financial asset market to clear it must be the case that

$$D_{t+1} = R_t B_t.$$

Government bonds are nominally non-state contingent, that is, $R_t B_t$ is in the information set of time t. This implies that D_{t+1} must also be in the information set of time t. In period 0, households are endowed with W_0 units of nominal wealth, which must be equal to the government's initial level of liabilities, A_0. Thus, in equilibrium equation (15) becomes

$$A_0 + E_0 \sum_{t=0}^{\infty} q_{t+1} P_t (1 - \tau_t) w_t h_t = E_0 \sum_{t=0}^{\infty} q_{t+1} \left[P_t (1 + \phi_t)(c_t^m + c_t^c) + (R_t - 1) M_t \right]. \tag{58}$$

For households to have well defined demand functions it must be the case that the nominal interest be non-negative, that is,

$$R_t \geq 1.$$

We can now define an equilibrium.

Definition 5 (Competitive Equilibrium in the Closed Economy) *A competitive equilibrium is a positive constant* λ *and set of stochastic processes* $\{P_t, r_{t+1}, M_t, A_{t+1}, c_t^c, c_t^m, h_t, q_t, w_t, y_t, \lambda_t\}_{t=0}^{\infty}$ *satisfying (10), (11), (14), (16)-(20), (30), and (55)-(58), given the exogenous stochastic processes* $\{g_t, z_t\}$*, policy sequences* τ_t*,* ϕ_t*, and* $R_t \geq 1$*, and the initial condition* A_0*.*

Competitive Equilibrium in Primal Form

The following proposition presents the primal form of the competitive equilibrium.

Proposition 12 *Given the initial conditions* A_0 *and* P_0*, and exogenous stochastic processes for* $\{z_t, g_t\}$*, contingent plans* $\{c_t^m, c_t^c, h_t\}$ *satisfying* $U_1(t)/U_2(t) \geq 1$ *and (57) are the same as those satisfying* $U_1(t)/U_2(t) \geq 1$*, (10), (11), (14), (16)-(20), (30), and (55)-(58) .*

Proof: We need to show that contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$ and (57) also satisfy (10), (11), (14), (16)-(20), (30), and (55)-(58). Choose R_t so that (18) holds, w_t so that (56) holds, and y_t such that (55) holds. Construct ϕ_0 so that the following equation holds for $t = 0$

$$\frac{A_t}{P_t}\frac{U_1(t)}{1+\phi_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[U_1(t+j)c_{t+j}^m + U_2(t+j)c_{t+j}^c + U_3(t+j)h_{t+j}\right] \tag{59}$$

This uniquely determines ϕ_0 for given plans $\{c_t^m, c_t^c, h_t\}$ because A_0/P_0 is given. Then choose τ_0 so that (19) is satisfied. Pick M_0 such that both (10) and (20) are satisfied. Find A_1 from (30). Pick $P_t(1+\phi_t)$ so as to satisfy (59) for $t = 1$. Then choose $(1-\tau_t)/(1+\phi_t)$ so that it satisfies (19). Use (30) and (57) to write $A_2 = R_1 A_1 + M_1(1-R_1) + P_1(1+\phi_1)[(1-\tau_1)/(1+\phi_1)z_1 h_1 - c_1^m - c_1^c]$. This expression defines A_2. Note that by construction (30) holds in period 1. Then, find $P_2(1+\phi_2)$ from (59). In this way one can construct time paths for A_t, $P_t(1+\phi_t)$ and $(1-\tau_t)/(1+\phi_t)$. Note that the price level and the consumption tax rate associated with a given path of $\{c_t^m, c_t^c, h_t\}$ is not unique. So one can choose, for example, a constant path for consumption tax rates, that is, $\phi_t = \phi_0$. Let $\lambda_t = U_1(t)/(1+\phi_t)$ so that (17) holds. Let $r_{t+1} = \beta(\lambda_{t+1}/P_{t+1})/(\lambda_t/P_t)$. Note that r_{t+1} is independent of the level of prices because λ_t/P_t depends only on $P_t(1+\phi_t)$, which is uniquely determined. Use (59) evaluated at t and $t+1$ to get

$$\frac{U_1(t)}{R_t P_t(1+\phi_t)}\left[R_t A_t - P_t(1+\phi_t)(R_t c_t^m + c_t^c + U_3/U_1 h_t\right] = \beta A_{t+1} E_t U_1(t+1)/(1+\phi_{t+1})/P_{t+1}$$

It follows that $R_t^{-1} = E_t r_{t+1}$. Now let $\lambda = \beta^t \lambda_t / P_t / q_t$. Note that λ is independent of the particular value taken by P_t. It again depends only on $P_t(1+\phi_t)$, which is unique. We need to check whether $\beta^{t+1}\lambda_{t+1}/P_{t+1}q_{t+1}$ is also equal to λ. This follows immediately from the definition of r_{t+1}. It is left to show that (58) holds. Express A_0 using (59) as follows:

$$\begin{aligned}\frac{A_0}{P_0}\lambda_0 &= E_0\sum_{t=0}^{\infty}\beta^t\frac{U_1(t)}{(1+\phi_t)R_t}\left[(1+\phi_t)R_tc_t^m+(1+\phi_t)c_t^c+(1+\phi_t)U_3(t)/U_1(t)R_th_t\right]\\ &= E_0\sum_{t=0}^{\infty}\beta^t\frac{\lambda_t}{P_tR_t}\left[(R_t-1)M_t+(1+\phi_t)P_t(c_t^m+c_t^c)-(1-\tau_t)w_tP_th_t\right]\\ A_0\lambda &= E_0\sum_{t=0}^{\infty}\beta^t\lambda q_{t+1}\left[(R_t-1)M_t+(1+\phi_t)P_t(c_t^m+c_t^c)-(1-\tau_t)w_tP_th_t\right]\end{aligned}$$

Eliminating λ from the left and right hand sides, we obtain (58), which is what we wanted to show. ∎

Proof of Proposition 10

The Ramsey problem consists in choosing c_t^m, c_t^c, h_t so as to maximize (9) subject to $U_1/U_2 \geq 1$ and (57). Consider a modified Ramsey problem that does not impose the condition $U_1(t)/U_2(t) \geq 1$. That is, consider the problem of maximizing (9) subject to (57). The first-order conditions associated with this problem are (57), $U_1(t)/U_2(t) = 1$, and $-U_3(t)/U_1(t) = z_t$. Note that the constraint $U_1(t)/U_2(t) \geq 1$ is satisfied. So the solution to the modified Ramsey problem is indeed the solution to the original Ramsey problem. So clearly, the Ramsey allocation is Pareto optimal. (Note that, because it achieves the first best, the Ramsey plan is time consistent.) Comparing the optimality conditions of the Ramsey problem with equilibrium conditions (18) and (19), it follows that the Ramsey policy associated with the Ramsey plan features $R_t = 1$ and $\tau_t = -\phi_t$. Moreover, it is clear from the proof of proposition 12 that in decentralizing the Ramsey plan, the government has some margin in choosing prices and consumption tax rates for $t > 0$. In particular, the after-tax consumption price level, $P_t(1+\phi_t)$ is uniquely pinned down, but neither P_t nor ϕ_t are individually determined for $t > 0$. It follows that the Ramsey plan can be supported by a constant tax policy of the form $\phi_t = -\tau_t = \phi_0$. ∎

The Case of a Predetermined Initial Consumption Tax Rate

Proposition 13 *Given initial conditions A_0, P_0, and ϕ_0, and exogenous stochastic processes for $\{z_t, g_t\}$, contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (57), and*

$$\frac{A_0}{P_0}\frac{U_1(0)}{1+\phi_0} = E_0\sum_{t=0}^{\infty}\beta^t\left[U_1(t)c_t^m+U_2(t)c_t^c+U_3(t)h_t\right] \tag{60}$$

are the same as those satisfying (10), (11), (14), (16)-(21), (24), (26) holding with equality, (27), (28), (30), and (47).

This primal form is the same as that of an economy with only labor income taxation (see Chari, et al. 1991). Thus, when P_0 and ϕ_0 cannot be chosen optimally by the Ramsey planner, the Ramsey real allocation is the same in the economy with labor income and consumption taxes as in the economy with labor income taxes only. In particular, the Ramsey real allocation is no longer Pareto optimal, and the distortion of the (credit good) consumption/leisure margin, $(1-\tau_t)/(1+\phi_t)$ is no longer nil. However, the optimality of the Friedman rule continues to hold.

Proof of Proposition 13

We first show that contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (10), (11), (14), (16)-(21), (24), (26) holding with equality, (27), (28), (30), and (47) also satisfy (57) and (60). Use (19) and (20) to eliminate $(1-\tau_t)w_t$ and $M_t/P_t(R_t-1)$ from (58). This yields:

$$\begin{aligned}
A_0 &= E_0 \sum_{t=0}^{\infty} q_{t+1} P_t \left[U_3(t)/U_1(t)(1+\phi_t)R_t h_t + (1+\phi_t)(c_t^m + c_t^c) + (R_t - 1)(1+\phi_t)c_t^m \right] \\
&= E_0 \sum_{t=0}^{\infty} q_t P_t r_{t+1} \frac{1+\phi_t}{U_1(t)} \left[U_3(t) R_t h_t + U_1(t) c_t^c + R_t U_1(t) c_t^m \right] \\
&= E_0 \sum_{t=0}^{\infty} q_t P_t r_{t+1} R_t \frac{1+\phi_t}{U_1(t)} \left[U_3(t) h_t + U_1(t)/R_t c_t^c + U_1(t) c_t^m \right] \\
&= E_0 \sum_{t=0}^{\infty} q_t P_t \frac{1+\phi_t}{U_1(t)} \left[U_3(t) h_t + U_2(t) c_t^c + U_1(t) c_t^m \right] \\
&= E_0 \sum_{t=0}^{\infty} q_t P_t \lambda_t^{-1} \left[U_1(t) c_t^m + U_2(t) c_t^c + U_3(t) h_t \right] \\
&= E_0 \sum_{t=0}^{\infty} \beta^t / \lambda \left[U_1(t) c_t^m + U_2(t) c_t^c + U_3(t) h_t \right] \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \frac{P_0(1+\phi_0)}{U_1(0)} \left[U_1(t) c_t^m + U_2(t) c_t^c + U_3(t) h_t \right]
\end{aligned}$$

where the second equality uses (14), the fourth equality uses the law of iterated expectations, (11), and (18), the fifth equality uses (17), the sixth equality uses (16), and finally, the seventh equality uses (17) and (16) evaluated at $t = 0$. Rearranging terms we obtain (60).

To show that contingent plans $\{c_t^m, c_t^c, h_t\}$ satisfying $U_1(t)/U_2(t) \geq 1$, (57), and (60) can be supported as a competitive equilibrium, proceed as follows. Let $\lambda = U_1(0)/[(1+\phi_0)P_0]$, thus (16) is satisfied for $t = 0$. Choose R_t such that (18) holds, w_t such that (56) holds, y_t such that (55) holds. For $t = 0$, construct τ_0 from (19). Pick M_0 such that both (10) and (20) are satisfied. Given these values, find A_1 from (30). Now, there are many options on how to set P_1 pointing to the fact that there are multiple price levels and consumption tax rates that support the Ramsey allocation as a competitive equilibrium. Specifically, any price level path and consumption tax rate path satisfying

$$\frac{A_t}{P_t(1+\phi_t)} U_1(t) = E_t \sum_{j=0}^{\infty} \beta^j \left[U_1(t+j) c_{t+j}^m + U_2(t+j) c_{t+j}^c + U_3(t+j) h_{t+j} \right] \tag{61}$$

can support the Ramsey allocation as a competitive equilibrium. That is, only the after tax price of consumption is uniquely pinned down by the Ramsey allocation. One possible price path is one in which $P_t = P_0$ for all t. That is, in this case there is no front-loading via surprise inflation. (There may still be front-loading via surprise consumption taxes.) Another possible competitive equilibrium is one in which $\phi_t = 0$ for all $t > 0$. That is, the consumption tax is not a necessary fiscal instrument to 'implement' the Ramsey allocation. Or put differently, the Ramsey allocation is the same regardless of whether the Ramsey planner has access to a consumption tax or not. Construct a time series for $P_t(1+\phi_t)$ and A_t for $t \geq 1$ as follows. Pick $P_t(1+\phi_t)$ so as to satisfy (61) for $t = 1$, then choose $(1-\tau_t)/(1+\phi_t)$ so that it satisfies (19). Choose $A_2 = R_1 A_1 + P_1(1+\phi_1)c_1^m(1-R_1) + P_1(1+\phi_1)[(1-\tau_1)/(1+\phi_1)z_1 h_1 - c_1^m - c_1^c]$. Note that by construction A_2 satisfies (30). Then, find $P_2(1+\phi_2)$ from (61). In this way one can construct time paths for A_t, $P_t(1+\phi_t)$ and $(1-\tau_t)/(1+\phi_t)$. Let $\lambda_t/P_t = U_1(t)/[P_t(1+\phi_t)]$ and let $q_t = \beta^t \lambda_t / P_t / \lambda$. Thus, (16) and (17) hold for all t and $q_0 = 1$ as required. Then let $r_{t+1} = q_{t+1}/q_t$ so that (14) is satisfied. It remains to be shown that (11) and (58) hold. The latter follows straightforward from (60). To show that (11) holds use (61). Note that by (60), equation (61) also holds for $t = 0$. Combining (61) evaluated at t and $t+1$ one can write

$$\frac{A_t}{P_t} = R_t^{-1}\left[\frac{M_t}{P_t}(R_t - 1) + \tau_t w_t h_t + \phi_t(c_t^m + c_t^c) - g_t\right] + \beta A_{t+1} E_t \frac{\lambda_{t+1}}{P_{t+1}\lambda_t}.$$

Using (30) to eliminate A_t yields

$$R_t^{-1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}.$$

Given our definition of r_{t+1}, this expression implies that (11) holds for all $t \geq 0$. ■

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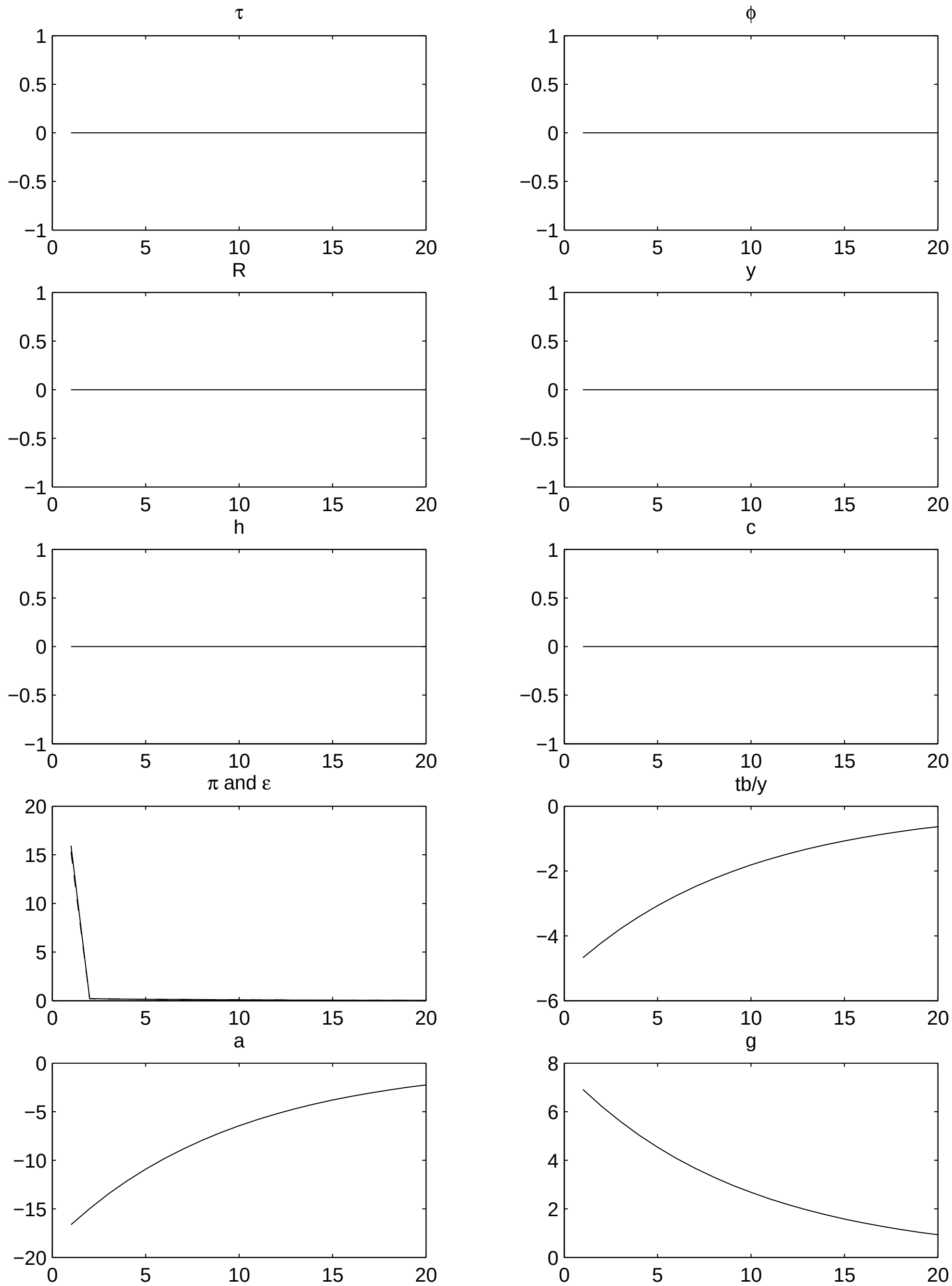
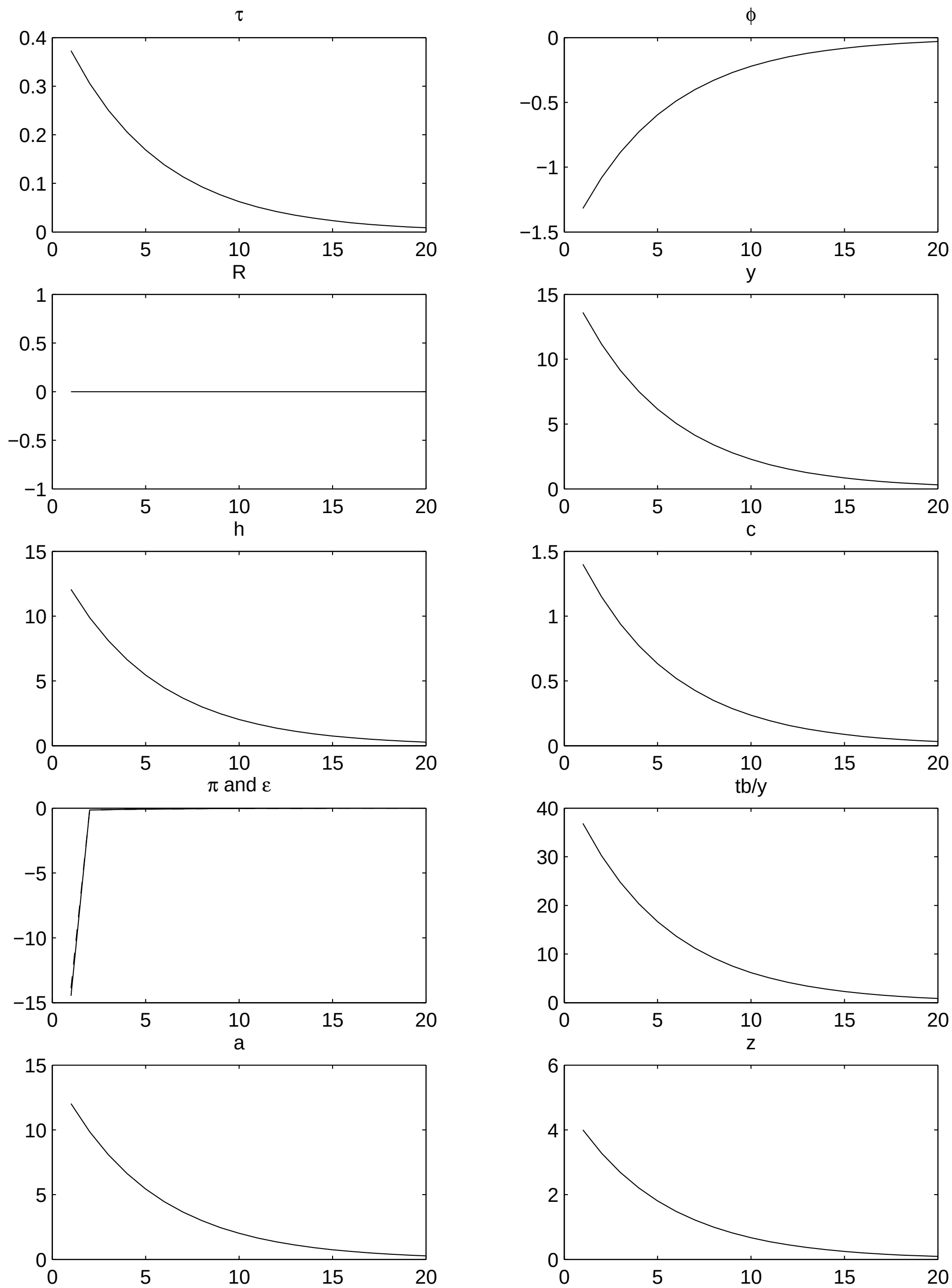
Figure 1: Impulse response of the Ramsey economy to a 1-standard-deviation positive innovation in government purchases



The impulse response of a variable x_{t+j} to a 1-standard-deviation positive innovation in g_t is computed as $E\{x_{t+j}|g_t = g^H \text{ and } a_0\} - E\{x_{t+j}|a_0\}$. Variables τ, ϕ, R, ϵ, and tb/y are expressed in percentage points. All other variables are expressed in percent deviations from their long-run mean. The impulse response of the devaluation rate, ϵ, is shown in a dashed line.

Figure 2: Impulse response of the Ramsey economy to a 1-standard-deviation positive innovation in labor productivity



The impulse response of a variable x_{t+j} to a 1-standard-deviation positive innovation in z_t is computed as $E\{x_{t+j}|z_t = z^H \text{ and } a_0\} - E\{x_{t+j}|a_0\}$. Variables τ, ϕ, R, and tb/y are expressed in percentage points. All other variables are expressed in percent deviations from their long-run mean. The impulse response of the devaluation rate, ϵ, is shown in a dashed line.