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REBELS, CONFORMISTS, CONTRARIANS AND MOMENTUM TRADERS

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ABSTRACT

We develop a model of optimal investment with two types of agents with different beliefs about the market dynamics. Market conformists agree with the true log-normal price distribution and rebels believe in price predictability. Depending on their exact beliefs, the rebels may follow either a momentum or a contrarian strategy. It is difficult to detect rebels' beliefs that are not far-fetched from the market perspective. The long-run investment portfolios of both conformist and rebels need not be biased towards equities.

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In a noisy capital market with imperfect information, there is room for rational agents with heterogenous beliefs. In this paper we compare the optimal portfolio strategies of two types of agents – conformists who believe in the log-normal market consensus and "rebels" whose alternative beliefs incorporate price predictability. Surprisingly, we find that momentum trading is optimal for rebels who believe that the market is grossly mispriced.

The empirical work of Lakonishok, Shleifer and Vishny (1994), Jegadeesh and Titman (1993) and Gatev, Goetzmann and Rouwenhorst (1999) documents profits from momentum and contrarian strategies that appear to exploit predictable price movements. These profits raise the question whether such momentum and contrarian strategies are, in fact, the optimal trading strategies of agents who incorporate price predictability in their alternative beliefs about the market process. The main contribution of our paper is to enable the empirical detection of non-conformist beliefs about the possibility of market timing.

Recent papers like DeLong et al. (1990), Barberis et al. (1997) and Hong and Stein (1999) merely postulate the existence of irrational agents who behave as momentum traders. However, alternative beliefs need not be irrational – Merton (1980), Summers (1986) and Poterba and Summers (1989) have argued that it is extremely difficult to distinguish statistically between randomness and certain forms of price-predictability.

Alternative specifications of the price process were explored in the early work of Merton (1971) who showed that an agent who believes in an exponential trend always invests more in the single risky asset than the log-normal agent. The recent work of Wachter (1998) also solves the portfolio-consumption problem for an alternative price dynamics in which the expected return is mean-reverting. Cox and Ross (1976) and Lo and Wang (1995) examine specifications that incorporate price predictability and show that they imply different option valuations.

In our model, the market is sequentially complete and the rebels remain price-takers at all times. Most importantly, the rebels' beliefs do not adapt as events unfold[2]. Presumably, the rebels either cannot learn the self-fulfilling market beliefs, because it takes too long to gather enough evidence to rationally change beliefs, or, alternatively, the rebels may dogmatically consider the market consensus wrong.

[2]Brennan and Cao (1996) have demonstrated that asymmetric information may lead to rational momentum trading when information is revealed gradually in the market.

We employ the Cox-Huang (1989) martingale approach to the terminal wealth portfolio problem of the agent, to address a collection of issues about the optimal trading strategy under the alternative beliefs that incorporate price predictability. What will the rebel do? How well will the rebel do? Can we distinguish the rebel from the typical market participant?

Our main results show that the rebels act as momentum traders when they believe that the current market price is far from the "right" price. In contrast, the conformists are contrarian or momentum traders depending on whether they are more or less risk averse than the representative agent. Thus, to an outsider, risk-averse rebels whose beliefs are far from the market's beliefs may appear as risk-tolerant conformists. The terminal wealth distribution of rebels whose beliefs are sufficiently different from those of the market are first-order dominated by certain conformist portfolios. But, in general, the properties of the terminal wealth distribution under different beliefs suggest that it is difficult to detect wrong beliefs from limited performance data. Finally, the long-run investment portfolios of both conformists and rebels need not be heavily biased towards equities. Both the conformist's and the rebel's long-run portfolios include a positive equity component, however equities need not dominate the portfolio.

Our conformist momentum results generalize and make empirically relevant the qualitative momentum aspect of the early portfolio insurance work of Leland (1980) and Brennan and Solanki (1981). Moreover, Leland (1980) considered the optimal portfolio insurance for agents who have different estimates of the mean holding period return.

Our paper differs from Leland (1980) in two major ways. First, we define alternative beliefs more generally, as beliefs about market *dynamics*, i.e. the specification of the market process. In contrast Leland defined differing expectations narrowly as different estimates of the mean within the same static log-normal specification. Second, we obtain in closed form empirically relevant results relating the number of shares traded to the form of the alternative beliefs in reversion. Leland's main stated objective is to characterize qualitatively the investors who would demand generalized portfolio insurance, i.e. convex payoffs. Ultimately, these two differences are necessary because they allow us to answer the following questions that have not been addressed before.

Our main objective is to enable the empirical detection of alternative beliefs in reversion and the feasibility of market timing. Our dynamic specification of the beliefs in reversion allows the investment opportunity set to depend on the current market conditions. Hence the more general *dynamic* definition of alternative beliefs is necessary in order to capture beliefs in the possibility of opportunistic market timing. We relate this alternative dynamic specification to observable trading patterns. In contrast, Leland (1980) emphasizes that his model does not allow for market timing, while the Brennan and Solanki (1981) model has no differences in expectations. These models remain within the static log-normal specification, and there are no perceived changes in the investment opportunity set. On the other hand, we are able to solve for the more general dynamic beliefs by using the Cox-Huang methodology, which was not available at the time of the earlier models. Our results explain observed counter-intuitive momentum trading with deviant beliefs in market reversion to a growth trend, while remaining within the attractive rational paradigm of utility-maximizing agents. The wrong beliefs are maintained because of market noise that prevents the timely unambiguous learning of the proper dynamic specification.

In section I we present the model. Section II analyzes the momentum and contrarian appearance of the optimal trading for different beliefs about the market dynamics. In section III we examine the performance of the strategies that are optimal under different beliefs. Section IV explores the feasibility of empirically discriminating between different beliefs and different risk aversion. Section V contains results on the long-run portfolios of the different agents. The concluding discussion of the results is in section VI.

I. The Model

A. Differential Beliefs

We assume that there is a single risky asset, *the market portfolio*, and a riskless asset with a constant risk-free rate r. The market is sequentially complete, with states distinguished by the price P of the market. The representative agents arrive in overlapping generations. There are two types of agents, who cannot affect the price individually. The *market conformists* agree with the representative agents' beliefs, but have different risk aversion.

The rebels are price takers, who have non-market beliefs. The rebels are assumed not to adapt to the self-fulfilling market beliefs.

We assume the usual continuous time primitives. Let B denote a standard one-dimensional Brownian motion living on Ω x [0,T], T ≤ ∞, where (Ω , , $\mathcal{P}$) is a complete probability space with the standard filtration = { $_t$: t ∈ [0,T]}. All the processes we consider are adapted to . When there is no ambiguity we will suppress the subscript for the current time t.

A.1 The Market

The price of the risky market follows a log-normal Ito process

$$\frac{dP}{P} = \mu dt + \sigma dB \tag{1}$$

with constant drift μ and local volatility σ which implies that the probability density π_M of the price P_T at time T conditional on price P at time t is

$$\pi_M(P_T|P) = \frac{1}{P_T\sigma\sqrt{\tau}}\,\Phi\left(\frac{\ln(P_T/P) - (\mu-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right) \tag{2}$$

where and $\tau = T - t$ and $\Phi(z) = 1/\sqrt{2\pi}\,\exp(-z^2/2)$ is the standard normal density.

A.2 The Rebels

The rebel's beliefs for the specification of the price dynamics incorporate return predictability in the form of mean-reversion of the log-price. Specifically, the log-price p follows an Ornstein-Uhlenbeck process

$$dp = \alpha(\gamma - p)dt + \sigma dB \tag{3}$$

so that the price $P = \exp(p)$ follows the process

$$\frac{dP}{P} = \alpha[(\gamma + \frac{\sigma^2}{2\alpha}) - \ln P]dt + \sigma dB \tag{4}$$

The analysis is easily modified to a price process that is deflated and adjusted for a growth trend. The conditional probability density π_α of the price P_T at time T conditional on price P at time t is (see appendix I)

$$\pi_\alpha(P_T|P) = \frac{1}{P_T\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\Phi\left(\frac{\ln P_T - \gamma - (\ln P - \gamma)e^{-\alpha\tau}}{\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\right) \tag{5}$$

We assume that all agents agree on the value of σ and can differ only in their estimate of the drift.

B. Arrow-Debreu Digital Options

Since there is a single risky asset and the market is complete, the Arrow-Debreu state-prices can be viewed as point-mass digital options, e.g. see Ingersoll (2000). Both the market conformists and the rebels value these digitals using the same Black-Scholes formula. The replicating portfolio for the digitals is the same for all agents.

Let $q[P;P_T]$ denote the time t price of the Arrow-Debreu security that pays \$1 if the price (state) at time T is P_T, conditional on price P at time t. Since there is a single risky asset in a complete market, the Arrow-Debreu security is a digital option whose value is uniquely determined as the discounted expected payoff under the risk-neutralized probability π_r e.g. see Cox and Ross (1976):

$$\begin{aligned} q[P;P_T] &= e^{-r\tau}\int_S \pi_r(S|P)\delta_{P_T} dS \\ &= e^{-r\tau}\pi_r(P_T|P) \end{aligned} \tag{6}$$

where δ_x is the point mass at x. The expected payoff of the Arrow-Debreu security for state P_T is just the conditional probability density of the price P_T. Armed with this result, we obtain the price of the Arrow-Debreu security for the two agents.

Proposition 1. The market and the rebels' valuation of the digital option is given by

$$q[P;P_T] = \frac{e^{-r\tau}}{P_T\sigma\sqrt{\tau}}\,\Phi\left(\frac{\ln(P_T/P) - (r-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right) \tag{7}$$

which is the usual Black-Scholes conditional price density.

Proof: from (6) using (2) with $\mu = r$ for the risk-neutralized density π_r. ❑

The digital options are replicated as follows.

Proposition 2. The number of market units in the hedge portfolio for a digital option $q[P;P_T]$ is

$$\begin{aligned}\Delta_{q[P;P_T]} &= \frac{\partial q[P;P_T]}{\partial P}\\ &= e^{-r\tau}\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{P_T P\sigma^3\sqrt{\tau^3}}\Phi\left(\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right)\end{aligned} \tag{8}$$

Proof: in appendix I. ❑

C. The Agent's Intertemporal Problem

All agents solve their portfolio problems in terms of Arrow-Debreu state-prices (see Cox-Huang (1989) or its exposition in Merton (1990). Since the market is complete, if the agent wants a security with payoffs schedule $\{c(P_T,T)\}$, this security can be constructed from Arrow-Debreu securities, and has price

$$S(t;c) \;=\; \int_t^\infty \int_{P_T} c(P_T,T)\, q[P;P_T]\, dP_T\, dT \tag{9}$$

In a sequentially complete market, each agent can achieve the optimum consumption schedule c for the agent's lifetime utility maximization.

$$\begin{aligned} &\max_{c(.)} \left\{ \int_{t_0}^\infty \int_{P_T} U[c(P_T,T)]\, \pi(P_T|P)\, dP_T\, dt \right\} \\ &\qquad s.t. \\ &W(t_0) \geq \int_{t_0}^\infty \int_{P_T} c(P_T,T)\, q[P;P_T]\, dP_T\, dt \\ &c(P_T,T) \geq 0 \;\; for \;\; all \;\; P \;\; and \;\; 0 \leq T \leq \infty \end{aligned} \tag{10}$$

where $\pi(P_T|\, P)$ is the agent's subjective transition probability density of the price of the market. Note that the budget constraint is computed with the market valuation for the Arrow-Debreu securities $q[P;P_T]$.

For any agent, the optimum consumption c^* is given by

$$c^*(P_T,T) \;=\; U_c^{-1}[\lambda \frac{q[P;P_T]}{\pi(P_T|P)}] \tag{11}$$

where the constant λ is found from

$$W(t_0) \;=\; \int_{t_0}^\infty \int_{P_T} U_C^{-1}[\lambda \frac{q[P;P_T]}{\pi(P_T|P)}]\, q[P;P_T]\, dP_T\, dt \tag{12}$$

Rewriting (11) as

$$U_c\;[c^*(P_T,T)] \frac{\pi(P_T|P)}{q[P;P_T]} \;=\; \lambda \tag{13}$$

we see that the risk averse agent equalizes across states the expected return $\pi(P_T| P)/q[P;P_T]$ in state P_T weighted by the marginal utility in that state.

The agent's demand for each Arrow-Debreu security $q[P;P_T]$ is such as to achieve the optimal consumption $c^*(P_T, T)$. Hence the optimum digital options portfolio $D^*_{q[P;PT]}$ is exactly $c^*(P_T, T)$. Because their subjective specification is not log-normal, the rebels believe that the expected return $\pi_\alpha(P_T|P)/q[P;P_T]$ of the Arrow-Debreu security is different from the market expected return $\pi_M(P_T|P)/q[P;P_T]$. Therefore, the digital options portfolio $D^*_{q[P;PT]}$ that is optimal for the rebels is different from that of the market conformists.

Since the market is sequentially complete, an agent replicates the Arrow-Debreu security $q[P;P_T]$ by holding a number of market units, n_q, equal to the option's delta $\Delta_{q[P;PT]}$

$$n_q = \Delta_{q[P;P_T]} = \frac{\partial q[P;P_T]}{\partial P} \tag{14}$$

Hence, the agent's optimal total demand n for the stock at time t is

$$\begin{aligned} n &= \int_t^\infty \int_{P_T} D^*_{q[P_0;P_T]} \Delta_{q[P;P_T]} dP_T dT \\ &= \int_t^\infty \int_{P_T} U_c^{-1}[\lambda \frac{q[P_0;P_T]}{\pi(P_T|P)}] \frac{\partial q[P;P_T]}{\partial P} dP_T dT \end{aligned} \tag{15}$$

Note that the option's delta $\Delta_{q[P;PT]}$ is the same for both agents even though the conditional price density $\pi_\alpha(P_T|P)$ of the rebels is different from the market conformist's conditional price density $\pi_M(P_T|P)$. The agents effectively perceive different investment opportunity sets.

The wealth process of the agent is given by

$$W(t) = \int_t^\infty \int_{P_T} D^*_{q[P_0;P_T]} q[P;P_T] dP_T dT \tag{16}$$

The time t wealth for the rebels is different from the market conformist's wealth, because their optimum digital options portfolios D^* are different. As long as the market is sequentially but not statically complete, there are two possibilities. Either the rebels do not know that they are wrong and assume that the market is mean-reverting, or the rebels know the market's log-normal beliefs and dogmatically presume that the market is wrong.

II. Optimal Momentum and Contrarian Trading

In order to determine whether an agent's demand for the market appears like momentum or contrarian trading, we focus on the number of physical units of the market held rather than the wealth fraction, or the dollar amount invested in the market. While a pristine economic definition of momentum would isolate the wealth effect, we note that in general only returns and volume are empirically observable, so that the following definitions are more empirically relevant.

Definition 1: An agent is a *contrarian* trader if the optimal number $n(t)$ of market units held satisfies

$$\frac{\partial n}{\partial P} < 0 \tag{17}$$

Ceteris paribus, an increase in the price causes a decrease in the number of market units held. In other words, the agent buys low and sells high.

Definition 2: An agent is a *momentum trader* if the optimal number $n(t)$ of market units held satisfies

$$\frac{\partial n}{\partial P} > 0 \tag{18}$$

Ceteris paribus, an increase in the price causes an increase in the number of market units held. In other words, the agent buys high and sells low.

Definition 3: An agent is a *momentum trader relative to the market* if the fraction w of the agent's current wealth that is invested in the market satisfies

$$\frac{\partial w}{\partial P} > \frac{\partial w_M}{\partial P} \tag{19}$$

where w_M is the fraction of the conformist's current wealth invested in the market. Conversely, an agent is a *contrarian relative to the market* if

$$\frac{\partial w}{\partial P} < \frac{\partial w_M}{\partial P} \tag{20}$$

Ceteris paribus, a price increase implies that the exposure change, net of the market conformist's exposure change, is positive for the relative momentum trader and negative for the relative contrarian. In other words, after a price increase, the relative contrarians decrease their excess exposure, while the relative momentum traders increase it.

The optimal market position at time t is exactly the number $n(t)$ of market units that are necessary in order to replicate all digital options positions that are optimal for the agent. Thus, to analyze the optimal trading of the agent, we translate the optimum demand schedule for digital options into optimum demand for market units. The hedge ratio $\Delta_{q[P;PT]}$ for the digital option given in Proposition 2, together with the optimal demand schedule $\{D^*_{q[P_0;PT]}\}$ for digital options, determine the number n of market units demanded by each agent at time t:

$$n(t) = \int_{P_T} D^*_{q[P_0;P_T]} \Delta_{q[P;P_T]} dP_T \tag{21}$$

Agents with different beliefs have different optimal demand schedules $D^*_{q[P_0;PT]}$ and consequently have different demands for market units.

For simplicity, we will assume that each agent solves a terminal wealth problem with time horizon T and power utility of terminal wealth, $U[W] = \frac{W^{1-\eta}}{1-\eta}$.

A. The Contrarian Conformists

In equilibrium, a representative agent holding the market of risky assets has relative risk aversion η^* that satisfies

$$\eta^* = \frac{\mu - r}{\sigma^2} \tag{22}$$

When agents conform to the market beliefs, we specialize (11) and (12) to get

Proposition 3. The market conformist's optimum demand schedule, $D^*_M(P_T)$, for digital options is given by

$$D^*_M(P_T) = m \; P_T^{\frac{\eta^*}{\eta}} \tag{23}$$

where

$$m = W_t \, P^{-\frac{\eta^*}{\eta}} e^{(1-\frac{\eta^*}{\eta})(r+\frac{\eta^*}{\eta}\frac{\sigma^2}{2})(T-t)} \tag{24}$$

Proof: in appendix II. □

Substituting in (21) the optimal demand schedule D^*_M for digital options and $\Delta_{q[P;PT]}$ from Proposition 2 , we obtain the following

Proposition 4. The market conformist demands $n_M(t)$ units of the market, where

$$n_M(t) = \frac{\eta^*}{\eta} \frac{W}{P} \tag{25}$$

where η^*/η is the constant fraction of current wealth invested in the market.

Proof: see appendix II. □

The individual conformists who are less risk-averse than the representative agent lever up their portfolios, borrowing from those conformists who are more risk-averse than the representative agent. The conformist's demand does *not* depend on the planning horizon T.

Now we are ready to characterize the trading of the market conformist.

Theorem 1: Market conformists who are more risk-averse than the representative agent are contrarian. Conversely, conformists who are more risk-tolerant than the representative agent are momentum traders.

Proof:

Writing the current wealth as a function of the current price, we have

$$\frac{\partial n_M(W(P),P)}{\partial P} = \frac{\partial n_M}{\partial W}\frac{\partial W}{\partial P} + \frac{\partial n_M}{\partial P} \tag{26}$$

where the first term is the wealth effect and the second term is a substitution effect. From proposition 4, we have

$$\begin{aligned} \frac{\partial n_M}{\partial P} &= -\frac{\eta^*}{\eta}\frac{W}{P^2} \\ \frac{\partial n_M}{\partial W} &= \frac{\eta^*}{\eta}\frac{1}{P} \end{aligned} \tag{27}$$

Now, from appendix II.4 we have

$$\frac{\partial W}{\partial P} = \frac{\eta^*}{\eta}\frac{W}{P} \tag{28}$$

Substituting (28) and (27) into (26) and using (25) we obtain

$$\begin{aligned} \frac{\partial n_M}{\partial P} &= \left(\frac{\eta^*}{\eta}\right)^2 \frac{W}{P^2} - \frac{\eta^*}{\eta}\frac{W}{P^2} \\ &= (\eta^* - \eta)\frac{\eta^*}{\eta^2}\frac{W}{P^2} \end{aligned} \tag{29}$$

Hence

$$\frac{\partial n_M}{\partial P} \gtrless 0 \Leftrightarrow \eta \lessgtr \eta^* \tag{30}$$

This is the result we wanted to prove. □

The result is driven by the balance between the wealth and the substitution effects. The substitution effect is always negative, with a higher price implying lower holding of the market. However, a higher market price also implies higher wealth and the agent has to buy extra units of the market in order to maintain the constant fraction of current wealth invested in the market. The two effects offset each other exactly for the conformist whose risk aversion is equal to that of the representative agent. Conformists who are more risk-tolerant than the representative agent maintain higher-than-representative fraction of wealth invested in the market and require more units than the representative agent. Conversely, the substitution effect dominates the wealth effect when the conformists are more risk-averse than the representative agent. The lesson from this result is that the appearance of the conformists as momentum or contrarian traders is driven exclusively by risk-aversion. As we will see shortly, this is not the case for the deviant rebels.

As a somewhat wistful aside, the risk-aversion explanation of conformist momentum trading is consistent with the recent phenomenon of momentum day-trading in risky Internet stocks. According to theorem 1, only conformists with high risk tolerance would rationally engage in the day-trading practices.

B. The Momentum Rebels

The rebels' demand schedules for digital options and for market units differ from those of the market conformists. We have the following

Proposition 5. The rebel's optimum demand schedule D^*_α for digital options is

$$D^*_\alpha(P_T;\lambda_\alpha(t)) = k\, P_T^{q \ln P_T + s} \tag{31}$$

where

$$k = W_t \sqrt{\frac{\theta-(1-\eta)}{\eta}}\; e^{r\tau - \frac{\theta(1-\eta)}{\theta-(1-\eta)}\frac{(L-Q)^2}{2\eta\sigma^2\tau}} \;,\; q = -\frac{\theta-1}{2\eta\sigma^2\tau} \;,\; s = \frac{\theta L - Q}{\eta\sigma^2\tau}$$

$$Q = \ln P + (r-\sigma^2/2)\tau,\; L = \gamma + (\ln P - \gamma)e^{-\alpha\tau},\; \tau = T-t,\; \theta = 2\alpha\tau/(1-e^{-2\alpha\tau}). \tag{32}$$

Proof: in appendix III. □

Similarly, the rebels' demand for market units is determined by their different beliefs.

Proposition 6. The rebel has optimum demand for the risky asset given by

$$n_\alpha = (a \ln P + b)\,\frac{W}{P} \tag{33}$$

where

$$a = -\frac{1-e^{-\alpha\tau}}{[1-(1-\eta)/\theta]\sigma^2\tau} \;,\; b = a[g-\gamma] \;,\; g = \frac{(r-\sigma^2/2)\tau}{1-e^{-\alpha\tau}} \tag{34}$$

Proof: in appendix III. □

Observe that unlike the conformist, the fraction $w_\alpha = a \ln P + b$ of the rebel's wealth that is invested in the market, is a function of the current price P. Intuitively, the current price level determines the expected adjustment of the price towards the "right" level, and hence the expected return over the holding period. In other words, the rebels' perceived investment opportunity set depends on the price, as the market may appear to be under- or over-priced relative to the "right" price.

In general, the rebels' deviant beliefs shape their unusual trading strategy.

Theorem 2: The rebels are momentum traders when they believe the market is grossly mispriced.

Proof:

Writing the wealth and substitution effects as before and using proposition 6, we have

$$\begin{aligned} \frac{\partial n_\alpha(W(P),P)}{\partial P} &= \frac{\partial n_\alpha}{\partial W}\frac{\partial W}{\partial P} + \frac{\partial n_\alpha}{\partial P} \\ &= \frac{1}{P}\left(w_\alpha \frac{\partial W}{\partial P} + [\frac{\partial w_\alpha}{\partial P} - \frac{w_\alpha}{P}]W \right) \end{aligned} \tag{35}$$

where w_α is the fraction of the rebel's wealth that is invested in the market. From Appendix III.4, we have

$$\frac{\partial W}{\partial P} = \frac{\eta^*}{\eta}\frac{W}{P} \tag{36}$$

so that

$$\frac{\partial n_\alpha}{\partial P} = \frac{1}{P}\left(\frac{w_\alpha}{P}(w_\alpha - 1) + \frac{\partial w_\alpha}{\partial P} \right) W \tag{37}$$

The sign of $\partial n_\alpha / \partial P$ is determined by the expression in the parentheses. Since we are interested in the sign of $\partial n_\alpha / \partial P$ as a function of the rebel's beliefs , we observe that

$$\begin{aligned} w_\alpha(\gamma) &= a[(p-\gamma) + g] \\ \frac{\partial w_\alpha}{\partial P}(\gamma) &= \frac{a}{P} \end{aligned} \tag{38}$$

where $p = \ln P$, $a = -\dfrac{\theta(1-e^{-\alpha\tau})}{[\theta-(1-\eta)]\sigma^2\tau}$, $g = \dfrac{(r-\sigma^2/2)\tau}{1-e^{-\alpha\tau}}$. Now we rewrite (37) as

$$\frac{\partial n_\alpha}{\partial P} = \frac{aW}{P^2}[\ ap^2 + [2a(g-\gamma)-1]p + a(g-\gamma)^2-(g-\gamma)+1\ ] \tag{39}$$

The quadratic equation in the brackets always has two real roots, p_L and p_H, and we have

$$
\begin{aligned}
&\frac{\partial n_\alpha}{\partial P} < 0 \Leftrightarrow p_L < p < p_H \\
&\frac{\partial n_\alpha}{\partial P} > 0 \Leftrightarrow p < p_L \ or \ p > p_H
\end{aligned}
\tag{40}
$$

When the rebels believe that the current mispricing is large, they act as momentum traders. Note that this behavior does not depend on the risk aversion of the rebels.

□

The properties of $\partial n_\alpha/\partial P$ are illustrated in Figure 1, which shows how the partial derivative $\partial n_\alpha/\partial P$, with the price level fixed at $P = 50$ at time $t = 0.5$, varies with different beliefs about the "right" price level $\exp(\gamma)$ and with risk aversion. The three lines correspond to rebels' risk aversion that is higher, equal to and lower than the risk aversion of the representative conformist. The most risk-tolerant rebels have the largest deviations.

Figure 1

The figure reveals the characteristic behavior of the derivative – it is positive when the rebels believe that the "right" price $\exp(\gamma)$ is much higher or lower than the current price, and it is negative for a range of values close to the current price. This behavior is independent of the risk aversion of the agent, but is driven by the deviant beliefs of the rebel.

The above result is rather surprising – the optimal trading for the rebels who believe that the market is grossly mispriced is to buy after the price went up, or to sell after it went down! Moreover, certain risk-averse rebels trade on the other side of equally risk-averse market conformists. To better understand the rationale of this optimal momentum trading, consider the fraction of the rebel's current wealth that is invested in the market. Figure 2 shows this fraction for the three rebels considered above, as their beliefs about the "right" price vary.

Figure 2

Observe that when the rebels believe that the market is currently overpriced they short it. Conversely, the rebels increase their market exposure when they believe that the "right" price is above the current price. This behavior is usually characterized as value investing. Intuitively, the rebels' beliefs in mean-reversion imply that the investment opportunity set is changing as the price moves. Had they believed in the log-normal market consensus, the rebels would have maintained their market exposure constant. The following theorem captures this distinction.

Theorem 3: The rebels are contrarian relative to the market conformists.

Proof:

We obtain from (33) above

$$\frac{\partial w_\alpha(t)}{\partial P} = \frac{a}{P} < 0 = \frac{\partial w_M(t)}{\partial P} \tag{41}$$

where we are using the same notation. This is the result we wanted to prove. ❑

Notice that $\partial w_\alpha/\partial P$ does not depend on the rebel's beliefs about the "right" price exp(γ). How contrarian relative to the market the rebels appear depends only on their risk aversion.

The above results show that the rebels can be momentum traders, while they are contrarian relative to the market. This behavior is due to a dominant wealth effect. As the current price moves away from the "right" price, the rebels act as relative contrarians, increasing their (possibly negative) excess exposure. However, they also believe that they are wealthier, because of the higher expected return from the expected price reversion. As a result, the exposure increase from the price change is more than the optimal one and the rebels trade market units as momentum traders.

We are now ready to compare the performance of the rebel's and the conformist's strategies. In the next section we consider the external observer-econometrician, who attempts to discriminate empirically between the agents' beliefs based on their performance.

III. Performance of the Rebels and the Conformists

Clearly, with their incorrect beliefs, the rebels cannot do as well as the market conformist. To examine the difference in performance, we will compare the wealth of the two types of agents in different states, as well as their terminal wealth distributions.

A. Wealth in Different States

The wealth of an agent at the terminal date is equal to the number of digital options held for the realized market state. Therefore, the terminal wealth as a function of the state is just the agent's demand schedule for digital options. More generally, we can solve for the current wealth in different states P at time $t \leq T$.

Proposition 7: The conformist's wealth at time $t \leq T$ is

$$W_M^*(P) = m_t \, P^{\frac{\eta^*}{\eta}} \tag{42}$$

where

$$m_t = W_0 \, P_0^{-\frac{\eta^*}{\eta}} e^{(1-\frac{\eta^*}{\eta})(r+\frac{\eta^*}{\eta}\frac{\sigma^2}{2})(t-t_0)} \tag{43}$$

Proof: in appendix II. □

We pointed out before that the critical parameter of the conformist's trading program is the risk aversion. Consequently we expect the conformist's performance to show strong dependence on risk-aversion. Figure 3 shows the terminal wealth of three conformists, who have different risk-aversion parameters.

Figure 3

Since risk-tolerant conformists have more market exposure than risk-averse ones, a market appreciation benefits them more *ex-post* and a market decline hurts them more.

Consider now the rebel's current wealth as a function of the state.

Proposition 8: The rebel's wealth at time $t \leq T$ is

$$W_{\alpha}^{*}(P) = \kappa \, P^{a_t \ln P + \beta_t} \tag{44}$$

where

$$a_t = -\frac{\theta_0 - 1}{2\nu_t}, \quad \beta_t = \frac{\theta_0 L_0 - Q_0 - (\theta_0 - 1)\tau}{\nu_t}, \quad \nu_t = \eta\sigma^2\tau_0 + (\theta_0 - 1)\sigma^2\tau$$

$$\kappa = \frac{W_0}{P_0^{a_0 \ln P_0 + \beta_0}} \sqrt{\frac{\nu_0}{\nu_t}} \, e^{\left[r - \frac{(r-\sigma^2/2)^2}{\sigma^2} - \frac{\sigma^2[\theta_0 L_0 - Q_0 + \eta(r-\sigma^2/2)\tau]^2}{\nu_0 \nu_t} \right](t-t_0)} \tag{45}$$

Proof: in appendix III. □

We emphasized before the key role that the rebels' beliefs play in their trading. Figure 4 shows the terminal wealth of four rebels, who have the same risk-aversion, but differ in their beliefs about the market. First, there are two types of rebels according to their beliefs about the "right" log-price level. Rebels of the first type believe that the "right" price is lower than the current price, so that the market is currently overpriced. The other type believes that the market is currently underpriced. The second aspect of the rebels' beliefs is the speed of the expected log-price correction, i.e. the reversion horizon. We consider rebels who either believe in fast, or in slow reversion, with expected log-price reversion over 1 and 5 years respectively.

Figure 4

The striking feature of the rebel's beliefs is the sharp concentration of maximum wealth around the state corresponding to the "right" price level exp(γ). Intuitively, the path realizations that are more consistent with reversion beliefs are more profitable for the rebels, whose trading program is predicated on the highest likelihood of these paths. The subjective speed of reversion also has a dramatic effect. When the rebels believe in faster reversion, they bet more aggressively on any perceived mispricing and their wealth has higher peak, and is more concentrated around the "right" price level.

Figures 3 and 4 also highlight the crucial qualitative difference between the wealth of the two types of agents in different states. While the conformist's wealth is monotonely increasing in the price, the rebel's wealth reaches a global maximum at some finite price, and then decreases as the price-state increases. Intuitively, when the log-price is above the "right" reversion level, the rebel believes that further increases are less likely. Accordingly, when the price keeps rising further, the rebels sell more of the market, realizing losses and consequently their terminal wealth is lower in these high market states. We now turn to the terminal wealth distributions of the two agents.

B. Wealth Distributions

Since P_T/P_t is log-normally distributed, we can compute the distribution of wealth from the terminal wealth functions in the previous section. The terminal wealth density for the conformist has analytical representation, but the inversion of the rebel's wealth in different states had to be carried out numerically, as described below.

Proposition 9: The market conformist's terminal wealth density is given by

$$f_{W_M(P_T)}(x) = \frac{1}{x\sigma\sqrt{T-t}}\Phi\left(\frac{\ln[W_M^{-1}(x)/P]-(\mu-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}\right) \tag{46}$$

where

$$W_M^{-1}(x) = P\ e^{(1-\frac{\eta}{\eta^*})(r+\frac{\eta^*}{\eta}\frac{\sigma^2}{2})(T-t)}\left(\frac{x}{W_t}\right)^{\frac{\eta}{\eta^*}} \tag{47}$$

Proof: see appendix II. □

The properties of the wealth density are inherited from the properties of the demand schedule for digital options. Figure 5 shows the wealth densities for the three conformists from Figure 3, who differ only in their risk aversion.

Figure 5

The more risk-tolerant conformists have a density of terminal wealth which is less concentrated, with the left-most peak and the fattest right tail. Intuitively, investing with higher risk-tolerance involves more leverage and a higher probability of losses – the left-most peak location, as well as a greater probability of high terminal wealth outcomes, as reflected in the fattest right tail. Conversely, the most risk-averse conformists are most concentrated, with the right-most peak location and the thinnest right tail. With the lowest leverage, they are less likely to incur large losses, or gains. Now, let us consider the rebels.

Proposition 10: The rebel's wealth distribution is given by

$$
\begin{aligned}
f_{W_\alpha(P_T)}(x) &= f_{P_T}[W_\alpha^{-1}(x)]W_\alpha^{-1}(x)' \\
&= \frac{1}{W_\alpha^{-1}(x)\sigma\sqrt{T-t}\;\; W'_\alpha(P_T)_{|P_T=W_\alpha^{-1}(x)}}\Phi\left(\frac{\ln(W_\alpha^{-1}(x)/P)-(\mu-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}\right)
\end{aligned}
\tag{48}
$$

where W_α is given in (44).

Proof: see appendix III. □

Consider the wealth densities of rebels who have different beliefs. Figure 6 shows the densities of the four rebels considered before (Figure 4), who believe in slow and fast reversion, and in a current under- and over-pricing respectively.

Figure 6

The densities for the two believers in slow reversion are concentrated over the shorter intervals to the right. The beliefs in long-term reversion imply that there are smaller bets on the unlikely short-term market collapse, so the rebels have small probability of ruin. Of these two rebels, the one who believes in current underpricing has the higher probability of achieving higher wealth. Intuitively, the slow upward reversion beliefs are most reasonable from the market's perspective.

Conversely, rebels who believe in short-term reversion have densities that are more dispersed and U-shaped, with high probability of ruin. Intuitively, the far-fetched beliefs lead to betting on highly-unlikely outcomes, and are characterized by more dispersed densities. The rebels who believe in current overpricing are most dispersed, as their heavy betting on near-term collapse implies higher wealth in that unlikely outcome. These rebels' beliefs are the most unreasonable from the market perspective. In the likely event of a market rally, their bets against the market are the largest so that they are penalized by the highest probability of ruin. Simply put, the rebels who believe in short-term reversion appear much less risk-averse than the ones who believe in long-term reversion.

Figure 6 also highlights the crucial qualitative difference between the terminal wealth densities of the conformists and the rebels. This difference can be better understood in conjunction with the hump-shaped rebel's wealth in different states in Figure 4. As the non-monotone function $W_\alpha *$ increases, it reaches a global maximum, so that it has a non-empty inverse function only over a finite interval. Using this inverse to compute the probability density for the terminal wealth implies that the density is discontinuous at the finite maximum wealth attainable by the rebel, jumping to zero after its maximum value. Thus, the rebel's wealth density is compactly supported. In contrast, the market conformist can reach an arbitrarily large wealth with non-zero probability.

We now compare the agents' performance. Is it always the case that the conformist has, in a meaningful way, better risk-adjusted performance? Our main result is stated in the following theorem.

Theorem 4: Market conformists' terminal wealth distributions do not first-order stochastically dominate the rebels' terminal wealth distributions for all parameter classes, but there are ranges for which the market conformist's distribution does first-order stochastically dominate the rebel's distribution.

Proof:

The easiest way to prove the theorem is to exhibit one case for each scenario. Figure 7 shows the two cases considered in the theorem.

Figure 7

The first panel shows that rebels who believe in long-term market reversion to a higher level are not dominated by risk-averse conformists, nor vice versa. Intuitively, the rebels' beliefs in this case are not so far-fetched from the market perspective, so that their trading strategy is predicated on the high likelihood of paths whose realization is in fact quite probable according to the market consensus distribution.

The second panel shows that the distribution of a risk-tolerant conformists dominates the distribution of fatalist rebels who believe that the market will tank fairly soon. Intuitively, as the market is likely to rise, the risk-tolerant conformist's bet on the chance for large gains is better aligned with the probable market paths than the doomsday rebel strategy that only pays off in the highly unlikely event of an imminent market crash.

□

IV. Different Beliefs or Different Risk Aversion?

Based on performance data alone, can econometricians determine whether an agent acted as a market conformist or a deviant rebel? To answer this, we consider some examples of conformist and rebel terminal wealth densities.

Figure 8 shows the densities of a risk-tolerant conformist and of a rebel who believes in short-term reversion to a much lower market level. The densities of the two agents are "close" over the most relevant range of terminal wealth outcomes.

Figure 8

Distinguishing between the two distributions is clearly difficult in the absence of extreme observations, but the qualitative difference between the two densities has an intuitive economic explanation. The conformists with low risk-aversion are likely to have losses if the market falls, but since they optimize based on the correct process, they cannot be bankrupt. If the market takes-off, their gains potential is unlimited. Contrast this with the pessimistic rebels, whose losses mount without limit as the market price increases towards infinity, while in the unlikely event of a market collapse, the fatalists have limited rewards.

To recapitulate, although the densities of the two agents are close, the same wealth outcome for the agents occurs in exactly the opposite market scenario realizations. This analysis suggests that very bearish rebels should fare poorly in good times, so that a time-series regression methodology may uncover their inadequate beliefs.

Figure 9 shows another case of similar-shaped densities. The densities of a conformist with high risk aversion and a rebel who believes in long-term upward reversion essentially overlap over the most probable states.

Figure 9

The two densities are qualitatively different. The conformist density has the feel of the underlying log-normal density, with infinite right tail and vanishing probability of ruin. The rebel density is compactly-supported, with the characteristic jump driven by the bounded maximum wealth. In this case, distinguishing between the two agent types with high confidence is likely to be infeasible with regression methods and limited data.

In general, the parameters of the rebel's beliefs affect the shape of the implied terminal wealth density. The density always retains its qualitative features – the compact support and the jump to zero – which are the result of the hump-shaped function for the wealth in different states. However, the parameters affect the location of the peak of the hump, as well as the hump dispersion. When the hump is massed around the outcomes which are more likely to occur for the log-normal market density, the rebel's beliefs appear more reasonable from the market perspective. Accordingly, the rebel's terminal wealth density overlaps more with the conformist's density, and the rebel's distribution is not dominated by the conformist's distribution. Intuitively, rebels whose beliefs are not unreasonable from the market perspective, invest in a way which makes more sense to the conformist. The above examples are summarized in the following

Observation: When the rebels' beliefs are not unreasonable from the market's perspective, their terminal wealth densities are not very different from the market conformists' terminal wealth densities.

The lesson from this section presents a challenge for the empiricist. In the absence of extreme observations beyond the compact support of the rebel's terminal wealth density, an econometrician may not be able to distinguish the agents' beliefs using only performance data. Not knowing the risk aversion of the agents further complicates the problem.

V. Investing for the Long Run

The common wisdom appears to be that agents who invest for the long run should hold portfolios that are heavily biased towards equities. This clearly depends on preferences and, more generally speaking, the optimal long-run investment is a function of what the agent believes. Agents who hold plausible beliefs have optimal long-run portfolio that need not be dominated by equities.

In this section we explore the limiting case of our previous results, as the investment horizon increases, i.e. letting terminal date $T \to \infty$.

A. The Conformists

A closer inspection of Proposition 4 reveals the well-known result that in the absence of changes in the investment opportunity set, the conformist's fractional investment in the market does not depend on the terminal date T. The fractional exposure to the market, ξ_M, is constant,

$$\xi_M = \frac{\eta^*}{\eta} \tag{49}$$

Increasing the investment horizon $T \to \infty$ does not affect the conformist's optimal exposure to the market. Given an opportunity set (μ, σ, r), the conformist's market exposure is completely determined by the risk aversion parameter η.

B. The Rebels

To find out how the long-term investment horizon affects the rebel's fractional investment in the market, we let $\tau \to \infty$ in Proposition 6. The market exposure ξ_α is

$$\xi_\alpha = a \ln P + b \to \frac{1}{2} - \frac{r}{\sigma^2} \tag{50}$$

where

$$a = -\frac{1-e^{-\alpha\tau}}{[1-(1-\eta)/\theta]\sigma^2\tau} \to 0 \ , \ b = \frac{\gamma(1-e^{-\alpha\tau})-(r-\sigma^2/2)\tau}{[1-(1-\eta)/\theta]\sigma^2\tau} \to \frac{1}{2} - \frac{r}{\sigma^2} \tag{51}$$

Figure 10 illustrates the effect of increasing the time horizon $\tau \to \infty$ on the fraction of the wealth invested in the market, for the four rebels with different beliefs that we considered in the previous sections.

Figure 10

Depending on whether they believe that the market is underpriced or overpriced, the rebels are long or short the market when the holding period is short. However, as we increase the investment horizon, the market exposure of all agents converges towards a fixed positive value. Intuitively, as the investment horizon increases, the rebels find that the market is more likely to revert to its intrinsic growth rate, which has a fixed risk-return tradeoff. The rebel's long-run portfolio balances this long-run risk-return tradeoff and the riskless investment according to the rebel's risk aversion.

Observe that the long-run portfolio is not dominated by equities, in the sense that the equity exposure is less than 0.5. Both the conformist's and the rebel's long-run portfolios include a positive equity component, but equities need not dominate even the very long-term investment portfolio.

VI. Conclusion

In this paper, we have developed a model of optimal investment under different beliefs in a noisy market. We have considered rebels who do not conform to the market beliefs. We assumed that instead of using the market's log-normal specification for the market price, the rebels believe in log-price reversion towards a "right" level. This setup can be reversed – we could posit log-price reversion of the market beliefs, while the non-conformist rebels would believe in a deviant log-normal specification. The analytical results that we have developed are sufficient to consider the 2 x 2 scenario table, which has log-normal or log-reverting, conformists or rebels. First, we have characterized the trading behavior of the agents, which is the same in either scenario. Next, the investment performance is symmetric, with market conformist wealth distributions that can dominate the terminal wealth distributions of rebel types, whose beliefs are far-fetched from the market perspective. Finally, the task of empirically distinguishing certain agents' beliefs using performance results alone, remains tenuous in all scenarios.

Focusing on the intuitive case of a log-normal market and a log-reverting rebel, our main conclusions are as follows. First, momentum trading is rational for rebels, who believe that the market is grossly mispriced. These rebels take the opposite side, trading with contrarian risk-averse conformists. Momentum trading is also optimal for the market conformists with high risk tolerance. These results are rather surprising, because momentum trading has long appeared counter-intuitive in a utility-maximization context. These optimal momentum strategies can be rationalized by interpreting the rebels as value investors, who buy in a subjectively underpriced market and sell when they believe the market is overpriced. On the other hand, the conformists' momentum is optimal when their risk aversion is higher than the risk aversion of the representative agent.

The performance of the conformists compares favorably to that of the rebels in the following sense. For a given utility function, the terminal wealth densities of the conformists are superior to the densities of the rebels. Moreover, if the rebel's beliefs are far-fetched from the market perspective, e.g. when a market crash is anticipated in the near future, the wealth distribution of risk-tolerant conformists first-order dominates the rebel's terminal wealth distribution.

The wealth densities of the conformists and the rebels are qualitatively different. The investment program which is optimal given the rebel's subjective beliefs in log-reversion allows for a limited upside potential. The bounded maximum terminal wealth implies that the rebels are characterized by a compactly-supported wealth density function.

Our results provide a theoretical foundation for the difficulties that lay in store for the econometrician attempting to distinguish agent types using only limited performance data. Consider, for example, the implications for the measurement of investment fund performance. Suppose that different beliefs are the result of different use of imperfect public information, e.g. asset allocation based on subjective interpretation of news about the global economy, or stock-picking based on proprietary financial analysis. Our results indicate that in the absence of more detailed information, the plausible translation of performance into superior ability is not feasible. Rebels whose beliefs are more reasonable from the market perspective usually appear as risk-averse conformists, while rebels with far-fetched beliefs may attempt to pose as conformists with high risk-tolerance.

Appendix I: The Basics

1. The α-density

Writing the Ornstein-Uhlenbeck process for p as

$$p(t) \;=\; \gamma \;+\; e^{-\alpha t} B[\frac{\sigma^2}{2\alpha}(e^{2\alpha t}-1)]$$

we obtain the conditional probability density

$$\pi_\alpha(p_T|p;\tau) \;=\; \frac{1}{\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\Phi\left(\frac{p_T-\gamma-(p-\gamma)e^{-\alpha\tau}}{\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\right)$$

Hence, $P = \exp(p)$ is lognormally distributed as

$$\pi_\alpha(P_T|P;\tau) \;=\; \frac{1}{P_T\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\Phi\left(\frac{\ln P_T-\gamma-(\ln P-\gamma)e^{-\alpha\tau}}{\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\right)$$

□

2. Digital Options

The Arrow-Debreu security's hedge ratio is

$$\begin{aligned}\Delta_{q[P;P_T]} \;&=\; \frac{\partial q[P;P_T]}{\partial P} \\ &=\; e^{-r\tau}\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{P_T P\sigma^3\sqrt{\tau^3}}\Phi\left(\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right)\end{aligned}$$

Set $Q \;=\; \ln P + (r-\sigma^2/2)\tau$ and $p_T \;=\; \ln P_T$, so that $dp_T \;=\; dP_T/P_T$. For the number $n(t)$ of units of the risky asset in the hedge portfolio, it follows from the above that

$$
\begin{aligned}
n(t) &= \Delta_{q[P;P_T]} dP_T \\
&= e^{-r\tau}\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{P_T P\sigma^3\sqrt{\tau^3}}\Phi\left(\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right)dP_T \\
&= e^{-r\tau}\frac{p_T-[\ln P+(r-\sigma^2/2)\tau]}{P\sigma^3\sqrt{\tau^3}}\Phi\left(\frac{p_T-[\ln P+(r-\sigma^2/2)\tau]}{\sigma\sqrt{\tau}}\right)dp_T \\
&= \frac{e^{-r\tau}}{P\sqrt{2\pi}\sigma^3\sqrt{\tau^3}}(p_T-Q)\exp\left(-\frac{[p_T-Q]^2}{2\sigma^2\tau}\right)dp_T
\end{aligned}
$$

□

Appendix II: The Conformists

1. The Constant $\lambda_M(t)$

Setting $\tau = T-t$, $a = \frac{\mu-r}{\eta\sigma^2}$, $b = \frac{\mu-r}{2\eta\sigma^2}[2\ln P+(\mu+r-\sigma^2)\tau]$, $Q = \ln P+(r-\frac{\sigma^2}{2})\tau$ and

$p_T = \ln P_T$, and using the defining expression for D_M^*, we obtain, for $0 \le t < T$

$$
\begin{aligned}
W_t &= \int_{P_T} D_M^*(P_T;\lambda_M(t))q[P;P_T]dP_T \\
&= (\lambda_M(t)e^{-r\tau})^{-\frac{1}{\eta}}\frac{e^{-r\tau}}{\sigma\sqrt{\tau}}\int_{-\infty}^{\infty} e^{\frac{1}{\eta}\frac{[\mu-r][2\ln(P_T/P)-(\mu+r-\sigma^2)\tau]}{2\sigma^2}}\Phi\left(\frac{p_T-Q}{\sigma\sqrt{\tau}}\right)dp_T \\
&= (\lambda_M(t)e^{-r\tau})^{-\frac{1}{\eta}}\frac{e^{-r\tau-b}}{\sigma\sqrt{\tau}}\int_{-\infty}^{\infty}\exp(ap_T)\Phi\left(\frac{p_T-Q}{\sigma\sqrt{\tau}}\right)dp_T
\end{aligned}
$$

so that

$$
\begin{aligned}
(\lambda_M(t)e^{-r\tau})^{\frac{1}{\eta}} &= \frac{e^{-r\tau-b}}{W_t\sqrt{2\pi\sigma^2\tau}}\int_{-\infty}^{\infty}\exp\left(ap_T-\frac{(p_T-Q)^2}{2\sigma^2\tau}\right)dp_T \\
&= \frac{e^{-r\tau-b}}{\sqrt{2\pi}W_t\sigma\sqrt{\tau}}\int_{-\infty}^{\infty}\exp\left(-\frac{p_T^2-2(Q+a\sigma^2\tau)p_T+Q^2}{2\sigma^2\tau}\right)dp_T \\
&= e^{-r\tau-b}W_t\exp\left(aQ+\frac{a^2\sigma^2\tau}{2}\right) \\
&= \frac{e^{-r\tau}}{W_t}e^{-\frac{\mu-r}{2\eta\sigma^2}[2\ln P+(\mu+r-\sigma^2)\tau]+\frac{\mu-r}{\eta\sigma^2}[\ln P+(r-\frac{\sigma^2}{2})\tau]+\frac{(\mu-r)^2}{2\eta^2\sigma^2}\tau}
\end{aligned}
$$

$$= \frac{e^{-r\tau}}{W_t} e^{\frac{(1-\eta)(\mu-r)^2}{2\eta^2\sigma^2}\tau}$$

and hence

$$\lambda_M(t)^{-\frac{1}{\eta}} = W_t\, e^{-\frac{1-\eta}{\eta}[r - \frac{(\mu-r)^2}{2\eta\sigma^2}]\tau}$$

□

2. Demand for Digital Options

For the market conformist, the optimum demand schedule D_M^* for digital options $q[P;P_T]$ with strike price P_T at time T is given by

$$D_M^*(P_T;\lambda_M(t)) = \{e^{-r\tau}\lambda_M(t)\frac{\Phi\left(\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right)}{\Phi\left(\frac{\ln(P_T/P)-(\mu-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right)}\}^{-\frac{1}{\eta}}$$

$$= W_t\, e^{r\tau - \frac{(1-\eta)(\mu-r)^2}{2\eta^2\sigma^2}\tau} \exp\left(\frac{\mu-r}{\eta\sigma^2}[\ln P_T/P - \frac{\mu+r-\sigma^2}{2}\tau]\right)$$

$$= W_t \left(\frac{P_T}{P}\right)^{\frac{\mu-r}{\eta\sigma^2}} e^{(r+\frac{\mu-r}{2\eta})(1-\frac{\mu-r}{\eta\sigma^2})\tau}$$

3. Demand for the Risky Asset

Set $a \equiv \frac{\mu-r}{\eta\sigma^2}$, $A \equiv \frac{1}{2\sigma^2\tau}$, $Q = \ln P + (r - \frac{\sigma^2}{2})\tau$, $B = Q + a\sigma^2\tau$, $C = Q^2$

$$k \equiv W_t\, e^{r\tau - \frac{\mu-r}{\eta\sigma^2}\{\ln P + [\frac{\mu-r}{2\eta} + r - \frac{\sigma^2}{2}]\tau\}} , \quad K \equiv \frac{k\, e^{-r\tau}}{P\sqrt{2\pi}\sigma^3\sqrt{\tau^3}}$$

For the conformist, the stock position $n_M(t)$ that replicates the digital options is

$$n_M(t) = \int_0^\infty D_M^*(P_T;\lambda_M(t))\Delta_{q[P;P_T]}(t)dP_T =$$

$$= k\int_0^\infty \exp(a \ln P_T)\Delta_{q[P;P_T]} dP_T$$

$$
\begin{aligned}
&= K\int_{-\infty}^{\infty}\exp\left(ap_T - \frac{(p_T - Q)^2}{2\sigma^2\tau}\right)(p_T - Q)dp_T \\
&= K\int_{-\infty}^{\infty}\exp(-Ap_T^2 - 2Bp_T + C)(p_T - Q)dp_T \\
&= K\sqrt{\pi A}(B - Q)\exp[A(B^2 - C)] \\
&= K\sqrt{2\pi\sigma^2\tau}\frac{\mu - r}{\eta\sigma^2}\sigma^2\tau \ \exp\left(\frac{\mu - r}{\eta\sigma^2}[\ln P + (r - \frac{\sigma^2}{2} + \frac{\mu - r}{2\eta})\tau]\right) \\
&= k\frac{e^{-r\tau}}{P}\frac{\mu - r}{\eta\sigma^2}e^{\frac{\mu - r}{\eta\sigma^2}\{\ln P + [r - \frac{\sigma^2}{2} + \frac{\mu - r}{2\eta}]\tau\}} \\
&= \frac{\mu - r}{\eta\sigma^2}\frac{W_t}{P}
\end{aligned}
$$

where we substituted $\lambda_M(t)^{-\frac{1}{\eta}}$ from section 1 of the appendix.

□

4. Wealth at time $t \le T$

Using the expression for D_M^* from part 2 of this appendix, we have,

$$
\begin{aligned}
W_t &= \int_{P_T} D_M^*(P_T;\lambda_M(0))q[P;P_T]dP_T \\
&= W_0\ P_0^{-\frac{\mu - r}{\eta\sigma^2}} e^{(r + \frac{\mu - r}{2\eta})(1 - \frac{\mu - r}{\eta\sigma^2})\tau_0} e^{-r\tau}\int_{-\infty}^{\infty}\exp(ap_T)\Phi\left(\frac{p_T - Q_t}{\sigma\sqrt{\tau}}\right)dp_T \\
&= W_0\ P_0^{-\frac{\mu - r}{\eta\sigma^2}} e^{(r + \frac{\mu - r}{2\eta})(1 - \frac{\mu - r}{\eta\sigma^2})\tau_0} P_0^{\frac{\mu - r}{\eta\sigma^2}} e^{-(r + \frac{\mu - r}{2\eta})(1 - \frac{\mu - r}{\eta\sigma^2})\tau} \\
&= W_0\ e^{(r + \frac{\mu - r}{2\eta})(1 - \frac{\mu - r}{\eta\sigma^2})(t - t_0)}
\end{aligned}
$$

where we used notation from part 1 of this appendix and a similar argument.

□

5. Terminal Wealth Density

The inverse to the conformist wealth function $W_M(P_T) = W_t\, e^{(r+\frac{\mu-r}{2\eta})(1-\frac{\mu-r}{\eta\sigma^2})(T-t)}\left(\frac{P_T}{P}\right)^{\frac{\mu-r}{\eta\sigma^2}}$ is

$$W_M^{-1}(x) = P\, e^{-\frac{\eta\sigma^2}{\mu-r}(r+\frac{\mu-r}{2\eta})(1-\frac{\mu-r}{\eta\sigma^2})(T-t)}\left(\frac{x}{W_t}\right)^{\frac{\eta\sigma^2}{\mu-r}}$$

so that

$$\begin{aligned} W_M^{-1}(x)' &= \frac{\eta\sigma^2}{\mu-r}\frac{P}{x}\, e^{-\frac{\eta\sigma^2}{\mu-r}(r+\frac{\mu-r}{2\eta})(1-\frac{\mu-r}{\eta\sigma^2})(T-t)}\left(\frac{x}{W_t}\right)^{\frac{\eta\sigma^2}{\mu-r}} \\ &= \frac{\eta\sigma^2}{\mu-r}\frac{W_M^{-1}(x)}{x} \end{aligned}$$

Hence the density $f_{W_M(P_T)}(x)$ of the wealth function $W_M(P_T)$ is

$$\begin{aligned} f_{W_M(P_T)}(x) &= f_{P_T}[W_M^{-1}(x)]W_M^{-1}(x)' \\ &= \frac{\eta\sigma^2}{\mu-r}\frac{W_M^{-1}(x)}{x}\frac{1}{W_M^{-1}(x)\sigma\sqrt{\tau}}\Phi\left(\frac{\ln[W_M^{-1}(x)/P]-(\mu-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}\right) \\ &= \frac{\eta\sigma^2}{\mu-r}\frac{1}{x\sigma\sqrt{\tau}}\Phi\left(\frac{\ln[W_M^{-1}(x)/P]-(\mu-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}\right) \end{aligned}$$

where the first equality follows from the next section.

□

Appendix III: The Rebels

1. The Constant $\lambda_\alpha(t)$

Set $\theta = 2\alpha\tau/(1-e^{-2\alpha\tau})$, $k = (\lambda_\alpha e^{-r\tau}\sqrt{1/\theta})^{-\frac{1}{\eta}}$, $Q = \ln P+(r-\sigma^2/2)\tau$, $L = \gamma+(\ln P-\gamma)e^{-\alpha\tau}$. Using the defining expression for D_α^*, the rebel's constant is found from

$$
\begin{aligned}
W_t &= \int_{P_T} D^*_\alpha(P_T;\lambda_\alpha(t))q[P;P_T]dP_T \\
&= k\frac{e^{-r\tau}}{\sigma\sqrt{\tau}}\int_{-\infty}^{\infty}\exp\left(\frac{[p_T-Q]^2-\theta[p_T-L]^2}{2\eta\sigma^2\tau}\right)\Phi\left(\frac{p_T-Q}{\sigma\sqrt{\tau}}\right)dp_T \\
&= k\frac{e^{-r\tau}}{\sqrt{2\pi\sigma^2\tau}}\int_{-\infty}^{\infty}\exp\left(-\frac{\theta[p_T-L]^2-(1-\eta)[p_T-Q]^2}{2\eta\sigma^2\tau}\right)dp_T \\
&= k\frac{e^{-r\tau-\frac{\theta L^2-(1-\eta)Q^2}{2\eta\sigma^2\tau}}}{\sqrt{2\pi\sigma^2\tau}}\int_{-\infty}^{\infty}\exp\left(-\frac{[\theta-(1-\eta)]p_T^2-2[\theta L-(1-\eta)Q]p_T}{2\eta\sigma^2\tau}\right)dp_T \\
&= ke^{-r\tau}\sqrt{\frac{\eta}{\eta+\theta-1}}\exp\left(\frac{[(1-\eta)Q-\theta L]^2}{2\eta(\eta+\theta-1)\sigma^2\tau}-\frac{\theta L^2-(1-\eta)Q^2}{2\eta\sigma^2\tau}\right)
\end{aligned}
$$

so that

$$
(\lambda_\alpha(t)e^{-r\tau}\sqrt{1/\theta})^{-\frac{1}{\eta}} = W_t\sqrt{(\theta-(1-\eta))/\eta}\; e^{r\tau-\frac{\theta(1-\eta)}{\theta-(1-\eta)}\frac{(Q-L)^2}{2\eta\sigma^2\tau}}
$$

$$
\lambda_\alpha^{-\frac{1}{\eta}} = (e^{r\tau}\sqrt{\theta})^{-\frac{1}{\eta}}W_t\sqrt{(\theta-(1-\eta))/\eta}\; e^{r\tau-\frac{\theta(1-\eta)}{\theta-(1-\eta)}\frac{(Q-L)^2}{2\eta\sigma^2\tau}}
$$

□

2. Demand for Digital Options

Using the same notation, the optimum demand schedule D^*_α for digital options $q[P;P_T]$ with strike price P_T at time T is given by

$$
\begin{aligned}
D^*_\alpha(P_T;\lambda_\alpha(t)) &= \left\{e^{-r\tau}\lambda_\alpha(t)\sqrt{(1-e^{-2\alpha\tau})/2\alpha\tau}\frac{\Phi\left(\frac{\ln(P_T/P)-(r-\sigma^2/2)\tau}{\sigma\sqrt{\tau}}\right)}{\Phi\left(\frac{\ln P_T-\gamma-(\ln P-\gamma)e^{-\alpha\tau}}{\sigma\sqrt{(1-e^{-2\alpha\tau})/2\alpha}}\right)}\right\}^{-\frac{1}{\eta}} \\
&= k\ \exp\left(\frac{[p_T-Q]^2-\theta[p_T-L]^2}{2\eta\sigma^2\tau}\right) \\
&= k\ \exp\left(\frac{(1-\theta)p_T^2-2(Q-\theta L)p_T+Q^2-\theta L^2}{2\eta\sigma^2\tau}\right)
\end{aligned}
$$

$$= W_t\, e^{r\tau}\sqrt{\frac{\eta+\theta-1}{\eta}}\,e^{\frac{(1-\theta)p_T^2-2(Q-\theta L)p_T+Q^2-\theta L^2}{2\eta\sigma^2\tau}+\frac{\theta L^2-(1-\eta)Q^2}{2\eta\sigma^2\tau}-\frac{[(1-\eta)Q-\theta L]^2}{2\eta(\eta+\theta-1)\sigma^2\tau}}$$

$$= W_t\, e^{r\tau}\sqrt{\frac{\eta+\theta-1}{\eta}}\,e^{\frac{(1-\theta)p_T^2-2(Q-\theta L)p_T+\eta Q^2}{2\eta\sigma^2\tau}-\frac{[(1-\eta)Q-\theta L]^2}{2\eta(\eta+\theta-1)\sigma^2\tau}}$$

$$= W_t\, e^{r\tau}\sqrt{\frac{\eta+\theta-1}{\eta}}\,e^{\frac{Q^2}{2\sigma^2\tau}-\frac{[(1-\eta)Q-\theta L]^2}{2\eta(\eta+\theta-1)\sigma^2\tau}}\,e^{\frac{(1-\theta)p_T^2-2(Q-\theta L)p_T}{2\eta\sigma^2\tau}}$$

$$= Ce^{\frac{(1-\theta)p_T^2-2(Q-\theta L)p_T}{2\eta\sigma^2\tau}}$$

where we set $C = W\sqrt{\frac{\theta-(1-\eta)}{\eta}}\,e^{r\tau+\frac{Q^2}{2\sigma^2\tau}-\frac{[\theta L-(1-\eta)Q]^2}{2\eta[\theta-(1-\eta)]\sigma^2\tau}}$

□

3. Demand for the Risky Asset

Using the notation in the previous sections, and also setting $Q = \ln P+(r-\frac{\sigma^2}{2})\tau$, the stock position $n_\alpha(t)$ that replicates the digital options positions for the rebel is

$$n_\alpha(t) = \int_t^\infty D_\alpha^*(P_T,\lambda_\alpha(t))\Delta_{q[P;P_T]}dP_T$$

$$= \frac{ke^{-r\tau}}{P\sqrt{2\pi\sigma^3}\sqrt{\tau^3}}\int_{-\infty}^{\infty}\exp\left(\frac{[p_T-Q]^2-\theta[p_T-L]^2}{2\eta\sigma^2\tau}-\frac{[p_T-Q]^2}{2\sigma^2\tau}\right)(p_T-Q)dp_T$$

$$= \frac{Ce^{-r\tau}e^{-\frac{Q^2}{2\sigma^2\tau}}}{P\sqrt{2\pi\sigma^3}\sqrt{\tau^3}}\int_{-\infty}^{\infty}e^{-\frac{1}{2\eta\sigma^2\tau}[(\theta-(1-\eta))p_T^2+2((1-\eta)Q-\theta L)p_T]}(p_T-Q)dp_T$$

$$= M\int_{-\infty}^{\infty}\exp[-\frac{bp_T^2+2cp_T}{a}](p_T-Q)dp_T$$

where $M = \frac{Ce^{-r\tau}e^{-\frac{Q^2}{2\sigma^2\tau}}}{P\sqrt{2\pi\sigma^3}\sqrt{\tau^3}}$, $a = 2\eta\sigma^2\tau$, $b = \theta-(1-\eta)$, $c = (1-\eta)Q-\theta L$

so that integrating we get

$$n_\alpha(t) = - M\sqrt{\pi a/b}\ \frac{c+bQ}{b}\ \exp(\frac{c^2}{ab})$$

$$= \frac{Ce^{-r\tau-\frac{Q^2}{2\sigma^2\tau}}}{P\sigma^2\tau}\sqrt{\frac{\eta}{\theta-(1-\eta)}}\frac{\theta(L-Q)}{\theta-(1-\eta)}e^{\frac{[\theta L-(1-\eta)Q]^2}{2\eta[\theta-(1-\eta)]\sigma^2\tau}}$$

$$= \frac{\theta(L-Q)}{[\theta-(1-\eta)]\sigma^2\tau}\ \frac{W}{P}$$

where we substituted λ_α from section 1 of the appendix.

□

4. Wealth at time $t \leq T$

Using the expression for D_M^* from part 2 of this appendix, we have,

$$W_t = \int_{P_T} D_\alpha^*(P_T,T;\lambda_\alpha)q[P;P_T]dP_T$$

$$= C\ \frac{e^{-r\tau}}{\sigma\sqrt{\tau_0}}\int_{-\infty}^{\infty}\exp\left(\frac{(1-\theta_0)p_T^2-2(Q_0-\theta_0L_0)p_T}{2\eta\sigma^2\tau_0}\right)\Phi\left(\frac{p_T-Q_t}{\sigma\sqrt{\tau}}\right)dp_T$$

$$= C\ \frac{e^{-r\tau-\frac{Q_t^2}{2\sigma^2\tau}}}{\sqrt{2\pi\sigma^2\tau_0}}\int_{-\infty}^{\infty}\exp\left(-\frac{(\eta\tau_0-(1-\theta_0)\tau)p_T^2-2(\eta\tau_0Q+(\theta_0L_0-Q_0)\tau)p_T}{2\eta\sigma^2\tau\tau_0}\right)dp_T$$

$$=C\ \sqrt{\eta/(\eta\tau_0+(\theta_0-1)\tau)}e^{-r\tau+\frac{1}{2\sigma^2\tau\tau_0}\left(-\tau_0Q^2+\frac{[\eta\tau_0Q+(\theta_0L_0-Q_0)\tau]^2}{\eta[\eta\tau_0+(\theta_0-1)\tau]}\right)}$$

$$=W_0\ \frac{\sqrt{\eta\tau_0+(\theta_0-1)\tau_0}}{\sqrt{\eta\tau_0+(\theta_0-1)\tau}}\frac{e^{-r\tau+\frac{1}{2\sigma^2\tau\tau_0}\left(-\tau_0Q^2+\frac{[\eta\tau_0Q+(\theta_0L_0-Q_0)\tau]^2}{\eta[\eta\tau_0+(\theta_0-1)\tau]}\right)}}{e^{-r\tau_0+\frac{1}{2\sigma^2\tau\tau_0}\left(-\tau Q_0^2+\frac{[\eta\tau Q_0+(\theta_0L_0-Q_0)\tau]^2}{\eta[\eta\tau+(\theta_0-1)\tau]}\right)}}$$

where we used notation from part 1 of this appendix and a similar argument.

❑

5. Terminal Wealth Density

This result also applies to the conformist discussed above. We have

$$Pr\{\ W_\alpha(P_T)\ \leq\ z\ \}\ =\ Pr\{\ P_T\ \leq\ W_\alpha^{-1}(z)\ \}$$

so that

$$\begin{aligned} f_{W_\alpha(P_T)}(x)\ &=\ \frac{d}{dz}_{|z=x} Pr\{\ W_\alpha(P_T)\ \leq\ z\ \}\ = \\ &=\ \frac{d}{dz}_{|z=x} Pr\{\ P_T\ \leq\ W_\alpha^{-1}(z)\ \}\ =\ f_{P_T}[W_M^{-1}(x)]W_M^{-1}(x)' \end{aligned}$$

□

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Figure 1

$\partial n_\alpha / \partial P$ for different beliefs and risk aversion

($P_0 = P_{t=0.5} = 50$)

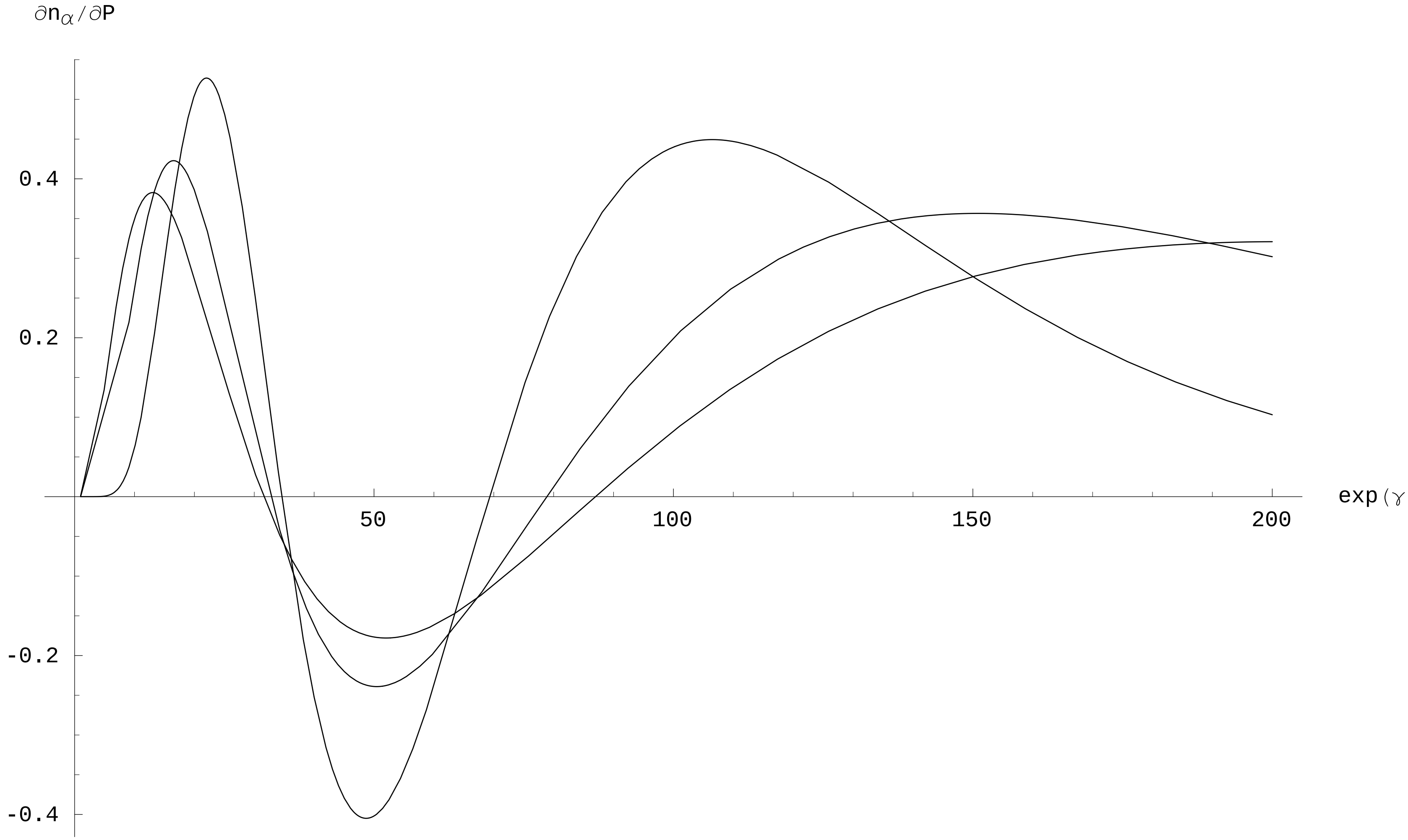


Figure 2

Rebels' exposure for different beliefs and risk aversion

($P_0 = P_{t=0.5} = 50$)

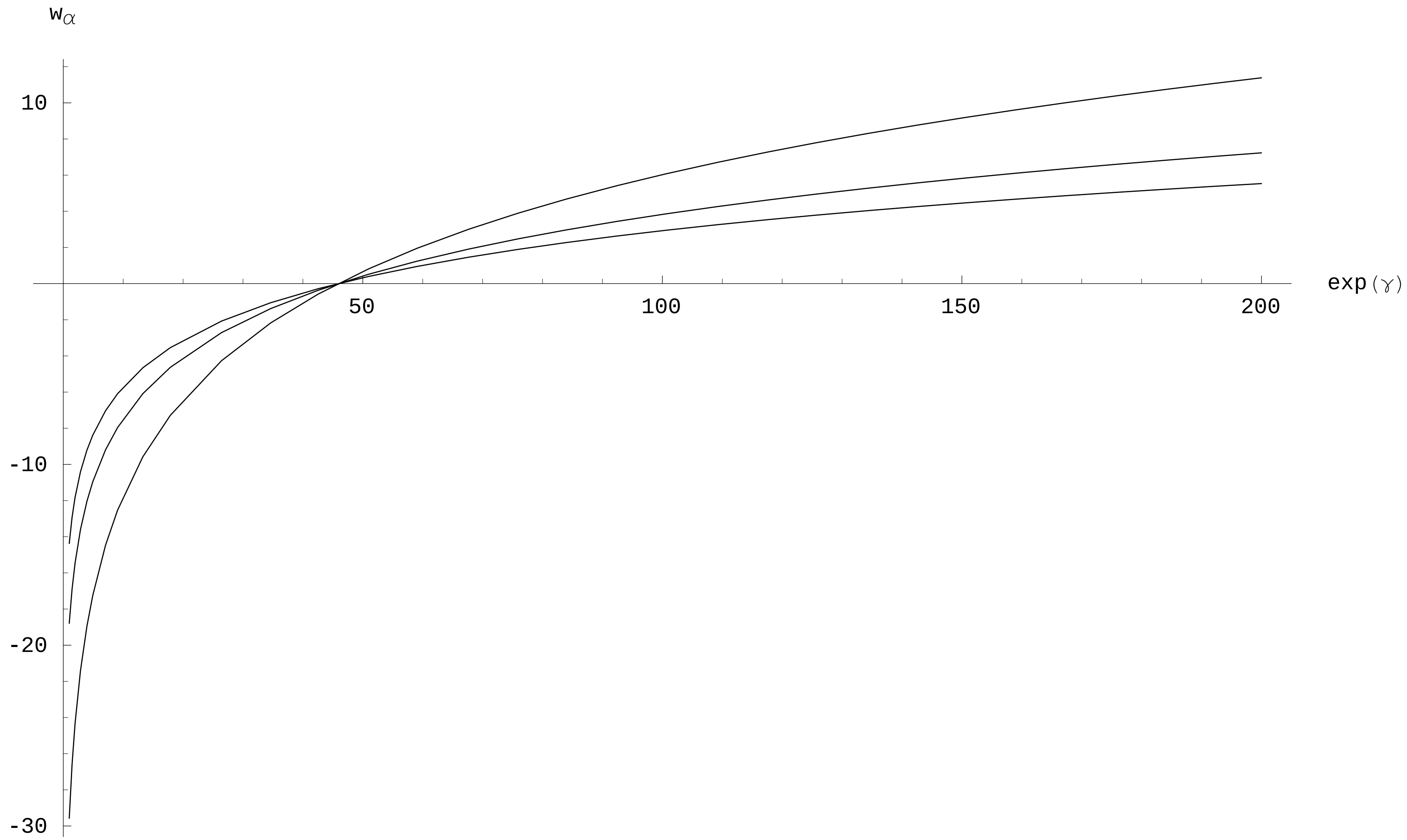


Figure 3
Conformist wealth in states P_T for different risk aversion
($\eta = 0.2$, 0.5 , 0.8)

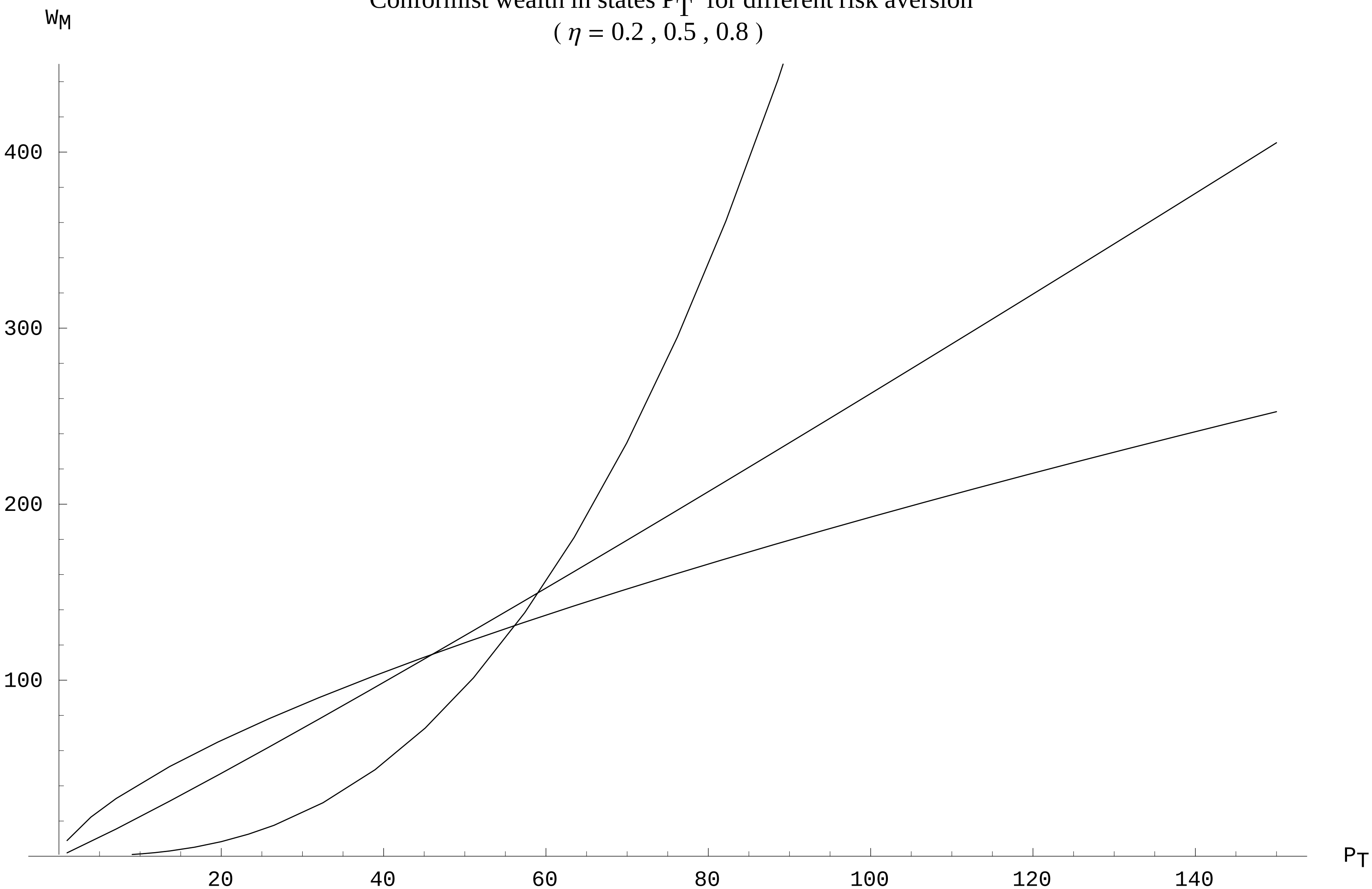


Figure 4
Rebels' wealth in states P_T for different beliefs
($\alpha = 0.2$, 1 and $\gamma = \text{Ln}[20]$, Ln[60])

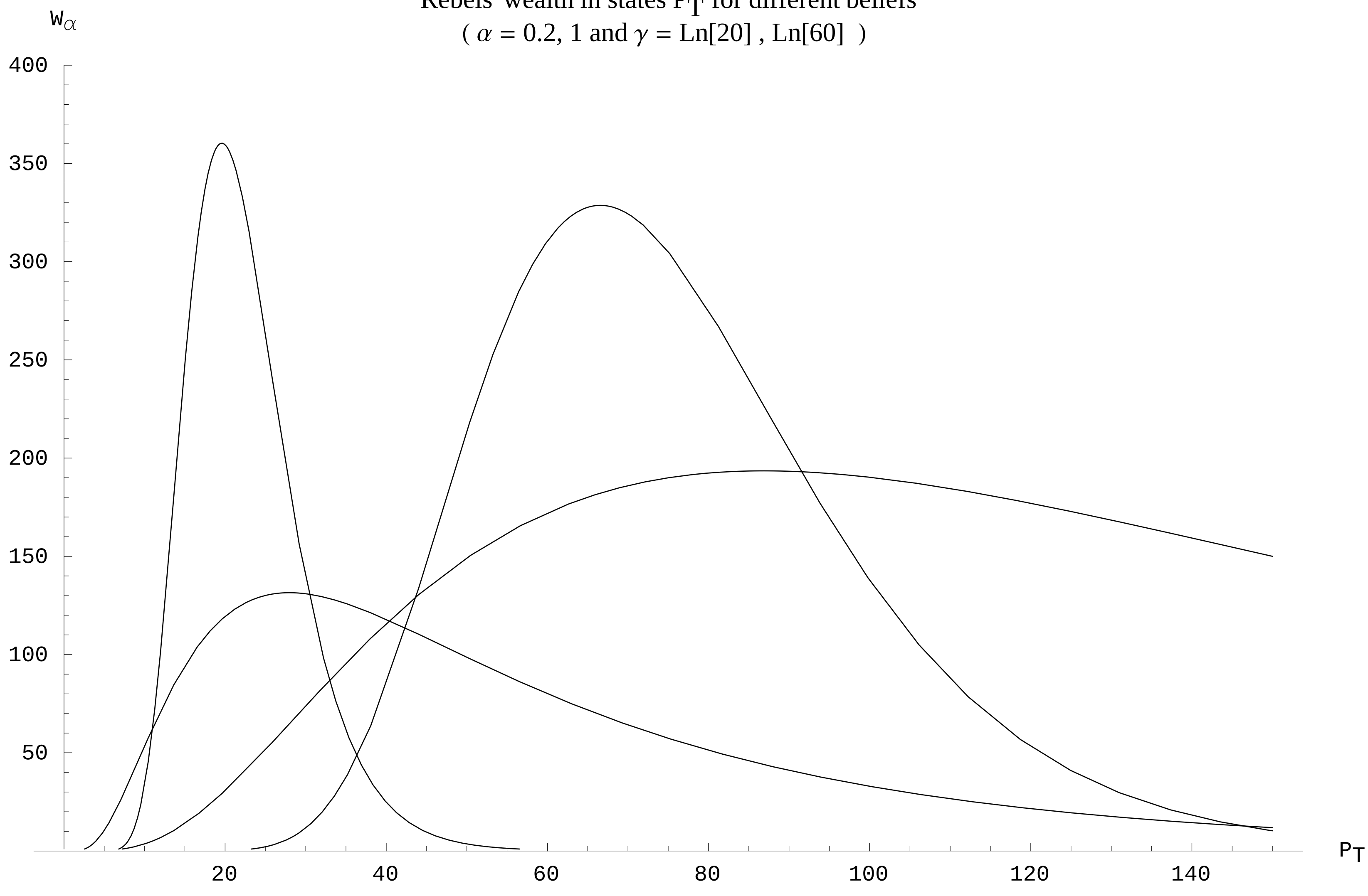


Figure 5
Conformist wealth density for different risk aversion
($\eta = 0.1 , 0.5 , 0.9$)

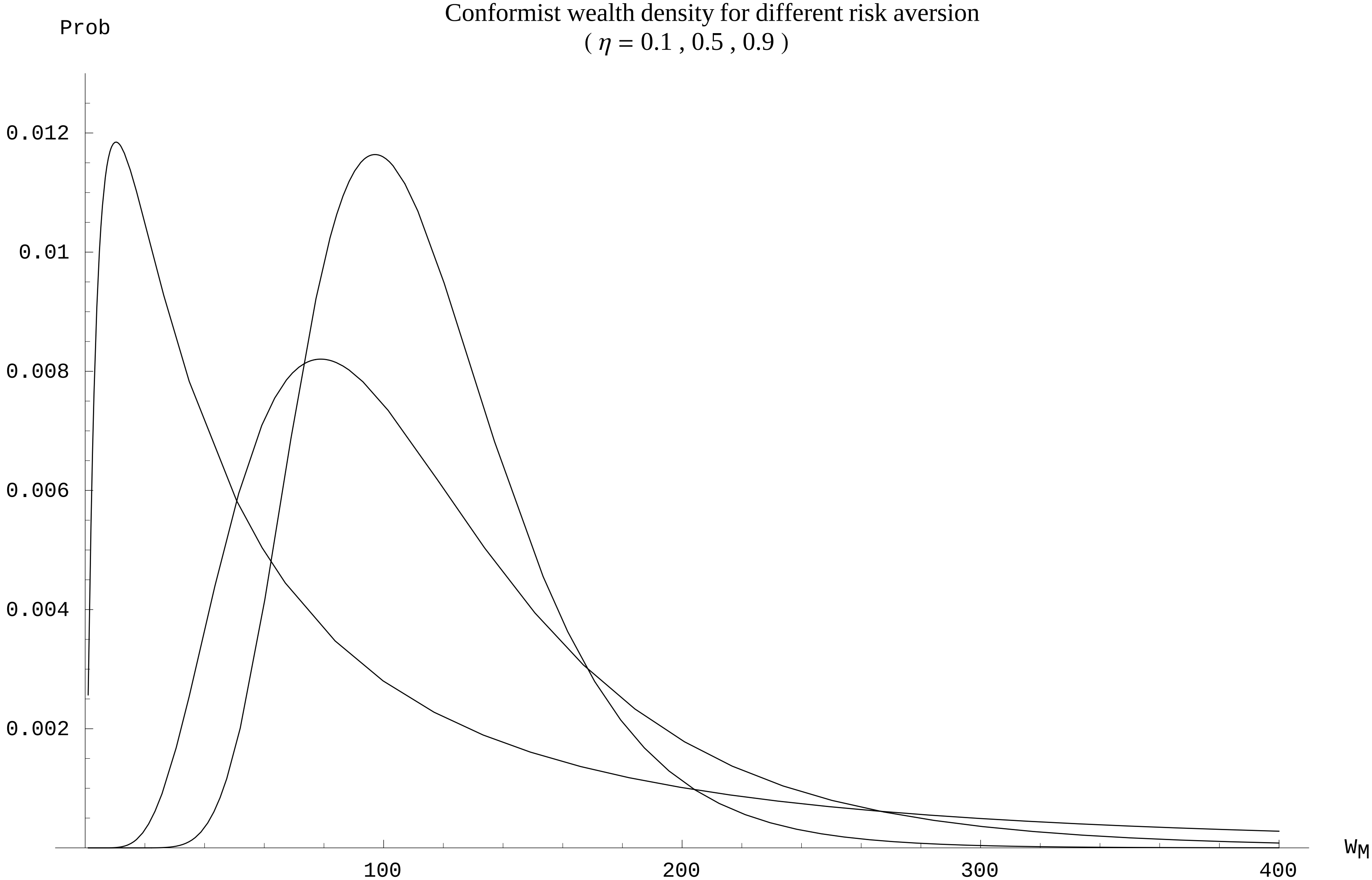


Figure 6
Rebels' wealth density for different beliefs
(α = 0.2, 1 and γ = Ln[20] , Ln[60])

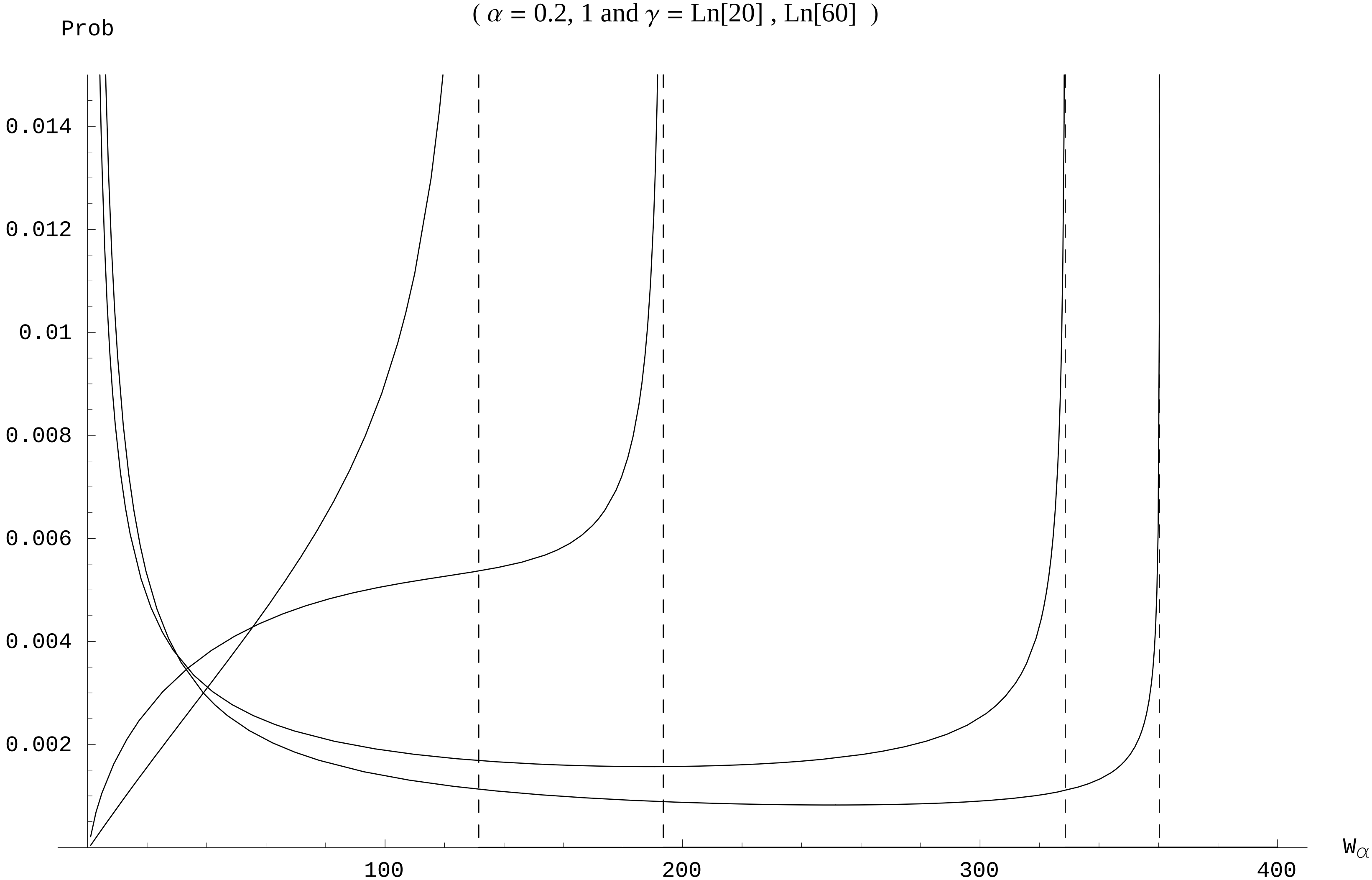
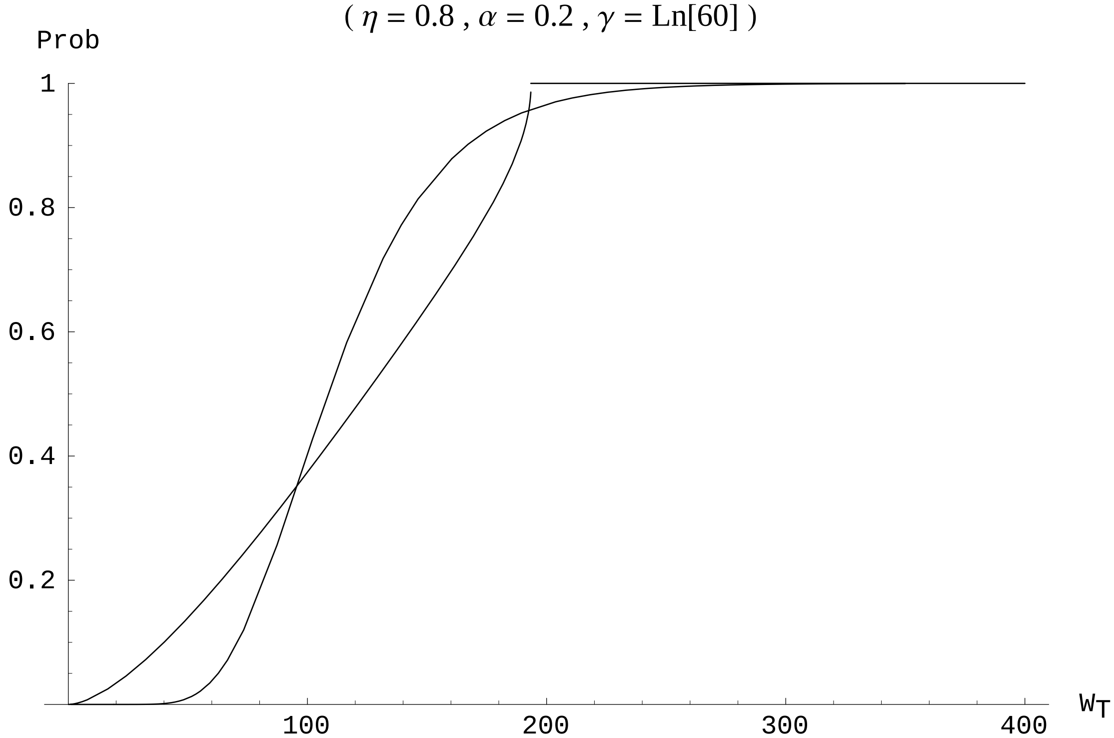


Figure 7

A. No first−order dominance
($\eta = 0.8$, $\alpha = 0.2$, $\gamma = \text{Ln}[60]$)



B. Conformists that first−order dominate the rebels
($\eta = 0.2$, $\alpha = 1$, $\gamma = \text{Ln}[20]$)

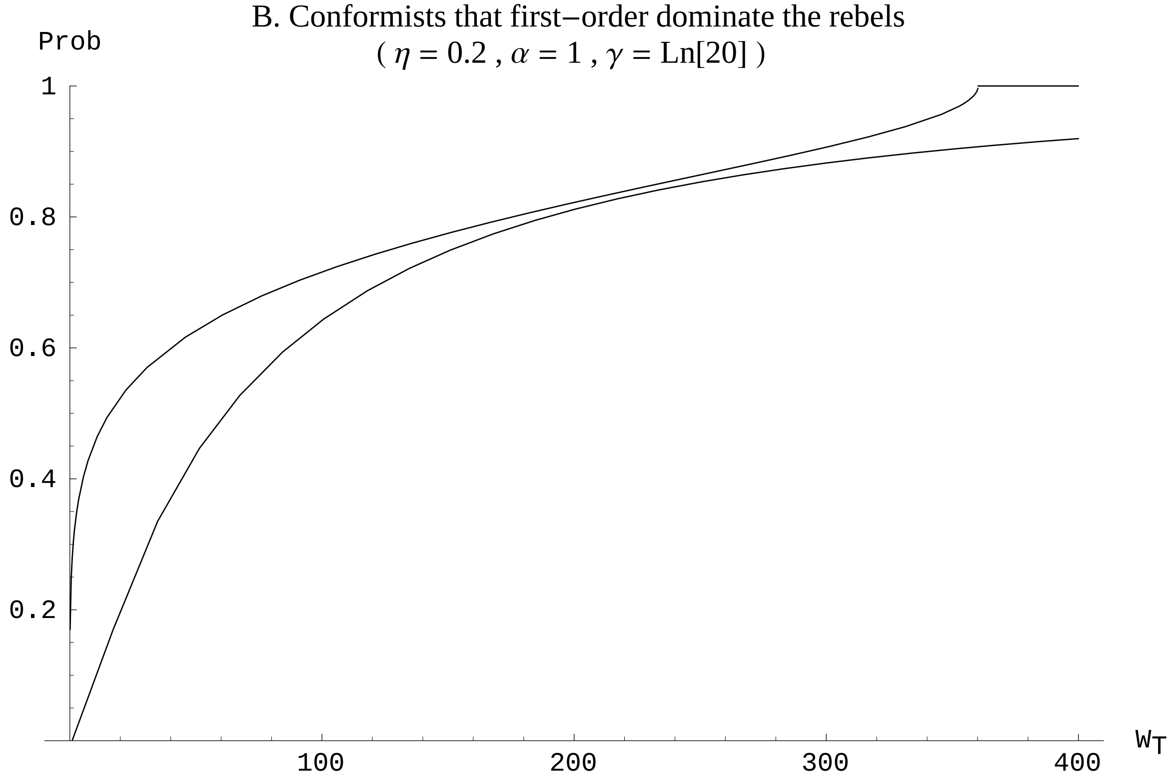


Figure 8
Terminal wealth densities for risk−tolerant conformists ($\eta = 0.2$)
and rebels who believe in near−term crash ($\alpha = 1$, $\gamma = \mathrm{Ln}[20]$)

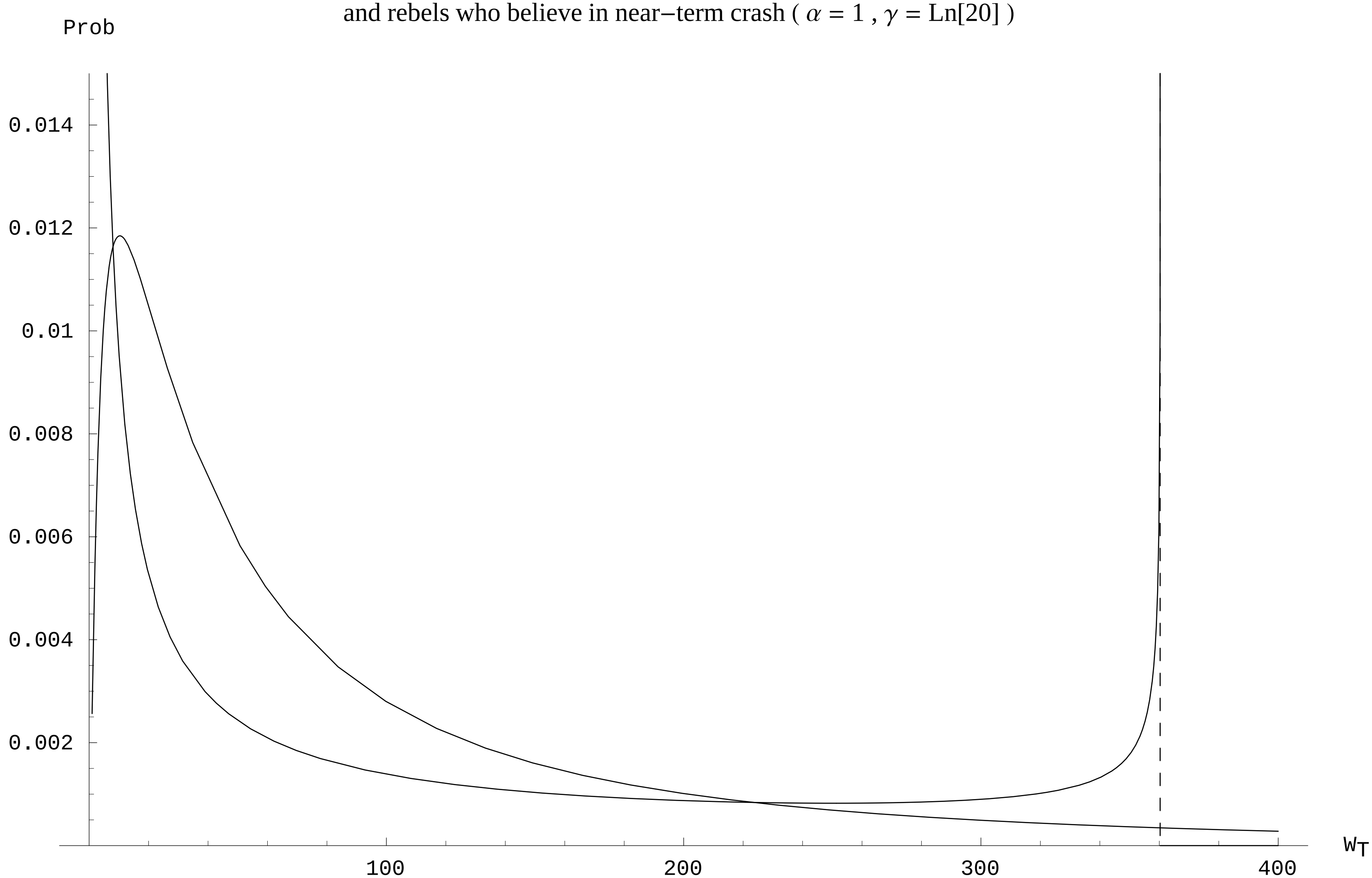


Figure 9

Terminal wealth densities for risk−averse conformists ($\eta = 0.8$) and rebels who believe in long−term appreciation ($\alpha = 0.2$, $\gamma = \mathrm{Ln}[60]$)

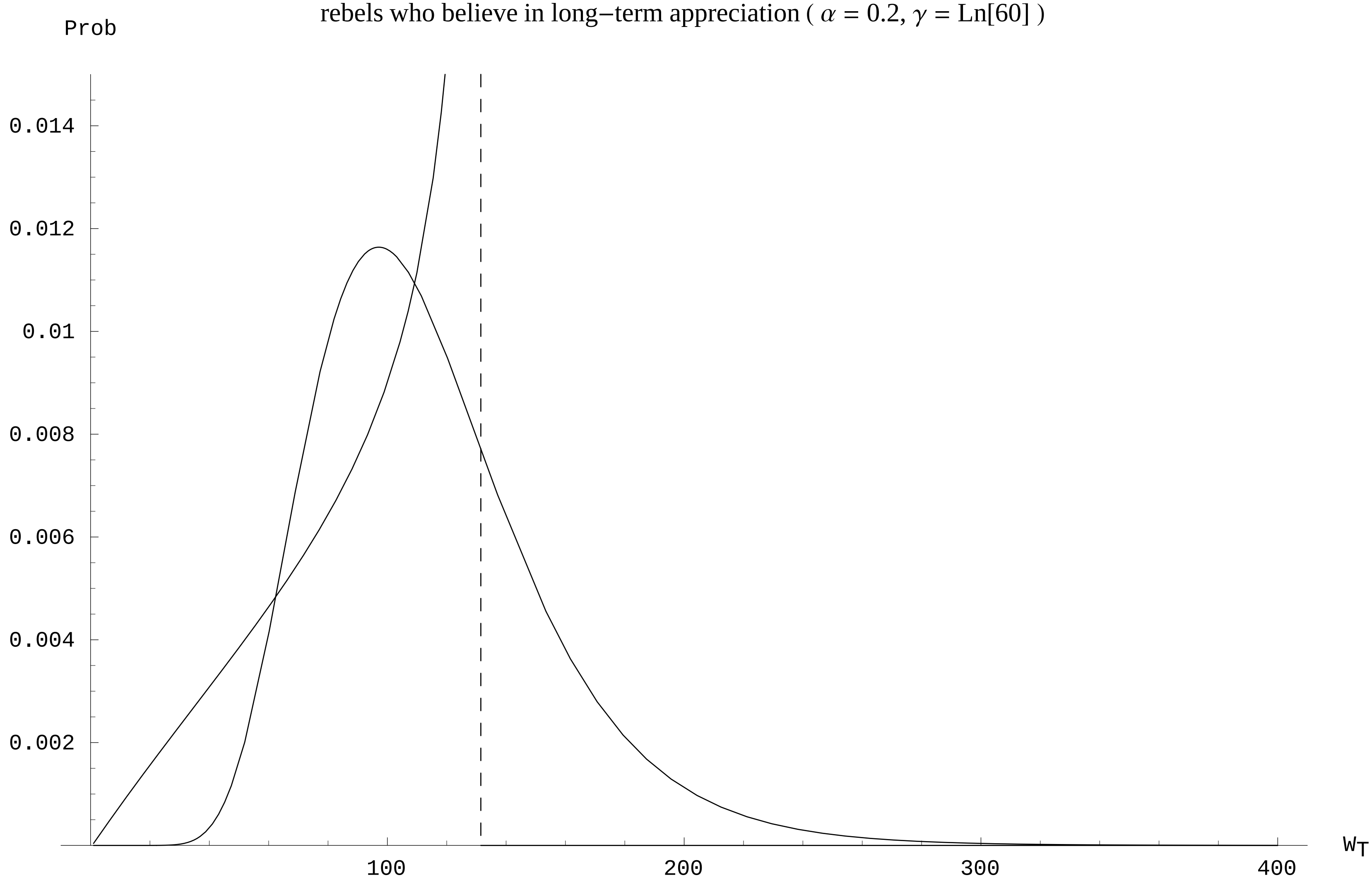


Figure 10
Rebels' initial market exposure for increasing investment horizon
($P_0 = 40$; $\alpha = 0.2$, 1 and $\gamma = Ln[20]$, $Ln[60]$)