NBER WORKING PAPER SERIES

MONEY AND THE OPEN ECONOMY BUSINESS CYCLE: A FLEXIBLE PRICE MODEL

Robert P. Flood

Robert J. Hodrick

Working Paper No. 1967

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 1986

We are grateful to the National Science Foundation for its support of this research through grant #SES-8319460. We also thank the participants of the 1983 NBER Summer Institute for useful discussions, especially Robert G. King and Alan C. Stockman, and we thank the participants in seminars at the University of Chicago, the University of Western Ontario, the University of Iowa, the University of California, San Diego, the State University of New York at Albany, Princeton University, and Northwestern University for their comments. We are very grateful to Jim Clouse for his efficient research assistance. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

Working Paper #1967
June 1986

Money and the Open Economy Business Cycle: A Flexible Price Model

ABSTRACT

This paper develops an open-economy model of the business cycle. The nominal prices in the model are flexible and monetary nonneutrality is developed using information confusion about the sources of disturbances to demand coupled with differential persistence of demand shocks. Firms use inventories to smooth their production, and consumers follow a stochastic permanent income expenditure function. The major implication of the model is that unperceived monetary disturbances improve the terms of trade and increase real output in contrast to sticky price models in which the terms of trade deteriorates. This implication of the model is examined empirically.

Robert P. Flood
Department of Economics
Northwestern University
Evanston, IL 60201

Robert J. Hodrick
Kellogg Graduate School
of Management
Northwestern University
Evanston, IL 60201

This paper develops an open-economy model of the business cycle that may be useful for distinguishing among the competing explanations of cyclical fluctuations. As is common with all modern theories of cyclical fluctuations, we seek an explanation of the observed fluctuations in real income and the comovements of real income and other aggregate time series that is based on rational behavior. The main advantage of our framework is its ability to encompass stylized versions of the most popular business cycle models in a common framework. This facilitates testing of the implications of the theories and highlights the additional implications that arise from development of the model in an open economy setting.

Most current explanations of business cycles that allow for nonneutralities of money are of one of two types: (a) the information-confusion models developed by Lucas (1972, 1975) and based on the Phelps (1969) "islands" paradigm and (b) the nominal precommitment models based on the work of Gray (1976), Fischer (1977), and Taylor (1979).[1] Although these models were developed for analysis of hypothetical closed economies, extensions of the models to open economies followed quite directly. The information-confusion models were first extended by Leiderman (1979) and early extensions of the nominal-precommitment models include Turnovsky (1976) and Flood (1979).[2] Unfortunately, the substantive differences between the two types of models occur in many dimensions other than information confusion versus nominal precommitment. Because the sets of models are not derived from the same underlying framework but are designed to explain the same set of empirical regularities, it is difficult to develop a test that can reject one set of models while failing to reject the other set.

For this reason our strategy in this paper is to develop an open economy business cycle model within a class of models capable of encompassing both the information-confusion models and the nominal-precommitment models. The model developed here is a flexible price model that is very much in the spirit of the information-confusion models but without the islands paradigm. The basic structure of the model, though, is similar to that of Flood and Hodrick (1985) where nominal precommitments are essential to the monetary portion of the business cycle. In this paper we develop a model that has monetary nonneutrality with flexible prices, but it can also be solved with nominal precommitments. Our goal is to derive restrictions on data that differ across the models, where the reason for the differing implications is essentially related to information confusion versus nominal precommitment.

I. An Informal Preview

Our model owes much to several recent studies. We use the cost-smoothing inventory model popularized by Blinder and Fischer (1981) and others as the center piece of the production side of the model.[3] The influence of money on real economic activity is modeled using information confusion about the sources of disturbances to demand coupled with the persistence of demand shocks as suggested by Lucas (1977). The importance of the persistence of demand shocks arises quite naturally in the cost-smoothing inventory model. Since firms must meet demand either out of inventories or out of new production, the persistence of a perceived shock to demand influences the firms' optimal responses. The more persistent is the perceived disturbance to demand, the more a firm responds with increases in its relative price and its output.[4]

The consumer side of the model draws on work by Flavin (1981) on the permanent income model of consumption. Flavin introduced a transitory component to the permanent income consumption function, and this aggregate disturbance in consumers' tastes between expenditure and saving plays an important role in our model. As Flavin noted, a disturbance today that causes consumption to be higher than permanent income depletes assets resulting in lower permanent income in the future and lower expected future consumption.[5]

In addition to the expenditure shock, we assume that aggregate demand for the home good contains a random disturbance that shifts demand between home and foreign goods. In our framework both shocks are white noise, but because the expenditure shock alters permanent income directly while the relative demand shock does not, the response of firms to an observed demand shock composed of the two unobserved disturbances depends on their perceptions of the composition of the shock. In particular, since a positive expenditure shock is expected to lower future expenditure and demand directly while a positive relative demand shock has no direct influence on expected future demand, the relative demand shock is more persistent than is the expenditure shock. Consequently, firms react more strongly with relative prices and output to a shock perceived to be a relative change in demand between home and foreign goods than to a shock perceived to be a disturbance to saving and expenditure.

Firms are assumed to know actual demand when they make their decisions for the period, but they do not know the composition of the stochastic component of demand. They use the information in their environment to infer values of the two disturbances. In addition to actual demand, firms are assumed to be able to observe all prices being charged by other firms, both foreign and domestic, and the quantities being produced by other firms.

Firms gain additional information from the economy-wide asset markets that are represented here by an economy-wide money market. There is useful additional information in the money market because the demand for money depends on the total level of expenditure by domestic residents, which in turn depends on the expenditure shock. Therefore, by combining their observations on interest rates and prices with the other variables in their information set, firms obtain a second signal concerning the expenditure disturbance.

This second signal is not, however, a direct observation of the expenditure disturbance because it is contaminated with other money market disturbances--most importantly the money-supply disturbance. Since we assume that firms do not see the actual money supply used by agents for transactions immediately, the asset-market signal is known to contain both the expenditure disturbance and a money-supply disturbance. Both a positive money-supply disturbance and a negative expenditure disturbance generate incipient excess supply in the money market and require equilibrating movements in observable interest rates and prices to bring the money market back to equilibrium. Since firms do not know the sources of the observed equilibrating movements, they remain less than completely informed about the expenditure disturbance.

An unperceived shock to the supply of money has real effects in our model because firms confuse the movements in interest rates and prices that are due to the money-supply disturbance with the unobservable expenditure shock. To understand the effect in more detail, suppose that the three disturbances, the expenditure shock, the relative demand shock, and the money-supply shock, are all uncorrelated white noises with zero means and finite variances. In a typical period some finite value of each disturbance will occur, but for illustration consider a hypothetical period where the realization of the expenditure and relative demand disturbances are both zero and the realization of the money-supply disturbance is positive. From the money market, the firms see a signal indicating a corrected excess supply of money. Because rational agents know that money-supply and expenditure disturbances normally both occur, the firms attribute part of the signal to a positive money supply disturbance and part of the signal to a negative expenditure disturbance. Since both the expenditure and relative demand shocks are actually zero in this example, the firms also observe that the composite demand shock is zero. Since the firms believe that the expenditure shock is negative, they perceive a positive relative demand shock toward the home good that exactly offsets the perceived negative expenditure shock.

Because firms respond more with relative price and output the more persistent is the demand shock, the firms' positive output and price responses to the comparatively persistent relative demand shock are larger than their negative relative price and output responses to the perceived negative expenditure shock. Thus, the firms' overall responses to their perceptions are positive relative price and output movements resulting from a positive money supply disturbance.

A formal presentation of our results follows in the next three sections and in the Appendix. In Section II we develop the real side of the model, and in Section III we present the monetary side of the model and combine the real and monetary sides of the model to give a complete reduced form solution for real aggregates. In Section IV we explore empirically the new implication of our model -- that monetary shocks are positively correlated with an economy's terms of trade. This implication stands in contrast to the implication of sticky-price models such as Dornbusch (1976) and Mussa (1982) and wage indexing models such as Flood and Marion (1982), where monetary shocks are negatively correlated with an economy's terms of trade. We present in the text only what is required to follow the argument. Most of the formalities are presented in the Appendix.

II . The Real Side

Our model is of a medium-sized open economy. The country is large in the markets for its own output and its own money, but the country is small in the markets for goods produced abroad. Domestic currency prices of all goods are endogenous as is the domestic nominal rate of interest and the exchange rate.[6]

We assume that J firms in the domestic economy produce output and hold inventories of that output. The goods of the firms are slightly differentiated from each other but possess a meaningful dimension along which they may be aggregated.[7]

The problem facing an individual firm is to maximize the discounted expected value of its real profits:

$$\max_{\{R^j_{t+i},\, N^j_{t+i}\}_{i=0}^{\infty}} E_t \sum_{i=0}^{\infty} \{Rev^j(t+i) - Cost^j(t+i)\}\sigma^i \tag{1}$$

where R^j_t is the relative price of the goods produced by firm j at time t in terms of the domestic price level, P_t, N^j_t is firm j's stock of inventory at time t and σ is the discount factor.[8] The expectation operator conditional on period t information is $E_t(\cdot)$. The revenue of firm j at time t is equal to the firm's relative price times the demand for its product, $Rev^j(t) = R^j_t D^j_t$. Demand for firm j's product, D^j_t, is equal to that firm's proportionate share of aggregate demand plus a term reflecting a negative (positive) response by consumers to higher (lower) than average relative price at firm j:

$$D_t^j = J^{-1}D_t - J\rho_3(R_t^j - \overline{R}_t) \quad , \rho_3 < 0, \tag{2}$$

where D_t is total demand for domestic output, and $\overline{R}_t$ is the average relative price of home goods in terms of the price level, $\overline{R}_t = J^{-1}\sum_{j=1}^{J} R_t^j$.

The total demand for domestic output, D_t, is the sum of domestic demand and foreign demand. We assume a linear form for aggregate demand,

$$D_t = \rho_0 - \rho_1\overline{R}_t + \rho_2 X_t + d_t \quad , \rho_i > 0, \ i = 0,1,2, \tag{3}$$

where X_t is aggregate real expenditure by domestic residents on all goods and d_t is a white noise demand disturbance between home and foreign goods. It is assumed that X_t is determined in accord with Flavin's (1981) development of Friedman's (1957) permanent income model.[9] Flavin (1981) defined permanent income at time t to be the constant resource flow that can be expected to be maintained for the remainder of the individual consumer's infinite horizon given time t information. If expenditure is set equal to permanent income plus a disturbance to the expenditure decision, Flavin (1981) demonstrated that the real expenditure process satisfies

$$X_t = X_{t-1} + u_t - \sigma^{-1}u_{t-1} + \varepsilon_t \tag{4}$$

where u_t is the expenditure disturbance at time t which is assumed to be white noise and ε_t is the revision at time t in the discounted value of real income,

$$\varepsilon_t = (E_t - E_{t-1})(1-\sigma)\sum_{i=0}^{\infty}(\overline{R}_{t+i}Y_{t+i})\sigma^i. \tag{5}$$

Aggregate output at time t is Y_t, and we assume that consumers use the same discount factor as firms, σ.

Notice in (4) that last period's expenditure shock, u_{t-1}, lowers current expenditure at the rate σ^{-1} as compared with last period's expenditure. If a positive u_{t-1} occurred last period, then expenditure was raised over permanent income by the amount of that disturbance which lowers future wealth by the amount of the disturbance. This lowers permanent income by the real rate of interest times the disturbance. Since σ^{-1} is equal to one plus the real rate of interest, the period t-1 expenditure disturbance causes expenditure at t to be lower than expenditure at t-1 by $\sigma^{-1}u_{t-1}$.

The two consumption shocks discussed earlier, the expenditure disturbance and the relative demand disturbance, are represented by the u_t and the d_t shocks, respectively. Notice that when both shocks have the same time series properties, the d_t shock will have a more persistent effect on demand than the u_t shock because the latter is "corrected" through a uniformly lower expected future level of expenditure.

The revision in discounted real income, ε_t in equation (5), is inherently nonlinear in R and Y. To keep the model linear we use the following approximation:

$$\overline{R}_t Y_t = \kappa_0 + \kappa_1 \overline{R}_t + \kappa_2 Y_t. \qquad (6)$$

No foreign elements appear explicitly in the demand for home goods. The relative price term in the demand function should, however, be interpreted as including both domestic and foreign responses to relative price. The relative demand disturbance may also be interpreted as including foreign influences.

Each firm's costs in each period consist of two elements, production costs and inventory-holding costs. Production costs for firm j are given by

$$C_1^j(t) = (\gamma_1 + k_t + \gamma_2 Y_t)Y_t^j + (\gamma_3/2)(Y_t^j)^2 . \tag{7}$$

Inventory-holding costs are

$$C_2^j(t) = \delta_1 N_{t-1} N_{t-1}^j + (\delta_2/2)(N_{t-1}^j)^2 . \tag{8}$$

The cost entering the firm's problem in (1) is the sum of the two costs in (7) and (8). The cost in (7) is a quadratic production cost function where the linear coefficient on firm output, Y_t^j, depends on a constant, γ_1, on an economy-wide cost shock, k_t, which we assume to be white noise and to be known by firms at t, and on aggregate output, Y_t. The cost shock is related through duality to the productivity shock popular in many macroeconomic models. Cost shocks allow the model to be consistent with the observed positive correlation of output and inventories documented by Blinder (1981).[10] The presence of aggregate output, $Y_t = \sum_{j=1}^{J} Y_t^j$, in the firm's cost function is intended to capture a presumed positive correlation of production costs with the business cycle. For example, if real wages rise with aggregate output, then this term would capture such correlation. The cost term in (8) is a quadratic inventory holding cost function. Holding costs at time t depend on beginning of period inventories, N_{t-1}^j. Further, the linear term in this function depends on aggregate inventories, $N_{t-1} = \sum_{j=1}^{J} N_{t-1}^j$, to reflect the presumed increase in storage-space rents when aggregate inventories are high. We allow negative inventories and interpret them as backlogged orders as in Blinder and Fischer (1981), Blinder (1982) and Eichenbaum (1983, 1984).[11].

Since sales are equal to demand in this framework, firms face the intertemporal constraint

$$Y_t^j = D_t^j + N_t^j - N_{t-1}^j, \tag{9}$$

which states that output either fills demand or is used for inventory accumulation. We use (9) to eliminate Y_t^j as a decision variable in the firm's problem. Hence, in the firm's maximization problem (1) the choice variables are current values and planned sequences of relative prices and inventories, R_{t+i}^j and N_{t+i}^j, $i = 0,1,\cdots$, with the initial inventory stock, N_{t-1}^j, as a predetermined state variable.

The solution of (1) can be characterized by a sequence of Euler equations having an intuitive interpretation. Maximization of (1) with respect to R_t^j and all expected future values of R_{t+i}^j yields the condition that in any period marginal revenue from sales should be equal to the marginal cost of production (but sales need not equal production). Maximization with respect to N_t^j and the sequence of expected future values of N_{t+i}^j yields the intertemporal condition that the marginal cost of production at time t+i plus the discounted cost of storage from time t+i until time t+i+1 must equal the expected discounted marginal cost of production at time t+i+1. Hence, the firm is indifferent at the margin between producing a unit and selling it this period or storing the unit, incurring the cost of storage, and expecting to sell the unit next period. In Appendix A we record these Euler equations for a typical firm.

Since our concern is with the aggregate of all firms, we assume that the cost coefficients are identical across firms. The linear demand functions combined with quadratic costs yield linear Euler equations that may be summed across firms to get the "aggregate Euler equations" reported below. The algebra involved in our problem is greatly simplified if we work with the special case of the aggregate Euler equations resulting when $J \to \infty$ (i.e., the number of firms becomes indefinitely large).[12]

The aggregate Euler equations at time t for $\lim J \to \infty$ are the following:

$$\overline{R}_t = \gamma_1 + k_t + \gamma_2 Y_t \tag{10}$$

and

$$(\gamma_1 + k_t + \gamma_2 Y_t) + \sigma\delta_1 N_t = E_t \sigma(\delta_1 + k_{t+1} + \gamma_2 Y_{t+1}). \tag{11}$$

Equation (10) sets marginal revenue from sales equal to marginal cost of production and equation (11) equates marginal cost of production plus discounted marginal storage cost to expected discounted future marginal cost. Since we assumed particular time series processes for the model's real disturbances, u_t, d_t and k_t, we may use the method of undetermined coefficients to obtain quasi-reduced form solutions for the real side variables, $\overline{R}_t$, N_t and Y_t. These solutions are linear functions of the firms' "state variables", N_{t-1}, X_{t-1}, u_{t-1}, k_t, $E_t u_t$, $E_t d_t$, and $E_{t-1} u_{t-1}$. The functions we report now are not full reduced forms because they contain endogenous variables, the conditional expectations of disturbances. The determination of the expected values of these shocks requires the explicit treatment of the inference problem described above involving the money market, which is addressed in the next section.

We define these quasi-reduced-form functions to be the following:

$$N_t = \underset{+}{\tau_{11}} N_{t-1} + \underset{-}{\tau_{12}} X_{t-1} + \underset{+}{\tau_{13}} u_{t-1} + \underset{-}{\tau_{14}} k_t + \underset{-}{\tau_{15}} E_t u_t + \underset{-}{\tau_{16}} E_t d_t + \underset{-}{\tau_{17}} E_{t-1} u_{t-1} \tag{12a}$$

$$\overline{R}_t = \underset{-}{\tau_{21}} N_{t-1} + \underset{+}{\tau_{22}} X_{t-1} + \underset{-}{\tau_{23}} u_{t-1} + \underset{+}{\tau_{24}} k_t + \underset{+}{\tau_{25}} E_t u_t + \underset{+}{\tau_{26}} E_t d_t + \underset{+}{\tau_{27}} E_{t-1} u_{t-1} \tag{12b}$$

$$Y_t = \underset{-}{\tau_{31}} N_{t-1} + \underset{+}{\tau_{32}} X_{t-1} + \underset{-}{\tau_{33}} u_{t-1} + \underset{-}{\tau_{34}} k_t + \underset{+}{\tau_{35}} E_t u_t + \underset{+}{\tau_{36}} E_t d_t + \underset{+}{\tau_{37}} E_{t-1} u_{t-1}. \tag{12c}$$

The signs of the coefficients in the quasi-reduced-forms are recorded below them. Expressions for these coefficients in terms of the behavioral parameters are reported in Appendix C. The appearance of three of the right-hand-side variables in these expressions requires some additional explanation. The terms $E_t u_t$ and $E_t d_t$ enter the decision rules for $N_t, \overline{R}_t$ and Y_t because firms make their decisions about inventories, pricing and production based on their beliefs about u_t and d_t. The term $E_{t-1} u_{t-1}$ enters because of the permanent income expenditure function. Recall that ε_t is the revision in beliefs about the discounted sum of future values of real income. This revision is across the time t-1 and the time t information sets. The perception of the value of discounted real income based on t-1 information depends on $E_{t-1} u_{t-1}$, which in turn must enter the quasi-reduced forms. Once we reach the level of full reduced forms in the next section, we will interpret the coefficients.

III. The Monetary Side and the Agents' Inference Problem

In the last section we derived quasi-reduced-form functions for the aggregate of firms' decisions concerning N_t, $\overline{R}_t$ and Y_t. These functions are not reduced forms because they depend on the currently determined endogenous variables $E_t u_t$ and $E_t d_t$. To derive the reduced form decision rules we must solve for $E_t u_t$ and $E_t d_t$ using all of the information available to the agents. That information consists of observations of demand and observations of the variables that appear in the money-market equilibrium.

We assume that only domestic agents hold domestic money, and aggregate money demand is given by the conventional specification

$$m_t^d - p_t = \alpha_0 - \alpha_1 i_t + \alpha_2 X_t \quad , \alpha_1, \alpha_2 > 0, \tag{13}$$

where m_t^d is the logarithm of money demand, p_t is the logarithm of the domestic price level and i_t is the level of the domestic interest rate. The logarithm of the money supply is given by

$$m_t^s = f(I_t) + v_t \quad , \tag{14}$$

where $f(I_t)$ is a function relating current money to variables in the agents' information set and v_t is the unsystematic portion of the money supply which may include shocks to a money muliplier.[13] The v_t shock is assumed to be a white noise disturbance with finite variance which is uncorrelated with the other disturbances in the model.

The agents know everything in I_t, which includes p_t, i_t, and part of X_t. The part of X_t they do not know exactly is u_t.[14] The agents do know that the money market is in equilibrium each period, and they use their information to observe the signal

$$a_t = v_t - \alpha_2 u_t. \tag{15}$$

This is the incipient excess supply of money that is corrected by movements of the endogenous variables to keep the money market in equilibrium.[15]

In addition, the agents see actual demand in the goods market, given in (3). Since the agents see D_t and the information set includes $\overline{R}_t$ and all components of X_t other than the exact value of u_t, the observation of demand gives the agents an observation of the signal

$$b_t = \rho_2 u_t + d_t. \tag{16}$$

The inference problem of firms is to form $E_t u_t$ and $E_t d_t$ from information on a_t and b_t. We assume that the agents use linear least squares projections to form $E_t u_t$ and $E_t d_t$.[16] Therefore,

$$E_t u_t = \phi_{ua} a_t + \phi_{ub} b_t \tag{17a}$$

and

$$E_t d_t = \phi_{da} a_t + \phi_{db} b_t \ . \tag{17b}$$

Reduced form expressions for $E_t u_t$ and $E_t d_t$ are obtained by substituting from (15) and (16) into (17a) and (17b) to yield

$$E_t u_t = \underset{+}{\omega_{uu}} u_t + \underset{-}{\omega_{uv}} v_t + \underset{+}{\omega_{ud}} d_t \tag{18a}$$

$$E_t d_t = \underset{+}{\omega_{du}} u_t + \underset{+}{\omega_{dv}} v_t + \underset{+}{\omega_{dd}} d_t \ , \tag{18b}$$

where the signs of the ω's are recorded below them and their expressions in terms of the underlying parameters are recorded in Appendix D. Notice that ω_{uv} is negative implying that a positive monetary shock, $v_t > 0$, contributes toward the perception of a negative u_t shock, as discussed above.

Full reduced form expressions for N_t, $\overline{R}_t$ and Y_t are obtained by substituting from (18a) and (18b) into (12a) - (12c). Also, use (18a) dated t-1 to substitute for $E_{t-1}u_{t-1}$. We write these final reduced forms in the following way:

$$N_t = \underset{+}{\lambda_{N1}} N_{t-1} + \underset{-}{\lambda_{N2}} X_{t-1} + \underset{+}{\lambda_{N3}} u_{t-1} + \underset{-}{\lambda_{N4}} k_t + \underset{-}{\lambda_{N5}} d_{t-1}$$
$$+ \underset{+}{\lambda_{N6}} v_{t-1} + \underset{-}{\lambda_{N7}} u_t + \underset{-}{\lambda_{N8}} d_t + \underset{\pm}{\lambda_{N9}} v_t \tag{19a}$$

$$\overline{R}_t = \underset{-}{\lambda_{R1}} N_{t-1} + \underset{+}{\lambda_{R2}} X_{t-1} + \underset{-}{\lambda_{R3}} u_{t-1} + \underset{+}{\lambda_{R4}} k_t + \underset{+}{\lambda_{R5}} d_{t-1}$$
$$+ \underset{-}{\lambda_{R6}} v_{t-1} + \underset{+}{\lambda_{R7}} u_t + \underset{+}{\lambda_{R8}} d_t + \underset{+}{\lambda_{R9}} v_t \tag{19b}$$

$$Y_t = \underset{-}{\lambda_{Y1}} N_{t-1} + \underset{+}{\lambda_{Y2}} X_{t-1} + \underset{-}{\lambda_{Y3}} u_{t-1} + \underset{-}{\lambda_{Y4}} k_t + \underset{+}{\lambda_{Y5}} d_{t-1}$$
$$+ \underset{+}{\lambda_{Y6}} v_{t-1} + \underset{+}{\lambda_{Y7}} u_t + \underset{+}{\lambda_{Y8}} d_t + \underset{+}{\lambda_{Y9}} v_t \ . \tag{19c}$$

The signs of the λ coefficients are recorded below them, and the expressions for the λ's in terms of the underlying parameters are recorded in Appendix

E. We turn now to an intuitive interpretation of the reduced form coefficients.

$\underline{N_{t-1}}$

The aggregate Euler equations can be expressed as a second order difference equation in N_t. Therefore, two possible solutions arise for λ_{N1}, one greater than one and the other between zero and one. The solution we have chosen for λ_{N1} is the one between zero and one implying that inventories evolve as a stable autoregression. All of the other coefficients we report are predicated on this choice. Relative price and output are both higher for lower values of N_{t-1} (i.e. λ_{R1}, $\lambda_{Y1} < 0$) since firms try to replenish inventory stocks when they are low by limiting demand and increasing production.

$\underline{X_{t-1}}$

An increase in X_{t-1} raises demand today and expected demand in all future periods. An increase in X_{t-1} lowers inventory at t ($\lambda_{N2} < 0$), since the inventory is used to meet part of the demand, and it increases relative price and output (λ_{R2}, $\lambda_{Y2} > 0$) to offset the increase in demand and to meet the remaining demand with current production of goods.

$\underline{u_{t-1}}$

A positive value of last period's expenditure disturbance has its primary impact through reducing expenditure and demand in the current period and in all future periods. Such a disturbance causes consumers to run down their asset stocks thereby lowering permanent income. There are other effects of u_{t-1} that arise because only the inferred value of u_{t-1} is available at time t-1, but these effects do not offset the primary impact. Consequently, u_{t-1} enters each of the reduced forms with coefficients whose signs are opposite to the signs due to X_{t-1}($\lambda_{N3} > 0$; λ_{R3}, $\lambda_{Y3} < 0$).

$\underline{k_t}$

The primary effect of a positive cost shock is to make production relatively expensive in the period when such a shock occurs. There are other effects of the shock through the revision in permanent income, but these do not offset the primary effect. A positive cost shock will cause firms to lower current output ($\lambda_{Y4} < 0$) and to try to limit some of current demand by raising their average relative price ($\lambda_{R4} > 0$). It is not optimal, however, to raise prices so high that demand falls by as much as does production. Firms therefore meet some of the discrepancy between production and demand by reducing inventories ($\lambda_{N4} < 0$).

$\underline{d_{t-1}, v_{t-1}}$

We work with a time-differenced version of the permanent income expenditure function in order to eliminate agents' asset stocks from explicit consideration. This differencing greatly simplifies some calculations, but at the cost of removing a relevant state variable, real asset stocks, from the final reduced forms. We were able to replace this state variable in the quasi reduced form with u_{t-1}, X_{t-1}, and the innovation in permanent income. This innovation is between the time t and the time t-1 information sets. Since the disturbances v_t, u_t and d_t are not known precisely at t and v_{t-1}, u_{t-1}, and d_{t-1} are not known precisely at t-1, the innovation in permanent income involves the term $u_{t-1} - E_{t-1}u_{t-1}$. The disturbances d_{t-1} and v_{t-1} enter the reduced form only through their roles in forming $E_{t-1}u_{t-1}$.[17]

$\underline{u_t \text{ and } d_t}$

The primary effects of positive values of the two demand shocks, u_t, the expenditure shock, and d_t, the relative demand shock, are to increase demand directly. Secondary effects of these shocks work through the revision in permanent income and through the firms' inference problem. Since the secondary effects never offset the primary effects, positive demand shocks cause higher output ($\lambda_{Y7}, \lambda_{Y8} > 0$), higher relative prices ($\lambda_{R7}, \lambda_{R8} > 0$), and lower inventories ($\lambda_{N7}, \lambda_{N8} < 0$). It is optimal to meet some of the demand shock with higher production, to limit some of the demand with higher relative prices, and to meet some of the demand out of inventory stocks.

$\underline{v_t}$

The net money supply shock is v_t. It may be composed of a shock to high-powered money plus a money multiplier shock less a money demand shock. Its real effects are due <u>entirely</u> to agents' use of information in the money market to sharpen perceptions about the two real demand shocks, u_t and d_t. A positive value of v_t makes agents believe that u_t is negative and that d_t is positive, with the direct effects of the two on demand being equal but opposite in sign. Since d_t is more persistent than is u_t, the net effect on firms' perceptions of the u_t and d_t shocks is an increase output ($\lambda_{Y9} > 0$).

The aggregated first order conditions equating marginal revenue from sales to marginal cost of production is (10). Therefore, when a positive v_t results in increased Y_t, it must also result in increased $\overline{R}_t$ ($\lambda_{R9} > 0$). Since marginal cost of production rises with Y_t, $\overline{R}_t$ must rise along with Y_t to give increased marginal revenue. Intuitively, when Y_t rises and $\overline{R}_t$ rises, cutting back demand, one would expect the difference between output and demand, the inventory change to be positive. We were never able to prove this result in our model. The problem, of course, is the very complex expenditure effect in demand. Once this effect is included, we cannot say what happens to inventories (λ_{N9} ?).

An interesting result in our model is $\lambda_{R9} > 0$. A monetary shock raises the price of domestic goods in terms of the price level. Sticky-price and sticky-wage open economy models contain the opposite prediction. For example, an important feature of the Dornbusch (1976) model is that nominal prices of goods are predetermined in units of the currency of the producing country. Unanticipated movements in exchange rates therefore change the relative prices of home and foreign goods. In the Dornbusch model a money shock depreciates the domestic currency raising the nominal and relative prices of foreign goods.

In open economy sticky wage models such as Flood and Marion (1982) a positive domestic money supply shock raises domestic output creating an incipient excess supply of the home good and requiring a deterioration in the domestic country's terms of trade. Therefore such models predict $\lambda_{R9} < 0$.

In our model the money shock raises the relative price of domestic goods in terms of the domestic price index. We assume the domestic price index, P_t, is a linear homogeneous function of the average domestic currency price of domestic goods, $\overline{H}_t$, and the average domestic currency price of foreign goods, $S_t\overline{H}_t^*$, where S_t is the exchange rate quoted as the domestic currency price of foreign currency, and $\overline{H}_t^*$ is the average foreign currency price of foreign goods,

$$P_t = P(\overline{H}_t, S_t\overline{H}_t^*) , \quad \frac{\partial P}{\partial \overline{H}}, \frac{\partial P}{\partial S_t\overline{H}_t^*} > 0. \tag{23}$$

It follows that

$$\bar{R}_t = \bar{H}_t / P(\bar{H}_t, S_t \bar{H}_t^*) = r(\bar{H}_t / S_t \bar{H}_t^*) \quad , \tag{24}$$

with $\partial r / \partial (\bar{H}_t / S_t \bar{H}_t^*) > 0$. Therefore, our model produces a potentially testable difference between models with nominal precommitments and flexible-price models.

The problem with producing convincing versions of such tests is that the results from the nominal precommitment models are predicated on money not feeding back from currently determined endogenous variables while our results are robust to such feedback as long as agents understand the nature of the feedback. In the next section we present some empirical work designed to shed light on the issue of the correlation between the terms of trade and a money shock under the strong condition that money does not feedback on currently determined endogenous variables.[18]

IV. Empirical Investigation

In this section we conduct a limited empirical investigation of the predictions of the model. As the previous section indicated, our investigation focuses on the prediction of our model that an innovation in the money supply is positively correlated with the relative price of the domestic product in terms of a basket of domestic and imported consumption goods while the other models predict a negative correlation.[19] All of the models predict that innovations in output will be positively correlated with innovations in the money supply.

Our investigation of these issues is in the context of a four variable unconstrained vector autoregression.[20] The four variables are the logarithm of real gross domestic product, the logarithm of real expenditure, a relative price measure, and the rate of growth of the monetary base. We use two alternative measures of the relative price, the export price index divided by the consumer price index, which is denoted R_t^1, and the export price index divided by the import price index, which is denoted R_t^2. As always, interpretation of residual correlations from vector autoregressions is difficult when contemporaneous values of the variables being studied are simultaneously determined. Consequently, we assume that no contemporaneous variables, other than the monetary disturbance, enter the money supply equation. Under this assumption we can interpret the residual from the money supply equation as the monetary innovation, and the correlations of the monetary innovation with the other innovations provide evidence concerning our model.

The results of our tests are presented in Table 1 for eight industrial countries. The sample period for each dependent variable spans the last quarter of 1973 to the last quarter of 1983. Each regression contains a constant, two lags of each of the four variables, and quarterly dummy variables. The statistical significance of the correlations is determined from Fisher's Transform test. See Kendall and Stuart (1977, p. 419) for details.

The overall impression from the contemporaneous correlations is the lack of statistically significant correlations across all variables. Of the eighty reported correlations other than between GDP and expenditure only sixteen have marginal levels of significance as low as 0.10. This represents twenty percent which is only slightly more than the ten percent that would be expected by chance from data that were uncorrelated. Consequently, these results provide very little evidence that can be used to differentiate between alternative explanations of business cycles. In terms of support for the nominal precommitment models versus our flexible price model, the results are somewhat mixed. The only residual correlations between the relative price variables and the monetary base that are statistically significant at the ten percent level are found in the cases of Japan and Germany, and they are positive as is predicted by our model. The same correlations are negative though for France and the United Kingdom at the twelve and fourteen percent levels of significance, respectively. Such mixed results do not lend strong support to either viewpoint.

If the demand shocks are predominate relative to supply or cost shocks in inducing variability in the data, the two types of models also give opposite predictions regarding the residual correlation between the terms of trade and real output. Our model predicts a positive correlation while the nominal precommitment models predict a negative correlation. Interpretation of these results is complicated though by the fact that cost shocks tend to produce negative correlation in both models. Hence, we present the contemporaneous correlations between R_t and Y_t merely in an effort to develop some potential stylized facts for future research. The only really strong result is the positive correlation between the terms of trade and output in Japan. The Italian case is particularly perplexing since the correlation changes sign and is significantly different from zero for the two alternative definitions of the terms of trade.

As we noted before, both sets of models predict significant positive correlation between the monetary base and real output. Since these correlations are generally insignificantly different from zero, only the results for the United Kingdom provide some weak support for this hypothesis. Such a finding may represent contemporaneous feedback from the real economy to the money supply that would cast doubt on the validity of the other interpretations as well.

V. Conclusions

In this paper we developed a model of the open-economy business cycle that may someday prove useful in distinguishing between alternative explanations of the transmission mechanism of monetary policy. The model is general enough to be solved in alternative ways which are representative of the information-confusion models and the nominal precommitment models that are the chief explanations of nonneutrality of money.

Our initial empirical investigation with relatively short quarterly time series suggested that there is not strong statistical significance between innovations in the monetary base and innovations in the terms of trade. This is unfortunate because the two types of models predict alternative signs for this correlation. One potential problem in conducting the empirical analysis is the need to make strong a priori restrictions on the lack of contemporaneous feedback from the real economy to the money supply. Relaxing this assumption is a difficult and challenging area for future empirical research.

Theoretical extensions of this analysis are somewhat more straightforward. Extension to a two country model can be accomplished by including foreign expenditure in domestic demand and domestic expenditure in foreign demand. Such an extension would add two disturbances to the model, the foreign expenditure disturbance and the foreign money supply disturbance. It would also add two markets to the model, the market for foreign money and the market for foreign goods. Consequently, while agents would observe additional informative aggregates, there would be additional disturbances added to the inference problem making the results of inference problems roughly similar to those in the text.

Table 1

Country	Contemporaneous Correlations						R^2			
	R,M	R,Y	R,X	M,Y	M,X	Y,X	R	M	Y	X
United States 1	.-13	-.19	-.26	-.18	-.15	.97	.92	.53	.98	.98
	(.43)	(.23)	(.10)	(.25)	(.36)	(.00)				
United States 2	-.02	-.05	-.07	-.10	.-10	.97	.96	.61	.98	.98
	(.89)	(.77)	(.64)	(.55)	(.55)	(.00)				
Germany 1	.09	-.22	-.16	.06	.11	.78	.88	.80	.99	.98
	(.60)	(.17)	(.31)	(.73)	(.50)	(.00)				
Germany 2	.28	-.15	-.07	.09	.08	.81	.92	.81	.99	.98
	(.08)	(.36)	(.68)	(.57)	(.64)	(.00)				
Japan 1	.03	-.25	-.12	.15	.27	.76	.96	.96	.99	.98
	(.86)	(.14)	(.46)	(.37)	(.10)	(.00)				
Japan 2	.39	.52	.28	.22	.20	.86	.94	.97	.99	.99
	(.02)	(.001)	(.09)	(.19)	(.23)	(.00)				
Italy 1	-.22	.36	.26	.02	-.02	.72	.80	.78	.99	.96
	(.19)	(.03)	(.12)	(.93)	(.89)	(.00)				
Italy 2	-.19	-.55	-.71	.19	.16	.73	.86	.77	.99	.96
	(.27)	(.00)	(.00)	(.26)	(.34)	(.00)				

Table 1 (cont'd.)

France 1	-.25 (.12)	.07 (.67)	.11 (.52)	.05 (.76)	.30 (.06)	.65 (.00)	.94	.58	.99	.99
France 2	-.21 (.19)	.16 (.32)	-.26 (.11)	.18 (.26)	.44 (.01)	.68 (.00)	.82	.57	.99	.99
Canada 1	.22 (.15)	-.09 (.58)	-.15 (.35)	-.06 (.73)	-.08 (.61)	.72 (.00)	.91	.93	.99	.98
Canada 2	.06 (.69)	-.19 (.23)	.22 (.17)	-.13 (.43)	-.02 (.93)	.78 (.00)	.80	.93	.99	.98
Switzerland 1	-.12 (.46)	.08 (.64)	.26 (.10)	-.06 (.69)	-.08 (.65)	.85 (.00)	.78	.73	.91	.92
Switzerland 2	.11 (.49)	-.07 (.66)	-.15 (.37)	-.03 (.86)	-.08 (.64)	.83 (.00)	.71	.70	.90	.91
United Kingdom 1	.22 (.17)	.15 (.35)	.29 (.07)	.25 (.12)	.32 (.04)	.83 (.00)	.86	.74	.93	.81
United Kingdom 2	-.23 (.14)	-.19 (.23)	-.37 (.02)	.20 (.22)	.29 (.07)	.85 (.00)	.96	.74	.92	.81

Notes: Numbers in parenthesis represent marginal levels of significance of a test that the estimated correlation coefficient is significantly different from zero. Two sets of results are reported for each country. The first set (1) uses the R_t^1 definition, and the second set (2) uses the R_t^2 definition. Descriptions of the data are given in the Appendix. There was no evidence of residual serial correlation in any of the regressions as judged by standard Q-tests.

Footnotes

1 The information-confusion models are often referred to as equilibrium theories whereas the nominal precommitment models are sometimes labeled disequilibrium. Both types of models are abstractions since they impose some assumptions about the opportunities available for mutually advantageous trade and postulate constraints faced by economic agents. The assumption that certain types of markets for transmitting information fail to exist seems to us no more fundamental than assuming what type of contractual structure agents employ in labor markets.

2 Other open-economy business cycle models employing the islands paradigm include Saidi (1980) and Kimbrough (1983). Other nominal-precommitment models include Weber (1981), Flood and Marion (1982), Marston (1982), and Aizenman and Frenkel (1983). Stockman and Koh (1984) examine the open-economy implications of the two types of models in a nonnested framework in an attempt to differentiate between the competing paradigms.

3 The cost smoothing inventory model was developed by Holt, et. al. (1960) and Lovell (1961). Recently, it has been used in industry analysis by Blanchard (1983) and Eichenbaum (1983, 1984). Blinder (1982) examines the microeconomics of the model in detail claiming that it serves as a basis for nominal price stickiness, but there is no money in his model. Hence, his analysis applies only to relative prices. A similar mistake plagues the analysis in Amihud and Mendelson (1982). Brunner, Cukierman, and Meltzer (1983) consider a complete macroeconomic framework that includes inventory holding by firms who are motivated to avoid stockouts. They also stress the importance of the permanence of shocks as an explanation of macroeconomic fluctuations. We note that the inventory mechanism per se is not crucial to the economics here. Very similar results could have been obtained with output adjustment costs or with investment in capital goods.

4 Our definition of persistence is given in Appendix B where we generalize Blinder's (1982) Theorem 1. Blinder demonstrated that a cost smoothing inventory holding firm responds more strongly with its price and output, the larger is the coefficient, ρ, in the first order autoregressive process for the demand shock, $\eta_t = \rho\eta_{t-1} + V_t$, where V_t is white noise.

5 Unlike Flavin who treated the real income process as exogenous, our model treats real income as an endogenous variable.

6 In this paper we are only concerned with the determination of real variables. Consequently, we do not need to make many assumptions about the exogeneity of foreign variables in order to obtain our results. In solving the model for its nominal variables, such assumptions are crucial. The only important exogenity assumption in our model is that the level of foreign expenditure is exogenous to domestic disturbances.

7 As a trivial example, consider each firm as producing goods of a color particular to that firm. Demand for a particular color of goods depends on the relative price of that color and may be identified with a particular firm, yet it is meaningful to discuss the total number of goods produced in the economy.

8 The domestic price level is a function of domestic currency prices of domestic and foreign goods. Therefore, R_t^j, for example, is a function of the relative price of firm j's output in terms of foreign goods.

9 We assume that each agent sets his expenditure from the permanent income model, knows his own expenditure shock and knows his own relative demand shock. At the individual level, however, each of these shocks is composed of two elements: an individual specific element and a contribution to the aggregate. Let e_t^i be individual i's expenditure shock, and let f_t^i be the individual i's relative demand shock. Suppose there are N domestic agents and N^* foreign agents where N and N^* are very large. We set $e_t^i = N^{-1}u_t + w_t^i$, and we impose $\sum_{i=1}^{N} w_t^i = 0$. We also set $f_t^i = (N+N^*)^{-1}d_t + z_t^i$ with $\sum_{i=1}^{N+N^*} z_t^i = 0$. It follows for large N and N^* and finite variances of all the disturbances that the individual's observations of e_t^i and f_t^i yield no useful information about the aggregate disturbances, u_t and d_t. This set of assumptions is, we think, not essential to the results of this paper. Our results follow <u>without</u> differential information across agents, and it seems unlikley to us that including differential information would reverse anything.

10 Since aggregate inventories include goods-in-process as well as final goods, one must be careful in interpreting the implications of this positive correlation as evidence for or against particular models. McCallum (1984) provides a simple demonstration that cost disturbances allow output and changes in final goods inventories to be positively correlated.

11 The cost structure implies that firms find backlogs to be optimal in a steady state. Since both firm and aggregate inventories can be negative, the cost structure also implies an incentive for each firm's inventory stock to be opposite in sign from average inventories, $\overline{N}_{t-1} = N_{t-1}/J$, since $\delta_1 N_{t-1} N_{t-1}^j = \delta_1 J \overline{N}_{t-1}^2 + \delta_1 J \overline{N}_{t-1}(N_{t-1}^j - \overline{N}_{t-1})$. Because a firm's costs also increase with $(N_{t-1}^j)^2$, the firm's choice of N_{t-1}^j is well defined.

12 The special case of $J \to \infty$ is in fact the only situation where our equilibrium is entirely satisfactory. The problem with a finite J is

that we neither have explicit costs of entry nor do we allow firms to account for reactions by other firms in their decision problem. Our set up with $J \to \infty$ is a competitive solution to the model. It is not surprising, however, that with linear demand and identical quadratic costs across firms, the reduced form equations for aggregates are essentially the same across a number of equilibrium concepts. In a problem much like ours, Eichenbaum (1983) has shown that decision rules at the industry level of an unknown number, J, of firms acting as (i) perfect competitors, (ii) a J-plant monopolist and (iii) Nash competitors are equivalent up to a term whose coefficient depends only on J. Consequently, we expect the qualitative properties of our results to be robust to a variety of industrial organizations of which the competitive case we examine should not be a very special case.

13 The v_t shock can also be interpreted as including a disturbance to money demand. Hence, it may be interpreted as the unsystematic part of the excess supply of money.

14 Examine (4) and recall that agents know X_{t-1}, u_{t-1} and ε_t by construction. The only potential part of X_t not in the agents' information set is some part of u_t.

15 Since we are unconcerned in this paper with the determination of nominal variables, we do not specify the exact functional relationship between p_t and the underlying nominal goods prices. All that is required to derive (15) is that agents know i_t, know how to form p_t from their observations on nominal prices, and know the parameters of the money demand function and $f(I_t)$.

16 See Sargent (1979) for a discussion of signal extraction and linear least squares projections. The ϕ coefficients in (17a) and (17b) are recorded in the Appendix.

17 Our model is classical in the terminology of McCallum (1979). McCallum argues that only the innovation in the money supply will affect real variables in a classical model. The fact that v_{t-1} appears in the reduced forms is attributable to the fact that we have eliminated a state variable. This effect is analyzed by McCallum.

18 Our empirical work is conditioned strongly not just to be consistent with the nominal precommitment models. If we were to allow feedback from current endogenous variables to money, we would have to model the money supply process with much more care in each of countries studied.

19 In our model only the unperceived part of the money shock has real effects whereas the full unanticipated shock has real effects in models with nominal precommitments. The empirical work of Barro and Hercowitz (1980) and Boschen and Grossman (1982) is addressed to this issue, and the evidence is interpreted as unfavorable to the unperceived money hypothesis. See Flood and Hodrick (1985 Section V) for a discussion of the issues and some criticisms of the methodology.

20 Although the solution of the model is a constrained vector autoregression, we were unable to estimate it because of lack of data on final goods inventories across countries. Elimination of N_{t-1} as a state variable induces the entire past history of the other endogenous variables as the new state. Estimation and inference require truncation of the AR process.

Appendixes

A. The Firm's Problem

The individual firm solves the problem stated in text (1) using the revenue definitions contained in (2)-(6) and the cost function definitions in (7) and (8), subject to an initial condition on N^j_{t-1}. When (9) is used to remove Y^j_{t+i} from the firm's problem, a pair of sequences of Euler equations result from the maximization of (1) with respect to the choice variables, R^j_{t+i}, i = 0, 1···, and N^j_{t+i}, i = 0, 1,···. These Euler equations are the following:

R^j_{t+i} i = 0, 1,···

$$E_t\{J^{-1}D_{t+i} - 2J\rho_3 R^j_{t+i} + J\rho_3\overline{R}_{t+i} + (\gamma_1 + k_{t+i} + \gamma_2 Y_{t+i})J\rho_3$$

$$+ \gamma_3 J\rho_3[J^{-1}D_{t+i} - J\rho_3(R^j_{t+i} - \overline{R}_{t+i}) + N^j_{t+i} - N^j_{t+i-1}]\} = 0 \qquad \text{(A1)}$$

N^j_{t+i} i = 0,1,···

$$E_t\{-(\gamma_1 + k_{t+i} + \gamma_2 Y_{t+i}) - \gamma_3[J^{-1}D_{t+i} - J\rho_3(R^j_{t+i} - \overline{R}_{t+i})$$

$$+ N^j_{t+i} - N^j_{t+i-1}] + \sigma(\gamma_1 + k_{t+i+1} + \gamma_2 Y_{t+i+1})$$

$$+ \sigma\gamma_3[J^{-1}D_{t+i+1} - J\rho_3(R^j_{t+i+1} - \overline{R}_{t+i+1}) + N^j_{t+i+1} - N^j_{t+i}]$$

$$- \sigma(\delta_1 N_{t+i} + \delta_2 N^j_{t+i})\} = 0 \quad . \qquad \text{(A2)}$$

These equations hold for each of the firms, $j = 1,2,\cdots J$. Equations (10) and (11) are obtained from (A1) and (A2) in two steps. First, since the information sets are identical across firms, we add the equations across firms. Second, we have reported the limiting versions $(J \to \infty)$ of our aggregate Euler equations for date t.

B. A Generalization of Blinder's Theorem

Since the generalization of Blinder's theorem does not involve cost shocks and does not require explicit treatment of the expenditure term in the demand function, we specialize our model by setting $k_{t+i} = 0$, $i = 0, 1,\cdots$ and by letting the demand for home goods have the simple form

$$D_t = -\rho_1\overline{R}_t + z_t \tag{A3}$$

where z_t includes all elements of equation (3) except the relative price term. Using (A3) in the aggregated Euler equations for $J \to \infty$ we obtain

$$E_t\overline{R}_{t+i} = (\gamma_1 + \gamma_2 E_t Y_{t+i}), \tag{A4}$$

$$E_t\{-(\gamma_1 + \gamma_2 Y_{t+i}) + \sigma(\gamma_1 + \gamma_2 E_t Y_{t+i+1}) - \sigma\delta_1 N_{t+i}\} = 0, \tag{A5}$$

for $i = 0,1,2,\cdots$

By substituting the demand function (A3) into the transition law for inventories, we obtain

$$Y_t = -\rho_1\overline{R}_t + z_t + N_t - N_{t-1}\ . \tag{A6}$$

Use (A6) to eliminate Y_{t+i} from (A4) and (A5), and use the revised (A4) to eliminate $E_t\overline{R}_{t+i}$ and $E_t\overline{R}_{t+i+1}$ from the revised (A5). The result is a second order linear difference equation involving N at three dates and z at two dates. The solution of this equation is

$$N_t = \pi_{10} + \pi_{11}N_{t-1} + \sum_{i=0}^{\infty} \lambda_i E_t z_{t+i} \qquad \text{(A7)}$$

where π_{10} is a constant, $0 < (\pi_{11} < 1$, (π_{11} is reported more fully in Appendix C) and

$$\lambda_0 = -\gamma_2/(1+\gamma_2\rho_1)\Delta \quad , \qquad -1 < \lambda_0 < 0 \ ,$$

$$\lambda_1 = [\sigma\gamma_2/(1+\gamma_2\rho_1)\Delta](1+\lambda_0),\ 0 < \lambda_1 < 1 \ ,$$

$$\lambda_i = [\sigma\gamma_2/(1+\gamma_2\rho_1)\Delta]\lambda_{i-1} \ , \ i = 2,3,\cdots$$

where $\Delta = \sigma\delta_1 + [\gamma_2 + (1-\pi_{11})\gamma_2\sigma]/(1+\gamma_2\rho_1)$.

It follows that

$$N_t = \pi_{10} + \pi_{11}N_{t-1} + \lambda_0 z_t + \lambda_1 \sum_{i=1}^{\infty} E_t z_{t+i}(\sigma\gamma_2/(1+\gamma_2\rho_1)\Delta)^i \ .$$

Therefore, using equation (A6) obtain

$$Y_t = (1+\rho_1\gamma_2)^{-1}[z_t + \pi_0 + (\pi_1 - 1)N_{t-1} + \lambda_0 z_t + \lambda_1 \sum_{i=1}^{\infty} E_t z_{t+i}(\sigma\gamma_2/(1+\gamma_2\rho_1)\Delta)^i].$$

A demand disturbance will alter z_t and the expected future values of the z's. In particular

$$\frac{\partial Y_t}{\partial z_t} = (1 + \rho_1\gamma_2)^{-1}[1 + \lambda_0 + \lambda_1 \sum_{i=1}^{\infty} \frac{\partial E_t z_{t+i}}{\partial z_t} (\sigma\gamma_2/(1+\gamma_2\rho_1)\Delta)^i].$$

Since $-1 < \lambda_0 < 0$, $(1 + \lambda_0) > 0$. Further, $\lambda_1 > 0$. Therefore, larger values of $\sum_{i=1}^{\infty} \frac{\partial E_t z_{t+i}}{\partial z_t} (\sigma\gamma_2/(1+\gamma_2\rho_1)\Delta)^i$ will magnify the effect of z_t on Y_t, and it is reasonable to define a z_t disturbance to be more persistent, the larger is its impact on this infinite sum.

C. Solutions to the Firms' Problems

When we solved the firm-level problem for the set of quasi-reduced forms (12a) - (12c) we accomplished the solution in two steps. First, we treated the permanent income innovation, $(E_t - E_{t-1})(1 - \sigma) \sum_{i=0}^{\infty} \overline{R}_{t+i} Y_{t+i} \sigma^i$, as a white noise disturbance, ε_t. Second, we solved for ε_t using the underlying disturbances. When solving for ε_t, we used the linearization (6) for real income, $\overline{R}_{t+i} Y_{t+i}$. For the convenience of readers we duplicate our two step solution procedure here.

Step One

$$N_t = \pi_{11} N_{t-1} + \pi_{12} X_{t-1} + \pi_{13} u_{t-1} + \pi_{14} k_t + \pi_{15} E_t u_t + \pi_{16} E_t d_t + \pi_{17} \varepsilon_t \quad (A8)$$

where

$$\pi_{11} = \frac{1}{2}\{A - [A^2 - (4/\sigma)]^{1/2}\}, \ 0 < \pi_{11} < 1, \ A \equiv [1 + (1/\sigma) + \delta_1(1+\gamma_2\rho_1)/\gamma_2]$$

$$\pi_{12} = -(1-\sigma)\rho_2\pi_{11}/(1-\pi_{11}\sigma), \qquad -\rho_2 < \pi_{12} < 0 ,$$

$$\pi_{13} = -\sigma^{-1}\pi_{12}, \qquad 0 < \pi_{13} ,$$

$$\pi_{14} = -\pi_{11}/\gamma_2, \qquad \pi_{14} < 0 ,$$

$$\pi_{15} = \pi_{12}(1-\pi_{11}) - \rho_2\pi_{11}, \qquad \pi_{15} < 0 ,$$

$$\pi_{16} = -\pi_{11}, \qquad \pi_{15} < \rho_2\pi_{16} < 0,$$

$$\pi_{17} = \pi_{12}, \qquad \pi_{17} < 0 ,$$

$$\overline{R}_t = \pi_{21}N_{t-1} + \pi_{22}X_{t-1} + \pi_{23}u_{t-1} + \pi_{24}k_t + \pi_{25}E_tu_t + \pi_{26}E_td_t + \pi_{27}\varepsilon_t \quad (A9)$$

where

$$\pi_{21} = (\pi_{11}-1)\gamma_2/(1+\gamma_2\rho_1), \qquad \pi_{21} < 0 ,$$

$$\pi_{22} = \gamma_2(\rho_2 + \pi_{12})/(1+\gamma_2\rho_1) , \qquad 0 < \pi_{22} ,$$

$$\pi_{23} = -\sigma^{-1}\pi_{22} \qquad \pi_{23} < 0 ,$$

$$\pi_{24} = (1-\pi_{11})/(1+\gamma_2\rho_1) \qquad 0 < \pi_{24} ,$$

$$\pi_{25} = \gamma_2(1-\pi_{11})(\rho_2 + \pi_{12})/(1+\gamma_2\rho_1), \quad 0 < \pi_{25} ,$$

$$\pi_{26} = \gamma_2(1 - \pi_{11})/(1 + \gamma_2\rho_1) \,, \qquad 0 < \pi_{26} \,,$$

$$\pi_{27} = \pi_{22}, \qquad 0 < \pi_{27} \,,$$

$$Y_t = \pi_{31}N_{t-1} + \pi_{32}X_{t-1} + \pi_{33}u_{t-1} + \pi_{34}k_t + \pi_{35}E_tu_t + \pi_{36}E_td_t + \pi_{37}\varepsilon_t \quad \text{(A10)}$$

$$\pi_{31} = (\pi_{11} - 1)/(1 + \gamma_2\rho_1) \,, \qquad \pi_{31} < 0 \,,$$

$$\pi_{32} = (\rho_2 + \pi_{12})/(1 + \gamma_2\rho_1) \qquad 0 < \pi_{32} \,,$$

$$\pi_{33} = - \sigma^{-1}\pi_{32}, \qquad \pi_{33} < 0 \,,$$

$$\pi_{34} = (\pi_{14} - \rho_1)/(1 + \gamma_2\rho_1) \qquad \pi_{34} < 0 \,,$$

$$\pi_{35} = (\rho_2 + \pi_{12})(1 - \pi_{11})/(1 + \gamma_2\rho_1), \quad 0 < \pi_{35} \,,$$

$$\pi_{36} = (1 - \pi_{11})/(1 + \gamma_2\rho_1) \,, \qquad 0 < \pi_{36} \,,$$

$$\pi_{37} = \pi_{32}, \qquad 0 < \pi_{37} \,,$$

<u>Step Two</u>

Recall that $\varepsilon_t \equiv (E_t - E_{t-1})\sum_{i=0}^{\infty}(\kappa_0 + \kappa_1\bar{R}_{t+i} + \kappa_2Y_{t+i})\sigma^i$. Since ε_t is a function only of the innovations in the time t information set, we postulate:

$$\varepsilon_t = \psi_1k_t + \psi_2E_tu_t + \psi_3E_td_t + \psi_4(u_{t-1} - E_{t-1}u_{t-1}) \,. \qquad \text{(A11)}$$

The values of the ψ coefficients may be obtained by the method of undetermined coefficients using the intermediate solutions (A8), (A9) and (A10) in the definition of ε_t with (A11) everywhere replacing ε_t. This results in

$$\psi_1 = (\gamma_2\psi)^{-1}(1 - \sigma)(1 - \pi_{11}\sigma)[-\kappa_2(1 - \sigma)\pi_{11}(1 + \gamma_2\rho_1) +$$

$$\gamma_2 (\kappa_1 - \kappa_2\rho_1)(1 - \pi_{11})], \ \psi_1 < 0 ,$$

$$\psi_2 = - (1 - \sigma^2)\pi_{11}(1 - \pi_{11})\rho_2\psi^{-1}(\kappa_2 + \gamma_2\kappa_1) , \ \psi_2 < 0 ,$$

$$\psi_3 = (1 - \sigma)(1 - \pi_{11})(1 - \pi_{11}\sigma)(\kappa_2 + \gamma_2\kappa_1)\psi^{-1} , \ \psi_3 > 0,$$

$$\psi_4 = -(1 - \sigma)\sigma^{-1}(1 - \pi_{11})(1 - \sigma^2\pi_{11}) \rho_2(\kappa_2 + \gamma_2\kappa_1)\psi^{-1} \ \psi_4 < 0 ,$$

where $\psi \equiv \{(1 - \kappa_2\rho_2)(1 - \pi_{11}\sigma)^2(1 + \gamma_2\rho_1) + \kappa_2\rho_2(1 - \sigma)^2\pi_{11}(1 + \gamma_2\rho_1)$

$$-(\kappa_1 - \kappa_2\rho_1)\gamma_2\rho_2(1 - \pi_{11})(1 - \sigma^2\pi_{11})\}.$$

To sign these coefficients we imposed two conditions: (i) that the demand curve be downward sloping with respect to $\overline{R}_t$ (the Marshall-Lerner condition) and (ii) that the marginal propensity to consume the home good be between zero and one. At the point of linearization income is equal to expenditure. Therefore the demand function may be written as $D = \rho_0 - \rho_1\overline{R} + \rho_2(\kappa_0 + \kappa_1\overline{R} + \kappa_2\overline{Y})$. The Marshall-Lerner condition requires $(\rho_2\kappa_1 - \rho_1) < 0$ or $(\kappa_2\rho_2)^{\cdot}\kappa_1 - \kappa_2\rho_1 < 0$. Since $\kappa_2\rho_2$ is the marginal propensity to consume the home good, we impose $0 \leq \kappa_2\rho_2 \leq 1$. It follows that $\kappa_1 - \kappa_2\rho_1 < 0$. To sign the expression for ψ we needed to know the sign of $1 - \kappa_2\rho_2$, which is non-negative by $0 \leq \kappa_2 \rho_2 \leq 1$, and the sign of $\kappa_1 - \kappa_2\rho_1$, which is negative.

Step Three

The next step in obtaining a solution is to use (A11) in (A8)-(A10) to obtain text equations (12a) - (12c). The following coefficients result from this substitution:

N_t- equation (12a)

$\tau_{11} = \pi_{11} > 0$, $\quad 0 < \pi_{21} < 1$,

$\tau_{12} = \pi_{12} < 0$, $\quad -\rho_2 < \tau_{12} < 0$,

$\tau_{13} = \pi_{13} + \pi_{17}\psi_4 > 0$,

$\tau_{14} = \pi_{14} + \pi_{17}\psi_1 < 0$,

$\tau_{15} = \pi_{15} + \pi_{17}\psi_2 < 0$,

$\tau_{16} = \pi_{16} + \pi_{17}\psi_3 < 0$,

$\tau_{17} = - \pi_{17}\psi_4 < 0$,

$\overline{R}_t$- equation (12b)

$\tau_{21} = \pi_{21} < 0$,

$\tau_{22} = \pi_{22} > 0$,

$$\tau_{23} = \pi_{23} + \pi_{27}\psi_4 < 0 ,$$

$$\tau_{24} = \pi_{24} + \pi_{27}\psi_1 > 0 ,$$

$$\tau_{25} = \pi_{25} + \pi_{27}\psi_2 > 0 ,$$

$$\tau_{26} = \pi_{26} + \pi_{27}\psi_3 > 0 ,$$

$$\tau_{27} = - \pi_{27}\psi_4 > 0 ,$$

<u>Y_t - equation (12c)</u>

$$\tau_{31} = \pi_{31} < 0 ,$$

$$\tau_{32} = \pi_{32} > 0 ,$$

$$\tau_{33} = \pi_{33} + \pi_{37}\psi_4 < 0 ,$$

$$\tau_{34} = \pi_{34} + \pi_{37}\psi_1 < 0 ,$$

$$\tau_{35} = \pi_{35} + \pi_{37}\psi_2 > 0 ,$$

$$\tau_{36} = \pi_{36} + \pi_{37}\psi_3 > 0 ,$$

$$\tau_{37} = - \pi_{37}\psi_4 > 0 .$$

For the coefficients τ_{13}, τ_{14}, τ_{15}, τ_{24}, τ_{25} and τ_{35} the terms involving the ψ's are of opposite signs than the original (e.g., $\pi_{13} > 0$, $\pi_{17}\,\psi_4 < 0$). In every case, however, the terms with ψ's attached do not influence the sign of the τ coefficients. (Proofs available on request.)

D. Signal Extraction

Agents in our model see two pieces of data, a_t and b_t, which are useful in forming beliefs about u_t and d_t. As defined in the text, $a_t = v_t - \alpha_2 u_t$ and $b_t = \rho_2 u_t + d_t$.

Linear least squares projections of u_t and d_t onto a_t and b_t result in

$$E_t u_t = \omega_{uu} u_t + \omega_{uv} v_t + \omega_{ud} d_t$$

$$E_t d_t = \omega_{du} u_t + \omega_{dv} v_t + \omega_{ud} d_t$$

where

$$\omega_{uu} = \Omega^{-1}(\rho_2^2 \sigma_u^2 \sigma_v^2 + \alpha_2^2 \sigma_u^2 \sigma_d^2) > 0 ,$$

$$\omega_{uv} = \Omega^{-1}(-\alpha_2 \sigma_u^2 \sigma_d^2) < 0 ,$$

$$\omega_{ud} = \Omega^{-1}(\rho_2 \sigma_u^2 \sigma_d^2) > 0 ,$$

$$\omega_{du} = \Omega^{-1}(\rho_2 \sigma_v^2 \sigma_d^2) > 0 ,$$

$$\omega_{dv} = \Omega^{-1}(\rho_2 \alpha_2 \sigma_u^2 \sigma_d^2) > 0 ,$$

$$\omega_{dd} = \Omega^{-1}(\sigma_v^2\sigma_d^2 + \alpha_2^2\sigma_u^2\sigma_d^2) > 0 ,$$

$$\Omega^{-1} = [\rho_2^2\sigma_u^2\sigma_v^2 + \sigma_v^2\sigma_d^2 + \alpha_2^2\sigma_u^2\sigma_d^2]^{-1} ,$$

and σ_j^2 is the variance of j for j = u, d, v. Recall also that u, d, and v are assumed to be mutually and serially uncorrelated.

E. Final Reduced Forms

The final reduced form solutions of the model are given in equations (19a) - (19c). They are obtained by substituting the results of the inference problem for E_tu_t, E_td_t and $E_{t-1}u_{t-1}$ in the semi-reduced forms given in equations (12a) - (12c). The reduced form coefficients are given by the following:

N_t - equation (19a)

$$\lambda_{N1} = \pi_{11} > 0 , \qquad 0 < \lambda_{N1} < 1 ,$$

$$\lambda_{N2} = \pi_{12} < 0 , \qquad -\rho_2 < \lambda_{N2} < 0 ,$$

$$\lambda_{N3} = \pi_{13} + \pi_{17}\psi_4(1 - \omega_{uu}) > 0 ,$$

$$\lambda_{N4} = \pi_{14} + \pi_{17}\psi_1 < 0 ,$$

$$\lambda_{N5} = - \pi_{17}\psi_4\omega_{ud} < 0 ,$$

$$\lambda_{N6} = - \pi_{17}\psi_4\omega_{uv} > 0 ,$$

$$\lambda_{N7} = (\pi_{15} + \pi_{17}\psi_2)\omega_{uu} + (\pi_{16} + \pi_{17}\psi_3)\omega_{du} < 0 ,$$

$$\lambda_{N8} = (\pi_{15} + \pi_{17}\psi_2)\omega_{ud} + (\pi_{16} + \pi_{17}\psi_3)\omega_{dd} < 0 ,$$

$$\lambda_{N9} = (\pi_{15} + \pi_{17}\psi_2)\omega_{uv} + (\pi_{16} + \pi_{17}\psi_3)\omega_{dv} \gtrless 0 ,$$

$\bar{R}_t$ - equation (19b)

$$\lambda_{R1} = \pi_{R1} < 0 ,$$

$$\lambda_{R2} = \pi_{R2} > 0 ,$$

$$\lambda_{R3} = \pi_{R3} + \pi_{R7}\psi_4(1 - \omega_{uu}) < 0 ,$$

$$\lambda_{R4} = \pi_{R4} + \pi_{R7}\psi_1 > 0 ,$$

$$\lambda_{R5} = - \pi_{R7}\psi_4\omega_{ud} > 0 ,$$

$$\lambda_{R6} = - \pi_{R7}\psi_4\omega_{uv} < 0 ,$$

$$\lambda_{R7} = (\pi_{R5} + \pi_{R7}\psi_2)\omega_{uu} + (\pi_{R6} + \pi_{R7}\psi_3)\omega_{du} > 0 ,$$

$$\lambda_{R8} = (\pi_{R5} + \pi_{R7}\psi_2)\omega_{ud} + (\pi_{R6} + \pi_{R7}\psi_3)\omega_{dd} > 0 ,$$

$$\lambda_{R9} = (\pi_{R5} + \pi_{27}\psi_2)\omega_{uv} + (\pi_{R6} + \pi_{R7}\psi_3)\omega_{dv} > 0 ,$$

Y_t - equation (19c)

$\lambda_{Y1} = (1/\gamma_2)\lambda_{R1} < 0$,

$\lambda_{Y2} = (1/\gamma_2)\lambda_{R2} > 0$,

$\lambda_{Y3} = (1/\gamma_2)(\lambda_{R3} - 1) < 0$,

$\lambda_{Y4} = (1/\gamma_2)(\lambda_{R4}^{-1}) < 0$,

$\lambda_{Y5} = (1/\gamma_2)\lambda_{R5} > 0$,

$\lambda_{Y6} = (1/\gamma_2)\lambda_{R6} > 0$,

$\lambda_{Y7} = (1/\gamma_2)\lambda_{R7} > 0$,

$\lambda_{Y8} = (1/\gamma_2)\lambda_{R8} > 0$,

$\lambda_{Y9} = (1/\gamma_2)\lambda_{R9} > 0$.

Data Appendix

All data were obtained from a magnetic tape of the International Monetary Funds' International Financial Statistics. The tape provides nominal and real (base 1980) figures for gross domestic product for four of our countries, the United Kingdom, Italy, France, and Switzerland. For the United States, Canada, Japan, and Germany only nominal GDP was available. Our output measure (Y_t) was real GDP where available and was constructed to be nominal GDP divided by the GNP deflator for the latter four countries. Real expenditure is real GDP plus imports minus exports. Real expenditure (X_t) was constructed by adding the nominal trade balance to nominal GDP and dividing by either the nominal GDP or GNP deflator. The relative price (R_t^1) was constructed as export prices divided by consumer prices while the alternative definition of the relative price or terms of trade (R_t^2) was constructed as the export price divided by the import price. All estimation uses the second quarter of 1973 as the earliest lag, and the final observation on the dependent variables is the fourth quarter of 1983 for the U.S., the U.K., Germany, and Canada. France and Switzerland end in 1983:3, Italy in 1982:4, and Japan in 1983:1.

References

Aizenman, Joshua and Frenkel, Jacob A. "Optimal Wage Indexation, Foreign Exchange Intervention, and Monetary Policy, "American Economic Review, June 1985, 75, 402-23.

Amihud, Yakov and Mendelson, Haim, "The Output-Inflation Relationship: An Inventory-Adjustment Approach," Journal of Monetary Economics, March 1982, 9, 163-84.

Barro, Robert J. and Hercowitz, Zvi, "Money Stock Revisions and Unanticipated Money Growth," Journal of Monetary Economics, April 1980, 6, 257-67.

Blanchard, Olivier J., "The Production and Inventory Behavior of the American Automobile Industry," Journal of Political Economy, June 1983, 91, 365-400.

Blinder, Alan, "Retail Inventory Behavior and Business Fluctuations," Brookings Papers on Economic Activity, 2:1981, 443-505.

________________, "Inventories and Sticky Prices: More on the Microfoundations of Macroeconomics," American Economic Review, June 1982, 72, 334-48.

Blinder, Alan and Fischer, Stanley, "Inventories, Rational Expectations, and the Business Cycles," Journal of Monetary Economics, November 1981, 8, 227-304.

Boschen, John A. and Grossman, Herschel I., "Tests of Equilibrium Macroeconomics Using Contemporaneous Monetary Data," Journal of Monetary Economics, November 1982, 10, 309-34.

Brunner, Karl, Cukierman, Alex, and Meltzer, Allan H. "Money and Economic Activity, Inventories and Business Cycles, " Journal of Monetary Economics, May 1983, 11, 281-320.

Dornbusch, Rudiger, "Expectations and Exchange Rate Dynamics," Journal of Political Economy, December 1976, 84, 1161-76.

Eichenbaum, Martin S., "A Rational Expectations Model of the Cyclical Behavior of Inventories of Finished Goods and Employment," Journal of Monetary Economics, August 1983, 12, 259-78.

________________, "Rational Expectations and the Smoothing Properties of Inventories of Finished Goods," Journal of Monetary Economics, July 1984, 14, 71-96.

Fischer, Stanley, "Long-Term Contracts, Rational Expectations and the Optimal Money Supply Rule," Journal of Political Economy, February 1977, 85, 191-205.

Flavin, Marjorie A., "The Adjustment of Consumption to Changing Expectations about Future Income," Journal of Political Economy, October 1981, 89, 974-1009.

Flood, Robert P., "Capital Mobility and the Choice of Exchange Rate System," International Economic Review, June 1979, 20, 405-16.

Flood, Robert P. and Hodrick, Robert J. "Optimal Price and Inventory Adjustment in an Open-Economy Model of the Business Cycle", Quarter Journal of Economics, Supplement 1985, 100, 887-914.

Flood, Robert P. and Marion, Nancy P., "The Transmission of Disturbances Under Alternative Exchange-Rate Regimes with Optimal Indexing," Quarterly Journal of Economics, February 1982, 97, 43-66.

Friedman, Milton, A Theory of the Consumption Function, Princeton: Princeton University Press (for the National Bureau of Economic Research 1957).

Gray, Jo Anna, "Wage Indexation: A Macroeconomic Approach," Journal of Monetary Economics, April 1976, 2, 221-36.

Holt, C., Modigliani, F. Muth, J., and Simon, H., Planning Production, Inventories, and Work Force, New York: Prentice Hall, 1960.

Kendall, Maurice and Alan Stuart, The Advanced Theory of Statistics, Vol. 1, London: Charles Griffen and Company, 1977.

Kimbrough, Kent P. "Exchange-Rate Policy and Monetary Information," Journal of International Money and Finance, December 1982, 2, 333-46.

Leiderman, Leonardo, "Expectations and Output-Inflation Tradeoffs in a Fixed-Exchange Rate Economy," Journal of Political Economy, December 1979, 87, 1285-1306.

Lovell, Michael, "Manufacturer's Inventories, Sales Expectations and the Acceleration Principle," Econometrica, March 1961, 29, 293-314.

Lucas, Robert E. Jr., "Expectations and the Neutrality of Money," Journal of Economic Theory, April 1972, 4, 103-24.

__________, "An Equilibrium Model of the Business Cycle, " Journal of Political Economy, December 1975, 83, 1113-44.

__________, "Understanding Business Cycles," in Karl Brummer and Allan H. Meltzer, editors, Stabilization of the Domestic and International Economy, Carnegie-Rochester Conference Series on Public Policy, 5, supplement to Journal of Monetary Economics, 1977, 7-30.

Marston, Richard . "Wages, Relative Prices and the Choice Between Fixed and Flexible Exchange Rates," Canadian Journal of Economics, February 1982, 15-87-103.

McCallum, Bennett T., "On the Observational Inequivalence of Classical and Keynesian Models," Journal of Political Economy, April 1979, 87, 395-402.

__________, "Inventory Fluctuations and Macroeconomic Analysis: A Comment," Carnegie-Mellon University, manuscript, 1984.

Mussa, Michael, "A Model of Exchange Rate Dynamics," Journal of Political Economy, February 1982, 90, 74-104.

Phelps, Edmund S., Microeconomic Foundations of Employment and Inflation Theory, New York: Norton, 1969.

Saidi, Nasser H., "Fluctuating Exchange Rates and the International Transmission of Economic Disturbances," Journal of Money, Credit and Banking, November 1980 Pt. 1, 12, 575-91.

Stockman, Alan C. and Koh, Ai Tee, "Open-Economy Implications of Two Models of Business Fluctuations," N.B.E.R. Working Paper No. 1317, March 1984.

Taylor, John B. "Estmation and Control of a Macroeconomic Model with Rational Expectations, "Econometrica, September 1979, 47, 1267-86.

Turnovsky, Stephen J., "The Relative Stability of Alternative Exchange Rate Systems in the Presence of Random Disturbances," Journal of Money, Credit and Banking February 1976, 8, 29-50.

Weber, Warren E., "Output Variability under Monetary Policy and Exchange Rate Rules," Journal of Political Economy August 1981, 89, 733-51.