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Autoregressive Spectrum Estimation Technique Applied to Quarterly Consumer Durables Expenditure Data

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Abstract

Classical spectral techniques can provide sharp insights into the cyclical patterns in a time series of economic data. Various problems in the application of classical spectral techniques, such as the choices of smoothing routine and bandwidth and the appearance of end-effects, inhibit the usefulness of spectral analysis. Alternatively, an autoregressive spectral technique does not share these problems, but does present the difficulty of the choice of the order of the autoregression. This paper applies classical and autoregressive spectral techniques to quarterly consumer durables expenditure data, discusses three approaches to the choice of the order of the autoregression, and compares the results of the different spectral techniques. Autoregressive spectral analysis provides a superior representation for this time series.

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Contents

Introduction

Spectral analysis, used to show how the variance of a stochastic process is distributed over cycle frequencies, has become a standard tool of econometricians. It is especially helpful in the development of structural models with lags and in the study of business cycles. However, the application of classical spectral analysis techniques based on the Fourier transform of the autocovariance function of a time series is by no means straight-forward. The researcher is typically confronted by two types of problems in the use of spectral techniques. First, which smoothing routine? Commonly used routines for smoothing are the Parzen, Tukey, Bartlett, rectangular, or triangular lag windows[1]. Second, which bandwidth or range for the window? With each of these decisions there exists no widely accepted optimality criterion[2]. Furthermore, the resulting smoothed periodogram is usually quite sensitive to these choices. The advantages of the power spectrum curve over the autocovariance function are tempered by these problems in the application of classical spectral techniques. This paper will show the desirability of using an autoregressive spectral estimation technique over the classical spectral techniques.

The autoregressive spectral analysis of quarterly consumer durables expenditure data from 1951:I to 1973:IV in this paper follows the procedure outlined by Richard Jones[3]. This paper divides into three sections. The first section displays the results of using a classical spectral technique on a stationary time series derived from the data. Some of the problems in the application of the technique and in the interpretation of the results are discussed. The second section deals with the estimation of the autoregression equations and discusses the choice of the order of the autoregression under three optimality criteria. The final section presents the results of the autoregressive spectral analysis and compares these results to those of the classical spectral technique. The basic findings is that, under the appropriate choice of the autoregression, the autoregressive spectral technique can provide a great deal of insight into the economic time series data and yields a representation far superior to that derived under the classical spectral technique.

I Results from a Classical Spectral Technique

Both classical and autoregressive spectral techniques require a stationary time series as input. The consumer durables expenditure data and a detrended quarterly first difference transformation of this series are shown in the appendix. This filter is recommended by Green[4] and Howrey[5]. To test the stationarity of the filtered series in either the strict[6] or wide[7] sense, the autocorrelation function, shown in the appendix, should be examined. The general damping found for lags of one through thirty means that this filtered series is a close approximation to a stationary stochastic process[8]:

In classical spectral techniques, the sample spectrum is simply the Fourier transform of the sample autocovariance function:

$$S(f) = \Delta \sum_{k=-(N-1)}^{(N-1)} c(k)\ \mathrm{EXP}(-i2\pi fk\Delta) \qquad -1/2\Delta \le f \le 1/2\Delta$$

with: $S(f)$ = sample spectrum for frequency $\underline{f}$
$c(k)$ = sample autocovariance function of filtered series
N = sample size
Δ = interval size

Various techniques have been proposed to smooth or reduce the variance of $S(f)$. These techniques focus on the choice of the function $w(k)$ such that:

$$S(f) = \Delta \sum_{k=-(N-1)}^{(N-1)} w(k)\ c(k)\ \mathrm{EXP}(-i2\pi fk\Delta) \qquad -1/2\Delta \le f \le 1/2\Delta$$

The lag window $w(k)$ must satisfy:

(1) $w(0) = 1$
(2) $w(k) = w(-k)$
(3) $w(k) = 0, \ |k| \le M, \ \ M \le N$

Condition (3), the truncation of the autocovariance function, allows covariances to be computed only up to lag M. The value of M is called the bandwidth of the estimation. Condition (2) makes S(f) an even function of frequency. It is only necessary to compute the sample spectrum over the range $0 \leq f \leq 1/2\Delta$. It is convenient to assume $\Delta=1$. The resulting form of the spectrum estimator is[9]:

$$S(f) = 2(c(0) + 2\sum_{k=1}^{M-1} w(k)\, c(k)\, \cos(2\pi fk)) \qquad 0 \leq f \leq 1/2$$

One additional attribute of the lag window is that the bandwidth varies inversely with the variance of the spectrum estimate, S(f). As the bandwidth increases, more smoothing is performed and the variance of S(f) decreases.

The application of the rectangular smoothing technique to the filtered series will point out some of the problems with classical spectral techniques[10]. Under the rectangular smoothing technique the lag window is:

$$w(k) = \begin{cases} 1 & |k| \leq M \\ 0 & |k| > M \end{cases}$$

Figures 1-4 show the resulting periodogram for bandwidths of 10, 20, 30, and 40. At this point in the paper we are only concerned with the solid curves. The dashed curves will be compared to the solid curves in section three. Note that the logs of the spectra were adjusted to be graphed on a uniform 0-5 scale.

Based on this illustration, two broad problems with classical spectral techniques, beyond the choice of the spectral window, will be discussed. More sophisticated techniques, such as the Parzen window applied by Howrey to filtered consumer durables expenditure data[11], possess the same difficulties.

Bandwidth: The usual approach in the application of classical spectral techniques is to present the spectrum estimates of a number of different bandwidths and then to leave to the reader's judgment the determination of which portrays the most credible pattern and hence is the "best" periodogram.

The choice of bandwidth is crucial for two reasons. First, the bandwidth affects the shape of the spectrum. Ideally, the bandwidth will correspond closely to the width of the peaks of the spectrum[12]. In the case of economic data, peaks will not all have the same width and may be quite diverse in widths. The NBER technical report on spectral analysis states[13]:

> The use of a smoothing process is not without problems. Depending on the range of the smoothing window and the variability of the spectrum, smoothing may obscure spectral peaks, or it may not average enough points to give a good estimate of the spectrum. In the former case, the spectrum will be very smooth; in the latter case, the spectrum may be very irregular. It is advisable to try several ranges or bandwidths to determine their effect on the spectrum.

Jenkins and Watts also emphasize the role of bandwidth[14]:

> In general, to achieve high fidelity the bandwidth of the window must be of the same order as the width of the narrowest important detail in the spectrum.

There is always a trade-off of decreasing bandwidth and encountering irregular detail against increasing bandwidth and smoothing over peaks. For example, Figure 1 with bandwidth of 10 shows very narrow peaks and troughs and a very jagged spectrum -- not acceptable as a representation of economic behavior. On the other hand, Figure 4 with bandwidth of 40 eliminates most of the variance of the estimator and shows a very flat, unrevealing spectrum at mid-range frequencies with no distinct peaks. Howrey encounters analogous difficulties with bandwidths for the Parzen window.

Second, only peaks in the spectrum that are separated by a frequency interval greater than the bandwidth are considered independent of each other. Figures 1-4 indicate the bandwidth scale near the tops of the plots. The desire to reveal independent peaks places a strong limit on the bandwidth choice and hence on the smoothing of the estimator's variance in most applications.

<u>End-Effects</u>: Note that in Figures 1-4 the spectrum estimates seem to indicate the domination of both very low and very high frequencies. Jenkins and Watts provide this interpretation of spectral patterns[15]:

> Smooth series are characterized by spectra which have most of their power at low frequencies...Quickly oscillating series are characterized by spectra which have most of their power at high frequencies.

Do Figures 1-4 mean that the filtered quarterly consumer durables expenditure series is both smooth and quickly oscillating? The explanation lies in the inability of smoothing techniques based on lag windows to operate efficiently near the endpoints of the spectrum. The bandwidth must be decreased to obtain an equal number of values on each side of the frequency being estimated, leading to a spectrum that often appears noisy near the endpoints. We see relatively unsmoothed values of the spectrum near the endpoint frequencies. It would be misleading to compare the spectrum values near the endpoints to those in the mid-range frequencies. Therefore, the classical spectral techniques give little indication of the comparative spectral power of very long and very short cycles. Yet, these frequencies are typically quite important in economic time series.

Due to these problems with classical spectral techniques, we now turn to an alternative spectral technique based on the coefficients of an autoregression equation for the filtered series.

II Choice of the Order of the Autoregression

Jones uses the following procedure for the calculation of the autoregressive spectrum in the scalar case based on an autoregression equation of order p[16]:

(1) Compute the one step prediction variance:

$$v = S_p/(N-1-p)$$

with: S_p = sum of squared residuals for autoregression of order p

N = length of data span

(2) Compute the spectral density function:

$$S(f) = v/\left|1 - \sum_{k=1}^{p} a_k \, \mathrm{EXP}(2\pi ikf)\right|^2 \qquad 0 \le f \le 1/2$$

with: a_k = coefficient for lag of length k in autoregression equation of order p

The identification of the order of the autoregression to be used in autoregressive spectral analysis is of comparable importance to the choice of bandwidth for the classical spectral techniques. Here, though, there do exist reasonable optimality criteria. Three criteria for the identification of the optimal order of the autoregression will be discussed: (1) Akaike's Information Criterion; (2) Stepwise Partial F-tests; and (3) Parzen's comparative whiteness F-test. We will find that Parzen's criterion gives the most useful results.

Autoregression equations of orders one through thirty were estimated for the filtered series using ordinary least squares with no constant term. Although stopping the estimation at order thirty was somewhat arbitrary, we will see later that further regressions would not have been helpful. Columns (2) and (3) of Table 1 show the maximum log likelihood and the sum of squared residuals for each autoregression equation.

Akaike's criterion is designed to minimize the average error for a one step ahead prediction. Akaike first proposed the final prediction error criterion (FPE)[17]. For the scalar case, this criterion is:

$$\min_{p} \ FPE_p = S_p \left(\frac{N + 1 + p}{N - 1 - p}\right)$$

Akaike later extended this model selection criterion to any maximum likelihood model. This more general criterion is called Akaike's Information Criterion (AIC)[18]:

$$\min_{p} \ AIC_p = -2L_p + 2p$$

with: L_p = maximum log likelihood for autoregression of order $\underline{p}$

Jones recommends the use of AIC in the selection of the order of the autoregression for autoregressive spectral analysis[19].

The results of computing AIC_p for autoregressions of order one through thirty are shown in Column (4) of Table 1. Note that although local minima are found at p=24 and p=26, no value appears lower than that at the boundary of p=30. However, in applications Akaike restricts the range of p*, the optimal order, with the following rule-of-thumb:

$$\text{Choose } p^* = \text{Min} \begin{cases} N/5 \\ \underline{p} \text{ corresponding to the global minimum of } AIC_p \end{cases}$$

With 91 observations in the filtered series, the maximum order of the autoregression recommended by Akaike is 18. We conclude that, possibly because of the relatively low number of observations, AIC does not identify an acceptable order of the autoregressions on the filtered series. In section III of this paper we shall compute the spectra corresponding to autoregressions of order 30 and of order less than 18 and find that there is strong evidence for the acceptance of Akaike's upper limit of N/5 on the order.

We next use stepwise partial F-tests to try to identify the optimal order of the autoregressions. A test comparing a restriction on the model (ω) to a maintained or accepted form of the model (Ω) which is distributed as the F-statistic with ($p_\Omega - p_\omega$, $n_\Omega - p_\Omega$) degrees of freedom takes the form:

$$F = \frac{((n_\Omega/n_\omega)S_{p_\omega} - S_{p_\Omega})/(p_\Omega - p_\omega)}{S_{p_\Omega}/(n_\Omega - p_\Omega)}$$

with: p_Ω, p_ω = orders of autoregressions corresponding to Ω and ω respectively
$S_{p_\Omega}, S_{p_\omega}$ = sum of squared residuals corresponding to autoregressions of orders p_Ω and p_ω respectively
n_Ω, n_ω = numbers of observations used in estimations of Ω and ω respectively

Stepwise partial F-tests are commonly used in the testing of nested hypotheses. In the case of the autoregression equations, $p_\omega < p_\Omega$. F-tests using this statistic starting with p_Ω=30 were executed. However, the first F-test with p_Ω=30 and p_ω=29 yielded a value of 3.32, resulting in the rejection of the restriction on the order of the autoregression ($F(1,61)_{.90} = 2.79$). We find that the stepwise partial F-tests do not allow the acceptance of orders of the autoregression lower than the boundary of 30.

The final criterion used for the identification of the optimal order of the autoregressions is Parzen's comparative whiteness F-test[20]. Parzen addressed the problem of how to reduce the number of terms in an autoregressive moving average (ARMA) model. Starting with an ARMA model Ω with a white noise error term, we seek a nested form ω with the fewest non-zero coefficients of the ARMA model such that the error term under ω is not significantly different than the white noise error term of Ω. Modifying Parzen's test statistic for the case of choosing an optimal order of autoregression equations, we have an F-test distributed with $(p_\Omega, n_\Omega - p_\Omega)$ degrees of freedom:

$$F = \left(\frac{S_{p_\omega} n_\Omega}{S_{p_\Omega} n_\omega} - 1\right) / \left(\frac{p_\Omega}{n_\Omega - p_\Omega}\right)$$

The results of the application of the above test statistic are shown in Column (5) of Table 1. This test, based on Parzen's comparative whiteness F-test and using the .90 level of significance recommended by Parzen, shows that all orders of autoregression down to and including 11 are acceptable restrictions on the autoregression of order 30, which was taken as the Ω model. Recall that Akaike's rule-of-thumb led to the need to identify an order less than or equal to 18. The order of the autoregression identified by Parzen's comparative whiteness F-test satisfies this condition.

In the next section two autoregressive spectra will be computed, based on orders thirty and eleven. The results confirm the need to be very careful in the identification of the order of the autoregression whose parameters are to be used as input to the autoregressive spectral analysis. The parameter values for orders thirty and eleven are shown in Table 2.

III Application of Autoregressive Spectral Analysis

The autoregressive spectral density function was evaluated at 64 frequencies between 0 and 0.5, spaced at even intervals of 0.007813 cycles/quarter. Figure 5 shows the autoregressive spectrum based on p=30. Figure 6 shows the autoregressive spectrum based on p=11. For ease of comparison, the solid line in

Figure 5 corresponds to the spectrum of p=30 and the dashed line in that figure corresponds to the spectrum of p=11.

The autoregressive spectrum corresponding to p=30 shows many signs of instability. The peaks and troughs are often far too narrow to be a valid representation of economic phenomena. Also, the spectrum shows a high variance, indicative of the presence of too much detail. For example, the difference between the peak and trough about 0.33 cycles/quarter is extremely sharp, unacceptably so. In the terms of classical spectral analysis, the spectrum appears to require additional smoothing.

On the other hand, the autoregressive spectrum corresponding to p=11 appears perfectly acceptable. The peaks and troughs are of credible widths and the spectral pattern is quite smooth. Although there is a strong emphasis on the very high frequencies, the spectrum does not show high power at the very low frequencies[21]. There does not exist the same concern over end-effects that pertained to the spectra obtained by the rectangular smoothing window or the autoregressive spectrum of p=30. Finally, the pattern of peaks closely corresponds to empirical observations about the behavior of business cycles[22]. The high power of the spectrum at the very high frequencies indicates a strongly oscillating series, very short cycles or fluctuations. The next highest concentration of power occurs at about an eleven quarter cycle. Much of the empirical literature emphasizes the three year cycle as a dominant force in business cycles. Other major peaks correspond to eight, four, three, and two and one-half quarter cycles.

If one accepts the autoregressive spectrum based on p=11 as a true representation, it is quite easy to see why classical spectral techniques may yield very poor results on this filtered series. Returning to the problem of finding a bandwidth which closely corresponds to the widths of the peaks of the spectrum, note there is a wide diversity of peak widths in the autoregressive spectrum based on p=11. Narrow peaks occur at about 0.06 and 0.4 cycles/quarter and rather wide peaks occur at frequencies of 0.25 and 0.31 cycles/quarter. Note also the very wide trough at 0.2 cycles/quarter. This diversity of widths makes the choice of bandwidth under classical spectral techniques extremely difficult. In fact, this diversity almost guarantees a substantial amount of bias in the application of classical spectral techniques with any bandwidth.

Comparison of the dashed lines, corresponding to the autoregressive spectrum with p=11, to the solid lines of the rectangular smoothing window in Figures 1-4 shows that none of the classical spectra is highly correlated with the autoregressive spectrum. The spectra of the lower bandwidths are far too irregular and the spectra of the higher bandwidths are too flat. Furthermore, all of the classical spectra show a concentration of power at the very low frequencies while the autoregressive spectrum shows a trough there.

In conclusion, this analysis has found that the autoregressive spectrum estimation technique, using the order identified by Parzen's comparative whiteness F-test, provides a great deal of insight into the filtered series of quarterly consumer durables expenditure data. Additionally, the autoregressive spectrum estimation technique was found far superior to the classical technique in the correspondence between the spectral representation and empirical understanding of the economic behavior. The autoregressive spectrum estimation technique also avoids many of the complications and pitfalls of the classical spectrum techniques, specifically dealing with bandwidths and end-effects.

Jones makes the following generalization about the use of the autoregressive spectrum estimation technique[23]:

> Experience gained from analyzing large amounts of data from the biological and physical sciences has indicated that using both autoregressive spectrum estimation and classical spectrum and superimposing the plots gives a much stronger feeling for the shape of the true spectrum being estimated.

The experience of this research using economic data leads to the conclusion that whereas the autoregressive technique revealed much about the data, the classical techniques had many serious problems and yielded no simple, clear interpretation of the data. Furthermore, superimposing the plots did not supplement the understanding gained from the autoregressive spectrum estimate.

Notes

1 G.M. Jenkins and D.G. Watts, Spectral Analysis and Its Applications. San Francisco: Holden Day, 1968. Chapter 6.3.

2 A.A. Lomnicki and Zaremba, "On Estimating the Spectral Density Function of a Stochastic Process," Journal of the Royal Statistical Society, 1957. Jenkins and Watts, p. 274-279.

3 R.H. Jones, "Identification and Autoregressive Spectrum Estimation," Technical Report No. 6 of the Department of Computer Science, State University of New York at Buffalo, Februray 1974.

4 G.R. Green, "Short- and Long-Term Simulations with the OBE Econometric Model," in B.G. Hickman (ed.), Econometric Models of Cyclical Behavior, Vol. 1. New York: Columbia University Press, 1972.

5 E.P. Howrey, "Dynamic Properties of a Condensed Version of the Wharton Model," in B.G. Hickman (ed.), Econometric Models of Cyclical Behavior, Vol. 2. New York: Columbia University Press, 1972. p. 602-603.

6 Jenkins and Watts, p. 149.

7 K. Astrom, Introduction to Stochastic Processes. New York: Academic Press, 1970. Chapter 2.

8 Jenkins and Watts, p. 149.

9 Jenkins and Watts, p. 260.

10 Rectangular smoothing was used because it is the standard spectral window of the spectral analysis package of the NBER-Project Troll system.

11 Howrey, p. 604.

12 Jenkins and Watts, p. 256.

13 "Documentation for Project Troll-- Spectral and Cross-Spectral Analysis," National Bureau of Economic Research, Computer Research Center for Economics and Management Science, 1974. p. 5.

14 Jenkins and Watts, p. 279.

15 Jenkins and Watts, p. 218.

16 Jones, p. 8-9.

17 H. Akaike, "Autoregressive Model Fitting for Control," Ann. Inst. Statist. Math, 1971.

18 H. Akaike, "Use of an Information Theoretic Quantity for Statistical Model Identification," Proceedings of the Fifth Hawaii International Conference on System Sciences, Western Periodical Co.

19 Jones, p. 2.

20 E. Parzen, "Sone Recent Advances in Time Series Modeling," Technical Report No. 10 of the Department of Computer Science, State University of New York at Buffalo, May 1974. p. 19-21. See also: E. Parzen, "Some Solutions to the Time Series Modeling and Prediction Problem," Technical Report No. 5 of the Department of Computer Science, State University of New York at Buffalo, February 1974.

21 Any detailed analysis and comparison of spectra requires the calculation of confidence intervals for the spectra. The present discussion is intended to explore how different spectral techniques and different parameters for the spectral techniques affect the resulting spectra for the same filtered series. No definitive statement about the spectra is intended.

22 G.H. Moore, "Tested Knowledge of Business Cycles," National Bureau of Economic Research, Inc., Forty-Second Annual Report, June 1962. A.F. Burns and W.C. Mitchell, Measuring Business Cycles. National Bureau of Economic Research, 1946. M. Friedman and A. Schwartz, A Monetary History of the United States. Princeton: Princeton University Press, 1963.

23 Jones, p. 1.

Figure 1

Rectangular Smoothing:Bandwidth=10

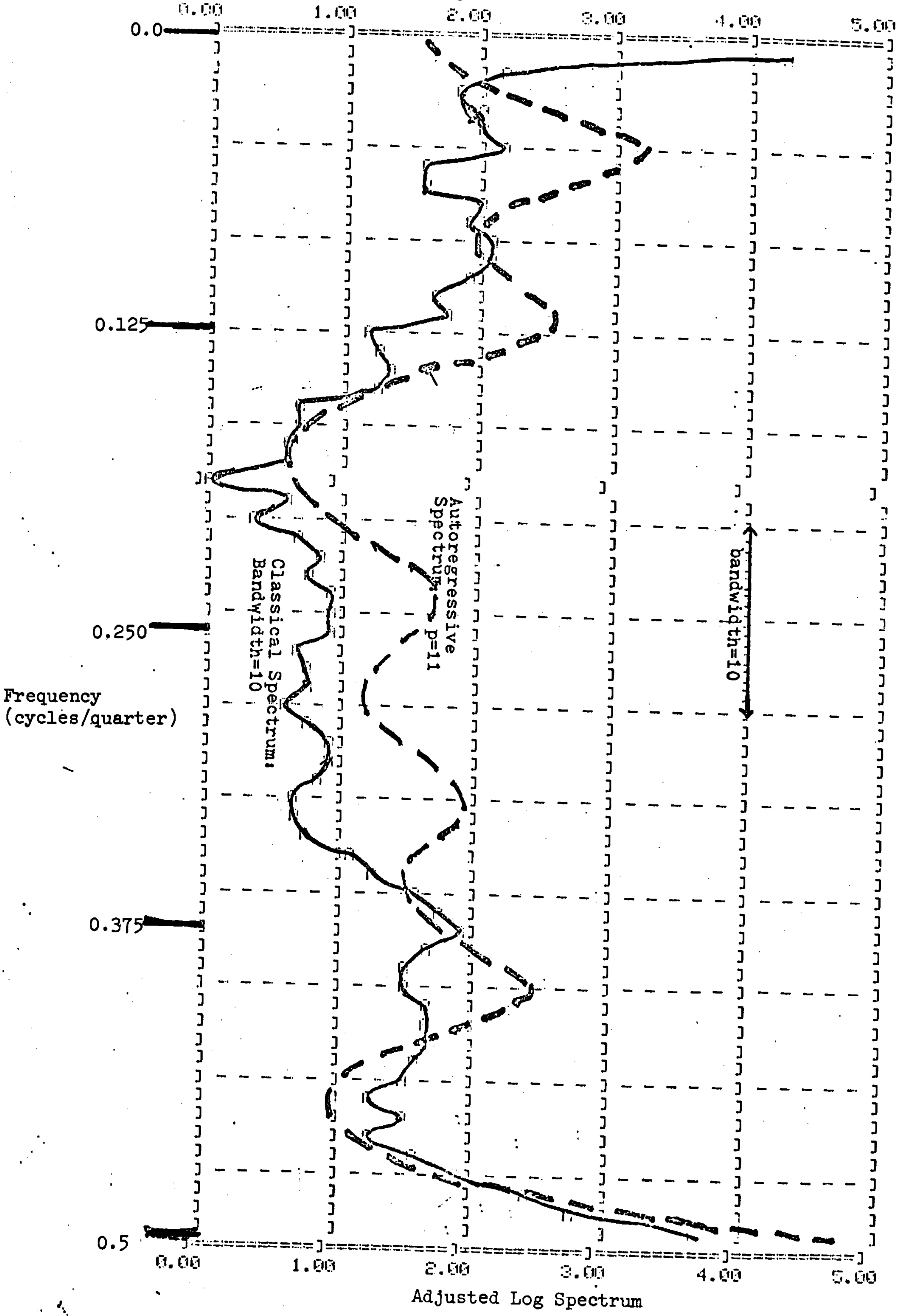


Figure 2
Rectangular Smoothing:Bandwidth=20

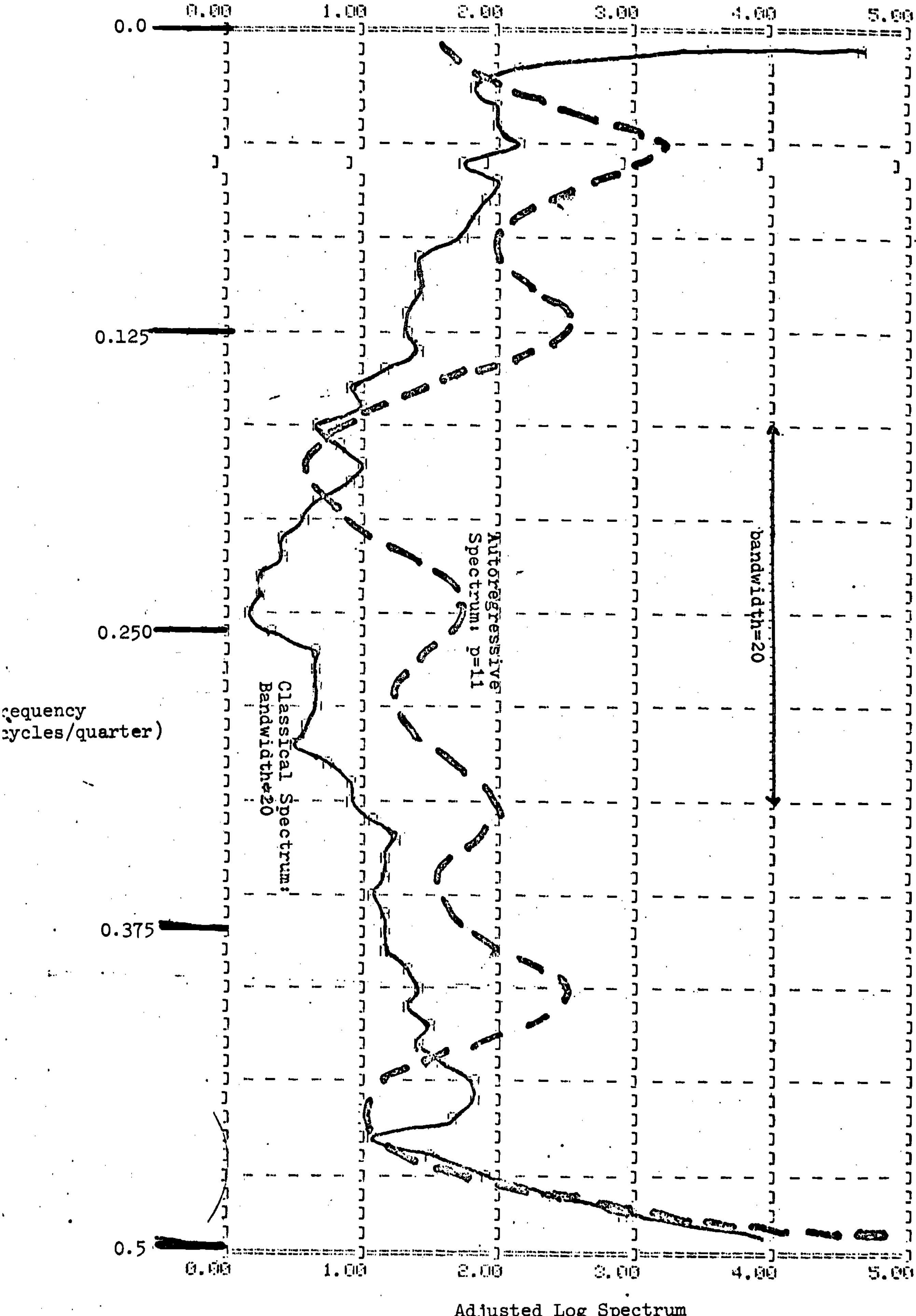


Figure 3

Rectangular Smoothing:Bandwidth=30

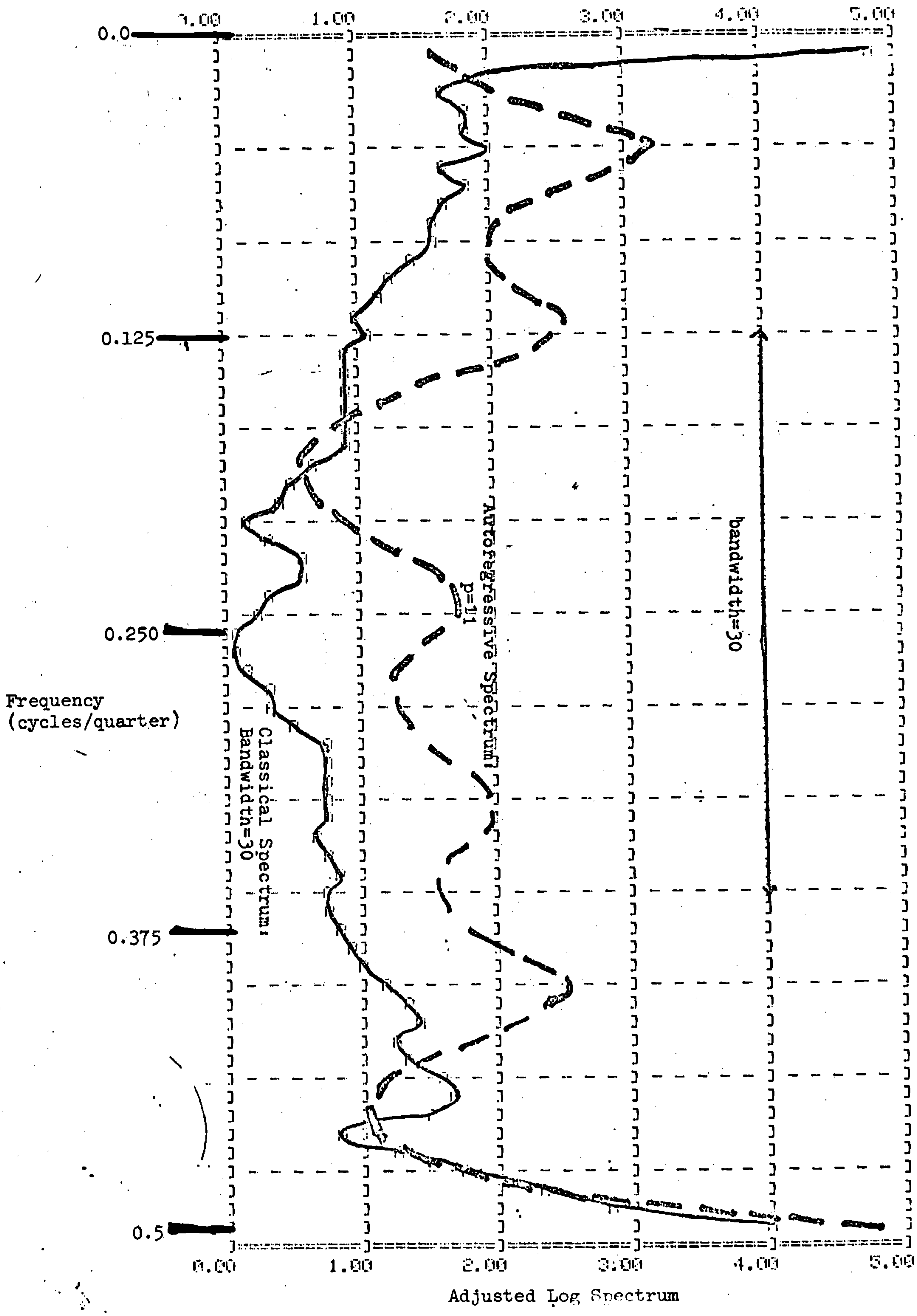


Figure 4
Rectangular Smoothing: Bandwidth=40

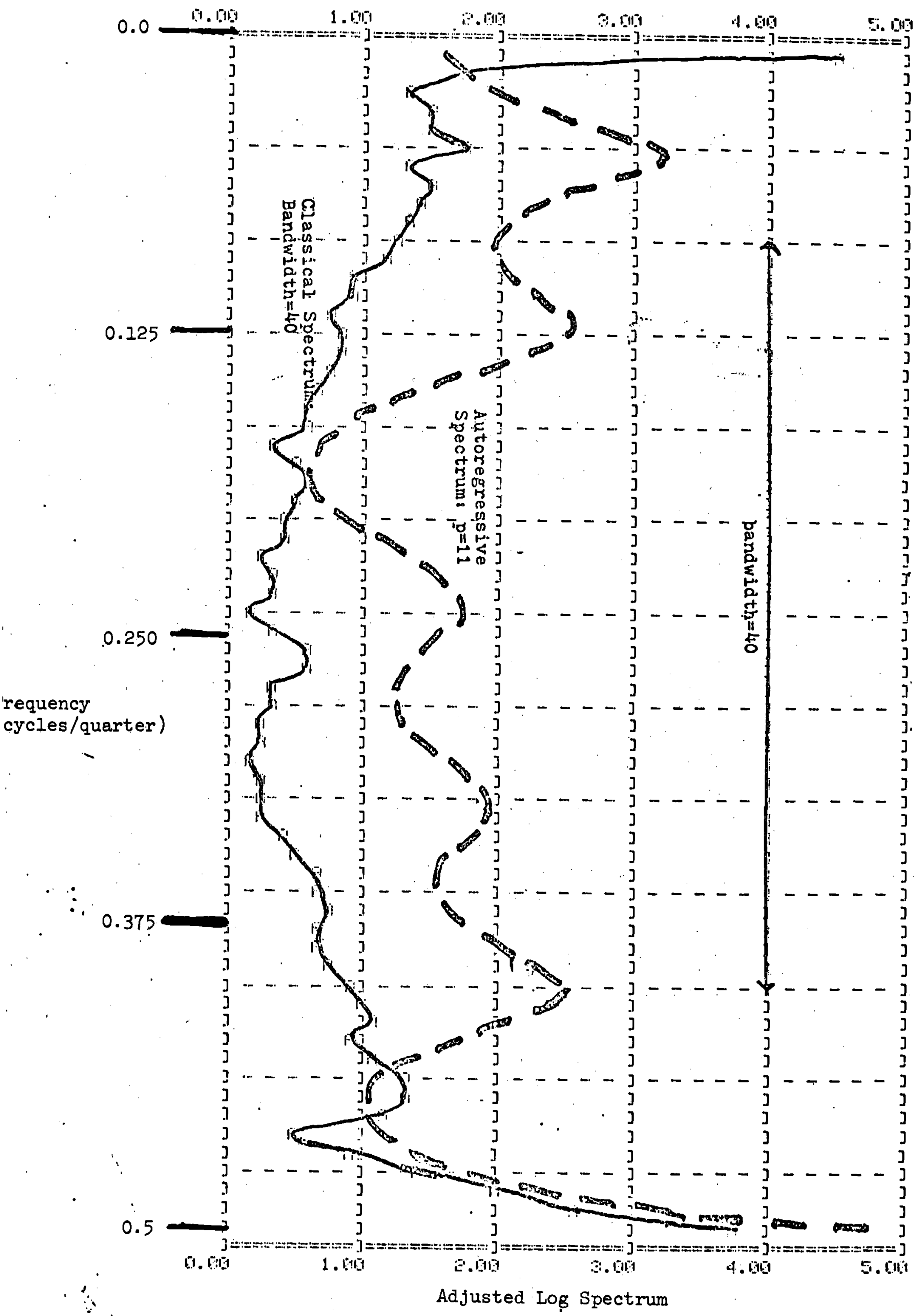


Table 1

Identifying the Optimal Order of the Autoregressions

(1) Order of Autoregression	(2) Maximum Log Likelihood	(3) Sum of Squared Residuals	(4) AIC	(5) F-Parzen ($F(30,61)_{.90}=1.48$)
1	-213.6	601.0	429.2	1.71
2	-210.3	574.3	424.6	1.58
3	-208.9	574.1	423.8	1.62
4	-207.5	573.4	423.0	1.66
5	-205.8	568.3	421.6	1.67
6	-203.5	555.6	419.0	1.63
7	-201.1	541.4	416.2	1.58
8	-198.7	527.6	413.4	1.53
9	-197.2	524.4	412.4	1.55
10	-195.4	517.7	410.8	1.54
11	-191.3	482.4	404.6	1.35
12	-187.4	450.4	398.8	1.16
13	-186.0	448.0	398.0	1.18
14	-184.8	447.6	397.6	1.22
15	-181.3	420.9	392.6	1.07
16	-177.5	393.0	387.0	0.91
17	-175.9	386.9	385.8	0.89
18	-172.2	360.7	380.4	0.74
19	-170.7	355.8	379.4	0.74
20	-167.9	338.5	375.8	0.64
21	-166.4	333.2	374.8	0.64
22	-165.2	330.4	374.4	0.65
23	-162.5	313.3	371.0	0.55
24	-158.2	283.1	364.41	0.34
25	-157.2	281.4	364.43	0.36
26	-155.7	274.7	363.3	0.34
27	-155.0	274.7	363.9	0.37
28	-151.0	247.1	357.9	0.17
29	-149.1	237.1	356.2	0.11
30	-146.5	221.1	352.9	----

Table 2

Parameters of Autoregressions of Orders 30 and 11

Length of lag	Order 30 Coefficient	Order 30 Standard Error	Order 11 Coefficient	Order 11 Standard Error
1	.0459	.1743	.0367	.1260
2	.1465	.1727	.2405	.1265
3	.0500	.1656	.1016	.1296
4	.0082	.1860	-.0639	.1347
5	-.1563	.1839	-.1699	.1336
6	.0071	.1905	-.0513	.1338
7	-.0130	.1801	-.0833	.1340
8	.1701	.1747	.1644	.1354
9	-.2254	.1777	-.0707	.1362
10	.0803	.1782	.0207	.1361
11	-.1204	.1854	-.2857	.1301
12	.5657	.2196		
13	.2284	.2457		
14	-.1743	.2525		
15	.1955	.2413		
16	.2845	.2410		
17	.1684	.2424		
18	.0774	.2478		
19	-.1774	.2476		
20	.2958	.2468		
21	.1569	.2555		
22	-.3299	.2473		
23	-.5922	.2529		
24	.5099	.2624		
25	-.0171	.2499		
26	.0066	.2548		
27	-.3469	.2657		
28	-.0956	.2835		
29	.3897	.2850		
30	-.2649	.2378		

Figure 5
Autoregressive Spectrum: p=30

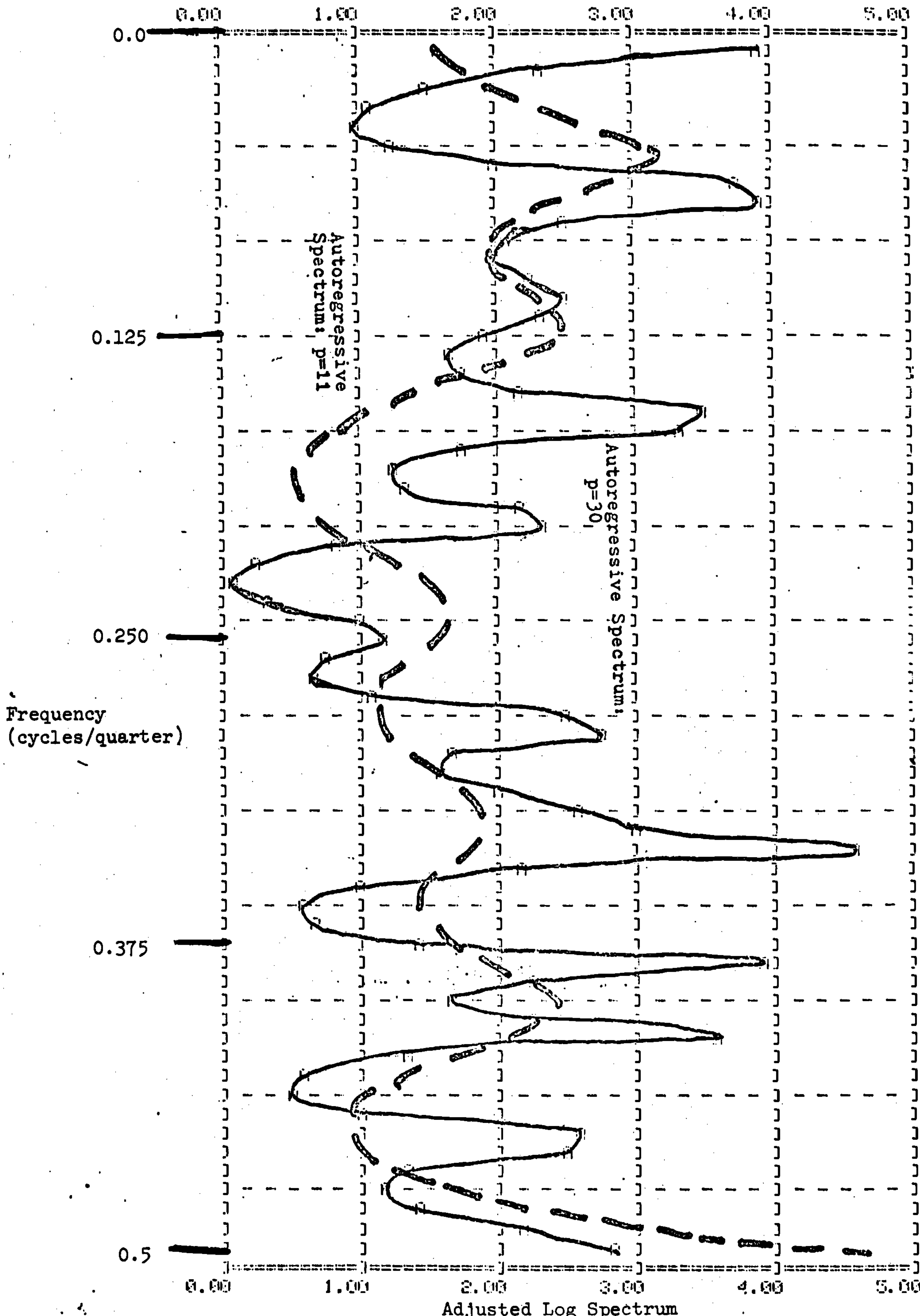
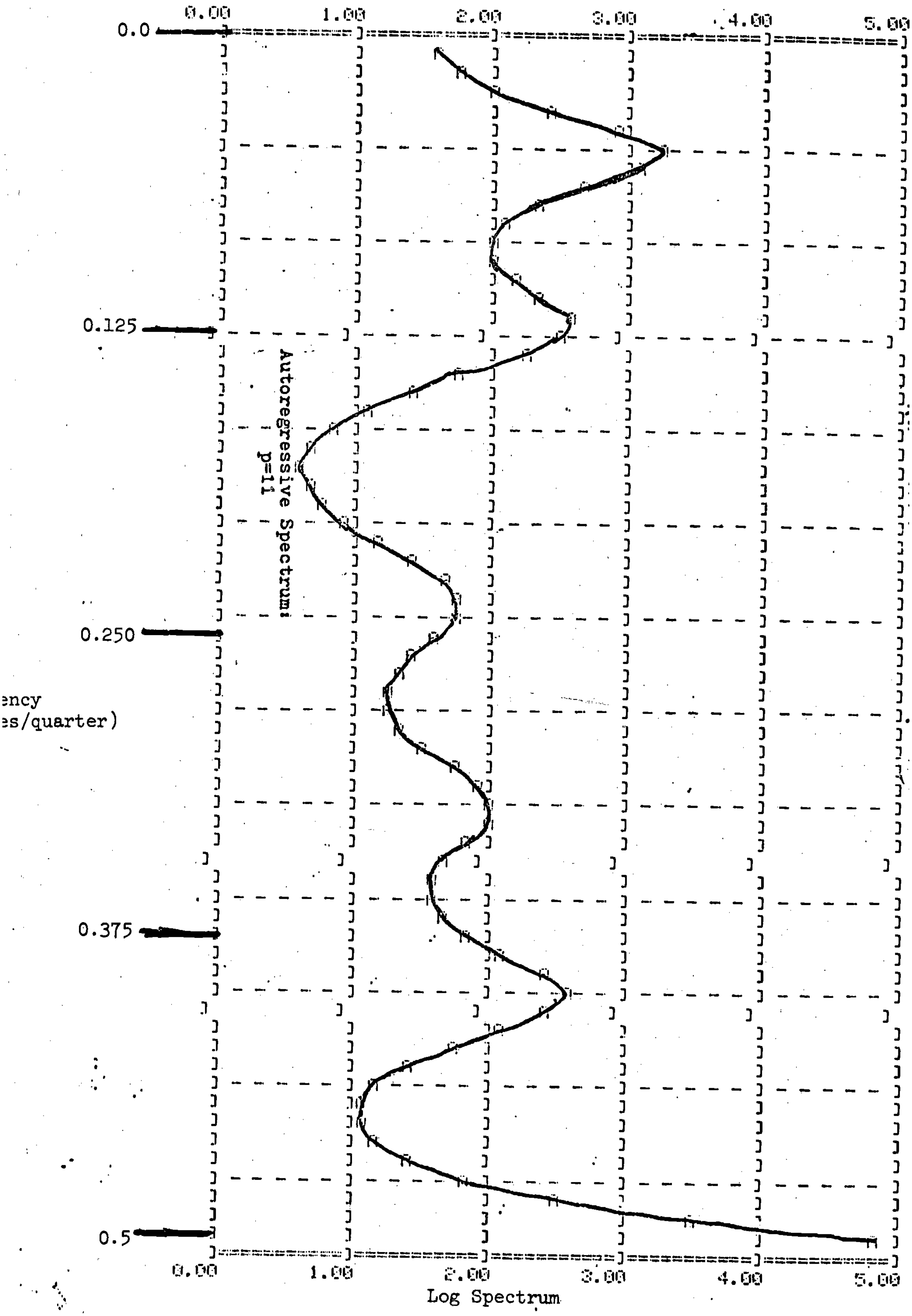
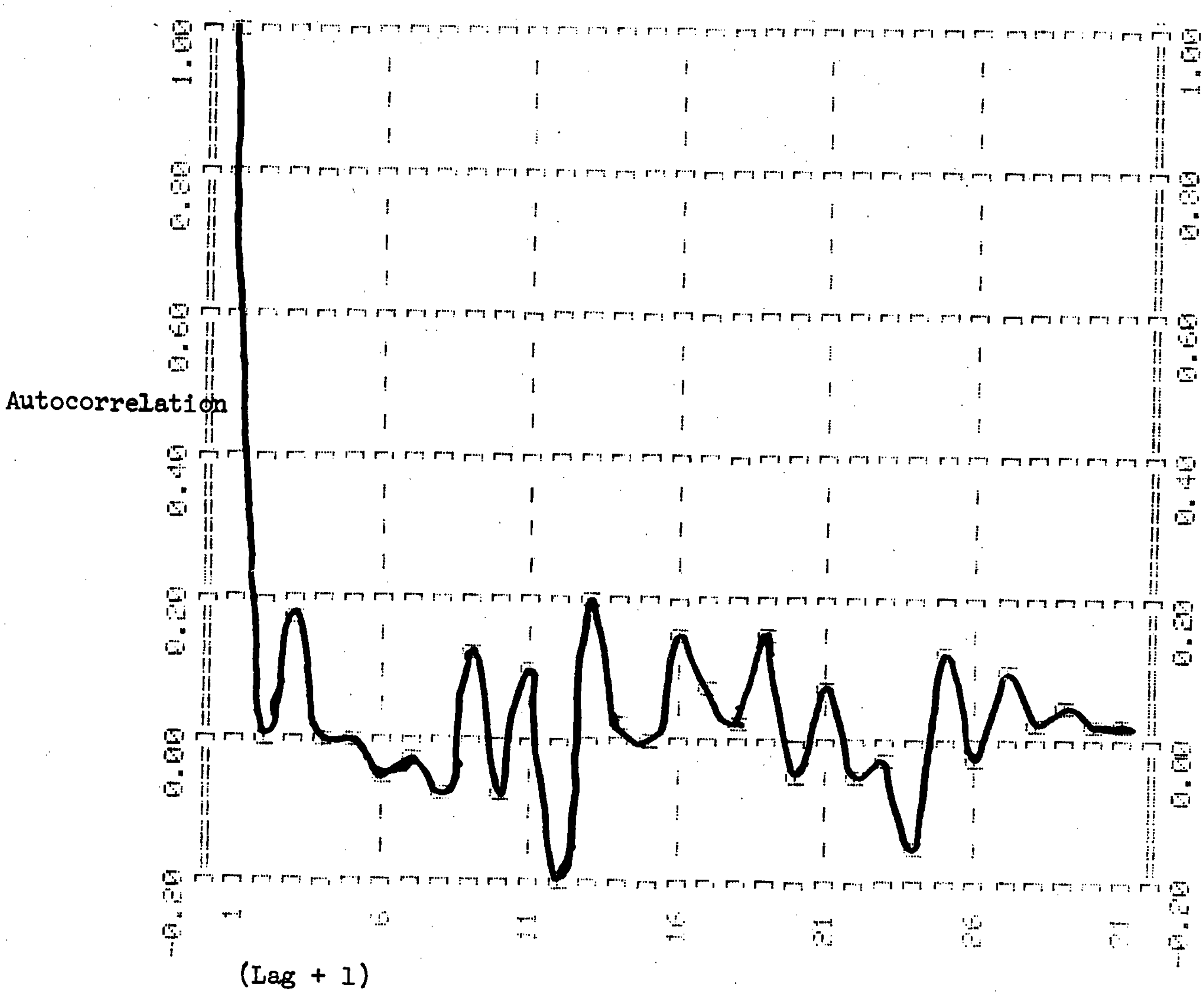


Figure 6
Autoregressive Spectrum: p=11



Appendix

Autocorrelation Function for Filtered Series



Appendix

Consumer Durables Expenditure Data and Filtered Series

Quarterly Consumer Durables
Expenditure Data ($ billions, nominal)

quarter	data	quarter	data
1951-I	33.6	1963-I	52.4
	28.6		53.2
	28.1		54.5
	28.3		55.6
1952-I	28.8	1964-I	57.9
	29.1		59.6
	27.5		60.7
	32.0		58.7
1953-I	33.5	1965-I	65.4
	33.5		64.4
	33.4		66.5
	32.6		68.9
1954-I	32.0	1966-I	71.2
	32.5		68.5
	32.5		71.3
	34.2		71.9
1955-I	37.4	1967-I	69.8
	39.6		73.6
	41.4		73.7
	40.1		75.3
1956-I	38.5	1968-I	80.4
	38.6		82.4
	38.4		86.3
	40.2		87.0
1957-I	41.4	1969-I	90.2
	40.9		91.0
	40.6		90.6
	40.2		91.4
1958-I	37.9	1970-I	90.9
	36.8		92.8
	37.7		93.4
	39.1		88.1
1959-I	42.8	1971-I	100.6
	45.0		102.1
	45.8		105.6
	43.6		107.4
1960-I	45.9	1972-I	112.1
	46.1		116.2
	45.3		121.2
	43.8		124.3
1961-I	41.9	1973-I	132.4
	43.4		132.1
	44.8		132.4
	46.6		124.3
1962-I	48.5		
	48.5		
	50.1		
	51.1		

Filtered Series

number	value	number	value
1	-6.0	47	0.0
2	-1.5	48	0.3
3	-0.8	49	-0.2
4	-0.5	50	0.3
5	-0.7	51	0.1
6	-2.6	52	1.3
7	3.5	53	0.7
8	0.5	54	0.1
9	-1.0	55	-3.0
10	-1.1	56	5.7
11	-1.8	57	-2.0
12	-1.6	58	1.1
13	-0.5	59	1.4
14	-1.0	60	1.3
15	0.7	61	-3.7
16	2.2	62	1.8
17	1.2	63	-0.4
18	0.8	64	-3.1
19	-2.3	65	2.8
20	-2.6	66	-0.9
21	-0.9	67	0.6
22	-1.2	68	4.1
23	0.8	69	1.0
24	0.2	70	2.9
25	-1.5	71	-0.3
26	-1.3	72	2.2
27	-1.4	73	-0.2
28	-3.3	74	-1.4
29	-2.1	75	-0.2
30	-0.1	76	-1.5
31	0.4	77	0.9
32	2.7	78	-0.4
33	1.2	79	-6.3
34	-0.2	80	11.5
35	-3.2	81	0.5
36	1.3	82	2.5
37	-0.8	83	0.8
38	-1.8	84	3.7
39	-2.5	85	3.1
40	-2.9	86	4.0
41	0.5	87	2.1
42	0.4	88	7.1
43	0.8	89	-1.3
44	0.9	90	-0.7
45	-1.0	91	-9.1
46	0.6		