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TAXES AND TRADING VERSUS INTENSITY STANDARDS:  
SECOND-BEST ENVIRONMENTAL POLICIES WITH INCOMPLETE REGULATION (LEAKAGE) OR MARKET POWER

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Taxes and Trading versus Intensity Standards: Second-Best Environmental Policies with Incomplete Regulation (Leakage) or Market Power

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**ABSTRACT**

This paper investigates whether an emissions tax (equivalent to an emissions cap) maximizes social welfare (defined as the sum of consumer and producer surplus) in the presence of incomplete regulation (leakage) or market power by analyzing an intensity standard regulating emissions per unit of output. With no other market failures, an intensity standard indeed yields lower welfare, although combining it with a consumption tax eliminates this discrepancy. For incomplete regulation, I show that under certain conditions an intensity standard can yield higher welfare than any emissions tax (including the optimal emissions tax). This result persists even with the addition of a consumption tax, which ameliorates output distortions and can sometimes help the intensity standard attain the first best (when an emissions tax/consumption tax combination cannot). Comparing intensity standards to output-based updating shows that the latter yields higher welfare because of its additional flexibility. Finally, I show that with market power an intensity standard can yield higher welfare than the optimal emissions tax.

The intuition of these results is relatively straightforward. The weakness of an intensity standard is that it relies more on substitution effects than output effects to reduce emissions. With incomplete regulation or market power, this disadvantage may be helpful since leakage may offset gains from reducing output and since market power already inefficiently reduces output.

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# 1 Introduction

Emissions taxes and emissions markets are widely regarded as the preferred policy instruments for regulation of environmental externalities. Since these instruments can impose the correct price on a missing market, the instruments can mimic the first-best market environment and implement the efficient control of the externality. This paper investigates whether these instruments indeed maximize social welfare (defined as the sum of consumer and producer surplus) in the presence of other market failures. Surprisingly, I show that these two instruments may yield lower welfare than a third instrument, an *intensity standard* regulating emissions per unit of output, in the presence of incomplete regulation (leakage) or market power.<sup>1</sup> In fact, since the analysis compares the second-best policies, the stronger result holds that under certain conditions any tax or any emissions cap yields lower welfare than an intensity standard.

Incomplete regulation or leakage can occur for two reasons.<sup>2</sup> First, a political jurisdiction may not be geographically consistent with the region that suffers environmental damages or with the product market. For example, since carbon is a global pollutant, regulating carbon emissions within any single country or set of countries may cause production and emissions to “leak” to countries which do not regulate carbon emissions. International leakage is especially troublesome since attempts to tax foreign-produced goods based on carbon content would likely violate international trade law. Second, within a political jurisdiction some sectors may use political clout to avoid regulation, and there may be costs to expanding the regulated base to cover 100% of the emissions.<sup>3</sup> For example, biofuels are largely exempt from proposed carbon legislation.<sup>4</sup> Once the scope of the regulation is set, production (and emissions) will tend to leak to the unregulated firms.

Market power’s effect on environmental regulation was first discussed by Buchanan (1969),

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<sup>1</sup>The analysis of intensity standards is parallel to some analyses of performance standards. Since performance standards are generally considered a command and control policy, I use “intensity standard” to reflect the market-based nature of recent policies such as the low carbon fuel standard.

<sup>2</sup>Fowlie (2007) investigates leakage with market power. See also Bushnell *et al.* (2007).

<sup>3</sup>Metcalf (2008) argues for a carbon tax base that covers 90% of the U.S.’s carbon emissions. Stavins (2008) states that “nearly all” U.S. CO<sub>2</sub> emissions could be captured by regulating 2,000 upstream entities. Bluestein (2005) estimates that “about 1,250 entities [are] required for 95%+ capture of domestic [CO<sub>2</sub>] production.” Proposed legislation is much less comprehensive.

<sup>4</sup>In particular, indirect land use effects are exempt from the Waxman-Markey bill.

and Barnett (1980) showed that the optimal emissions tax for a monopoly should generally be less than the marginal damages.<sup>5</sup> These theoretical concerns are important as many polluting industries are likely subject to market power.<sup>6</sup>

The inefficiency of intensity standards was established by Helfand (1991) and Fischer (2001).<sup>7</sup> More recently Holland *et al.* (2009) argued that California's Low Carbon Fuel Standard (LCFS), an intensity standard regulating carbon emissions per unit of transportation fuel, cannot attain the first best, could increase carbon emissions, and has much higher abatement costs than an efficient policy.<sup>8</sup> However, optimal intensity standards and optimal emissions taxes (or trading) have not been compared under leakage or market power.<sup>9</sup>

While the main result of this paper, that an intensity standard can yield higher welfare than the second-best tax or emissions cap, is surprising in the light of the prior literature, the intuition is relatively straightforward. Environmental market mechanisms reduce emissions through both substitution and output effects. Substitution effects reduce emissions by employing additional capital (*e.g.*, emissions control technology) or more costly fuel inputs (*e.g.*, switching to a cleaner fuel source). Output effects reduce emissions by reducing consumption of the polluting good (*e.g.*, through car-pooling or investments in energy efficiency). Intensity standards fail because they mainly induce substitution effects and not output effects. For example, Holland *et al.* argue that the LCFS does not directly encourage carpooling, reduced driving, or vehicular fuel efficiency. Alternatively, a tax or emissions cap efficiently reduces emissions through both substitution and output effects.

With leakage or market power, an intensity standard can yield higher welfare because output effects may not efficiently reduce emissions. With leakage, higher marginal costs of the

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<sup>5</sup>Mansur (2007) compares taxes and emissions trading in concentrated industries. Optimal policy design is even more complicated with multiple heterogeneous firms with market power.

<sup>6</sup>For example, electricity is usually provided by regulated monopolies, world oil markets are affected by the OPEC cartel, and petroleum refining, coal mines, railroad transport of coal and ethanol, and cement and steel production are highly concentrated.

<sup>7</sup>Fullerton and Heutel (2007) analyze the incidence of intensity standards. Fischer (2003) characterizes the imperfect competition equilibrium with intensity standards.

<sup>8</sup>Newell and Pizer (2008) show that adjusting a regulation to an exogenous index (such as GDP) can decrease abatement costs. An exogenous index avoids the additional distortions of an intensity standard.

<sup>9</sup>The cost-effectiveness of a variety of policy instruments, including intensity standards, has been analyzed in the presence of pre-existing distortionary taxes (Goulder *et al.* 1999) and in the presence of industry compensation requirements (Bovenberg *et al.* 2008).

regulated sector are not translated directly into reduced consumption (and reduced emissions) since production can increase from the unregulated sector. If the supply of the unregulated sector is elastic enough and dirty enough, leakage may even increase total emissions. Similarly, with market power, firms inefficiently restrict output. Thus a policy that additionally restricts output may increase abatement costs unnecessarily. An intensity standard can yield higher welfare since it distorts output decisions less than a tax or emissions cap.

This paper is closely related to work on output-based updating (also called output-based allocations) of emissions permits.<sup>10</sup> Recent cap-and-trade legislation addressing climate change includes output-based updating for sectors susceptible to leakage.<sup>11,12</sup> Both intensity standards and output-based updating pursue two objectives (penalizing emissions and encouraging output) with one instrument. Clearly two instruments would be superior, and I show that an optimal combined emissions tax and production subsidy (for the covered sector) attain higher welfare than the optimal intensity standard. Whether output-based updating can attain higher welfare than the optimal intensity standard depends on the degree to which the updating can mimic the optimal combined emissions tax and output subsidy.

The advantage/disadvantage of intensity standards is that they subsidize output, which can lead to too much consumption. However, excess consumption can be remedied with a consumption tax.<sup>13</sup> With leakage, the advantage of a consumption tax is that it can apply equally to domestic and foreign production and thus complies with trade laws. This advantage is important since I derive conditions under which an intensity standard combined with a consumption tax can attain the first best whereas an emissions tax combined with a consumption tax cannot.<sup>14</sup>

Section 2 presents the basic model with pollution as the only market failure and illustrates the solution techniques. I show that in the absence of additional market failures an emissions tax (emissions cap) attains the first best, but an intensity standard does not. However, I show that

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<sup>10</sup>See Fischer and Fox (2007, 2009) and Bushnell and Chen (2009).

<sup>11</sup>Output-based updating is included in the House version of the American Clean Energy and Security Act of 2009 (ACES Act), H.R. 2454, also known as Waxman-Markey.

<sup>12</sup>Stavins (2007) argues instead for requiring that imports of carbon-intensive products—such as iron and steel, aluminum, cement, bulk glass, and paper, and possibly a very limited set of other particularly CO<sub>2</sub> emissions-intensive goods—carry with them CO<sub>2</sub> allowances. This scheme may conflict with trade law.

<sup>13</sup>This consumption tax is not meant as a broad-based tax on all consumption, but rather on consumption of this good.

<sup>14</sup>See also Fullerton (1997) and Fullerton and Mohr (2003) on similar combined instruments.

this deficiency can be corrected with the addition of a consumption tax.

Section 3 extends the model to analyze leakage from a covered (domestic) sector to an uncovered (foreign) sector. I characterize the second-best emissions tax and intensity standard and show that neither policy can attain efficiency unless uncovered emissions are also taxed at marginal damages. The second proposition shows that an intensity standard can attain higher welfare than the second-best emissions tax and derives a sufficient condition for this result. I then address combining the policies with a consumption tax applied to both covered and uncovered production. This additional instrument is useful since it can allow the intensity standard to attain the first-best even though an emissions tax/consumption tax combination cannot.

Section 4 extends the basic model to analyze market power and shows that an intensity standard can attain higher welfare than an emissions tax. Section 5 concludes.

## 2 First-best regulation

To introduce the model and solution methods, I first analyze regulation where the sole market failure arises from the externality. After describing the market equilibrium subject to an emissions tax or intensity standard, the optimal policy is characterized.

Emissions reductions are generally modeled as abatement from counterfactual emissions, which depend on output and are endogenous. To disentangle the relationship between output and emissions, I model emissions as an input (or “netput”) in the production process, *i.e.*, emissions are modeled the same as labor or capital.<sup>15</sup> The firm may demand more or less of the emissions input depending on prices. Modeling emissions as an input allows for rich substitution possibilities and places at our disposal all of the usual tools of production theory.

Assume a (representative) firm produces output,  $q$ , with a concave production function  $f(k, e)$  with non-negative marginal products ( $f_k \geq 0$  and  $f_e \geq 0$ ) where  $k$  is a vector of market inputs (*e.g.*, labor, capital, fuel, etc.) with price vector  $w$  and  $e$  is an unpriced input (*e.g.*, emissions).<sup>16</sup> Let  $U$  be the benefit function, where  $U' > 0$  and  $U'' < 0$ , and let damages from

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<sup>15</sup>Although not common, a similar approach is used for example in Fullerton and Heutel (2007).

<sup>16</sup>The  $i$ th market input, market input price, and marginal product are represented by  $k_i$ ,  $w_i$ , and  $f_{k_i}$ .

pollution be  $\tau e$ .<sup>17</sup>

Suppose the firm is subject to an emissions tax. The firm's cost function depends on the emissions tax,  $t$ , and is given by  $c(q; w, t) = \min_{k,e} wk + te + \lambda[q - f(k, e)]$  where  $\lambda$  is the Lagrange multiplier. Cost minimization implies that  $w = \lambda f_k(k, e)$  and  $t = \lambda f_e(k, e)$ , and the envelope theorem implies that marginal cost is the shadow value, *i.e.*,  $c_q(q; w, t) = \lambda$ . The four endogenous variables in the equilibrium— $q$ ,  $\lambda$ ,  $k$ , and  $e$ —are completely determined by the two first-order conditions from cost minimization; the production function:  $q = f(k, e)$ ; and the market clearing condition:  $U'(q) = \lambda$ .

The regulator chooses the tax to maximize net social benefits

$$\max_t U(q) - c(q; w, t) - \tau e + te. \quad (1)$$

Note that the tax revenue is counted as a cost for the firm and thus must be added to the objective.<sup>18</sup> The first order condition (FOC) is then

$$[U'(q) - c_q] \frac{\partial q}{\partial t} - \frac{\partial c}{\partial t} + e - (\tau - t) \frac{\partial e}{\partial t} = 0 \quad (2)$$

Since the first term is zero by the market clearing condition and the second and third terms are additive inverses by applying the envelope theorem to the regulated firm's cost function, the FOC implies the well-known result that  $t = \tau$ , *i.e.*, the optimal tax is simply marginal damages. This tax attains the efficient allocation, which is characterized by  $U'(q) = w_i/f_{k_i} = \tau/f_e$ . Similarly, the optimal emissions cap can reduce emissions efficiently as shown in the appendix. Note that the tax reduces pollution through both substitution and output effects.<sup>19</sup>

Now suppose the firm is subject to an intensity standard  $\sigma$  such that  $e/q$  must be less than  $\sigma$ . The firm's cost function depends on the intensity standard and is given by  $c(q; w, \sigma) = \min_{k,e} wk + \lambda[q - f(k, e)] + \gamma[e - \sigma q]$  where  $\lambda$  and  $\gamma$  are Lagrange multipliers. Cost minimization implies that  $w = \lambda f_k(k, e)$  and  $\gamma = \lambda f_e(k, e)$ . Note that the envelope theorem implies that the

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<sup>17</sup>Marginal damages are assumed constant, but the results are easily extended to increasing marginal damages.

<sup>18</sup>The regulator does not receive any benefit from the tax revenue. To see this, note that the objective is equivalent to  $U(q) - wk - \tau e$ . To model a benefit (cost) from tax revenue (for example, from off-setting other distortionary taxes) a multiplier could be included on the  $te$  term.

<sup>19</sup>Substitution and output effects can be made precise by analyzing Slutsky-like equations from the input demands.

marginal cost is  $c_q(q; w, \sigma) = \lambda - \gamma\sigma = w_i/f_{k_i} \cdot (1 - f_e\sigma)$ .<sup>20</sup> The five endogenous variables in the equilibrium— $q$ ,  $\lambda$ ,  $k$ ,  $e$ , and  $\gamma$ —are completely determined by the two first-order conditions from cost minimization; the production function,  $q = f(k, e)$ ; the market clearing condition,  $U'(q) = \lambda - \gamma\sigma$ ; and the binding intensity standard,  $e = \sigma q$ .

The regulator chooses the intensity standard to maximize net social benefits

$$\max_{\sigma} U(q) - c(q; w, \sigma) - \tau e. \quad (3)$$

The first order condition is then

$$[U'(q) - c_q] \frac{\partial q}{\partial \sigma} - \frac{\partial c}{\partial \sigma} - \tau \frac{\partial e}{\partial \sigma} = 0. \quad (4)$$

Since the first term is zero by the market clearing condition and since  $\frac{\partial c}{\partial \sigma} = \gamma q = qw_i f_e(k, e)/f_{k_i}(k, e)$ , this FOC implies that  $\tau \frac{\partial e}{\partial \sigma} = qw_i f_e(k, e)/f_{k_i}(k, e)$  which can be written:

$$\frac{f_e(k, e)}{f_{k_i}(k, e)} = \frac{\tau}{w_i} \left( \frac{\partial e}{\partial \sigma} \cdot \frac{\sigma}{e} \right) = \frac{\tau}{w_i} \left( 1 + \frac{\partial q}{\partial \sigma} \frac{\sigma}{q} \right) \quad (5)$$

where the last equation follows since  $\partial e/\partial \sigma = q + \sigma \partial q/\partial \sigma$ .

Unlike the emissions tax, the intensity standard cannot attain the first-best regulation of the externality. First note that the best intensity standard has the wrong combination of inputs since [5] implies that  $\frac{\tau}{w_i} \neq \frac{f_e(k, e)}{f_{k_i}(k, e)}$  unless the elasticity of output with respect to the standard is zero. Even if this elasticity were zero, so that the input combination were correct, the output level would still be wrong since  $c_q(q; w, \sigma) = \frac{w_i}{f_{k_i}} [1 - f_e\sigma]$  which is less than the efficient level  $\frac{w_i}{f_{k_i}}$ . Intuitively, the intensity standard acts as a subsidy to output since additional output relaxes the constraint on emissions; hence, marginal cost is too low.

An intensity standard does not distort the relative prices faced by market inputs. With market inputs  $i$  and  $j$ , the marginal rate of technical substitution (MRTS) between  $e$  &  $k_i$  and between  $e$  and  $k_j$  is in [5]. Dividing the results shows that the MRTS between the two market inputs,  $k_i$  and  $k_j$ , equals the market input price ratio, *i.e.*,  $f_{k_i}/f_{k_j} = w_i/w_j$ . Thus the intensity

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<sup>20</sup>The intuition of this condition describes the output subsidy effect. With an emissions tax, the cost of increasing output by one (marginal) unit is the cost of the additional input required,  $1/f_{k_i}$ , times its price,  $w_i$ . Cost minimization insures that the firm equates this cost across all market inputs. With an intensity standard, the cost of increasing output by one (marginal) unit is reduced. Increasing output by one unit relaxes the standard which allows  $\sigma$  additional units of the unpriced input and  $f_e\sigma$  additional units of free output. Thus increasing output requires additional market inputs for only the proportion  $1 - f_e\sigma$  of additional output.



standard does not distort the relative prices faced by market inputs, but distorts the price faced by emissions relative to market inputs and distorts the marginal cost.

Since the inefficiency of the intensity standard arises primarily from its failure to price output correctly, the inefficiency might be reduced by combining the intensity standard with a consumption tax. In fact, the result is much stronger: the inefficiency can be eliminated. Since the analysis of the consumption tax is quite similar to that above, it is relegated to the appendix (as are proofs of all results). To summarize:

**Proposition 1** *i) An emissions tax or emissions cap can attain the efficient level of emissions, input usage, and production. ii) An intensity standard cannot attain efficiency. iii) An intensity standard coupled with a consumption tax can attain the efficient level of emissions, input usage, and production.*

The inefficiency of an intensity standard in (ii) is analogous to results in Fischer (2001), Holland *et al.* and in Helfand. As in Holland *et al.* the result depends on the differentiability of  $U$ . Intuitively, if demand is perfectly inelastic, an intensity standard can attain the first best since there is no output distortion and the standard corrects the relative input prices.<sup>21</sup>

The efficiency of the combined consumption tax and intensity standard in (iii) is analogous to, but more general than, a result in Holland *et al.* which showed that an LCFS combined with a gasoline tax could be efficient.<sup>22</sup> As shown in the appendix, the optimal intensity standard sets  $f_e/f_{k_i} = \tau/w_i$  and the optimal consumption tax is  $\tau\sigma$ . This result suggests that the combined consumption tax and intensity standard may hold promise for correcting the inefficiencies of intensity standards.

### 3 Second-best regulation with leakage

To extend the model to analyze leakage, consider a covered (regulated or domestic) firm, which produces  $q^C$ , and an uncovered (unregulated or foreign) firm, which produces  $q^U$ .<sup>23</sup> Each firm has access to the same production technology described by the concave production function  $f(k^i, e^i)$  for  $i \in \{C, U\}$  where  $k^i$  is a vector of market inputs (*e.g.*, labor, capital, fuel, etc.) with prices

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<sup>21</sup>This can be seen since the elasticity in [5] is zero, *i.e.*, perfectly inelastic demand implies  $\partial q/\partial\sigma = 0$ . Moreover, although marginal cost is too low, consumption is not too great since demand is perfectly inelastic.

<sup>22</sup>See also Fullerton and Mohr (2003).

<sup>23</sup>Hopefully there is no confusion between the utility function  $U$  and the superscript  $U$  for “uncovered.”

$w$ , and  $e^i$  is an unpriced input (*e.g.*, emissions). Let  $U(Q)$  be the benefit from consumption of the two perfect substitutes, *i.e.*,  $Q = q^C + q^U$ , where  $U' > 0$  and  $U'' < 0$ .<sup>24</sup> Let damages from pollution be  $\tau(e^C + e^U)$ .<sup>25</sup> Assume emissions of the uncovered firm are subject to an *uncovered emissions charge*,  $t_U$ , (possibly zero), so its cost function is given by  $c^U(q^U; w, t_U) = \min_{k,e} wk^U + t_U e^U + \lambda^U [q^U - f(k^U, e^U)]$  where  $\lambda^U$  is the Lagrange multiplier.<sup>26</sup> Cost minimization implies the two first order conditions:  $w = \lambda^U f_k(k^U, e^U)$  and  $t_U = \lambda^U f_e(k^U, e^U)$ . As above, the envelope theorem implies that the marginal cost is  $c_q^U(q^U; w) = \lambda^U$ .

### 3.1 Second-best emissions tax with leakage

If the covered firm is subject to an emissions tax,  $t$ , its cost function is  $c^C(q^C; w, t)$ ; cost minimization implies that  $w = \lambda^C f_k(k^C, e^C)$  and  $t = \lambda^C f_e(k^C, e^C)$  where  $\lambda^C$  is the Lagrange multiplier; and the envelope theorem implies that  $c_q^C(q^C; w, t) = \lambda^C$ . The eight endogenous variables in the equilibrium— $q^i$ ,  $\lambda^i$ ,  $k^i$ , and  $e^i$ —are completely determined by the four first-order conditions from cost minimization; the two production functions; and the two market clearing conditions:  $U'(Q) = \lambda^C = \lambda^U$ .

The regulator chooses the tax to maximize net social benefits<sup>27</sup>

$$\max_t U(Q) - c^C(q^C; w, t) - c^U(q^U; w, t_U) - \tau(e^C + e^U) + te^C + t_U e^U.$$

Again the tax revenue is counted as a cost for the firms and thus must be added to the objective.

The FOC is then

$$[U'(Q) - c_q^C] \frac{\partial q^C}{\partial t} + [U'(Q) - c_q^U] \frac{\partial q^U}{\partial t} - \frac{\partial c^C}{\partial t} + e^C - (\tau - t) \frac{\partial e^C}{\partial t} - (\tau - t_U) \frac{\partial e^U}{\partial t} = 0.$$

Since the first two terms are zero by the market clearing conditions and the third and fourth terms are additive inverses by applying the envelope theorem to the covered firm's cost function, the

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<sup>24</sup>The model is readily extended to imperfect substitutes. If the goods are not substitutable (*i.e.*, are additive separable), then there is no leakage, and the emissions tax dominates. As goods become closer substitutes, leakage increases, and the emissions tax may be dominated.

<sup>25</sup>Marginal damages are assumed constant and independent of the source, but the results are easily extended to increasing marginal damages and different transfer coefficients.

<sup>26</sup>If  $t_U = 0$  and  $f_e > 0$ , demand for emissions is infinite. If  $t_U > 0$ , emissions are finite. Note that  $t_U$  could model an implicit or implied tax on emissions.

<sup>27</sup>This objective is quite general and can model leakage within and across political jurisdictions depending on who “the regulator” is and on what benefits/costs enter the objective. An appendix explicitly models international leakage where the regulator is concerned solely with domestic benefits. As above, the revenue from the emissions tax or the uncovered emissions charge provides no benefit as explained in footnote 18.

FOC implies that

$$t = \tau + (\tau - t_U) \frac{\partial e^U / \partial t}{\partial e^C / \partial t}. \quad (6)$$

To interpret this optimal emissions tax, consider two extremes.<sup>28</sup> If the uncovered emissions charge is equal to marginal damages, *i.e.*, if  $t_U = \tau$ , then the optimal emissions tax is equal to marginal damages and the first-best is attained. At the other extreme, if  $t_U = 0$ , the best tax is less than social damages if the tax decreases covered emissions and increases uncovered emissions. Here the MRTS of the covered firm is less than the input price ratio  $\tau/w_i$ , and covered emissions are too high relative to control technology.<sup>29</sup>

### 3.2 Second-best intensity standard with leakage

If the covered firm is subject to an intensity standard  $\sigma$ , the firm's cost function is  $c^C(q^C; w, \sigma)$ ; cost minimization implies  $w = \lambda^C f_k(k^C, e^C)$  and  $\gamma = \lambda^C f_e(k^C, e^C)$  where  $\lambda^C$  and  $\gamma$  are Lagrange multipliers; and the envelope theorem implies that  $c_q^C(q^C; w, \sigma) = \lambda^C - \gamma\sigma$ . The nine endogenous variables in the equilibrium— $q^i$ ,  $\lambda^i$ ,  $k^i$ ,  $e^i$ , and  $\gamma$ —are completely determined by the four first-order conditions from cost minimization; the two production functions; the two market clearing conditions:  $U'(Q) = \lambda^C - \gamma\sigma = \lambda^U$ ; and the binding intensity standard:  $e^C = \sigma q^C$ .

The regulator chooses the intensity standard to maximize net social benefits

$$\max_{\sigma} U(Q) - c^C(q^C; w, \sigma) - c^U(q^U; w, t_U) - \tau(e^C + e^U) + t_U e^U.$$

The first order condition is then

$$[U'(Q) - c_q^C] \frac{\partial q^C}{\partial \sigma} + [U'(Q) - c_q^U] \frac{\partial q^U}{\partial \sigma} - \frac{\partial c^C}{\partial \sigma} - \tau \frac{\partial e^C}{\partial \sigma} + (\tau - t_U) \frac{\partial e^U}{\partial \sigma} = 0.$$

Since the first two terms are zero by the market clearing conditions and since  $\frac{\partial c^C}{\partial \sigma} = \gamma q^C = w_i q^C f_e(k^C, e^C) / f_{k_i}(k^C, e^C)$ , this FOC implies that

$$\frac{f_e(k^C, e^C)}{f_{k_i}(k^C, e^C)} = \frac{\tau}{w_i} \left(1 + \frac{\partial q^C}{\partial \sigma} \frac{\sigma}{q^C}\right) + \frac{\tau - t_U}{w_i} \left(\frac{\partial e^U}{\partial \sigma} \frac{\sigma}{e^C}\right). \quad (7)$$

<sup>28</sup> Another interesting extreme is when the goods are not substitutable (*i.e.*, additive separable). In this case, [6] is unchanged but  $\partial e^U / \partial t = 0$ , so the optimal emissions tax is  $\tau$ .

<sup>29</sup> Due to the equivalence of emissions caps and taxes, the second-best emissions cap attains the same allocation as the second-best tax.

Note that the optimal intensity standard does not equate the MRTS with the social input price ratio  $\tau/w_i$  and the deviation is greater *i*) for a larger magnitude elasticity of output with respect to the standard; *ii*) for a greater responsiveness of uncovered emissions with respect to the standard; and *iii*) for greater deviation of the uncovered emissions charge from marginal damages. Also, note that even if the uncovered emissions charge is equal to marginal damages, the optimal standard does not attain the first best.

### 3.3 Comparison of emissions taxes and intensity standards with leakage

With leakage, neither an emissions tax nor an intensity standard will generally attain the first best. Thus either may attain higher welfare. Although the second-best net social benefits are difficult to compare analytically, the main result simply compares the possibilities and is easy to state and prove.<sup>30</sup>

**Proposition 2** *Under incomplete regulation, the following hold: i) if the uncovered emissions charge equals marginal damages, i.e., if  $t_U = \tau$ , then the optimal emissions tax attains the first best, but the optimal intensity standard does not; ii) if  $t_U < \tau$ , an intensity standard can attain higher welfare than the second-best emissions tax; and iii) additionally assuming Cobb-Douglas technology and constant returns to scale, i.e.,  $f(k, e) = K \prod_i k_i^{\alpha_i} e^\beta$  and  $\sum_i \alpha_i = 1 - \beta$ , the optimal intensity standard attains higher welfare than the second-best emissions tax if the uncovered emissions charge is low, i.e., if  $t_U/\tau \leq 1 - (1 - \beta)^{(1-\beta)/\beta}$ .*

The result in *(i)* is a corollary to Proposition 1. If  $t_U = \tau$ , the optimal emissions tax simply mimics the uncovered emissions charge and emissions are correctly priced. The earlier analysis in Section 2 showed that the intensity standard does not generally attain the first best.<sup>31</sup> The result in *(ii)* is a possibility result. If  $t_U < \tau$ , the analysis in section 3.1 showed that the optimal emissions tax is less than  $\tau$  and thus does not attain the first best. Although the intensity standard does not attain the first best either, the proof shows a number of examples where an intensity standard attains higher welfare than the best emissions tax. The result in *(iii)* derives a sufficient condition for an intensity standard to attain higher welfare than the best emissions tax with Cobb-Douglas technologies and constant returns to scale. Intuitively, the intensity standard attains higher welfare

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<sup>30</sup>The difficulty lies in deriving the optimal second-best policy, since it generally depends on how emissions change with the policy, i.e.,  $\partial e/\partial t$  in [6] and  $\partial e/\partial \sigma$  in [7]. On the other hand, it is quite easy to solve for the equilibrium for a given tax or intensity standard. Rather than using [6] or [7] directly, the numerical examples derive equilibrium net social benefits for a given policy and then choose the policy to optimize net social benefits.

<sup>31</sup>Under constant returns to scale where  $t_U = \tau$ , an intensity standard can attain the first best by setting  $\sigma = 0$ , so all production leaks to uncovered firms where emissions are correctly priced.

if the uncovered emissions charge is sufficiently below marginal damages.<sup>32</sup> The appendix on international leakage derives an analogous condition showing that an intensity standard attains higher welfare if the second-best emissions tax is sufficiently below marginal damages, *i.e.*, if the import price is sufficiently below the price that would result from a domestic emissions tax  $\tau$ .

The intuition of Proposition 2 can be illustrated with the special case involving constant returns to scale production functions. It is well known that with constant returns to scale, the marginal cost function is constant. In this case, leakage is extreme: any attempt to tax emissions leads to an increase in the marginal cost of covered firms and production shifts entirely to uncovered firms. Thus the second-best emissions tax is  $t_U$ , *i.e.*, simply matches the uncovered emissions charge, and has no effect.

Can an intensity standard do better? The appendix shows that the marginal cost function with an intensity standard is also constant for constant returns to scale. Moreover, the marginal cost function is decreasing in  $\sigma$ .<sup>33</sup> Thus, the regulator can adjust the intensity standard such that the marginal cost of the covered firm does not exceed the marginal cost of the uncovered firm, thereby preventing leakage and mimicking the marginal cost (and output level) of an emissions tax. This intensity standard will result in different inputs for producing the same level of output and may have lower social costs. The sufficient condition insures that this intensity standard has lower social costs and hence attains higher welfare than the optimal emissions tax.

The right hand side of the sufficient condition is decreasing in  $\beta$ . Thus it is more likely that the intensity standard attains higher welfare than an emissions tax if  $\beta$  is smaller. For example, if  $\beta = 0.1$ , the intensity standard attains higher welfare if  $t_U < 0.6\tau$ . However, if  $\beta = 0.9$ , the intensity standard only attains higher welfare if the uncovered emissions charge is much more lax, *i.e.*, if  $t_U < 0.2\tau$ .<sup>34</sup>

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<sup>32</sup>The condition is not necessary since it only demonstrates that the intensity standard which mimics the second-best emissions tax has lower social costs. Even if this intensity standard has higher social costs, the optimal intensity standard can have lower deadweight loss.

<sup>33</sup>Total costs are decreasing in  $\sigma$ , hence marginal cost (equal to average cost) is also decreasing.

<sup>34</sup>Since Cobb-Douglas assumes that all inputs are substitutes, simply estimating  $\beta$  from the expenditure shares on emissions would be misleading. A more accurate estimate of  $\beta$  might come from the expenditure share of all inputs which are complements to emissions.

Table 1 illustrates Proposition 2 for a simple numerical example.<sup>35</sup> Panel A illustrates the case where the uncovered emissions charge is lax. The assumption of constant returns to scale implies that the best emissions tax matches the uncovered emissions charge. This tax is ineffective, and it attains lower welfare than an intensity standard which leads to the same level of output but at lower social costs. Panel B illustrates a more stringent uncovered emissions charge and shows that the optimal emissions tax does not necessarily attain lower welfare. For  $\beta = 0.8$ , the sufficient condition fails and the optimal intensity standard ( $\sigma = 1.21$ ) yields lower welfare. For  $\beta = 0.5$ , the sufficient condition holds with equality, so the intensity standard that mimics the best emissions tax does not reduce deadweight loss. However, the optimal intensity standard, which is slightly more lax, does yield higher welfare than the optimal emissions tax. For  $\beta = 0.2$ , the optimal emissions tax yields lower welfare.<sup>36</sup>

The intensity standard and consumption tax combination seems quite promising especially given the result in Proposition 1. The following proposition shows that the intensity standard can still yield higher welfare than an emissions tax even if both instruments are combined with a consumption tax. Moreover, the proposition describes conditions under which a combined intensity standard and consumption tax can attain the first best.

**Proposition 3** *Under incomplete regulation, assume  $t_U < \tau$ . i) A combined intensity standard and consumption tax can yield higher welfare than the second-best combination of an emissions tax and a consumption tax. ii) With Cobb-Douglas technology and constant returns to scale, a combined intensity standard and consumption tax attain the first best iff  $t_U/\tau \geq (1 - \beta)^{1/\beta}$ .*

With complete regulation, the intensity standard corrects the relative price of inputs, the consumption tax corrects the relative price of output, and the combined policy attains the first best. However, with incomplete regulation the stringency of the intensity standard may be constrained by the marginal cost of the uncovered firm. If the uncovered emissions charge is lax, the regulator would like to make the intensity standard more stringent but cannot since this would raise the marginal cost of the covered firm above the marginal cost of the uncovered firm (causing leakage).<sup>37</sup> However, if the uncovered emissions charge is not lax, *e.g.*, if  $t_U/\tau \geq (1 - \beta)^{1/\beta}$ , the constraint does

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<sup>35</sup>Appendix Table 2 shows similar results for a constant elasticity of substitution production function.

<sup>36</sup>The intensity standard can still yield higher welfare with decreasing returns to scale. See Appendix Table 1.

<sup>37</sup>In this case, the intensity standard would be too lax and the consumption tax, given by  $t_c = \tau\sigma$  would be too high to attain the first best.

not bind, so the regulator can set the intensity standard and consumption tax at their optimal levels and can attain the first best.<sup>38,39</sup> Note that attainment of the first best requires constant returns to scale.<sup>40</sup>

Table 2 illustrates Proposition 3. Notice that the addition of the consumption tax reduces deadweight loss for all policies relative to Table 1 primarily by reducing output. Moreover, the dominance of the intensity standard is maintained in Panel A with a lax uncovered emissions charge although efficiency is not attained. In Panel B the advantage of the emissions tax from Table 1 disappears, and the intensity standard/consumption tax combination attains the first best, even though the emissions tax/consumption tax combination does not.<sup>41</sup>

### 3.4 Comparison with output-based updating

The advantage of the intensity standard is that it implicitly taxes emissions while subsidizing production. Similarly, output-based updating of emissions permits implicitly subsidizes production while capping emissions. It is difficult to compare these two instruments directly since the subsidy effects of the updating scheme depend on its details. For example, Fischer and Fox (2007) and Fischer (2001) assume that each firm is allocated an exogenous proportion of the available permits based on its output. The strength of the output subsidy depends on the proportion of the permits that are updated in this way, and also on the discount factor and firms' expectations.

Since output-based updating can vary the strength of the output subsidy, a simpler comparison for the intensity standard is with a combined emissions tax and output subsidy to the covered firm. Intuitively, the combination of two instruments is likely to yield higher welfare than the single instrument. This intuition is correct, and the following result is proved in the appendix:

**Proposition 4** *Under incomplete regulation, the second-best combination of an emissions tax and a output subsidy for covered firms yields higher welfare than the second-best intensity standard.*

This result suggests that output-based updating is superior to an intensity standard if the subsidy inherent in the output-based updating is sufficiently flexible to mimic the optimal output

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<sup>38</sup>The right hand side of the necessary and sufficient condition is decreasing in  $\beta$ . Thus as  $\beta$  increases it is more likely that the intensity standard/consumption tax combination attains the first best.

<sup>39</sup>The combined emissions tax and consumption tax cannot attain the first best if  $t_U < \tau$ .

<sup>40</sup>With increasing marginal costs, some uncovered production occurs. Since the uncovered production is under-taxed, the first best is not attained.

<sup>41</sup>The necessary and sufficient condition does not hold in Panel A but holds in Panel B.

subsidy. On the other hand, the result also suggests that simply subsidizing output (combined with an emissions tax) might be superior to output-based updating since it clearly yields higher welfare than an intensity standard and does not suffer from some of the other problems of output-based updating.<sup>42</sup>

## 4 Second-best regulation with market power

As noted above, the advantage of an intensity standard is that it simultaneously taxes emissions and subsidizes output. This feature is also a potential advantage in markets subject to market power. To analyze market power, return to the model and notation introduced in Section 2 with no leakage. For simplicity, the model analyzes a monopoly producer.<sup>43</sup>

First, suppose the monopoly firm is subject to an emissions tax. As above, the firm's cost function depends on the emissions tax,  $t$ , and is given by  $c(q; w, t)$  derived from the concave production function  $f(k, e)$  with Lagrange multiplier,  $\lambda$ . The four endogenous variables in the equilibrium— $q$ ,  $\lambda$ ,  $k$ , and  $e$ —are completely determined by the two first-order conditions from cost minimization; the production function:  $q = f(k, e)$ ; and the market equilibrium condition setting marginal revenue equal to marginal cost:  $U'(q) + qU''(q) = \lambda$ .

The regulator chooses the tax to maximize net social benefits. The objective is as in [1], and the first-order condition is as in [2]. The second-best emissions tax is then

$$t = \tau + qU''(q) \frac{\partial q / \partial t}{\partial e / \partial t}. \quad (8)$$

Since production and emissions are decreasing in the emissions tax and since  $U'' < 0$ , the optimal emissions tax is less than damages.<sup>44</sup>

Now suppose the monopoly is subject to an intensity standard  $\sigma$  such that  $e/q \leq \sigma$ , and the firm's cost function is  $c(q; w, \sigma)$ . The five endogenous variables in the equilibrium— $q$ ;  $\lambda$ , the shadow value on production;  $\gamma$ , the shadow value of the intensity standard;  $k$ ; and  $e$ —are completely determined by the two first-order conditions from cost minimization; the production

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<sup>42</sup>Two problems that have been identified include permit price “inflation” and firms’ expectations about the linkage between output and the subsidy.

<sup>43</sup>The model easily extends to oligopoly. See Fischer (2003).

<sup>44</sup>This result is also shown in Barnett (1980).



function,  $q = f(k, e)$ ; the market equilibrium condition,  $U'(q) + qU''(q) = \lambda - \gamma\sigma$ ; and the binding intensity standard,  $e = \sigma q$ .

The regulator chooses the intensity standard to maximize the objective as in [3] and the first-order condition is as in [4]. The following condition holds under the second-best intensity standard:

$$\frac{f_e(k, e)}{f_k(k, e)} = \frac{\tau}{w} \left( 1 + \frac{\partial q}{\partial \sigma} \frac{\sigma}{q} \right) + \frac{U''(q)}{w} \frac{\partial q}{\partial \sigma}$$

This condition is equivalent to the condition in [5] with the additional term capturing the slope of the demand curve. Since  $U'' < 0$  and  $\frac{\partial q}{\partial \sigma}$  is generally positive, comparing this equation with [5] shows that (conditional on output) the monopoly leads to more emissions relative to control technology under the intensity standard. The two instruments can now be compared:

**Proposition 5** *Under monopoly, the second-best intensity standard can yield higher welfare than the second-best emissions tax.*

Table 3 illustrates Proposition 5. For all the examples, the intensity standard yields higher welfare than the optimal emissions tax through higher output produced with less emissions.

## 5 Conclusion

This paper demonstrates that emissions taxes (equivalently emissions trading) may not be the best instruments for correcting environmental externalities in the presence of incomplete regulation (leakage) or market power. In fact, since I analyze the second-best policies, my results show that with leakage or market power any emissions tax may yield lower welfare than an intensity standard. A sufficient condition shows that the dominance is more likely if the second-best emissions tax is sufficiently below marginal damages.

With leakage, the optimal intensity standard leads to too much consumption. An additional consumption tax can lead to correct consumption and attain the first best (even though an emissions tax/consumption tax combination would not). The consumption tax is a particularly useful instrument in this context since it does not violate trade law.

The optimal intensity standard yields lower welfare than a combined emissions tax and output subsidy for covered firms. If output-based updating can mimic a general output subsidy

then updating can yield higher welfare than an intensity standard. The analysis emphasizes the importance of flexibility in the subsidy portion of the updating scheme.

With multiple market failures the policy choice is whether to use a potentially inferior instrument hoping other market failures can be addressed with other instruments or to use a superior instrument and accept the second-best world. This paper provides a framework for analyzing these policy instruments and suggests that an intensity standard should not be neglected.

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## Appendices

### Equivalence of emissions tax and emissions cap

Suppose the firm is subject to an emissions cap. The firm's cost function depends on the emissions cap,  $\bar{e}$ , and is given by  $c(q; w, \bar{e}) = \min_k wk + \lambda[q - f(k, \bar{e})]$  where  $\lambda$  is the Lagrange multiplier. Cost minimization implies that  $w = \lambda f_k(k, \bar{e})$ . Applying the envelope theorem implies that marginal cost is the shadow value, *i.e.*,  $c_q(q; w, \bar{e}) = \lambda$ . The three endogenous variables in the equilibrium— $q$ ,  $\lambda$ , and  $k$ —are completely determined by the first-order condition from cost minimization; the production function:  $q = f(k, e)$ ; and the market equilibrium condition:  $U'(q) = \lambda$ .

The regulator chooses the emissions cap to maximize net social benefits

$$\max_{\bar{e}} U(q) - c(q; w, \bar{e}) - \tau \bar{e}.$$

The first order condition is then

$$[U'(q) - c_q] \frac{\partial q}{\partial \bar{e}} - \frac{\partial c}{\partial \bar{e}} - \tau = 0$$

Since the first term is zero by the market clearing condition, the FOC implies that  $\tau = -\frac{\partial c}{\partial \bar{e}} = w_i f_e / f_{k_i}$ , and the efficient allocation is attained, *i.e.*,  $U'(q) = w_i / f_{k_i} = \tau / f_e$ .

### Proof of Proposition 1

The results in (i) and (ii) are proved in the text. To demonstrate the result in (iii), suppose the firm is subject to a consumption tax  $t_c$  and to an intensity standard  $\sigma$ . As above, the firm's cost function depends on the intensity standard and is given by  $c(q; w, \sigma) = \min_{k,e} wk + \lambda[q - f(k, e)] + \gamma[e - \sigma q]$  where  $\lambda$  and  $\gamma$  are Lagrange multipliers. The five endogenous variables in the equilibrium— $q$ ,  $\lambda$ ,  $k$ ,  $e$ , and  $\gamma$ —are completely determined by the two first-order conditions from cost minimization; the production function,  $q = f(k, e)$ ; the market equilibrium condition which incorporates the consumption tax:  $U'(q) - t_c = \lambda - \gamma\sigma$ ; and the binding intensity standard,  $e = \sigma q$ .

The regulator chooses  $t_c$  and  $\sigma$  to maximize net social benefits

$$\max_{t_c, \sigma} U(q) - c(q; w, \sigma) - \tau e.$$

The first order conditions for  $t_c$  and  $\sigma$  are

$$[U'(q) - c_q] \frac{\partial q}{\partial t_c} - \tau \frac{\partial e}{\partial t_c} = 0 \tag{9}$$

and

$$[U'(q) - c_q] \frac{\partial q}{\partial \sigma} - \frac{\partial c}{\partial \sigma} - \tau \frac{\partial e}{\partial \sigma} = 0$$

Since  $U'(q) - c_q = t_c$  by the market clearing condition, the first condition implies that  $t_c \partial q / \partial t_c = \tau \partial e / \partial t_c = \tau \sigma \partial q / \partial t_c$  which implies that  $t_c = \tau \sigma$ . Similarly the second condition implies that  $t_c \partial q / \partial \sigma + q w_i f_e / f_{k_i} = \tau \partial e / \partial \sigma = \tau q + \tau \sigma \partial q / \partial \sigma$  which implies that  $f_e / f_{k_i} = \tau / w_i$ , *i.e.*, the MRTSs are correct. By noting that  $\tau = \gamma$ , the efficient allocation is attained, *i.e.*,  $U'(q) = \lambda = w_i / f_{k_i} = \tau / f_e$ .

## Derivation of cost functions for Cobb-Douglas

Here I derive the costs functions  $c(q; w, t)$  and  $c(q; w, \sigma)$  for the case of the Cobb-Douglas production function with three inputs  $f(k, e) = Kk_1^{\alpha_1}k_2^{\alpha_2}e^\beta$ .<sup>45</sup> Since demand for any Cobb-Douglas input approaches infinity as its price approaches zero, any numerical computations assume a positive price for emissions in the unregulated market.

With a tax, the emissions input is priced by the tax, and the cost function is well known:<sup>46</sup>

$$c(q; w, t) = (\alpha_1 + \alpha_2 + \beta)Mq^{\frac{1}{\alpha_1 + \alpha_2 + \beta}}$$

where  $M$  is given by

$$M = \left[ \frac{1}{K} \left( \frac{w_1}{\alpha_1} \right)^{\alpha_1} \left( \frac{w_2}{\alpha_2} \right)^{\alpha_2} \left( \frac{t}{\beta} \right)^\beta \right]^{\frac{1}{\alpha_1 + \alpha_2 + \beta}}, \quad (10)$$

so the marginal cost is  $c_q(q; w, t) = Mq^{\frac{1 - \alpha_1 - \alpha_2 - \beta}{\alpha_1 + \alpha_2 + \beta}}$ . Recall that with constant (decreasing, increasing) returns to scale, the marginal cost is constant (increasing, decreasing).<sup>47</sup>

With an intensity standard, the emissions input is unpriced but is subject to the standard. The cost function is derived from the cost minimization  $c(q; w, \sigma) = \min_{k, e} wk + \lambda[q - f(k, e)] + \gamma[e - \sigma q]$ . For the market inputs, the FOC implies  $k_i w_i = \alpha_i \lambda q$  which shows that the MRTS, which is a function of the input ratio, equals the input price ratio. If the intensity standard is binding, the production function can be written  $q = Kk_1^{\alpha_1 + \alpha_2} (k_2/k_1)^{\alpha_2} (\sigma q)^\beta$  which implies

$$q^{1 - \beta} = K\sigma^\beta k_1^{\alpha_1 + \alpha_2} \left( \frac{\alpha_2 w_1}{\alpha_1 w_2} \right)^{\alpha_2}. \quad (11)$$

This equation allows output to be expressed as a function of  $k_1$ , and the cost function is

$$c(q; w, \sigma) = wk = (\alpha_1 + \alpha_2)\lambda q = (\alpha_1 + \alpha_2) \frac{w_1}{\alpha_1} k_1 = (\alpha_1 + \alpha_2) N q^{\frac{1 - \beta}{\alpha_1 + \alpha_2}}$$

where  $N$  is given by

$$N = \left[ \frac{1}{K\sigma^\beta} \left( \frac{w_1}{\alpha_1} \right)^{\alpha_1} \left( \frac{w_2}{\alpha_2} \right)^{\alpha_2} \right]^{\frac{1}{\alpha_1 + \alpha_2}}.$$

The marginal cost is  $c_q(q; w, \sigma) = (1 - \beta)Nq^{\frac{1 - \alpha_1 - \alpha_2 - \beta}{\alpha_1 + \alpha_2}}$ . Note that this implies that with constant (decreasing, increasing) returns to scale, both marginal cost functions  $c_q(q; w, t)$  and  $c_q(q; w, \sigma)$  are constant (increasing, decreasing).<sup>48</sup>

## Cost function with constant returns to scale

**Appendix Lemma 1:** *If returns to scale are constant, marginal costs are constant under an emissions tax or under an intensity standard.*

Proof: If all inputs are priced, it is well known that marginal costs are constant under constant returns to scale, *i.e.*,  $c(q; w, t) = qc(1; w, t)$ .

<sup>45</sup>To simplify to two inputs, simply set  $\alpha_2 = 0$ . The analysis generalizes readily to many inputs.

<sup>46</sup>See for example Nicholson's text.

<sup>47</sup>For the numerical analysis, it is useful to note that the conditional factor demand for  $e$  is given by  $e =$

$$\left[ \frac{q}{K} \left( \frac{w_1}{\alpha_1} \right)^{\alpha_1} \left( \frac{w_2}{\alpha_2} \right)^{\alpha_2} \left( \frac{\beta}{t} \right)^{\alpha_1 + \alpha_2} \right]^{\frac{1}{\alpha_1 + \alpha_2 + \beta}}.$$

<sup>48</sup>For the numerical analysis, it is useful to note that the conditional factor demand for  $e$  is given by  $e = \sigma q$  and the conditional factor demand for  $k_1$  can be found from [11].

Under a binding intensity standard, let  $k(1)$  and  $e(1)$  be the cheapest input combination for producing one unit of output. To show that  $qk(1)$  and  $qe(1)$  are the cheapest input combination for producing  $q$ , first note that the intensity standard still binds, *i.e.*,  $qe(1)/q = e(1) = \sigma$ , and note that the cost minimization condition still holds:  $w_i/f_{k_i}(qk(1), qe(1)) \cdot (1 - f_e(qk(1), qe(1))) = w_i/f_{k_i}(k(1), e(1)) \cdot (1 - f_e(k(1), e(1)))$  for each market input  $i$  since marginal products are homogeneous of degree zero. Thus  $c(q; w, \sigma) = wqk(1) = qc(1; w, \sigma)$ .

## Proof of Proposition 2

The result in (i) is proved in the text. The possibility result (ii) is proved by the example that follows in (iii) as well as by the numerical examples that appear in the text and in the additional appendices.

To demonstrate (iii), note that with constant returns to scale, the marginal cost functions under both an emissions tax and intensity standard are constant as shown in Appendix Lemma 1. Since an emissions tax greater than  $t_U$  would increase the covered firm's marginal cost and cause complete spillovers, the second-best emissions tax is  $t_U$ .

A binding intensity standard can mimic the second-best emissions tax. To prove the sufficient condition, let the equilibrium with the second-best emissions tax be given by  $e^{C^*}$ ,  $k^{C^*}$ ,  $e^{U^*} = 0$ , and  $k^{U^*} = 0$ .<sup>49</sup> Note that this equilibrium is characterized by  $U'(Q^*) = M^*$  where  $M^*$  is defined in [10] for  $t = t_U$ . Thus  $e^{C^*} = M^*\beta Q^*/t_U$  and  $k_i^{C^*} = M^*\alpha_i Q^*/w_i$ . Let  $e^{C'}$ ,  $k^{C'}$  define the binding intensity standard which mimics the second-best emissions tax. This equilibrium is characterized by  $M^* = w_i/f_{k_i} \cdot (1 - \sigma f_e) = w_i k_i^{C'}/(\alpha_i Q^*) \cdot (1 - \beta)$  which implies that  $(1 - \beta)k_i^{C'} = k_i^{C^*}$ . Furthermore  $e^{C'} = e^{C^*} \prod_i (k_i^{C^*}/k_i^{C'})^{\alpha_i/\beta} = e^{C^*} (1 - \beta)^{(1-\beta)/\beta}$  since output must be equal. Now compare the social costs. Note that  $wk^{C'} + \tau e^{C'} \leq wk^{C^*} + \tau e^{C^*}$  iff  $M^*Q^* \sum_i \alpha_i/(1 - \beta) + \tau M^*\beta Q^*/t_U \cdot (1 - \beta)^{(1-\beta)/\beta} \leq M^*Q^* \sum_i \alpha_i + \tau M^*\beta Q^*/t_U$  iff  $t_U/\tau \leq 1 - (1 - \beta)^{(1-\beta)/\beta}$ . If this sufficient condition holds, mimicking the second-best emissions tax with an intensity standard reduces (does not increase) social costs, so the second-best emissions tax is dominated.<sup>50</sup>

## Proof of Proposition 3

The possibility result in (i) is demonstrated by the example that follows in (ii) as well as by numerical examples in the text.

To demonstrate (ii), consider an intensity standard consumption tax combination which would attain the first best in the absence an uncovered firm. In particular, let  $\sigma = 1/K \cdot \prod_i (\beta w_i/\tau \alpha_i)^{\alpha_i}$  and let  $t_c = \tau \sigma$ . Since the production function implies that  $\sigma = e/q = 1/K \cdot \prod_i (e/k_i)^{\alpha_i}$ , equilibrium will have  $\beta w_i/\tau \alpha_i = e/k_i$  for every  $i$  which implies  $f_e/f_{k_i} = \tau/w_i$ , *i.e.*, MRTSs are equal to the social price ratios.<sup>51</sup> Now note that the marginal cost is

$$c_q(q; w, \sigma) = \frac{w_i}{f_{k_i}} (1 - \sigma f_e) = \frac{w_i k_i}{\alpha_i q} (1 - \beta) = \frac{\tau \sigma}{\beta} (1 - \beta) = \frac{1 - \beta}{K} \left(\frac{\tau}{\beta}\right)^\beta \prod_i \left(\frac{w_i}{\alpha_i}\right)^{\alpha_i} \quad (12)$$

where the first equality follows from cost minimization, the second equality from substitution of the marginal products, the third since MRTSs are correct, and the fourth by substitution for  $\sigma$ .<sup>52</sup> If this marginal cost is less than the marginal cost of the uncovered firm then there is no leakage,

<sup>49</sup>Note “\*” and “’” are defined only for this proof.

<sup>50</sup>A sufficient condition can be similarly derived for decreasing returns to scale by a slight modification of the proof. The more general sufficient condition, which depends on the endogenous  $t^*$  is  $t^*/\tau \leq 1 - (1 - \beta)^{\sum \alpha_i/\beta} \cdot (1 - \beta)/\sum \alpha_i$ .

<sup>51</sup>For this  $\sigma$ , an equilibrium exists with efficient MRTSs. If other equilibria exist, this may not hold.

<sup>52</sup>Note that  $U'(q) - tc = c_q = \tau \sigma/\beta - \tau \sigma$  so  $U'(q) = \tau/f_e$ , *i.e.*, output is correct as in Proposition 1.

and this policy combination attains the first best. The uncovered marginal cost is given by  $M^*$  which is defined in [10] for  $t = t_U$ . Comparing [10] and [12], shows that  $c_q(q; w, \sigma) \leq M^*$  iff  $(1 - \beta)\tau^\beta \leq t_U^\beta$  iff  $t_U/\tau \geq (1 - \beta)^{1/\beta}$ .

#### Proof of Proposition 4

Consider the second-best intensity standard,  $\sigma^*$  under incomplete regulation. Denote the resulting equilibrium values by  $e^{C^*}$ ,  $k^{C^*}$ ,  $e^{U^*}$ , and  $k^{U^*}$ .<sup>53</sup> Note that this equilibrium is completely characterized by  $e^{C^*} = \sigma^* q^{C^*}$  and the equations

$$U'(Q^*) = \frac{w_i}{f_{k_i}(k^{C^*}, e^{C^*})} - \frac{w_i f_e(k^{C^*}, e^{C^*}) \sigma^*}{f_{k_i}(k^{C^*}, e^{C^*})} = \frac{w_i}{f_{k_i}(k^{U^*}, e^{U^*})} = \frac{t_U}{f_e(k^{U^*}, e^{U^*})}$$

Now consider the emissions tax  $t$  and output subsidy  $s$  to the covered firm where  $t = w_i f_e(k^{C^*}, e^{C^*}) / f_{k_i}(k^{C^*}, e^{C^*})$  and  $s = t \sigma^*$ .<sup>54</sup> Note that this equilibrium is characterized by the equations:

$$U'(Q) = \frac{w_i}{f_{k_i}(k^C, e^C)} - s = \frac{t}{f_e(k^C, e^C)} - s = \frac{w_i}{f_{k_i}(k^U, e^U)} = \frac{t_U}{f_e(k^U, e^U)} \quad (13)$$

It is straightforward to verify that  $e^{C^*}$ ,  $k^{C^*}$ ,  $e^{U^*}$ , and  $k^{U^*}$  are equilibrium values for this  $t$  and  $s$ , *i.e.*, the equilibria are identical. First, the second equation of [13] implies that  $w_i f_e(k^C, e^C) / f_{k_i}(k^C, e^C) = t = w_i f_e(k^{C^*}, e^{C^*}) / f_{k_i}(k^{C^*}, e^{C^*})$ , so the MRTSs of the covered firm are identical for all inputs. Second, the first equation of [13] shows that  $Q^*$  is the equilibrium output level. Since output and the uncovered and covered MRTSs are identical, the equilibria are identical.

To complete the proof, note that the equilibrium with the second-best intensity standard is mimicked by the equilibrium with this  $t$  and  $s$ . Since the second-best emissions tax and output subsidy to the covered firm can do no worse, the second-best intensity standard is dominated.

#### Proof of Proposition 5

Proposition 5 is a possibility result and thus is proved by the examples in Table 3.<sup>55</sup>

#### Model of international leakage

To analyze international leakage, model the domestic (covered) sector as in Section 3, and assume imports (which are a perfect substitute for domestic production) can be purchased at price  $p_I$ .<sup>56</sup> Foreign emissions  $e^I$  are a function of import production  $q^I$ . The domestic regulator then chooses a policy to maximize domestic benefits less domestic costs.

Using the notation developed in the paper for the covered sector, the optimal emissions tax maximizes

$$\max_t U(q^C + q^I) - c^C(q^C; w, t) - p^I q^I - \tau(e^C + e^I) + t e^C.$$

The FOC then implies that the optimal (domestic) emissions tax is

$$t = \tau \left( 1 + \frac{\partial e^I / \partial t}{\partial e^C / \partial t} \right)$$

<sup>53</sup>Note “\*” and “U” are defined only for this proof.

<sup>54</sup> $t$  is well-defined since  $w_i / f_{k_i} \cdot (1 - f_e \sigma) = w_j / f_{k_j} \cdot (1 - f_e \sigma)$  implies  $w_i / f_{k_i} = w_j / f_{k_j}$ .

<sup>55</sup>To date, I have neither been unable to find a counter example to Proposition 5 nor to prove the result in more generality.

<sup>56</sup>The model is easily extended to model an import supply which is not perfectly elastic.



which is analogous to [6].

Similarly, the optimal intensity standard maximizes

$$\max_{\sigma} U(q^C + q^I) - c^C(q^C; w, \sigma) - p^I q^I - \tau(e^C + e^I).$$

The FOC then implies that the optimal (domestic) intensity standard is characterized by

$$\frac{f_e(k^C, e^C)}{f_{k_i}(k^C, e^C)} = \frac{\tau}{w_i} \left(1 + \frac{\partial q^C}{\partial \sigma} \frac{\sigma}{q^C}\right) + \frac{\tau}{w_i} \left(\frac{\partial e^I}{\partial \sigma} \frac{\sigma}{e^C}\right)$$

which is analogous to [7].

The results in Proposition 2 are readily extended for the international leakage model. In particular with Cobb-Douglas and constant returns to scale as in Proposition 2(iii), there is complete leakage if the domestic marginal cost is driven above  $p_I$ . Thus the optimal emissions tax  $t^*$  is implicitly defined by  $p_I = \frac{1}{K} \left(\frac{t^*}{\beta}\right)^\beta \prod_i \left(\frac{w_i}{\alpha_i}\right)^{\alpha_i}$ .<sup>57</sup> The proof of Proposition 2(iii) is then easily extended to derive the sufficient condition  $t^*/\tau \leq 1 - (1 - \beta)^{(1-\beta)/\beta}$  which is analogous to the condition in the proposition.

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<sup>57</sup>If foreign production is cleaner than domestic production at  $t^*$ , then the best emissions tax causes complete leakage, and the best intensity standard can do no worse.

## Additional Referee's Appendices

### Numerical examples of superiority of intensity standard under incomplete regulation

Two examples demonstrate the potential superiority of an intensity standard over an emissions tax. To simplify computations, the Cobb-Douglas production technologies are assumed with one market input where  $K = 1$ . To avoid infinite demand for emissions from the uncovered firm, the uncovered firm is assumed to face the uncovered emissions charge  $t_U$ . Consumer surplus is  $U(Q) = 100Q - Q^2/2$ , so that the marginal benefit of consumption is  $U'(Q) = 100 - Q$ .

*Example with constant returns to scale*

For constant returns to scale, assume  $\alpha = 1/2$  and  $\beta = 1/2$ . To complete the parameterization, assume factor prices  $w = 1/2$  &  $t_U = 1/20,000$  and damages  $\tau = 1/2$ . In this case, the parameters that define the marginal costs (written as a function of the relevant policy) are  $M(t) = \sqrt{2t}$  and  $N(\sigma) = 1/\sigma$ , and marginal costs are  $c_q(q; w, t) = \sqrt{2t}$  and  $c_q(q; w, \sigma) = 1/2 \cdot 1/\sigma$ .

An emissions tax is completely ineffective with constant returns to scale since any positive tax would increase the covered marginal cost above the uncovered marginal cost, and all production would leak to the uncovered firms. Thus the optimal tax equilibrium is equivalent to the unregulated equilibrium. In the unregulated equilibrium, the marginal cost is  $M(t_U) = 0.01$ . Production is  $Q = 100 - 0.01 = 99.99$  and consumer surplus is  $\sim \$5,000$ .<sup>58</sup> Inputs are found from the factor demand:  $e = 100q = 9,999$  and  $k = q/100 = 0.9999$ . Net social benefits are then consumer surplus ( $\$5,000$ ) less production costs ( $\sim \$0$ ), less damages from pollution ( $\$5,000$ ) for net social surplus of zero.<sup>59</sup> Note that the emissions intensity in the unregulated equilibrium is  $e/Q = 100$ .<sup>60</sup>

The optimal intensity standard is set such that the marginal cost of the covered firm is equal to (slightly below) the marginal cost of the uncovered firm. With  $\sigma = 50$ , the marginal cost is 0.01, and production and consumer surplus are as in the unregulated equilibrium, but emissions are cut in half ( $9,999/2$ ) and  $k$  doubles.<sup>61</sup> Net social benefits are then consumer surplus ( $\sim \$5,000$ ) less production costs ( $\sim \$1$ ), less damages from pollution ( $\$2,500$ ) for net social surplus of  $\$2,499$ . Thus the second-best intensity standard dominates the second-best emissions tax.

For an emissions tax combined with a consumption tax  $t_c$ , an emissions tax is again completely ineffective. The consumption tax, however, can reduce damages from emissions. Subject to a consumption tax (and no emissions tax), equilibrium production is found by solving  $100 - Q - t_c = \sqrt{2t_U}$ . By graphing welfare as a function of the consumption tax, the optimal consumption tax is seen to be  $\$49.995$ . Production is  $\sim 50$  with emissions of 5,000 and market inputs of 0.5. Net social benefits are then consumer surplus ( $\$3,750$ ) less production costs ( $\sim \$0.25$ ), less damages from pollution ( $\$2,500$ ) for net social surplus of  $\$1,250$ .<sup>62</sup>

For an intensity standard combined with a consumption tax, the stringency of the intensity standard is limited by the marginal cost of the unregulated firms. Thus the optimal standard is  $\sigma = 50$  as above. The optimal consumption tax is then  $t_c = \tau\sigma = 25$ .<sup>63</sup> Equilibrium production is then 75, emissions are 3,750 and market inputs are 1.5. Net social benefits are then consumer surplus ( $\$4,687$ ) less production costs ( $\sim \$0.75$ ), less damages from pollution ( $\$1,875$ ) for net social

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<sup>58</sup>Since covered and uncovered firms are identical, production can be allocated between them arbitrarily without affecting welfare. Thus, only aggregate quantities are derived.

<sup>59</sup>Zero net surplus is an artifact of the parameterization, *e.g.*, higher damages would lead to negative net surplus.

<sup>60</sup>The efficient equilibrium has marginal cost determined by  $M(\tau) = 1$ , so  $Q = 99$  and  $e = k = 99$ . The efficient net social surplus is consumer surplus ( $\sim \$5,000$ ) less production costs ( $\$49.50$ ) less damages ( $\$49.50$ ) for net social surplus of  $\$4,901$ .

<sup>61</sup>The intensity standard is binding since the unregulated emissions intensity is 100.

<sup>62</sup>For this parameterization, an emissions tax combined with a consumption tax is dominated by an intensity standard alone.

<sup>63</sup>This optimal consumption tax conditional on  $\sigma$  is derived above from [9].

surplus of \$2,812. Note that the intensity standard combined with a consumption tax does not attain the first best and that production is too low. However, this combination dominates an emissions tax with a consumption tax.

*Example with decreasing returns to scale*

For decreasing returns to scale, assume all parameters are as above, except:  $\alpha = 1/3$ ,  $\beta = 1/3$ ,  $w = 1/3$ ,  $\tau = 1/3$ , and  $t_U = 1/30,000$ . In this case, the parameters that define the marginal costs (written as a function of the relevant policy) are  $M(t) = \sqrt{3t}$  and  $N(\sigma) = 1/\sigma$  and marginal costs are  $c_q(q; w, t) = \sqrt{3tq}$  and  $c_q(q; w, \sigma) = 2/3 \cdot 1/\sigma \cdot q$ .

In the emissions tax equilibrium, the uncovered and covered marginal costs are equal, so  $q_d = t_U q_f / t$ . The equilibrium uncovered production is found by solving  $100 - q_f(1 + t_U/t) = \sqrt{3t_U q_f}$  for  $q_f$ . Unfortunately, this equation is non-linear even with this simple parameterization.<sup>64</sup> However, the equation can be easily solved using numerical simulation software.<sup>65</sup> I characterize the equilibria for several emissions taxes. It is easy to verify that these are indeed equilibria.<sup>66</sup> The second-best tax that maximizes net social benefits is found by graphing.

First, if  $t = 1/30,000$ , the tax equilibrium is equivalent to the unregulated equilibrium. For this tax,  $q_d = q_f = 49.96$  so  $Q = 99.93$  and the marginal cost is \$0.07. Emissions are  $e_d = e_f = 35,318$  and market inputs are  $k_d = k_f = 3.53$ . Welfare is consumer surplus ( $\sim$ \$5000), less input costs (\$2.35) less damages (\$23,545) for net social surplus of -\$18,548. Note that the unregulated emissions intensity is 707.

By numerically solving the tax equilibrium and graphing the net social benefits as a function of  $t$ , the second-best tax can be seen to be approximately,  $t = 0.000051 > t_U$ . For this equilibrium, production is  $Q = 99.92$ , and the marginal cost is \$0.08. Production and emissions shift to the uncovered firm:  $q_f = 60.66$  &  $q_d = 39.26$  and  $e_f = 47,244$  and  $e_d = 19,792$ ; however the covered production uses more capital  $k_f = 4.72$  &  $k_d = 7.29$ . Welfare is consumer surplus ( $\sim$ \$5000), less input costs (\$4.01) less damages (\$22,346) for net social surplus of -\$17,350. Note that this is a slight improvement over the unregulated equilibrium.

The intensity standard equilibrium can be found by equating marginal costs, which implies that  $2/3 \cdot 1/\sigma q_d = \sqrt{3t_U q_f}$ . Solving for  $q_d$  and substituting into the equation which equates marginal utility and uncovered marginal costs implies that  $q_f$  can be solved from  $100 - q_f = \sqrt{3t_U q_f}(1 + 3\sigma/2)$ . By graphing the equilibrium welfare as a function of  $\sigma$ , the second-best intensity standard is approximately  $\sigma = 441$ . Note that this standard is binding. Marginal cost is \$0.07, and the production of 99.93 is more evenly split between covered and uncovered production:  $q_f = 52.16$  and  $q_d = 47.77$ . Emissions are also more evenly shared:  $e_f = 37,666$  and  $e_d = 21,067$  but covered market inputs are higher:  $k_f = 3.77$  &  $k_d = 5.18$ . Welfare is consumer surplus ( $\sim$ \$5000), less input costs (\$2.98) less damages (\$19,478) for net social surplus of -\$14,581.

Note that the efficient equilibrium has market production of 93.17 which is equally divided between the two firms, and marginal cost is \$6.83. Emissions and market inputs are equal, *i.e.*,  $e_f = e_d = k_f = k_d = 318$  and the emissions intensity is 6.8. Efficient social surplus is then consumer surplus (\$4976), less input costs (\$212) less damages (\$212) for net social surplus of \$4,553.

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<sup>64</sup>With constant elasticity of demand, the equation could be more easily solved analytically. However, the equilibrium for the intensity standard would still require numerical techniques.

<sup>65</sup>Code is available upon request.

<sup>66</sup>The factor demand for emissions is  $e = q^{1/3}(1/3t)^{1/2}$  and for the market input is  $k = 3te$ .

### Derivation of cost functions for constant elasticity of substitution

Assume the production function has the constant elasticity of substitution following form:  $q = K(k_1^\alpha + k_2^\alpha + e^\alpha)^{\frac{\beta}{\alpha}}$  where the elasticity of substitution is  $1/(1 - \alpha) > 0$ . The cost function for an emissions tax is well known and is given by:

$$c(q; w_1, w_2, t) = q^{\frac{1}{\beta}} K^{\frac{-1}{\beta}} \left[ t^{\frac{\alpha}{\alpha-1}} + w_1^{\frac{\alpha}{\alpha-1}} + w_2^{\frac{\alpha}{\alpha-1}} \right]^{\frac{\alpha-1}{\alpha}}.$$

Note that the returns to scale depends on  $\beta$ , and the marginal cost is constant if  $\beta = 1$ .<sup>67</sup>

To derive  $c(q; w_1, w_2, \sigma)$ , first note that  $w_2/w_1 = (k_2/k_1)^{\alpha-1}$ , which implies that  $R \equiv (k_2/k_1)^\alpha = (w_2/w_1)^{\frac{\alpha}{\alpha-1}}$  and that  $w_2 k_2 = w_1 k_1 R$ . The production function then implies that

$$(q/K)^{\frac{\alpha}{\beta}} = k_1^\alpha (1 + R) + \sigma^\alpha q^\alpha,$$

so the contingent factor demand is

$$k_1 = (1 + R)^{-\frac{1}{\alpha}} q \left[ q^{\frac{\alpha-\alpha\beta}{\beta}} K^{\frac{-\alpha}{\beta}} - \sigma^\alpha \right]^{\frac{1}{\alpha}}.$$

The cost function can then be written:

$$c(q; w_1, w_2, \sigma) = w_1 k_1 + w_2 k_2 = w_1 k_1 (1 + R) = w_1 (1 + R)^{\frac{\alpha-1}{\alpha}} q \left[ q^{\frac{\alpha-\alpha\beta}{\beta}} K^{\frac{-\alpha}{\beta}} - \sigma^\alpha \right]^{\frac{1}{\alpha}}$$

which can be written

$$c(q; w_1, w_2, \sigma) = \left( w_1^{\frac{\alpha}{\alpha-1}} + w_2^{\frac{\alpha}{\alpha-1}} \right)^{\frac{\alpha-1}{\alpha}} q \left[ q^{\frac{\alpha-\alpha\beta}{\beta}} K^{\frac{-\alpha}{\beta}} - \sigma^\alpha \right]^{\frac{1}{\alpha}}.$$

Note that if  $\beta = 1$  (*i.e.*, the production function has constant returns to scale), then this function is linear in  $q$  and marginal cost is constant.

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<sup>67</sup>See for example Nicholson's text. The cost functions for one market input have  $w_2 = 0$ .

## Tables

Table 1. Single policies under incomplete regulation: comparing optimal emissions taxes with intensity standards under Cobb-Douglas and constant returns to scale.

Panel A. Lax uncovered emissions charge.  $t_U = 0.01$

| $\beta$ | Emissions tax ( $t=t_f$ ) | Intensity Standard | DWL  | Output | Emissions | Standard dominates? |
|---------|---------------------------|--------------------|------|--------|-----------|---------------------|
| 0.8     | 0.01                      | -                  | 1.54 | 1.96   | 5.67      | Yes                 |
|         | -                         | 1.93               | 0.66 | 1.96   | 3.79      |                     |
| 0.5     | 0.01                      | -                  | 5.21 | 1.86   | 13.14     | Yes                 |
|         | -                         | 3.54               | 2.06 | 1.86   | 6.57      |                     |
| 0.2     | 0.01                      | -                  | 5.37 | 1.62   | 12.24     | Yes                 |
|         | -                         | 3.09               | 1.88 | 1.62   | 5.01      |                     |

Panel B. Stringent uncovered emissions charge.  $t_U = 0.25$

| $\beta$ | Emissions tax ( $t=t_f$ ) | Intensity Standard | DWL  | Output | Emissions | Standard dominates? |
|---------|---------------------------|--------------------|------|--------|-----------|---------------------|
| 0.8     | 0.25                      | -                  | 0.10 | 1.53   | 2.31      | No                  |
|         | -                         | 1.01               | 0.30 | 1.53   | 1.55      |                     |
|         | -                         | 1.21               | 0.20 | 1.77   | 2.15      |                     |
| 0.5     | 0.25                      | -                  | 0.12 | 1.29   | 1.83      | Yes                 |
|         | -                         | 0.71               | 0.12 | 1.29   | 0.91      |                     |
|         | -                         | 0.84               | 0.10 | 1.40   | 1.18      |                     |
| 0.2     | 0.25                      | -                  | 0.05 | 1.28   | 0.74      | Yes                 |
|         | -                         | 0.24               | 0.02 | 1.28   | 0.30      |                     |
|         | -                         | 0.30               | 0.01 | 1.32   | 0.40      |                     |

Notes: Parameterization:  $U(q)=2-q$ ;  $f(k,e)=k^{1-\beta}e^\beta$ ;  $\tau=0.5$ ; and  $w=0.5$ .  
Efficient social surplus (quantity, emissions) is 0.69 (1.18, 1.55); 0.5 (1, 1); and 0.69 (1.18, 0.39).

Table 2. Combined policies under incomplete regulation: comparing consumption taxes with emissions taxes or intensity standards under Cobb-Douglas and constant returns to scale.

Panel A. Lax uncovered emissions charge.  $t_U = 0.01$

| $\beta$ | Emissions tax ( $t=t_f$ ) | Intensity Standard | Consumption tax | DWL   | Output | Emissions | Standard dominates? |
|---------|---------------------------|--------------------|-----------------|-------|--------|-----------|---------------------|
| 0.8     | 0.01                      | -                  | 1.41            | 0.54  | 0.55   | 1.59      | Yes                 |
|         | -                         | 1.93               | 0.96            | 0.19  | 1.00   | 1.93      |                     |
| 0.5     | 0.01                      | -                  | 1.86            | 0.500 | 0.00   | 0.00      | Yes                 |
|         | -                         | 3.54               | 1.77            | 0.496 | 0.09   | 0.32      |                     |
| 0.2     | 0.01                      | -                  | 1.62            | 0.691 | 0.00   | 0.00      | Yes                 |
|         | -                         | 3.09               | 1.54            | 0.688 | 0.08   | 0.24      |                     |

Panel B. Stringent uncovered emissions charge.  $t_U = 0.25$

| $\beta$ | Emissions tax ( $t=t_f$ ) | Intensity Standard | Consumption tax | DWL  | Output | Emissions | Standard dominates? |
|---------|---------------------------|--------------------|-----------------|------|--------|-----------|---------------------|
| 0.8     | 0.25                      | -                  | 0.38            | 0.03 | 1.15   | 1.74      | Yes/ First best     |
|         | -                         | 1.32               | 0.66            | 0.00 | 1.18   | 1.55      |                     |
| 0.5     | 0.25                      | -                  | 0.35            | 0.06 | 0.94   | 1.33      | Yes/ First best     |
|         | -                         | 1.00               | 0.50            | 0.00 | 1.00   | 1.00      |                     |
| 0.2     | 0.25                      | -                  | 0.14            | 0.04 | 1.14   | 0.65      | Yes/ First best     |
|         | -                         | 0.33               | 0.16            | 0.00 | 1.18   | 0.39      |                     |

Notes: Parameterization:  $U(q)=2-q$ ;  $f(k,e)=k^{1-\beta}e^\beta$ ;  $\tau=0.5$ ; and  $w=0.5$ .

Efficient social surplus (quantity, emissions) is 0.69 (1.18, 1.55); 0.5 (1, 1); and 0.69 (1.18, 0.39).

Table 3. Monopoly: comparing emissions taxes with intensity standards under Cobb-Douglas and constant returns to scale.

| $\beta$ | Emissions tax ( $t=t_f$ ) | Intensity Standard | DWL  | Output | Emissions | Standard dominates? |
|---------|---------------------------|--------------------|------|--------|-----------|---------------------|
| 0.8     | 0.14                      | -                  | 0.12 | 0.85   | 1.43      | Yes                 |
|         | -                         | 1.43               | 0.04 | 0.94   | 1.34      |                     |
| 0.5     | 0.25                      | -                  | 0.10 | 0.65   | 0.91      | Yes                 |
|         | -                         | 1.22               | 0.04 | 0.80   | 0.97      |                     |
| 0.2     | 0.31                      | -                  | 0.16 | 0.63   | 0.30      | Yes                 |
|         | -                         | 0.46               | 0.12 | 0.70   | 0.32      |                     |

Notes: Parameterization:  $U(q)=2-q$ ;  $f(k,e)=k^{1-\beta}e^\beta$ ;  $\tau=0.5$ ; and  $w=0.5$ .  
 Efficient social surplus (quantity, emissions) is 0.69 (1.18, 1.55); 0.5 (1, 1); and 0.69 (1.18, 0.39).

## Appendix Tables

Appendix Table 1. Incomplete regulation with decreasing returns: comparing optimal emissions taxes with intensity standards under Cobb-Douglas and decreasing returns to scale.

Panel A. Lax uncovered emissions charge.  $t_U = 0.01$

| $\alpha, \beta$ | Emissions tax | Intensity Standard | DWL  | Output | Emissions | Standard dominates? |
|-----------------|---------------|--------------------|------|--------|-----------|---------------------|
| 0.16, 0.64      | 0.011         | -                  | 1.62 | 1.95   | 5.58      | Yes                 |
|                 | -             | 2.38               | 1.31 | 1.96   | 4.93      |                     |
| 0.4, 0.4        | 0.013         | -                  | 4.88 | 1.82   | 12.14     | Yes                 |
|                 | -             | 4.48               | 3.87 | 1.83   | 10.06     |                     |
| 0.64, 0.16      | 0.023         | -                  | 4.05 | 1.53   | 9.33      | Yes                 |
|                 | -             | 2.76               | 3.46 | 1.55   | 8.12      |                     |

Panel B. Stringent uncovered emissions charge.  $t_U = 0.25$

| $\alpha, \beta$ | Emissions tax | Intensity Standard | DWL   | Output | Emissions | Standard dominates? |
|-----------------|---------------|--------------------|-------|--------|-----------|---------------------|
| 0.16, 0.64      | 0.28          | -                  | 0.10  | 1.43   | 1.99      | No                  |
|                 | -             | 1.07               | 0.11  | 1.45   | 1.81      |                     |
| 0.4, 0.4        | 0.31          | -                  | 0.09  | 1.19   | 1.42      | Yes                 |
|                 | -             | 0.77               | 0.08  | 1.22   | 1.22      |                     |
| 0.64, 0.16      | 0.37          | -                  | 0.032 | 1.18   | 0.53      | Yes                 |
|                 | -             | 0.28               | 0.026 | 1.22   | 0.47      |                     |

Panel C. Lax uncovered emissions charge.  $t_U = 0.01$ , plus consumption tax.

| $\alpha, \beta$ | Emissions tax | Intensity Standard | Consumption tax | DWL  | Output | Emissions | Standard dominates? |
|-----------------|---------------|--------------------|-----------------|------|--------|-----------|---------------------|
| 0.16, 0.64      | 0.011         | -                  | 1.32            | 0.43 | 0.64   | 1.39      | Yes                 |
|                 | -             | 1.88               | 1.22            | 0.35 | 0.75   | 1.48      |                     |
| 0.4, 0.4        | 0.013         | -                  | 1.84            | 0.63 | 0.08   | 0.23      | Yes                 |
|                 | -             | 2.40               | 1.77            | 0.60 | 0.14   | 0.40      |                     |
| 0.64, 0.16      | 0.022         | -                  | 1.71            | 0.78 | 0.07   | 0.19      | Yes                 |
|                 | -             | 1.52               | 1.66            | 0.77 | 0.11   | 0.29      |                     |

Panel D. Stringent uncovered emissions charge.  $t_U = 0.25$ , plus consumption tax.

| $\alpha, \beta$ | Emissions tax | Intensity Standard | Consumption tax | DWL   | Output | Emissions | Standard dominates? |
|-----------------|---------------|--------------------|-----------------|-------|--------|-----------|---------------------|
| 0.16, 0.64      | 0.26          | -                  | 0.40            | 0.03  | 1.09   | 1.41      | Yes                 |
|                 | -             | 1.06               | 0.42            | 0.02  | 1.09   | 1.24      |                     |
| 0.4, 0.4        | 0.29          | -                  | 0.33            | 0.042 | 0.92   | 1.04      | Yes                 |
|                 | -             | 1.00               | 0.42            | 0.041 | 0.95   | 0.95      |                     |
| 0.64, 0.16      | 0.35          | -                  | 0.12            | 0.03  | 1.09   | 0.48      | Yes                 |
|                 | -             | 0.29               | 0.15            | 0.02  | 1.09   | 0.42      |                     |

Notes: Parameterization:  $U(q)=2-q$ ;  $f(k,e)=k^\alpha e^\beta$ ;  $\tau=0.5$ ; and  $w=0.5$ .



Appendix Table 2. Incomplete regulation with constant elasticity of substitution: comparing optimal emissions taxes with intensity standards under CES and constant returns to scale.

Panel A. Lax uncovered emissions charge.  $t_U = 0.01$

| $\alpha$ | Emissions tax | Intensity Standard | DWL  | Output | Emissions | Standard dominates? |
|----------|---------------|--------------------|------|--------|-----------|---------------------|
| 0.2      | 0.01          | -                  | 0.14 | 2.00   | 0.40      | Yes                 |
|          | -             | 0.11               | 0.06 | 2.00   | 0.22      |                     |
| 0.5      | 0.01          | -                  | 0.49 | 1.99   | 1.91      | Yes                 |
|          | -             | 0.74               | 0.29 | 1.99   | 1.47      |                     |
| 0.8      | 0.01          | -                  | 0.24 | 1.99   | 1.99      | Yes                 |
|          | -             | 0.95               | 0.21 | 1.99   | 1.88      |                     |

Panel B. Stringent uncovered emissions charge.  $t_U = 0.25$

| $\alpha$ | Emissions tax | Intensity Standard | DWL    | Output | Emissions | Standard dominates? |
|----------|---------------|--------------------|--------|--------|-----------|---------------------|
| 0.2      | 0.25          | -                  | 0.0047 | 1.98   | 0.09      | Yes                 |
|          | -             | 0.03               | 0.0001 | 1.98   | 0.06      |                     |
| 0.5      | 0.25          | -                  | 0.05   | 1.83   | 0.81      | Yes                 |
|          | -             | 0.23               | 0.01   | 1.87   | 0.44      |                     |
| 0.8      | 0.25          | -                  | 0.12   | 1.75   | 1.63      | Yes                 |
|          | -             | 0.35               | 0.02   | 1.75   | 0.62      |                     |

Panel C. Lax uncovered emissions charge.  $t_U = 0.01$ , plus consumption tax.

| $\alpha$ | Emissions tax | Intensity Standard | Consumption tax | DWL  | Output | Emissions | Standard dominates? |
|----------|---------------|--------------------|-----------------|------|--------|-----------|---------------------|
| 0.2      | 0.01          | -                  | 0.10            | 0.14 | 1.90   | 0.38      | Yes                 |
|          |               | 0.11               | 0.06            | 0.05 | 1.94   | 0.22      |                     |
| 0.5      | 0.01          | -                  | 0.47            | 0.38 | 1.52   | 1.46      | Yes                 |
|          |               | 0.74               | 0.37            | 0.22 | 1.62   | 1.20      |                     |
| 0.8      | 0.01          | -                  | 0.49            | 0.12 | 1.50   | 1.50      | Yes                 |
|          |               | 0.95               | 0.47            | 0.10 | 1.52   | 1.43      |                     |

Panel D. Stringent uncovered emissions charge.  $t_U = 0.25$ , plus consumption tax.

| $\alpha$ | Emissions tax | Intensity Standard | Consumption tax | DWL   | Output | Emissions | Standard dominates? |
|----------|---------------|--------------------|-----------------|-------|--------|-----------|---------------------|
| 0.2      | 0.25          | -                  | 0.01            | 0.005 | 1.97   | 0.09      | Yes/First           |
|          | -             | 0.03               | 0.02            | 0.000 | 1.97   | 0.06      | Best                |
| 0.5      | 0.25          | -                  | 0.11            | 0.05  | 1.72   | 0.77      | Yes/First           |
|          | -             | 0.25               | 0.13            | 0.00  | 1.75   | 0.44      | Best                |
| 0.8      | 0.25          | -                  | 0.23            | 0.09  | 1.52   | 1.41      | Yes/First           |
|          | -             | 0.42               | 0.21            | 0.00  | 1.58   | 0.66      | Best                |

Notes: Parameterization:  $U(q)=2-q$ ;  $f(k,e)=(k^\alpha+e^\alpha)^{1/\alpha}$ ;  $\tau=0.5$ ; and  $w=0.5$ .