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FAMILIAL LOVE AND INTERTEMPORAL OPTIMALITY

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SUMMARY

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This paper analyzes the intertemporal efficiency and optimality of steady states within overlapping-generations models in which the utility of individual working couples depends on the consumption of their parents and children as well as their own consumption. The analysis considers both a basic model in which altruistic behavior can take only the form of gifts of consumption goods from working couples to their retired parents and an extended model in which altruistic behavior also can take the form of bequests from parents to their surviving children. In the basic model, saving only involves storing consumption goods, whereas the extended model includes capital and neoclassical production.

The following conclusions from the analysis apply to both models: An altruistic utility function promotes intertemporal efficiency. However, altruism creates an externality that implies that satisfying the conditions for efficiency does not insure intertemporal optimality. Nevertheless, if the utility of working couples is appropriately sensitive at the margin to their own consumption, their parents' consumption, and their children's consumption, the steady state that is consistent with individual behavior is both efficient and optimal.

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The specification of efficiency and optimality conditions for intertemporal resource allocation and the possible inconsistency between these conditions and the consequences of private decision making within a context of overlapping generations have traditionally been central issues in the theoretical analysis of the long-run evolution of the economy--see, for example, Samuelson (1958), Diamond (1965), and Cass and Yaari (1966). Recently, the development of more general behavioral models that allow for voluntary private intergenerational giving--for example, support of retired parents by working children and bequests from deceased parents to surviving children--has provided a new perspective for analyzing these issues--see, for example, Barro (1974; 1976), Drazen (1978), and Buiter (1979). The present paper continues this recent line of inquiry by considering intertemporal efficiency and optimality within models of familial love, formalized in the assumption that the utility of individual working couples depends on the consumption of their parents and children as well as their own consumption.

The main conclusions from this analysis are the following: An intertemporally altruistic utility function promotes intertemporal efficiency. However, with such an altruistic utility function, in contrast to the models of Cass and Yaari and Diamond, satisfying the conditions for efficiency does not insure intertemporal optimality. Nevertheless, there exist strong but plausible restrictions on an altruistic utility function that make the results of private behavior both efficient and optimal.

1. Children Love Parents

The analysis in the first part of the paper extends the model of Cass and Yaari by assuming that a working couple's utility depends positively on the past and current consumption of each set of its retired parents. This form of familial love creates the possibility of altruistic behavior in the form of voluntary gifts of consumption goods from working couples to their retired parents. Aside from this extension, this basic model follows the Cass and Yaari setup. Specifically, the analysis abstracts from stochastic factors and assumes that the life cycle consists of one working period and one retired period, that labor services are the only productive input, and that consumption goods, which can be storable, are the only product.

Historical and heuristic interest in the Cass and Yaari model provides motivation for using this setup. More importantly, however, analysis of this basic model yields general results about the way in which familial love affects the relation between private behavior and intertemporal efficiency and optimality. The second part of the paper extends these results by incorporating into the analysis love of parents for their surviving children, the resulting possibility of altruistic behavior in the form of bequests, and neoclassical production relations involving both capital services and labor services.

1.1 Specification of the Basic Model

The lifetime utility of each couple who is working during the current period, denoted by U_t, is given by

$$U_t = U(c_t^1, c_t^2, c_{t-1}^1, c_{t-1}^2),$$

where c represents consumption per couple, the superscripts denote working and retired periods, and the subscripts identify a generation by the date of its working period. Specifically, according to this notation,

c_t^1 is the current consumption of the current working couple,

c_t^2 is its prospective future consumption during retired years,

c_{t-1}^1 is the past consumption of each set of its retired parents during their working years, and

c_{t-1}^2 is the current consumption of each set of its retired parents.

Each of these arguments is nonnegative. The function U is increasing in each argument, concave, and twice differentiable. Each first partial derivative approaches infinity as the value of the respective argument approaches zero.

The specification of children's utility as depending on parents' consumption, rather than on parents' utility as in the formulations of Drazen and Buiter, seems plausible in that it relates the child's utility directly to the objective events of parents' consumption, rather than to the parents' subjective evaluation of these events. In addition, the dependence of utility on parents' consumption avoids the convergence problems that Buiter and Drazen encounter in defining the utility maximand. The present formulation effectively links all generations through the constraints on consumption, given by equations (1.1-4), rather than through the utility function.

The arguments of the utility function are determined as follows:

(1.1) $c_t^1 = 2w - s_t - 2g_t;$

(1.2) $c_t^2 = rs_t + ng_{t+1},$

(1.3) $c_{t-1}^1 = 2w - s_{t-1} - 2g_{t-1},$ and

(1.4) $c_{t-1}^2 = rs_{t-1} + ng_t,$

where w is real wage income per capita,

s is saving, which takes the form of accumulation of consumption goods during working years,

g is the amount of consumption goods given by a working couple to each set of its retired parents,

r is the physical rate of return plus unity on stored consumption goods, and

n is the number of children per set of parents.

Each of these variables is nonnegative. Note that r equal to zero would represent full depreciation in storage, that r equal to unity would represent no depreciation in storage, and r greater than unity would represent appreciation in storage. Note also that $n/2$ equals unity plus the rate of population growth, which, it is relevant to recall, Samuelson denotes as the biological rate of return. The basic model assumes that w, r, and n are exogenously determined and constant across generations. In the extended model of neoclassical production in the second part of the paper, wage income and the physical rate of return become endogenous variables.

1.2 Efficiency and Optimality of Steady States

Denote steady-state values by the absence of the time subscript. Thus, by definition,

(2.1) $c^1 = 2w - s - 2g$ and

(2.2) $c^2 = rs + ng.$

Following standard usage, define as <u>efficient</u> steady states in which the values of s and g maximize c^2 for given values of c^1, subject to equations (2.1-2). Note that efficiency involves specifying a consumption possibilities frontier, but does not concern choosing among points on that frontier. From the form of equations (2.1-2) and the nonnegativity restrictions on c^1, c^2, s, and g, we can infer the following efficiency properties:

(Ia) If $r = n/2$, any steady state that conforms to the non-negativity restrictions is efficient. Specifically, if $r = n/2$, efficiency implies $2w \geq s + 2g \geq 0$, with $s \geq 0$ and $g \geq 0$.

(Ib) If $r \neq n/2$, efficiency puts additional restrictions or either s or g. Specifically, if $r < n/2$, efficiency implies $s = 0$ and $w \geq g \geq 0$. Alternatively, if $r > n/2$, efficiency implies $g = 0$ and $2w \geq s \geq 0$.

These results indicate that, as a way to provide for retirement consumption, saving is efficient if and only if the physical rate of return is not less than the biological rate of return and gifts from working children are efficient if and only if the biological rate of return is not less than the physical rate of return.

Again following standard usage, define as optimal a steady state in which s and g takes values that maximize $U(c^1,c^2,c^1,c^2)$, subject to equations (2.1-2). Given the non-negativity restrictions and the assumption that the partial derivatives of U approach infinity as c^1 and c^2 approach zero, the first-order conditions for the solution to the optimality problem require that c^1, c^2, s, and g satisfy a pair of the following possible restrictions:

either (3.1) $-(U_1 + U_3) + r(U_2 + U_4) = 0$ and $2w - 2g > s > 0$,

or (3.2) $-(U_1 + U_3) + r(U_2 + U_4) \leq 0$ and $s = 0$, and

either (3.3) $-(U_1 + U_3) + \frac{n}{2}(U_2 + U_4) = 0$ and $w - \frac{s}{2} > g > 0$,

or (3.4) $-(U_1 + U_3) + \frac{n}{2}(U_2 + U_4) \leq 0$ and $g = 0$,

but not both (3.2) and (3.4). For brevity, the full functional form for each partial derivative, e.g., $U_1(c^1,c^2,c^1,c^2)$ is not explicitly written out.

Which of the eligible pairs of conditions (3.1-4) is applicable depends on the relation between r and $n/2$. Specifically, the implications of these first-order conditions for the optimality properties of a steady state are the following:

(IIa) If $r = n/2$, the pairs of conditions (3.1) and (3.3), (3.1) and (3.4), or (3.2) and (3.3) can be relevant. In this case, optimality restricts only the sum, $s + 2g$. The optimal value, $(s + 2g)^*$, has an interior value, $2w > (s + 2g)^* > 0$, such that c^1 and c^2 satisfy the equivalent conditions (3.1) and (3.3), with $s \geq 0$ and $g \geq 0$.

(IIb) If $r > n/2$, the pair of conditions (3.1) and (3.4) is relevant. In this case, the optimal value, s^*, has an interior value, $2w > s^* > 0$, such that c^1 and c^2 satisfy condition (3.1), and the optimal value, g^*, has the boundary value, $g^* = 0$, that satisfies condition (3.4).

(IIc) If $r < n/2$, the pair of conditions (3.2) and (3.3) is relevant. In this case, s^* has the boundary value, $s^* = 0$, that satisfies condition (3.2), and g^* has an interior value, $w > g^* > 0$, such that c^1 and c^2 satisfy condition (3.3).

Note that these results indicate that efficiency is necessary but not sufficient for optimality.

It is worth stressing that these concepts of efficiency and optimality relate only to a normative comparison of possible steady states. A normative comparison of nonsteady-state paths would require additional criteria, such as the standard of Pareto optimality. Although an analysis of such criteria is beyond the scope of the present paper, two interesting observations can readily be made: First, an efficient steady state is Pareto optimal. Second, a path that went from an arbitrary starting point to an efficient steady state in a single generation generally would not be Pareto optimal. Specifically, an immediate change from $g > 0$ and $s = 0$ to $g = 0$ and $s > 0$ generally would make either current working couples or then parents worse off, whereas an immediate change from $g = 0$ and $s > 0$ to $g > 0$ and $s = 0$ generally would make the terminal generations worse off.

1.3 Individual Behavior, Efficiency, and Optimality

Consider if, and under what conditions, individual behavior is consistent with a steady state that is efficient or optimal. The choice problem for an individual current working couple is to select values for s_t and g_t to maximize $U(c_t^1, c_t^2, c_{t-1}^1, c_{t-1}^2)$, subject to equations (1.1-4). In addition to the choice variables, these constraints involve the exogenous variables, w, r, and n, the predetermined variables, s_{t-1} and g_{t-1}, and the expectational variable, g_{t+1}.

The formation of g_{t+1} is important for the results of the analysis. One simple possibility would be to treat g_{t+1} as exogenous, an assumption that would mean that the individual couple believes that its decisions about s_t and g_t will not affect the future decision about g_{t+1} made by its children. The problem with this assumption is that in general it implies that the couple does not expect its children to perform the same choice calculus that it itself is performing.

A more satisfactory treatment is to assume that the couple relates its expectation, g_{t+1}, to s_t and g_t in the same way that its own decision about g_t depends on s_{t-1} and g_{t-1}. In general, the value of g_t that maximizes $U(c_t^1, c_t^2, c_{t-1}^1, c_{t-1}^2)$ depends on s_{t-1}, because s_{t-1} enters into c_{t-1}^1 and c_{t-1}^2, and depends on g_{t-1}, because g_{t-1} enters into c_{t-1}^1. If, however, the function U is additively separable in c_{t-1}^1, the chosen value of g_t does not depend on g_{t-1}. With this simplification, we focus attention on the important consideration that parents' saving, s_{t-1}, and giving to parents, g_t, are substitutes in the provision of parents' current consumption, c_{t-1}^2. This substitutability implies that, for an interior solution, the chosen value of g_t is inversely related to s_{t-1}. (The lower bound on the derivative of g_t with respect to s_{t-1} would be $-r/n$, a value that would mean that an increase in s_{t-1} induces a decrease in g_t sufficient to leave c_{t-1}^2 unchanged.)

A reasonable specification of rational behavior would seem to be that the functional relation between g_{t+1} and s_t is the same as the functional relation between g_t and s_{t-1}. Thus, rationality means that the individual couple expects that, if it saves more, the amount that it receives from its children during retired years will be smaller, provided that this expected amount is not already zero. This belief implies that the private return from saving can be less than the physical rate of return, represented by r. Let λ denote the value of the derivative of g_{t+1} with respect to s_t.

Some other aspects of the individual couple's choice problem are worth noting. For example, equation (1.4), which determines c^2_{t-1}, effectively abstracts from any free rider problem among siblings. Specifically, this specification of the choice problem assumes that in selecting s_t and g_t the individual couple presumes that its siblings also are choosing to give to their parents an amount equal to g_t. A possible rationalization for this assumption is that the siblings have a familial understanding according to which they act as a single decision maker in choosing g_t.

This formulation of the individual couple's choice problem also makes no explicit allowance for borrowing and lending. This simplification, however, is not consequential. Because the only feasible transactions in a loan market would be between members of the current working generation, such a market would not alter the form of the constraints faced by the representative working couple.

In light of the nonnegativity restrictions and the assumption that the partial derivatives of U approach infinity as c^1_t, c^2_t, c^1_{t-1}, and c^2_{t-1} approach zero, the first-order conditions for the solution to the individual couple's choice problem require that s_t and g_t satisfy a pair from the following possible restrictions:

either (4.1) $-U_1 + (r+n\lambda)U_2 = 0$ and $2w - 2g_t > s_t > 0$,

or (4.2) $-U_1 + (r+n\lambda)U_2 \leq 0$ and $s_t = 0$, and either (4.3) $-U_1 + \frac{n}{2} U_4 = 0$ and $w - \frac{s_t}{2} > g_t > 0$,

or (4.4) $-U_1 + \frac{n}{2} U_4 \le 0$ and $g_t = 0$.

Again, for brevity, the full functional form for each partial derivative, e.g., $U(c_t^1, c_t^2, c_{t-1}^1, c_{t-1}^2)$, and for λ is not explicitly written out. The above discussion of the nature of λ implies that $r+n\lambda$ is positive, but less than or equal to r.

Let S, G, C^1, C^2, and Λ denote the steady-state values that are consistent with these first-order conditions and equations (1.1-4). Assuming that in a steady state the expectation, g_{t+1}, is correct and, hence, equals G, the pair of conditions (4.2) and (4.4) would imply $C^2 = 0$ and cannot be relevant in a steady state. Which of the other pairs is relevant in the steady state depends on the relation between rU_2 and $\frac{n}{2} U_4$. Specifically, the implications of the first-order conditions (4.1-4) for the characteristics of steady states are the following:

(IIIa) If C^1 and C^2 satisfy $(r+n\Lambda)U_2 = \frac{n}{2} U_4$, the pairs of conditions (4.1) and (4.3), (4.1) and (4.4), or (4.2) and (4.3) can be relevant. In this case, the sum, $S + 2G$, has an interior value, $2w > S + 2G > 0$, that satisfies the equivalent conditions (4.1) and (4.3), with $S \ge 0$ and $G \ge 0$.

(IIIb) If C^1 and C^2 satisfy $(r+n\Lambda)U_2 > \frac{n}{2} U_4$, the pair of conditions (4.1) and (4.4) is relevant. In this case, S has an interior value, $2w > S > 0$, that satisfies condition condition (4.4). Because G does not take an interior value, this case also has $\Lambda = 0$.

(IIIc) If C^1 and C^2 satisfy $(r+n\Lambda)U_2 < \frac{n}{2} U_4$, the pair of conditions (4.2) and (4.3) is relevant. In this case, S has the boundary value, $S = 0$, that satisfies condition (4.2), and G has an interior value, $W > G > 0$, that satisfies condition (4.3).

Note that Drazen's paper considers what amounts to a special case of this analysis that assumes $\Lambda = 0$, $U_1 = rU_2$, $S > 0$, and $n = 2$, and, in which, consequently, $G > 0$ requires $rU_2 = U_4$.

Comparing the characteristics of individual behavior with the efficiency and optimality properties derived above indicate that the efficiency and optimality of S and G depend on the relation between r and $n/2$ and on the form of the function U. With regard to efficiency, we can draw the following conclusions:

(IVa) From (Ia), $r = n/2$ is a sufficient condition for S and G to be efficient.

(IVb) If $r \neq n/2$, either inefficient positive saving or inefficient giving to parents is possible. Specifically, from (IIIa-c), a necessary condition for $S > 0$ is that C^1 and C^2 satisfy $(r+n\Lambda)U_2 \geq \frac{n}{2} U_4$, whereas, from (Ib), a necessary and sufficient condition for $S > 0$ to be inefficient is that $r < n/2$. Similarly, from (IIIa-c), a necessary condition for $G > 0$ is that C^1 and C^2 satisfy $(r+n\Lambda)U_2 \leq \frac{n}{2} U_4$, whereas from (Ib), a necessary and sufficient condition for $G > 0$ to be inefficient is that $r > n/2$.

(V) If $r \neq n/2$, from (IVb), a necessary and sufficient condition for S and G to be efficient is that $(r+n\Lambda)U_2 \gtrless \frac{n}{2} U_4$ as $r \gtrless \frac{n}{2}$. A sufficient condition for this outcome is that C^1 and C^2 satisfy $(1+2\Lambda)U_2 = U_4$.

The possibility of inefficient positive saving, indicated in (IVb), generalizes a result obtained by Cass and Yaari. Cass and Yaari stress that, if $r < n/2$, saving is an inefficient way to provide retirement consumption. Their model, moreover, implicitly assumes that working couples associate zero marginal utility with the consumption of their retired parents. Given this assumption, only the pair of conditions (4.1) and (4.4) is relevant Λ equals zero, and S is positive for all positive values of r. The analysis in the present section has derived more general conditions for $S > 0$ that require $(r+n\Lambda)U_2 \geq \frac{n}{2} U_4$, but do not require $U_4 = 0$.

The possibility of inefficient positive giving to parents, also indicated in (IVb), is a further extension of the results of Cass and Yaari. This possibility arises if $r > n/2$, in which case efficiency requires each working generation to set aside current output to provide for its own future consumption and $G > 0$ would represent inefficient altruism. The present analysis indicates the $G > 0$ can obtain, regardless of the relation between r and $n/2$, if $(r+n\Lambda)U_2 \leq \frac{n}{2} U_4$. This condition in turn requires that working couples associate sufficiently high marginal utility with the current consumption of their retired parents.

More importantly, the present analysis reveals that the efficiency of individual behavior does not depend only on the relation between r and $n/2$. Specifically, (V) indicates that, whatever the sign of the difference between r and $n/2$, S and G are efficient if the difference between $(r+n\Lambda)U_2$ and $\frac{n}{2} U_4$ has the same sign. In the special case given in (V), if the marginal utility that working couples associate with their parents' current retirement consumption is equal to the marginal utility that they associate with their own prospective retirement consumption, with the latter weighted by a factor involving the effect of current saving on future giving, individual behavior is efficient regardless of the relation between r and $n/2$. Note that the present model incorporates two factors, the positive value of U_4 and the associated possibility of a negative value of Λ, that mitigate in favor of efficiency, but were not present in the model of Cass and Yaari.

Turning to question of optimality, the critical factor is the degree of correspondence between conditions (3.1-4) and conditions (4.1-4). As noted above, efficiency is necessary but not sufficient for optimality. Specifically, assuming efficiency, we can draw from (IIa-c) and (IIIa-c) the following conclusions about the optimality of S and G:

(VIa) If $r = n/2$, implying efficiency, and if C^1 and C^2 satisfy $(r+n\Lambda)U_2 \leq \frac{n}{2} U_4$, then $S + 2G \gtrless (s + 2g)^*$ as C^1 and C^2 satisfy $\frac{U_1}{U_4} \lessgtr \frac{U_1+U_3}{U_2+U_4}$.

(VIb) If $r = n/2$, implying efficiency, and if C^1 and C^2 satisfy $(r+n\Lambda)U_2 \geq \frac{n}{2} U_4$, then $S + 2G \gtrless (s + 2g)^*$ as C^1 and C^2 satisfy $\frac{U_1}{(1+2\Lambda)U_2} \lessgtr \frac{U_1+U_2}{U_2+U_4}$.

(VIc) If $r < n/2$ and if C^1 and C^2 satisfy $(r+n\Lambda)U_2 < \frac{n}{2} U_4$, implying efficiency, then $S = s^* = 0$, but $G \gtrless g^*$ as C^1 and C^2 satisfy $\frac{U_1}{U_4} \lessgtr \frac{U_1+U_3}{U_2+U_4}$.

(VId) If $r > n/2$ and if C^1 and C^2 satisfy $(r+n\Lambda)U_2 > \frac{n}{2} U_4$, implying efficiency, then $G = g^* = 0$, but $S \gtrless s^*$ as C^1 and C^2 satisfy $\frac{U_1}{(1+2\Lambda)U_2} \lessgtr \frac{U_1+U_3}{U_2+U_4}$.

(VII) Sufficient conditions, from (VIa-d), for S and G to be both efficient and optimal are that C^1 and C^2 satisfy $(1+2\Lambda)U_2 = U_4$ and $U_1 = (1+2\Lambda)U_3$.

Note that these equalities refer only to the chosen values C^1 and C^2, although they would clearly be satisfied if the U function were such that $(1+2\Lambda)U_2 = U_4$ and $U_1 = (1+2\Lambda)U_3$ for all values of c^1 and c^2.

The possibilities for efficient but not optimal values of S and G, indicated in (VIa-d), illustrate that satisfying the conditions for efficiency generally does not insure optimality. This conclusion also generalizes the results obtained by Cass and Yaari. Given their setup, in which $U_3 = U_4 = 0$ and $\lambda = 0$ for all combinations of c^1 and c^2, condition (3.1) is equivalent to condition (4.1), and, hence, if $r > n/2$ and $S > 0$ is efficient, S also equals s^*. More generally, however, even with efficiency, if working couples associate positive marginal utility with both their own consumption and their parents' consumption, an externality arises that can cause non-optimality. Specifically, s* or g* satisfy conditions involving U_1, U_2, U_3, and U_4, whereas S satisfies a condition involving only U_1 and U_2 and G satisfies a condition involving only U_1 and U_4.

Concern at the margin for parents' consumption, nevertheless, does not preclude optimality. Specifically, (VIa-d) indicate that how closely the specification of optimality corresponds to the outcome of individual behavior depends on the relation between Λ, U_1, U_2, U_3, and U_4. As a special case, indicated in (VII), if the sufficient condition for efficiency given in (V) is satisfied, and if, in addition, the marginal utility working couples associate with their own current consumption equals the marginal utility they associate with their parents' past consumption, with the latter marginal utility weighted by a factor involving Λ, individual behavior is optimal.

2. Parents Also Love Children

This part of the paper extends the analysis by assuming that a working couple's utility depends positively on the prospective future consumption of each of its children, as well as on the consumption of its parents. This familial love of parents for their surviving children creates the possibility of altruistic behavior in the form of bequests. Allowing for bequests, the specification of meaningful efficiency conditions requires the introduction of a technology that incorporates the services of capital goods, assumed for simplicity to be interchangeable with consumption goods, as well as labor services, and that exhibits diminishing returns to capital and labor. The following analysis, like Diamond's model, specifically assumes neoclassical production.

2.1 Specification of the Extended Model

Using the same notation developed above, the lifetime utility of each couple who is working during the current period now is given by

$$U_t = V(c_t^1, c_t^2, c_{t-1}^1, c_{t-1}^2, c_{t+1}^1, c_{t+1}^2)$$

where the new arguments, c_{t+1}^1 and c_{t+1}^2, are the prospective future consumption of each of its children during their working years and retired years, respectively. The function V is increasing in each argument, concave, and twice differentiable. Again, each first partial derivative approaches infinity as the value of the respective argument approaches zero.

The arguments of this utility function are determined as follows:

(5.1) $c_t^1 = 2w_t - s_t - 2g_t,$

(5.2) $c_t^2 = r_t(s_t+2h_t) + ng_{t+1} - nh_{t+1},$

(5.3) $c_{t-1}^1 = 2w_{t-1} - s_{t-1} - 2g_{t-1},$

(5.4) $c_{t-1}^2 = r_{t-1}(s_{t-1}+2h_{t-1}) + ng_t - nh_t,$

(5.5) $c_{t+1}^1 = 2w_{t+1} - s_{t+1} - 2g_{t+1},$ and

(5.6) $c_{t+1}^2 = r_{t+1}(s_{t+1}+2h_{t+1}) + ng_{t+2} - nh_{t+2},$

where the new variable, h, is the bequest of consumption goods received by a couple from each set of its parents. To avoid the possibility of time inconsistency in the individual couple's choice problem, assume that the amount of this bequest is fixed before the couple begins working and its parents retire, although the bequest is not received until the couple retires and its parents die. Note that the sum, $s_{t-1} + 2h_{t-1}$, represents the amount of capital, denoted by k_t, that each current retired couple is making available to provide capital services that cooperate in current production.

The production function is linear homogeneous in capital and labor services, concave, and twice differentiable. Consequently, current product per worker depends on the current amount of capital per worker in the form

(6.1) $$w_t + (r_{t-1}-1)\frac{k_t}{n} = f(\frac{k_t}{n}), \; f' > 0 \text{ and } f'' < 0.$$

Assuming that the market for capital equates the rate of return received by owners of capital to the marginal product of capital implies

(6.2) $$r_{t-1} = 1 + f'(\frac{k_t}{n}).$$

Substitution of equation (6.2) into equation (6.1) yields a positive relation between w_t and k_t/n:

(6.3) $$w_t = f(\frac{k_t}{n}) - f'(\frac{k_t}{n})\frac{k_t}{n}.$$

2.2 Efficiency and Optimality in the Extended Model

The steady-state levels of consumption in the extended model are given by

(7.1) $$c^1 = 2w - s - 2g \text{ and}$$

(7.2) $$c^2 = r(s+2h) + ng - nh = (r - \frac{n}{2})k + \frac{n}{2}(s+2g).$$

Note that the equality, $k = s + 2h$, and the production relations (6.1-2) imply that, for given n, c^1 and c^2 depend only on k and the sum, $s + 2g$.

In efficient steady states, the values of k and $s + 2g$ maximize c^2 for given values of c^1, subject to equations (7.1-2). Assuming that $f'(0) + 1 \geq n/2$, and given the production relations (6.1-2) and the nonnegativity restrictions, we can readily show that efficiency implies

(8) $$r = 1 + f'(k/n) = n/2 \text{ and } 2w \geq s + 2g \geq 0.$$

Condition (8) combines the standard Golden Rule with the requirement that c^1 and c^2 be nonnegative. This result indicates that, with neoclassical production, efficiency involves a capital stock such that the physical rate of return equals the biological rate of return. This equality implies that the sum of the bequests received by a couple plus the income earned on these bequests, 2rh, equals the bequests left by this couple, nh, and, from equation (7.2), that $c^2 = rs + ng$, as in the basic model. Beyond a prescription for k and the nonnegativity conditions, efficiency involves no other restrictions on s, g, or h.

This latter conclusion is consistent with the analysis of efficiency in the basic model for the case of $r = n/2$. The difference in this regard between the models is that in the basic model equality between r and $n/2$ would be fortuitous, whereas according to condition (8) this equality is required for efficiency. Note that, if the importance of capital in production were so small that the production function did not imply $f'(0) + 1 \geq n/2$, condition (8) would not apply. Instead, efficiency would require $k = 0$ and, hence, $s = h = 0$. In this situation, the efficiency properties derived in the analysis of the basic model for the case of $r < n/2$ would be relevant. Note also that, if, as in the basic model, r and w were independent of k, and r were larger than $n/2$, the efficient value of k would be undefined. This property is what necessitates the introduction of neoclassical production when bequests are allowed.

In an optimal steady state, the values of k and $s + 2g$ maximize $V(c^1,c^2,c^1,c^2,c^1,c^2)$, subject to conditions (7.1-2). Assuming that $f'(0) + 1 \geq n/2$, and given the production relations (6.1-2) and the nonnegativity restrictions, the first-order conditions for the solution to the optimality problem are that the optimal value, k*, satisfies the Golden Rule efficiency condition,

(9.1) $$r = 1 + f'(k/n) = n/2,$$

and that the optimal value, $(s + 2g)^*$, has an interior value, $2w > (s + 2g)^* > 0$, such that c^1 and c^2 satisfy

$$(9.2) \qquad -(V_1+V_3+V_5) + \frac{n}{2}(V_2+V_4+V_6) = 0.$$

These optimality conditions are consistent with the analysis of optimality in the basic model for the case of $r = n/2$. Note that optimality restricts only the sums, $s + 2h = k$ and $s + 2g$, and does not imply restrictions beyond nonnegativity on s, g, or h individually. For example, anyone of s, g, or h can be zero, but also all three can be positive.

2.3 Individual Behavior in the Extended Model

The choice problem for an individual current working couple in the extended model is to select values for s_t, g_t, and h_{t+1} to maximize $V(c^1_t, c^2_t, c^1_{t-1}, c^2_{t-1}, c^1_{t+1}, c^2_{t+1})$, subject to equations (5.1-6). In addition to the choice variables, these constraints involve the exogenous variable, n, the predetermined variables--w_t, h_t, w_{t-1}, s_{t-1}, g_{t-1}, r_{t-1}, and h_{t-1}--and the expectational variables--r_t, g_{t+1}, w_{t+1}, s_{t+1}, r_{t+1}, g_{t+2}, and h_{t+2}.

With regard to expectations, it seems appropriate to treat r_t, w_{t+1}, and r_{t+1} as exogenous, because the individual couple would not expect its decisions to affect these market-determined remunerations. The treatment of other expectational variables should be analogous to the treatment of g_{t+1} in the basic model. Specifically, the simplifying assumption that the function U is additively separable in c_{t-1} would imply that g_{t+1}, s_{t+1}, g_{t+2}, and h_{t+2} are independent of the chosen value of g_t. The relations between these expectational variables and the chosen values of s_t and h_{t+1}, however, could be a significant consideration. In particular, for the cases of interior solutions, rationality would seem to imply that an increase in s_t would cause decreases in g_{t+1} and g_{t+2} and increases in s_{t+1} and h_{t+2}, whereas an increase in h_{t+1} would cause decreases in s_{t+1} and g_{t+2} and increases in g_{t+1} and h_{t+2}. These relations, which the interested reader can easily rationalize, all reflect the substitutability between saving, giving to parents, and bequests to children as ways to provide consumption during retired years.

Accounting for variability in the expectational variables, unfortunately, makes the algebra involved is describing individual behavior much more complex, analogously to the complication resulting from introducing the variables λ and Λ in the basic model, without providing much additional insight into the workings of the model. Consequently, the analysis that follows treats all of the expectational variables as constants. The interested reader can work out the more complete case. With this simplification, and given the nonnegativity restrictions and the assumption that the partial derivatives of V approach infinity as c_t^1, c_t^2, c_{t-1}^1, c_{t-1}^2, c_{t+1}^1, and c_{t+1}^2 approach zero, the first-order conditions for the solution to this problem require that s_t, g_t, and h_{t+1} satisfy a triple of the following possible restrictions:

either (10.1) $-V_1 + r_t V_2 = 0$ and $2w_t - 2g_t > s_t$

$$> \max\left[0, \frac{n}{r_t}(h_{t+1} - g_{t+1}) - 2h_t\right],$$

or (10.2) $-V_1 + r_t V_2 \leq 0$ and $s_t = 0$, and

either (10.3) $-V_1 + \frac{n}{2} V_4 = 0$ and $w_t - \frac{s_t}{2} > g_t$

$$> \max\left[0, h_t - \frac{r_{t-1}}{n}(s_{t-1} + 2h_{t-1})\right],$$

or (10.4) $-V_1 + \frac{n}{2} V_4 \leq 0$ and $g_t = 0$, and

either (10.5) $-\frac{n}{2} V_2 + r_{t+1} V_6 = 0$ and $\frac{r_t}{n}(s_t + 2h_t) + g_{t+1} > h_{t+1}$

$$> \max\left[0, \frac{h}{2r_{t+1}}(n_{t+2} - g_{t+2}) - \frac{s_{t+1}}{2}\right],$$

or (10.6) $-\frac{n}{2} V_2 + r_{t+1} V_6 \leq 0$ and $h_{t+1} = 0$.

Let S, G, H, K, R, W, C^1, and C^2 now denote the steady-state values that are consistent with the first-order conditions (10.1-6), equations (5.1-6), and the productivity relations (6.1-2). Assuming that in a steady state the expectations--r_t, g_{t+1}, w_{t+1}, s_{t+1}, r_{t+1}, g_{t+2}, and h_{t+2}--are correct and, hence, are equal, as appropriate, to R, H, G, W, and S, conditions (10.1-6) imply that R, C^1, and C^2 satisfy a pair from the following possible restrictions:

either (11.1) $RV_2 = \frac{n}{2} V_4$,

or (11.2) $RV_2 > \frac{n}{2} V_4$,

or (11.3) $RV_2 < \frac{n}{2} V_4$, and

either (11.4) $\frac{n}{2} V_2 = RV_6$,

or (11.5) $\frac{n}{2} V_2 > RV_6$.

The applicable pair of these relations implies which of the triples from conditions (10.1-6) can be relevant in the steady state.

The important observation to be stressed in the present context is that the sufficient conditions for efficiency and optimality in the extended model are similar to the analogous conditions in the basic model. Specifically, conditions (8), (10.1-6), and (11.1-5) imply the following:

(VIII) Sufficient conditions for K and S + 2G to be efficient are that C^1 and C^2 satisfy $V_2 = V_4 = V_6$.

To derive (VIII), observe that, if C^1 and C^2 satisfy $V_2 = V_4 = V_6$, the pairs of conditions (11.1) and (11.5), (11.2) and (11.4), (11.2) and (11.5), and (11.3) and (11.4) are obviously inconsistent, and the pair of conditions (11.3) and (11.5) is also inconsistent, because it would require $R < n/2$, but, from conditions (10.1-6), it also implies $K = 0$ and, hence, $R > n/2$. Consequently, the only possible case from conditions (11.1-5) is that C^1 and C^2

satisfy the pair of conditions (11.1) and (11.4) and, moreover, that K and R satisfy $R = n/2$. In this case, the sum, $S + 2H = K$, satisfies the efficiency condition, $1 + f'(K/n) = n/2$, and the sum, $S + 2G$, has an efficient interior value, $2W > S + 2G > 0$, that satisfies the equivalent conditions (10.1) and (10.3), with $S \geq 0$, $G \geq 0$, and $H \geq 0$.

With regard to optimality, conditions (9.1-2), (10.1-6), and (11.1-5) imply the following:

(IX) Sufficient conditions for K and $S + 2G$ to be both efficient and optimal are that C^1 and C^2 satisfy $V_2 = V_4 = V_6$ and $V_1 = V_3 = V_5$.

To derive (IX), recall the derivation of (VIII) and observe that, if C^1 and C^2 satisfy $V_2 = V_4 = V_6$ and $V_1 = V_3 = V_5$, conditions (10.1) and (10.3) are equivalent in the steady state to condition (9.2).

These results say that if the marginal utilities that working couples associate with their own prospective retirement consumption, their parents' current retirement consumption, and their children's prospective retirement consumption are equal, individual behavior is efficient. Specifically, the steady state that is consistent with individual behavior has the Golden Rule capital stock. Moreover, if, in addition, the marginal utilities that working couples associate with their own current consumption, their parents' past consumption, and their children's prospective consumption during their working years are equal, individual behavior is also optimal.

3. Concluding Remarks

This paper is preliminary in that it does consider a number of important issues that arise naturally in the present context. First, the analysis has dealt only with the description and evaluation of steady states. It has not considered the stability of steady states or the normative properties of nonsteady-state paths. Second, the analysis has assumed fertility to be an exogenous constant, rather than a choice variable. Willis (1979) analyzes the choice of fertility in a model in which giving to parents is positive, but exogenously determined, and there is no saving. Third, the analysis has not considered the possible effects of government financial policies, such as debt issue and social security. Questions about the impact of these policies have motivated the recent work of Barro, Drazen, and Buiter, referred to above. Fourth, the analysis has abstracted from investment in human capital, a phenomenon on which Drazen's paper focuses. Finally, this paper has made no attempts to compare the present interpretation that gifts and bequests involve altruistic behavior with the hypothesis, recently advanced by Kotlikoff and Spivak (1979), that gifts and bequests reflect risk-sharing behavior. Each of these issues is the subject of currently ongoing research.

What this paper has accomplished can be summarized as follows: The paper has analyzed the intertemporal efficiency and optimality of steady states within overlapping-generations models in which the utility of individual working couples depends on the consumption of their parents and children as well as their own consumption. The analysis has considered both a basic model in which altruistic behavior can take only the form of gifts of consumption goods from working couples to their retired parents and an extended model in which altruistic behavior also can take the form of bequests from parents to their surviving children. In the basic model, saving only involves storing consumption goods, whereas the extended model includes capital and neoclassical production.

The following conclusions from the analysis apply to both models: An altruistic utility function promotes intertemporal efficiency. However, altruism creates an externality that implies that satisfying the conditions for efficiency does not insure intertemporal optimality. Nevertheless, if the utility of working couples is appropriately sensitive at the margin to their own consumption, their parents' consumption, and their children's consumption, the steady state that is consistent with individual behavior is both efficient and optimal.

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