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OPERATIONAL TIME AND SEASONALITY IN DISTRIBUTED LAG ESTIMATION

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The following paper discusses the analysis of some types of economic time series using an altered time scale, or operational time. It is argued that for some series, observations that are ordinarily thought of as equidistant in time are actually irregularly spaced in a more natural time scale. Section A discusses point or impulse sampling of related series and the estimation of distributed lag relationships between them. Section B discusses time-aggregated sampling. In Section C, operational-time methods are used to calculate the distributed lag relationship between starts and completions for single-family dwellings in the United States. The results are statistically compared with those of ordinary distributed lag methods.

A. Point-Sampled Time Series

Assume $y(t) = b*x(t) + u(t)$ (1)

$$\text{where } b*x(t) = \int_{-\infty}^{\infty} b(s)\, x(t-s)\, ds \tag{2}$$

and $E(u(t)\, x(s)) = 0$, all t and s.

Not let y(t) and x(t) be sampled at discrete time intervals, according to the impulse process:

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-t_n) \tag{3}$$

where the numbers ..., t_{-1}, t_0, t_1, t_2... form a stationary point process (SPP)[1] independent of both x(t) and y(t), and $\delta(.)$ is the Dirac delta-function. Let $\phi_y(w)$, $\phi_x(w)$, $\phi_s(w)$[2] be the spectral densities for the processes y(t), x(t), and s(t). x(t) and y(t) are assumed to be covariance stationary.

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[1]See Beutler and Leneman[1] for a definition of an SSP.

[2]See Beutler and Leneman[2] for the method of calculating $\phi_x(w)$. $\phi_s(w)$ is a generalized function as is s(t); generalized functions will not be distinguished from ordinary ones in notation.

Let $Y(t) = s(t) \cdot y(t)$ and $X(t) = s(t) \cdot x(t)$ be the sampled series. The spectral densities for Y and X are then given by:

$$\phi_X(w) = \phi_s * \phi_x(w); \ \phi_Y(w) = \phi_s * \phi_y(w)$$

If we define $\tilde{f}(w)$ as the fourier transform of $f(t)$, ϕ_{yx} as the cross-spectrum of y and x, and ϕ_u as the spectrum of u (and assume all these exist),

$$\phi_{yx} = \tilde{b}\,\phi_x$$

$$\phi_y = |b|^2 \phi_x + \phi_u.$$

If we define:

$$\tilde{B} = \frac{\phi_s * (\tilde{b} \cdot \phi_x)}{\phi_s * \phi_x} \tag{4}$$

$$\phi_U = \phi_s * \phi_u + \phi_s * (|\tilde{b}|^2 \cdot \phi_x) - |\tilde{B}|^2 \ \phi_s * \phi_x$$

Then $\quad \phi_y = |\tilde{B}|^2 \phi_X + \phi_U$, and $\phi_{YX} = \tilde{B}\,\phi_X$.

Thus $\quad Y(t) = B * X(t) + U(t) \qquad (5)$

where $\quad E[X(s)\,U(t)] = 0$, all t,s.[3]

The relationship between B and b has been examined in some detail by Sims [5] for periodic sampling.

Note that (4) and (5) provide very little practical information when the structure of the sample points $\{t_n\}$ is complicated or random. This is because estimates of ϕ_Y, ϕ_{YX}, and ϕ_X will converge to the values represented above only when the time scale is taken into account. A consistent estimation procedure when $\{t_n\}$ is Poisson (and the autocovariance function R_x is continuous enough) would be to divide the sample into lag intervals, and calculate sample autocovariances in these intervals. As the sample size increased, the number of lag intervals

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[3] This exposition is identical to Sims [5], but is more general. Periodic sampling is a particular stationary point process.

could be increased more slowly and the width of the lag intervals decreased.

A sampling scheme of interest to economists is a periodic one, with period larger than the distance between sample points. An example of this is quarterly observations, with the operational time distance between every fourth observation a constant. Observations on the inventories of a commodity whose retail sales are seasonal might be an example of this. Operational time could be considered to move slowly during quarters of low turnover, and faster during the quarter of high turnover.[4]

For this sampling scheme, the relationship between B and b is identical to that given by Sims [5], namely:

$$B(t) = b * R_x * R_X^{-1}(t) \tag{6}$$

Sims' argument must be modified some; for instance, it is clear only that the filter $R_x * R_X^{-1}(t)$ has the value 1 at $t = 0$ and 0 at $t = 4, 8, 12, \ldots$. One would expect something like Sims' Proposition G[5] to still be valid; that is if b is "smooth" and x has its power concentrated in the low frequencies, then estimating B correctly gives values of B near b.

Again it must be mentioned that the above paragraph applies only when operational time intervals are known, and are used in the estimation process. It is interesting to ask what happens if operational time considerations are ignored, and the quarterly observations are mistakenly treated as being equally spaced in time. Some analysis of this problem for univariate time series autocovariance analysis is presented in Clark [4]. The central analytic result presented there was that the incorrectly estimated autocovariance would converge in probability to a weighted average of the true autocovariances. That is, if

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[4]If the lead time for inventory replacement did not follow the same pattern, this scheme would not be valid.

[5]Sims [5], page 552.

$F_n(t)$ is the distribution function of the operational time to the n^{th} observation, given that there is an observation at $t = 0$, then[6]

$$\operatorname*{plim}_{T \to \infty} \left(\tilde{R}_x^T(n) \equiv \frac{1}{T-n} \sum_{S=1}^{T-n} X(s)\, X(s+n)\right) = \int R_x(t)\, dF_n(t) . \qquad (7)$$

$F_n(t)$ is pictured in Figure 1 for quarterly operational time intervals .1, .2, .3, .4.

Least squares estimates $\tilde{B}(s)$ would converge to an average: $1/4[B_1(s) + B_2(s) + B_3(s) + B_4(s)]$ where $B_i(s)$ is calculated by using only the observations on $y(t)$ at the end of the ith quarter. Since there are 4 possible patterns of observation, occurring with equal frequency, it is intuitively appealing that the estimated lag structure is an average of these.

Again, in this case, if b is "smooth," $X(t)$ has its power at low frequencies, and the variation of operational time between quarters is small compared to the total length of $b(s)$, then the estimated $B(s)$ should be close to $b(s)$. The averaging here should further complicate the problem of estimating the detailed structure of $b(s)$ when it is not smooth.

B. Time-Aggregated Sampling

Section A discussed the instantaneous measurement of a stochastic economic variable. This is the measurement of a point in time, rather than a flow during an interval of time. The money supply of U.S. population on December 29, 1973 would be an example of such an observation. Monthly retail sales or quarterly housing starts are most naturally considered as a flow accumulated over a period

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[6] Clark [4], page 3. This formula must be modified if x does not have zero mean.

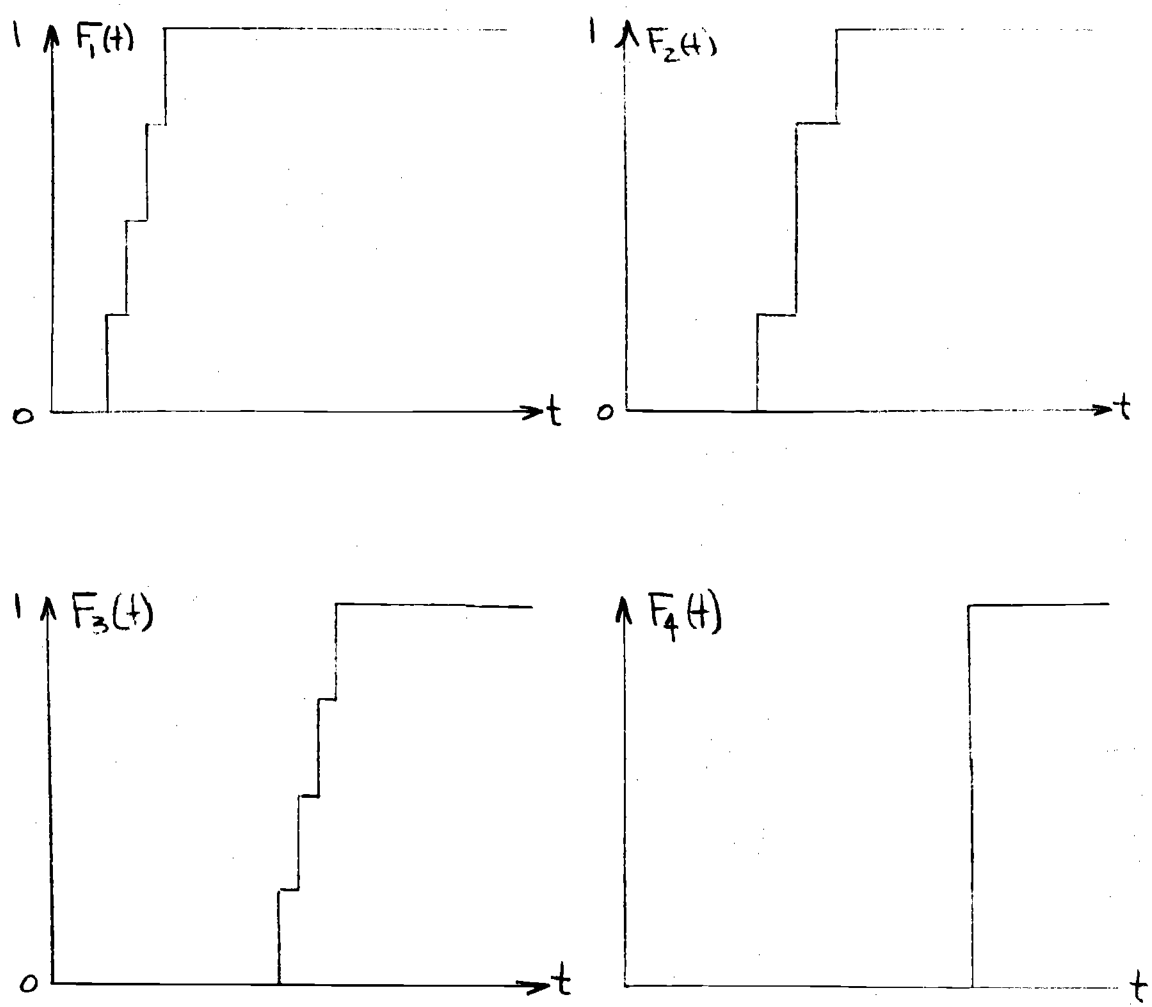


Figure 1. Distribution of operational time for quarterly data with different operational time in each quarter

of time.[7] If the operational time for the particular system being studied varies between observations, and the flow variable is positive, a seasonally varying pattern of observations should be produced. Operational time, then, can be an alternative method for dealing with seasonal variation. In the context of distributed lag estimation, operational time will be most useful when the whole system evolves more slowly at some times than at others. Then the different distributed lags explaining the dependent variable at different points in the seasonal cycle will be "squashed in" or "stretched out" versions of the distributed lag for other seasons.

Mathematically, one operational time hypothesis is as follows:

Let $y(t) = b * x(t) + u(t)$; $E(x(t)\, u(s)) = 0$ all t,s.

Observations $Y(1), Y(2), Y(3), \ldots\ldots.$

$X(1), X(2), X(3), \ldots\ldots.$

are generated:

$$X(1) = \int_{\tau_o}^{\tau_1} x(u)du; \quad X(j) = \int_{\tau_{j-1}}^{\tau_j} x(u)du$$

$$Y(1) = \int_{\tau_o}^{\tau_1} y(u)dn; \quad Y(j) = \int_{\tau_{j-1}}^{\tau_j} y(u)du\ ,$$

[7] As Sims has pointed out to me, his article [5] deals to some extent with processes that are "unit-averaged" and then "point-sampled." Note that in when sampling is at equal intervals, this is equivalent to time-aggregated sampling. In a system where operational time intervals vary, the original series must be considered as filtered through a time-dependent averaging process before "point-sampling" and "time-aggregated sampling" are equivalent.

τ_1, τ_2, τ_3,... are the operational times ending the observation (sum) of y and x. Thus X(2), the number of housing starts in February, is the sum of all the housing starts during the operational time which elapsed during February, namely $\tau_2 - \tau_1$.

One possible alternative is a model in which the distributed lag changes with the season:

$$Y(t) = \sum b(s,t)\, X(t-s) + u(t)$$

$$\text{and} \quad Y(j) = \int_{j-1}^{j} y(u)du; \; X(j) = \int_{j-1}^{j} X(u)du$$

In discrete time, one must estimate a distributed lag for each different season. This may be difficult due to data limitations.

Another alternative is that the underlying model relates seasonally adjusted flows:

$$\tilde{Y}(t) = \sum b(s)\tilde{X}(t-s) + \tilde{U}(t)$$

$$\tilde{Y}(t) = Y(t) \cdot a(t) \; , \; \tilde{X}(t) = X(t) \cdot c(t) \; .$$

$$Y(j) = \int_{j-1}^{j} y(u)du \;\; X(j) = \int_{j-1}^{j} x(u)du \; .$$

a(t) and c(t) are seasonal flow adjustment factors. This model implies that the Y and X series should be seasonally adjusted, and then the distributed lag estimated. Predictions of Y would be made by predicting seasonally adjusted Y, and then dividing by the appropriate constant to seasonally unadjust it.

C. Applications of Various Techniques to Housing Starts and Completions Data

As a practical test of the use of the operational time concept, the start-to-completion lag for privately owned single-family dwellings in the United States was estimated using monthly data from January 1968 (the beginning date for the completions series) to December 1972.[8] Six additional months of data (Jan.-June 1973) were used as a specification test for the models. Mean-square prediction error was calculated with this data, which was not used in estimation. As well as the possible seasonal variation models mentioned above, a model with no allowance for seasonal variation was estimated. In all models, since lag length was long enough to make unrestricted least squares estimates impractical, the lag distributions were restricted to lie in a family of connected broken line distributions. This restriction still yields linear regressions, but is closer to the present author's a priori notions than Lagrange polynomials. Since it takes some time to complete a house, the distributions were also restricted to be zero at both the start and end. A possible distribution of the start-to-completion lag is given in Figure 2.

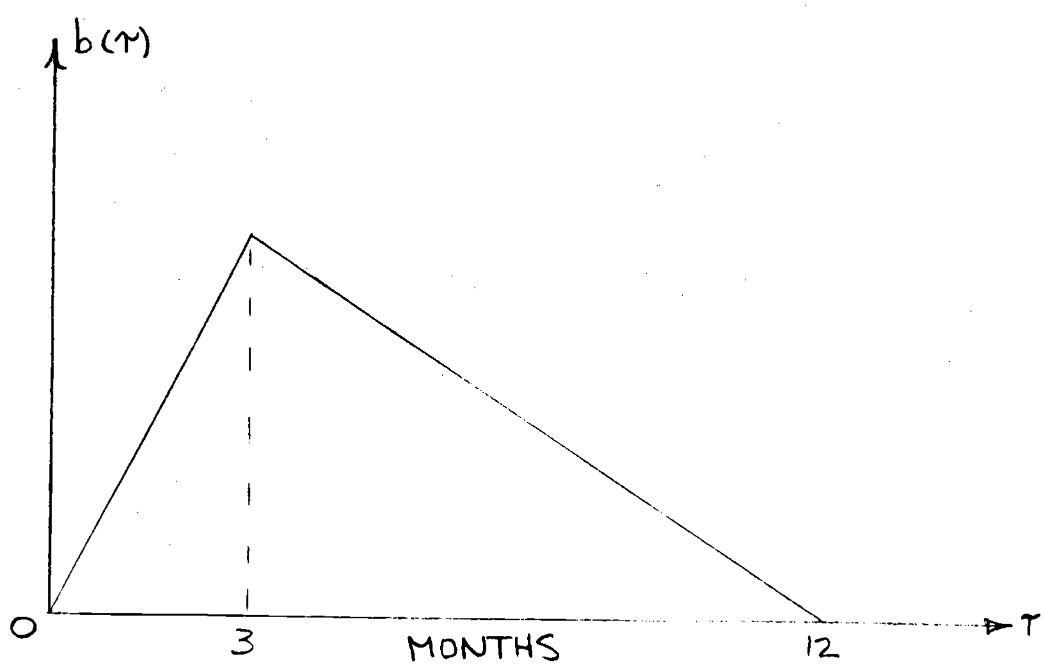


Figure 2

"Broken-line" Lag Distribution with One Corner

[8] Sources for this data were [9] and [10].

The more corners that are allowed, the more independent variables appear in the regressions. A further restriction was imposed: $b(\tau) \geq 0$.

C.1. No seasonal adjustment

Ignoring the seasonal variation in both V(t) and X(t) leads to results which are better than one might have expected in explaining a seasonal dependent variable. This is because the seasonality in starts explains some seasonality in completions. The results are poor compared with those obtained with more complicated techniques, as we will see in later sections. The model was as follows:

$$Y(t) = \sum_{s=0}^{12} b(s)\, X(t-s) + U(t)$$

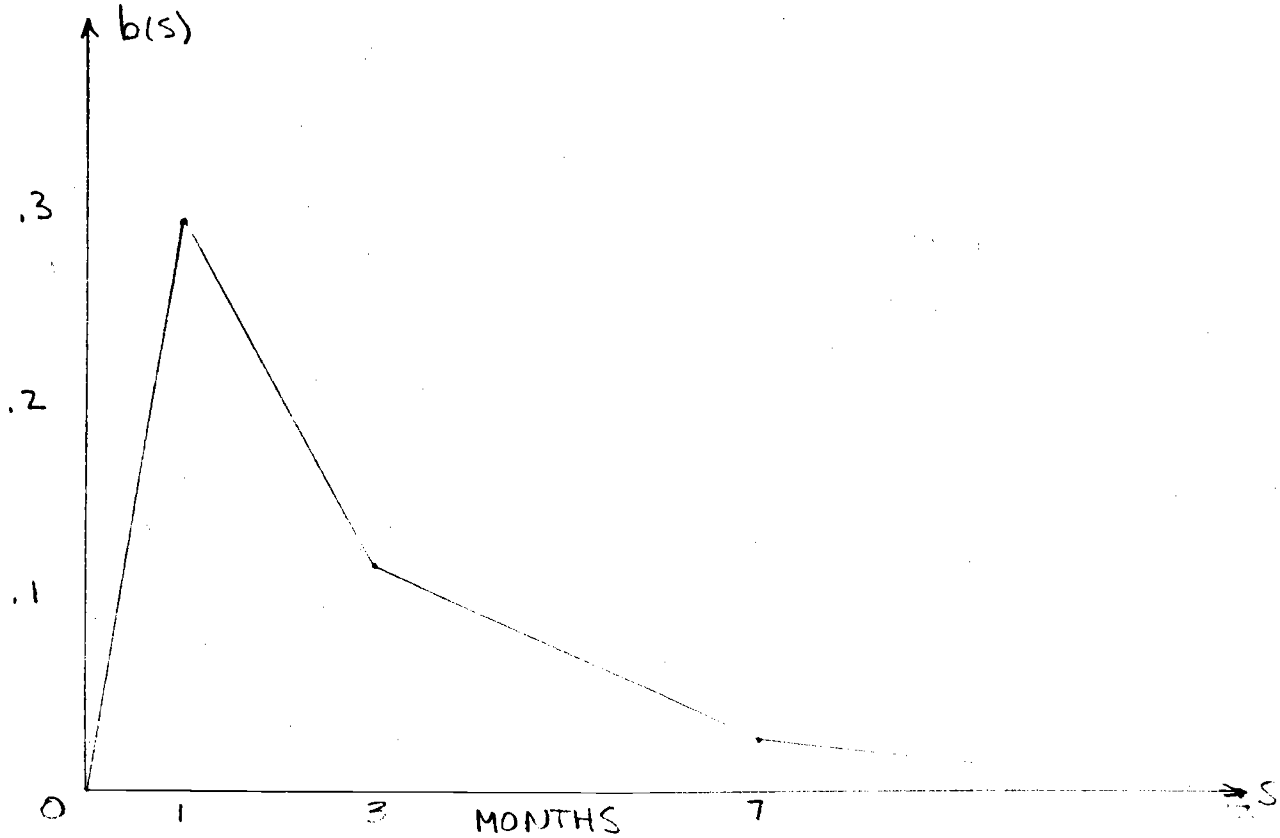
Y(t) = single family privately owned houses completed in month t.

X(t) = single family privately owned houses started in month t.

The lag distribution in Figure 3 was found using a 3-cornered lag distribution and ordinary least squares after the position of the corners was shifted[9] to obtain the best fit.

The value of the prediction sum of squares may be compared to that for a naive estimator, $\hat{Y}(t) = Y(t-1)$. The prediction sum of squares for this estimator is 411.9.

[9] The statistical properties of such a procedure are not obvious.



R^2 = .9936
Durbin-Watson (D-W) = 1.56
Prediction Sum of Squares (Jan.-June 1973) = 155.6

Figure 3

Start-to-completion Lag for Privately Owned Single Family Houses, 1968-72. No Seasonal Adjustment.

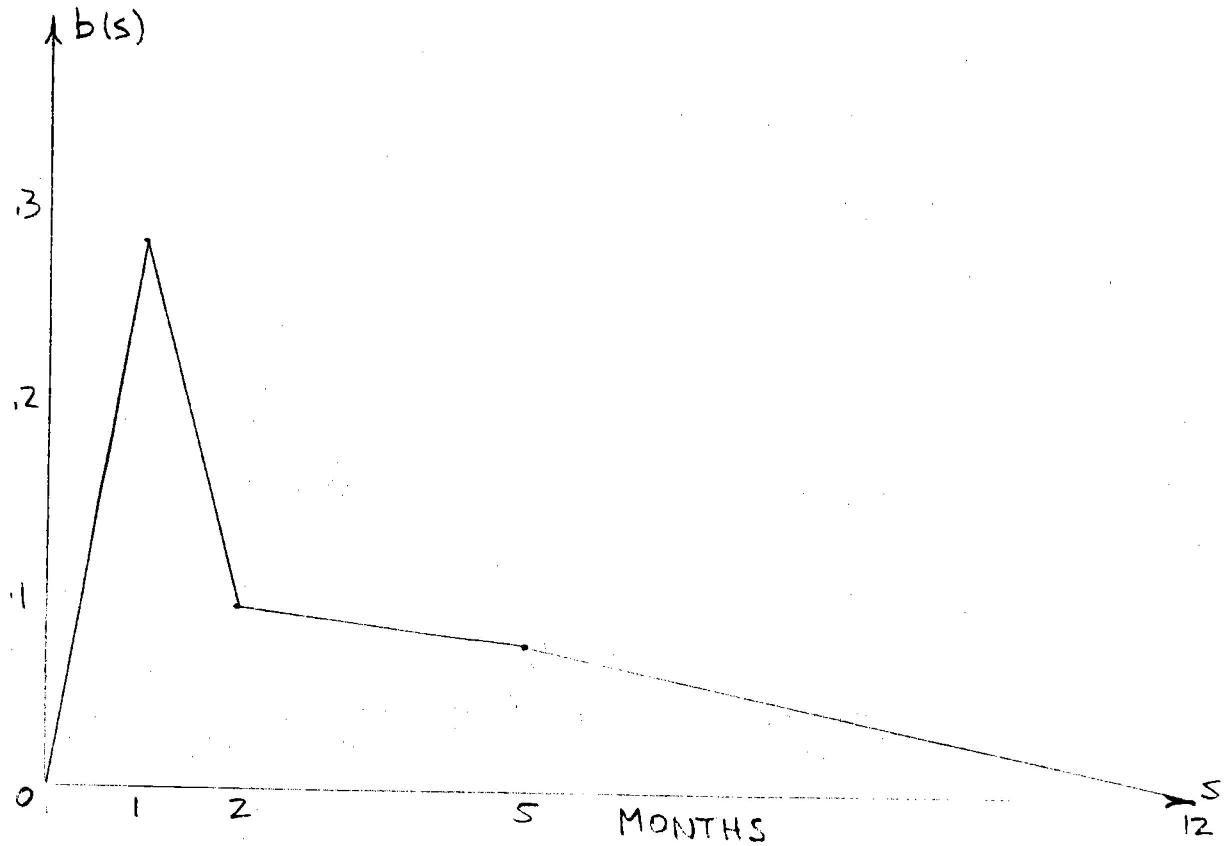
C.2. Both Series Seasonally Adjusted

Let $\tilde{Y}(t)$ and $\tilde{X}(t)$ be seasonally adjusted monthly values for the housing as in C.1. The seasonal adjustments for housing starts were calculated using 1959-67 housing start data. Since completions data for earlier than 1968 is unavailable, seasonal adjustment coefficients are calculated from the 1968-72 data.

$$\tilde{Y}(t) = \sum_{s=o}^{12} b(s)\, \tilde{X}(t-s) + u(t) \tag{8}$$

The Durbin-Watson statistic indicates that the seasonal adjustment of both series has not been at all beneficial. It seems like the seasonal flow model (9) which relates relative rates of starts and completions is less well specified than one that relates actual starts and completions. The increased sum of squared prediction errors (our specification test) verifies this.

Also note that this model does not have the clear interpretation that C.1. does. In C.1., b(s) may be interpreted as the density function for a housing completion in month s, given that there is a housing start in month 0. After seasonal adjustment, such an interpretation is not valid until b(s) has been multiplied by the relevant ratio of seasonal weights. Thus an estimation technique which imposes "smoothness" on b(s) does not impose "smoothness" on the conditional probability of a completion given a start. The poor results may also be caused by the seasonal filter for Y(t) which was constructed with the data that were used in regression.



$R^2 = .9946$
D-W = .6433
Prediction Sum of Squares = 191.1

Figure 4

Start-to-Completion Lag for Single-Family Privately Owned Houses, 1968-1972, Seasonally Adjusted Data

C.3. Time-Varying Lags

$$Y(t) = \sum_{s=o}^{12} b(s,t)\ X(t) + u(t) \tag{10}$$

$$b(s,t) = B(s,t + 12),\ \text{all } t.$$

The above scheme allows separate lag structures for different months. The sensibility of such a model is clear; if a house was completed in October, the probability that it was started three months earlier might be different than the same probability for a house completed in March. Note that any "smoothness" restriction used in estimating the lag of completions on starts is a restriction on the proportion of starts in previous months that will be completed in a given month. Since the lag distribution varies from month to month, this smoothness may not imply smoothness in the distribution of completions, conditional on the number of starts in a given month. If b(s) is stationary, smoothness in one sense implies smoothness in the other.

It seems like this latter type of smoothness would be more reasonable in this case. Such smoothness could be incorporated in an estimation procedure which estimated all 12 lags at once, with restrictions on the coefficients. A further reasonable restriction might be that the sum of weights to completion for starts in any given month would be constant over all months. Thus the likelihood of an unfinished house would be the same for all months. Such restricted regressions were not attempted.

The results for (10) are presented in Table 1. Since there are only 4 data points for each regression, the estimated lag is restricted to start and end at value zero, rising linearly to a single peak. Estimation can be accomplished with univariate regressions. The month of the peak and the month of the end point were chosen by minimizing squared residuals. An à priori restriction that the end point should be between six and twelve months was used.

Table 1

Separate Monthly Completion/Start Lags

Month	Months to Peak	Months to End of Lag	Height of Peak at Lag	Sum of Lag Weights
January	8	9	.1593	.7169
February	4	10	.1558	.7800
March	5	7	.2538	.8883
April	1	6	.3365	1.0095
May	1	12	.1647	.9882
June	9	10	.2211	1.1055
July	1	12	.1586	.9516
August	1	12	.1717	1.0302
September	2	6	.2939	.8817
October	1	8	.2423	.8481
November	1	8	.2161	.7567
December	1	10	.1918	.9590

DW = .43

R^2 = .9967

Prediction Sum of Squares = 81.40

Although some evidence of the shortening of the lag appears in the early months of the year, the most evident feature is the unreliability of using four data points even in univariate regressions. Many more observations are necessary to estimate the variable lag well. Despite the erratic jumps in the lag structure, this formulation does well on our specification test, cutting the sum of squared prediction errors for January-June completions by 50%, and the sum of squared residuals by 50%, also. The Durbin-Watson is even worse for this model, indicating some unexplainable autocovariance in the errors. Clearly the lag does vary from month to month.

C.4. Tinsley Variable Lag Distribution

Tinsley [7] has proposed a method for estimation of a variable distributed lag, and it is reasonable to ask how such a lag scheme would work in this present situation. Here we are placing restrictions on the lag coefficient estimates in C3; if these restrictions are true (or approximately so), improved estimates and predictions should be the result.

Tinsley's method consists of allowing the lag distribution to vary linearly with another exogenous variable, say Z(t), in such a way that the timing of the reaction of Y(t) to changes in X(t) varies, but the overall reaction does not. The lag distribution changes while the sum of the lag weights remains constant. The distributed lag model for housing completions and starts becomes:

$$Y(t) = \sum_{s=o}^{12} b(s,t)\ X(t-s) + U(t)$$

$$b(s,t) = b(s) + c(s)\ Z(t-s)$$

$$\sum_{s=o}^{12} c(s) = 0$$

$Z(t)$ for this model might reasonably be temperature, or $Z(t) = \cos((\pi t + \phi)/6)$, where ϕ is chosen to make $Z(t)$ smallest in January and largest in July. Other values ($\phi = 1,2,..,6$) were also tried, primarily because it was found that there was a "delayed reaction" to temperatures in the operational time estimations in C.5. Again, "broken line" distributions were fit for both $b(s)$ and $c(s)$, with both constrained to zero at $s=0$ and $s=12$. $b(s)$ was allowed three corners, with $c(s)$ allowed two, which leaves four coefficients to be estimated in a linear regression when the constraint on $c(s)$ is used. Table 2 gives the estimates of $b(s,t)$ after some experimentation with phase shifts and corner placement.

Notice that the lag distribution is distinctly bimodal, possibly indicating that tract and prefabricated houses are put up quickly, while "custom built" homes are completed over a longer, more uncertain period. This result could also mean that some builders announce a "start" only when a house is complete or almost complete.

If the distribution is really bimodal, the low Durbin-Watson statistic in C.3 can be at least partially explained; our specification did not allow for anything but unimodal distributions. It is also worthwhile to note that the results here were best when the "temperature cosine" was allowed to have its minimum in February rather than January. An "ad hoc" explanation for this will be given in the next section.

C.5. A Completion/Start Lag Incorporating Operational Time

An alternative set of restrictions on the separate monthly lags in C.3 is given by the "operational time" hypothesis discussed in sections A and B. The speed of evolution for $y(t)$, $x(t)$, and $u(t)$ processes within months varies from month to month. This will allow the completion/start lag to shrink in the

Table 2

Tinsley Variable Completion/Start Lags

	lag index (s)											
	1	2	3	4	5	6	7	8	9	10	11	12
January	.301	.172	.029	.033	.048	.065	.057	.048	.038	.026	.013	0
February	.290	.164	.028	.037	.049	.064	.054	.045	.036	.025	.013	0
March	.286	.158	.027	.043	.053	.065	.053	.043	.034	.024	.013	0
April	.290	.156	.026	.048	.057	.068	.054	.043	.033	.023	.012	0
May	.301	.158	.025	.052	.062	.072	.057	.043	.032	.022	.011	0
June	.315	.164	.026	.053	.065	.076	.060	.045	.033	.021	.011	0
July	.330	.172	.027	.052	.067	.079	.063	.048	.034	.022	.011	0
August	.341	.180	.028	.048	.065	.080	.066	.051	.036	.023	.011	0
September	.345	.185	.029	.043	.062	.079	.067	.053	.038	.024	.011	0
October	.341	.188	.030	.037	.057	.076	.066	.053	.040	.025	.012	0
November	.330	.185	.031	.033	.053	.072	.063	.053	.040	.026	.013	0
December	.315	.180	.030	.032	.049	.068	.060	.051	.040	.027	.013	0

R-squared = .9937

Durbin-Watson = 1.74

Prediction Sum of Squares = 75.2

summer months when construction costs are lower due to good weather, and lengthen in the winter when costs are higher.[10]

To use the operational time hypothesis, two problems must be dealt with. First, an operational time scale must be constructed. This is again the "temperature cosine"; operational time τ is given by

$$\tau = t(t + \alpha \cos ((\pi t + \phi)/6).$$

The coefficient α was found by nonlinear least squares, while ϕ was expected to be chosen so that January gave -1 for the value of the cosine. Some experimenting showed that a February minimum gave better results. This is possibly due to a lagged response of contractors to true temperature conditions. It is also possible that true "operational time" might include a measure of precipitation or snow accumulation, both of which would move the minimum operational time toward February.

The second problem is the interpolation of continuous flow values for starts and completions when monthly accumulations are given as data. This problem was solved in a very simple (and almost surely suboptimal) way by assuming constant flows over each month. The flow value can then be calculated by dividing the accumulation of starts and completions during each month by the operational time difference for that month. Thus the accumulation of these flows over the operational time during each month would give the observed values for monthly starts and completions. Thus, the model is as follows:

$$y(t) = \int_0^\infty b(s)\ x\ (t - s) + u(t)$$

$y(t)$ = flow of completions at operational time t.

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[10] This is a statement about costs for a given rate of output. Presumably more workers and/or overtime would be used in the summer so that costs of construction would be equalized over the seasons.

$x(t)$ = flow of starts at operational time t.

$$X(i) = \int_{\tau_{i-1}}^{\tau_i} x(u)\, du, \quad Y(i) = \int_{\tau_{i-1}}^{\tau_i} y(u)\, du$$

$X(i)$, $Y(i)$ = monthly accumulations of starts and completions in month i.

τ_i = operational time at the end of month i

$$\hat{y}(t) = Y(i)/(\tau_i - \tau_{i-1}),$$

$$\hat{x}(t) = X(i)/(\tau_i - \tau_{i-1}), \text{ for } \tau_{i-1} \leq t < \tau_i$$

The interpolated flow values $\hat{x}(t)$ and $\hat{y}(t)$ could be constructed more efficiently by using their estimated autocovariances,[11] but the present simple method is significantly easier. b(s) is estimated by a linear regression of $\hat{y}(t)$ on $\hat{x}(t)$, where 100 observations (equally spaced in operational time) are given for each calendar year. The results of this estimation are given in Figure 5 and Table 3.

The results again are bimodal, even more strongly than those for the Tinsley lags in C.4. The value for the prediction sum of squares is particularly encouraging; the standard deviation of forecast error has been reduced about 40% below its values in C.3. (separate lags) and C.4. (Tinsley lags).

If a long time series for monthly starts and completions were available, an adequate test of the operational time hypothesis against the alternative of separate lags could be made; operational time would probably be rejected. As we have seen, however, operational time is a good approximate restriction to use with the present limited set of data.

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[11] See Whittle [8] for a discussion of these techniques.

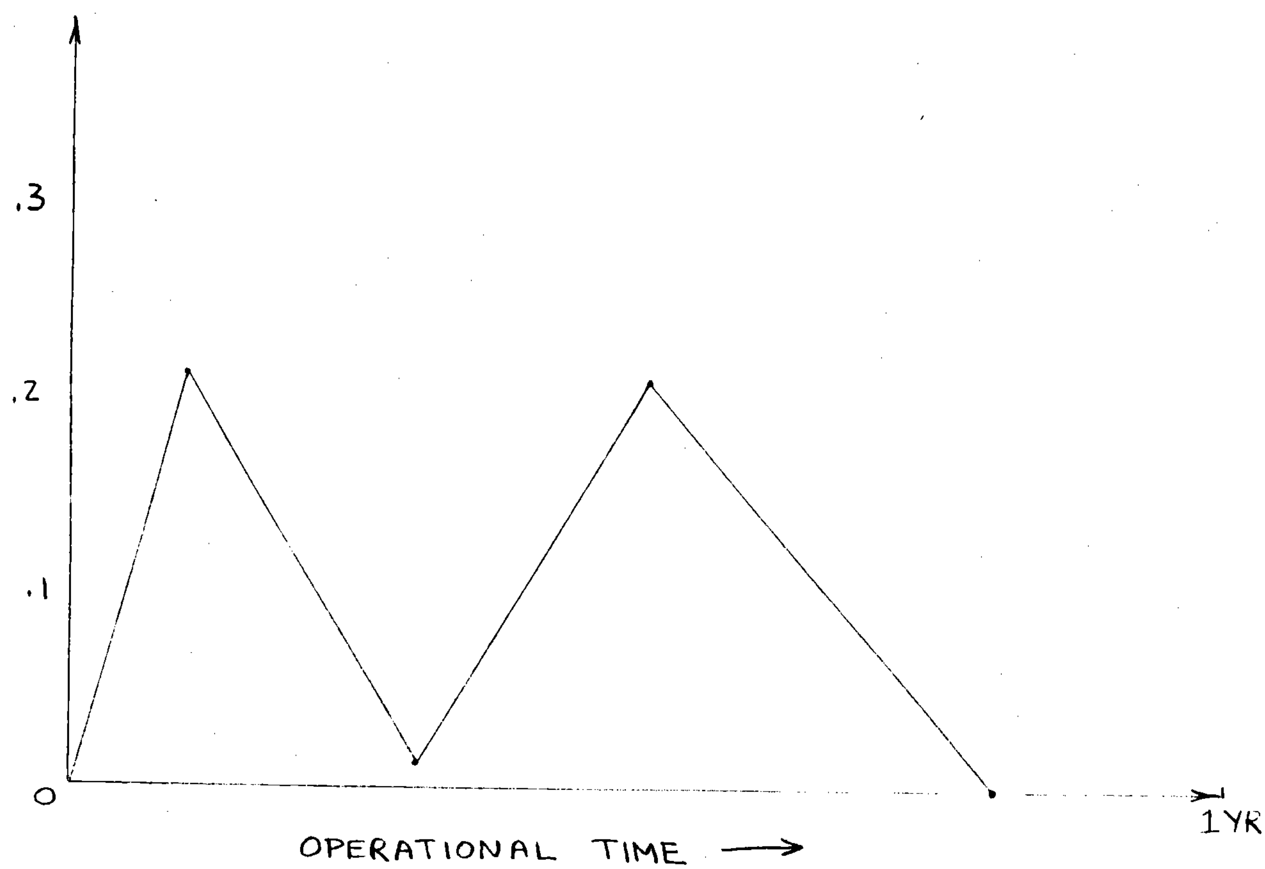

Figure 5

Estimated Start-to-Completion Lag in Operational Time

Table 3

Completion/Start Lag Using Operational Time[12]

Month (t)	lag index (s) 1	2	3	4	5	6	7	8	9	10	11	11
January	.060	.131	.091	.048	.055	.112	.151	.112	.060	.009	0	0
February	.059	.138	.098	.049	.064	.122	.140	.094	.045	.005	0	0
March	.063	.153	.107	.051	.080	.138	.130	.081	.036	.003	0	0
April	.071	.170	.115	.056	.102	.156	.125	.073	.031	.002	0	0
May	.080	.184	.118	.064	.124	.171	.126	.073	.032	.003	0	0
June	.089	.190	.116	.069	.136	.183	.134	.080	.038	.005	0	0
July	.095	.187	.110	.068	.135	.186	.148	.094	.049	.009	0	0
August	.095	.177	.103	.060	.119	.177	.164	.113	.063	.015	0	0
September	.090	.164	.097	.051	.096	.156	.179	.132	.078	.020	0	0
October	.081	.150	.092	.045	.075	.134	.181	.144	.087	.022	0	0
November	.072	.139	.089	.045	.060	.118	.174	.143	.086	.020	0	0
December	.064	.132	.088	.046	.053	.111	.163	.130	.075	.015	0	0

R^2 = .9938

Durbin-Watson = 1.56

Prediction Sum of Squares = 26.3

[12] Since these lags are estimated for "continuous time" (actually, increments smaller than months) the numbers represent the relationship between Y(t) and X(t) implied by the estimated lag structure at the end of each month only.

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