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MEASURING CAPITAL

W. Erwin Diewert

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ABSTRACT

The paper revisits Harper, Berndt and Wood (1989) and calculates Canadian reproducible capital services aggregates under alternative assumptions about the form of depreciation, the opportunity cost of capital and the treatment of capital gains. Five different models of depreciation are considered: (1) one hoss shay; (2) straight line depreciation; (3) declining balance or geometric depreciation; (4) linearly declining efficiency profiles and (5) linearly increasing maintenance profiles. The latter form of depreciation does not seem to have been considered in the literature before. This model assumes that there is a known time profile of maintenance expenditures that can be associated with each asset and the optimal time of retirement of the asset as well as the profile of used asset prices becomes endogenous under this specification. It turns out if the maintenance profile increases linearly, then the linearly declining efficiency profile model emerges; see (4) above. We consider 3 alternative assumptions about the interest rate and the treatment of capital gains so that we evaluate 15 models in all and compare their differences. Following Hill (2000), we also consider the differences between cross section and time series depreciation and anticipated time series depreciation (which adds anticipated obsolescence of the asset to normal cross section depreciation of the asset). Finally, we follow the suggestion made by Diewert and Lawrence (2000) that a superlative index number formula be used to aggregate up vintages of capital rather than the usual assumption of linear aggregation, which implicitly assumes that the capital services yielded by each vintage of a homogeneous type of capital are perfectly substitutable.

W. Erwin Diewert
Department of Economics
University of British Columbia
Vancouver, Canada, V6N 1Z1
and NBER
diewert@econ.ubc.ca

MEASURING CAPITAL by W. Erwin Diewert April 18, 2002

Email: diewert@econ.ubc.ca

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1. Introduction[1]

"Capital (I am not the first to discover) is a very large subject, with many aspects; wherever one starts, it is hard to bring more than a few of them into view. It is just as if one were making pictures of a building; though it is the same building, it looks quite different from different angles." John Hicks (1973; v).

"Perhaps a more realistic motive for reading earlier writers is not to rediscover forgotten truths, but to gain a perspective of how present day ideas have evolved and, perhaps, by reading the original statements of important ideas, to see them more vividly and understand them more clearly." Geoffrey Whittington (1980; 240).

In this paper, we discuss some of the problems involved in constructing price and quantity series for both capital stocks and the associated flows of services when there is general (and specific) price change in the economy.

In section 2, we discuss some of the problems that occur when an economy is experiencing very high inflation. Under these conditions, it will be necessary for the national price statistician to shorten the accounting period (or give up price measurement altogether).

[1] The author is indebted to Kevin Fox, Peter Hill, Ulrich Kohli, Alice Nakamura and Paul Schreyer for helpful comments.

In section 3, we present the basic equations relating stocks and flows of capital assuming that data on the prices of vintages of a homogeneous capital good are available. This framework is not applicable under all circumstances[2] but it is a framework that will allow us to disentangle the effects of general price change, asset specific price change and depreciation.

Section 4 continues the theoretical framework that was introduced in section 3. We show how information on vintage asset prices, vintage rental prices and vintage depreciation rates are all equivalent under certain assumptions; i.e., knowledge of any one of these three sequences or profiles is sufficient to determine the other two.

Section 5 discusses alternative sets of assumptions on nominal interest rates and anticipated asset price changes. We specify three different sets of assumptions that we will use in our empirical illustration of the suggested methods.

Section 6 is a section that is not used in the remaining sections but it discusses the significance of our assumptions made in the previous section and relates them to controversies in national income accounting. In particular, we discuss whether anticipated asset price decline should be an element of depreciation as understood by national income accountants.

Section 7 discusses the problems involved in aggregating over vintages of capital, both in forming capital stocks and capital services. Instead of the usual perpetual inventory method for aggregating over vintages, which assumes perfectly substitutable vintages of the same stock, we suggest the use of a superlative index number formula to do the aggregation.

Sections 8 to 11 show how the general algebra presented in sections 3 and 4 can be adapted to deal with four specific models of depreciation. The four models considered are the one hoss shay model, the straight line depreciation model, the geometric model of depreciation and the linear efficiency decline model. In each of these sections, we illustrate the method by computing the corresponding stocks and flows using Canadian data on two asset classes.

Section 12 considers a fifth type of depreciation model, one that is based on the assumption that each vintage of the asset has a specific maintenance and operating cost requirement associated with it. We show that this type of model can lead to the linear efficiency decline model studied in section 11. However, the main use of the analysis presented in section 12 is to suggest a reason why accelerated depreciation assumptions are quite reasonable and likely to occur empirically.

Section 13 presents a graphical summary of the 12 empirical models considered in the Annex.

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[2] Most notably, our framework cannot deal with unique or one of a kind assets, which by definition, do not have vintages.

Section 14 concludes.

A data appendix lists the Canadian data we used in sections 8-11.

2. Inflation and the Measurement of Economic Activity

Our goal in this paper is twofold: (1) to measure the price and quantity of the *stock* of reproducible capital held by a production unit (an establishment, a firm, an industry or an entire economy) at a *point* in time and (2) to measure the price and quantity of the *flow* of reproducible capital services utilized by a production unit over a *period* of time. In particular, we want to extend the procedures for measuring capital stocks and flows to cover situations where there is general price level change or *inflation*. In this section, we shall review some of the general measurement problems that arise when inflation is high.

When capital flows are measured, the normal period of time is either a year or a quarter. Under conditions of high inflation, the aggregation of homogeneous commodity flows within a quarter or a year is complicated by the fact that the within period transactions are valued at very different prices. The recent national income accounting literature explains the problem as follows:

"Conventional index number theory is mostly concerned with comparisons between *points* of time whereas, in national accounts, price and quantity comparisons have to be made between discrete *periods* of time. Significant changes in price and quantity flows may occur not only between different periods but also within a single accounting period, especially one as long as a year. Indeed, the central problem of accounting under high inflation is that prices are much higher at the end of the accounting period than at the beginning." Peter Hill (1996; 11).

"The underlying problem is not a traditional index number problem. It stems from the use of current value data as inputs into the calculation of indirect price or quantity measures under high inflation. Current accounts permit identical quantities of the same homogeneous product to be valued at very different prices during the course of the same year. Implicitly, quantities sold at higher prices later in the year are treated as if they were superior qualities when they are not." Peter Hill (1996; 12).

"Under high inflation, the monetary value of flows of goods and services at different points of time within the same accounting period are not commensurate with each other because the unit of currency used as the *numeraire* is not stable. Adding together different quantities of the same good valued at different prices is equivalent, from a scientific point of view, to using different units of measurement for different sets of observations on the same variable. In the case of physical data, however, it is rather more obvious that adding quantities measured in grams to quantities measured in ounces is a futile procedure." Peter Hill (1996; 32).

"Before the preparation of the 1993 SNA, issues connected with high or significant inflation had not been dealt with at all in international recommendations concerning national accounts. Uneasiness especially with the recording of nominal interest had been often expressed, for instance in Europe and North America at the time of two digit inflation and above all in countries, like in Latin America, experiencing high or hyper inflation. In relation with the latter situations, uneasiness extended to the whole set of accounts, because, due to the significant rate of inflation within each year, annual accounts in current values could no longer be deemed homogeneous as regards the level of prices in each year. They combine intra-annual flows that are valued at very different prices and are not, strictly speaking, additive. The effect of the intra-annual change in the general price level can be neglected for the sake of simplicity only when the rate of inflation is low. When it is high, the meaning of annual accounts in current values becomes fuzzy." André Vanoli (1998).

"When inflation is high, the aggregation of flows from different periods becomes very much a case of 'adding apples and bananas'— the flows at the end of the period will carry a much greater weight than the flows at the beginning of the period, so that the change on average will reflect development at the end of the period disproportionately. Annual national accounts at current prices become virtually meaningless and computation of national accounts at constant prices becomes very problematic." Ezra Hadar and Soli Peleg (1998; 2).

Of course, concern over the effects of general price level change has a much longer history in the general cost accounting literature; see Baxter (1984), Tweedie and Whittington (1984) and Whittington (1992) for example.[3]

We now discuss in more detail the accounting problems caused by high inflation that are referred to in the above quotations. The basic problem is this: all discrete time economic theories and most of index number theory assumes that all of the transactions of a production unit in a homogeneous commodity within the accounting period can be represented by a *single price* and a *single quantity*. It is natural to let the single quantity be the *sum* of the quantities sold (in the case of an output) or the *sum* of the quantities purchased (in the case of an input). But then, if we want the single price times the single quantity to equal the *value* of transactions for the commodity in the period, the single price must equal the value of transactions divided by the sum of quantities purchased or sold; i.e., the single price must equal a *unit value*.[4] But when there is substantial inflation within the accounting period, unit values give a much higher weight to transactions that occur near the end of the period compared to transactions that occurred near the beginning; it is as if the end of period transactions are being *artificially quality adjusted* to be more valuable than the beginning of the period transactions.

The obvious solution to this artificial implicit weighting problem is to choose the accounting period to be small enough so that the general inflation within the period is small enough to be ignored. This is precisely the solution suggested by the index number theorist Fisher[5] and the measurement economist Hicks: the length of the accounting period should be the Hicksian "week":

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[3] The inflation accounting literature extends back to Middleditch: "Today's dollar is, then, a totally different unit from the dollar of 1897. As the general price level fluctuates, the dollar is bound to become a unit of different magnitude. To mix these units is like mixing inches and centimeters or measuring a field with a rubber tape-line." Livingston Middleditch (1918; 114-115).

[4] The early index number theorists Walsh (1901; 96) (1921; 88), Fisher (1922; 318) and Davies (1924; 96) all suggested unit values as the prices that should be inserted into a bilateral index number formula. Walsh nicely sums up the case for unit values as follows: "Some nice questions arise as to whether only what is consumed in the country, or only what is produced in it, or both together are to be counted; and also there are difficulties as to the single price quotation that is to be given at each period to each commodity, since this, too, must be an average. Throughout the country during the period a commodity is not sold at one price, nor even at one wholesale price in its principle market. Various quantities of it are sold at different prices, and the full value is obtained by adding all the sums spent (at the same stage in its advance towards the consumer), and the average price is found by dividing the total sum (or the full value) by the total quantities." Correa Moylan Walsh (1921; 88).

[5] "Essentially the same problem enters, however, whenever, as is usually the case, the data for prices and quantities with which we start are averages instead of being the original market quotations. Throughout this book, 'the price' of any commodity or 'the quantity' of it for any one year was assumed given. But what is such a price or quantity? Sometimes it is a single quotation for January 1 or July 1, but usually it is

"I shall define a week as that period of time during which variations in price can be neglected." John R. Hicks (1946; 122).

Thus it seems that there is a simple solution to the problem of constructing meaningful accounting period prices and quantities for homogeneous commodities when there is high inflation: simply shorten the accounting period!

Hill (1996) however notes that there are at least three classes of problems associated with the above solution:

"In order to keep these issues in perspective, it is useful to summarise the problems created by continually shortening the accounting period.

1. The compilation of accounts for shorter time periods requires more information about the times at which various transactions take place. Enquiries may have to be conducted more frequently thereby creating additional costs for the data collectors. More burdens are also placed on the respondents supplying the information. In many cases, they may be unable to supply the necessary information because their own internal records and accounts do not permit them to do so, especially when they traditionally report their accounts for longer time periods, such as a year.
2. As production is a process which can extend over a considerable period of time, its measurement becomes progressively more difficult the shorter the accounting period. The problem is not confined to agriculture or forestry where many production processes take a year or more. The production of large fixed assets such as large ships, bridges, power stations, dams or the like can extend over several years. The output produced over shorter periods of time then has to be measured on the basis of work in progress completed each period. …
3. Because many transactions, especially large transactions, are not completed within the day, there are typically many receivables and payables outstanding at any given moment of time. They assume greater importance in relation to the flows as the accounting period is reduced. This makes it more difficult to reconcile the values of different flows in the accounts, especially if the two parties to the transaction perceive it as taking place at different times from each other and do not record it in the same way required by the system. … Peter Hill (1996; 34-35).

Thus shortening the accounting period leads to increased costs for the statistical agency and the businesses being surveyed. Moreover, firm accounting is geared to years and quarters and it may not be possible for production units to provide complete accounting information for periods shorter than a quarter. As the accounting period becomes shorter, it is less likely that production, shipment, billing and payment for the same commodity will all coincide within the accounting period. Also as the accounting period becomes shorter, work in progress will tend to become ever more important relative to final sales, creating difficult valuation problems.[6] Put another way, more and more inputs will shift from being intermediate inputs (inputs that are used up within the accounting period) to being durable inputs (inputs whose contribution to production extends over more than

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an average of several quotations scattered through the year. The question arises: On what principle should this average be constructed? The *practical* answer is *any* kind of average since, ordinarily, the variations during a year, so far, at least, as prices are concerned, are too little to make any perceptible difference in the result, whatever kind of average is used. Otherwise, there would be ground for subdividing the year into quarters or months until we reach a small enough period to be considered practically a point." Irving Fisher (1922; 318).

[6] There are very few price indexes for work in progress! This is to be expected since there are very few transactions involving partially completed products.

one period). In addition to these difficulties, there are others. For example, as the accounting period becomes shorter, transactions tend to become more erratic and sporadic. Many goods will not be sold in a supermarket in a particular day or week. Normal index number theory breaks down under these conditions: it is difficult to compare a positive amount of a good sold in one period with a zero amount sold in the next period. A related difficulty is that many commodities are produced or demanded on a seasonal basis. If the accounting period is a year, then there are no seasonal commodity difficulties but as we shorten the period from a year, we will run into the problem of seasonal fluctuations in prices and quantities. In many cases, a seasonal commodity will not be available in all seasons and we again run into the problem of comparing positive values with zero values in the periods when the commodity is out of season. Even if the seasonal commodity does not disappear, the application of standard index number theory is not straightforward.[7]

Nevertheless, even in the face of the above difficulties, it seems that the only possible solution to the artificial implicit weighting problem that is generated by high inflation is to shorten the accounting period so that normal index number theory can be applied in order to construct meaningful economic aggregates.[8]

In addition to the above general problems associated with economic measurement of flow variables under conditions of high inflation, there are some additional problems associated with the measurement of capital. These additional problems are associated with the stock and flow aspects of capital. We will conclude this section by explaining these problems.

Given an accounting period of some predetermined length, we can associate with it at least three separate points in time:

- The beginning of the accounting period;
- The middle of the accounting period; and
- The end of the accounting period.

In interpreting the national accounts or the accounts of a business unit, we generally think of all flow variables as being concentrated in the middle of the period. If we follow this convention in the context of high inflation, then we require one (nominal) interest rate to index the value of money or financial capital going from the beginning of the period to the middle of the period and we require another (nominal) interest rate to index the value of money going from the middle of the period to the end of the period. Given these two interest rates, we could construct *centered user costs of capital* for each type of

[7] Hadar and Peleg (1998; 5) comment on the importance of seasonal adjustment procedures in the context of high inflation: "As a by-product of the emphasis on quarterly estimates at constant prices the seasonal adjustment got large attention and many resources were spent to improve the adjustment." Diewert (1996) (1998) (1999) reviews possible approaches to the problems involved in treating seasonal commodities (and suggests solutions) when there is high inflation.

[8] Our discussion in the previous paragraph indicates that this cannot be done if the economy is experiencing a hyperinflation. Thus meaningful economic measurement becomes impossible under very high inflation. This is a hidden cost of inflation that is not discussed very much in the literature on the costs of inflation.

reproducible capital, which would be the appropriate flow variables that would match up with the other flow variables in the production accounts of the production unit. However, in order to reduce the notational complexity of this annex, we do not construct centered user costs in what follows. Instead, for each type of asset, we construct either a *beginning of the period user cost* (which measures the cost of using the asset for the period under consideration from the perspective of the price level prevailing at the beginning of the period) or an *end of the period user cost* (which measures the cost of using the asset for the period under consideration from the perspective of the price level prevailing at the end of the period). Of course, armed with a knowledge of the appropriate half period interest rates, it is easy to convert these "bookend" user costs into centered user costs.

In the following section, we explain the fundamental equations relating stocks and flows of capital.

3. The Fundamental Equations Relating Stocks and Flows of Capital

Before we begin with our algebra, it seems appropriate to explain why accounting for the contribution of capital to production is more difficult than accounting for the contributions of labour or materials. The main problem is that when a reproducible capital input is purchased for use by a production unit at the beginning of an accounting period, we cannot simply charge the entire purchase cost to the period of purchase. Since the benefits of using the capital asset extend over more than one period, the initial purchase cost must be distributed somehow over the useful life of the asset. This is *the fundamental problem of accounting*.[9] Hulten (1990) explains the consequences for accountants of the durability of capital as follows:

"Durability means that a capital good is productive for two or more time periods, and this, in turn, implies that a distinction must be made between the value of using or renting capital in any year and the value of

[9] "The difficulty of imputing expenses to individual sales or even to the gross earnings of the accounting period, the month or year, is an ever present problem for the accountant in the periodic determination of enterprise income. The longer the period for which the income is to be determined, the smaller the relative amount of error. Absolute accuracy can be attained only when the venture is completed and the enterprise terminated." William T Crandell (1935; 388-389).
"Early enterprises and partners working in the main in isolated trading ventures, needed only an irregular determination of profit. But before the business corporation had been very long in operation it was evident that it needed to be treated as a continuing enterprise. For example, calculating dividends by separate voyages was found impractical in the East India Company by 1660. Profit calculation therefore became a matter of periodic estimates in place of the known results of completed ventures." A.C. Littleton (1933; 270).
"The third convention is that of the annual accounting period. It is this convention which is responsible for most of the difficult accounting problems. Without this convention, accounting would be a simple matter of recording completed and fully realized transactions: an act of primitive simplicity." Stephen Gilman (1939; 26).
"*All* the problems of income measurement are the result of our desire to attribute income to arbitrarily determined short periods of time. Everything comes right in the end; but by then it is too late to matter." David Solomons (1961; 378). Note that these authors do not mention the additional complications that are due to the fact that future revenues and costs must be discounted to yield values that are equivalent to present dollars.

owning the capital asset. This distinction would not necessarily lead to a measurement problem if the capital services used in any given year were paid for in that year; that is, if all capital were rented. In this case, transactions in the rental market would fix the price and quantity of capital in each time period, much as data on the price and quantity of labor services are derived from labor market transactions. But, unfortunately, much capital is utilized by its owner and the transfer of capital services between owner and user results in an implicit rent typically not observed by the statistician. Market data are thus inadequate for the task of directly estimating the price and quantity of capital services, and this has led to the development of indirect procedures for inferring the quantity of capital, like the perpetual inventory method, or to the acceptance of flawed measures, like book value." Charles R. Hulten (1990; 120-121).

In a noninflationary environment, the value of an asset at the beginning of an accounting period is equal to the discounted stream of future rental payments that the asset is expected to yield. Thus the *stock value* of the asset is set equal to the discounted future *service flows*[10] that the asset is expected to yield in future periods. Let the price of a new capital input purchased at the beginning of period t be P_0^t. In a noninflationary environment, it can be assumed that the (potentially observable) sequence of (cross sectional) vintage rental prices prevailing at the beginning of period t can be expected to prevail in future periods. Thus there was no need to have a separate notation for future expected rental prices for a new asset as it ages. However, in an inflationary environment, it is necessary to distinguish between the observable rental prices for the asset at different ages or vintages at the beginning of period t and future *expected* rental prices for assets of various vintages.[11] Thus let f_0^t be the (observable) rental price of a new asset at the beginning of period t, let f_1^t be the (observable) rental price of a one period old asset at the beginning of period t, let f_2^t be the (observable) rental price of a 2 period old asset at the beginning of period t, etc. Then *the fundamental equation* relating the *stock value of a new asset* at the beginning of period t, P_0^t, to the sequence *of cross sectional vintage rental prices* prevailing at the beginning of period t, $\{f_n^t : n = 0,1,2,\ldots\}$[12] is:

$$(1)\ P_0^t = f_0^t + [(1+i_1^t)/(1+r_1^t)]\, f_1^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)]\, f_2^t + \ldots$$

In the above equation,[13] $1+i_1^t$ is the *rental price escalation factor* that is *expected* to apply to a one period old asset going from the beginning of period t to the end of period t (or equivalently, to the beginning of period t+1), $(1+i_1^t)(1+i_2^t)$ is *the rental price escalation*

[10] Walras (1954) (first edition published in 1874) was one of the earliest economists to state that capital stocks are demanded because of the future flow of services that they render. Although he was perhaps the first economist to formally derive a user cost formula as we shall see, he did not work out the explicit discounting formula that Böhm-Bawerk (1891; 342) was able to derive.

[11] Note that these future expected rental prices are not generally observable due to the lack of futures markets for these vintage future rentals of the asset.

[12] The sequence of (cross sectional) vintage rental prices $\{f_n^t\}$ is called the *age-efficiency profile* of the asset.

[13] It should be noted that Irving Fisher (1897; 365) seemed to be well aware of the complexities that are imbedded in equation (1): "There is not space here to discuss the theory in greater detail, nor to apply it to economic problems. A full treatment would take account of the various standards in which income is or may be expressed, of the case in which the rates of interest at different dates and for different periods does not remain constant, of the fact that the services of capital which are discounted in its value are only *expected* services, not those which actually materialise, and of the consequent discrepancy between income anticipated and income realised, of the propriety or impropriety of including man himself as a species of income-bearing capital, and so on."

factor that is *expected* to apply to a 2 period old asset going from the beginning of period t to the beginning of period t+2, etc. Thus the i_n^t *are expected vintage rental price inflation rates* that are formed at the beginning of period t. The term $1+r_1^t$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period t+1, the term $(1+r_1^t)(1+r_2^t)$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period t+2, etc. Thus the r_n^t are one period *nominal interest rates* that represent the *term structure of interest rates* at the beginning of period t.[14]

We now generalize equation (1) to relate the *stock value of an n period old asset* at the beginning of period t, P_n^t, to the sequence *of cross sectional vintage rental prices* prevailing at the beginning of period t, $\{f_n^t\}$; thus for n = 0,1,2,…, we assume:

$$(2)\ P_n^t = f_n^t + [(1+i_1^t)/(1+r_1^t)]\, f_{n+1}^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)]\, f_{n+2}^t + \ldots$$

Thus older assets discount fewer terms in the above sum; i.e., as n increases by one, we have one less term on the right hand side of (2). However, note that we are applying the same price escalation factors $(1+i_1^t)$, $(1+i_1^t)(1+i_2^t)$, …, to escalate the cross sectional rental prices prevailing at the beginning of period t, f_1^t, f_2^t,…, and to form estimates of future expected rental prices for each vintage of the capital stock that is in use at the beginning of period t.

The vintage rental prices prevailing at the beginning of period t, f_0^t, f_1^t, … are potentially observable.[15] These cross section rental prices reflect the relative efficiency of the various vintages of the capital good under consideration at the beginning of period t. It is assumed that these rentals are paid (explicitly or implicitly) by the users at the beginning of period t. Note that the sequence of vintage asset stock prices at the beginning of period t, P_0^t, P_1^t, … is not affected by general inflation provided that the general inflation affects the expected asset inflation rates i_n^t and the nominal interest rates r_n^t in a proportional manner. We will return to this point later.

The physical productivity characteristics of a unit of capital of each vintage are determined by the sequence of cross sectional rental prices. Thus a brand new asset is characterized by the vector of current vintage rental prices, f_0^t, f_1^t, f_2^t, … , which are interpreted as "physical" contributions to output that the new asset is expected to yield during the current period t (this is f_0^t), the next period (this is f_1^t), and so on. An asset

[14] Peter Hill has noted a major problem with the use of equation (1) as the starting point of our discussion: namely, *unique assets* will by definition not have vintages and so the cross sectional vintage rental prices f_n^t will not exist for these assets! In this case, the f_n^t should be interpreted as expected future rentals that the unique asset is expected to generate at today's prices. The $(1+i_n^t)$ terms then summarize expectations about the amount of asset specific price change that is expected to take place. This reinterpretation of equation (1) is more fundamental but we chose not to make it our starting point because it does not lead to a workable method for national statisticians to form reproducible estimates of these future rental payments. However, in many situations (e.g., the valuation of a new movie), the statistician will be forced to attempt to implement Hill's (2000) more general model.

[15] This is the main reason that we use the vintage approach to capital measurement rather than the more fundamental discounted future expected rentals approach advocated by Hill.

which is one period old at the start of period t is characterized by the vector f_1^t, f_2^t, …, etc.[16]

We have not explained how the expected rental price inflation rates i_n^t are to be estimated. We shall deal with this problem in section 5 below. However, it should be noted that there is no guarantee that our expectations about the future course of rental prices are correct.

At this point, we make some simplifying assumptions about the expected rental inflation rates i_n^t and the interest rates r_n^t. We assume that these anticipated vintage rental inflation factors at the beginning of each period t are all equal; i.e., we assume:

$$(3)\ i_n^t = i^t\ ; \qquad n = 1,2,\ldots$$

We also assume that the term structure of interest rates at the beginning of each period t is constant; i.e., we assume:

$$(4)\ r_n^t = r^t\ ; \qquad n = 1,2,\ldots$$

However, note that as the period t changes, r^t and i^t can change.

Using assumptions (3) and (4), we can rewrite the system of equations (2), which relate the sequence or profile of *vintage stock prices* at the beginning of period t $\{P_n^t\}$ to the sequence or profile of (cross sectional) *vintage rental prices* at the beginning of period t $\{f_n^t\}$, as follows:

$$\begin{aligned}
(5)\ P_0^t &= f_0^t + [(1+i^t)/(1+r^t)]\, f_1^t + [(1+i^t)/(1+r^t)]^2\, f_2^t + [(1+i^t)/(1+r^t)]^3\, f_3^t + \ldots \\
P_1^t &= f_1^t + [(1+i^t)/(1+r^t)]\, f_2^t + [(1+i^t)/(1+r^t)]^2\, f_3^t + [(1+i^t)/(1+r^t)]^3\, f_4^t + \ldots \\
P_2^t &= f_2^t + [(1+i^t)/(1+r^t)]\, f_3^t + [(1+i^t)/(1+r^t)]^2\, f_4^t + [(1+i^t)/(1+r^t)]^3\, f_5^t + \ldots \\
&\ldots \\
P_n^t &= f_n^t + [(1+i^t)/(1+r^t)]\, f_{n+1}^t + [(1+i^t)/(1+r^t)]^2\, f_{n+2}^t + [(1+i^t)/(1+r^t)]^3\, f_{n+3}^t + \ldots
\end{aligned}$$

On the left hand side of equations (5), we have the sequence of vintage asset prices at the beginning of period t starting with the price of a new asset, P_0^t, moving to the price of an asset that is one period old at the start of period t, P_1^t, then moving to the price of an asset that is 2 periods old at the start of period t, P_2^t, and so on. On the right hand side of equations (5), the first term in each equation is a member of the sequence of vintage rental prices that prevails in the market (if such markets exist) at the beginning of period t. Thus f_0^t is the rent for a new asset, f_1^t is the rent for an asset that is one period old at the beginning of period t, f_2^t is the rent for an asset that is 2 periods old, and so on. This sequence of current market rental prices for the assets of various vintages is then extrapolated out into the future using the anticipated price escalation rates $(1+i^t)$, $(1+i^t)^2$, $(1+i^t)^3$, etc. and then these future expected rentals are discounted back to the beginning of period t using the discount factors $(1+r^t)$, $(1+r^t)^2$, $(1+r^t)^3$, etc. Note that given the period t expected asset inflation rate i^t and the period t nominal discount rate r^t, we can go from

[16] Triplett (1996; 97) used this characterization for capital assets of various vintages.

the (cross sectional) sequence of vintage rental prices $\{f_n^t\}$ to the sequence of vintage asset prices $\{P_n^t\}$ using equations (5). We shall show below how this procedure can be reversed; i.e., we shall show how given the sequence of vintage asset prices, we can construct estimates for the sequence of vintage rental prices.

It seems that Böhm-Bawerk was the first economist to use the above method for relating the future service flows of a durable input to its stock price:

"If the services of the durable good be exhausted in a short space of time, the individual services, provided that they are of the same quality— which, for simplicity's sake, we assume— are, as a rule, equal in value, and the value of the material good itself is obtained by multiplying the value of one service by the number of services of which the good is capable. But in the case of many durable goods, such as ships, machinery, furniture, land, the services rendered extend over long periods, and the result is that the later services cannot be rendered, or at least cannot be rendered in a normal economic way, before a long time has expired. As a consequence, the value of the more distant material services suffers the same fate as the value of future goods. A material service, which, technically, is exactly the same as a service of this year, but which cannot be rendered before next year, is worth a little less than this year's service; another similar service, but obtainable only after two years, is, again, a little less valuable, and so on; the values of the remote services decreasing with the remoteness of the period at which they can be rendered. Say that this year's service is worth 100, then next year's service— assuming a difference of 5 % per annum— is worth in today's valuation only 95.23; the third year's service is worth only 90.70; the fourth year's service, 86.38; the fifth, sixth and seventh year's services, respectively, worth 82.27, 78.35, 74.62 of present money. The value of the durable good in this case is not found by multiplying the value of the current service by the total number of services, but is represented by a sum of services decreasing in value." Eugen von Böhm-Bawerk (1891; 342).

Thus Böhm-Bawerk considered a special case of (5) where all service flows f_n were equal to 100 for n = 0,1,…,6 and equal to 0 thereafter, where the asset inflation rate was expected to be 0 and where the interest rate r was equal to .05 or 5 %.[17] This is a special case of what has come to be known as the *one hoss shay model* and we shall consider it in more detail in section 8 of this below.

Note that equations (5) can be rewritten as follows:[18]

$$
\begin{aligned}
(6)\quad P_0^t &= f_0^t + [(1+i^t)/(1+r^t)]\, P_1^t \quad ; \\
P_1^t &= f_1^t + [(1+i^t)/(1+r^t)]\, P_2^t \quad ; \\
P_2^t &= f_2^t + [(1+i^t)/(1+r^t)]\, P_3^t \quad ; \\
&\ldots \\
P_n^t &= f_n^t + [(1+i^t)/(1+r^t)]\, P_{n+1}^t \ ; \ldots
\end{aligned}
$$

[17] Böhm-Bawerk (1891; 343) went on and constructed the sequence of vintage asset prices using his special case of equations (5).

[18] Christensen and Jorgenson (1969; 302) do this for the geometric depreciation model except that they assume that the rental is paid at the end of the period rather than the beginning. Variants of the system of equations (6) were derived by Christensen and Jorgenson (1973), Jorgenson (1989; 10), Hulten (1990; 128) and Diewert and Lawrence (2000; 276). Irving Fisher (1908; 32-33) derived these equations in words as follows: "Putting the principle in its most general form, we may say that for any arbitrary interval of time, the value of the capital at its beginning is the discounted value of two elements: (1) the actual income accruing within that interval, and (2) the value of the capital at the close of the period."

The first equation in (6) says that the value of a new asset at the start of period t, P_0^t, is equal to the rental that the asset can earn in period t, f_0^t,[19] plus the expected asset value of the capital good at the end of period t, $(1+i^t)\, P_1^t$, but this expected asset value must be divided by the discount factor, $(1+r^t)$, in order to convert this future value into an equivalent beginning of period t value.[20]

Now it is straightforward to solve equations (6) for the sequence of period t vintage rental prices, $\{f_n^t\}$, in terms of the vintage asset prices, $\{P_n^t\}$:

$$
\begin{aligned}
(7)\quad f_0^t &= P_0^t - [(1+i^t)/(1+r^t)]\, P_1^t &&= (1+r^t)^{-1}\, [P_0^t\, (1+r^t) - (1+i^t)\, P_1^t] \\
f_1^t &= P_1^t - [(1+i^t)/(1+r^t)]\, P_2^t &&= (1+r^t)^{-1}\, [P_1^t\, (1+r^t) - (1+i^t)\, P_2^t] \\
f_2^t &= P_2^t - [(1+i^t)/(1+r^t)]\, P_3^t &&= (1+r^t)^{-1}\, [P_2^t\, (1+r^t) - (1+i^t)\, P_3^t] \\
&\ldots \\
f_n^t &= P_{n+1}^t - [(1+i^t)/(1+r^t)]\, P_{n+1}^t &&= (1+r^t)^{-1}\, [P_n^t\, (1+r^t) - (1+i^t)\, P_{n+1}^t]\ ;\ \ldots
\end{aligned}
$$

Thus equations (5) allow us to go from the sequence of vintage rental prices $\{f_n^t\}$ to the sequence of vintage asset prices $\{P_n^t\}$ while equations (7) allow us to reverse the process.

Equations (7) can be derived from elementary economic considerations. Consider the first equation in (7). Think of a production unit as purchasing a unit of the new capital asset at the beginning of period t at a cost of P_0^t and then using the asset throughout period t. However, at the end of period t, the producer will have a depreciated asset that is expected to be worth $(1+i^t)\, P_1^t$. Since this offset to the initial cost of the asset will only be received at the end of period t, it must be divided by $(1+r^t)$ to express the benefit in terms of beginning of period t dollars. Thus the net cost of *using* the new asset for period t[21] is $P_0^t - [(1+i^t)/(1+r^t)]\, P_1^t$.

The above equations assume that the actual or implicit period t rental payments f_n^t for assets of different vintages n are made at the *beginning* of period t. It is sometimes convenient to assume that the rental payments are made at the *end* of each accounting period. Thus we define the *end of period t vintage rental price or user cost* for an asset that is n periods old at the beginning of period t, u_n^t, in terms of the corresponding *beginning of period t rental price* f_n^t as follows:

$$(8)\quad u_n^t \equiv (1+r^t)\, f_n^t\ ;\ \ n = 0,1,2,\ldots$$

---

[19] Note that we are implicitly assuming that the rental is paid to the owner at the beginning of period t.

[20] Another way of interpreting say the first equation in (6) runs as follows: the purchase cost of a new asset P_0^t less the rental f_0^t (which is paid immediately at the beginning of period t) can be regarded as an investment, which must earn the going rate of return r^t. Thus we must have $[P_0^t - f_0^t](1+r^t) = (1+i^t)P_1^t$ which is the (expected) value of the asset at the end of period t. This line of reasoning can be traced back to Walras (1954; 267): “A man who buys a house for his own use must be resolved by us into two individuals, one making an investment and the other consuming directly the services of his capital.”

[21] This explains why the rental prices f_n^t are sometimes called *user costs*. This derivation of a user cost was used by Diewert (1974; 504), (1980; 472-473), (1992a; 194) and by Hulten (1996; 155).

Thus if the rental payment is made at the end of the period instead of the beginning, then the beginning of the period rental f_n^t must be escalated by the interest rate factor $(1+r^t)$ in order to obtain the end of the period user cost u_n^t.[22]

Using equations (8) and the second set of equations in (7), it can readily be shown that the sequence of end of period t user costs $\{u_n^t\}$ can be defined in terms of the period t sequence of vintage asset prices $\{P_n^t\}$ as follows:

$$
\begin{aligned}
(9)\quad u_0^t &= P_0^t\,(1+r^t) - (1+i^t)\,P_1^t \\
u_1^t &= P_1^t\,(1+r^t) - (1+i^t)\,P_2^t \\
u_2^t &= P_2^t\,(1+r^t) - (1+i^t)\,P_3^t \\
&\ldots \\
u_n^t &= P_n^t\,(1+r^t) - (1+i^t)\,P_{n+1}^t\ ;\ \ldots
\end{aligned}
$$

Equations (9) can also be given a direct economic interpretation. Consider the following explanation for the user cost for a new asset, u_0^t. At the end of period t, the business unit expects to have an asset worth $(1+i^t)\,P_1^t$. Offsetting this benefit is the beginning of the period asset purchase cost, P_0^t. However, in addition to this cost, the business must charge itself either the explicit interest cost that occurs if money is borrowed to purchase the asset or the implicit opportunity cost of the equity capital that is tied up in the purchase. Thus offsetting the end of the period benefit $(1+i^t)\,P_1^t$ is the initial purchase cost and opportunity interest cost of the asset purchase, $P_0^t\,(1+r^t)$, leading to a end of period t net cost of $P_0^t\,(1+r^t) - (1+i^t)\,P_1^t$ or u_0^t.

It is interesting to note that in both the accounting and financial management literature of the past century, there was a reluctance to treat the opportunity cost of *equity capital* tied up in capital inputs as a genuine cost of production.[23] However, more recently, there is an acceptance of an imputed interest charge for equity capital as a genuine cost of production.[24]

In the following section, we will relate the vintage asset price profiles $\{P_n^t\}$ and the user cost profiles $\{u_n^t\}$ to *depreciation profiles*. However, before turning to the subject of depreciation, it is important to stress that the analysis presented in this section is based on

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[22] It is interesting that Böhm-Bawerk (1891; 343) carefully distinguished between rental payments made at the beginning or end of a period: “These figures are based on the assumption that the whole year’s utility is obtained all at once, and, indeed, obtained in anticipation at the beginning of the year; e.g., by hiring the good at a year’s interest of 100 payable on each 1st January. If, on the other hand, the year’s use can only be had at the end of the year, a valuation undertaken at the beginning of the year will show figures not inconsiderably lower. … That the figures should alter according as the date of the valuation stands nearer or farther from the date of obtaining the utility, is an entirely natural thing, and one quite familiar in financial life.”

[23] This literature is reviewed in Diewert and Fox (1999; 271-274).

[24] Stern Stewart & Co. has popularized this concept of EVA, Economic Value Added. In a newspaper advertisement in the Financial Post in 1999, it described this “new” concept as follows: “EVA measures your company’s after tax profits from operations minus the cost of all the capital employed to produce those profits. What makes EVA so revealing is that it takes into account a factor no conventional measures include: the cost of the operation’s capital— not just the cost of debt but the cost of equity capital as well.”

a number of restrictive assumptions, particularly on future price expectations. Moreover, we have not explained how these asset price expectations are formed and we have not explained how the period t nominal interest rate is to be estimated (we will address these topics in section 7 below). We have not explained what should be done if the sequence of second hand asset prices $\{P_n^t\}$ is not available and the sequences of vintage rental prices or user costs, $\{f_n^t\}$ or $\{u_n^t\}$, are also not available (we will address this problem in later sections as well). We have also assumed that asset values and user costs are independent of how intensively the assets are used. Finally, we have not modeled uncertainty (about future prices and the useful lives of assets) and attitudes towards risk on the part of producers. Thus the analysis presented in this paper is only a start on the difficult problems associated with measuring capital input.

4. Cross Section Depreciation Profiles

Recall that in the previous section, P_n^t was defined to be the price of an asset that was n periods old at the beginning of period t. Generally, the decline in asset value as we go from one vintage to the next oldest is called *depreciation*. More precisely, we define the *cross section depreciation* D_n^t [25] of an asset that is n periods old at the beginning of period t as

$$(10)\ D_n^t \equiv P_n^t - P_{n+1}^t \quad ; n = 0,1,2,\ldots$$

Thus D_n^t is the value of an asset that is n periods old at the beginning of period t, P_n^t, minus the value of an asset that is n+1 periods old at the beginning of period t, P_{n+1}^t.[26]

Obviously, given the sequence of period t vintage asset prices $\{P_n^t\}$, we can use equations (10) to determine the period t sequence of declines in vintage asset values, $\{D_n^t\}$. Conversely, given the period t cross section depreciation sequence or *profile*, $\{D_n^t\}$, we can determine the period t vintage asset prices by adding up amounts of depreciation:

[25] This terminology is due to Hill (1999) who distinguished the decline in second hand asset values due to aging (cross section depreciation) from the decline in an asset value over a period of time (time series depreciation). Triplett (1996; 98-99) uses the cross section definition of depreciation and shows that it is equal to the concept of capital consumption in the national accounts but he does this under the assumption of no expected real asset inflation. We will examine the relationship of cross section to time series depreciation in section 6 below.

[26] Of course, the objections to the use of second hand market data to determine depreciation rates are very old: "We readily agree that where a market is sufficiently large, generally accessible, and continuous over time, it serves to coordinate a large number of subjective estimates and thus may impart a moment of (social) objectivity to value relations based on prices forced on it. But it can hardly be said that the second-hand market for industrial equipment, which would be the proper place for the determination of the value of capital goods which have been in use, satisfies these requirements, and that its valuations are superior to intra-enterprise valuation." L.M. Lachmann (1941; 376-377). "Criticism has also been voiced about the viability of used asset market price data as an indicator of in use asset values. One argument, drawing on the Ackerlof Lemons Model, is that assets resold in second hand markets are not representative of the underlying population of assets, because only poorer quality units are sold when used. Others express concerns about the thinness of resale markets, believing that it is sporadic in nature and is dominated by dealers who under-bid." Charles R. Hulten and Frank C. Wykoff (1996; 17-18).

$$\begin{aligned}(11)\ P_0^t &= D_0^t + D_1^t + D_2^t + \ldots \\ P_1^t &= D_1^t + D_2^t + D_3^t + \ldots \\ &\ldots \\ P_n^t &= D_n^t + D_{n+1}^t + D_{n+2}^t + \ldots\end{aligned}$$

Rather than working with first differences of vintage asset prices, it is more convenient to reparameterize the pattern of cross section depreciation by defining the *period t depreciation rate* δ_n^t for an asset that is n periods old at the start of period t as follows:

$$(12)\ \delta_n^t \equiv 1 - [P_{n+1}^t/P_n^t] = D_n^t / P_n^t\ ;\ \ n = 0,1,2,\ldots$$

In the above definitions, we require n to be such that P_n^t is positive.[27]

Obviously, given the sequence of period t vintage asset prices $\{P_n^t\}$, we can use equations (12) to determine the period t sequence of vintage *cross section depreciation rates*, $\{\delta_n^t\}$. Conversely, given the cross section sequence of period t depreciation rates, $\{\delta_n^t\}$, as well as the price of a new asset in period t, P_0^t, we can determine the period t vintage asset prices as follows:

$$\begin{aligned}(13)\ P_1^t &= (1 - \delta_0^t)\, P_0^t \\ P_2^t &= (1 - \delta_0^t)(1 - \delta_1^t)\, P_0^t \\ &\ldots \\ P_n^t &= (1 - \delta_0^t)(1 - \delta_1^t)\ldots(1 - \delta_{n-1}^t)\, P_0^t\ ;\ \ldots\end{aligned}$$

The interpretation of equations (13) is straightforward. At the beginning of period t, a new capital good is worth P_0^t. An asset of the same type but which is one period older at the beginning of period t is less valuable by the amount of depreciation $\delta_0^t\, P_0^t$ and hence is worth $(1 - \delta_0^t)\, P_0^t$, which is equal to P_1^t. An asset which is two periods old at the beginning of period t is less valuable than a one period old asset by the amount of depreciation $\delta_1^t\, P_1^t$ and hence is worth $P_2^t = (1 - \delta_1^t)\, P_1^t$ which is equal to $(1 - \delta_1^t)(1 - \delta_0^t)\, P_0^t$ using the first equation in (13) and so on. Suppose $L - 1$ is the first integer which is such that δ_{L-1}^t is equal to one. Then P_n^t equals zero for all $n \geq L$; i.e., at the end of L periods of use, the asset no longer has a positive rental value. If L = 1, then a new asset

[27] This definition of depreciation dates back to Hicks (1939) at least and was used extensively by Hulten and Wykoff (1981a) (1981b), Diewert (1974; 504) and Hulten (1990; 128) (1996; 155): "If there is a perfect second hand market for the goods in question, so that a market value can be assessed for them with precision, corresponding to each particular degree of wear, then the value-loss due to consumption can be exactly measured…" John R. Hicks (1939; 176). Current cost accountants have also advocated the use of second hand market data (when available) to calculate "objective" depreciation rates: "But as a practical matter the quantification and valuation of asset services used is not a simple matter and we must fall back on estimated patterns as a basis for current cost as well as historic cost depreciation. For those fixed assets which have active second hand markets the problem is not overly difficult. A pattern of service values can be obtained at any time by comparing the market values of different ages or degrees of use. The differences so obtained, when related to the value of a new asset, yield the proportions of asset value which are normally used up or foregone in the various stages of asset life." Edgar O. Edwards and Philip W. Bell (1961; 175).

of this type delivers all of its services in the first period of use and the asset is in fact a nondurable asset.

Now substitute equations (12) into equations (9) in order to obtain the following formulae for the sequence of the *end of the period vintage user costs* $\{u_n^t\}$ in terms of the price of a new asset at the beginning of period t, P_0^t, and the sequence of cross section depreciation rates, $\{\delta_n^t\}$:

$$
\begin{aligned}
(14)\ u_0^t &= [(1+r^t) - (1+i^t)(1-\delta_0^t)]\, P_0^t \\
u_1^t &= (1-\delta_0^t)[(1+r^t) - (1+i^t)(1-\delta_1^t)]\, P_0^t \\
&\ldots \\
u_n^t &= (1-\delta_0^t)\ldots(1-\delta_{n-1}^t)[(1+r^t) - (1+i^t)(1-\delta_n^t)]\, P_0^t\ ;\ \ldots
\end{aligned}
$$

Thus given P_0^t (the beginning of period t price of a new asset), i^t (the new asset inflation rate that is expected at the beginning of period t), r^t (the one period nominal interest rate that the business unit faces at the beginning of period t) and given the sequence of cross section vintage depreciation rates prevailing at the beginning of period t (the δ_n^t), then we can use equations (14) to calculate the sequence of vintage end of the period user costs for period t, the u_n^t. Of course, given the u_n^t, we can use equations (8) to calculate the beginning of the period user costs (the f_n^t) and then use the f_n^t to calculate the sequence of vintage asset prices P_n^t using equations (5) and finally, given the P_n^t, we can use equations (12) in order to calculate the sequence of vintage depreciation rates, the δ_n^t. Thus *given any one of these sequences or profiles, all of the other sequences are completely determined*. This means that assumptions about depreciation rates, the pattern of vintage user costs or the pattern of vintage asset prices *cannot be made independently of each other*.[28]

It is useful to look more closely at the first equation in (14), which expresses the user cost or rental price of a new asset at the beginning of period t, u_0^t, in terms of the depreciation rate δ_0^t, the one period nominal interest rate r^t, the new asset inflation rate i^t that is expected to prevail at the beginning of period t and the beginning of period t price for a new asset, P_0^t:

$$(15)\ u_0^t = [(1+r^t) - (1+i^t)(1-\delta_0^t)]\, P_0^t = [r^t - i^t + (1+i^t)\delta_0^t]\, P_0^t.$$

Thus the user cost of a new asset u_0^t that is purchased at the beginning of period t (and the actual or imputed rental payment is made at the end of the period) is equal to $r^t - i^t$ (a nominal interest rate minus an asset inflation rate which can be loosely interpreted[29] as a

[28] This point was first made explicitly by Jorgenson and Griliches (1967; 257): "An almost universal conceptual error in the measurement of capital input is to confuse the aggregation of capital stock with the aggregation of capital service." See also Jorgenson and Griliches (1972; 81-87). Much of the above algebra for switching from one method of representing vintage capital inputs to another was first developed by Christensen and Jorgenson (1969; 302-305) (1973) for the geometrically declining depreciation model. The general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989) and Hulten (1990; 127-129) (1996; 152-160).
[29] We will provide a more precise definition of a real interest rate later.

real interest rate) times the initial asset cost P_0^t plus $(1+ i^t)\delta_0^t P_0^t$ which is *depreciation* on the asset at beginning of the period prices, $\delta_1^t P_0^t$, times *the inflation escalation factor*, $(1+ i^t)$.[30] If we further assume that the expected asset inflation rate is 0, then (15) further simplifies to:

(16) $u_0^t = [r^t + \delta_0^t]\, P_0^t$.

Under these assumptions, the user cost of a new asset is equal to the interest rate plus the depreciation rate times the initial purchase price.[31] This is essentially the user cost formula that was obtained by Walras in 1874:

"Let P be the price of a capital good. Let p be its gross income, that is, the price of its service inclusive of both the depreciation charge and the insurance premium. Let μP be the portion of this income representing the depreciation charge and νP the portion representing the insurance premium. What remains of the gross income after both charges have been deducted, $\pi = p - (\mu+\nu)P$, is the *net* income.
We are now able to explain the differences in gross incomes derived from various capital goods having the same value, or conversely, the differences in values of various capital goods yielding the same gross incomes. It is, however, readily seen that the values of capital goods are rigorously proportional to their net incomes. At least this would have to be so under certain normal and ideal conditions when the market for capital goods is in equilibrium. Under equilibrium conditions the ratio $[p - (\mu+\nu)P]/P$, or the rate of net income, is the same for all capital goods. Let i be this common ratio. When we determine i, we also determine the prices of all landed capital, personal capital and capital goods proper by virtue of the equation $p - (\mu+\nu)P = iP$ or $P = p/[i+\mu+\nu]$." Léon Walras (1954; 268-269).

However, the basic idea that a durable input should be charged a period price that is equal to a depreciation term plus a term that would cover the cost of financial capital goes back much further[32]. For example, consider the following quotation from Babbage:

"Machines are, in some trades, let out to hire, and a certain sum is paid for their use, in the manner of rent. This is the case amongst the frame-work knitters: and Mr. Hensen, in speaking of the rate of payment for the use of their frames, states, that the proprietor receives such a rent that, besides paying the full interest for his capital, he clears the value of his frame in nine years. When the rapidity with which improvements succeed each other is considered, this rent does not appear exorbitant. Some of these frames have been worked for thirteen years with little or no repair." Charles Babbage (1835; 287).

Babbage did not proceed further with the user cost idea. Walras seems to have been the first economist who formalized the idea of a user cost into a mathematical formula. However, the early industrial engineering literature also independently came up with the user cost idea; Church described how the use of a machine should be charged as follows:

---

[30] This formula was obtained by Christensen and Jorgenson (1969; 302) for the geometric model of depreciation but it is valid for any depreciation model. Griliches (1963; 120) also came very close to deriving this formula in words: "In a perfectly competitive world the annual rent of a machine would equal the marginal product of its services. The rent itself would be determined by the interest costs on the investment, the deterioration in the future productivity of the machine due to current use, and the expected change in the price of the machine (obsolescence)."

[31] Using equations (13) and (14) and the assumption that the asset inflation rate $i^t = 0$, it can be shown that the user cost of an asset that is n periods old at the start of period t can be written as $u_n^t = (r^t + \delta_n^t)P_n^t$ where P_n^t is the beginning of period t second hand market price for the asset.

[32] Solomons (1968; 9-17) indicates that interest was regarded as a cost for a durable input in much of the nineteenth century accounting literature. The influential book by Garcke and Fells (1893) changed this.

"No sophistry is needed to assume that these charges are in the nature of rents, for it might easily happen that in a certain building a number of separate little shops were established, each containing one machine, all making some particular part or working on some particular operation of the same class of goods, but each shop occupied, not by a wage earner, but by an independent mechanic, who rented his space, power and machinery, and sold the finished product to the lessor. Now in such a case, what would be the shop charges of these mechanics? Clearly they would comprise as their chief if not their only item, just the rent paid. And this rent would be made up of: (1) Interest. (2) Depreciation. (3) Insurance. (4) Profit on the capital involved in the building, machine and power-transmitting and generating plant. There would also most probably be a separate charge for power according to the quantity consumed.
Exclude the item of profit, which is not included in the case of a shop charge, and we find that we have approached most closely to the new plan of reducing any shop into its constituent production centres. No one would pretend that there was any insuperable difficulty involved in fixing a just rent for little shops let out in this plan." A. Hamilton Church (1901; 907-908).

"A production centre is, of course, either a mechanic, or a bench at which a hand craftsman works. Each of these is in the position of a little shop carrying on one little special industry, paying rent for the floor space occupied, interest for the capital involved, depreciation for the wear and tear, and so on, *quite independently of what may be paid by other production centres* in the same shop." A. Hamilton Church (1901; 734).

Church was well aware of the importance of determining the "right" rate to be charged for the use of a machine in a multiproduct enterprise. This information is required not only to price products appropriately but to determine whether an enterprise should make or purchase a particular commodity. Babbage and Canning were also aware of the importance of determining the right machine rate charge:[33]

[33] Under moderate inflation, the difficulties with traditional cost accounting based on historical cost and no proper allowance for the opportunity of capital, the proper pricing of products becomes very difficult. Diewert and Fox (1999; 271-274) argued that this factor contributed to the great productivity slowdown that started around 1973 and persisted to the early 1990's. The traditional method of cost accounting can be traced back to a book first published in 1887 by the English accountants, Garcke and Fells, who suggested allocating the "indirect costs" of producing a good proportionally to the amount of labour and materials costs used to make the item: "In some establishments the direct expenditures in wages and materials only is considered to constitute the cost; and no attempt is made to allocate to the various working or stock orders any portion of the indirect expenses. Under this system the difference between the sum of the wages and materials expended on the articles and their selling price constitutes the gross profit, which is carried in the aggregate to the credit of profit and loss, the indirect factory expenses already referred to, together with the establishment expenses and depreciation, being particularised on the debit side of that account. This method has certainly simplicity in its favour, but a more efficient check upon the indirect expenses would be obtained by establishing a relation between them and the direct expenses. This may be done by distributing all the indirect expenses, such as wages of foremen, rent of factory, fuel, lighting, heating, and cleaning, etc. (but not the salaries of clerks, office rent, stationery and other establishment charges to be referred to later), over the various jobs, as a percentage, either upon the wages expended upon the jobs respectively, or upon the cost of both wages and materials." Emile Garcke and John Manger Fells (1893; 70-71). Compare this rather crude approach to cost accounting to the masterful analysis of Church! Garcke and Fells endorsed the idea that deprecation was an admissible item of cost that should be allocated in proportion to the prime cost (i.e., labour and materials cost) of manufacturing an article but they explicitly ruled out interest as a cost: "The item of Depreciation may, for the purpose of taking out the cost, simply be included in the category of the indirect expenses of the factory, and be distributed over the various enterprises in the same way as those expenses may be allocated; or it may be dealt with separately and more correctly in the manner already alluded to and hereafter to be fully described. The establishment expenses and interest on capital should not, however, in any case form part of the cost of production. There is no advantage in distributing these items over the various transactions or articles produced. They do not

"The great competition introduced by machinery, and the application of the principle of the subdivision of labour, render it necessary for each producer to be continually on the watch, to discover improved methods by which the cost of the article he manufactures may be reduced; and, with this view, it is of great importance to know the precise expense of every process, as well as of the wear and tear of machinery which is due to it." Charles Babbage (1835; 203).

"The question of 'adequate' rates of depreciation, in the sense that they will ultimately adjust the valuations to the realities, is often discussed as though it had no effect upon ultimate profit at all. Of some modes of valuing, it is said that they tend to overvalue some assets and to undervalue others, but the aggregate of book values found is nearly right. If the management pay no attention at all to the unit costs implied in such valuations, no harm is done. But if the cost accountant gives effect to these individually bad valuations through a machine-rate burden charge, and if the selling policy has regard for apparent unit profits, the valuation may lead to the worst rather than to the best possible policy." John B. Canning (1929; 259-260).

The above equations relating vintage asset prices P_n^t, beginning of the period vintage user costs f_n^t, end of the period vintage user costs u_n^t and the (cross section) vintage depreciation rates δ_n^t are the fundamental ones that we will specialize in subsequent sections in order to measure both wealth capital stocks and capital services under conditions of inflation. In the following section, we shall consider several options that could be used in order to determine empirically the interest rates r^t and the asset inflation rates i^t.

5. The Empirical Determination of Interest Rates and Asset Inflation Rates

We consider initially three broad approaches[34] to the determination of the nominal interest rate r^t that is to be used to discount future period value flows by the business units in the aggregate under consideration:

- Use the ex post rate of return that will just make the sum of the user costs exhaust the gross operating surplus of the production sectors in the aggregate under consideration.
- Use an aggregate of nominal interest rates that the production sectors in the aggregate might be facing at the beginning of each period.

vary proportionately with the volume of business. … The establishment charges are, in the aggregate, more or less constant, while the manufacturing costs fluctuate with the cost of labour and the price of material. To distribute the charges over the articles manufactured would, therefore, have the effect of disproportionately reducing the cost of production with every increase, and the reverse with every diminution, of business. Such a result is greatly to be deprecated, as tending to neither economy of management nor to accuracy in estimating for contracts. The principles of a business can always judge what percentage of gross profit upon cost is necessary to cover fixed establishment charges and interest on capital." Emile Garcke and John Manger Fells (1893; 72-73). The aversion of accountants to include interest as a cost can be traced back to this quotation.

[34] Other methods for determining the appropriate interest rates that should be inserted into user cost formulae are discussed by Harper, Berndt and Wood (1989) and in Chapter 5 of Schreyer (2001). Harper, Berndt and Wood (1989) evaluate empirically 5 alternative rental price formulae using geometric depreciation but making different assumptions about the interest rate and the treatment of asset price change. They show that the choice of formula matters.

- Take a fixed real interest rate and add to it actual ex post consumer price inflation or anticipated consumer price inflation.

The first approach was used for the entire private production sector of the economy by Jorgenson and Griliches (1967; 267) and for various sectors of the economy by Christensen and Jorgenson (1969; 307). It is also widely used by statistical agencies. It has the advantage that the value of output for the sector will exactly equal the value of input in a consistent accounting framework. It has the disadvantages that it is subject to measurement error and it is an *ex post rate of return* which may not reflect the economic conditions facing producers at the beginning of the period.

The second approach suffers from aggregation problems. There are many interest rates in an economy at the beginning of an accounting period and the problem of finding the "right" aggregate of these rates is not a trivial one.

The third approach works as follows. Let the consumer price index for the economy at the beginning of period t be c^t say. Then the ex post general consumer inflation rate for period t is ρ^t defined as:

$$(17)\ 1 + \rho^t \equiv c^{t+1}/c^t .$$

Let the production units under consideration face the real interest rate r^{*t}. Then by the Fisher (1896) effect, the relevant nominal interest rate that the producers face should be approximately equal to r^t defined as follows:

$$(18)\ r^t \equiv (1+r^{*t})(1+\rho^t) - 1.$$

The Australian Bureau of Statistics assumes that producers face a real interest rate of 4 %. This is consistent with long run observed economy wide real rates of return for most OECD countries which fall in the 3 to 5 per cent range. We shall choose this third method for defining nominal interest rates and choose the real rate of return to be 4 % per annum; i.e., we assume that the nominal rate r^t is defined by (18) with the real rate defined by

$$(19)\ r^{*t} \equiv .04$$

assuming that the accounting period chosen is a year.[35]

We turn now to the determination of the asset inflation rates, the i^t, which appear in most of the formulae derived in the preceding sections of this annex. There are three broad approaches that can be used in this context:

- Use actual ex post asset inflation rates over each period.

[35] If we are in a high inflation situation so that the accounting period becomes a quarter or a month, then r^{*t} must be chosen to be appropriately smaller.

- Assume that each asset inflation rate is equal to the general inflation rate for each period.
- Estimate anticipated asset inflation rates for each period.

In what follows, we will compute vintage user costs using Canadian data on investments for two broad classes of assets (nonresidential construction and machinery and equipment) for 4 different sets of assumptions about depreciation or the relative efficiency of vintage assets. We will undertake these computations in an inflationary environment and make each of the three sets of assumptions about the asset inflation rates listed above for each of the 4 depreciation models, giving 12 models in all that will be compared. If the various models give very different results, this indicates that the statistical agency computing capital stocks and service flows under inflation must choose its preferred model with some care.

When we assume that each asset inflation rate is equal to the general inflation rate ρ^t defined by (17), the equations presented earlier simplify. Thus if we replace $1+i^t$ by $1+\rho^t$ and $1+r^t$ by $(1+r^*)(1+\rho^t)$, equations (5), which relate the vintage asset prices $P_n^{\ t}$ to the vintage rental prices $f_n^{\ t}$, become:

$$
\begin{aligned}
(20)\ P_0^{\ t} &= f_0^{\ t} + [1/(1+r^*)]\ f_1^{\ t} + [1/(1+r^*)]^2\ f_2^{\ t} + [1/(1+r^*)]^3\ f_3^{\ t} + \ldots \\
P_1^{\ t} &= f_1^{\ t} + [1/(1+r^*)]\ f_2^{\ t} + [1/(1+r^*)]^2\ f_3^{\ t} + [1/(1+r^*)]^3\ f_4^{\ t} + \ldots \\
&\ldots \\
P_n^{\ t} &= f_n^{\ t} + [1/(1+r^*)]\ f_{n+1}^{\ t} + [1/(1+r^*)]^2\ f_{n+2}^{\ t} + [1/(1+r^*)]^3\ f_{n+3}^{\ t} + \ldots
\end{aligned}
$$

Note that only the constant real interest rate r^* appears in these equations.

If we replace $1+i^t$ by $1+\rho^t$ and $1+r^t$ by $(1+r^*)(1+\rho^t)$, equations (14), which relate *the end of period vintage user costs* $u_n^{\ t}$ to the vintage depreciation rates $\delta_n^{\ t}$, become:

$$
\begin{aligned}
(21)\ u_0^{\ t} &= (1+\rho^t)[(1+r^*) - (1-\delta_0^{\ t})]\ P_0^{\ t} && = (1+\rho^t)[r^* + \delta_0^{\ t}]\ P_0^{\ t} \\
u_1^{\ t} &= (1+\rho^t)(1-\delta_0^{\ t})[(1+r^*) - (1-\delta_1^{\ t})]\ P_0^{\ t} && = (1+\rho^t)(1-\delta_0^{\ t})[r^* + \delta_1^{\ t}]\ P_0^{\ t} \\
&\ldots \\
u_n^{\ t} &= (1+\rho^t)(1-\delta_0^{\ t})\ldots(1-\delta_{n-1}^{\ t})[(1+r^*) - (1-\delta_n^{\ t})]\ P_0^{\ t} \\
& && = (1+\rho^t)(1-\delta_0^{\ t})\ldots(1-\delta_{n-1}^{\ t})\ [r^* + \delta_n^{\ t}]\ P_0^{\ t}.
\end{aligned}
$$

Now use equations (8) and $1+r^t = (1+r^*)(1+\rho^t)$ and substitute into (21) to obtain the following equations, which relate the *beginning of period vintage user costs* $f_n^{\ t}$ to the vintage depreciation rates $\delta_n^{\ t}$:

$$
\begin{aligned}
(22)\ f_0^{\ t} &= (1+r^*)^{-1}[r^* + \delta_0^{\ t}]\ P_0^{\ t} \\
f_1^{\ t} &= (1+r^*)^{-1}(1-\delta_0^{\ t})[r^* + \delta_1^{\ t}]\ P_0^{\ t} \\
&\ldots \\
f_n^{\ t} &= (1+r^*)^{-1}(1-\delta_0^{\ t})\ldots(1-\delta_{n-1}^{\ t})\ [r^* + \delta_n^{\ t}]\ P_0^{\ t}.
\end{aligned}
$$

Note that only the constant real interest rate r^* appears in equations (22) but equations (21) also have the general inflation rate $(1+\rho^t)$ as a multiplicative factor.

As mentioned above, in our third class of assumptions about asset inflation rates, we want to estimate *anticipated inflation rates* and use these estimates as our i^t in the various formulae we have exhibited. Unfortunately, there are any number of forecasting methods that could be used to estimate the anticipated inflation asset inflation rates. We will take a somewhat different approach than a pure forecasting one: we will simply *smooth* the observed ex post inflation rates and use these smoothed rates as our estimates of anticipated asset inflation rates.[36] A similar forecasting problem arises when we use ex post actual consumer price index inflation rates (recall (17) and (18) above) in order to generate anticipated general inflation rates. Thus in our third set of models, we will use both smoothed asset inflation rates and smoothed general inflation rates as our estimates for anticipated rates. In our first class of models, we will use actual ex post rates in both cases.

Before we proceed to consider our four specific depreciation models, we briefly consider in the next section a topic of some current interest: namely the interaction of (foreseen) obsolescence and depreciation. We also discuss cross section versus time series depreciation.

6. Obsolescence and Depreciation

We begin this section with a definition of the time series depreciation of an asset. Define the *ex post time series depreciation* of an asset that is n periods old at the beginning of period t, E_n^t, to be its second hand market price at the beginning of period t, P_n^t, less the price of an asset that is one period older at the beginning of period t+1, P_{n+1}^{t+1}; i.e.,

(23) $E_n^t \equiv P_n^t - P_{n+1}^{t+1}$; $n = 0,1,2,\ldots$

Definitions (23) should be contrasted with our earlier definitions (10), which defined the *cross section amounts of depreciation* for the same assets at the beginning of period t, $D_n^t \equiv P_n^t - P_{n+1}^t$.

We can now explain why we preferred to work with the cross section definition of depreciation, (10), over the time series definition, (23). The problem with (23) is that time series depreciation captures the effects of changes in *two* things: changes in *time* (this is the change in t to t+1)[37] and changes in the *age* of the asset (this is the change in n to n+1).[38] Thus time series depreciation aggregates together two effects: the asset

[36] Unfortunately, different analysts may choose different smoothing methods so there may be a problem of a lack of reproducibility in our estimating procedures. Harper, Berndt and Wood (1989; 351) note that the use of time series techniques to smooth ex post asset inflation rates and the use of such estimates as anticipated price change dates back to Epstein (1977).

[37] This change could be captured by either $P_n^t - P_n^{t+1}$ or $P_{n+1}^t - P_{n+1}^{t+1}$.

[38] This change could be captured by either $P_n^t - P_{n+1}^t$ or $P_n^{t+1} - P_{n+1}^{t+1}$.

specific price change that occurred between time t and time t+1 *and* the effects of asset aging (depreciation). Thus the time series definition of depreciation combines together two distinct effects.

The above definition of ex post time series depreciation is the original definition of depreciation and it extends back to the very early beginnings of accounting theory:

"[There are] various methods of estimating the Depreciation of a Factory, and of recording alteration in value, but it may be said in regard to any of them that the object in view is, so to treat the nominal capital in the books of account that it shall always represent as nearly as possible the real value. Theoretically, the most effectual method of securing this would be, if it were feasible, to revalue everything at stated intervals, and to write off whatever loss such valuations might reveal without regard to any prescribed rate… The plan of valuing every year instead of adopting a depreciation rate, though it might appear the more perfect, is too tedious and expensive to be adopted … The next best plan, which is that generally followed … is to establish average rates which can without much trouble be written off every year, to check the result by complete or partial valuation at longer intervals, and to adjust the depreciation rate if required." Ewing Matheson (1910; 35).

Hotelling, in the first mathematical treatment of depreciation in continuous time, defined time series depreciation in a similar manner:

"Depreciation is defined simply as rate of decrease of value." Harold Hotelling (1925; 341).

However, what has to be kept in mind that these early authors who used the concept of time series depreciation were implicitly or explicitly assuming *that prices were stable across time*, in which case, time series and cross section depreciation coincide.

Hill (2000) recently argued that a form of time series depreciation was to be preferred over cross section depreciation for national accounts purposes:

"The basic cost of using an asset over a certain period of time consists of depreciation, the decline in the value of that asset, plus the associated financial, or capital cost. An alternative definition of depreciation has been proposed in recent years in what may be described as the vintage accounting approach to depreciation. In the context of vintage accounting, depreciation is defined as the difference between the value of an asset of age k and one of age k+1 *at the same point of time*, the two assets *being identical except for their ages*. This concept, although superficially the same as the traditional concept, is in fact radically different because it effectively rules out obsolescence from depreciation *by definition*.
The issue is not a factual one about whether obsolescence does or does not cause the value of assets to decline over time. The question is how should such a decline be interpreted and classified. Whereas the traditional concept of depreciation treats expected obsolescence as an integral part of depreciation, in the vintage approach it is treated as a separate item, a revaluation, which has to be treated as real holding loss in the SNA. Reclassifying part of what has always been treated as depreciation in both business and economic accounting as a holding loss would reduce depreciation and increase every balancing item in the SNA from Net National Product and Income to net saving." Peter Hill (2000; 6).

Since the depreciation rates δ_n^t defined by (12) are cross section depreciation rates and they play a key role in the beginning and end of period t user costs f_n^t and u_n^t defined by (14), (21) and (22), it is necessary to clarify their use in the context of Hill's point that these depreciation rates should *not* be used to measure depreciation in the national accounts.

Our response to the Hill critique is twofold:

- Cross section depreciation rates as we have defined them *are* affected by anticipated obsolescence in principle but
- Hill is correct in arguing that cross section depreciation will not generally equal ex post time series depreciation or anticipated time series depreciation.

Let us consider the first point above. Provisionally, we define *anticipated obsolescence* as a situation where the expected new asset inflation rate (adjusted for quality change) i^t is *negative*.[39] For example, everyone anticipates that the quality adjusted price for a new computer next quarter will be considerably lower than it is this quarter.[40] Now turn back to equations (5) above, which define the profile of vintage asset prices P_n^t at the start of period t. It is clear that the negative i^t plays a role in defining the sequence of vintage asset prices as does the sequence of vintage rental prices that is observed at the beginning of period t, the f_n^t. Thus in this sense, cross sectional depreciation rates certainly embody assumptions about anticipated obsolescence. In fact, Zvi Griliches had a nice verbal description of the factors which explain the pattern of used asset prices:

"The net stock concept is motivated by the observed fact that the value of a capital good declines with age (and/or use). This decline is due to several factors, the main ones being the decline in the life expectancy of the asset (it has fewer work years left), the decline in the physical productivity of the asset (it has poorer work years left) and the decline in the relative market return for the productivity of this asset due to the availability of better machines and other relative price changes (its remaining work years are worth less). One may label there three major forces as exhaustion, deterioration and obsolescence." Zvi Griliches (1963; 119).

Thus for an asset that has a finite life, as we move down the rows of equations (5), the number of discounted rental terms decline and hence asset value declines, which is Griliches' concept of *exhaustion*. If the cross sectional rental prices are monotonically declining (due to their declining efficiency), then as we move down the rows of equations (5), the higher rental terms are being dropped one by one so that the asset values will also decline from this factor, which is Griliches' concept of *deterioration*. Finally, a negative anticipated asset inflation rate will cause all future period rentals to be discounted more

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39 Paul Schreyer and Peter Hill noted a problem with this provisional definition of anticipated obsolescence as a negative value of the expected asset inflation rate: it will not work in a high inflation environment. In a high inflation environment, the nominal asset inflation rate i^t will generally be positive but we will require this nominal rate to be less than general inflation in order to have anticipated obsolescence. Thus our final definition of *anticipated obsolescence* is that the real asset inflation rate i^{*t} defined later by (28) be negative; see the discussion just above equation (30) below.

40 Our analysis assumes that the various vintages of capital are adjusted for quality change (if any occurs) as they come on the market. In terms of our Canadian empirical example to follow, we are assuming that Statistics Canada correctly adjusted the published investment price deflators for machinery and equipment and nonresidential construction for quality change. We also need to assume that the form of quality change affects all future efficiency factors (i.e., the f_n^t) in a proportional manner. This is obviously only a rough approximation to reality: technical change may increase the durability of a capital input or it may decrease the amount of maintenance or fuel that is required to operate the asset. These changes can lead to nonproportional changes in the f_n^t.

heavily, which could be interpreted as Griliches' concept of *obsolescence*.[41] Thus all of these explanatory factors are imbedded in equations (5).[42]

Now let us consider the second point: that cross section depreciation is not really adequate to measure time series depreciation in some sense to be determined.

Define the *ex ante time series depreciation* of an asset that is n periods old at the beginning of period t, $\Delta_n{}^t$, to be its second hand market price at the beginning of period t, $P_n{}^t$, less the *anticipated price* of an asset that is one period older at the beginning of period t+1, $(1+i^t)\,P_{n+1}{}^t$; i.e.,

$$(23)\ \Delta_n{}^t \equiv P_n{}^t - (1+i^t)\,P_{n+1}{}^t \quad ; n = 0,1,2,\ldots$$

Thus anticipated time series depreciation for an asset that is t periods old at the start of period t, $\Delta_n{}^t$, differs from the corresponding cross section depreciation defined by (10), $D_n{}^t \equiv P_n{}^t - P_{n+1}{}^t$, in that the anticipated new asset inflation rate, i^t, is missing from $D_n{}^t$. However, *note that the two forms of depreciation will coincide if the expected asset inflation rate i^t is zero*.

We can use equations (12) and (13) in order to define the ex ante depreciation amounts $\Delta_n{}^t$ in terms of the cross section depreciation rates $\delta_n{}^t$. Thus using definitions (23), we have:

$$\begin{aligned}
(24)\ \Delta_n{}^t &\equiv P_n{}^t - (1+i^t)\,P_{n+1}{}^t && ; n = 0,1,2,\ldots \\
&= P_n{}^t - (1+i^t)(1-\delta_n{}^t)\,P_n{}^t && \text{using (12)} \\
&= [1 - (1+i^t)(1-\delta_n{}^t)]\,P_n{}^t && \\
&= (1-\delta_1{}^t)(1-\delta_2{}^t)\ldots(1-\delta_{n-1}{}^t)[1 - (1+i^t)(1-\delta_n{}^t)]\,P_0{}^t && \text{using (13)} \\
&= (1-\delta_1{}^t)(1-\delta_2{}^t)\ldots(1-\delta_{n-1}{}^t)[\,\delta_n{}^t - i^t(1-\delta_n{}^t)]\,P_0{}^t. &&
\end{aligned}$$

We can compare the above sequence of ex ante time series depreciation amounts $\Delta_n{}^t$ with the corresponding sequence of cross section depreciation amounts:

$$\begin{aligned}
(25)\ D_n{}^t &\equiv P_n{}^t - P_{n+1}{}^t && ; n = 0,1,2,\ldots \\
&= P_n{}^t - (1-\delta_n{}^t)\,P_n{}^t && \text{using (12)} \\
&= [1 - (1-\delta_n{}^t)]\,P_n{}^t &&
\end{aligned}$$

[41] However, it is more likely that what Griliches had in mind was Hill's second point; i.e., that time series depreciation will be larger than cross section depreciation in a situation where i^{*t} is negative.

[42] "Normal wear-and-tear in the course of production is clearly a reason why the value of a capital instrument should be greater at the beginning of a year than at the end, even if the final value was foreseen accurately. Normal wear-and-tear is therefore an element of true depreciation. So is exceptional wear-and-tear, due to exceptionally heavy usage; if the exceptionally heavy usage had been foreseen, the gap between the beginning-value and the end-value would have been larger. On the other hand, any deterioration which the machine undergoes outside its utilisation does *not* give rise to true depreciation; if such deterioration had been foreseen, the initial capital value would have been written down in consequence; the deterioration which it undergoes is therefore not depreciation, but a capital loss." John R. Hicks (1942; 178). In our view, foreseen price declines in future rentals *are* reflected in initial capital values.

$$= (1-\delta_1^t)(1-\delta_2^t) \ldots (1-\delta_{n-1}^t)[\,\delta_n^t]\, P_0^t \qquad \text{using (13).}$$

Of course, if the anticipated asset inflation rate i^t is zero, then (24) and (25) coincide and ex ante time series depreciation equals cross section depreciation. If we are in the provisional expected obsolescence case with i^t negative, then it can be seen comparing (24) and (25) that

(26) $\Delta_n^t > D_n^t$ for all n such that $D_n^t > 0$;

i.e., if i^t is negative (and $0 < \delta_n^t < 1$), then ex ante time series depreciation exceeds cross section depreciation over all in use vintages of the asset. If i^t is *positive* so that the rental price of each vintage is expected to rise in the future , then ex ante time series depreciation is *less* than the corresponding cross section depreciation for all assets that have a positive price at the end of period t. This corresponds to the usual result in the vintage user cost literature where capital gains or an ex post price increase for a new asset leads to a negative term in the user cost formula (plus a revaluation of the cross section depreciation rate). Here we are restricting ourselves to *anticipated* capital gains rather than the *actual* ex post capital gains and we are focussing on depreciation concepts rather than the full user cost.

This is not quite the end of the story in the high inflation context. National income accountants often readjust asset values at either the beginning or end of the accounting period to take into account general price level change. At the same time, they also want to decompose nominal interest payments into a real interest component and another component that compensates lenders for general price change.

Recall (17), which defined the general period t inflation rate ρ^t and (18), which related the period t nominal interest rate r^t to the real rate r^{*t} and the inflation rate ρ^t. We rewrite (18) as follows:

(27) $1 + r^{*t} \equiv (1 + r^t)/(1 + \rho^t)$.

In a similar manner, we define the period t anticipated real asset inflation rate i^{*t} as follows:

(28) $1 + i^{*t} \equiv (1 + i^t)/(1 + \rho^t)$.

Recall definition (23), which defined the *ex ante time series depreciation* of an asset that is n periods old at the beginning of period t, Δ_n^t. The first term in this definition reflects the price level at the beginning of period t while the second term in this definition reflects the price level at the end of period t. We now express the second term in terms of the beginning of period t price level. Thus we define the *ex ante real time series depreciation* of an asset that is n periods old at the beginning of period t, Π_n^t, as follows:

(29) $\Pi_n^t \equiv P_n^t - (1+i^t)\, P_{n+1}^t/(1+\rho^t)$; $n = 0,1,2,\ldots$

$$= P_n^t - (1+i^t)(1-\delta_n^t)\, P_n^t/(1+\rho^t) \qquad \text{using (12)}$$

$= [1 - (1+i^{*t})(1+\rho^t)(1-\delta_n{}^t)]\, P_n{}^t/(1+\rho^t)$ using (28)
$= (1-\delta_0{}^t)(1-\delta_1{}^t) \ldots (1-\delta_{n-1}{}^t)[1 - (1+i^{*t})(1-\delta_n{}^t)]\, P_0{}^t$ using (13)
$= (1-\delta_0{}^t)(1-\delta_1{}^t) \ldots (1-\delta_{n-1}{}^t)[\delta_n{}^t - i^{*t}(1-\delta_n{}^t)]\, P_0{}^t$.

The ex ante real time series depreciation amount $\Pi_n{}^t$ defined by (29) can be compared to its cross section counterpart $D_n{}^t$, defined by (25) above. Of course, if the real anticipated asset inflation rate i^{*t} is zero, then (29) and (25) coincide and real ex ante time series depreciation equals cross section depreciation.

We are now in a position to provide a more satisfactory definition of expected obsolescence, particularly in the context of high inflation. We now define *expected obsolescence* to be the situation where the *real* asset inflation rate i^{*t} is negative. If the real asset inflation rate is negative, then it can be seen comparing (29) and (25) that

(30) $\Pi_n{}^t > D_n{}^t$ for all n such that $D_n{}^t > 0$;

i.e., real anticipated time series depreciation exceeds the corresponding cross section depreciation provided that i^{*t} is negative.

Thus the general user cost formulae that we have developed from the vintage accounts point of view can be reconciled to reflect the point of view of national income accountants. We agree with Hill's point of view that cross section depreciation is not really adequate to measure time series depreciation as national income accountants have defined it since Pigou:

"Allowance must be made for such part of capital depletion as may fairly be called 'normal'; and the practical test of normality is that the depletion is sufficiently regular to be foreseen, if not in detail, at least in the large. This test brings under the head of depreciation all ordinary forms of wear and tear, whether due to the actual working of machines or to mere passage of time— rust, rodents and so on— and all ordinary obsolescence, whether due to technical advance or to changes of taste. It brings in too the consequences of all *ordinary* accidents, such as shipwreck and fire, in short of all accidents against which it is customary to insure. But it leaves out capital depletion that springs from the act of God or the King's enemies, or from such a miracle as a decision tomorrow to forbid the manufacture of whisky or beer. These sorts of capital depletion constitute, not depreciation to be made good before current net income is reckoned, but capital losses that are irrelevant to current net income." A.C. Pigou (1935; 240-241).

Pigou (1924) in an earlier work had a more complete discussion of the obsolescence problem and the problems involved in defining time series depreciation in an inflationary environment. Pigou first pointed out that the national dividend or net annual income (or in modern terms, real net output) should subtract depreciation or capital consumption:

"For the dividend may be conceived in two sharply contrasted ways: as the flow of goods and services which is *produced* during the year, or as the flow which is *consumed* during the year. Dr. Marshall adopts the former of these alternatives …. Naturally, since in every year plant and equipment wear out and decay, what is produced must mean what is produced on the whole when allowance has been made for this process of attrition. … In concrete terms, his conception of the dividend includes an inventory of all the new things that are made [i.e., gross investment], accompanied as a negative element, by an inventory of all the decay and demolition of old things [i.e., capital consumption]. A.C. Pigou (1924; 34-35).

Pigou then went on to discuss the roles of obsolescence and general price change in measuring depreciation:

"The concrete content of the dividend is, indeed, unambiguous— the inventory of things made and (double counting being eliminated) and services rendered, *minus*, as a negative element, the inventory of things worn out during the year. But how are we to *value* this negative element? For example, if a machine originally costing £1000 wears out and, owing to a rise in the general price level, can only be replaced at a cost of £1500, is £1000 or £1500 the proper allowance? Nor is this the only, or, indeed, the principle difficulty. For depreciation is measured not merely by the physical process of wearing out, and capital is not therefore maintained intact when provision has been made to replace what is thus worn out. Machinery that has become obsolete because of the development of improved forms is not really left intact, however excellent its physical condition; and the same thing is true of machinery for whose products popular taste has declined. If, however, in deference to these considerations, we decide to make an allowance for obsolescence, this concession implies that the value, and not the physical efficiency, of instrumental goods [i.e., durable capital inputs] is the object to be maintained intact. But, it is then argued, the value of instrumental goods, being the present value of the services which they are expected to render in the future, necessarily varies with variations in the rate of interest. Is it really a rational procedure to evaluate the national dividend by a method which makes its value in relation to that of the aggregated net product of the country's industry depend on an incident of that kind? If that method is adopted, and a great war, by raising the rate of interest, depreciates greatly the value of existing capital, we shall probably be compelled to put, for the value of the national dividend in the first year of that war, a very large *negative* figure. This absurdity must be avoided at all costs, and we are therefore compelled, when we are engaged in evaluating the national dividend, to leave out of account any change in the value of the country's capital equipment that may have been brought about by broad general causes. This decision is arbitrary and unsatisfactory, but it is one which it is impossible to avoid. During the period of the war, a similar difficulty was created by the general rise, for many businesses, in the value of the normal and necessary holding of materials and stocks, which was associated with the general rise of prices. On our principles, this increase of value ought not to be reckoned as an addition to the income of the firms affected, or, consequently, to the value of the national dividend." A.C. Pigou (1924; 39-41).

The above quotation indicates that Pigou was responsible for many of the conventions of national income accounting that persist down to the present day.[43] He essentially argued that (unanticipated) capital gains or losses be excluded from income and that the effects of general price level change be excluded from estimates of depreciation. He also argued for the inclusion of (foreseen) obsolescence in depreciation. Unfortunately, he did not spell out *exactly* how all of this could be done in the accounts. Our algebra above can be regarded as an attempt to formalize these Pigovian complications.

It should be noted that the early industrial engineering literature also stressed that the possibility of obsolescence meant that depreciation allowances should be larger than those implied by mere wear and tear:

"Machinery for producing any commodity in great demand, seldom actually wears out; new improvements, by which the same operations can be executed either more quickly or better, generally superceding it long before that period arrives: indeed, to make such an improved machine profitable, it is usually reckoned that

[43] Colin Clark (1940; 31) echoed Pigou's recommendations: "The appreciation in value of capital assets and land must not be treated as an element in national income. Depreciation due to physical wear and tear and obsolescence must be treated as a charge against current income, but not the depreciation of the money value of an asset which has remained physically unchanged. Appreciation and depreciation of capital were included in the American statistics of national income prior to 1929, but now virtually the same convention has been adopted in all countries."

in five years it ought to have paid itself, and in ten to be superceded by a better." Charles Babbage (1835; 285).

"The possibility of New Inventions, processes, or machines coming into use, which may supercede or render an existing plant Obsolete, is a contingency that presses on most manufacturing trades, principally those which have long established, but sometimes also in new concerns where old methods have been adopted or imitated just as they were being superceded elsewhere." Ewing Matheson (1910; 38).

"A reserve beyond the ordinary depreciation above described may then become necessary, because the original plant, when once superceded by such inventions, may prove unsaleable as second-hand plant, except in so far as it may have a piecemeal or scrap value. … This risk sometimes arises, not from improvements in the machinery, but from alterations in the kind of product, rendering new machines necessary to suit new patterns or types. Contingencies such as these should encourage an ample reduction of nominal value in the early years of working, so as to bring down the book value of the plant to a point which will allow even of dismantling without serious loss. In such trades, profits should be large enough to allow for a liberal and rapid writing off of capital value, which is in effect the establishment of a reserve-fund as distinct from depreciation for wear and tear." Ewing Matheson (1910; 39-40).

Thus Matheson considered obsolescence that could arise not only from new inventions but also from shifts in demand.

We will end this section by pointing out another important use for the concept of real anticipated time series depreciation. However, before doing this, it is useful to rewrite equations (5), which define the beginning of period t vintage asset prices P_n^t in terms of the vintage beginning of period t rental prices f_n^t, and equations (7), which define the vintage user costs f_n^t in terms of the vintage asset prices P_n^t, using definitions (27) and (28), which define the period t real interest rate r^{*t} and expected asset inflation rate i^{*t} respectively in terms of the corresponding nominal rates r^t and i^t and the general inflation rate ρ^t. Substituting (27) and (28) into (5) yields the following system of equations:

$$
\begin{aligned}
(31)\ P_0^t &= f_0^t + [(1+i^{*t})/(1+r^{*t})]\ f_1^t + [(1+i^{*t})/(1+r^{*t})]^2\ f_2^t + [(1+i^{*t})/(1+r^{*t})]^3\ f_3^t + \ldots \\
P_1^t &= f_1^t + [(1+i^{*t})/(1+r^{*t})]\ f_2^t + [(1+i^{*t})/(1+r^{*t})]^2\ f_3^t + [(1+i^{*t})/(1+r^{*t})]^3\ f_4^t + \ldots \\
&\ldots \\
P_n^t &= f_n^t + [(1+i^{*t})/(1+r^{*t})]\ f_{n+1}^t + [(1+i^{*t})/(1+r^{*t})]^2\ f_{n+2}^t + [(1+i^{*t})/(1+r^t)]^3\ f_{n+3}^t + \ldots
\end{aligned}
$$

Similarly, substituting (27) and (28) into (7) yields the following system of equations:

$$
\begin{aligned}
(32)\ f_0^t &= P_0^t - [(1+i^{*t})/(1+r^{*t})]\ P_1^t &&= (1+r^{*t})^{-1}\ [P_0^t\,(1+r^{*t}) - (1+i^{*t})\ P_1^t] \\
f_1^t &= P_1^t - [(1+i^{*t})/(1+r^{*t})]\ P_2^t &&= (1+r^{*t})^{-1}\ [P_1^t\,(1+r^{*t}) - (1+i^{*t})\ P_2^t] \\
&\ldots \\
f_n^t &= P_n^t - [(1+i^{*t})/(1+r^{*t})]\ P_{n+1}^t &&= (1+r^{*t})^{-1}\ [P_n^t\,(1+r^{*t}) - (1+i^{*t})\ P_{n+1}^t]\ ;\ \ldots
\end{aligned}
$$

Note that the nominal interest and inflation rates have entirely disappeared from the above equations. In particular, the beginning of the period vintage user costs f_n^t can be defined in terms or real variables using equations (32) if this is desired. On the other hand, entirely equivalent formulae for the vintage user costs can be obtained using the initial set of equations (7), which used only nominal variables. Which set of equations is used in practice can be left up to the judgement of the statistical agency or the user.[44] The point is that the *careful and consistent use of discounting should eliminate the effects of general inflation from our price variables*; discounting makes comparable cash flows received or paid out at different points of time.

Recall definition (29), which defined Π_n^t as the *ex ante real time series depreciation* of an asset that is n periods old at the beginning of period t. It is convenient to convert this amount of depreciation into a *percentage* of the initial price of the asset at the beginning of period t, P_n^t. Thus we define the *ex ante time series depreciation rate for an asset that is n periods old at the start of period t*, π_n^t, as follows:[45]

$$
\begin{aligned}
(33)\quad \pi_n^t &\equiv \Pi_n^t / P_n^t && ;\ n = 0,1,2,\ldots \\
&= [P_n^t - (1+i^t)\, P_{n+1}^t/(1+\rho^t)] / P_n^t && \text{using (29)} \\
&= [P_n^t - (1+i^t)(1-\delta_n^t)\, P_n^t/(1+\rho^t)] / P_n^t && \text{using (12)} \\
&= [1 - (1+i^{*t})(1-\delta_n^t)] && \text{using (28).}
\end{aligned}
$$

Now substitute definition (12) for the cross section depreciation rate δ_n^t into the nth equation of (32) and we obtain the following expression for the beginning of period t user cost of an asset that is n periods old at the start of period t:

$$
\begin{aligned}
(34)\quad f_n^t &= (1+r^{*t})^{-1}\, [P_n^t\,(1+r^{*t}) - (1+i^{*t})\, P_{n+1}^t] && n = 0,1,2,\ldots \\
&= (1+r^{*t})^{-1}\, [P_n^t\,(1+r^{*t}) - (1+i^{*t})(1-\delta_n^t)\, P_n^t] && \text{using (12)} \\
&= (1+r^{*t})^{-1}\, [(1+r^{*t}) - (1+i^{*t})(1-\delta_n^t)]\, P_n^t && \\
&= (1+r^{*t})^{-1}\, [r^{*t} + \pi_n^t]\, P_n^t && \text{using (33).}
\end{aligned}
$$

Thus the period t vintage user cost for an asset that is n periods old at the start of period t, f_n^t, can be decomposed into the sum of two terms. Ignoring the discount factor, $(1+r^{*t})^{-1}$, the first term is $r^{*t}\, P_n^t$, which represents the *real interest cost* of the financial capital that is tied up in the asset, and the second term is $\pi_n^t\, P_n^t = \Pi_n^t$, which represents a concept of *national accounts depreciation*.

The last line of (34) is important if at some stage statistical agencies decide to switch from measures of gross domestic product to measures of net domestic product. If this change occurs, then the user cost for each vintage of capital, f_n^t, must be split up into two

---

[44] In particular, it is not necessary for the statistical agency to convert all nominal prices into real prices as a preliminary step before "real" user costs are calculated. The above algebra shows that our nominal vintage user costs f_n^t can also be interpreted as "real" user costs that are expressed in terms of the value of money prevailing at the beginning of period t.

[45] To see that there can be a very large difference between the cross section depreciation rate δ_n^t and the corresponding ex ante time series depreciation rate π_n^t, consider the case of an asset whose vintages yield exactly the same service for each period in perpetuity. In this case, all of the vintage asset prices P_n^t would be identical and the cross section depreciation rates δ_n^t would all be zero. Now suppose a marvelous new invention is scheduled to come on the market next period which would effectively drive the price of this class of assets down to zero. In this case, i^{*t} would be -1 and substituting this expected measure of price change into definitions (33) shows that the ex ante time series depreciation rates would all equal one; i.e., under these conditions, we would have $\pi_n^t = 1$ and $\delta_n^t = 0$ for all vintages n.

terms as in (34). The first term, $(1+r^{*t})^{-1}\, r^{*t}\ P_n^{\,t}$ times the number of units of that vintage of capital in use, (the real opportunity cost of financial capital) could remain as a primary input charge while the second term, $(1+r^{*t})^{-1}\, \pi_n^{\,t}\, P_n^{\,t}$ times the number of units of that vintage of capital in use, (real national accounts depreciation) could be treated as an intermediate input charge (similar to the present treatment of imports). The second term would be an offset to gross investment.[46]

This completes our discussion of the obsolescence problem.[47] In the next section, we turn our attention to the problem of aggregating across vintages of the same capital good.

7. Aggregation over Vintages of a Capital Good

In previous sections, we have discussed the beginning of period t *stock price* $P_n^{\,t}$ of an asset that is n periods old and the corresponding beginning and end of period *user costs*, $f_n^{\,t}$ and $u_n^{\,t}$. The stock prices are relevant for the construction of *real wealth measures* of capital and the user costs are relevant for the construction of *capital services measures*. We now address the problems involved in obtaining quantity series that will match up with these prices.

Let the period t−1 investment in a homogeneous asset for the sector of the economy under consideration be I^{t-1}. We assume that the starting capital stock for a new unit of capital stock at the beginning of period t is $K_0^{\,t}$ and this stock is equal to the new investment in the asset in the previous period; i.e., we assume:

$$(35)\ K_0^{\,t} \equiv I^{t-1}.$$

Essentially, we are assuming that the length of the period is short enough so that we can neglect any contribution of investment to current production; a new capital good becomes productive only in the period immediately following its construction. In a similar manner, we assume that the capital stock available of an asset that is n periods old at the start of period t is $K_n^{\,t}$ and this stock is equal to the gross investment in this asset class during period t−n−1; i.e., we assume:

$$(36)\ K_n^{\,t} \equiv I^{t-n-1}\,; \qquad\qquad n = 0,1,2,\ldots$$

Given these definitions, the value of the capital stock in the given asset class for the sector of the economy under consideration (*the wealth capital stock*) at the start of period t is

[46] Using this methodology, we would say that capital is being *maintained intact* for the economy if the value of gross investments made during the period (discounted to the beginning of the period) is equal to or greater than the sum of the real national accounts depreciation terms over all assets. This is *a maintenance of financial capital concept* as opposed to Pigou's *maintenance of physical capital concept*: "Net income consists of the whole of the annual output minus what is needed to maintain the stock of capital intact; and this stock is kept intact provided that its physical state is held constant." A.C. Pigou (1935; 235).

[47] It should be noted that our discussion of the obsolescence issue only provides an introduction to the many thorny issues that make this area of inquiry quite controversial. For further discussion, see Oulton (1995), Scott (1995) and Triplett (1996) and the references in these papers.

$$(37)\ W^t \equiv P_0{}^t K_0{}^t + P_1{}^t K_1{}^t + P_2{}^t K_2{}^t + \ldots$$
$$= P_0{}^t I^{t-1} + P_1{}^t I^{t-2} + P_2{}^t I^{t-3} + \ldots \qquad \text{using (36).}$$

Turning now to the capital services quantity, we assume that the quantity of services that an asset of a particular vintage at a point in time is proportional (or more precisely, is equal) to the corresponding stock. Thus we assume that the quantity of services provided in period t by a unit of the capital stock that is n periods old at the start of period t is $K_n{}^t$ defined by (36) above. Given these definitions, the value of capital services for all vintages of asset in the given asset class for the sector of the economy under consideration (*the productive services capital stock*) during period t using the end of period user costs $u_n{}^t$ defined by equations (8) above is

$$(38)\ S^t \equiv u_0{}^t K_0{}^t + u_1{}^t K_1{}^t + u_2{}^t K_2{}^t + \ldots$$
$$= u_0{}^t I^{t-1} + u_1{}^t I^{t-2} + u_2{}^t I^{t-3} + \ldots \qquad \text{using (36).}$$

Now we are faced with the problem of decomposing the value aggregates W^t and S^t defined by (37) and (38) into separate price and quantity components. If we assume that each new unit of capital lasts only a finite number of periods, L say, then we can solve this value decomposition problem using normal index number theory. Thus define the period *t vintage stock price and quantity vectors*, P^t and K^t respectively, as follows:

$$(39)\ P^t \equiv [P_0{}^t,P_1{}^t,\ldots,P_{L-1}{}^t]\ ;\ K^t \equiv [K_0{}^t,K_1{}^t,\ldots,K_{L-1}{}^t] = [I^{t-1},I^{t-2},\ldots,I^{t-L-1}]\ ;\ t = 0,1,\ldots,T.$$

Fixed base or chain indexes may be used to decompose value ratios into price change and quantity change components. In the empirical work which follows, we have used the chain principle.[48] Thus the value of the capital stock in period t, W^t, relative to its value in the preceding period, W^{t-1}, has the following index number decomposition:

$$(40)\ W^t / W^{t-1} = P(P^{t-1},P^t,K^{t-1},K^t)\, Q(P^{t-1},P^t,K^{t-1},K^t)\ ; \qquad t = 1,2,\ldots,T$$

where P and Q are *bilateral price and quantity indexes* respectively.

In a similar manner, we define the period *t vintage end of the period user cost price and quantity vectors*, u^t and K^t respectively, as follows:

$$(41)\ u^t \equiv [u_0{}^t,u_1{}^t,\ldots,u_{L-1}{}^t]\ ;\ K^t \equiv [K_0{}^t,K_1{}^t,\ldots,K_{L-1}{}^t] = [I^{t-1},I^{t-2},\ldots,I^{t-L-1}]\ ;\ t = 0,1,\ldots,T.$$

We ask that the value of capital services in period t, S^t, relative to its value in the preceding period, S^{t-1}, has the following index number decomposition:

$$(42)\ S^t / S^{t-1} = P(u^{t-1},u^t,K^{t-1},K^t)\, Q(u^{t-1},u^t,K^{t-1},K^t)\ ; \qquad t = 1,2,\ldots,T$$

[48] Given smoothly trending price and quantity data, the use of chain indexes will tend to reduce the differences between Paasche and Laspeyres indexes compared to the corresponding fixed base indexes and so chain indexes are generally preferred; see Diewert (1978; 895) for a discussion.

where again P and Q are *bilateral price and quantity indexes* respectively.

There is now the problem of choosing the functional form for either the price index P or the quantity index Q.[49] In the empirical work that follows, we used the *Fisher* (1922) *ideal price and quantity indexes*. These indexes appear to be "best" from the axiomatic viewpoint[50] and can also be given strong economic justifications.[51]

It should be noted that our use of an index number formula to aggregate both vintage stocks and vintage services is more general than the usual vintage aggregation procedures, which essentially assume that the different vintages of the same capital good are perfectly substitutable so that linear aggregation techniques can be used.[52] However, as we shall see in subsequent sections, the more general mode of aggregation suggested here frequently reduces to the traditional linear method of aggregation provided that the vintage prices all move in strict proportion over time.

Many researchers and statistical agencies relax the assumption that an asset lasts only a fixed number of periods, L say, and make assumptions about the distribution of retirements around the average service life, L. In our empirical work that follows, for simplicity, we will stick to the sudden death assumption; i.e., that all assets in the given asset class are retired at age L. However, this simultaneous retirement assumption can readily be relaxed (at the cost of much additional computational complexity) using the following methodology developed by Hulten:

> "We have thus far taken the date of retirement T to be the same for all assets in a given cohort (all assets put in place in a given year). However, there is no reason for this to be true, and the theory is readily extended to allow for different retirement dates. A given cohort can be broken into components, or subcohorts, according to date of retirement and a separate T assigned to each. Each subcohort can then be characterized by its own efficiency sequence, which depends among other things on the subcohort's useful life T_i." Charles R. Hulten (1990; 125).

We now have all of the pieces that are required in order to decompose the capital stock of an asset class and the corresponding capital services into price and quantity components. However, in order to construct price and quantity components for capital services, we need information on the relative efficiencies f_n^t of the various vintages of the capital input or equivalently, we need information on cross sectional vintage depreciation rates δ_n^t in order to use (42) above. The problem is that we do not have accurate information on either of these series so in what follows, we will assume a standard asset life L and make additional *assumptions* on the either the pattern of vintage efficiencies or depreciation

[49] Obviously, given one of these functional forms, we may use (40) to determine the other.
[50] See Diewert (1992b; 214-223).
[51] See Diewert (1976; 129-134).
[52] This more general form of aggregation was first suggested by Diewert and Lawrence (2000). For descriptions of the more traditional linear method of aggregation, see Jorgenson (1989; 4) or Hulten (1990; 121-127) (1996; 152-165).

rates. Thus in a sense, we are following the same somewhat mechanical strategy that was used by the early cost accountants:[53]

"The function of depreciation is recognized by most accountants as the provision of a means for spreading equitably the cost of comparatively long lived assets. Thus if a building will be of use during twenty years of operations, its cost should be recognized as operating expense, not of the first year, nor the last, but of all twenty years. Various methods may be proper in so allocating cost. The method used, however, is unimportant in this connection. The important matter is that at the time of abandonment, the cost of the asset shall as nearly as possible have been charged off as expense, under some systematic method." M.B. Daniels (1933; 303).

However, our mechanical strategy is more complex than that used by early accountants in that we translate assumptions about the pattern of cross section depreciation rates into implications for the pattern of vintage rental prices and asset prices, taking into account the complications induced by discounting and expected future asset price changes.

In the following sections, we will consider 4 different sets of assumptions and calculate the resulting aggregate capital stocks and services using Canadian data.[54] We *illustrate* how the various depreciation models differ from each other using annual Canadian data on two broad classes of asset:[55]

- machinery and equipment and
- nonresidential structures.

We use Canadian data on gross investment in these two asset classes (in current and in constant dollars) because it extends back to 1926 and hence vintage capital stocks can be formed without making arbitrary starting value assumptions.

Our first problem is to decide on the average age of retirement for each of these asset classes. One source is the OECD (1993) where average service lives for various asset classes were reported for 14 or so OECD countries. For machinery and equipment (excluding vehicles) used in manufacturing activities, the average life ranged from 11 years for Japan to 26 years for the United Kingdom. For vehicles, the average service lives ranged from 2 years for passenger cars in Sweden to 14 years in Iceland and for road freight vehicles, the average life ranged from 3 years in Sweden to 14 years in Iceland. For buildings, the average service lives ranged from 15 years (for petroleum and gas buildings in the US) to 80 years for railway buildings in Sweden. Faced with this wide range of possible lives, we decided to follow the example of Angus Madison (1993) and assume an average service life of 14 years for machinery and equipment and 39 years for nonresidential structures. The Canadian data that we used may be found in Appendix 1 below.

[53] Canning (1929; 204) criticized this strategy as follows: "The interminable argument that has been carried on by the text writers and others about the relative merits of the many formulas for measuring depreciation has failed, not only to produce the real merits of the several methods, but, more significantly, it has failed to produce a rational set of criteria of excellence whereby to test the aptness of any formula for any sub-class of fixed assets."
[54] We also consider a fifth set of assumptions but we do not empirically implement this last model.
[55] More realistic models would work with more disaggregated investment series.

We turn now to our first efficiency and depreciation model.

8. The One Hoss Shay Model of Efficiency and Depreciation

In section 3 above, we noted that Böhm-Bawerk (1891; 342) postulated that an asset would yield a constant level of services throughout its useful life of L years and then collapse in a heap to yield no services thereafter. This has come to be known as the one hoss shay or light bulb model of depreciation. Hulten notes that this pattern of relative efficiencies has the most intuitive appeal:

"Of these patterns, the one hoss shay pattern commands the most intuitive appeal. Casual experience with commonly used assets suggests that most assets have pretty much the same level of efficiency regardless of their age— a one year old chair does the same job as a 20 year old chair, and so on." Charles R. Hulten (1990; 124).

Thus the basic assumptions of this model are that the period t efficiencies and hence cross sectional rental prices f_n^t are all equal to say f^t for vintages n that are less than L periods old and for older vintages, the efficiencies fall to zero. Thus we have:

$$(43)\ f_n^t = f^t \quad \text{for } n = 0,1,2,...,L-1; \qquad = 0 \quad \text{for } n = L, L+1,L+2,....$$

Now substitute (43) into the first equation in (5) and get the following formula[56] for the rental price f^t in terms of the price of a new asset at the beginning of year t, P_0^t:

$$(44)\ f^t = P_0^t/[1 + (\gamma^t) + (\gamma^t)^2 + ...+ (\gamma^t)^{L-1}]$$

where the period t discount factor γ^t is defined in terms of the period t nominal interest rate r^t and the period t expected asset inflation rate i^t as follows:

$$(45)\ \gamma^t \equiv (1 + i^t)/(1 + r^t).$$

Now that the period t rental price f^t for an unretired asset has been determined, substitute equations (43) into equations (5) and determine the sequence of period t vintage asset prices, P_n^t:

$$(46)\ P_n^t = f^t\, [1 + (\gamma^t) + (\gamma^t)^2 + ...+ (\gamma^t)^{L-1-n}] \quad \text{for } n = 0,1,2,...,L-1; \qquad = 0 \quad \text{for } n = L, L+1,L+2,...$$

Finally, use equations (8) to determine the end of period t rental prices, u_n^t, in terms of the corresponding beginning of period t rental prices, f_n^t:

[56] This formula simplifies to $P_0^t[1-(\gamma^t)^L]/[1-\gamma^t]$ provided that γ^t is less than 1 in magnitude. This last restriction does not hold for our Canadian data, since for some years, i^t exceeds r^t. However, (44) is still valid under these conditions.

(47) $u_n^t = (1 + r^t)f_n^t$;   n = 0,1,2,...

Given the vintage asset prices defined by (46), we could use equations (12) above to determine the corresponding vintage cross section depreciation rates δ_n^t. We will not table these depreciation rates since our focus is on constructing measures of the capital stock and of the flow of services that the stocks yield.

We have data in current and constant dollars for investment in nonresidential structures and for machinery and equipment in Canada for the years 1926 to 1999 inclusive; see Appendix 1 below for a description of these data. As was mentioned in the previous section, we follow the example set by Madison (1993) and assume an average service life of 14 years for machinery and equipment and 39 years for nonresidential structures. Thus 1965 is the first year for which we will have data on all 39 vintages of nonresidential structures. Now it is a straightforward matter to use the vintage asset prices defined by (46) above (where L equals 39) and apply (40) in the previous section to aggregate over the 39 vintages of nonresidential capital using the Fisher (1922) ideal index number formula and form aggregate price and quantity series for the nonresidential construction (wealth) capital stock, P_{NR}^t and K_{NR}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can be found in Table 1 at 5 year intervals. Similarly, we use (46) above (where L equals 14) and apply (40) in the previous section to aggregate over the 14 vintages of machinery and equipment using the Fisher ideal index number formula and form aggregate price and quantity series for the machinery and equipment (wealth) capital stock, P_{ME}^t and K_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Table 1 at 5 year intervals. In this *first model*, we assume that producers exactly anticipate the asset inflation rates, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively; these ex post inflation rates are listed in Table A2 in Appendix 1 below.[57] Having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(1)^t$ and $K(1)^t$, where the 1 indicates that this is our first model in a grand total of 12 alternative aggregate capital stock models.

Table 1: Model 1 Capital Stocks and Prices Assuming Perfect Foresight

Year	P_{NR}	P_{ME}	P(1)	K_{NR}	K_{ME}	K(1)
1965	1.0000	1.0000	1.0000	37.2	21.4	58.5
1970	1.3255	1.1141	1.2483	49.3	28.6	77.9
1975	2.1252	1.4242	1.8598	62.6	39.5	101.8
1980	2.9598	2.4246	2.7743	80.4	56.9	135.5
1985	4.4893	2.2414	3.5921	101.3	84.5	179.4
1990	5.5418	2.0559	4.0632	118.2	128.1	226.1
1995	5.6645	1.8213	3.9886	131.2	168.9	263.5
1999	6.1071	1.8966	4.2533	141.8	222.1	302.7

[57] Moreover, we assume that producers extrapolate the current asset inflation rates into the future.

Annual Growth Rates	1.0547	1.0190	1.0435	1.0402	1.0713	1.0495

The quantity units in Table 1 are in billions of 1965 Canadian dollars. It can be seen that the aggregate (over vintages) stock price for nonresidential construction increased approximately six fold from 1965 to 1999 while the aggregate machinery and equipment stock price increased only approximately 90% over this period. The Fisher ideal aggregate price for these two capital stock components increased from 1 to 4.2533 over this period. The average annual (geometric) growth rate factors are listed in the last row of Table 1. It can be seen that the price of a unit of nonresidential construction capital increased by 5.47% per year and the price of a unit of machinery and equipment capital increased by only 1.90% per year on average. The average rate of price increase for the capital aggregate was 4.35% per year. On the quantity side, the stock of nonresidential construction capital increased from \$37.2 billion to \$141.8 billion (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 4.02% while the stock of machinery and equipment capital increased from \$21.4 billion to \$222.1 billion (constant 1965) Canadian dollars, for an annual average growth rate of 7.13%. The capital aggregate grew at an annual average growth rate of 4.95%.

Using equations (43), (44) and (47) along with the data tabled in Appendix 1 (see Tables A1 and A2), we can construct the end of the period user costs for each of our 39 vintages of nonresidential construction capital. Now use equation (38) to construct the service flow aggregate for nonresidential construction for each year. Then we use (42) in the previous section (where L equals 39) to aggregate over the 39 vintages of nonresidential capital using the Fisher (1922) ideal index number formula and form the aggregate rental price for nonresidential construction, u_{NR}^t, and the corresponding services aggregate, k_{NR}^t, for the years 1965-1999.[58] These series, along with their annual average (geometric) growth rates, can be found in Table 2 at 5 year intervals. Similarly, we use (42) above (where L equals 14) and aggregate over the 14 vintages of machinery and equipment using the Fisher ideal index number formula and form aggregate capital services price and quantity series, u_{ME}^t and k_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Table 2 at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(1)^t$ and $k(1)^t$, where the 1 again indicates that this is our first model in a grand total of 15 alternative aggregate capital stock models.

Table 2: Model 1 Capital Service Prices and Quantities Assuming Perfect Foresight

[58] Since all of the vintage rental prices are equal, it turns out that the aggregate rental price is equal to this common vintage rental price and the service aggregate is equal to the simple sum over the vintages. This result is an application of Hicks' (1939; 312-313) aggregation theorem; i.e., if all prices in the aggregate move in strict proportion over time, then any one of these prices can be taken as the price of the aggregate.

Year	u_{NR}	u_{ME}	u(1)	k_{NR}	k_{ME}	k(1)
1965	1.0000	1.0000	1.0000	1572	3216	4789
1970	1.7182	1.1462	1.3363	2019	4069	6086
1975	3.2780	1.5913	2.1440	2638	5355	8008
1980	3.9591	4.7094	4.5559	3456	7810	11076
1985	7.8561	3.1499	4.6700	4449	11475	15224
1990	10.4455	2.8093	5.0818	5340	17290	20534
1995	9.5433	2.1670	4.3039	6102	23622	25424
1999	9.7617	2.8492	5.0056	6667	30973	30631
Annual Growth Rates	1.0693	1.0313	1.0485	1.0434	1.0689	1.0561

The quantity units in Table 2 are in millions of 1965 Canadian dollars. It can be seen that the aggregate (over vintages) capital services price for nonresidential construction increased approximately ten fold from 1965 to 1999 while the aggregate machinery and equipment services price increased approximately three fold over this period. These are much greater increases than were exhibited by the corresponding stock prices in Table 1 above. The Fisher ideal aggregate price for these two capital services components increased from 1 to 5.0056 over this period. The average annual (geometric) growth rate factors are listed in the last row of Table 2. It can be seen that the price of a unit of nonresidential construction capital services increased by 6.93% per year and the price of a unit of machinery and equipment capital services increased by 3.13% per year on average. The average rate of price increase for the capital services aggregate was 4.85% per year, which is higher than our earlier estimate of 4.35% per year increase for the price of a unit of the corresponding stock. On the quantity side, the flow of nonresidential construction capital services increased from $1572 million to $6667 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 4.34% while the flow of machinery and equipment capital services increased from $3216 million to $30,973 million (constant 1965) Canadian dollars, for an annual average growth rate of 6.89%. The capital services aggregate grew at an annual average growth rate of 5.61% compared to the 4.95% annual average growth rate for the aggregate capital stock. The fact that capital services grew faster than the corresponding stock is explained by the fact that in constructing the services aggregate, the faster growing machinery and equipment component gets a much larger price weight than in the corresponding capital stock measure.

In order to save space, we have reported our estimates of the one hoss shay capital stocks and service flows (using the perfect foresight assumption) for only every fifth year. Unfortunately, this space saving benefit does come at a cost: the year to year volatility in the user cost series is much greater than the numbers in Table 2 indicate. To get an indication of this volatility, see Figure 3 below where the aggregate price of capital services u(1) is graphed.

We turn now to our *second one hoss shay depreciation model*, which will eliminate the volatility problem mentioned in the last paragraph. In this model, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, it is assumed that producers use the current CPI inflation rate as estimators of anticipated asset inflation rates. This model turns out to be equivalent to the constant real interest rate model that is frequently used by statistical agencies.[59] In terms of computations, we simply replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by the CPI inflation rate ρ^t listed in Table A2 of the Appendix and then repeat all of the computations made to implement Model 1 above. The counterparts to Tables 1 and 2 are Tables 3 and 4 below; Table 3 lists the *Model 2* prices and quantities for the Canadian capital stock at 5 year intervals over the period 1965-1999 while Table 4 lists the rental prices and flows of capital services over this same period.

Table 3: Model 2 Capital Stocks and Prices Assuming Constant Real Rates

Year	P_{NR}	P_{ME}	P(2)	K_{NR}	K_{ME}	K(2)
1965	1.0000	1.0000	1.0000	41.5	22.1	63.6
1970	1.2476	1.0932	1.1937	55.0	29.8	84.8
1975	1.9319	1.3898	1.7400	70.4	41.1	111.0
1980	2.8122	1.9324	2.4968	90.3	59.3	147.6
1985	3.8861	1.9516	3.1498	113.5	88.2	194.7
1990	4.6936	1.8136	3.5358	132.4	134.2	244.6
1995	4.9227	1.6696	3.5758	146.7	176.4	284.4
1999	5.3871	1.5889	3.7569	158.8	232.2	325.9
Annual Growth Rates	1.0508	1.0137	1.0397	1.0403	1.0716	1.0492

The constant real interest rate capital stocks in Table 3 are somewhat larger than their counterparts in Table 1 and their rate of price growth is smaller; for example, the average geometric rate of growth for the constant real interest rate capital stocks in Table 3 is 3.97% per year compared to the 4.35% per year rate of price increase reported in Table 1. However, the average overall geometric growth rates for the capital stock aggregates are similar: 4.92% per year in Table 3 (Model 2) versus 4.95% per year in Table 1 (Model 1).

Table 4: Model 2 Capital Service Prices and Quantities Assuming Constant Real Interest Rates

Year	u_{NR}	u_{ME}	u(2)	k_{NR}	k_{ME}	k(2)
1965	1.0000	1.0000	1.0000	2727	3588	6316
1970	1.2645	1.1079	1.1763	3502	4539	8040
1975	2.1003	1.5109	1.7693	4576	5975	10535
1980	3.0339	2.0847	2.4974	5995	8713	14557
1985	3.9615	1.9895	2.8136	7717	12803	19919
1990	4.8220	1.8632	3.0440	9263	19289	26480
1995	4.9283	1.6715	2.9366	10584	26354	32764

[59] The nominal interest rate is still used in forming the end of the period user costs; otherwise, only real interest rates are used in this model.

1999	5.3718	1.5843	3.0023	11564	34556	38926
Annual Growth Rates	1.0507	1.0136	1.0329	1.0434	1.0689	1.0549

When we compare the service prices and quantities in Table 2 (Model 1, the perfect foresight model) with the corresponding service prices and quantities in Table 4 (Model 2, the constant real interest rate model), a number of things stand out:

- The Model 2 user costs are much less volatile;
- The Model 1 user costs grow much more quickly;
- The Model 2 levels of capital services are much higher but
- The Model 1 and 2 average growth rates for capital services are very similar.

Thus the two models give very different results overall. The average rate of price increase for the Model 2 capital services aggregate was 3.29% per year, which is much lower than our earlier Model 1 estimate of 4.85% per year. On the quantity side, the Model 2 flow of nonresidential construction capital services increased from \$2727 million to \$11,564 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 4.34% while the Model 2 flow of machinery and equipment capital services increased from \$3588 million to \$34,556 million (constant 1965) Canadian dollars, for an annual average growth rate of 6.89%. The Model 2 capital services aggregate grew at an annual average growth rate of 5.49% compared to the Model 1 5.61% capital services annual average growth rate.

We turn now to our *third one hoss shay depreciation model*. In this model (*Model 3*), instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, we assume that they can anticipate the *trends* in asset inflation rates. In the Data Appendix below, we describe in detail how these trends were determined. In terms of computations, we use exactly the same program that we used to implement Model 1 except that we replace the rather volatile nominal interest rate r^t that was listed in Table A2 of the Appendix by the smoothed nominal interest rate that is listed in Table A3 of the Appendix. We also replace the two ex post asset inflation rates, $i_{NR}{}^t$ and $i_{ME}{}^t$, by their smoothed counterparts listed in Table A3 in the Appendix. The counterparts to Tables 1 and 2 are Tables 5 and 6 below; Table 5 lists the Model 3 prices and quantities for the Canadian capital stock at 5 year intervals over the period 1965-1999 while Table 6 lists the rental prices and flows of capital services over this same period.

Table 5: Model 3 Capital Stocks and Prices using Smoothed Interest Rates and Inflation Rates

Year	P_{NR}	P_{ME}	P(3)	K_{NR}	K_{ME}	K(3)
1965	1.0000	1.0000	1.0000	37.8	22.3	60.1
1970	1.3178	1.1050	1.2387	50.0	30.0	80.0
1975	2.0209	1.4318	1.7977	63.6	41.3	104.4
1980	3.1309	2.1395	2.7548	81.7	59.6	139.1

1985	4.5069	2.0939	3.5277	103.0	88.5	184.1
1990	5.5886	2.0006	4.0531	120.2	134.0	231.9
1995	5.2068	1.7171	3.6929	133.6	177.1	270.7
1999	5.8415	1.7278	4.0058	144.4	232.6	310.8
Annual Growth Rates	1.0533	1.0162	1.0417	1.0402	1.0714	1.0495

Comparing the numbers in Tables 1, 3 and 5, it can be seen that there are some small differences between the capital stocks generated by our three variants of the one hoss shay model of depreciation but the average growth rates are virtually identical. There is more variation across the three models in the movement of the stock prices with Model 1 giving the highest rate of price growth for the capital aggregate (4.35% per year), followed by Model 3 (4.17% per year) and then Model 2 (3.97% per year). The Model 1,2 and 3 aggregate prices of capital are graphed in Figure 1 below and the corresponding aggregate quantities are graphed in Figure 3 below.

Table 6: Model 3 Capital Service Prices and Quantities using Smoothed Interest Rates and Smoothed Asset Inflation Rates

Year	$\mathbf{u_{NR}}$	$\mathbf{u_{ME}}$	**u(3)**	$\mathbf{k_{NR}}$	$\mathbf{k_{ME}}$	**k(3)**
1965	1.0000	1.0000	1.0000	1706	3707	5413
1970	1.6639	1.1410	1.3075	2191	4689	6880
1975	2.5619	1.6352	1.9287	2863	6172	9036
1980	5.0119	2.9973	3.6525	3751	9001	12533
1985	7.9464	2.5273	4.1699	4828	13226	17216
1990	10.7287	2.6100	4.9265	5795	19927	23177
1995	6.7931	1.7946	3.2684	6621	27225	28711
1999	8.1325	2.0749	3.8404	7234	35698	34606
Annual Growth Rates	1.0636	1.0217	1.0404	1.0434	1.0689	1.0561

Comparing the numbers in Tables 2, 4 and 6, it can be seen that there are large differences in the levels of capital services generated by the 3 models but the growth rates are virtually identical. However, there is much more variation across the three models in the movement of the service prices with Model 1 giving the highest rate of price growth for the capital services aggregate (4.85% per year), followed by Model 3 (3.847% per year) and then Model 2 (3.29% per year). The Model 1, 2 and 3 aggregate prices of capital services are graphed in Figure 2 below and the corresponding aggregate quantities of services are graphed in Figure 4 below.

We finish off this section by graphing the aggregate price and quantity series for the Canadian capital stocks 1965-1999 (Figures 1 and 2 below) and for capital services (Figures 3 and 4 below) for our three one hoss shay models.

Insert Figures 1,2,3,4.

Viewing Figure 1, it can be seen that the Model 1 stock prices, P(1), are more volatile than the Model 2 and 3 prices and the Model 3 prices represent a smoothed version of the Model 1 prices. The Model 2 prices, which assume a constant real interest rate of 4 per cent and make no allowance for anticipated asset price changes, generally lie below the other two series.

The tremendous volatility of the Model 1 rental prices, u(1), is evident from viewing Figure 3. Thus the use of ex post asset inflation rates as ex ante or anticipated inflation rates leads to user costs that are unreasonable. The Model 3 aggregate user costs, u(3), while still more volatile than the constant real interest rate user costs, u(2), are reasonable and smooth out the fluctuations in the u(1) series. The u(2) series lies below the other two user cost series because the constant real interest rate user costs make no allowance for the extra depreciation that arises from the anticipated price declines that are due to obsolescence; i.e., the u(2) series ignores the systematic real price declines in the price of machinery and equipment.

Examination of Figure 2 shows that all three one hoss shay models give rise to much the same aggregate capital stocks. The constant real interest rate capital stocks K(2) are the biggest, followed by the smoothed anticipated inflation stocks K(3) and the fully anticipated inflation stocks K(1) are the smallest. The aggregate capital services graphed in Figure 4 show much the same pattern but with more dispersion. The constant real interest rate aggregated capital services k(2) are the biggest, followed by the smoothed anticipated inflation capital services k(3) and the fully anticipated inflation capital services k(1) are the smallest.

We turn now to our second model of depreciation and efficiency.

9. The Straight Line Depreciation Model

The straight line method of depreciation is very simple in a world without price change: one simply makes an estimate of the most probable length of life for a new asset, L periods say, and then the original purchase price P_0^t is divided by L to yield as estimate of period by period depreciation for the next L periods. In a way, this is the simplest possible model of depreciation, just as the one hoss shay model was the simplest possible model of efficiency decline.[60] The accountant Canning summarizes the straight line depreciation model as follows:

"*Straight Line Formula* ... In general, only two primary estimates are required to be made, viz., scrap value at the end of n periods and the numerical value of n. ... Obviously the number of periods of contemplated use of an asset can seldom be intelligently estimated without reference to the anticipated conditions of use. I the formula is to be respectable at all, the value of n must be the most probable number of periods that will yield the most economical use." John B. Canning (1929; 265-266).

[60] In fact, it can be verified that if the nominal interest rate r^t and the nominal asset inflation rate i^t are both zero, then the one hoss shay efficiency model will be entirely equivalent to the straight line depreciation model.

The following quotations indicate that the use of straight line depreciation dates back to the 1800's at least:

"Sometimes an equal installment is written off every year from the original value of the plant; sometimes each machine or item of plant is considered separately; but it is more usual to write off a percentage, not of the original value, but from the balance of the plant account of the preceding year." Ewing Matheson (1910; 55).

"In some instances the amount charged to revenue account for depreciation is a fixed sum, or an arbitrary percentage on the book value." Emile Garcke and John Manger Fells (1893; 98).

The last two quotations indicate that the declining balance or geometric depreciation model (to be considered in the next section) also dates back to the 1800's as a popular method for calculating depreciation.

We now set out the equations which describe the straight line model of depreciation in the general case when the anticipated asset inflation rate i^t is nonzero. Assuming that the asset has a life of L periods and that the cross sectional amounts of depreciation $D_n^t \equiv P_n^t - P_{n+1}^t$ defined by (10) above are all equal for the assets in use, then it can be seen that the beginning of period t vintage asset prices P_n^t will decline linearly for L periods and then remain at zero; i.e., the P_n^t will satisfy the following restrictions:

$$(48)\ P_n^t = P_0^t\,[L - n]/L \qquad n = 0,1,2,...,L$$
$$= 0 \qquad n = L+1,L+2,...$$

Recall definition (12) above, which defined the cross sectional depreciation rate for an asset that is n periods old at the beginning of period t, δ_n^t. Using (48) and the nth equation in (13), we have:

$$(49)\ (1 - \delta_0^t)(1 - \delta_1^t)\ldots(1 - \delta_{n-1}^t) = P_n^t / P_0^t = 1 - (n/L) \qquad \text{for } n = 1,2,...,L.$$

Using (49) for n and n+1, it can be shown that

$$(50)\ (1 - \delta_n^t) = [L - (n+1)]/[L - n] \qquad n = 0,1,2,...,L - 1.$$

Now substitute (49) and (50) into the general user cost formula (14) in order to obtain the *period t end of the period straight line user costs*, u_n^t:[61]

$$(51)\ u_n^t = (1 - \delta_0^t)\ldots (1 - \delta_{n-1}^t)[(1+r^t) - (1+i^t)(1 - \delta_n^t)]\ P_0^t \qquad n = 0,1,2,...,L - 1$$
$$= [1 - (n/L)][(1+r^t) - (1+i^t)([L - (n+1)]/[L - n])]\ P_0^t.$$

Equations (48) give us the sequence of vintage asset prices that are required to calculate the wealth capital stock while equations (51) give us the vintage user costs that are required to calculate capital services for the asset. It should be noted that if the

[61] The user costs for n = L, L+1,L+2,... are all zero.

anticipated asset inflation rate i^t is large enough compared to the nominal interest rate r^t, then the user cost u_n^t can be negative. This means that the corresponding asset becomes an output rather than an input for period t.[62]

At this point, we can proceed in much the same manner as in the previous section. We use the vintage asset prices defined by (48) above (where L equals 39) and apply (40) in section 7 to aggregate over the 39 vintages of nonresidential capital using the Fisher (1922) ideal index number formula and we form aggregate price and quantity series for the nonresidential construction (wealth) capital stock, P_{NR}^t and K_{NR}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can be found in Table 7 at 5 year intervals. Similarly, we use (48) above (where L equals 14) and apply (40) to aggregate over the 14 vintages of machinery and equipment using the Fisher ideal index number formula and we form aggregate price and quantity series for the machinery and equipment (wealth) capital stock, P_{ME}^t and K_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Table 7 at 5 year intervals. In this *fourth model*, we assume that producers exactly anticipate the asset inflation rates, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively; these ex post inflation rates are listed in Table A2 in Appendix 1 below. Having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(4)^t$ and $K(4)^t$.

Table 7: Model 4 Capital Stocks and Prices Assuming Perfect Foresight

Year	**P_{NR}**	**P_{ME}**	**P(4)**	**K_{NR}**	**K_{ME}**	**K(4)**
1965	1.0000	1.0000	1.0000	36.4	20.5	56.9
1970	1.2476	1.0932	1.1916	48.0	27.8	75.8
1975	1.9319	1.3898	1.7313	60.8	38.5	98.7
1980	2.8122	1.9324	2.4822	77.5	55.5	131.1
1985	3.8861	1.9516	3.1132	97.0	82.4	172.8
1990	4.6936	1.8136	3.4765	112.0	125.9	216.9
1995	4.9227	1.6696	3.5060	122.5	164.3	250.3
1999	5.3871	1.5889	3.6713	131.7	217.0	287.3
Annual Growth Rates	1.0508	1.0137	1.0390	1.0385	1.0719	1.0488

The quantity units in Table 7 are in billions of 1965 Canadian dollars. It can be seen that the aggregate (over vintages) stock price for nonresidential construction increased approximately 5.4 fold from 1965 to 1999 while the aggregate machinery and equipment stock price increased only approximately 59% over this period. The Fisher ideal aggregate price for these two capital stock components increased from 1 to 3.6713 over

[62] However, one is led to wonder if the model is reasonable if some vintages of the asset have negative user costs while other vintages have positive one.

this period. The average annual (geometric) growth rate factors are listed in the last row of Table 7. It can be seen that the price of a unit of nonresidential construction capital increased by 5.08% per year and the price of a unit of machinery and equipment capital increased by only 1.37% per year on average. The average rate of price increase for the capital aggregate was 3.9% per year. This should be compared to the average rate of price increase for the one hoss shay capital aggregate which was much higher at 4.35% per year; see Table 1 above. On the quantity side, the stock of nonresidential construction capital increased from \$36.4 billion to \$131.7 billion (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.85% while the stock of machinery and equipment capital increased from \$20.5 billion to \$217.0 billion (constant 1965) Canadian dollars, for an annual average growth rate of 7.19%. The capital aggregate grew at an annual average growth rate of 4.88%. This was fairly close to the average rate of growth of the one hoss shay capital aggregate, which grew at 4.95% per year; see Table 1 above.

Using equations (51) along with the data tabled in Appendix 1 (see Tables A1 and A2), we can construct the end of the period user costs for each of our 39 vintages of nonresidential construction capital. Now use equation (38) to construct the service flow aggregate for nonresidential construction for each year. Then we use (42) in the previous section (where L equals 39) to aggregate over the 39 vintages of nonresidential capital using the Fisher (1922) ideal index number formula and form the aggregate rental price for nonresidential construction, $u_{NR}{}^{t}$, and the corresponding services aggregate, $k_{NR}{}^{t}$, for the years 1965-1999.[63] These series, along with their annual average (geometric) growth rates, can be found in Table 8 at 5 year intervals. Similarly, we use (42) above (where L equals 14) and aggregate over the 14 vintages of machinery and equipment using the Fisher ideal index number formula and we form aggregate capital services price and quantity series, $u_{ME}{}^{t}$ and $k_{ME}{}^{t}$, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Table 8 at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, $u_{NR}{}^{t}$ and $k_{NR}{}^{t}$ respectively, and the aggregate price and quantity of machinery and equipment services, $u_{ME}{}^{t}$ and $k_{ME}{}^{t}$ respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(4)^{t}$ and $k(4)^{t}$.

Table 8: Model 4 Capital Service Prices and Quantities Assuming Perfect Foresight

Year	u_{NR}	u_{ME}	u(4)	k_{NR}	k_{ME}	k(4)
1965	1.0000	1.0000	1.0000	1621	3209	4830
1970	1.7715	1.1506	1.3611	2106	4086	6196
1975	3.2446	1.6214	2.1802	2813	5349	8164

[63] It turned out that some of our rental prices were negative. This may not be a major theoretical problem since in this case, the corresponding capital input becomes a net output. However, the computations were carried out using the econometrics computer program SHAZAM and the index number option fails when any price is negative. In this case, it was necessary to write up a subroutine that would compute the Fisher indexes when some prices were negative. The four inner products that are building blocks into the Fisher indexes must all be positive in order to take the positive square root. This condition was satisfied by the data in all cases.

1980	3.9663	3.9947	4.0790	3631	8214	11574
1985	7.5567	3.2111	4.6746	4593	10989	14974
1990	9.8516	3.0996	5.2371	5402	15533	19356
1995	9.1905	2.5327	4.5915	6043	20106	23187
1999	9.5117	3.1134	5.1915	6557	26578	27953
Annual Growth Rates	1.0685	1.0340	1.0496	1.0420	1.0642	1.0530

The quantity units in Table 8 are in millions of 1965 Canadian dollars. It can be seen that the aggregate (over vintages) capital services price for nonresidential construction increased approximately 9.5 times from 1965 to 1999 while the aggregate machinery and equipment services price increased approximately 3.1 times over this period. These are much greater increases than were exhibited by the corresponding stock prices in Table 7 above. The Fisher ideal aggregate price for these two capital services components increased from 1 to 5.2 over this period. The average annual (geometric) growth rate factors are listed in the last row of Table 8. It can be seen that the price of a unit of nonresidential construction capital services increased by 6.85% per year and the price of a unit of machinery and equipment capital services increased by 3.40% per year on average. The average rate of price increase for the capital services aggregate was 4.96% per year, which is higher than our earlier estimate of 3.90% per year increase for the price of a unit of the corresponding stock. The straight line depreciation rate of capital services price growth of 4.96% per year should be compared to the average rate of price increase for the one hoss shay capital services aggregate which was somewhat lower at 4.85% per year; see Table 2 above. The use of ex post asset inflation rates again leads to user costs that are extremely volatile; see Figure 7 below. On the quantity side, the flow of nonresidential construction capital services increased from \$1621 million to \$6557 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 4.20% while the flow of machinery and equipment capital services increased from \$3209 million to \$26,578 million (constant 1965) Canadian dollars, for an annual average growth rate of 6.42%. The capital services aggregate grew at an annual average growth rate of 5.3% compared to the 4.9% annual average growth rate for the aggregate capital stock. . The straight line depreciation rate of capital services growth of 5.30% per year should be compared to the average rate of growth for the one hoss shay capital services aggregate which was higher at 5.61% per year; see Table 2 above.

We turn now to our *second straight line depreciation model*, which will eliminate the volatility problem mentioned in the last paragraph. In this *Model 5*, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, it is assumed that producers use the current CPI inflation rate as estimators of anticipated asset inflation rates. In terms of computations, we simply replace the two ex post asset inflation rates, $i_{NR}{}^{t}$ and $i_{ME}{}^{t}$, by the CPI inflation rate ρ^{t} listed in Table A2 of the Appendix and then repeat all of the computations made to implement Model 4 above.

It turns out that the Model 5 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 4 counterparts in Table 7. This follows from equations (48), which describe the pattern of vintage asset prices: in both Models 4 and 5 (and 6 to be considered shortly), these asset prices do not depend on r^t or i^t and hence the resulting asset prices and capital stocks will be identical. Hence there is no need to table the capital stocks and prices for Model 5. However, the Model 5 vintage user costs and capital service flows are very different from their Model 4 counterparts. Table 9 lists the Model 5 rental prices and flows of capital services for the Canadian capital stock at 5 year intervals over the period 1965-1999.

Table 9: Model 5 Capital Service Prices and Quantities Assuming Constant Real Interest Rates

Year	u_{NR}	u_{ME}	u(5)	k_{NR}	k_{ME}	k(5)
1965	1.0000	1.0000	1.0000	2857	3542	6400
1970	1.2645	1.1122	1.1808	3718	4544	8262
1975	2.1003	1.5242	1.7830	4778	6026	10780
1980	3.0339	2.1377	2.5377	6175	8620	14645
1985	3.9615	2.1646	2.9410	7835	11971	19365
1990	4.8220	2.1693	3.2782	9222	16915	24758
1995	4.9283	2.0080	3.2045	10317	22136	29737
1999	5.3718	1.9887	3.3398	11185	27829	34561
Annual Growth Rates	1.0507	1.0204	1.0361	1.0410	1.0625	1.0508

When we compare the service prices and quantities in Table 8 (Model 4, the perfect foresight model) with the corresponding service prices and quantities in Table 9 (Model 5, the constant real interest rate model), a number of things stand out:

- The Model 5 user costs are much less volatile;
- The Model 4 user costs grow much more quickly;
- The Model 5 levels of capital services are much higher but
- The Model 4 and 5 average growth rates for capital services are fairly similar.

This is the same pattern of differences that we found when we compared the capital services aggregates generated by Models 1 and 2. The average rate of price increase for the Model 5 capital services aggregate was 3.61% per year, which is much lower than our earlier Model 4 estimate of 4.96% per year. On the quantity side, the Model 5 flow of nonresidential construction capital services increased from $2857 million to $11,185 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 4.1% while the Model 5 flow of machinery and equipment capital services increased from $3542 million to $27,829 million (constant 1965) Canadian dollars, for an annual average growth rate of 6.25%. The Model 5 capital services aggregate grew at an annual average growth rate of 5.08% compared to the Model 4 5.30% capital services annual average growth rate.

We turn now to our *third straight line deprecation model*, which we call *Model 6*. In this model, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, we assume that they can anticipate the *trends* in asset inflation rates. In terms of computations, we use exactly the same program that we used to implement Model 4 except that we replace the rather volatile nominal interest rate r^t that was listed in Table A2 of the Appendix by the smoothed nominal interest rate that is listed in Table A3 of the Appendix. We also replace the two ex post asset inflation rates, $i_{NR}{}^t$ and $i_{ME}{}^t$, by their smoothed counterparts listed in Table A3 in the Appendix.

As mentioned earlier, the Model 6 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 4 counterparts in Table 7. Hence there is no need to table the capital stocks and prices for Model 6. However, the Model 6 vintage user costs and capital service flows are very different from their Model 4 and 5 counterparts. Table 10 lists the Model 6 rental prices and flows of capital services for the Canadian capital stock at 5 year intervals over the period 1965-1999.

Table 10: Model 6 Capital Service Prices and Quantities using Smoothed Interest Rates and Smoothed Asset Inflation Rates

Year	u_{NR}	u_{ME}	u(6)	k_{NR}	k_{ME}	k(6)
1965	1.0000	1.0000	1.0000	1787	3646	5433
1970	1.6894	1.1511	1.3303	2321	4648	6969
1975	2.5902	1.6322	1.9465	3008	6241	9236
1980	4.9631	2.9035	3.5908	3885	8946	12604
1985	7.6227	2.7076	4.2643	4913	12226	16545
1990	10.0280	2.9027	5.0611	5773	17540	21498
1995	6.9624	2.1156	3.6134	6443	23087	25932
1999	8.2162	2.4257	4.1992	6991	29709	30841
Annual Growth Rates	1.0639	1.0264	1.0431	1.0409	1.0636	1.0524

On the quantity side, Model 6 gives much the same results as the other two straight line depreciation models, Models 4 and 5. In particular, the average annual (geometric) rate of growth of aggregate capital services for Models 4, 5 and 6 was 5.30 %, 5.08% and 5.24% per year respectively. However, on the user cost side, the three models give very different results. The perfect foresight model, Model 4, gave the highest annual average growth rate for the aggregate price of capital services, 4.96% per year, while the constant real interest rate model, Model 5, gave the lowest average growth rate, 3.61% per year. The smoothed anticipated prices model, Model 6, gave an intermediate growth rate for the price of capital services, 4.31% per year. As can be seen from Figure 7 below, the Model 5 and 6 aggregate user costs were much smoother than the volatile Model 4 user costs.

We finish off this section by graphing the aggregate price and quantity series for the Canadian capital stocks 1965-1999 (Figures 5 and 6 below) and for capital services

(Figures 7 and 8 below) for our three straight line depreciation models. Since the price and quantity series for the capital stocks are all the same for our three straight line models (i.e., $P(4)^t = P(5)^t = P(6)^t$ and $K(4)^t = K(5)^t = K(6)^t$), in Figure 5, we graph P(4) with the three one hoss shay capital stock prices and in Figure 6, we graph K(4) with the three one hoss shay aggregate capital stocks, K(1), K(2), and K(3).

Insert Figures 5, 6, 7 and 8

We turn now to our third class of depreciation and efficiency models.

10. The Declining Balance or Geometric Depreciation Model

The declining balance method of depreciation dates back to Matheson (1910; 55) at least as we noted in the previous section.[64] In terms of the algebra presented in section 4 above, the method is very simple: all of the cross sectional vintage depreciation rates δ_n^t defined by (12) are assumed to be equal to the same rate δ, where δ a positive number less than one; i.e., we have for all time periods t:

$$(52)\ \delta_n^t = \delta\ ; \qquad n = 0,1,2,\ldots .$$

Substitution of (52) into (14) leads to the following formula for the sequence of period t vintage user costs:

$$\begin{aligned}(53)\ u_n^t &= (1-\delta)^{n-1}\,[(1+r^t) - (1+i^t)(1-\delta)]\,P_0^t\ ; && n = 0,1,2,\ldots \\ &= (1-\delta)^{n-1}\,u_0^t\ ; && n = 1,2,\ldots .\end{aligned}$$

The second set of equations in (53) says *that all of the vintage user costs are proportional to the user cost for a new asset*. This proportionality means that we do not have to use an index number formula to aggregate over vintages to form a capital services aggregate. To see this, using (53), the period t services aggregate S^t defined earlier by (38) can be rewritten as follows:

$$\begin{aligned}(54)\ S^t &\equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \ldots \\ &= u_0^t\,[K_0^t + (1-\delta)\,K_1^t + (1-\delta)^2 K_2^t + \ldots\,] \\ &= u_0^t K_A^t\end{aligned}$$

where the *period t capital aggregate* K_A^t is defined as

$$(55)\ K_A^t \equiv K_0^t + (1-\delta)\,K_1^t + (1-\delta)^2 K_2^t + \ldots$$

If the depreciation rate δ and the vintage capital stocks are known, then K_A^t can readily be calculated using (55). Then using the last line of (54), we see that the value of capital services (over all vintages), S^t, decomposes into the price term u_0^t times the quantity term K_A^t. Hence, it is not necessary to use an index number formula to aggregate over vintages using this depreciation model.

A similar simplification occurs when calculating the wealth stock using this depreciation model. Substitution of (52) into (13) leads to the following formula for the sequence of period t vintage asset prices:

[64] Matheson (1910; 91) used the term "diminishing value" to describe the method. Hotelling (1925; 350) used the term "the reducing balance method" while Canning (1929; 276) used the term the "declining balance formula".

$$(56)\ P_n^t = (1-\delta)^{n-1} P_0^t\ ; \qquad\qquad n = 1,2,\ldots .$$

Equations (56) say *that all of the vintage asset prices are proportional to the price of a new asset*. This proportionality means that again, we do not have to use an index number formula to aggregate over vintages to form a capital stock aggregate. To see this, using (56), the period t wealth aggregate W^t defined earlier by (37) can be rewritten as follows:

$$\begin{aligned}(57)\ W^t &\equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \ldots \\ &= P_0^t [K_0^t + (1-\delta) K_1^t + (1-\delta)^2 K_2^t + \ldots] \\ &= P_0^t K_A^t\end{aligned}$$

where K_A^t was defined by (55). Thus K_A^t can serve as both a capital stock aggregate or a flow of services aggregate, which is a major advantage of this model.[65]

There is a further simplification of the model which is useful in applications. If we compare equation (55) for period t+1 and period t, we see that the following formula results using equations (39):

$$(58)\ K_A^{t+1} \equiv K_0^{t+1} + (1-\delta) K_A^t .$$

Thus the period t+1 aggregate capital stock, K_A^{t+1}, is equal to the investment in new assets that took place in period t, which is K_0^{t+1}, plus $1-\delta$ times the period t aggregate capital stock, K_A^t. This means that given a starting value for the capital stock, we can readily update it just using the depreciation rate δ and the new investment in the asset during the prior period.

We now need to address the problem of determining the depreciation rate δ for a particular asset class. Matheson was perhaps the first engineer to address this problem. On the basis of his experience, he simply postulated some approximate rates that could be applied:

"In most [brick or stone] factories an average of 3 per cent for buildings will generally be found appropriate, if due attention is paid to repairs. Such a rate will bring down a value of £ 1000 to £ 400 in thirty years." Ewing Matheson (1910; 69).

"Buildings of wood or iron would require a higher rate, ranging from 5 to 10 per cent, according to the design and solidity of the buildings, the climate, the care and the regularity of the painting, and according also, to the usage they are subjected to." Ewing Matheson (1910; 69).

"Contractors' locomotives working on imperfect railroads soon wear out, and a rate of 20 per cent is generally required, bringing down the value of an engine costing £ 1000 to £ 328 in five years." Ewing Matheson (1910; 86).

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[65] This advantage of the model has been pointed out by Jorgenson (1989) (1996b) and his coworkers. Its early application dates back to Jorgenson and Griliches (1967) and Christensen and Jorgenson (1969) (1973).

"In engineering factories, where the work is of a moderate kind which does not strain the machines severely, and where the hours of working do not average more than fifty per week, 5 per cent written off each year from the diminishing value will generally suffice for the wear-and-tear of machinery, cranes and fixed plant of all kinds, if steam engines and boilers be excluded." Ewing Matheson (1910; 82).

"The high speed of the new turbo generators introduced since 1900, and their very exact fitting, render them liable to certain risks from variations in temperature and other causes. Several changes in regard to speed and methods of blading have occurred since their first introduction and if these generators are taken separately, only after some longer experience has been acquired can it be said that a depreciation rate of 10 per cent on the dimishing value will be too much for maintaining a book-figure appropriate to their condition. Such a rate will reduce £ 1000 to £ 349 in ten years." Ewing Matheson (1910; 91).

How did Matheson arrive at his estimated depreciation rates? He gave some general guidance as follows:

"The main factors in arriving at a fair rate of depreciation are:

1. The Original value.
2. The probable working Life.
3. The Ultimate value when worn out or superceded.

Therefore, in deciding upon an appropriate rate of depreciation which will in a term of years provide for the estimated loss, it is not the original value or cost which has to be so provided for, but that cost less the ultimate or scrap value." Ewing Matheson (1910; 76).

The algebra corresponding to Matheson's method for determining δ was explicitly described by the accountant Canning (1929; 276). Let the initial value of the asset be V_0 and let its scrap value n years later be V_n. Then V_0, V_n and the depreciation rate δ are related by the following equation:

(59) $V_n = (1 - \delta)^n V_0$.

Canning goes on to explain that $1 - \delta$ may be determined by solving the following equation:

(60) $\log (1 - \delta) = [\log V_n - \log V_0]/n$.

It is clear that Matheson used this framework to determine depreciation rates even though he did not lay out formally the above straightforward algebra.

However, Canning had a very valid criticism of the above method:

"This method can be summarily rejected for a reason quite independent or the deficiencies of formulas 1 and 2 above [(59) and (60) above]. Overwhelming weight is given to V_n in determining book values. ... Thus the least important constant in reality is given the greatest effect in the formula." John B. Canning (1929; 276).

Thus Canning pointed out that the scrap value, V_n, which is not determined very accurately from an a priori point of view, is the tail that is wagging the dog; i.e., this poorly determined value plays a crucial role in the determination of the depreciation rate.[66]

An effective response to Canning's criticism of the declining balance method of depreciation did not emerge until relatively recently when Hall (1971), Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b) used an entire array of used asset prices at point in time in order to determine the geometric depreciation rate which best matched up with the data.[67] Another theoretical possibility would be to use information on vintage rental prices in order to deduce the depreciation rate.[68] Hulten and Wykoff summarize their experience in estimating depreciation rates from used asset prices by concluding that the assumption of geometric or declining balance depreciation described their data relatively well:

We have used the approach to study the depreciation patterns of a variety of fixed business assets in the United States (e.g., machine tools, construction equipment, autos and trucks, office equipment, office buildings, factories, warehouses, and other buildings). The straight line and concave patterns [i.e., one hoss shay patterns] are strongly rejected ; geometric is also rejected, but the estimated patterns are extremely lose to (though steeper than) the geometric form, even for structures. Although it is rejected statistically, the geometric pattern is far closer than either of the other two candidates. This leads us to accept the geometric pattern as a reasonable approximation for broad groups of assets, and to extend our results to assets for which no resale markets exist by imputing depreciation rates based on an assumption relating the rate of geometric decline to the useful lives of assets." Charles C. Hulten and Frank C. Wykoff (1996; 16).

This brings us to our next problem: how should we convert our estimated asset lives of 39 years for structures and 14 years for machinery and equipment into comparable geometric rates?

One possible method for converting an average asset life, L periods say, into a comparable geometric depreciation rate is to argue as follows. Suppose that we believe that the straight line model of depreciation is the correct one and the asset under consideration has a useful life of L periods. Suppose further that investment in this type of asset is constant over time at one unit per period and asset prices are constant over

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[66] "There may be cases in which the formula fits the facts, but ... the chance of its being a formula of close fit is remote indeed. Its chief usefulness seems to be to furnish drill in the use of logarithms for students in accounting." John B. Canning (1929; 277).

[67] Jorgenson (1996a) has a nice review of most of the empirical studies of depreciation. It should be noted that Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1996; 22) showed that equation (59) must be adjusted to correct for the early retirement of assets. The accountant Schmalenbach (1959; 91) (the first German edition was published in 1919) also noticed this problem: "The mistake should not be made, however, of drawing conclusions about useful life from those veteran machines which are to be seen in most businesses. Those which one sees are but the rare survivors; the many dead have long lain buried. This can be the source of serious errors."

[68] This possibility is mentioned by Hulten and Wykoff (1996; 15): "In other words, if there were active rental markets for capital services as there are for labor services, the observed prices could be used to estimate the marginal products. And the rest of the framework would follow from these estimates. But, again, there is bad news: most capital is owner utilized, like much of the stock of single family houses. This means that owners of capital , in effect, rent it to themselves, leaving no data track for the analyst to observe."

time. Under these conditions, the long run equilibrium capital stock for this asset would be[69]:

$$(61)\ 1 + [(L-1)/L] + [(L-2)/L] + ... + [2/L] + [1/L] = L(L+1)/2L = (L+1)/2.$$

Under the same conditions, the long run equilibrium geometric depreciation capital stock would be equal to the following sum:

$$(62)\ 1 + (1-\delta) + (1-\delta)^2 + ... = 1/[1-(1-\delta)] = 1/\delta.$$

Now find the depreciation rate δ which will make the two capital stocks equal; i.e., equate (61) to (62) and solve for δ. The resulting δ is:

$$(63)\ \delta = 2/(L+1).$$

Obviously, there are a number of problematical assumptions that were made in order to derive the depreciation rate δ that corresponds to the length of life L[70] but (63) gives us at least a definite method of conversion from one model to the other.

Since we assumed that the average length of life for nonresidential construction was L equal to 39 years, applying the conversion formula (63) implies that δ_{NR} equals .05; i.e., we assume that the declining balance or geometric depreciation rate for nonresidential construction in Canada is 5%. Similarly, our assumed life of 14 years for machinery and equipment translates into a δ_{ME} equal to a 13 1/3% geometric depreciation rate for this asset class.

There is one remaining problem to deal with and then we can proceed to table the results for three geometric depreciation models for Canada. The problem is this: before 1926, we do not have reliable investment data but the effects of investments made prior to 1926 live on forever in the infinite lived geometric depreciation model that we considered in equations (54) to (58) above. In the case of machinery and equipment investments made before 1926, by the time we get to 1965, what is left of the original investments is negligible. However, in the case of a \$1000 investment in nonresidential structures made in 1925, \$128.50 of it would still be available as a productive input in 1965, assuming a

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[69] Recall equations (48), which imply that the vintage asset prices are proportional. Hence Hicks' Aggregation Theorem will imply that the capital aggregate will be the simple sum on the left hand side of (61).

[70] The two assumptions that are the least justified are: (1) the assumption that the straight line depreciation model is the correct model to do the conversion and (2) the assumption that investment has been constant back to minus infinity. Hulten and Wykoff (1996; 16) made the following suggestions for converting an L into a δ: "Information is available on the average service life, L, from several sources. The rate of depreciation for non-marketed assets can be estimated using a two step procedure based on the 'declining balance' formula $\delta = X/L$. Under the 'double declining balance' formula, X = 2. The value of X can be estimated using the formula $X = \delta L$ for those assets for which these estimates are available. In the Hulten-Wykoff studies, the average value for of X for producer's durable equipment was found to be 1.65 (later revised to 1.86). For nonresidential structures, X was found to be 0.91. Once X is fixed, δ follows for other assets whose average service life is available."

5% geometric depreciation rate. Hence we need a method for estimating the geometric capital stock that is available at the start of 1926 in order to not bias downward our estimates of the geometric capital stock for nonresidential construction for the period 1965-1999. We decided to assume that nonresidential investment for the period prior to 1926 grew at the same rate that it grew during the years 1926-1999.[71] Thus for the years 1927 to 1999, we took investment in nonresidential construction during the current year divided by the corresponding investment in the prior year (both in constant dollars) as our dependent variable and regressed this variable on a constant. The estimated constant turned out to be 1.0509. Hence, for the prior to 1926 period, we assumed that investments in nonresidential construction grew at the rate $g \equiv .05$; i.e., a 5% growth rate. Thus if I_{NR}^{1926} was the investment in 1926, we assumed that the investments in prior years were:

$$(64)\ I_{NR}^{1926}/(1+g),\ I_{NR}^{1926}/(1+g)^2,\ I_{NR}^{1926}/(1+g)^3, \ldots .$$

Using assumption (64), we can calculate an estimate of the starting capital stock for nonresidential construction at the start of 1927 as

$$\begin{aligned}(65)\quad K_{NR}^{1927} &\equiv I_{NR}^{1926} \{1 + [(1-\delta)/(1+g)] + [(1-\delta)/(1+g)]^2 + [(1-\delta)/(1+g)]^3 + \ldots \} \\ &= I_{NR}^{1926} \{1/(1 - [(1-\delta)/(1+g)]\} \\ &= I_{NR}^{1926} (1 + g)/(g + \delta)\end{aligned}$$

where $g = .05$ and $\delta = .05$. Now we can use formula (58) above, starting at the year $t = 1927$, to build up the capital stock for each of our two asset classes. For nonresidential construction, our starting 1927 capital stock was defined by (65) and for machinery and equipment, it was simply the 1926 investment in machinery and equipment, I_{ME}^{1926} say.

At this point, we can proceed in much the same manner as in the previous section. We have already explained how we can use equations (58) to form the aggregate capital stocks for nonresidential construction and machinery and equipment. From (57), it can be seen that the corresponding capital stock price is P_0^t, the price of a new vintage at the beginning of year t. These series, along with their annual average (geometric) growth rates, can be found in Table 11 at 5 year intervals. In this *seventh model*, having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(7)^t$ and $K(7)^t$.

Table 11: Model 7 (Geometric Depreciation) Capital Stocks and Asset Prices

Year	P_{NR}	P_{ME}	P(7)	K_{NR}	K_{ME}	K(7)
1965	1.0000	1.0000	1.0000	32.8	19.1	51.9

[71] This method for obtaining a starting value for the geometric capital stock is due to Kohli (1982); see also Fox and Kohli (1998).

1970	1.2476	1.0932	1.1896	42.6	26.3	68.9
1975	1.9319	1.3898	1.7238	53.3	35.9	88.7
1980	2.8122	1.9324	2.4701	68.0	51.1	117.4
1985	3.8861	1.9516	3.0881	85.0	75.0	154.4
1990	4.6936	1.8136	3.4404	98.0	115.4	194.5
1995	4.9227	1.6696	3.4655	107.1	149.3	224.1
1999	5.3871	1.5889	3.6243	115.9	199.7	259.7
Annual Growth Rates	1.0508	1.0137	1.0386	1.0378	1.0715	1.0485

The quantity units in Table 11 are in billions of 1965 Canadian dollars. It can be seen that the aggregate (over vintages) stock price for nonresidential construction increased approximately 5.4 fold from 1965 to 1999 while the aggregate machinery and equipment stock price increased only approximately 59% over this period. Comparing these columns in Table 11 with the corresponding columns in Table 7, it can be verified that these numbers are exactly the same. This is because in both the straight line depreciation model and the geometric model, the price of a new asset acts as the aggregate stock price over all vintages. However, when we use the Fisher formula to aggregate the two types of capital together to get either P(4) or P(7), we get slightly different numbers because the aggregate quantities of the two types of asset differ in the two models. The Fisher ideal aggregate price for these two capital stock components increased from 1 to 3.6243 over this period. The average annual (geometric) growth rate factors are listed in the last row of Table 11. It can be seen that the price of a unit of nonresidential construction capital increased by 5.08% per year and the price of a unit of machinery and equipment capital increased by only 1.37% per year on average. The average rate of price increase for the capital aggregate was 3.86% per year. This should be compared to the average rate of price increase for the one hoss shay capital aggregate which was much higher at 4.35% per year; see Table 1 above. On the quantity side, the stock of nonresidential construction capital increased from \$32.8 billion to \$115.9 billion (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.78% (3.85% for the straight line model) while the stock of machinery and equipment capital increased from \$19.1 billion to \$199.7 billion (constant 1965) Canadian dollars, for an annual average growth rate of 7.15% (7.19% for the straight line model). The capital aggregate grew at an annual average growth rate of 4.85%. The corresponding aggregate growth rates for the one hoss shay and straight line models were 4.95% and 4.88% per year respectively; see Tables 1 and 7 above.

We turn now to the service flow part of our seventh model, where we assume that producers exactly anticipate the asset inflation rates, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively; these ex post inflation rates are listed in Table A2 in Appendix 1 below. The user cost for a new asset at the start of period t, u_0^t, is defined in equations (53). Equation (54) shows that this user cost matches up with the corresponding aggregated over vintages capital stock so the computations are simplified in this model. Denote these user costs by u_{NR}^t and u_{ME}^t for our two assets and denote the corresponding service aggregates by k_{NR}^t and k_{ME}^t respectively. We renormalize these series so that both user costs are unity in 1965.[72] These series, along with their annual average (geometric) growth rates, can be found in Table 12 at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(7)^t$ and $k(7)^t$.

Table 12: Model 7 Capital Service Prices and Quantities Assuming Perfect Foresight

Year	u_{NR}	u_{ME}	u(7)	k_{NR}	k_{ME}	k(7)
1965	1.0000	1.0000	1.0000	1916	3069	4985
1970	1.6479	1.1424	1.3337	2490	4231	6700
1975	3.0509	1.5827	2.0991	3113	5761	8868
1980	3.8265	3.8772	3.9280	3967	8214	11973
1985	6.8722	2.8839	4.2359	4963	12039	16249
1990	8.9110	2.6052	4.5646	5720	18541	21748
1995	8.4034	2.0818	3.9808	6255	23985	25747
1999	8.7449	2.5168	4.4776	6764	32069	31237
Annual Growth Rates	1.0659	1.0275	1.0451	1.0378	1.0715	1.0555

The quantity units in Table 12 are in millions of 1965 Canadian dollars as usual. The growth rates for the two types of capital services, k_{NR} and k_{ME}, in Table 12 are exactly equal to the corresponding growth rates for the two types of stock, K_{NR} and K_{ME}, in Table 11; this follows from the fact that the corresponding quantity series are proportional to each other after normalization. Comparison of the growth rates in Table 12 with the corresponding growth rates in Table 8 (the corresponding straight line depreciation model) shows that there are some substantial differences. For example, the average annual geometric rate of growth for the user cost of machinery and equipment was 3.40% per year for the straight line model versus 2.75% per year for the geometric model. The geometric model rate of capital services price growth of 4.51% per year should be compared to the straight line model rate of capital services price growth of 4.96% per year which in turn can be compared to the average rate of price increase for the one hoss shay capital services aggregate which was somewhat higher at 4.85% per year; see Tables 2 and 8 above. The use of ex post asset inflation rates again leads to user costs that are extremely volatile; see Figure 11 below. On the quantity side, the flow of nonresidential construction capital services increased from \$1916 million to \$6764 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.78% while the flow of machinery and equipment capital services increased from \$3069 million to \$32,069 million (constant 1965) Canadian dollars, for an annual average growth rate of 7.15%. The capital services aggregate grew at an annual average growth rate of 5.55%

[72] Before normalization, the service flow aggregates k_{NR}^t and k_{ME}^t are exactly equal to the corresponding stock aggregates. Thus the rates of growth of the corresponding stock and flow variables will be the same.

compared to the 4.85% annual average growth rate for the aggregate capital stock. The geometric model average rate of capital services growth rate of 5.55% per year can be compared to the straight line growth rate of capital services of 5.30% per year and to the average rate of growth for the one hoss shay capital services aggregate of 5.61% per year; see Tables 2 and 8 above.

We turn now to our *second geometric depreciation model*, which will eliminate the volatility problem mentioned in the last paragraph. In this *Model 8*, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, it is assumed that producers use the current CPI inflation rate as estimators of anticipated asset inflation rates. In terms of computations, we simply replace the two ex post asset inflation rates, $i_{NR}{}^t$ and $i_{ME}{}^t$, by the CPI inflation rate ρ^t listed in Table A2 of the Appendix and then repeat all of the computations made to implement Model 7 above.

It turns out that the Model 8 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 7 counterparts in Table 7. This follows from equations (57), which show that the aggregate (over vintages) stock price is equal to the price of a new asset, which in turn does not depend on our assumptions about interest rates or expected asset inflation rates. Hence there is no need to table the capital stocks and prices for Model 8 (or Model 9 below). However, the Model 8 vintage user costs and capital service flows are very different from their Model 2 counterparts and slightly different from their Model 5 counterparts. Table 13 lists the Model 8 rental prices and flows of capital services for the geometric depreciation (constant real interest rate) Canadian capital stocks at 5 year intervals over the period 1965-1999.

Table 13: Model 8 Capital Service Prices and Quantities Assuming Constant Real Interest Rates

Year	$\mathbf{u_{NR}}$	$\mathbf{u_{ME}}$	**u(8)**	$\mathbf{k_{NR}}$	$\mathbf{k_{ME}}$	**k(8)**
1965	1.0000	1.0000	1.0000	3014	3379	6393
1970	1.2645	1.1079	1.1804	3917	4659	8568
1975	2.1003	1.5109	1.7786	4896	6344	11171
1980	3.0339	2.0847	2.5109	6241	9045	15050
1985	3.9615	1.9895	2.8254	7807	13257	20282
1990	4.8220	1.8632	3.0471	8997	20416	26723
1995	4.9283	1.6715	2.9316	9839	26411	31599
1999	5.3718	1.5843	2.9879	10640	35313	37855
Annual Growth Rates	1.0507	1.0136	1.0327	1.0378	1.0715	1.0537

When we compare the service prices and quantities in Table 13 (geometric depreciation model with constant real interest rates) with the corresponding service prices and quantities in Table 9 (straight line depreciation model with constant real rates), we see that the rental price series for each type of asset, $u_{NR}{}^t$ and $u_{ME}{}^t$, are identical. In both of these models, the vintage user costs are all proportional to the user cost for a new asset and hence the equality follows. However, the corresponding stocks that aggregate over vintages are not identical: the average annual geometric growth rate for nonresidential structures was 4.10% for the straight line model and 3.78% for the geometric model and the average annual growth rate for machinery and equipment was 6.96% for the straight line model and 7.15% for the geometric model. The overall annual rate of growth for capital services for the straight line model was 5.41% per year compared to 5.37% per year for the geometric model where both models assumed constant real interest rates. This is not a large difference. In Figure 11 below, it will be seen that the user cost that corresponds to the geometric model with constant real interest rates, u(8), is much less volatile than the corresponding geometric model that assumes perfect foresight, u(7).

We turn now to our *third geometric deprecation model*, which we call *Model 9*. In this model, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, we assume that they can anticipate the *trends* in asset inflation rates. In terms of computations, we use exactly the same program that we used to implement Model 7 except that we replace the rather volatile nominal interest rate r^t that was listed in Table A2 of the Appendix by the smoothed nominal interest rate that is listed in Table A3 of the Appendix. We also replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by their smoothed counterparts listed in Table A3 in the Appendix.

As mentioned earlier, the Model 9 constant real interest rate capital stocks (and prices) are *exactly* equal to their Model 7 counterparts in Table 7. Hence there is no need to table the capital stocks and prices for Model 9. However, the Model 9 vintage user costs are somewhat different from their Model 7 and 8 counterparts. Table 14 lists the Model 9 rental prices and flows of capital services for the Canadian capital stock at 5 year intervals over the period 1965-1999.

Table 14: Model 9 Capital Service Prices and Quantities using Smoothed Interest Rates and Smoothed Asset Inflation Rates

Year	u_{NR}	u_{ME}	u(9)	k_{NR}	k_{ME}	k(9)
1965	1.0000	1.0000	1.0000	2066	3477	5543
1970	1.5910	1.1349	1.3037	2685	4793	7449
1975	2.4785	1.6061	1.9186	3357	6527	9800
1980	4.5384	2.7596	3.4025	4278	9306	13254
1985	6.8189	2.4039	3.8579	5352	13639	17958
1990	8.9179	2.4171	4.4108	6168	21005	23981
1995	6.4151	1.7680	3.2143	6745	27172	28407
1999	7.5181	1.9554	3.6505	7294	36331	34482
Annual Growth Rates	1.0611	1.0199	1.0388	1.0378	1.0715	1.0552

When we compare the two capital services, k_{NR}^t and k_{ME}^t across the last three tables, it can be seen that they are identical and hence so are their growth rates. Hence when we aggregate across these two assets to form the Model 7,8 and 9 capital services aggregates, we find that the average annual geometric growth rates are quite similar: 5.55%, 5.37% and 5.52% respectively. When we compare the service prices and quantities in Table 14 (geometric depreciation model with smoothed asset inflation rates) with the corresponding service prices and quantities in Table 10 (straight line depreciation model with smoothed asset inflation rates), we see that the rental price series for each type of asset, $u_{NR}{}^t$ and $u_{ME}{}^t$, are no longer identical across the two models. Th geometric aggregate rental price grew at an annual geometric rate of 3.88% per year while the straight line aggregate rental price grew at a 4.31% per year rate. In Figure 11 below, it will be seen that the user cost that corresponds to the geometric model with smoothed asset inflation rates, u(9), is much less volatile than the corresponding geometric model that assumes perfect foresight, u(7), but the trend in each series is similar.

We finish off this section by graphing the aggregate price and quantity series for the Canadian capital stocks 1965-1999 (Figures 9 and 10 below) and for capital services (Figures 11 and 12 below) for our three geometric depreciation models. Since the price and quantity series for the capital stocks are all the same for our three geometric models (i.e., $P(7)^t = P(8)^t = P(9)^t$ and $K(7)^t = K(8)^t = K(9)^t$, in Figure 9, we graph P(7) with the three one hoss shay capital stock prices, P(1)-P(3) along with the single straight line depreciation capital stock price P(4) and in Figure 10, we graph K(7) with the three one hoss shay aggregate capital stocks, K(1), K(2), and K(3), along with the single straight line capital stock K(4).

Insert Figures 9,10,11,and 12.

Figure 9 provides a comparison of aggregate capital stock prices across all 9 models considered so far, but note that the 3 straight line models all have the same aggregate stock price P(4) and the 3 geometric models all have the same price P(7). It can be seen that there is a huge amount of dispersion in the prices of the 3 one hoss shay models: the P(1) series is the highest jagged curve (which uses ex post asset inflation rates), followed by P(3) (which uses smoothed ex post asset inflation rates), followed by P(2) (which uses a constant real interest rate). It can be seen that the P(3) curve is a smoothed version of the very nonsmooth P(1) curve. The constant real interest rate one hoss shay aggregate capital stock price series, P(2), is only slightly above the straight line depreciation price series, P(4), and the geometric depreciation rate capital stock price series, P(5). The last two series are virtually identical. Thus the aggregate capital stock prices for 7 of our 9 models are very close, with two of the one hoss shay models generating quite different numbers.

Figure 10 compares the aggregate capital stocks across the 9 models considered thus far. Again, we note that that the 3 straight line models all generate the same aggregate stocks K(4) and the 3 geometric models all generate the same stocks K(7). The 3 one hoss shay models generate the biggest aggregate capital stocks: the constant real interest rate series K(2) is the highest curve, followed by the smoothed asset inflation rates series K(3) and the ex post inflation rates series K(1) is just slightly below the corresponding smoothed series K(3). The straight line depreciation model aggregate capital stocks K(4) are next and finally, the geometric depreciation aggregate stocks K(7) are considerably below the other stock series.

Figures 11 and 12 plot the user costs and aggregate capital services that correspond to the 3 geometric depreciation models considered in this section. Figure 11 shows the tremendous volatility of the geometric user cost, u(7), that uses ex post asset inflation rates. The user cost model that uses smoothed asset inflation rates, u(9), is much smoother (but still quite volatile) and it captures the trends in the u(7) series. The geometric depreciation model that uses the assumption of constant real interest rates, u(8), lies considerably below the other two series. On the quantity side of things, the constant real interest rate model gives the highest capital services series, k(8), followed by the smoothed asset inflation rates series, k(9), and the perfectly anticipated asset inflation model generates the lowest curve, k(7).

We turn now to our fourth class of depreciation and relative efficiency model.

11. The Linear Efficiency Decline Model

Recall that our first class of models (the one hoss shay models) assumed that the efficiency (or cross section user cost) of the asset remained constant over the useful life of the asset. In our second class of models (the straight line depreciation models), we assumed that the cross section depreciation of the asset declined at a linear rate. In our third class of models (the geometric depreciation models), we assumed that cross section depreciation declined at a geometric rate. Comparing the third class with the second class of models, it can be seen that geometric depreciation is more *accelerated* than straight line depreciation; i.e., depreciation is relatively large for new vintages compared to older ones.[73] In this section, we will consider another class of models that gives rise to an accelerated pattern of depreciation: the class of models that exhibit *a linear decline in efficiency*.[74]

It is relatively easy to develop the mathematics of this model. Let f_0^t be the period t rental price for an asset that is new at the beginning of period t. If the useful life of the asset is L years and the efficiency decline is linear, then the sequence of period t cross sectional user costs f_n^t is defined as follows:

$$(66)\quad f_n^t \equiv f_0^t\,[L-n]/L\ ; \qquad n = 0,1,2,\ldots,L-1\ ;$$
$$\equiv 0 \qquad ; \qquad n = L,L+1,L+2,\ldots\,.$$

Now substitute (66) into the first equation in (5) and get the following formula for the rental price f_0^t in terms of the price of a new asset at the beginning of year t, P_0^t:

$$(67)\quad f_0^t = LP_0^t/[L + (L-1)(\gamma^t) + (L-2)(\gamma^t)^2 + \ldots + 1(\gamma^t)^{L-1}]$$

[73] See Figure 2 in Chapter 6 of the *Manual*.
[74] This class of models was also considered in Chapter 6 of the *Manual*. It was described as an asset whose age efficiency falls by a constant amount each year.

where the period t discount factor γ^t is defined in terms of the period t nominal interest rate r^t and the period t expected asset inflation rate i^t in the usual way:

$$(68)\ \gamma^t \equiv (1 + i^t)/(1 + r^t).$$

Now that f_0^t has been determined, substitute (67) into (66) and substitute the resulting equations into equations (5) and determine the sequence of period t vintage asset prices, P_n^t:

$$(69)\ P_n^t = P_0^t\,[(L-n) + (L-n-1)(\gamma^t) + ...+ 1(\gamma^t)^{L-1-n}]/[L + (L-1)(\gamma^t) + ...+ 1(\gamma^t)^{L-1}] \quad \text{for } n = 0,1,2,...,L-1$$
$$= 0 \quad \text{for } n = L, L+1,L+2,... .$$

Finally, use equations (8) to determine the end of period t rental prices, u_n^t, in terms of the corresponding beginning of period t rental prices, f_n^t:

$$(70)\ u_n^t = (1 + r^t)f_n^t\,; \qquad n = 0,1,2,...$$

Given the vintage asset prices defined by (69), we could use equations (12) above to determine the corresponding vintage cross section depreciation rates δ_n^t. We will not table these depreciation rates since our focus is on constructing measures of the capital stock and of the flow of services that the stocks yield. However, we will note that if we recall definition (10) for the period t cross section depreciation of an asset of vintage n, $D_n^t \equiv P_n^t - P_{n+1}^t$, and assume that the nominal interest rate r^t and the nominal asset inflation rate i^t are both zero, then using (69), it can be shown that

$$(71)\ D_n^t \equiv P_n^t - P_{n+1}^t = P_0^t\,[L - n]/[L(L+1)/2] \quad \text{for } n = 0,1,2,...,L;$$

i.e., when $r^t = i^t = 0$, *depreciation declines at a linear rate* for the linear efficiency decline model. When depreciation declines at a linear rate, the resulting formula for depreciation is called *the sum of the year digits formula*.[75] Thus just as the one hoss shay and straight line depreciation models coincide when $r^t = i^t = 0$, so too do the linear efficiency decline and sum of the digits depreciation models coincide.

In our *tenth Model*, we assume that producers exactly anticipate the asset inflation rates, i_{NR}^t and i_{ME}^t, for nonresidential construction and for machinery and equipment respectively. We use the Fisher ideal index to aggregate over vintages using formula (69) above for the vintage asset prices. Having constructed the aggregate price and quantity of nonresidential capital, P_{NR}^t and K_{NR}^t respectively, and the aggregate price and quantity of machinery and equipment, P_{ME}^t and K_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for the wealth stock, which we denote by $P(10)^t$ and $K(10)^t$; see Table 15 below.

[75] See Chapter 6 of the *Manual*. Canning (1929; 277) describes the method in some detail so it was already in common use by that time.

Table 15: Model 10 Capital Stocks and Prices Assuming Perfect Foresight

Year	P_{NR}	P_{ME}	P(10)	K_{NR}	K_{ME}	K(10)
1965	1.0000	1.0000	1.0000	29.6	15.0	44.7
1970	1.2877	1.1030	1.2234	38.6	21.4	60.0
1975	2.0296	1.4061	1.8021	48.0	29.8	77.3
1980	2.8906	2.1564	2.6312	60.8	43.0	102.0
1985	4.1906	2.0871	3.3621	75.7	63.5	133.7
1990	5.1187	1.9271	3.7770	86.1	98.2	166.8
1995	5.2982	1.7414	3.7552	92.5	124.8	188.4
1999	5.7521	1.7291	3.9621	98.5	166.6	215.7
Annual Growth Rates	1.0528	1.0162	1.0413	1.0360	1.0733	1.0474

The quantity units in Table 10 are in billions of 1965 Canadian dollars. The average rate of price increase for the linear efficiency decline capital aggregate was 4.13% per year, which is lower than the corresponding rate of aggregate price increase for the one hoss shay aggregate of 4.35% per year. On the quantity side, the stock of nonresidential construction capital increased from \$29.6 billion to \$98.5 billion (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.60% while the stock of machinery and equipment capital increased from \$15.0 billion to \$166.6 billion (constant 1965) Canadian dollars, for an annual average growth rate of 7.33%. Of course the levels of the capital aggregate are only about 2/3 to 3/4 of the corresponding one hoss shay levels due to the accelerated form of depreciation for the former model. The linearly declining efficiency capital aggregate grew at an annual average growth rate of 4.74%, which is lower than the corresponding rate of growth for the one hoss shay aggregate of 4.95%.

Using equations (66), (67) and (70) along with the data tabled in Appendix 1 (see Tables A1 and A2), we can construct the end of the period user costs for each of our 39 vintages of nonresidential construction capital. As usual, use equation (38) to construct the service flow aggregate for nonresidential construction for each year. Then we use (42) (where L equals 39) to aggregate over the 39 vintages of nonresidential capital using the Fisher (1922) ideal index number formula and form the aggregate rental price for nonresidential construction, u_{NR}^t, and the corresponding services aggregate, k_{NR}^t, for the years 1965-1999.[76] These series, along with their annual average (geometric) growth rates, can be found in Table 16 at 5 year intervals. Similarly, we use (42) above (where L equals 14) and aggregate over the 14 vintages of machinery and equipment using the Fisher ideal index number formula and form aggregate capital services price and quantity series, u_{ME}^t and k_{ME}^t, for the years 1965-1999. These series, along with their annual average (geometric) growth rates, can also be found in Table 16 at 5 year intervals. Having constructed the aggregate price and quantity of nonresidential capital services, u_{NR}^t and k_{NR}^t respectively, and the aggregate price and quantity of machinery and

[76] Since all of the vintage rental prices are proportional to each other, again Hicks' (1939; 312-313) aggregation theorem implies that all of the usual indexes are equal to each other.

equipment services, u_{ME}^t and k_{ME}^t respectively, we may again use the Fisher ideal formula and aggregate these two series into a single aggregate price and quantity series for capital services, which we denote by $u(10)^t$ and $k(10)^t$.

Table 16: Model 10 Capital Service Prices and Quantities Assuming Perfect Foresight

Year	u_{NR}	u_{ME}	u(10)	k_{NR}	k_{ME}	k(10)
1965	1.0000	1.0000	1.0000	2066	3162	5227
1970	1.5517	1.1333	1.2978	2721	4305	7012
1975	2.8288	1.5639	2.0336	3444	5949	9366
1980	3.6236	3.6046	3.6849	4394	8582	12716
1985	6.2794	2.7018	3.9775	5498	12744	17337
1990	8.1234	2.4488	4.3007	6346	19468	23072
1995	7.6853	1.9864	3.7872	6945	25405	27418
1999	8.0210	2.3442	4.2092	7467	33554	32915
Annual Growth Rates	1.0632	1.0254	1.0432	1.0385	1.0719	1.0556

The quantity units in Table 16 are in millions of 1965 Canadian dollars. It can be seen that the price of a unit of nonresidential construction capital services increased by 6.32% per year and the price of a unit of machinery and equipment capital services increased by 2.54% per year on average. The average rate of price increase for the linearly declining efficiency capital services aggregate was 4.32% per year, which is much less than the corresponding rate of price increase for the one hoss shay aggregate capital services price, which was 4.85% per year. On the quantity side, the flow of nonresidential construction capital services increased from \$2066 million to \$7467 million (constant 1965) Canadian dollars, for an annual average (geometric) growth rate of 3.85% while the flow of machinery and equipment capital services increased from \$3162 million to \$33,554 million (constant 1965) Canadian dollars, for an annual average growth rate of 7.19%. The capital services aggregate grew at an annual average growth rate of 5.56% compared to the 5.61% annual average growth rate for the corresponding one hoss shay capital services. As usual, the linear efficiency decline user costs u(10) that are based on the assumption of perfect foresight are very volatile; see Figure 15 below.

We turn now to our *second linear efficiency decline model*, which will eliminate the volatility problem mentioned in the last paragraph. In this *Model 11*, instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, it is assumed that producers use the current CPI inflation rate as estimators of anticipated asset inflation rates. This model turns out to be equivalent to the constant real interest rate model that was used throughout the main *Manual*. As usual, in terms of computations, we simply replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by the CPI inflation rate ρ^t listed in Table A2 of the Appendix and then repeat all of the computations made to implement Model 10 above. The counterparts to Tables 15 and 16 are Tables 17 and 18 below; Table 17 lists the *Model 11* prices and quantities for the Canadian capital stock at

5 year intervals over the period 1965-1999 while Table 18 lists the rental prices and flows of capital services over this same period.

Table 17: Model 11 Capital Stocks and Prices Assuming Constant Real Interest Rates

Year	P_{NR}	P_{ME}	P(11)	K_{NR}	K_{ME}	K(11)
1965	1.0000	1.0000	1.0000	31.4	15.3	46.7
1970	1.2476	1.0932	1.1953	41.0	21.8	62.7
1975	1.9319	1.3898	1.7421	51.0	30.4	80.8
1980	2.8122	1.9324	2.4990	64.6	43.8	106.6
1985	3.8861	1.9516	3.1443	80.3	64.8	139.5
1990	4.6936	1.8136	3.5189	91.4	100.5	173.7
1995	4.9227	1.6696	3.5512	98.1	127.4	195.9
1999	5.3871	1.5889	3.7215	104.5	170.3	224.0
Annual Growth Rates	1.0508	1.0137	1.0394	1.0360	1.0734	1.0472

The Model 11 capital stock quantities are very similar to the Model 10 quantities. The overall average growth rate for the price of the aggregate stock is a bit higher for Model 10 (4.13% per year) than for Model 11 (3.93% per year).

Table 18: Model 11 Capital Service Prices and Quantities Assuming Constant Real Interest Rates

Year	u_{NR}	u_{ME}	u(11)	k_{NR}	k_{ME}	k(11)
1965	1.0000	1.0000	1.0000	2988	3400	6388
1970	1.2645	1.1079	1.1806	3936	4630	8561
1975	2.1003	1.5109	1.7797	4982	6398	11311
1980	3.0339	2.0847	2.5124	6357	9229	15334
1985	3.9615	1.9895	2.8262	7954	13706	20798
1990	4.8220	1.8632	3.0462	9181	20937	27339
1995	4.9283	1.6715	2.9299	10047	27323	32487
1999	5.3718	1.5843	2.9851	10802	36086	38591
Annual Growth Rates	1.0507	1.0136	1.0327	1.0385	1.0719	1.0543

The one hoss shay capital services aggregate that assumes constant real interest rates, k(2), is quite close to the linear efficiency decline capital services aggregate that assumes constant real interest rates, k(11), and their average annual geometric growth rates are also close: 5.49% for k(2) versus 5.43% for k(11). However, k(11) is 15 to 20% bigger in levels than the first linear efficiency decline capital services aggregate k(10), which assumed that anticipated asset inflation rates were equal to ex post rates. The average annual geometric growth rate for k(10) was somewhat higher at 5.56% per year.

We turn now to our *third linear efficiency decline model*. In this model (*Model 12*), instead of assuming that producers correctly anticipate each year's ex post asset inflation rates, we assume that they can anticipate the *trends* in asset inflation rates. In terms of computations, we use exactly the same program that we used to implement Model 10 except that we replace the rather volatile nominal interest rate r^t that was listed in Table A2 of the Appendix by the smoothed nominal interest rate that is listed in Table A3 of the Appendix. We also replace the two ex post asset inflation rates, i_{NR}^t and i_{ME}^t, by their smoothed counterparts listed in Table A3 in the Appendix. The counterparts to Tables 15 and 16 are Tables 19 and 20 below; Table 19 lists the Model 12 prices and quantities for the Canadian capital stock at 5 year intervals over the period 1965-1999 while Table 20 lists the rental prices and flows of capital services over this same period.

Table 19: Model 12 Capital Stocks and Prices using Smoothed Interest Rates and Inflation Rates

Year	P_{NR}	P_{ME}	P(12)	K_{NR}	K_{ME}	K(12)
1965	1.0000	1.0000	1.0000	29.9	15.4	45.2
1970	1.2840	1.0987	1.2194	38.9	21.9	60.7
1975	1.9774	1.4102	1.7719	48.4	30.4	78.2
1980	2.9749	2.0299	2.6291	61.3	43.9	103.3
1985	4.1990	2.0190	3.3350	76.3	64.9	135.4
1990	5.1418	1.9017	3.7765	86.8	100.4	168.8
1995	5.0738	1.6921	3.6153	93.4	127.7	190.8
1999	5.6233	1.6546	3.8497	99.4	170.3	218.4
Annual Growth Rates	1.0521	1.0149	1.0404	1.0360	1.0733	1.0474

Comparing the numbers in Tables 15, 17 and 19, it can be seen that there are some small differences between the capital stocks generated by our three variants of the linear efficiency decline model but the average growth rates are virtually identical. There is more variation across the three models in the movement of the stock prices with Model 10 giving the highest rate of price growth for the capital aggregate (4.13% per year), followed by Model 12 (4.04% per year) and then Model 11 (3.94% per year). The Model 10, 11 and 12 aggregate prices of capital are graphed in Figure 13 below and the corresponding aggregate quantities are graphed in Figure 14 below.

Table 20: Model 12 Capital Service Prices and Quantities using Smoothed Interest Rates and Smoothed Asset Inflation Rates

Year	u_{NR}	u_{ME}	u(12)	k_{NR}	k_{ME}	k(12)
1965	1.0000	1.0000	1.0000	2186	3481	5667
1970	1.5179	1.1288	1.2788	2879	4740	7601
1975	2.3823	1.5817	1.8813	3643	6551	10122
1980	4.2462	2.6535	3.2554	4649	9450	13766

1985	6.3349	2.3309	3.7107	5817	14033	18747
1990	8.2766	2.3269	4.2349	6714	21436	24900
1995	6.0823	1.7447	3.1581	7348	27974	29607
1999	7.0720	1.8905	3.5360	7900	36947	35553
Annual Growth Rates	1.0592	1.0189	1.0378	1.0385	1.0719	1.0555

Comparing the numbers in Tables 16, 18 and 20, it can be seen that there are large differences in the levels and small differences in the growth rates for capital services generated by the 3 models: the average annual geometric growth rates for k(10), k(11) and k(12) are 5.56%, 5.43% and 5.55% per year. 4.32%, 3.27% and 4.04% per year respectively. However, there is much more variation across the three models in the movement of the service prices with Model 10 giving the highest rate of price growth for the capital services aggregate (4.32% per year), followed by Model 12 (3.78% per year) and then Model 11 (3.27% per year). The Model 10, 11 and 12 aggregate prices of capital services are graphed in Figure 15 below and the corresponding aggregate quantities of services are graphed in Figure 16 below.

Insert Figures 13, 14 ,15 and 16

Viewing Figure 13, we see that the P(10) curve is the highest and the most wiggly; this is the aggregate capital stock price series that assumes that anticipated asset inflation rates are equal to the actual ex post rates. Smoothing these volatile asset inflation rates leads to the P(12) curve, which is much smoother and captures the trend in P(10). The constant real interest aggregate capital stock price series P(11) lies substantially below the other two series.

Viewing Figure 14, we see that all three of the linear efficiency decline models generate the same aggregate wealth capital stock to a high degree of approximation. For the record, the K(11) series is the top curve, the K(12) series is in the middle and the K(10) series is the lowest curve.

Viewing Figure 15, we see that the aggregate linear efficiency decline user cost series u(10), which assumes that anticipated asset inflation rates are equal to the actual ex post rates, is the highest very volatile curve. Smoothing these volatile asset inflation rates leads to the u(12) curve, which is much smoother and captures the trend in u(10). The constant real interest rate user cost series, u(11), lies far below the other two aggregate user cost series for much of the sample period.

Figure 16 plots the three linear efficiency decline aggregate capital services series, k(10)-k(12). Each of these series is reasonably smooth but note that they are spread out much more than the corresponding aggregate capital stock series, K(10)-K(12), that were plotted in Figure 14. *Thus the different assumptions on anticipated asset price movements generate substantially different measures of capital services for these linear efficiency decline models*. The constant real interest rate series, k(11), is the top curve, followed by the smoothed asset inflation rates model, k(12), and the ex post asset inflation rates model, k(10), is the lowest curve.

We turn now to a class of models that has not been formally considered in the literature but which relates in a very interesting way to the models presented in this section.

12. The Linearly Increasing Maintenance Expenditures Model

Many years ago, the accountant Canning raised the following interesting problem that bears on our topic:[77]

> "By spending enough for parts replacements (repairs), it is possible to keep any machine running for an indefinitely great length of time, but it does not *pay* to do so. Query: How does one know just when a machine is worn out?" John B. Canning (1929; 251).

In other words, Canning notes that the choice of when to retire an asset is really an *endogenous* decision rather than an *exogenous* one as we have assumed up to now. In this section, we attempt to model the retirement decision in a preliminary way using the concept of a *maintenance profile*.

For most new machines and new structures, engineers are able to devise a *maintenance schedule* that will ensure that the asset delivers its services during the period under consideration. Thus in the Queensland Competition Authority (2000; Chapter 13),[78] a schedule of costs per kilometer of rail track that is required to keep the rails in working order as a function of the age of the track is laid out. These maintenance expenditures will enable the track to deliver transportation services over its lifetime. This schedule has a fixed cost aspect to it and then as the track ages, the maintenance expenditures increase linearly up to a certain point and then flatten out. Similarly, a new truck will have a schedule of recommended maintenance operations that the owner is urged to follow. In addition to these maintenance expenditures, we could also include operating costs like fuel and driver inputs since these inputs are necessary to deliver ton miles of output. Finally, an office building will also have maintenance expenditures associated with it and some operating expenditures such as heat since the renters of offices typically want square meters of space maintained at a comfortable temperature. In any case, we assume that at the beginning of period t, we know the period t maintenance and operating expenditures necessary to operate an asset that is n periods old at the beginning of period t, m_n^t, n = 0,1,2,... .We say that $\{m_n^t\}$ is the *period t (cross section) maintenance profile*.

---

[77] Matheson (1910; 76-77) raises the same sort of issues in a less focussed manner: "But this principle has to be applied with considerable qualification where repairs really renew the life of a machine and prolong greatly its period of useful work. For instance, a locomotive during its life may have its wheel tires renewed four times, its boiler three times, and be painted seven times, so that before the framework, the wheels and other more durable parts fail, and the engine is broken up, much more than its original cost will have been expended on it. The value of any such serious renewals of this kind should be duly credited in a proper system of depreciation. Another course, followed more often in the United States than in Great Britain, is to prefer the substitution of new machines with all modern improvements rather than to renew or repair old plant, which even rendered serviceable may not be so economical in working."

[78] Euan Morton of the Queensland Competition Authority brought this work to the author's attention.

We now have to distinguish between the *gross and net rental prices* of an asset that is n periods old at the beginning of period t, g_n^t and f_n^t, respectively. An office or an apartment is typically rented on a gross basis; i.e., the tenant rents an office that is t periods old at the beginning of period t and pays the gross rent g_n^t at the beginning of the period and the landlord is responsible for the period t maintenance costs m_n^t.[79] On the other hand, a truck (on a long term lease) is usually rented on a net basis; i.e., the user of the truck is responsible for operating costs and maintenance. In any case, the relationship between the gross and net rental prices is:

$$(72)\ g_n^t = f_n^t + m_n^t \ ; \qquad\qquad n = 0,1,2,\ldots .$$

Thus our present notation is consistent with our previous notation where we valued an asset by the discounted stream of its net rentals; i.e., previously, we used the cross section profile of net rentals f_n^t (extrapolated to future periods) in order to value vintage assets.

Our new vintage asset valuation equation is the following equation, which gives the value of a new asset at the beginning of period t, assuming that the asset will be retired after L periods of use:

$$\begin{aligned}(73)\ P_0^t(L) \equiv\ & g_0^t + (\gamma^t)\, g_1^t + (\gamma^t)^2\, g_2^t + \ldots + (\gamma^t)^{L-1}\, g_{L-1}^t \\ & - [m_0^t + (\beta^t)\, m_1^t + (\beta^t)^2\, m_2^t + \ldots + (\beta^t)^{L-1}\, m_{L-1}^t]\end{aligned}$$

where the discount factors γ^t and β^t are defined as follows:[80]

$$(74)\ \gamma^t \equiv (1+i^t)/(1+r^t)\ ;\ \beta^t \equiv (1+\alpha^t)/(1+r^t).$$

As in the previous sections, i^t is *the one period anticipated inflation rate for the services of the asset* at the beginning of period t, r^t is *the period t nominal interest rate* and hence γ^t is the same discount factor that has appeared in previous sections. A new parameter is α^t, which is *the one period anticipated inflation rate for maintenance (and operating cost) services* and so β^t is the counterpart to γ^t except that β^t is the discount factor that applies to future anticipated *costs* while γ^t is the discount factor that applies to future anticipated *gross revenues*.

The interpretation of equation (73) is straightforward: a new asset that is to be used for L periods is equal to the discounted stream of the gross rentals that it is expected to yield minus the discounted stream of expected maintenance and operating costs.

We now evaluate equation (73) for L = 1,2,... , and pick the L which gives the highest value of $P_0^t(L)$. We call this optimal value L*. Once the optimal L* has been

[79] We assume that these costs are converted to beginning of period t costs using present values if necessary.
[80] We could have presented a more general equation than (73); i.e., we could have presented a counterpart to the very general equation (2) in section 3 above, involving the anticipated asset inflation rates i_n^t, the one period nominal interest rates r_n^t and another set of anticipated maintenance cost inflation rates α_n^t. However, we would soon be forced to impose the simplifying assumptions (3) and (4) along with a new set of simplifying assumptions, $\alpha_n^t = \alpha^t$.

determined, then if used asset markets are in equilibrium, the sequence of period t vintage asset prices P_n^t , the sequence of period t vintage gross rental prices g_n^t and the sequence of period t cross section maintenance costs m_n^t should satisfy the following system of equations:

$$
\begin{aligned}
(75)\quad P_0^t &= g_0^t + (\gamma^t)\, g_1^t + (\gamma^t)^2\, g_2^t + ...+ (\gamma^t)^{L-1}\, g_{L*-1}^t \\
&\quad - [m_0^t + (\beta^t)\, m_1^t + (\beta^t)^2\, m_2^t + ...+ (\beta^t)^{L-1}\, m_{L*-1}^t] \\
P_1^t &= g_1^t + (\gamma^t)\, g_2^t + (\gamma^t)^2\, g_3^t + ...+ (\gamma^t)^{L-1}\, g_{L*-2}^t \\
&\quad - [m_1^t + (\beta^t)\, m_2^t + (\beta^t)^2\, m_3^t + ...+ (\beta^t)^{L-1}\, m_{L*-2}^t] \\
&\ldots \\
P_{L*-2}^t &= g_{L*-2}^t + (\gamma^t)\, g_{L*-1}^t - [m_{L*-2}^t + (\beta^t)\, m_{L*-1}^t\,] \\
P_{L*-1}^t &= g_{L*-1}^t - m_{L*-1}^t.
\end{aligned}
$$

Equations (75) are the counterparts to our earlier system of equations (5). Given the cross section gross rental prices g_n^t and the cross section maintenance costs m_n^t (and the discount factors γ^t and β^t), we can determine the period t vintage asset prices P_n^t using equations (75). Given the vintage asset prices P_n^t and the cross sectional maintenance costs m_n^t, we can also use equations (75) to determine the vintage gross rental prices g_n^t: start with the last equation in (75) and determine g_{L*-1}^t; then move up to the second last equation and determine g_{L*-2}^t; etc. Of course, once the g_n^t have been determined, then we may use equations (72) to determine the net rental prices (or vintage user costs) f_n^t and then we can use these f_n^t as weights for the vintage capital stocks and construct a measure of capital services as in the previous sections. Thus equations (75) are indeed the key equations in this section.

Unfortunately, in general, we cannot derive counterparts to equations (6) using equations (75). To see why this is so, look at the first equation in (75) and try to convert it into a counterpart to the first equation in (6):

$$
\begin{aligned}
(76)\quad P_0^t &= g_0^t - m_0^t + \gamma^t\, [g_1^t + \gamma^t g_2^t + ...+ (\gamma^t)^{L-2} g_{L*-1}^t] - \beta^t[m_1^t + \beta^t m_2^t + ...+ (\beta^t)^{L-2} m_{L*-1}^t] \\
&\neq g_0^t - m_0^t + \gamma^t\, P_1^t.
\end{aligned}
$$

It is also the case that we no longer have the simple relationship between anticipated time series depreciation that we derived in equations (24) above. Put another way, suppose all expectations held at the beginning of period t turned out to be true. Under this assumption, we can derive the following relationship between the price of a one period old asset at the beginning of period t, P_1^t, and the price of a one period old asset at the beginning of period t+1, P_1^{t+1}, as follows:

$$
\begin{aligned}
(77)\quad P_1^{t+1} &= (1+i^t)[g_1^t + \gamma^t g_2^t + ...+ (\gamma^t)^{L-2} g_{L*-1}^t] - (1+\alpha^t)[m_1^t + \beta^t m_2^t + ...+ (\beta^t)^{L-2} m_{L*-1}^t] \\
&\neq \quad (1+i^t)P_1^t.
\end{aligned}
$$

However, if we assume that the period t anticipated rental price escalation factor $1+i^t$ is equal to the period t anticipated maintenance cost escalation factor $1+\alpha^t$ so that γ^t is equal to β^t, then the two inequalities (76) and (77) become equalities. Hence, to make further progress[81], we make the following simplifying assumption:

(78) $\alpha^t = i^t$ or $\gamma^t = \beta^t$.

Using assumption (78), we can rewrite equations (75) as follows:

$$
\begin{aligned}
(79)\quad P_0{}^t &= g_0{}^t - m_0{}^t + \gamma^t P_1{}^t && = f_0{}^t + [(1+i^t)/(1+r^t)]P_1{}^t \\
P_1{}^t &= g_1{}^t - m_1{}^t + \gamma^t P_2{}^t && = f_1{}^t + [(1+i^t)/(1+r^t)]P_2{}^t \\
&\ldots && \ldots \\
P_{L^*-2}{}^t &= g_{L^*-2}{}^t - m_{L^*-2}{}^t + \gamma^t P_{L^*-1}{}^t && = f_{L^*-2}{}^t + [(1+i^t)/(1+r^t)]P_{L^*-1}{}^t \\
P_{L^*-1}{}^t &= g_{L^*-2}{}^t - m_{L^*-2}{}^t + 0 && = f_{L^*-1}{}^t + 0
\end{aligned}
$$

where the second set of equations follows using equations (72). Note that equations (79) are exact counterparts to equations (6) in section 3 above.

Obviously, equations (79) can be rewritten to give us explicit formulae for the gross rental prices $g_n{}^t$ in terms of the period t vintage asset prices $P_n{}^t$ and the period t vintage maintenance costs $m_n{}^t$:

$$(80)\ g_n{}^t = m_n{}^t + P_n{}^t - [(1+i^t)/(1+r^t)]P_{n+1}{}^t\ ; \qquad n = 0,1,2,\ldots,L^*-1.$$

Equations (80) are exact counterparts to our earlier system of equations (7) for the period t user costs, $f_n{}^t$.

In order to get a useful, explicit depreciation model, we make some further assumptions:

(81) $\gamma^t \equiv (1+i^t)/(1+r^t) = \gamma$ for all periods t;
(82) $g_n{}^t = \lambda^t g$ for all periods t and $n = 0,1,2,\ldots$;
(83) $m_n{}^t = \lambda^t [b+nc]$ for all periods t and $n = 0,1,2,\ldots$

where g, b and c are positive parameters with $g > b$; i.e., the gross rental must be greater than the fixed maintenance cost. We now explain the meaning of assumptions (81)-(83). Assumption (81) means *that the real interest rate is constant* over all periods. Assumption (82) is *a one hoss shay type assumption* except that it is applied to the gross output of the asset; i.e., (82) means that the gross services yielded by a properly maintained asset of any vintage in period t is the same across all vintages. Assumption (83) says that the period t vintage maintenance costs have a *fixed cost component* that is the same across all vintages, $\lambda^t b$, plus another component that *increases linearly* in the age of the asset, $\lambda^t nc$ for an asset that is n periods old at the start of period t. The presence of the scalar factor λ^t in both (82) and (83) means that we are assuming that period t rental prices $g_n{}^t$ and maintenance costs $m_n{}^t$ are essentially constant except for a common period t *inflation factor* λ^t.

[81] We do this in order to obtain a simpler set of relations between $g_n{}^t$, $m_n{}^t$ and $P_n{}^t$ than the rather complex system of relations defined by equations (75).

Now substitute assumptions (78) and (81)-(83) into (73) and obtain the following expression for the function $P_0^t(L)$, which gives the anticipated asset value of a new asset as a function of the number of periods L that it is used:[82]

$$(84)\ P_0^t(L) \equiv g_0^{\,t} + (\gamma^t)\, g_1^{\,t} + (\gamma^t)^2\, g_2^{\,t} + ...+ (\gamma^t)^{L-1}\, g_{L-1}^{\,t} - [m_0^{\,t} + (\beta^t)\, m_1^{\,t} + (\beta^t)^2\, m_2^{\,t} + ...+ (\beta^t)^{L-1}\, m_{L-1}^{\,t}]$$
$$= \lambda^t\,[g - b][1 + \gamma + \gamma^2 + ...+ \gamma^{L-1}] - \lambda^t\, c[1 + 2\gamma + 3\gamma^2 + ...+ (L-1)\gamma^{L-2}].$$

Now reparameterize the positive parameter c as follows:

$$(85)\ c \equiv [g - b]d$$

where d is another positive parameter. Substitute (85) into (84) and (84) can be rewritten as follows:

$$(86)\ P_0^t(L) = \lambda^t\,[g - b]h(L)$$

where the function h(L) is defined as

$$(87)\ h(L) \equiv [1 + \gamma + \gamma^2 + ...+ \gamma^{L-1}] - d\,[1 + 2\gamma + 3\gamma^2 + ...+ (L-1)\gamma^{L-2}].$$

Provided that d is small enough and the real interest rate escalation factor γ is close to one, a positive integer L* that maximizes h(L) will exist. This exercise determines the optimal age of retirement of a new asset.

However, we now reverse the argument: given an L*, we look for a positive parameter d such that h(L) will be at a maximum when L = L*. This can be done numerically. For $\gamma = 1/(1.04)$ (this corresponds to a real interest rate of 4%, our standard assumption), we found that the d that corresponded to L* = 39 was approximately equal to 0.002. This in turn corresponds to the assumption that maintenance costs for nonresidential structures were rising at the rate of 0.2 percentage points per year. The d that corresponded to L* = 14 was approximately equal to 0.012. This in turn corresponds to the assumption that maintenance costs for machinery and equipment were rising at the rate of 1.2 percentage points per year.

Once the d* that corresponds to the desired asset life L* has been found, then the function h(L) defined by (87) is known. Now set the right hand side of equation (86) (evaluated at L = L*) equal to the price of a new asset at the beginning of period t, P_0^t, and solve the resulting equation for $\lambda^t\,[g - b]$. The solution is:

[82] By examining (84), it can be seen that as b and c increase (so that either the fixed cost component or the rate of increase in maintenance costs increases), then the optimal age of retirement L* will decrease. Conversely, as g increases (so that the gross revenue yielded by the asset exogenously increases), then the optimal age of retirement will increase.

(88) $\lambda^t [g - b] = P_0{}^t/h(L^*)$.

We now have enough information to evaluate the sequence of period t net rental prices, $f_n{}^t$, as follows:[83]

$$
\begin{aligned}
(89)\ f_n{}^t &= g_n{}^t - m_n{}^t && n = 0,1,2,\ldots,L-1 \text{ using (72)} \\
&= \lambda^t g - \lambda^t [b+nc] && \text{using (82) and (83)} \\
&= \lambda^t [g - b] - \lambda^t n[g - b]d && \text{using (85)} \\
&= \lambda^t [g - b] [1 - nd] && \text{rearranging terms} \\
&= P_0{}^t [1 - nd]/h(L^*) && \text{using (88).}
\end{aligned}
$$

If the price $P_0{}^t$ of a new asset is known, then everything on the right hand side of (89) is known and the sequence of net rental prices $f_n{}^t$ can be calculated. Once the $f_n{}^t$ are known, then the second set of equations in (79) can be used in order to obtain the sequence of vintage asset prices, $P_n{}^t$: given $P_0{}^t$ and $f_0{}^t$, use the first equation in (79) to determine $P_1{}^t$; then use $f_1{}^t$ and the second equation to determine $P_2{}^t$ and so on.

We will not table the results of this linearly increasing maintenance expenditures model because it turned out to be virtually equivalent to Model 11 in the previous section. The explanation for this result is contained in equations (89): these equations show that *the (net) user costs decline linearly as older assets are used*. This is precisely the assumption made in Model 11 in the previous section, which also assumed constant real interest rates.[84]

The importance of the model presented in this section is that it casts some light on the conditions under which we might expect net rental prices to decline in a linear fashion even though we know the asset is of the *gross* one hoss shay type; i.e., an older truck can deliver the same ton miles in a period as a younger one provided that we spend enough on maintenance. Thus the simplified model presented at the end of this section provides a justification for assuming a quite accelerated form of depreciation, even though the asset essentially delivers one hoss shay type services. Put another way, in the context of assets that are capable of delivering the same services as they age, then if maintenance costs rise as the asset ages, accelerated depreciation is inevitable.[85] The more general model presented at the beginning of this session could also be used in regulatory contexts where maintenance schedules often exist and the determination of "economic" depreciation is a matter of some importance.

[83] We do not have enough information to obtain the sequence of gross rental prices, $g_n{}^t$, but, fortunately, it is the net rental prices that we need in order to calculate a capital services aggregate.

[84] It is not clear that the user costs defined by (89) decline down to zero as we reach age L*. Empirically, this proved to be the case and so the results of this model were virtually the same as those of Model 11.

[85] The results in this section also enable us to reinterpret the geometric depreciation model, which is often interpreted as an asset evaporation model; i.e., each period, a fraction of the existing stock of assets simply "evaporates". However, now we see that the asset may in fact be delivering a constant amount of gross services but a certain pattern of increasing maintenance costs is in fact causing used asset prices to have the profile implied by geometric depreciation (up to some limiting age).

The simple model presented at the end of this section may also help to explain why there is tremendous diversity in the ages at which identical assets are retired in different countries. For example, if maintenance costs are higher or are expected to rise more quickly in a particular country, then the model implies that identical assets in that country will be retired at an earlier age. This observation can help to explain why well maintained assets in developing countries are used much longer than in developed countries. Conversely, assets employed in a country enjoying a boom so that gross rental prices are relatively high will be retired at a later age than assets employed in a country experiencing relatively low rental rates, other things being equal. If future asset rental rates are expected to decline or increase less rapidly than future maintenance costs (i.e., i^t increases less than α^t), then the expected future gross revenues will decline or grow less rapidly than expected future operating costs and the asset will be retired earlier. Thus the models presented in this section can cast some light on why the same asset is retired at different ages across countries and uses.[86]

In the following section, we make some graphical comparisons across our 12 models.

13. A Comparison of the Twelve Models

In this section, we will compare stock prices and user costs across our four types of model that are based on alternative assumptions about the structure of depreciation or asset efficiency, holding constant our assumptions about nominal interest rates and anticipated asset price movements. We will also compare capital stocks and service flows across depreciation and relative efficiency models, holding constant our assumptions about nominal interest rates and anticipated asset price movements.

Figure 17 plots the aggregate capital stock prices generated by our four classes of depreciation and efficiency models, assuming that ex post asset price movements are perfectly anticipated. Note the volatility of these series. The one hoss shay stock prices P(1) are the highest, followed by the linear efficiency decline prices P(10). The straight line and geometric depreciation prices, P(4) and P(7), are the lowest and are very close to each other.

Figure 18 plots the aggregate capital stock prices generated by our four classes of depreciation and efficiency models, assuming that ex post asset price changes are equal to changes in the consumer price index. This model assumes a constant real interest rate of 4 per cent. These stock prices are much smoother than those exhibited in Figure 17 and they are also much closer to each other. The one hoss shay and linear efficiency decline prices, P(2) and P(11), are virtually indistinguishable on the top, followed by the straight line depreciation prices P(5) and then followed very closely by the geometric stock prices P(8).

[86] We have just scratched the surface in exploring the implications of this class of models. These models should also be imbedded in a more formal intertemporal production model; see Hicks (1939; Chapter 15) and Diewert (1980; 472-475).

Figure 19 plots the aggregate capital stock prices generated by our four classes of depreciation and efficiency models, assuming that anticipated asset price changes are equal to smoothed ex post asset price changes. These stock price series smooth out considerably the much rougher series exhibited in Figure 17. The one hoss shay stock prices P(3) are the highest, followed by the linear efficiency decline prices P(12). The straight line and geometric depreciation prices, P(6) and P(9), are the lowest and are very close to each other.

Figure 20 plots the aggregate capital stocks that correspond to the smoothed asset price change model; i.e., Figure 20 is the quantity counterpart to Figure 19. The one hoss shay capital stock curve K(3) is the highest, followed by the straight line depreciation curve K(6), which in turn is followed by the geometric depreciation curve K(9). The linear efficiency decline stock K(12) is the lowest curve. These results are intuitively plausible: the one hoss shay model has the least accelerated form of depreciation, followed by the straight line model, followed by the geometric depreciation model and the linear efficiency decline model generates the most accelerated form of depreciation. In an economy where investment is growing over time, the capital stocks corresponding to the least accelerated form of depreciation will grow the quickest, followed by the more accelerated forms and the capital stock corresponding to the most accelerated form of depreciation will grow the slowest. Figures 25 and 26 plot the aggregate capital stocks that correspond to the perfectly anticipated asset price change and the constant real interest rate models: the results are much the same as those exhibited in Figure 20.

Insert Figures 17, 18, 19 and 20

Figure 21 plots the aggregate user costs generated by our four classes of depreciation and efficiency models, assuming that ex post asset price movements are perfectly anticipated. Note that the user cost series in Figure 21 are even more volatile that the capital stock prices charted in Figure 17. The one hoss shay and straight line depreciation user costs, u(1) and u(4), are the highest, followed by the geometric depreciation and linear efficiency decline user costs, u(7) and u(10).

Figure 22 plots the aggregate user costs generated by our four classes of depreciation and efficiency models, assuming that ex post asset price changes are equal to changes in the consumer price index. This model assumes a constant real interest rate of 4 per cent. These user costs are much smoother than those exhibited in Figure 21 and they are also much closer to each other. The straight line depreciation user costs u(5) are on top, followed by the one hoss shay, geometric and linear efficiency decline user costs, u(2), u(8) and u(11), which are too close to each other to be distinguished visually.

Figure 23 plots the aggregate user costs generated by our four classes of depreciation and efficiency models, assuming that anticipated asset price changes are equal to smoothed ex post asset price changes. These user cost series smooth out considerably the much rougher series exhibited in Figure 21. The straight line and one hoss shay user costs, u(6) and u(3), are very close to each other on top but near the end of our sample period, the one hoss shay user costs u(3) dip below the straight line depreciation user costs u(6). The geometric depreciation and linear efficiency decline user costs, u(9) and u(12), are fairly close to each other on the bottom. These two models represent the most accelerated forms of depreciation.

Figure 24 plots the aggregate capital services that correspond to the smoothed asset price change model; i.e., Figure 24 is the quantity counterpart to Figure 23. The linear efficiency decline capital services curve k(12) is the highest, followed closely by the geometric depreciation and one hoss shay curves, k(9) and k(3), which are very close to each other. The straight line depreciation curve k(6) is the lowest curve.

Insert Figures 21, 22, 23 and 24

Figures 27 and 28 plot the aggregate capital services that correspond to the perfectly anticipated asset price change and the constant real interest rate models. The aggregate services using ex post asset price changes plotted in Figure 27 are more volatile and more widely dispersed than the aggregate services plotted in Figures 24 and 28, as one might expect. The linear efficiency decline services are the top curve k(10), followed by the geometric depreciation services k(7), followed by the one hoss shay services k(1) and the straight line depreciation capital services k(4) are the bottom curve. The aggregate services using constant real interest rates plotted in Figure 28 are fairly similar to the smoothed capital services exhibited in Figure 24. For the constant real interest rate services in Figure 28, the one hoss shay and linear efficiency decline services, k(2) and k(11), are at the top followed very closely by the geometric depreciation services k(8) and the straight line depreciation capital services k(5) are the bottom curve. Thus overall, three of our four depreciation and efficiency models give rise to much the same measures of capital services, holding constant the assumptions about asset price changes and the reference interest rate. However, the straight line depreciation capital services seem to be consistently below the corresponding services generated by the other three classes of models.

Insert Figures 25, 26, 27 and 28.

14. Conclusion

We have considered the problems involved in constructing price and quantity measures for both the capital stock and the flow of services yielded by the stock in an inflationary environment. In order to accomplish these tasks, the statistician will have to make decisions in a number of dimensions:

- What length of life L best describes the asset?
- What form of depreciation or asset efficiency is appropriate?
- What assumptions should be made about the reference interest rate and the treatment of anticipated asset price change?

In this paper, we focused on the last two questions. We considered four classes of depreciation or efficiency and three types of assumption on the nominal interest rate r^t

and on the anticipated asset inflation rate i^t, giving 12 models in all. We evaluated these 12 models using aggregate Canadian data on two asset classes over the period 1926-1999. We found that the assumptions on the form of depreciation or vintage asset efficiency were less important than the assumptions made about the reference interest rate and the treatment of anticipated asset price changes.[87]

We consider the third question above first. In order to answer this question, it is necessary to ask about the purpose for which the capital data will be used. For some purposes, it may be useful to use ex post asset price changes as anticipated price changes. For example, this approach may be useful in constructing estimates of taxable business income if capital gains are taxable. It may also be useful if we want to evaluate the ex post efficiency of a firm, industry or economy. However, for most other uses, assuming that anticipated price changes are equal to actual ex post price changes is very unsatisfactory since it is unlikely that producers could anticipate all of the random noise that seems to be inherent in series of actual ex post asset price changes. Moreover, this approach generates tremendous volatility in user costs and statistical agencies would face credibility questions if this approach were used.

Thus we restrict our attention to the choice between assuming a constant real interest rate or using smoothed ex post asset price changes as estimates of anticipated asset price changes. The assumption of constant real interest rates has a number of advantages:

- The resulting price and quantity series tend to be very smooth.
- The estimates are *reproducible*; i.e., any statistician given the same basic price and quantity data along with an assumed real interest rate will be able to come up with the same aggregate price and quantity measures.

However, the use of smoothed ex post asset price changes as measures of anticipated asset price changes has some advantages as well:

- Longer run trends in relative asset prices can be accommodated.
- The anticipated obsolescence phenomenon can be captured.

Each individual statistical agency will have to weigh the costs and benefits of the two approaches in order to decide which approach to use.

We now discuss which of our four sets of assumptions on the form of depreciation or vintage asset efficiency decline is “best”.

The one hoss shay model of efficiency decline, while seemingly a priori attractive, does not seem to work well empirically; i.e., vintage depreciation rates tend to be much more accelerated than the rates implied by the one hoss shay model. We also saw in Section 11, that the simple one hoss shay model does not take into account the implications of

[87] Harper, Berndt and Wood (1989) also found that differing assumptions on r^t and i^t made a big difference empirically using U.S. data. However, they considered only geometric depreciation. Our paper can be viewed as an extension of their work to consider also variations in the form of depreciation.

rising maintenance and operating costs for an asset as it ages. Thus if maintenance costs are linearly rising over time, a "gross" one hoss shay model gives rise to a linearly declining efficiency model, which of course, is a model that exhibits very accelerated depreciation.

The straight line depreciation model, while not as inconsistent with the data as the one hoss shay model, also does not generate the pattern of accelerated depreciation that seems to characterize many used asset markets. However, given the simplicity of this model (to explain to the public), it could be used by statistical agencies.

The geometric depreciation model seems to be most consistent with the empirical studies on used assets of the four simple classes of model that we considered.[88] Of course, geometric depreciation has the disadvantage that it will never exhaust the full value of the asset.[89]

Finally, a good alternative to the geometric depreciation model is the linear efficiency decline model. However, this model may have a pattern of "overaccelerated" depreciation relative to the geometric model. What is required is more empirical work so that the actual pattern of depreciation can be determined.

We conclude by noting some limitations of the analysis presented in this paper:

- We have not dealt with the problems posed by *unique* assets.
- We have not dealt with the problems posed by assets that depreciate by *use* rather than by *age*.[90]
- We have neglected property taxes, income taxes and insurance premiums as additional components of user costs.
- We have neglected the problems posed by indirect commodity taxes on investment goods; this complication can lead to differences between investment prices and asset stock prices.
- We have neglected many forms of capital in our empirical work including inventories, land, knowledge capital, resource stocks and infrastructure capital.
- We have not discussed the many complexities involved in making quality adjustments for new types of capital.

Appendix : Canadian Data for 1926-1999

We list our primary data on gross business investment in Canada for the years 1926 to 1999 for two assets: machinery and equipment and nonresidential construction. From

[88] See Hulten and Wykoff (1981a) (1981b) and Jorgenson (1996a).

[89] Some statistical agencies solve this problem by "scrapping" the depreciated value of the asset when it reaches a certain age. This solves one problem but it introduces two additional problems: (i) the truncation age has to be decided upon and (ii) the theoretical simplicity of the model is lost.

[90] Our reason for neglecting use is simple: usually, the national statistician will not have data on the use of machines available.

Leacy (1983), Series F23 and F24, we have nominal investments (in millions of Canadian dollars) for these two asset classes for the years 1926 to 1976. The corresponding constant 1971 dollar series are F43 and F44. We only use the data for the years 1926 to 1961 inclusive. For the more recent data, we use Table 1 in Statistics Canada (2000; 4-6), which lists business investment in nonresidential structures and in machinery and equipment for the years 1961 to 1999 in current millions of dollars and in constant 1992 prices. These series are linked through the year 1961 and the prices are renormalized to equal 1 in the year 1965. The resulting price series, P_{NR} and P_{ME}, and quantity series, I_{NR} and I_{ME}, are listed in Table A1 below.

Table A1: Price and Quantity Data for Two Types of Investment in Canada, 1926-1999; Quantities in Millions of 1965 Dollars

Year	P_{NR}	P_{ME}	I_{NR}	I_{ME}
1926	0.4058	0.3515	596.3	742.6
1927	0.4065	0.3400	745.4	961.8
1928	0.4154	0.3384	1001.5	1105.0
1929	0.4290	0.3455	1146.8	1276.5
1930	0.4120	0.3318	929.7	1060.8
1931	0.3804	0.3216	701.8	618.8
1932	0.3640	0.3215	337.9	335.9
1933	0.3567	0.3134	218.7	261.7
1934	0.3592	0.3109	256.1	373.1
1935	0.3657	0.3151	322.6	457.0
1936	0.3751	0.3189	399.8	561.4
1937	0.4046	0.3425	467.1	820.4
1938	0.3968	0.3455	433.5	793.0
1939	0.3917	0.3423	421.2	739.1
1940	0.4047	0.3713	521.4	1096.2
1941	0.4267	0.4006	679.7	1372.9
1942	0.4604	0.4176	723.2	1216.4
1943	0.4832	0.4425	649.8	784.1
1944	0.4896	0.4374	529.0	850.4
1945	0.4921	0.4281	534.4	1051.1
1946	0.5216	0.4324	864.7	1336.7
1947	0.5824	0.4825	1047.4	2033.3
1948	0.6505	0.5428	1276.0	2107.5
1949	0.6716	0.5776	1389.1	2173.0
1950	0.6908	0.6103	1521.4	2190.6
1951	0.7811	0.6890	1668.2	2390.4
1952	0.8265	0.7019	1904.4	2560.2
1953	0.8336	0.7146	2093.2	2734.3
1954	0.8222	0.7233	2042.0	2419.6
1955	0.8429	0.7353	2210.2	2483.3
1956	0.8855	0.7806	2922.7	3129.5
1957	0.8841	0.8219	3505.3	3156.0

1958	0.8808	0.8352	3188.0	2683.1
1959	0.8856	0.8539	2933.4	2820.1
1960	0.8938	0.8658	2902.1	2916.4
1961	0.8905	0.8714	2939.8	2460.4
1962	0.8954	0.8998	2858.9	2631.8
1963	0.9210	0.9250	2956.6	2856.3
1964	0.9473	0.9623	3418.1	3360.6
1965	1.0000	1.0000	3728.0	3947.0
1966	1.0605	1.0326	4278.2	4700.8
1967	1.1063	1.0256	4055.9	4803.2
1968	1.1185	1.0246	4022.2	4471.0
1969	1.1819	1.0520	4026.7	4894.5
1970	1.2421	1.1018	4417.4	4989.1
1971	1.3174	1.1315	4556.6	5138.1
1972	1.3923	1.1767	4537.8	5556.4
1973	1.5433	1.2087	4896.0	6800.7
1974	1.8301	1.3374	5195.8	7660.9
1975	2.0449	1.4981	5876.0	8096.9
1976	2.1615	1.5873	5740.1	8450.2
1977	2.2770	1.6831	6087.1	8429.8
1978	2.4426	1.7516	6244.9	9121.9
1979	2.6641	1.8595	7048.6	10614.4
1980	2.9878	1.7650	7851.7	12811.5
1981	3.3250	1.7869	8449.3	15488.6
1982	3.5731	1.9192	7681.0	13059.7
1983	3.5487	1.9009	7045.9	12815.4
1984	3.6813	1.8781	6971.4	13677.6
1985	3.7921	1.8300	7322.6	15753.9
1986	3.8483	1.8277	6898.9	17463.1
1987	4.0287	1.7861	7165.6	20155.9
1988	4.2590	1.7525	7893.2	23908.5
1989	4.4463	1.7453	8135.8	25750.4
1990	4.5839	1.7309	8154.6	24607.8
1991	4.4856	1.5730	7890.8	24740.5
1992	4.4531	1.5229	6659.2	25380.0
1993	4.5097	1.5568	6694.9	24202.1
1994	4.6633	1.6067	7291.4	26493.7
1995	4.7260	1.5984	7335.9	29083.4
1996	4.8643	1.5241	7474.9	31886.6
1997	4.9931	1.5412	8718.3	39385.3
1998	5.1033	1.5290	8870.8	42915.3
1999	5.1754	1.4180	9047.3	49613.0

Note that the price of machinery and equipment more or less declines from 1982 onward. This reflects the effects of the Statistics Canada quality adjustments for information technology equipment.

Recall that the ex post consumer price index for the economy at the beginning of year t was defined as c^t and the ex post general inflation rate for period t, ρ^t, was defined by equation (17) above as $\rho^t \equiv [c^{t+1}/c^t] - 1$. We approximate c^t by the Total Goods Consumer Price Index for Canada for the previous year; see Table 12 in Statistics Canada (2000; 50), which covers the years 1961 to 1999. We list the resulting general year t ex post inflation rates ρ^t for the years 1965 to 1999 in Table A2 below. Given ρ^t, we can use equation (18) above to calculate the ex post nominal interest rate that producers face in year t as $r^t \equiv (1+r^{*t})(1+\rho^t) - 1$ where we set the real interest rate r^{*t} equal to 4 %. The ex post nominal interest rates r^t are also listed in Table A2 below.

The beginning of year t price for a new asset P_0^t is approximated by the corresponding investment price in the previous year; i.e., we assume P_0^t equals either P_{NR}^{t-1} or P_{ME}^{t-1}. Since inflation in Canada was not all that high over the period, this is probably an acceptable approximation for illustrative purposes.[91] Once the beginning of the period new asset prices are determined, we can define the corresponding ex post nominal asset inflation rates i^t by $[P_0^{t+1} / P_0^t] - 1$. The two ex post nominal asset inflation rates i_{NR}^t and i_{ME}^t are also listed in Table A2. Finally, we use equation (28) and the nominal inflation rates i_{NR}^t, i_{ME}^t and ρ^t in order to define the two real ex post nominal asset inflation rates, i_{NR}^{*t} and i_{ME}^{*t}, as follows:

(A1) $1 + i_{NR}^{*t} \equiv (1 + i_{NR}^t)/(1 + \rho^t)$; $1 + i_{ME}^{*t} \equiv (1 + i_{ME}^t)/(1 + \rho^t)$.

These two ex post real asset inflation rates are also listed in Table A2.

Table A2: Ex Post Nominal Interest Rates and Nominal and Real Inflation Rates

Year	ρ^t	r^t	i_{NR}^t	i_{ME}^t	i_{NR}^{*t}	i_{ME}^{*t}
1965	0.0204	0.0612	0.0556	0.0392	0.0345	0.0184
1966	0.0400	0.0816	0.0605	0.0326	0.0197	-0.0071
1967	0.0337	0.0750	0.0432	-0.0068	0.0092	-0.0391
1968	0.0419	0.0835	0.0111	-0.0009	-0.0296	-0.0411
1969	0.0446	0.0864	0.0566	0.0267	0.0115	-0.0171
1970	0.0342	0.0756	0.0510	0.0474	0.0162	0.0127
1971	0.0289	0.0701	0.0606	0.0270	0.0308	-0.0019
1972	0.0482	0.0901	0.0568	0.0399	0.0083	-0.0079
1973	0.0766	0.1197	0.1085	0.0272	0.0296	-0.0459
1974	0.1068	0.1510	0.1859	0.1065	0.0715	-0.0002
1975	0.1093	0.1537	0.1174	0.1201	0.0072	0.0097
1976	0.0754	0.1184	0.0570	0.0595	-0.0171	-0.0147
1977	0.0782	0.1213	0.0534	0.0603	-0.0229	-0.0165
1978	0.0900	0.1336	0.0728	0.0407	-0.0158	-0.0452
1979	0.0917	0.1354	0.0907	0.0616	-0.0010	-0.0276

[91] For countries with really high inflation, more care would need to be taken in constructing consistent beginning, middle and end of period prices.

1980	0.1008	0.1449	0.1215	-0.0509	0.0188	-0.1378
1981	0.1240	0.1690	0.1129	0.0124	-0.0099	-0.0993
1982	0.1087	0.1530	0.0746	0.0740	-0.0307	-0.0313
1983	0.0582	0.1005	-0.0068	-0.0095	-0.0614	-0.0640
1984	0.0434	0.0852	0.0374	-0.0120	-0.0058	-0.0531
1985	0.0402	0.0818	0.0301	-0.0256	-0.0097	-0.0633
1986	0.0413	0.0830	0.0148	-0.0012	-0.0255	-0.0409
1987	0.0435	0.0853	0.0469	-0.0228	0.0032	-0.0635
1988	0.0405	0.0821	0.0572	-0.0188	0.0160	-0.0570
1989	0.0495	0.0915	0.0440	-0.0041	-0.0053	-0.0511
1990	0.0483	0.0902	0.0309	-0.0082	-0.0166	-0.0539
1991	0.0557	0.0980	-0.0214	-0.0912	-0.0731	-0.1392
1992	0.0152	0.0558	-0.0073	-0.0319	-0.0222	-0.0464
1993	0.0180	0.0587	0.0127	0.0222	-0.0052	0.0042
1994	0.0020	0.0420	0.0341	0.0321	0.0320	0.0300
1995	0.0216	0.0624	0.0134	-0.0052	-0.0080	-0.0262
1996	0.0163	0.0570	0.0293	-0.0465	0.0127	-0.0618
1997	0.0161	0.0567	0.0265	0.0112	0.0103	-0.0048
1998	0.0093	0.0497	0.0221	-0.0079	0.0127	-0.0170
1999	0.0175	0.0582	0.0141	-0.0726	-0.0033	-0.0885
Average	0.0511	0.0932	0.0505	0.0121	-0.0005	-0.0368

The arithmetic average of the interest rates and inflation rates over the 35 years is listed in the last row of Table A2. Thus the average Consumer Price inflation was 5.11 % per year, which was close to the average inflation rate for the price of nonresidential construction. The average inflation rate for machinery and equipment was much lower at 1.21 % per year on average. From the last column of Table A2, it can be seen that after 1980, the real inflation rate for machinery and equipment was predominantly negative and the average *real* rate of price change was − 3.68 % per year. It can also be seen that the general inflation rate ρ^t surged to rates in the 7 to 12 % range for the years 1972 through 1982 and remained in the 4 to 5 % range until 1992 when it collapsed to the 1 to 2 % range. Note also the volatility of all of the listed ex post inflation rates.

The volatility of ex post asset inflation rates makes it unlikely that producers could perfectly anticipate these rates at the beginning of each year. Thus for our third class of models, we used *smoothed ex post inflation rates* as our estimators of ex ante expected price change. Since the ex post changes in the Consumer Price Index (the ρ^t) are also quite volatile, we also smooth these rates so that our nominal interest rate series (the r^t) will also be less volatile. We used Cleveland's (1979) and Cleveland and Devlin's (1988) locally weighted regression method as our smoothing method as implemented by White's (1997; 261-262) SHAZAM computer program.[92] In order to implement the smoothing method, one has to choose a smoothing parameter, f , which controls how much smoothing is done. We considered only 3 values for this smoothing parameter: f = .20, .25 or .30. We chose one of these 3 values for f by using the cross validation score

[92] This is the METHOD=LOWESS option on the NONPAR command in SHAZAM.

as our model choice criterion.[93] For smoothing the ex ante ρ^t series, we ended up choosing f = .2 while for smoothing the $i_{NR}{}^t$ and $i_{ME}{}^t$ series, we ended up choosing f = .2 and f = .3 respectively. These smoothed counterparts to the interest rates and inflation rates listed in Table A2 are reported in Table A3.

Table A3: Smoothed Interest Rates and Nominal and Real Inflation Rates

Year	ρ^t	r^t	$i_{NR}{}^t$	$i_{ME}{}^t$	$i_{NR}{}^{*t}$	$i_{ME}{}^{*t}$
1965	0.0264	0.0675	0.0574	0.0217	0.0302	-0.0046
1966	0.0313	0.0725	0.0504	0.0215	0.0185	-0.0095
1967	0.0360	0.0774	0.0449	0.0219	0.0086	-0.0135
1968	0.0395	0.0811	0.0399	0.0225	0.0004	-0.0164
1969	0.0386	0.0801	0.0426	0.0227	0.0038	-0.0153
1970	0.0377	0.0792	0.0515	0.0272	0.0133	-0.0101
1971	0.0416	0.0832	0.0616	0.0381	0.0192	-0.0033
1972	0.0550	0.0972	0.0834	0.0512	0.0269	-0.0036
1973	0.0756	0.1186	0.1112	0.0609	0.0331	-0.0137
1974	0.0907	0.1343	0.1226	0.0664	0.0293	-0.0223
1975	0.0936	0.1373	0.1122	0.0696	0.0170	-0.0219
1976	0.0894	0.1330	0.0860	0.0688	-0.0031	-0.0189
1977	0.0850	0.1284	0.0694	0.0585	-0.0143	-0.0244
1978	0.0870	0.1305	0.0757	0.0435	-0.0104	-0.0401
1979	0.0955	0.1393	0.0925	0.0326	-0.0028	-0.0574
1980	0.1042	0.1483	0.1018	0.0251	-0.0021	-0.0716
1981	0.1043	0.1485	0.0911	0.0165	-0.0120	-0.0795
1982	0.0924	0.1360	0.0645	0.0082	-0.0255	-0.0771
1983	0.0721	0.1150	0.0411	0.0034	-0.0290	-0.0641
1984	0.0527	0.0948	0.0256	0.0001	-0.0257	-0.0500
1985	0.0435	0.0852	0.0260	-0.0065	-0.0167	-0.0478
1986	0.0417	0.0834	0.0334	-0.0126	-0.0080	-0.0521
1987	0.0425	0.0842	0.0393	-0.0152	-0.0030	-0.0553
1988	0.0445	0.0862	0.0443	-0.0197	-0.0002	-0.0615
1989	0.0469	0.0888	0.0378	-0.0242	-0.0087	-0.0680
1990	0.0464	0.0883	0.0197	-0.0241	-0.0256	-0.0674
1991	0.0391	0.0806	0.0056	-0.0194	-0.0322	-0.0563
1992	0.0283	0.0694	0.0022	-0.0158	-0.0254	-0.0429
1993	0.0173	0.0580	0.0097	-0.0142	-0.0075	-0.0310
1994	0.0139	0.0544	0.0187	-0.0097	0.0048	-0.0232
1995	0.0143	0.0549	0.0240	-0.0052	0.0096	-0.0192
1996	0.0155	0.0561	0.0243	-0.0126	0.0087	-0.0277
1997	0.0150	0.0556	0.0224	-0.0218	0.0073	-0.0363
1998	0.0148	0.0553	0.0204	-0.0313	0.0055	-0.0454
1999	0.0143	0.0548	0.0179	-0.0413	0.0036	-0.0548

[93] Using this choice also led to the smallest amount of autocorrelation for each series.

Average	0.0510	0.0931	0.0506	0.0116	-0.0003	-0.0373

Comparing Table A3 with Table A2, it can be seen that the smoothing process has made the underlying trends much more visible. Note that the arithmetic averages for each series listed in the last row of Table A3 are very close to the corresponding arithmetic averages listed in the last row of Table A2. This is as it should be: the smoothing process should eliminate the random noise in our series without distorting the overall trends.

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Measuring Capital: Charts

Figure 1: Alternative One Hoss Shay Stock Prices

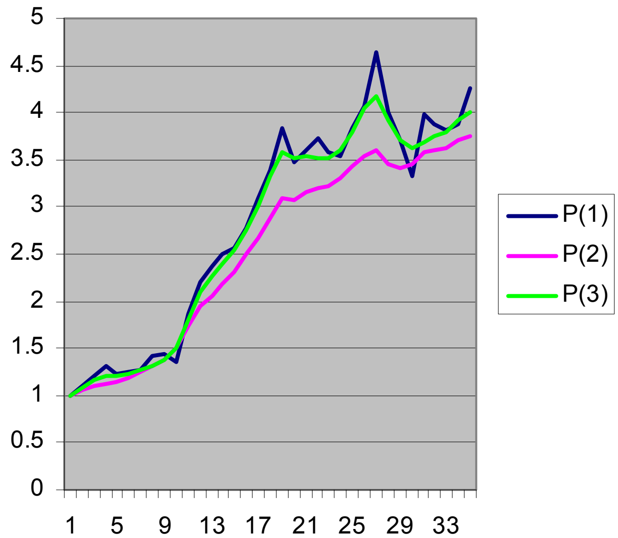


Figure 2: Alternative One Hoss Shay Capital Stocks

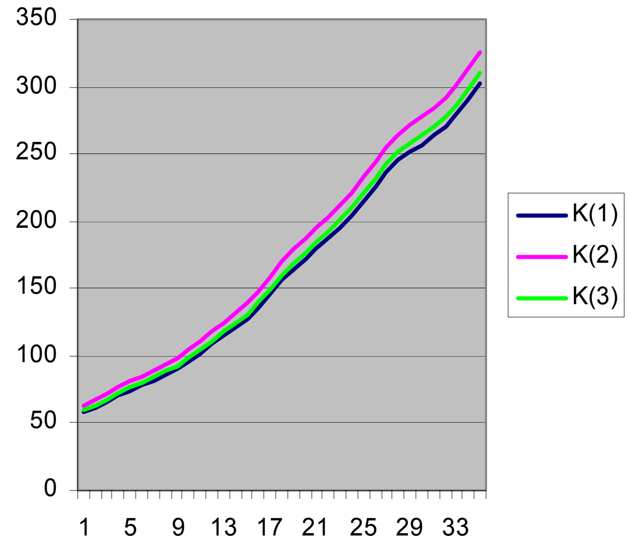


Figure 3: Alternative One Hoss Shay Aggregate User Costs

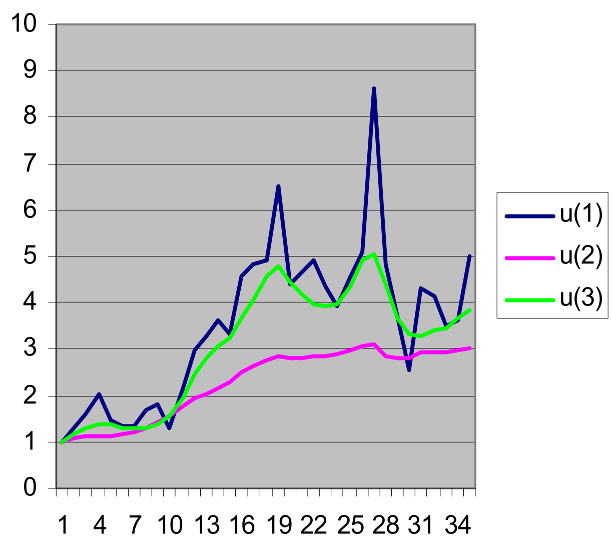
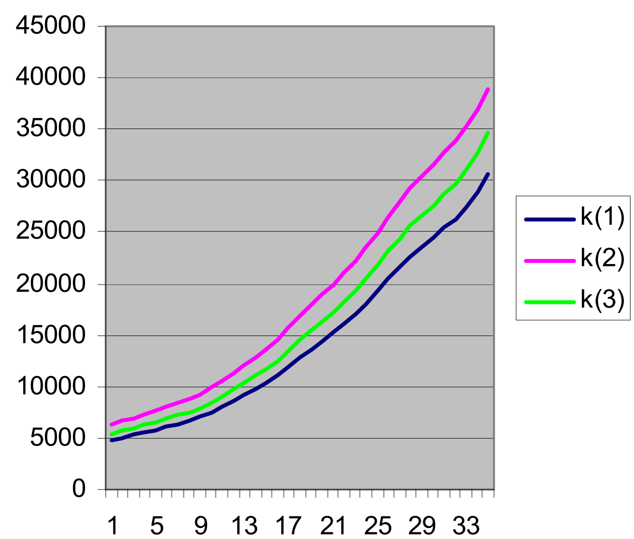


Figure 4: Alternative One Hoss Shay Aggregate Capital Services



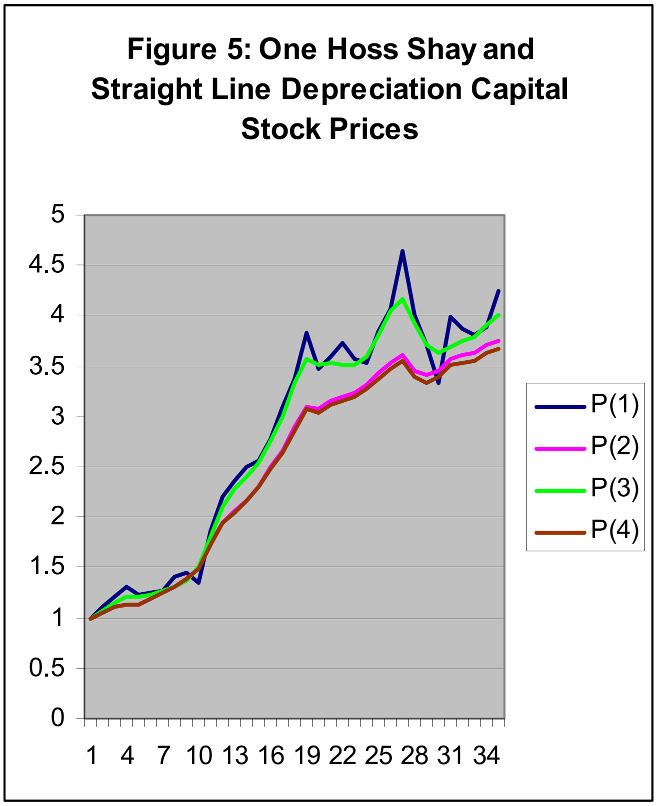


Figure 5: One Hoss Shay and Straight Line Depreciation Capital Stock Prices
P(1)
P(2)
P(3)
P(4)

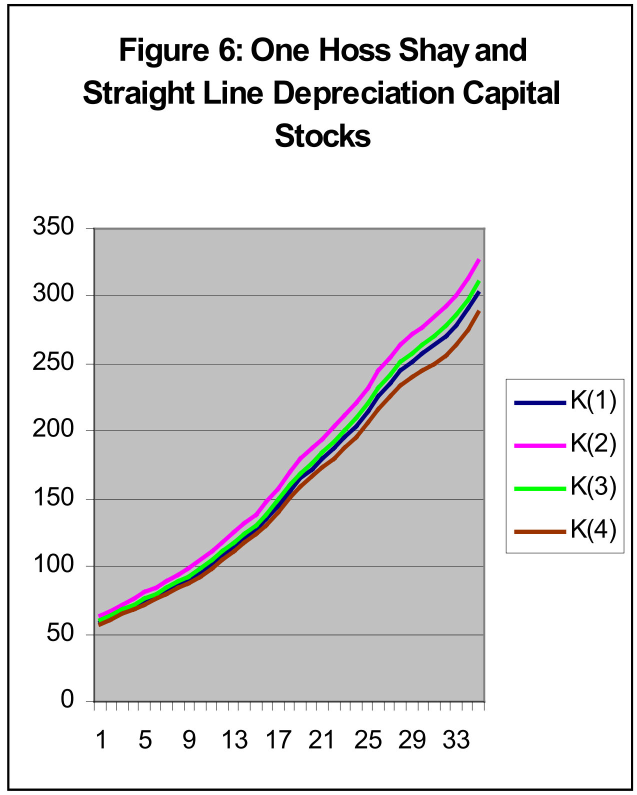


Figure 6: One Hoss Shay and Straight Line Depreciation Capital Stocks
K(1)
K(2)
K(3)
K(4)

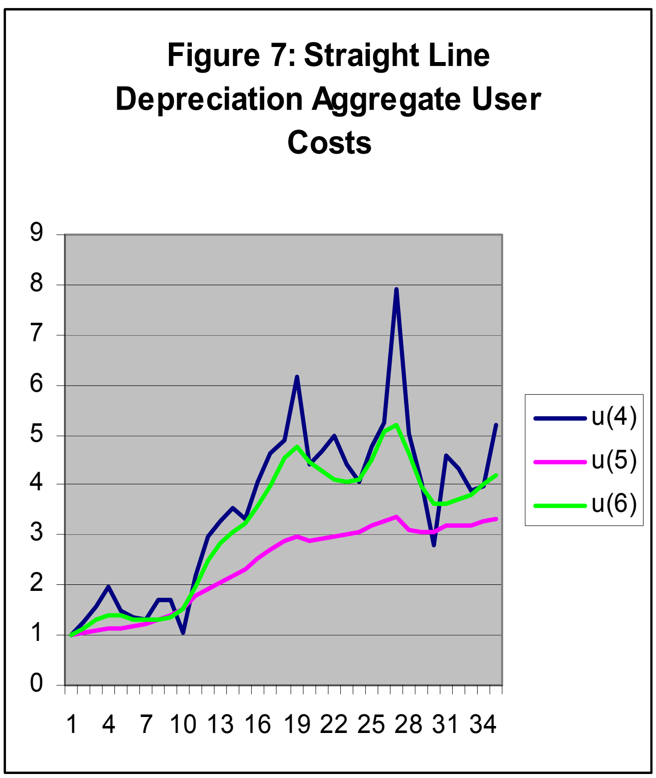


Figure 7: Straight Line Depreciation Aggregate User Costs
u(4)
u(5)
u(6)

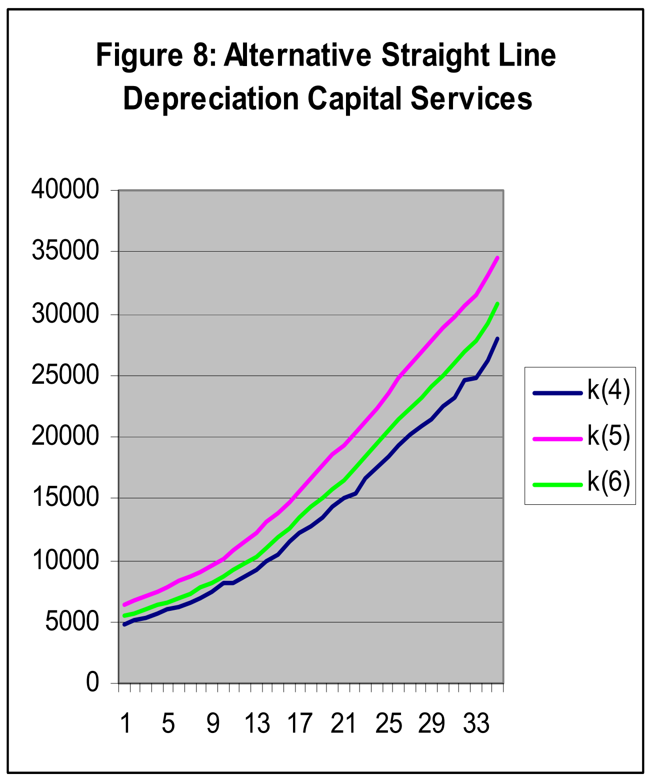


Figure 8: Alternative Straight Line Depreciation Capital Services
k(4)
k(5)
k(6)

Figure 9: Aggregate Capital Stock Prices for Five Models

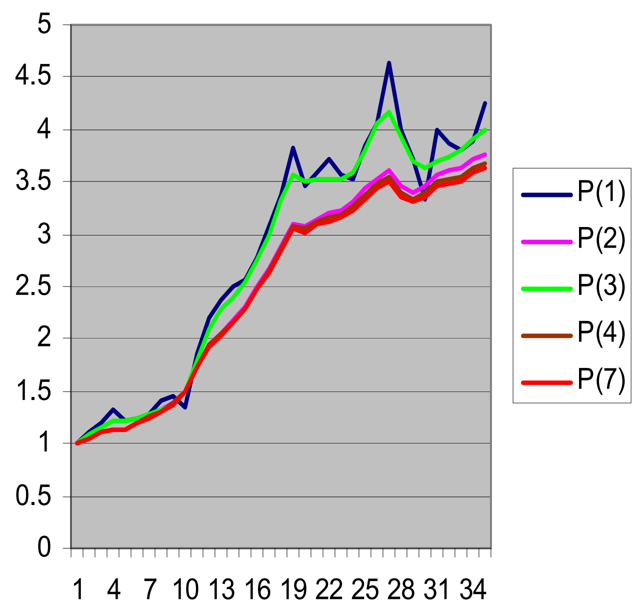


Figure 10: Aggregate Capital Stocks for Five Models

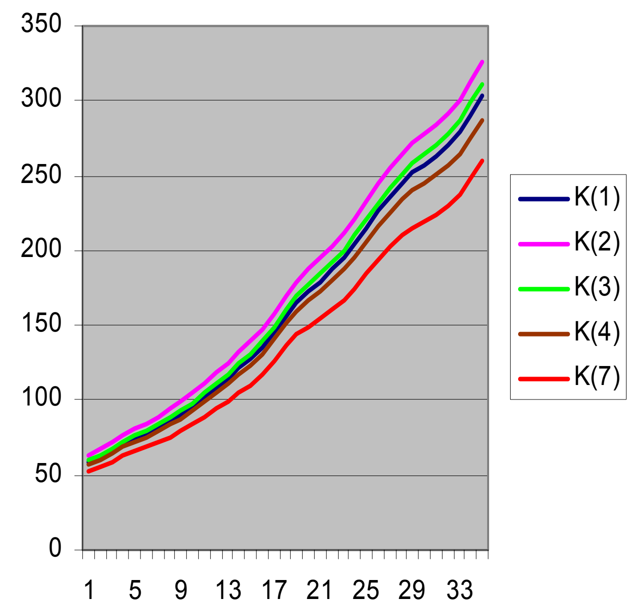


Figure 11: Alternative Geometric Depreciation User Costs

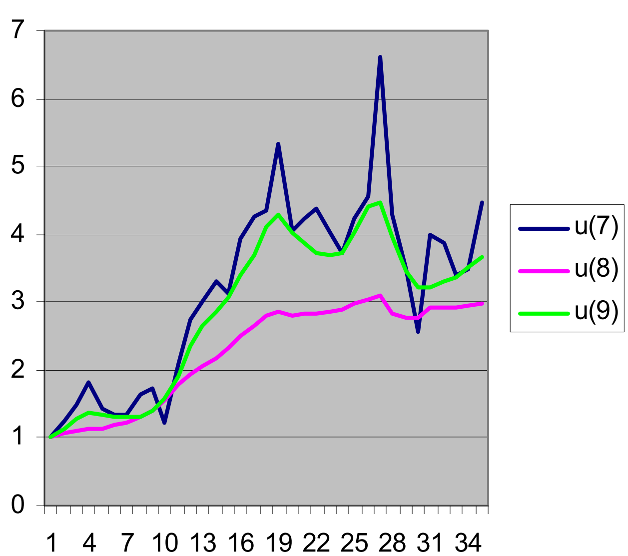


Figure 12: Alternative Geometric Depreciation Capital Services

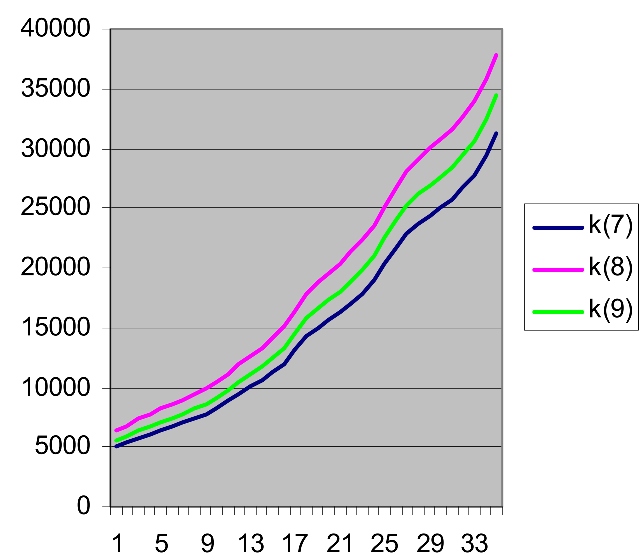


Figure 13: Linear Efficiency Decline Aggregate Capital Stock Prices

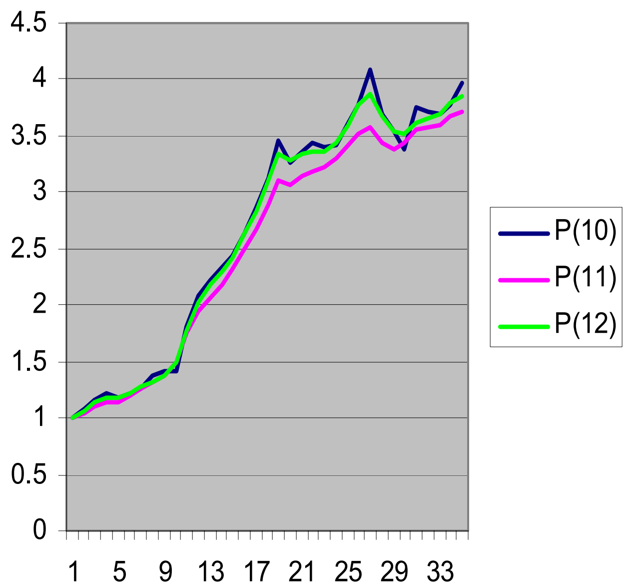


Figure 14: Alternative Linear Efficiency Decline Capital Stocks

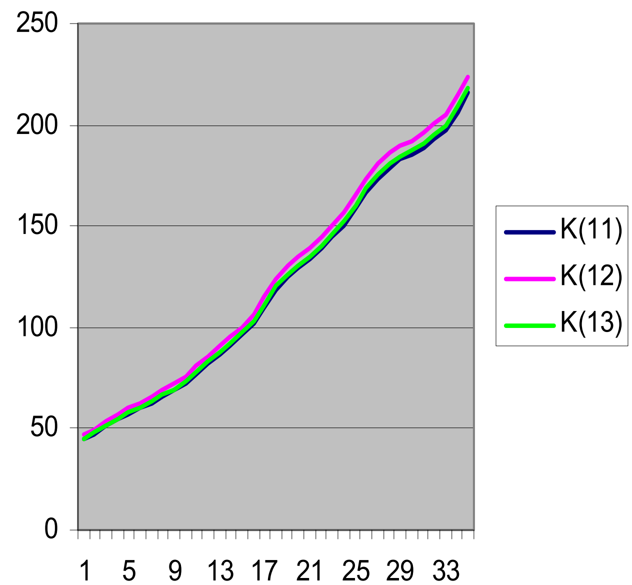


Figure 15: Alternative Linear Efficiency Decline User Costs

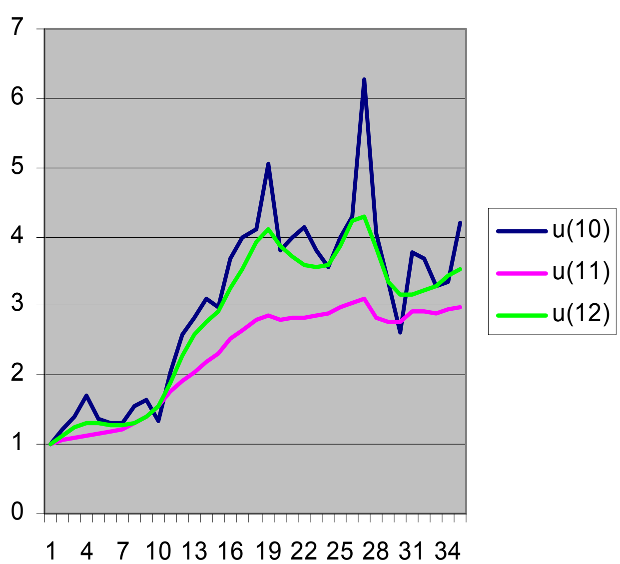


Figure 16: Alternative Linear Efficiency Decline Capital Services

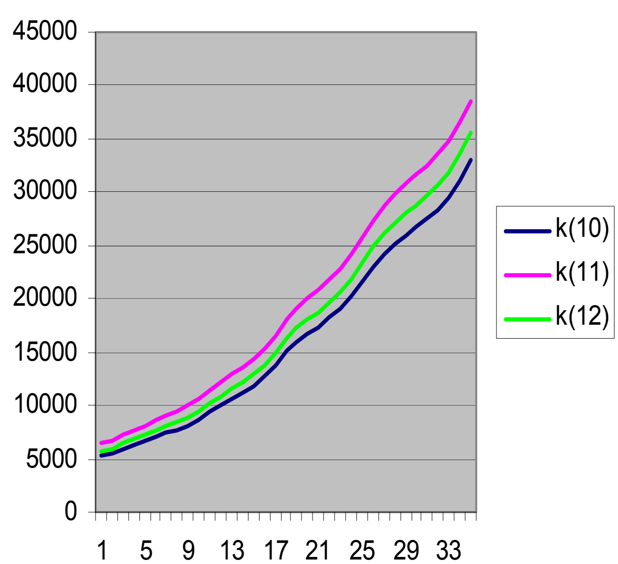


Figure 17: Capital Stock Prices using Ex Post Price Changes

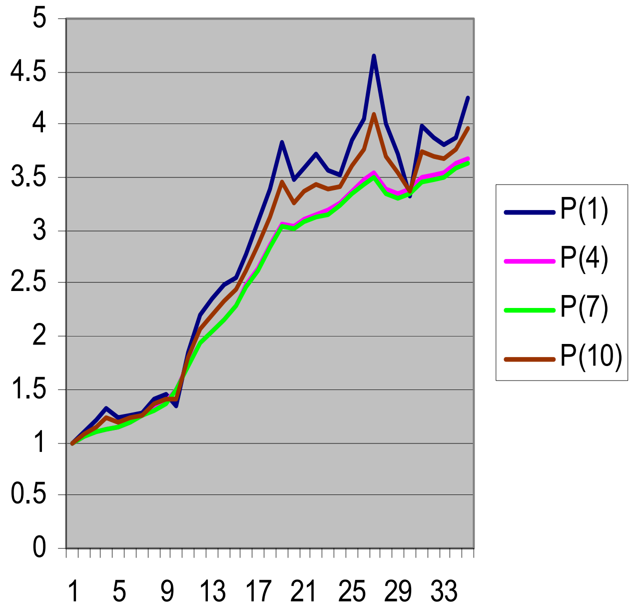


Figure 18: Alternative Stock Prices with Constant Real Interest Rates

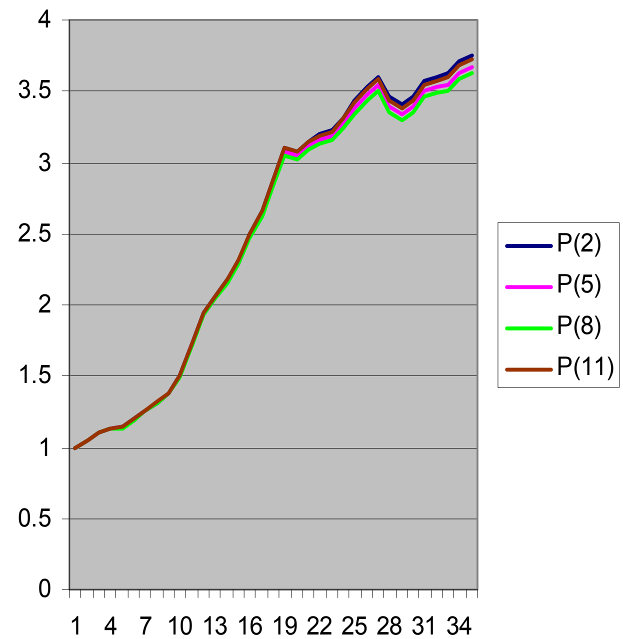


Figure 19: Alternative Stock Prices Using Smoothed Asset Price Changes

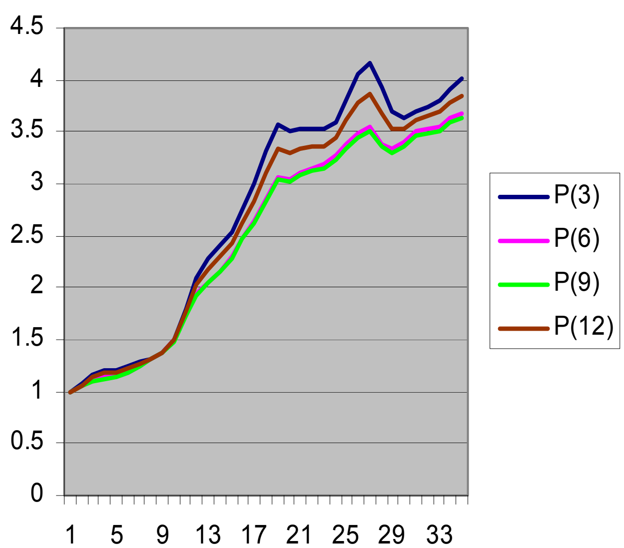


Figure 20: Alternative Capital Stocks Using Smoothed Asset Price Changes

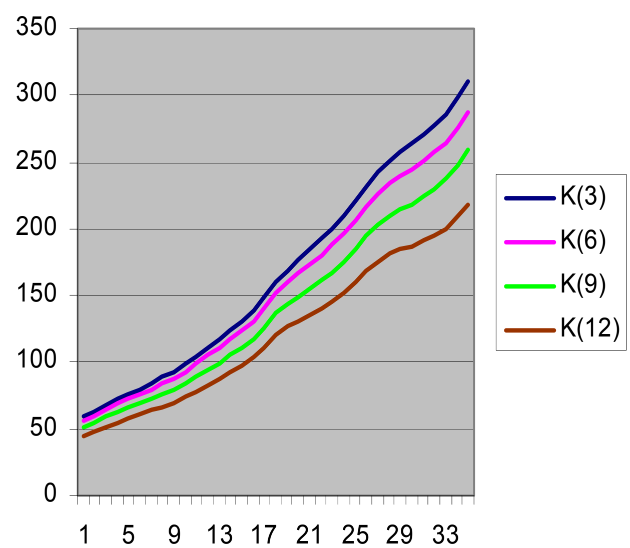


Figure 21: Alternative User Costs Using Ex Post Asset Price Changes

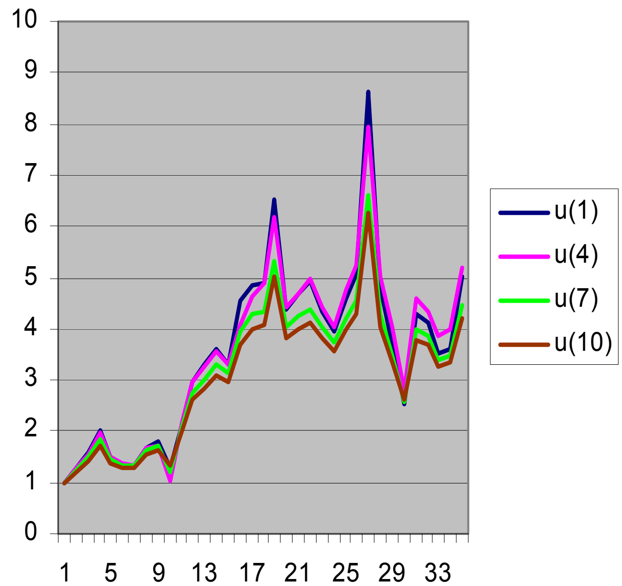


Figure 22:Alternative User Costs Using Constant Real Interest Rates

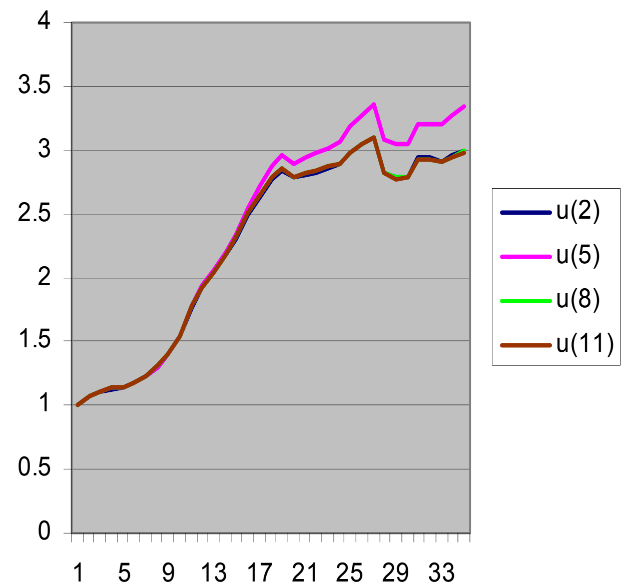


Figure 23: Alternative User Costs Using Smoothed Asset Price Changes

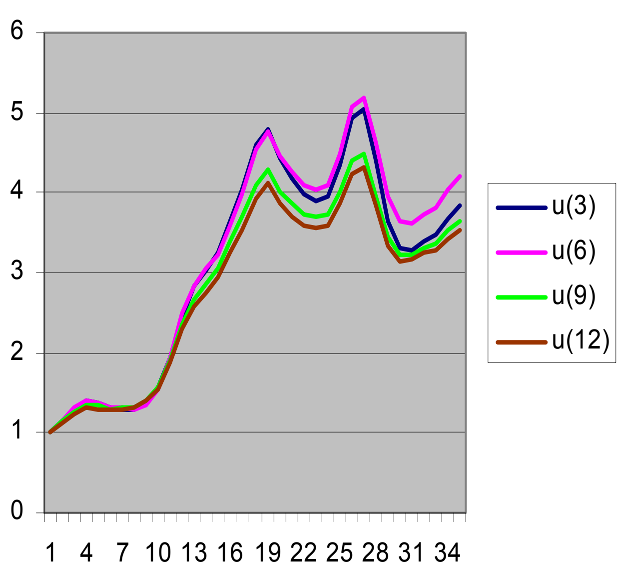


Figure 24: Alternative Capital Services Using Smoothed Asset Price Changes

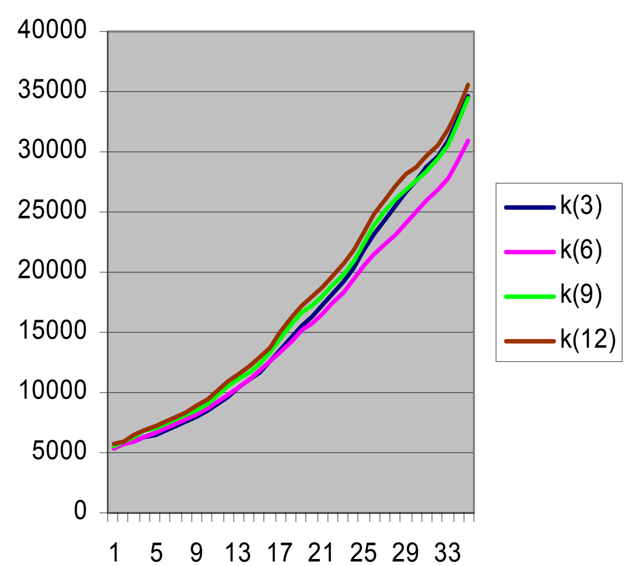


Figure 25: Alternative Capital Stocks Using Ex Post Price Changes

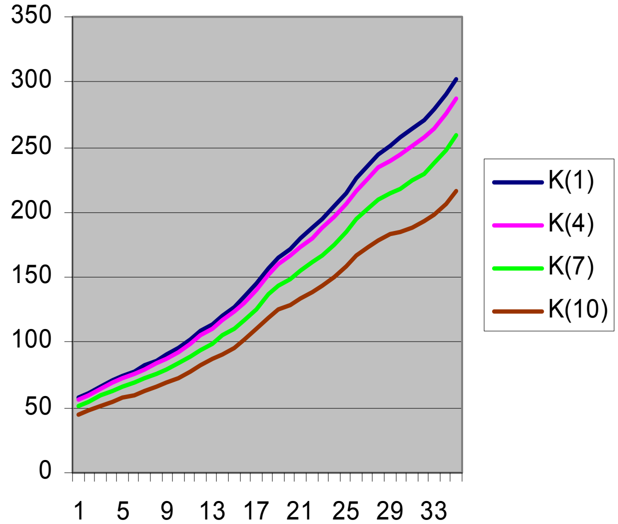


Figure 26: Alternative Capital Stocks Using Constant Real Rates

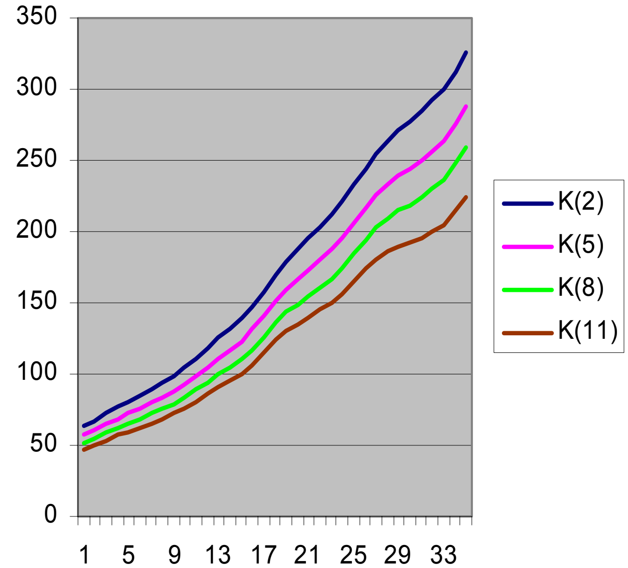


Figure 27: Alternative Capital Services Using Ex Post Price Changes

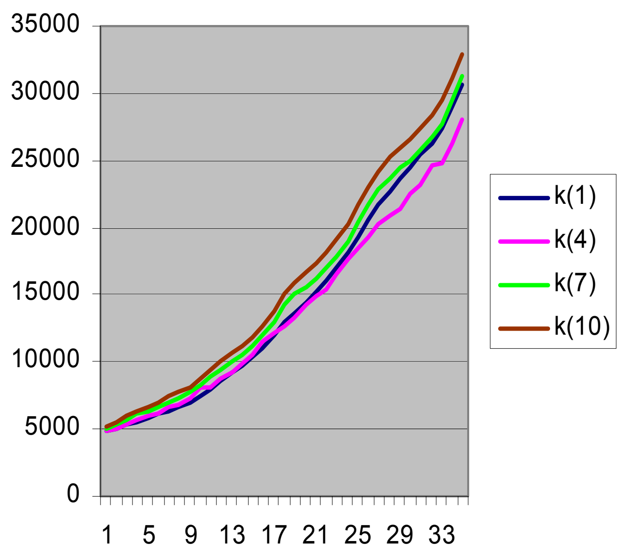


Figure 28: Alternative Capital Services Using Constant Real Rates