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A THEORY OF LARGE ELECTIONS

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ABSTRACT

We propose a new game theoretic approach to modeling large elections that overcomes the "paradox of voting" in a costly voting framework, without reliance on the assumption of ad hoc preferences for voting. The key innovation that we propose is the adoption of a "smooth" policy rule under which the degree to which parties favor their own interests is increasing in their margin of victory. In other words, mandates matter. We argue that this approach is an improvement over the existing literature as it is consistent with the empirical evidence. Incorporating this policy rule into a costly voting model with paternalistic voters yields a parsimonious model with attractive properties. Specifically, the model predicts that when the size of the electorate grows without bound, limiting turnout is strictly positive both in terms of numbers and proportions. Further, the model preserves the typical comparative statics predictions that have been identified in the extant costly voting models such as the underdog effect and the competition effect. Finally, under the case of selfish agents, we are able to extend Palfrey and Rosenthal's (1985) zero turnout result to a general class of smooth policy rules. Thus, this new approach reconciles the predictions of standard costly voting, both in terms of positive turnout and comparative statics predictions with the assumption of a large electorate environment.

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1. Introduction

In the original formulation of his rational choice theory, Anthony Downs (1957) drew attention to what has come to be known as the "paradox of voting": if voting is costly, turnout in large elections should be negligible. Downs formulated the problem in a decision theoretic framework, relying on the fact that the probability of being pivotal is exogenous. Ledyard (1981, 1984) and Palfrey and Rosenthal (1983, 1985) questioned the validity of this approach and recast the problem in game theoretic terms. As a result of these efforts, in their 1985 paper, Palfrey and Rosenthal demonstrated that in large electorates, provided uncertainty about relative costs, voters with positive net voting costs abstain even in a game theoretic framework: "We have come full circle and are once again beset by the paradox of not voting."[1] (Palfrey and Rosenthal, 1985, p. 64). In spite of this paradox, game theoretic costly voting models have been, and continue to be, prominent (see, for instance, Campbell, 1999; Börgers, 2004; Goeree and Grosser, 2007; Krasa and Polborn, 2009; Krishna and Morgan, 2010; Taylor and Yildirim 2010, among others). Their popularity is driven by two factors: first, these models are consistent with the notion that voters behave strategically (e.g. Riker and Ordeshook, 1968; Franklin et al., 1994); second, they are able to generate intuitive, and empirically supported, comparative statics predictions (e.g. Shachar and Nalebuff, 1999; Blais, 2000; Levine and Palfrey, 2007) – at least with a finite electorate.

To date, the prediction that turnout converges to zero as the size of the electorate grows has

[1] Curiously, the literature has referred to this notion as both the "paradox of voting" and the "paradox of not voting".

typically been overcome in the costly voting model through the incorporation of either *ad hoc* preferences for voting or coordination mechanisms that lack a solid micro-foundation (see for instance Riker and Ordeshook, 1968; Harsanyi, 1977, 1992; Feddersen and Sandroni, 2006; Coate and Conlin, 2004). We instead propose a new approach to modeling large elections that overcomes this paradox in a costly voting framework, without reliance on the assumption of direct psychic rewards from casting one's ballot.

We consider a two-party system. Citizens are characterized by their political preference and cost of voting, both being private information. We modify the standard model in two ways. First, we drop the usual winner-take-all assumption and instead include a "smooth" policy rule under which the degree to which parties (or elected officials) favor their own party's interests is increasing in their margin of victory. In other words, mandates matter. Specifically, we assume that the benefit from government action is distributed across members of the two parties according to a continuous function that is strictly increasing (tilted toward members of the winning party) in the proportion of votes received by the winning party. Thus, politicians are "smooth" in the sense that, for members of a given party, benefits are strictly increasing in the vote percentage received by said party.

Second, following in the tradition of recent work by Feddersen and Sandroni (2006), we assume that voters are paternalistic. Citizens not only receive a private benefit from having their preferred policies adopted, but they also receive spillover benefits from the impact that these policies have on other individuals.

To motivate our assumption about smooth politicians, we first argue that this is a logical expectation in an environment where candidates serve at the will of their electorate. We additionally provide empirical evidence of this type of behavior in the U.S. Congress. In particular, we use a panel data model with member and Congress fixed effects to demonstrate that the degree to which members of congress adopt partisan voting records is increasing in the margin of victory in their most recent election.

Using our proposed theoretical framework we show that, when the size of the electorate grows without bound, limiting turnout is strictly positive (in terms of proportion) if the supporters of both parties are paternalistic. This relies critically on the assumption that mandates matter. Indeed, we show that if the election is decided by winner-take-all majority rule, turnout still converges to zero even if voters are paternalistic. The intuition is as follows. For a paternalistic individual in a winner-take-all election, while the benefit from voting increases with the size of the population, her probability of being pivotal goes to zero at an even faster rate. While, under the marriage of paternalism *and* a smooth policy function, the rate at which policy impact decreases with population is such that it is precisely offset by the increase in paternalistic benefits arising from the increase in the population. Conversely, when we relax the paternalism assumption and assume that the supporters of at least one party are purely selfish, we are able to extend Palfrey and Rosenthal's (1985) famous zero turnout result to a general class of smooth policy rules. Thus, we require both paternalism and the assumption that mandates matter to overcome the paradox of voting.

In analyzing the comparative statics properties of our model, we distinguish between two different types of paternalism: Exclusive and Inclusive Paternalism. While Exclusive agents are solely concerned about members of their own party, Inclusive voters also care about the supporters of the other party. Under this dichotomy, our framework allows us to discern between two competing comparative statics effects associated with the relative size of the two parties. On the one hand, increasing the size of one party makes its supporters less likely to vote (the well-known *free-riding effect*). On the other hand, changing the relative composition of the electorate varies the spillovers received by a voter. We refer to this second channel of impact as the *spillover effect*. While *free-riding* and *spillover effects* perfectly offset each other in the case of Exclusive voters, the latter is less pronounced when voters are Inclusive. As a consequence, when agents display Inclusive Paternalism, under the assumption of identical cost distributions and identical overall levels of paternalism, the incentive to free-ride leads members of the minority party to turn out to vote at higher rates than do the majority (the so-called *underdog effect* -- see Levine and Palfrey, 2007); nevertheless, the majority never loses its initial advantage and receives a higher share of the votes. Furthermore, assuming the same cost distribution for both parties and symmetric paternalism, the model generates the prediction that the closer the election the higher turnout (the so-called *competition effect*--see Levine and Palfrey, 2007). Both the *underdog* and *competition effects* have been well documented empirically (see, for instance, Shachar and Nalebuff, 1999; Blais, 2000; Levine and Palfrey, 2007) and characterized as theoretical features of standard costly voting models with finite electorate and identical cost distributions (see Taylor and Yildirim, 2010). Key however, as shown by Taylor and Yildirim (2010), is the fact that these effects disappear when the electorate size grows without bound. Thus, a main contribution of our approach is its ability to reconcile the attractive properties of standard costly voting models of small elections with the assumption of a large electorate environment.

2. Background and Motivation

The first attempt to solve the paradox of voting in costly elections relied on the assumption that citizens receive a direct benefit from voting that is independent of the election outcome. When this benefit exceeds the cost of voting ($D > c$) citizens vote (see Riker and Ordeshook, 1968). While overcoming the problem of zero turnout, this model abstracts from strategic interactions and provides no comparative statics predictions regarding voter turnout. Other scholars approached the problem from a different perspective, assuming instead that leaders can mobilize party supporters (e.g. Morton, 1991; Shachar and Nalebuff, 1999). As a consequence, the game becomes one with a small number of players and equilibrium turnout is positive. Although they are able to provide comparative statics results, such models do not provide a micro-foundation to explain how leaders mobilize followers.

More recently, following in the tradition of Riker and Ordeshook's civic duty model, Feddersen and Sandroni (2006) develop a model of ethical voters in the spirit of Harsanyi's (1977) rule utilitarian framework. Under their specification, the exogenously given voting rule of Riker and Ordeshook is replaced by an endogenously determined group-specific utilitarian voting rule that accounts for the benefits and voting costs of all individuals.

In the model of Feddersen and Sandroni, within each of two types (parties) there are two kinds of agents: ethical voters and abstainers, with the latter abstaining under all circumstances. Ethical voters on the other hand receive a benefit D greater than the maximum voting cost $\bar{c}$ for "doing their part", where "doing their part" consists of following the threshold voting strategy that would be adopted by a social planner maximizing a utilitarian social welfare function. The social planner's objective function adopted by a given type (party) puts equal weight on the benefits and voting costs of members of each type (party). Hence, ethical voters exhibit other-regarding preferences: while they are altruistic with regards to other voters' costs, their specification is paternalistic in that an individual of a given type projects her own preferred election outcome onto the entire measure of voters.

Feddersen and Sandroni's model provides a strong set of comparative statics predictions and represents a major step forward in the quest for an appropriate theory of large elections. A version of their specification is also used by Coate and Conlin (2004) to explain turnout in Texas liquor referenda. Nevertheless, the model relies on the assumption that ethical voters of a given group do not interact strategically among themselves and, for this reason, the game is actually isomorphic to a two-player game. This leaves open the question of how ethical voters of the same type can coordinate and behave as a single actor, instead of free-riding on each other. While there exists ample evidence that individuals exhibit other-regarding preferences (see, for instance, Fehr and Schmidt, 2006), and such assumptions are typically employed to explain why agents cooperate in prisoner's dilemma problems, the (group) rule utilitarian approach is non-standard outside Harsanyi's and Feddersen and Sandroni's specifications and lacks empirical support. Moreover, the model crucially depends on the assumption that (at least some) ethical voters care about the social cost of the election. If this was not the case, all ethical voters would turn out to vote, as already predicted in Riker and Ordeshook. While this assumption makes sense from the social planner's perspective it is not clear why this is an accurate description of a strategic voter's preference.

Most recently, in a current working paper, Evren (2011) shows how the rule utilitarian assumption of Feddersen and Sandroni's model can be relaxed when there is uncertainty over the proportion of other-regarding voters. In doing so, he is able to replicate Feddersen and Sandroni's results in a game theoretic framework.

In this paper we take a different approach to the problem. Building on the classic work of Palfrey and Rosenthal (1985), we extend their specification to include the paternalistic behavior posited by those working in the tradition of Feddersen and Sandroni. Additionally, critical to our approach is the assumption that mandates matter and election outcomes are mapped into policy decisions through a smooth policy function that is continuous in election margins (smooth politicians).

The remainder of the paper is structured as follows. We begin, in Section 3, with the case for replacing traditional winner-take-all models of election outcomes with our proposed smooth policy approach. In Section 4, assuming smooth politicians, we construct a parsimonious and tractable model that yields positive turnout when voters are paternalistic, while we extend Palfrey and Rosenthal's (1985) zero turnout result to a general class of smooth policy rules when the supporters of at least one party are purely selfish. Section 5 provides comparative statics predictions: we show that, under the assumption of identical cost distributions, our model generates both the *underdog effect* and the *competition effect*. Section 6 concludes.

3. Smooth Politicians: Empirical Evidence

The key innovation of our modeling approach is the incorporation of a smooth policy rule under which the degree of partisanship following an election is a continuous function of the relative vote shares of the two parties. Here we argue that elections determine not only who gets elected, but also what kind of policies the winners will enact. The basic reasoning is that candidates who win by larger margins are likely to adopt more partisan policies than are candidates who win by small margins.

Several arguments exist to support this type of behavior and have received attention in the political science literature.[2] For example, candidates who win by a landslide can claim they have a mandate for their party's policies while those who win by small margins cannot claim such a

[2] A thorough summary of this literature is provided in Smirnov and Fowler (2007).

mandate. Additionally, when elections are repeated games and candidates are uncertain about the distribution of voter preferences, close elections may cause candidates to eschew highly partisan policies to avoid the risk of losing the next election to a more moderate candidate. Finally, candidates may be intrinsically motivated to legislate from the center of their electorate. In this case, close elections signal a moderate electoral center while large margins signal a more partisan electoral center.

A number of studies have provided empirical support for the link between margin of victory and policy outcomes. Examples include work by Fowler (2005), Somer-Topcu (2009) and Peterson et al. (2003). Fowler (2005) finds that in U.S. Senate races candidates respond to increases in the previous election's Republican vote by adopting more conservative positions in the current race. Somer-Topcu (2009), using data from the Comparative Manifesto Project to evaluate the impact of election outcomes on party policy positions in a sample of 23 Democracies between 1945 and 1998, finds that parties respond to declining vote shares by changing the policies they support. Finally, Peterson et al. (2003) analyze newspaper coverage to identify U.S. Presidential and off-year elections that were perceived as providing a "mandate" to the winning party. They then provide empirical evidence that, following a "mandate" election, members of congress deviate from their historical voting pattern in the direction of the mandate – with this effect attenuating over time.

While this extant literature is suggestive of smooth behavior by politicians, more direct evidence can be obtained through an analysis of congressional voting behavior. Table 1 provides an analysis of the impact of margin of victory on the voting behavior of U.S. Members of Congress for the 105th through 111th Congress. For a given Member of Congress in a given year, we identify their degree of partisanship using the first dimension of Poole and Rosenthal's well-known DW-Nominate Score.[3] This component of the DW-Nominate Score provides a one dimensional ranking on the liberal-conservative axis for each Member of Congress in each Congressional Session – based on an analysis of all roll-call votes during each session. For a given Congressperson in a given year, we measure degree of partisanship as the distance between that Representative's D-Nominate Score and the "center" which we identify as the midpoint between the scores of the most liberal Republican and the most conservative Democrat. The first seven columns of Table 1 report the results from a regression of degree of partisanship on Margin of victory and years in office for each of seven sessions of Congress – with the eighth column presenting repeated cross-sectional results that are pooled across all seven Congresses. The final row of the table reports the marginal effect of a one standard deviation increase in margin of victory on the level of partisanship, expressed in terms of standard deviations in partisanship. For example for the 105th Congress, the model predicts that a one standard deviation increase in margin of victory will be associated with a .2 standard deviation increase in partisanship. The estimated marginal effect is highly significant for all seven congresses and ranges between .11 and .44 – with a pooled estimate of .25.

Of course, these results do not necessarily imply a direct causal link. One reasonable possibility is that they reflect the selection of more ideologically extreme candidates into districts with larger partisan majorities. However, if these results are solely driven by candidate selection, given that election outcomes are the most direct measure of a given district's voter ideology, voters would still benefit by increasing their candidate's margin of victory (decreasing

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[3] See Poole and Rosenthal (2001) for a discussion of the construction of their D-W Nominate Score. A complete set of scores for the 1st through 111th Congress are available online at: http://pooleandrosenthal.com/dwnominate.asp. In the current work we restrict our analysis to the 105th thru the 111th congress because of limitations in the availability of digitized data and the difficulty in constructing panel data that accounts for re-districting in earlier periods.

Table 1: Margin of Victory and Extreme Voting

	Congress Specific							Pooled	
Congress	105	106	107	108	109	110	111	105 - 111	105 - 111
Margin of Victory	0.1368 ***	0.0572 **	0.1754 ***	0.0599 **	0.1431 ***	0.2443 ***	0.2604 ***	0.1507 ***	0.0746 ***
S.E.	0.0338	0.0263	0.0343	0.0271	0.0275	0.0257	0.0299	0.0111	0.0095
Years in Office	-0.0008	-0.0010	-0.0007	-0.0013	-0.0001	0.0001	0.0011	-0.0002	0.0049 ***
S.E.	0.0010	0.4261	0.3867	0.0009	0.0009	0.0008	0.0009	0.0004	0.0006
rep F.E.	.	.	.	.	.	.	.	N	Y
Congress F.E.	.	.	.	.	.	.	.	Y	Y
N	423	431	428	430	431	424	432	2999	2999
# Legislators	423	431	428	430	431	424	432	753	753
Mean (Extreme)	0.4253	0.4426	0.4496	0.4661	0.4793	0.4881	0.5017	0.4647	0.4647
S.D. (Extreme)	0.1498	0.1551	0.1778	0.1439	0.1445	0.1465	0.1705	0.1578	0.1578
Mean (MOV)	0.3033	0.4264	0.3953	0.4253	0.4070	0.3603	0.3740	0.3847	0.3847
S.D. (MOV)	0.2201	0.2876	0.2487	0.2597	0.2503	0.2602	0.2608	0.2590	0.2590
Marginal Effect	0.2011	0.1060	0.2452	0.1082	0.2479	0.4338	0.3981	0.2473	0.1224

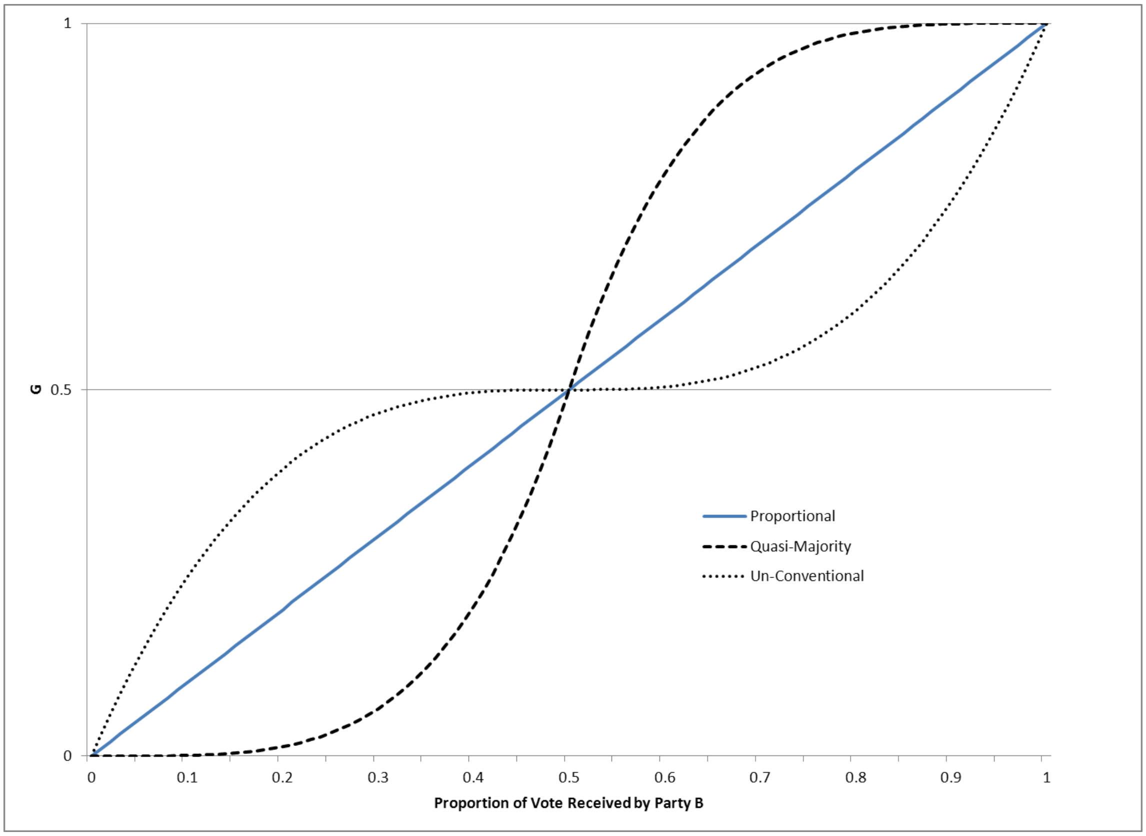
their candidate's loss margin) because the selection mechanism would in the long-run lead to preferred policy outcomes. While such a direct causal link is not necessary to motivate the assumption of smooth politicians, it is possible to use the panel nature of the congressional data to test for a more causal link. Specifically, by pooling the data and including candidate and congress fixed effects, we can more directly evaluate the way that the voting behavior of a given candidate evolves over time in response to changes in their margin of victory. The final column of Table 1 reports the results from this analysis. Again, margin of victory is highly significant, and the estimated marginal effect is .12. Thus, even when we focus solely on the within candidate variation in ideological voting patterns, there is clear evidence in support of the smooth politician assumption.

To incorporate the notion of smooth politicians in the context of a costly voting model, we assume that policy outcomes can be mapped to the interval $(0,1)$. Members of party A strictly prefer outcomes closer to 0 and members of party B strictly prefer outcomes closer to 1. Election outcomes (expressed as the proportion voting for party B) are mapped into policy outcomes via a policy function $G(\cdot)$. The only restrictions on the policy function are that it be strictly increasing in the proportion of votes for party B and that there is symmetric treatment of the two parties.[4] This specification provides a great deal of flexibility; as is shown in Figure 1, which presents 3 different possible policy functions. The solid line presents a "proportional" policy function which is consistent with the proportional representation rules that operate in many parliamentary democracies. The dashed line represents a "quasi-majority" policy function which approximates policy outcomes under direct election regimes such as those that operate in the U.S. Congress. Our model allows such a "quasi-majority" policy function to be arbitrarily

[4] We formalize these notions below in Section 4.

close to the type of step function that characterizes the winner-take-all assumption that has been typical in costly voting models.

Figure 1.



Lastly, the dotted line represents an "un-conventional" policy rule that, while likely of little empirical relevance, is still admitted by the model.

We conclude this section by noting that our goal is not to provide a theoretical treatment that explains why politicians are smooth but rather we seek to show how incorporating this type observed behavior into the costly voting framework yields attractive theoretical results.

4. The Model

We consider a model of costly voting with two parties: $P = A, B$. Society is composed of $N + 1$

citizens. Each individual has the same ex ante independent probability $\lambda \in (0,1)$ of being a supporter of party A and $1-\lambda$ of supporting party B. Citizens decide simultaneously whether to vote or to abstain. If they decide to participate in the election they bear a cost. We assume that for a generic individual i, supporter of party P, there is a cost to voting $c_{i\in P} \in \left[\underline{c}_P, \overline{c}_P\right] \subset \mathbb{R}_+$. Members of party P draw their voting costs independently from the differentiable distribution $F_P(c_P)$, with $F_P{}'(c_P) > 0$ on the entire support. While λ, F_A and F_B are common knowledge, each citizen's preference and cost to voting are private information. If at least one individual votes, then each member of party P receives a benefit from government action according to the electoral rule $G: [0,1] \rightarrow [0,1]$, which is a function of the proportion of total votes z_P obtained by party P. We assume $G'(z_P) > 0$ and finite, $G(0) = 0$, $G(1) = 1$ and $G(z_P) = 1 - G(1 - z_P)$. This last assumption is an anonymity condition that guarantees symmetric treatment of the two parties. If no one votes the benefit from government action received by each individual is equal to $\frac{1}{2}$.

Individuals receive direct benefits from government allocations, but they also exhibit paternalism. They receive spillovers from the benefits obtained by the other members of their party and may enjoy positive utility from each member of the other party being subject to their own party's policy. If a supporter of party P receives a direct benefit b from government action, she enjoys an additional benefit equal to $\gamma_P^P b$ for each member of her own party, and a benefit $\gamma_P^{\bar{P}} b$ for every supporter of the alternative party. We assume that γ_P^P and $\gamma_P^{\bar{P}}$ are common knowledge and that $\gamma_P^P, \gamma_P^{\bar{P}} \geq 0$.

Consider individual i, with cost c_i, belonging to party P. Let N_P and V_P be the number of supporters of party P and of votes cast for P, both exclusive of individual i, respectively.

Moreover, call $V_{\bar{P}}$ the number of votes cast for the other party. Notice that for a supporter of party P voting for the other party is dominated by abstaining, hence citizens' actions boil down to abstain or vote for their preferred alternative. If individual i decides to abstain her benefit is given by:

$$\left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right] G\left(\frac{V_P}{V_P + V_{\bar{P}}}\right) \qquad \text{if } V_P + V_{\bar{P}} > 0 \text{, and by}$$

$$\frac{1}{2}\left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right] \qquad \text{if } V_P + V_{\bar{P}} = 0.$$

If i decides to vote, she receives a gross benefit equal to:

$$\left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right] G\left(\frac{V_P + 1}{V_P + V_{\bar{P}} + 1}\right) \qquad \text{if } V_P + V_{\bar{P}} > 0 \text{, and}$$

$$1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}} \qquad \text{if } V_P + V_{\bar{P}} = 0,$$

and pays the cost c_i. Therefore, i's net benefit from voting is given by:

$$u_{i\in P} = \begin{cases} \left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right]\left[G\left(\dfrac{V_P + 1}{V_P + V_{\bar{P}} + 1}\right) - G\left(\dfrac{V_P}{V_P + V_{\bar{P}}}\right)\right] - c_i & if\ V_P + V_{\bar{P}} > 0 \\ \dfrac{1}{2}\left[1 + N_P\gamma_P^P + (N - N_P)\gamma_P^{\bar{P}}\right] - c_i & if\ V_P + V_{\bar{P}} = 0 \end{cases}.$$

The solution concept that we employ is Bayesian-Nash equilibrium (BNE). As it is customary in this literature, we restrict our attention to type-symmetric Bayesian-Nash equilibria, in the sense that all citizens supporting the same alternative choose the same strategy. In turn, participation decisions depend on the realization of the individual voting cost. Formally, a strategy is a mapping $s_P\colon [\underline{c}_P, \overline{c}_P] \rightarrow \{0,1\}$, where $s_P(c_i) = 0$ means that individual i supporting party P abstains and votes otherwise. A strategy profile $\{s_A, s_B\}$ is a type-symmetric BNE of the game if $s_P(c_i)$ maximizes every individual's expected payoff, given that all other individuals adhere to s_P.

We start by exploring voters' behavior when $N + 1$ is finite. It is possible to characterize citizens' strategies through cut-off values c_P^* such that

$$s_P(c_i) = \begin{cases} 1 & if\, c_i \leq c_P^* \\ 0 & if\, c_i > c_P^* \end{cases}. \tag{1}$$

Proposition 1 *There exists a pure strategy type-symmetric BNE characterized by the voting strategy in (1) and thresholds c_A^* and c_B^*.*[5]

Having characterized the equilibrium for $N+1$ finite, we now turn to analyze voters' behavior in large elections. In order to do this we need to introduce some extra notation. First of all, call $\nu_A \geq 0$ the probability that an A-supporter votes in the election. Equally, $\nu_B \geq 0$ is the probability that a member of party B votes. Moreover, given individual i, let $\hat{\lambda}$ and $(1-\hat{\lambda})$ be the realized proportions of the electorate, exclusive of i, that support party A and B, respectively. Finally, denote by $\hat{\nu}_A$ and $\hat{\nu}_B$ the realized proportions of the electorate, exclusive of i, that vote for party A and B, respectively. For simplicity of notation let us define $\Gamma_A = \gamma_A^A \lambda + \gamma_A^B(1-\lambda)$ and $\hat{\Gamma}_A = \gamma_A^A \hat{\lambda} + \gamma_A^B(1-\hat{\lambda})$. Similarly, for a B-supporter, $\Gamma_B = \gamma_B^A \lambda + \gamma_B^B(1-\lambda)$ and $\hat{\Gamma}_B = \gamma_B^A \hat{\lambda} + \gamma_B^B(1-\hat{\lambda})$.

With no loss of generality, consider a supporter of party A. Given a sample of size $N+1$, conditional on the decisions of all other individuals, the expected gross benefit from voting for an A-supporter is given by Equation (2).

$$E[B_A] = E\left[\left(1+N\hat{\Gamma}_A\right)\left[G\left(\frac{\hat{\nu}_A\hat{\lambda}N+1}{\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N+1}\right) - G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}\right)\right]\middle|\, \hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B(1-\hat{\lambda})N > 0\right] *$$

$$P\left[\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B(1-\hat{\lambda})N > 0\right] + E\left[\frac{1}{2}\left(1+N\hat{\Gamma}_A\right)\middle|\, \hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B(1-\hat{\lambda})N = 0\right] * P\left[\hat{\nu}_A\hat{\lambda}N + \right.$$

[5] The proofs of all propositions, corollaries, and lemmas are provided in the Appendix.

$\hat{\nu}_B(1-\hat{\lambda})N = 0]$. (2)

Notice that the expectation in equation (2) is taken over the random variables $\hat{\lambda}$, $\hat{\nu}_A$ and $\hat{\nu}_B$. Next, note that the limiting distributions of $\hat{\lambda}$, $\hat{\nu}_A\hat{\lambda}$ and $\hat{\nu}_B(1-\hat{\lambda})$ are $N\left[\lambda, \frac{\lambda(1-\lambda)}{N}\right]$, $N\left[\nu_A\lambda, \frac{\nu_A\lambda(1-\nu_A\lambda)}{N}\right]$ and $N\left[\nu_B(1-\lambda), \frac{\nu_B(1-\lambda)[1-\nu_B(1-\lambda)]}{N}\right]$, respectively. As a result we have $Plim\ \hat{\lambda} = \lambda$, $Plim\ \hat{\nu}_A\hat{\lambda} = \nu_A\lambda$ and $Plim\ \hat{\nu}_B(1-\hat{\lambda}) = \nu_B(1-\lambda)$. Hence, supposing an equilibrium characterized by thresholds c_A^* and c_B^*, limiting turnout in equilibrium is equal $\lambda F_A(c_A^*) + (1-\lambda)F_B(c_B^*)$.

The following proposition summarizes the main result of the paper.

Proposition 2 *When $N \to \infty$, limiting turnout is positive if and only if $\Gamma_A > 0$ and $\Gamma_B > 0$. In the case where $\Gamma_A, \Gamma_B > 0$ the expected gross benefit from voting for an A-member and a B-member is given respectively by:*

$$\Gamma_A \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)}\right) = c_A^* > \underline{c}_A, \text{ and}$$

$$\Gamma_B \frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)}\right) = c_B^* > \underline{c}_B.$$

Conversely, if $\Gamma_A = \Gamma_B = 0$, $\underline{c}_A \geq \frac{1}{2}$ and $\underline{c}_B \geq \frac{1}{2}$ then turnout is equal to zero. Finally, in all other cases, turnout for both parties converges to zero.

Proposition 2 states that, if both groups are paternalistic, equilibrium turnout is strictly positive when the population grows without bound. All A-supporters with a cost less than $c_A^* = \Gamma_A \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)}\right)$, and all B-supporters with a cost less than $c_B^* = \Gamma_B \frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)}\right)$ vote in equilibrium. Crucially, this result does not depend on the values of $\underline{c}_A$ and $\underline{c}_B$. The intuition behind this finding is that, although the weight of an individual vote becomes negligible as N tends to infinity, the benefit grows with the size of the electorate.

If all agents are selfish, Proposition 2 extends Palfrey and Rosenthal's (1985) zero turnout result to a general class of smooth policy rules. Indeed, when everyone is selfish our model predicts that turnout is either zero, if $\underline{c}_A, \underline{c}_B \geq \frac{1}{2}$, or converges to zero otherwise. Finally, turnout tends to zero if the supporters of only one party are paternalistic; since selfish players will abstain, there cannot be a mass of paternalistic voters casting a ballot in equilibrium.

One extension to this result is Corollary 1 which states that, ceteris paribus, limiting turnout in equilibrium can be arbitrarily high if voters are sufficiently paternalistic.

Corollary 1 *Given $G(\cdot)$ and λ, there exists a pair $(\tilde{\Gamma}_A, \tilde{\Gamma}_B)$ such that $c_A^* = \overline{c}_A$ and $c_B^* = \overline{c}_B$ for any (Γ_A, Γ_B) with $\Gamma_A \geq \tilde{\Gamma}_A$ and $\Gamma_B \geq \tilde{\Gamma}_B$. Given $G(\cdot)$, λ and $\Gamma_{\overline{P}} > 0$ there exists $\check{\Gamma}_P$ such that $c_P^* = \overline{c}_P$ $\forall$ $\Gamma_P \geq \check{\Gamma}_P$.*

5. Equilibrium Analysis: Exclusive vs Inclusive Paternalism

We now focus on large elections where limiting turnout is positive, that is $\Gamma_\mathrm{A}, \Gamma_\mathrm{B} > 0$. We consider two different types of paternalistic voters. If members of party P are such that $\gamma_P^P > 0$, while $\gamma_P^{\overline{P}} = 0$, we will say that they are Exclusive, as they only care about the members of their own party. On the other hand, we will call them Inclusive if $\gamma_P^P, \gamma_P^{\overline{P}} > 0$. We limit our analysis to the cases where all citizens are either Exclusive or Inclusive.[6] We begin by presenting the following lemma.

Lemma 1

If voters are Exclusive then $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial\lambda} = 0$*, and thus:* $sign\left[\frac{\partial c_A^*}{\partial\lambda}\right] = sign\left[\frac{\partial c_B^*}{\partial\lambda}\right]$.

If voters are Inclusive then $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial\lambda} < 0$*, and thus it cannot be the case that both* $\frac{\partial c_A^*}{\partial\lambda} > 0$ *and* $\frac{\partial c_B^*}{\partial\lambda} < 0$.

Lemma 1 is instructive, as it illustrates the two effects of a marginal change in λ. On the one hand we have the well-known *free-riding effect*: increasing λ makes A-members less likely to vote, while inducing B-supporters to vote more often. On the other hand, by changing the composition of the electorate, an increase in λ varies the spillovers received by a voter. We call this relationship the *spillover effect*.

Suppose that voters are Exclusive. An increase in λ produces a positive *spillover effect* for A-supporters, and a negative one for members of party B. *Spillover* and *free-riding effect* perfectly offset each other and, as a result, the ratio $\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)}$ is unchanged. As a consequence, the equilibrium thresholds for the two parties move in the same direction. If voters are Inclusive the *spillover effect* is less pronounced. Following an increase in λ, an A-supporter receives higher

[6] Given some additional conditions, most results also hold when the supporters of one party are Exclusive, while the members of the other party are Inclusive. However, the study of the "mixed" case would complicate the analysis without adding any interesting insights. Similarly, the key results do not greatly change if $\gamma_P^{\bar{P}} > 0$, but $\gamma_P^P = 0$, that is when voters only care about the supporters of the opposite party. Nevertheless, we do not analyze this case, as it is not particularly realistic.

spillovers from members of her own party, but lower spillovers from B-members. The opposite happens for a supporter of party B. Hence, the ratio $\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)}$ decreases with λ, implying that the equilibrium threshold for party A cannot increase while the equilibrium threshold for B decreases.

Lemma 1 allows us to establish the following result.

Proposition 3

i. *If voters are Exclusive then* $\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)} = \frac{\gamma_A^A}{\gamma_B^B}$, $\lim_{\lambda\to 0} F_A(c_A^*) = 0$, $\lim_{\lambda\to 0} F_B(c_B^*) = 0$, $\lim_{\lambda\to 1} F_A(c_A^*) = 0$ *and* $\lim_{\lambda\to 1} F_B(c_B^*) = 0$.

ii. *If voters are Inclusive then* $\lim_{\lambda\to 0} F_A(c_A^*) = 1$ *and* $\lim_{\lambda\to 0} F_B(c_B^*) = 0$, *while* $\lim_{\lambda\to 1} F_A(c_A^*) = 0$ *and* $\lim_{\lambda\to 1} F_B(c_B^*) = 1$.

Thus, under Exclusive Paternalism, the *free-riding effect* is offset by the *spillover effect* and voter turnout goes to zero as λ goes to zero. Conversely, under Inclusive Paternalism, the *free-riding effect* dominates and as λ goes to zero, minority turnout goes to 100% and majority turnout goes to zero.

5.1 Identical Cost Distributions

In the remainder of the paper we focus on the case where all citizens draw their costs from the same cost distribution, i.e. we assume $F_A = F_B = F$. Proposition 4 establishes that if voters are Exclusive the group with the higher degree of paternalism will always vote more often than the other, while the same relative percentage will cast a ballot when $\gamma_A^A = \gamma_B^B$.

Proposition 4 *Suppose Exclusive voters and* $F_A = F_B$.

i. *If* $\gamma_A^A = \gamma_B^B$ *then* $c_A^*(\lambda) = c_B^*(\lambda) \forall \lambda$.

ii. *If* $\gamma_A^A > \gamma_B^B$ *then* $c_A^*(\lambda) > c_B^*(\lambda) \forall \lambda$.

iii. *If* $\gamma_A^A < \gamma_B^B$ *then* $c_A^*(\lambda) < c_B^*(\lambda) \forall \lambda$.

On the other hand, Proposition 5 demonstrates that if voters are Inclusive, there exists a critical value $\tilde{\lambda}$ such that A-supporters vote more often than B-supporters for any $\lambda < \tilde{\lambda}$, while the opposite is true when $\lambda > \tilde{\lambda}$.

Proposition 5 (Underdog Effect) *Suppose Inclusive voters and* $F_A = F_B$. *There exists* $\tilde{\lambda}$ *such that* $c_A^*(\lambda) > c_B^*(\lambda) \forall \lambda < \tilde{\lambda}$, $c_A^*(\lambda) < c_B^*(\lambda) \forall \lambda > \tilde{\lambda}$ *and* $c_A^*(\tilde{\lambda}) = c_B^*(\tilde{\lambda})$.

i. *If* $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ *then* $\tilde{\lambda} = \frac{1}{2}$ *(Pure Underdog Effect).*

ii. *If* $\gamma_A^A + \gamma_A^B > \gamma_B^A + \gamma_B^B$ *then* $\tilde{\lambda} > \frac{1}{2}$ *(Asymmetric Underdog Effect).*

iii. *If* $\gamma_A^A + \gamma_A^B < \gamma_B^A + \gamma_B^B$ *then* $\tilde{\lambda} < \frac{1}{2}$ *(Asymmetric Underdog Effect).*

The value of $\tilde{\lambda}$ depends on the relative degree of paternalism of the two groups. If both groups exhibit the same overall level of Inclusive paternalism then $\tilde{\lambda} = \frac{1}{2}$, meaning that the minority supporters vote with a strictly higher probability than do the members of the majority party. This phenomenon, called *underdog effect*[7], has been discussed in several papers and has been formalized in a general framework by Taylor and Yildirim (2010) for the case of small electorate and same cost distribution. Crucially, in Taylor and Yildirim's model the underdog effect disappears when $N \to \infty$, as turnout converges to zero.

The *underdog effect* is caused by the relatively higher incentive to free-ride experienced by the majority supporters. This is why it arises only when voters are Inclusive. As shown in Lemma 1, if voters are Exclusive the *spillover effect* is sufficiently strong to offset the *free-riding effect*, and the equilibrium thresholds of the two groups move in the same direction when λ changes. Moreover, notice that the *underdog effect* can only arise if both groups display the same overall level of paternalism, that is if $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$. If that is not the case, the game is no longer symmetric, implying that one group votes with a strictly higher probability when the electorate is evenly split. Hence, the switching point $\tilde{\lambda}$ cannot be equal to $\frac{1}{2}$. Thus, we draw a distinction between a *pure underdog effect* and an *asymmetric underdog effect*.

When both groups are characterized by the same overall level of paternalism, despite the *underdog effect*, the majority supporters never lose their initial advantage, provided that individuals care about their fellow group members at least as much as do those supporting the alternative party. As shown by Taylor and Yildirim (2010) for the small electorate case, in equilibrium the majority party receives a higher share of the votes. As we prove in the next proposition, this is also true when voters are Exclusive.[8] Define $\theta_A^* = \frac{\lambda F(c_A^*)}{\lambda F(c_A^*) + (1-\lambda) F(c_B^*)}$ as the

[7] The term *underdog effect* was introduced by Levine and Palfrey (2007), who document it with experimental evidence. Shachar and Nalebuff (1999) and Blais (2000), among others, provide empirical evidence for the case of large elections.

[8] Supposing that an individual is at least as much concerned for her own group as is a member of the other party is a realistic, and not very restrictive, assumption. Moreover, by definition, it is always true for Exclusive voters.

share of the votes cast for party A in equilibrium.

Proposition 6 *Suppose that* $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ *and* $F_A = F_B$. *If* $\gamma_A^A \geq \gamma_B^A$ *and* $\gamma_B^B \geq \gamma_A^B$ *then* $\theta_A^* < \frac{1}{2}\ \forall\ \lambda < \frac{1}{2}$, *while* $\theta_A^* > \frac{1}{2}\ \forall\ \lambda > \frac{1}{2}$.

Another important phenomenon that has received significant attention in the voting literature is the so-called *competition effect*, which states that turnout is higher the closer is the election. Levine and Palfrey (2007), who coined the term *competition effect*, provide evidence for it from laboratory experiments. This has been a difficult property to demonstrate in models of large elections (see Krasa and Polborn, 2009). Indeed, as reported by Taylor and Yildirim "the widely held intuition that elections with a more evenly split electorate should generate a greater expected turnout appears to be a property of small elections" (Taylor and Yildirim, 2010, p. 464). Nevertheless, the reason why their model does not predict higher turnout in closer elections is because in a standard costly voting framework turnout tends to zero as $N \to \infty$. Yet, the *competition effect* is well documented in large elections (see, for example, Blais, 2000; Shachar and Nalebuff, 1999).

In Proposition 7 we provide analytical evidence of the *competition effect* for the case of Exclusive voters, provided the following two conditions hold: $\gamma_A^A = \gamma_B^B$ and $G''(z_P) \geq 0$ for $z_P < \frac{1}{2}$ (implying $G''(z_P) \leq 0$ when $z_P > \frac{1}{2}$). Both conditions are rather weak and intuitive. Firstly, if $\gamma_A^A \neq \gamma_B^B$ the game is asymmetric and turnout for the two parties is not the same when $\lambda = \frac{1}{2}$: hence a more evenly split electorate does not imply a closer election. Second, notice that so far we have not made any restrictions on the shape of the function $G(z_P)$. If $G''(z_P)$ is greater than (or equal to) zero for $z_P < \frac{1}{2}$, an individual vote has more (or equal) weight, and the benefit from voting is greater (or equal), when it closes the gap with the other party than when it increases it. This assumption encompasses both the proportional and quasi-majority policy rules discussed above in Section 3.

Proposition 7 (Competition Effect) *Suppose Exclusive voters and $F_A = F_B$. If $\gamma_A^A = \gamma_B^B$ and $G''(z_P) \geq 0$ for $z_P < \frac{1}{2}$ then turnout is higher the closer λ is to $\frac{1}{2}$.*

We can only provide an analytical proof for the case of Exclusive voters. However, in our model the competition effect is also a feature of large elections with Inclusive voters – provided that $\gamma_A^A = \gamma_B^B, \gamma_A^B = \gamma_B^A$ and $G''(z_P) \geq 0$ for $z_P < \frac{1}{2}$. Figures 2 and 3 illustrate predicted turnout as a function of λ for a proportional policy rule and a quasi-majority policy rule, respectively. In both examples, $\gamma_A^A = \gamma_B^B = 0.5$, $\gamma_A^B = \gamma_B^A = 0.25$ and costs are uniformly distributed. $G(z_P)= z_P$ in Figure 2, while in Figure 3 $G(z_P)=2\ z_P^2$ for $z < \frac{1}{2}$.

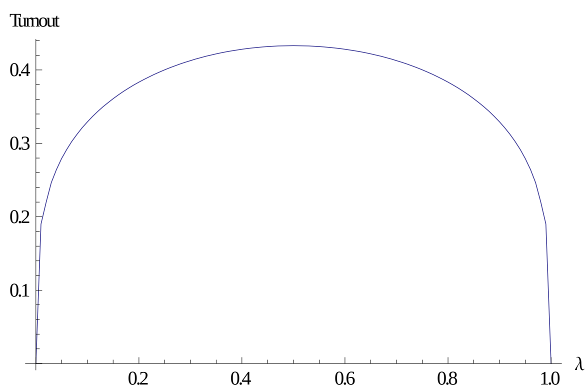


Figure 2.

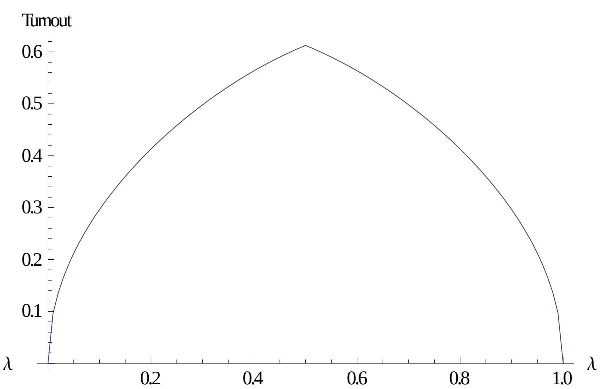


Figure 3.

While, we only present two examples here, an exhaustive search over specifications satisfying the conditions $\gamma_A^A = \gamma_B^B$, $\gamma_A^B = \gamma_B^A$ and $G''(z_P) \geq 0$ for $z_P < \frac{1}{2}$ has failed to yield a counter-example.

Propositions 2 thru 7 and their associated lemmas and corollaries serve to highlight the large election properties of a costly voting model that incorporates paternalism (Exclusive and Inclusive) and smooth politicians – the key strength of the model being its ability to overcome the paradox of voting, and to capture *free-riding* and *spillover effects*, the *underdog effect*, and the *competition effect* in a purely game theoretic formulation of a large election.

As we showed in Proposition 2, if voters are selfish our model predicts that turnout converges to zero, even assuming that mandate matters. Although we argue that a smooth policy rule is a much more realistic assumption than considering elections as winner-take-all, it is a fair question to ask what would happen if voters were paternalistic, but the outcome of the election was decided by majority rule. Proposition 8 provides the answer to this question, for the case of symmetric overall paternalism and assuming, as we did when discussing vote shares, that $\gamma_A^A \geq \gamma_B^A$ and $\gamma_B^B \geq \gamma_A^B$.

Proposition 8 *Assume $F_A = F_B$. Suppose also that $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$, with $\gamma_A^A \geq \gamma_B^A$ and $\gamma_B^B \geq \gamma_A^B$. Under majority rule, $c_A^* = \overline{c}_A$ and $c_B^* = \overline{c}_B$ if $\lambda = \frac{1}{2}$, while limiting turnout is equal to zero for any $\lambda \neq \frac{1}{2}$.*

The above proposition tells us that if the election is decided by majority rule then, even with paternalistic voters, turnout converges to zero, unless the electorate is evenly split, in which case everyone votes. Interestingly, Evren (2010) obtains the same result in a model in which a part of the electorate has other-regarding preferences, under the assumption that their proportion is known. Proposition 8 highlights the necessity of assuming a smooth policy rule in our framework in order to overcome the paradox of voting in a general sense, that is for any possible composition of the electorate. The intuition is that, under majority rule, if the electorate is unevenly split the higher benefit of voting that a paternalistic individual enjoys in a larger population is dominated by the lower probability of being pivotal, with the latter going to zero faster then N goes to infinity.

6. Discussion and Conclusion

Since the seminal work of Ledyard (1981, 1984) and Palfrey and Rosenthal (1983, 1985), economists have commonly modeled elections as participation games where voters pay a cost to vote. The prominence of such models can be explained by their game theoretic micro foundations, and their ability to generate predictions which are consistent with notions of strategic voting behavior and the comparative statics results that have been documented in empirical studies of election outcomes. While popular, current formulations of the costly voting model typically confront one major drawback – the paradox of voting which was first described in decision theoretic terms by Anthony Downs (1957).

Several attempts have been made to solve the paradox (see Feddersen, 2004, for a review) and thus reconcile costly voting models with the evidence that a substantial fraction of the population turns out to vote. Among all of these approaches, Feddersen and Sandroni's work is perhaps the most successful example. Unfortunately, while their framework yields strong comparative statics predictions and positive turnout, their model is inherently non-strategic with regards to members of the same party.

In this paper we approach the problem from a different perspective, developing a parsimonious game theoretic model that builds on the basic framework of Palfrey and Rosenthal (1985). We extend their analysis in two ways. Our primary innovation is the adoption of a smooth politician framework under which election outcomes are mapped into policy decisions through a policy function that is continuous in election margins. Second, similar in some ways to the work of Feddersen and Sandroni, we assume paternalistic voters. Under this new model, when citizens are paternalistic, large elections yield strictly positive turnout both in terms of numbers and proportions. Conversely, the model predicts that, if the supporters of at least one party are purely selfish, turnout will converge to zero as the electorate grows; thus, we extend Palfrey and Rosenthal's (1985) theorem to a general class of smooth policy functions. A similar result holds for the case of paternalism & majority rule. Indeed, under the winner-take-all assumption, if the electorate is unevenly split, turnout converges to zero – even when voters are paternalistic, while everyone votes if the electorate is evenly split. Finally, our framework yields sensible comparative statics predictions, namely the *underdog* and *competition effect*, that were believed to be only properties of small elections (see Taylor and Yildirim, 2010).

Thus, our analysis demonstrates that by reconceptualizing a paternalistic model of costly voting to incorporate a smooth policy function – an arguably more realistic description of the policy process – it is possible to reconcile the small sample predictions of the costly voting framework with a game theoretic large electorate environment.

Finally, we note that in a concurrent working paper, Evren (2010) also overcomes the paradox of voting in a costly voting framework. Similar to the model of Feddersen and Sandroni, Evren assumes altruistic voters and a winner-take-all election. Evren's definition of altruism partially corresponds to our notion of paternalism, in that a voter does not incorporate other individuals' benefits in her own utility function (as altruism is typically understood), but rather projects her own preference onto other citizens; he also assumes, though, that individuals care about the voting costs of others, thus following in the spirit of Feddersen and Sandroni. In this framework, Evren shows that, if there is uncertainty about the proportion of the population who are altruistic voters, the rule utilitarian assumption is not necessary to generate positive turnout in large elections.

It is an open question about which assumption is a more appropriate description of the driving force behind voter behavior: expectations regarding the importance of mandate or uncertainty regarding the composition of the electorate. We consider Evren's paper and ours to be complements and view an exploration of their relative merits as fertile ground for future research.

Appendix

Proof of Proposition 1

Suppose citizens play according to the strategy defined by (1) and thresholds $\hat{c}_A$ and $\hat{c}_B$. Then for a supporter of party A the expected gross benefit from voting is given by

$$E[B_A(\hat{c}_A,\hat{c}_B)] = \sum_{N_A=0}^{N} \binom{N}{N_A} \lambda_A^{N_A}(1-\lambda_A)^{(N-N_A)} \sum_{V_A=0}^{N_A} \sum_{V_B=0}^{N-N_A} \binom{N_A}{V_A} \binom{N-N_A}{V_B} F(\hat{c}_A)^{V_A}\big(1-F(\hat{c}_A)\big)^{N_A-V_A} F(\hat{c}_B)^{V_B}\big(1-F(\hat{c}_B)\big)^{N-N_A-V_B}(\pi_A(V_A,V_B,N_A)),$$

where

$$\pi_A(V_A,V_B,N_A) = \begin{cases} [1+\gamma_A^A N_A + \gamma_A^B(N-N_A)]\left[G\left(\frac{V_A+1}{V_A+V_B+1}\right) - G\left(\frac{V_A}{V_A+V_B}\right)\right] & if\ V_A+V_B>0 \\ \frac{1}{2}[1+\gamma_A^A N_A + \gamma_A^B(N-N_A)] & if\ V_A+V_B=0 \end{cases}.$$

Similarly $E[B_B(\hat{c}_A,\hat{c}_B)]$ represents the expected gross benefit from voting for a supporter of party B and is calculated in an analogous way. For this to be an equilibrium, it must be the case that the supporter of A with cost $\hat{c}_A$ and the supporter of B with cost $\hat{c}_B$ must be indifferent between voting and abstaining when all other players adopt the same strategy.

To prove the existence of such equilibrium we construct the function

$$\Phi(\hat{c}_A,\hat{c}_B) = \left(\max\left\{\min\{E[B_A(\hat{c}_A,\hat{c}_B)],\overline{c}_A\},\underline{c}_A\right\},\max\left\{\min\{E[B_B(\hat{c}_A,\hat{c}_B)],\overline{c}_B\},\underline{c}_B\right\}\right).$$

As both $E[B_A(\hat{c}_A,\hat{c}_B)]$ and $E[B_B(\hat{c}_A,\hat{c}_B)]$ are continuous in $\hat{c}_A$ and $\hat{c}_B$, then $\Phi(\hat{c}_A,\hat{c}_B)$ is also continuous. Hence, by Brower's fixed point theorem there must exist a pair (c_A^*,c_B^*) such that $\Phi(c_A^*,c_B^*) = (c_A^*,c_B^*)$. If $c_P^* \in (\underline{c}_P,\overline{c}_P)$, then all supporters of party P with cost less than c_P^* will vote and those with higher costs will abstain. Similarly, if $c_P^* = \underline{c}_P$ all members of party P will abstain, while if $c_P^* = \overline{c}_P$ they will all vote.■

Proof of Proposition 2

In order to evaluate the limit of Equation (2), we must consider *two cases*.

i) In the *first case*, suppose that $c_A^* = \underline{c}_A$ and $c_B^* = \underline{c}_B$, implying that $\nu_A = \nu_B = 0$. Here, all other citizens choose *not* to vote with probability 1. As a result, the returns to voting for a supporter of party A are given by:

$$E[B_A|\nu_A + \nu_B = 0] = E\left[\frac{1}{2}\left(1 + N\hat{\Gamma}_A\right)\right].$$

Given that $\hat{\lambda} \to \lambda$, in the limit, as $N \to \infty$, then $\hat{\Gamma}_A$ converges to Γ_A. Hence the returns to voting become infinite if $\Gamma_A > 0$, while they are equal to $\frac{1}{2}$ when $\Gamma_A = 0$. Clearly, the same holds for a supporter of party B, with the gross benefit from voting equal to either infinity or $\frac{1}{2}$ when $\Gamma_B > 0$ or $\Gamma_B = 0$, respectively. If $\Gamma_A = \Gamma_B = 0$, this implies that $c_A^* = \underline{c}_A$ and $c_B^* = \underline{c}_B$ can only be an equilibrium if $\underline{c}_A \geq \frac{1}{2}$ and $\underline{c}_B \geq \frac{1}{2}$. On the other hand, if at least one between Γ_A and Γ_B is strictly positive, then it cannot be the case that $c_A^* = \underline{c}_A$ and $c_B^* = \underline{c}_B$ in equilibrium.

ii) In the *second case*, suppose $c_P^* > \underline{c}_P$ for at least for one party. As a result, for at least one type the probability of voting is strictly positive, which translates into $\nu_A + \nu_B > 0$. We begin by showing that for this case the Plim of the second term in Equation (2) is zero and can thus be ignored. To see this, first note that the second half of this term is the probability that no one votes. Following the approach taken by Taylor and Yildirim (2010) it is easy to show that the limiting marginal distributions of $\left\{\hat{\nu}_A\hat{\lambda}N, \hat{\nu}_B\left(1 - \hat{\lambda}\right)N\right\}$ are independent Poisson distributions with means equal to $\{\nu_A\lambda N, \nu_B(1 - \lambda)N\}$.[9] As a result,

$$\lim_{N\to\infty} P\left[\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B\left(1 - \hat{\lambda}\right)N = 0\right] = \frac{(\nu_A\lambda N)^0}{0!e^{\nu_A\lambda N}}\frac{[\nu_B(1-\lambda)N]^0}{0!e^{\nu_B(1-\lambda)N}} = \frac{1}{e^{N[\nu_A\lambda+\nu_B(1-\lambda)]}}.$$

Next assume, without loss of generality, that $\gamma_A^A \geq \gamma_A^B$ and note that $\lambda < 1$. Thus,

$$\lim_{N\to\infty} E\left[\frac{1}{2}\left(1 + N\hat{\Gamma}_A\right)\middle|\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B\left(1 - \hat{\lambda}\right)N = 0\right] * P\left[\hat{\nu}_A\hat{\lambda}N + \hat{\nu}_B\left(1 - \hat{\lambda}\right)N = 0\right] \leq$$

[9] See Taylor & Yildirim (2010), Lemma 5.

$$\lim_{N\to\infty} \frac{1+\gamma_A^A N}{2e^{N[\nu_A\lambda+\nu_B(1-\lambda)]}} = 0.$$

We now turn to the first term in Equation (2). For simplicity of notation let us define

$$\Delta = \frac{\hat{\nu}_B(1-\hat{\lambda})}{[\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N+1][\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})]}.$$

We can re-write the expression over which we are taking the expectation as:

$$\left(\frac{1}{N}+\hat{\Gamma}_A\right)N\Delta\frac{\left[G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}+\Delta\right)-G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}\right)\right]}{\Delta}.$$

Recall that, by construction, this term is limited to outcomes where $\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N>0$.

Moreover, notice that, by definition:

$$\lim_{N\to\infty}\frac{\left[G\left(\frac{\hat{\nu}_A\hat{\lambda}N}{\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N}+\Delta\right)-G\left(\frac{\hat{\nu}_A\hat{\lambda}N}{\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N}\right)\right]}{\Delta} = G'\left(\frac{\hat{\nu}_A\hat{\lambda}N}{\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N}\right).$$

Thus we have the result of Equation (A1).

$$\lim_{N\to\infty}\left(\frac{1}{N}+\hat{\Gamma}_A\right)N\Delta\frac{\left[G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}+\Delta\right)-G\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}\right)\right]}{\Delta}\Bigg|\,\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N>0 =$$

$$\hat{\Gamma}_A\frac{\hat{\nu}_B(1-\hat{\lambda})}{[\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})]^2}G'\left(\frac{\hat{\nu}_A\hat{\lambda}}{\hat{\nu}_A\hat{\lambda}+\hat{\nu}_B(1-\hat{\lambda})}\right). \tag{A1}$$

Recall that the $Plim\ \hat{\lambda}=\lambda$, $Plim\ \hat{\nu}_A\hat{\lambda}=\nu_A\lambda$ and $Plim\ \hat{\nu}_B(1-\hat{\lambda})=\nu_B(1-\lambda)$. This fact, combined with the result that the second term of Equation (2) converges to zero, implies that, conditional on $\hat{\nu}_A\hat{\lambda}N+\hat{\nu}_B(1-\hat{\lambda})N>0$, in the limit Equation (2) collapses to

$$\Gamma_A\frac{\nu_B(1-\lambda)}{[\nu_A\lambda+\nu_B(1-\lambda)]^2}G'\left(\frac{\nu_A\lambda}{\nu_A\lambda+\nu_B(1-\lambda)}\right)$$

Finally, recalling that in equilibrium individuals use a threshold voting strategy, the probability ν_P that a member of the generic party P votes is equal to $F_P(c_P^*)$. Hence, we can re-write the limiting benefit for A-members as

$$\Gamma_A \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \tag{A2}$$

The limiting benefit for B-members can be calculated in an analogous way and is equal to

$$\Gamma_B \frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \tag{A3}$$

This implies that, in equilibrium, we have

$$c_A^* = \Gamma_A \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right), \tag{A4}$$

and

$$c_B^* = \Gamma_B \frac{F_A(c_A^*)\lambda}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \tag{A5}$$

Suppose $\Gamma_A, \Gamma_B > 0$. We are going to show that equilibrium turnout is bounded away from zero.

Notice first that, by definition,

$$G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right) = G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right).$$

Therefore, from equations (A4) and (A5) we know that, in equilibrium, the following holds

$$\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)} = \frac{\Gamma_A(1-\lambda)}{\Gamma_B \lambda} = \frac{\gamma_A^A \lambda\,(1-\lambda)+\gamma_A^B(1-\lambda)^2}{\gamma_B^A\,\lambda^2+\gamma_B^B \lambda(1-\lambda)}. \tag{A6}$$

From (A6) it follows that $\frac{\partial c_A^*}{\partial c_B^*} > 0$ and, as a consequence, there exists a function ϕ such that

$c_B^* = \phi(c_A^*)$ and $\frac{\partial \phi(c_A^*)}{\partial c_A^*} > 0$. Moreover, $F_B\big(\phi(c_A^*)\big) \to 0$ when $c_A^* \to \underline{c}_A$.

Suppose that $c_A^* \to \underline{c}_A$ when $N \to \infty$. This implies that, at the limit, the benefit from voting for an A-member is less than or equal to $\underline{c}_A$. Notice from (A2) that, since $G'(\cdot)$ is finite and strictly positive, this is only possible if $\lim\limits_{c_A^* \to \underline{c}_A} \frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2} < \infty$. Because $\frac{F_B(c_B^*)(1-\lambda)}{\left[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)\right]^2}$ can be written as $\frac{(1-\lambda)}{\frac{F_A(c_A^*)^2}{F_B(B_A^*)}\lambda^2+F_B(C_B^*)(1-\lambda)^2+2\lambda(1-\lambda)F_A(C_A^*)}$ and equation (A6) implies that c_B^* goes to

$\underline{c}_B$ as c_A^* goes to $\underline{c}_A$, it must be the case that:

$$\lim_{c_A^* \to \underline{c}_A} \frac{F_B\left(\phi(c_A^*)\right)}{\left(F_A(c_A^*)\right)^2} \frac{(1-\lambda)}{\lambda^2} = k < \infty. \tag{A7}$$

Working with Equation (A3), by similar logic, as c_A^* goes to $\underline{c}_A$ the benefit from voting for a B-member tends to:

$$\frac{F_A(c_A^*)}{\left(F_B\left(\phi(c_A^*)\right)\right)^2} \frac{\lambda}{(1-\lambda)^2} \Gamma_B G'(\cdot). \tag{A8}$$

Note that, if the condition outlined in (A7) holds, it must be the case that $F_B\left(\phi(c_A^*)\right)$ goes to zero faster than $F_A(c_A^*)$. However, this implies that expression (A8) tends to infinity and thus each B-supporter votes in equilibrium. Analogously, it can be shown that if $c_B^* \to \underline{c}_B$ then the benefit from voting for an A-member tends to infinity, proving that turnout is bounded away from zero in equilibrium. Moreover, notice from (A6) that if $c_B^* > \underline{c}_B$ then $c_A^* > \underline{c}_A$, and vice versa, which means that in equilibrium turnout is positive for both parties.

Finally, suppose $\Gamma_P = 0$. In this case, the limiting benefit from voting for a member of party P is equal to zero, which means that c_P^* cannot be greater than $\underline{c}_P$ when $N \to \infty$. This in turns implies that the benefit from voting for a member of party $\overline{P}$ tends to zero, even if $\Gamma_{\overline{P}} > 0$, and as a consequence $c_{\overline{P}}^* \to \underline{c}_{\overline{P}}$. Therefore, unless both Γ_A and Γ_B are strictly positive, equilibrium turnout is either zero, if $\Gamma_A = \Gamma_B = 0$, $\underline{c}_A \geq \frac{1}{2}$ and $\underline{c}_B \geq \frac{1}{2}$, or converges to zero otherwise. ■

Proof of Corollary 1

This result follows immediately from the proof of Proposition 2. ■

Proof of Lemma 1

From (A6) we calculate $\frac{\partial(c_A^* F_A(c_A^*)/c_B^* F_B(c_B^*))}{\partial \lambda}$ which is equal to

$$\frac{\gamma_A^A(1-2\lambda)-2\gamma_A^B(1-\lambda)}{\lambda[\gamma_B^A\lambda+\gamma_B^B(1-\lambda)]}-\frac{(1-\lambda)[\gamma_A^A\lambda+\gamma_A^B(1-\lambda)][2\gamma_B^A\lambda+\gamma_B^B(1-2\lambda)]}{\lambda^2[\gamma_B^A\lambda+\gamma_B^B(1-\lambda)]^2}.$$

The above expression reduces to

$$\frac{-\gamma_A^B\gamma_B^B(1-\lambda)^2-2\gamma_A^B\gamma_B^A\lambda(1-\lambda)-\gamma_A^A\gamma_B^A\lambda^2}{\lambda^2[\gamma_B^A\lambda+\gamma_B^B(1-\lambda)]^2}. \tag{A9}$$

From (A9) we can see that the sign of the derivative is zero if $\gamma_A^B=\gamma_B^A=0$ (i.e. citizens are Exclusive), but it is negative otherwise.

Notice that $\frac{\partial(c_A^*F_A(c_A^*)/c_B^*F_B(c_B^*))}{\partial\lambda}$ equals

$$\frac{1}{\left(c_B^*F_B(c_B^*)\right)^2}\left[\frac{\partial c_A^*}{\partial\lambda}\left(\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right)-\frac{\partial c_B^*}{\partial\lambda}\left(\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right)\right].$$

Thus, if citizens are Exclusive we have

$$\frac{\partial c_A^*}{\partial\lambda}\left(\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right)=\frac{\partial c_B^*}{\partial\lambda}\left(\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right).$$

Therefore, it must be the case that $sign\left[\frac{\partial c_A^*}{\partial\lambda}\right]=sign\left[\frac{\partial c_B^*}{\partial\lambda}\right]$.

Finally, if voters are Inclusive we know that

$$\frac{\partial c_A^*}{\partial\lambda}\left(\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}\right)<\frac{\partial c_B^*}{\partial\lambda}\left(\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}\right).$$

Notice that $\frac{1}{c_A^*}+\frac{\partial F_A(c_A^*)}{\partial c_A^*}\frac{1}{F_A(c_A^*)}>0$ and $\frac{1}{c_B^*}+\frac{\partial F_B(c_B^*)}{\partial c_B^*}\frac{1}{F_B(c_B^*)}>0$. Suppose $\frac{\partial c_A^*}{\partial\lambda}>0$ and $\frac{\partial c_B^*}{\partial\lambda}<0$.

The left hand side of the above inequality would be positive, while the right hand side would be negative, which cannot be. ■

Proof of Proposition 3

i. We know from Proposition 2 that

$$c_B^*[F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)]^2=\Gamma_B F_A(c_A^*)\lambda G'\left(\frac{F_B(c_B^*)(1-\lambda)}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right), \tag{A10}$$

and

$$c_A^*[F_A(c_A^*)\lambda + F_B(c_B^*)(1-\lambda)]^2 = \Gamma_A F_B(c_B^*)(1-\lambda)G'\left(\frac{F_A(c_A^*)\lambda}{F_A(c_A^*)\lambda+F_B(c_B^*)(1-\lambda)}\right). \quad \text{(A11)}$$

When $\lambda \to 0$, Equation (A10) reduces to $c_B^* F_B(c_B^*)^2 = 0$ and therefore

$$\lim_{\lambda\to 0} F_B(c_B^*) = 0. \quad \text{(A12)}$$

Analogously, Equation (A11) reduces to $c_A^* F_A(c_A^*)^2 = 0$ when $\lambda \to 1$, implying that

$$\lim_{\lambda\to 1} F_A(c_A^*) = 0. \quad \text{(A13)}$$

When voters are Exclusive, we know from (A6) that $\frac{c_A^* F_A(c_A^*)}{c_B^* F_B(c_B^*)} = \frac{\gamma_A^A}{\gamma_B^B}$. This, together with (A12), implies that $\lim_{\lambda\to 0} F_B(c_B^*) = \lim_{\lambda\to 0} F_A(c_A^*) = 0$. Similarly, given (A13), we conclude that $\lim_{\lambda\to 1} F_A(c_A^*) = \lim_{\lambda\to 1} F_B(c_B^*) = 0$.

ii. If voters are Inclusive, the benefit for an A-member converges to $\gamma_A^B \frac{1}{F_B(c_B^*)} G'\left(\frac{0}{F_B(c_B^*)}\right)$ when $\lambda \to 0$. Given (A12), since $G'(\cdot)$ is always positive and finite, the latter tends to infinity and $\lim_{\lambda\to 0} F_A(c_A^*) = 1$. Similarly, when $\lambda \to 1$ the benefit for a B-member converges to $\gamma_B^A \frac{1}{F_A(c_A^*)} G'\left(\frac{0}{F_A(c_A^*)}\right)$ and, given (A13), $\lim_{\lambda\to 1} F_B(c_B^*) = 1$.■

Proof of Proposition 4

Recall from Proposition 3 that, when voters are Exclusive, $\frac{c_A^* F(c_A^*)}{c_B^* F(c_B^*)} = \frac{\gamma_A^A}{\gamma_B^B}$. It follows immediately that if $\gamma_A^A > \gamma_B^B$ then $c_A^*(\lambda) > c_B^*(\lambda)\forall\lambda$, while $c_A^*(\lambda) < c_B^*(\lambda)\forall\lambda$ if $\gamma_A^A < \gamma_B^B$ and $c_A^*(\lambda) = c_B^*(\lambda)\forall\lambda$ if $\gamma_A^A = \gamma_B^B$. ■

Proof of Proposition 5

Given Proposition 3, we know that, by continuity, there must exist at least one value $\tilde{\lambda}$ such that $c_A^*(\tilde{\lambda}) = c_B^*(\tilde{\lambda})$. Moreover, we know from Lemma 1 that $\frac{\partial\left(\frac{c_A^* F(c_A^*)}{c_B^* F(c_B^*)}\right)}{\partial\lambda} < 0$ and, therefore, when $c_A^* = c_B^*$ it must be the case that $\frac{\partial c_B^*}{\partial\lambda} > \frac{\partial c_A^*}{\partial\lambda}$. However, if there were more than one crossing point, it would imply $\frac{\partial c_B^*}{\partial\lambda} < \frac{\partial c_A^*}{\partial\lambda}$ for at least one of these points, which cannot be. This proves that there exists a unique $\tilde{\lambda}$ such that $c_A^*(\tilde{\lambda}) = c_B^*(\tilde{\lambda})$, $c_A^*(\lambda) > c_B^*(\lambda)\forall\lambda < \tilde{\lambda}$ and $c_A^*(\lambda) < c_B^*(\lambda)\forall\lambda > \tilde{\lambda}$.

Suppose $\lambda = \frac{1}{2}$. From (A6) we know that $\frac{c_A^* F(c_A^*)}{c_B^* F(c_B^*)} = \frac{\gamma_A^A + \gamma_A^B}{\gamma_B^A + \gamma_B^B}$. Hence, if $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ then $c_A^*\left(\lambda = \frac{1}{2}\right) = c_B^*\left(\lambda = \frac{1}{2}\right)$, implying that $\tilde{\lambda} = \frac{1}{2}$. If $\gamma_A^A + \gamma_A^B < \gamma_B^A + \gamma_B^B$ then $c_A^*\left(\lambda = \frac{1}{2}\right) < c_B^*\left(\lambda = \frac{1}{2}\right)$, which implies that $\tilde{\lambda} < \frac{1}{2}$. Finally, if $\gamma_A^A + \gamma_A^B > \gamma_B^A + \gamma_B^B$ then $c_A^*\left(\lambda = \frac{1}{2}\right) > c_B^*\left(\lambda = \frac{1}{2}\right)$, meaning that $\tilde{\lambda} > \frac{1}{2}$. ■

Proof of Proposition 6

Consider first the case of Exclusive voters. Since $\gamma_A^B = \gamma_B^A = 0$, assuming $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ implies $\gamma_A^A = \gamma_B^B$. We know from Proposition 4 that in this case $c_A^*(\lambda) = c_B^*(\lambda)\forall\lambda$. It follows that $\theta_A^*(\lambda) < \frac{1}{2}\ \forall\lambda < \frac{1}{2}$ and $\theta_A^*(\lambda) > \frac{1}{2}\ \forall\lambda > \frac{1}{2}$.

Consider now the case of Inclusive voters. From (A6) we know that

$$\lambda c_A^* F(c_A^*)[\lambda\gamma_B^A + (1-\lambda)\gamma_B^B] = (1-\lambda)c_B^* F(c_B^*)[\lambda\gamma_A^A + (1-\lambda)\gamma_A^B]. \tag{A14}$$

Let us re-arrange $\gamma_A^A + \gamma_A^B = \gamma_B^A + \gamma_B^B$ as $\gamma_A^A - \gamma_B^A = \gamma_B^B - \gamma_A^B$. As we are assuming $\gamma_A^A \geq \gamma_B^A$ and $\gamma_B^B \geq \gamma_A^B$, this implies that $\lambda(\gamma_A^A - \gamma_B^A) \leq (1-\lambda)(\gamma_B^B - \gamma_A^B)$ when $\lambda < \frac{1}{2}$, while $\lambda(\gamma_A^A - \gamma_B^A) \geq (1-\lambda)(\gamma_B^B - \gamma_A^B)$ when $\lambda > \frac{1}{2}$. Re-arranging we have $\lambda\gamma_B^A + (1-\lambda)\gamma_B^B \geq \lambda\gamma_A^A + (1-\lambda)\gamma_A^B$ if $\lambda < \frac{1}{2}$ and $\lambda\gamma_B^A + (1-\lambda)\gamma_B^B \leq \lambda\gamma_A^A + (1-\lambda)\gamma_A^B$ for $\lambda > \frac{1}{2}$. In addition, we know that $c_A^* > c_B^*$ for $\lambda < \frac{1}{2}$, while $c_A^* < c_B^*$ when $\lambda > \frac{1}{2}$. Hence, from (A14) we can conclude that $\lambda F(c_A^*) < (1-\lambda)F(c_B^*)$ when $\lambda < \frac{1}{2}$ and $\lambda F(c_A^*) > (1-\lambda)F(c_B^*)$ when $\lambda > \frac{1}{2}$. As a consequence $\theta_A^*(\lambda) < \frac{1}{2}\ \forall\lambda < \frac{1}{2}$ and $\theta_A^*(\lambda) > \frac{1}{2}\ \forall\lambda > \frac{1}{2}$.■

Proof of Proposition 7

Notice that

$$\partial \frac{\lambda F(c_A^*)+(1-\lambda)F(c_B^*)}{\partial \lambda} = \lambda \frac{\partial F(c_A^*)}{\partial c_A^*}\frac{\partial c_A^*}{\partial \lambda} + F(c_A^*) + (1-\lambda)\frac{\partial F(c_B^*)}{\partial c_B^*}\frac{\partial c_B^*}{\partial \lambda} - F(c_B^*). \tag{A15}$$

We know from Proposition 4 that $c_A^*(\lambda) = c_B^*(\lambda) \forall \lambda$. Together with Lemma 1, this implies that $\frac{\partial c_A^*}{\partial \lambda} = \frac{\partial c_B^*}{\partial \lambda}$. Therefore, (A15) reduces to $\frac{\partial F(c_A^*)}{\partial c_A^*}\frac{\partial c_A^*}{\partial \lambda}$, implying that the sign of $\partial \frac{\lambda F(c_A^*)+(1-\lambda)F(c_B^*)}{\partial \lambda}$ depends entirely on $\frac{\partial c_A^*}{\partial \lambda}$. From Proposition 2 we calculate $\frac{\partial c_A^*}{\partial \lambda}$

$$\frac{\partial c_A^*}{\partial \lambda} = \frac{\gamma_A^A}{F^2(c_A^*)}\left[(1-2\lambda)G'(\lambda)F(c_A^*) + \lambda(1-\lambda)G''(\lambda)F(c_A^*) - \lambda(1-\lambda)G'(\lambda)\frac{\partial F(c_A^*)}{\partial c_A^*}\frac{\partial c_A^*}{\partial \lambda}\right].$$

Re-arranging the above equation we obtain

$$\frac{\partial c_A^*}{\partial \lambda} = \frac{\gamma_A^A F(c_A^*)}{F^2(c_A^*)+\lambda(1-\lambda)G'(\lambda)\frac{\partial F(c_A^*)}{\partial c_A^*}}[(1-2\lambda)G'(\lambda) + \lambda(1-\lambda)G''(\lambda)]. \tag{A16}$$

Notice that (A16) is positive for $\lambda < \frac{1}{2}$ and negative for $\lambda > \frac{1}{2}$.■

Proof of Proposition 8

Consider a sequence $G_m(z_P)$ such that all the properties of $G(\cdot)$ hold for any m and, moreover, ${G_m}''(z_P) > 0$ for $z_P < \frac{1}{2}$, $G_{m+1}(z_P) < G_m(z_P) \forall z_P \in \left(0, \frac{1}{2}\right)$ and

$$\lim_{m\to\infty} G_m(z_P) = \begin{cases} 0 & if\ z_P \in [0, 1/2) \\ 1/2 & if\ z_P = 1/2 \\ 1 & if\ z_P \in (1/2, 1] \end{cases}.$$

When $m \to \infty$, G_m converges to a unit step function, thus representing majority rule.

Let us first focus on $z_P \in [0, \frac{1}{2})$. From $\lim_{m\to\infty} G_m(z_P) = 0\ \forall z_P \in [0, \frac{1}{2})$ it follows that $\forall z_P < \frac{1}{2}$, given $\varepsilon > 0$, there exist $\mu > 0$ and M such that $\frac{G_m(z_P+\mu)-G_m(z_P)}{\mu} < \varepsilon\ \forall\ m > M$. Since ${G_m}''(z_P) >$

0 for $z_P < \frac{1}{2}$, we know that $\frac{G_m(z_P+\mu)-G_m(z_P)}{\mu} > {G_m}'(z_P)$. Hence, $\forall z_P \in [0, \frac{1}{2})$, given $\varepsilon > 0$, there exists M such that ${G_m}'(z_P) < \varepsilon \; \forall \; m > M$.

Moreover, since $G_m\left(\frac{1}{2}\right) = \frac{1}{2}$ and $\lim_{m\to\infty} G_m(z_P) = 0 \; \forall z_P \in [0, \frac{1}{2})$ it also follows that $\forall z_P < \frac{1}{2}$, given K arbitrarily large, there exist $\sigma > 0$ and $\dot{M}$ such that $\frac{G_m\left(\frac{1}{2}\right)-G_m\left(\frac{1}{2}-\sigma\right)}{\sigma} > K \; \forall \; m > \dot{M}$. As ${G_m}''(z_P) > 0$ for $z_P < \frac{1}{2}$, this implies that, given K arbitrarily large, there exists $\dot{M}$ such that ${G_m}'\left(\frac{1}{2}^-\right) > K \; \forall \; m > \dot{M}$.

Since G_m is symmetric around $z_P = \frac{1}{2} \; \forall \; m$, it follows that: $\forall z_P \in (\frac{1}{2}, 1]$, given $\varepsilon > 0$, there exists M such that ${G_m}'(z_P) < \varepsilon \; \forall \; m > M$; and, given K arbitrarily large, there exists $\dot{M}$ such that ${G_m}'\left(\frac{1}{2}^+\right) > K \; \forall \; m > \dot{M}$.

This proves that

$$\lim_{m\to\infty} {G_m}'(z_P) = \begin{cases} 0 & if \; z_P \in [0, 1/2) \\ \infty & if \; z_P = 1/2 \\ 0 & if \; z_P \in (1/2, 1] \end{cases}. \tag{A17}$$

From Proposition 6 we know that in equilibrium the share of the votes for the two parties will only be equal when $\lambda = \frac{1}{2}$. Hence, if $\lambda = \frac{1}{2}$, (A17) implies that $\lim_{m\to\infty} {G_m}'\left(\frac{F(c_A^*)\lambda}{F(c_A^*)\lambda+F(c_B^*)(1-\lambda)}\right) = \lim_{m\to\infty} {G_m}'\left(\frac{F(c_B^*)(1-\lambda)}{F(c_A^*)\lambda+F(c_B^*)(1-\lambda)}\right) = \infty$ and, from Proposition 2 follows that, under majority rule, $c_A^* = \overline{c}_A$ and $c_B^* = \overline{c}_B$. Whenever $\lambda \neq \frac{1}{2}$, the equilibrium share of the votes for the two parties will be different, and thus (A17) implies that $\lim_{m\to\infty} {G_m}'\left(\frac{F(c_A^*)\lambda}{F(c_A^*)\lambda+F(c_B^*)(1-\lambda)}\right) = \lim_{m\to\infty} {G_m}'\left(\frac{F(c_B^*)(1-\lambda)}{F(c_A^*)\lambda+F(c_B^*)(1-\lambda)}\right) = 0$. As a consequence, Proposition 2 implies that, under majority rule, limiting turnout is equal to zero. ■

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