Wohltmann, Hans-Werner; Winkler, Roland C.

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by Hans-Werner Wohltmann and Roland Winkler

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Anticipated and Unanticipated Oil Price Shocks and Optimal Monetary Policy[◇]

Hans-Werner Wohltmann[*] and Roland Winkler[‡]

Department of Economics, Christian-Albrechts-University of Kiel, Olshausenstr. 40, D-24098 Kiel, Germany

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Abstract

This paper studies the welfare effects of several monetary policy rules in the presence of anticipated and unanticipated oil price shocks. Our analysis is based on a stylized New Keynesian model of a small open economy. Our main findings are the following: i) Standard interest rate rules amplify the welfare loss compared to neutral monetary policies. ii) The optimal policy under commitment, by contrast, dampens the welfare loss. iii) Optimized simple rules can replicate the outcome under the optimal unrestricted rule if they are history-dependent, contain the exchange rate and, in the anticipated case, forward-looking elements. iv) Anticipated oil shocks lead to a higher welfare loss than unanticipated shocks.

JEL classification: E32, E52, F41

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* Corresponding author: Phone: ++49-431-880-1449, Fax: ++49-431-880-3150, E-mail: wohltmann@economics.uni-kiel.de

‡ E-mail: winkler@economics.uni-kiel.de

1 Introduction

The purpose of this paper is to study the interplay between oil price shocks and monetary policy. We evaluate the welfare effects of several monetary policy rules in the presence of unanticipated and anticipated oil price shocks. Bhandari and Turnovsky (1984) emphasize that most of the oil price increases in the 1970's and early 1980's were anticipated. In our opinion this notion holds more than ever for the sharp hike in oil prices during the last years because of the increasing worldwide demand for this exhaustible resource. Therefore, it seems necessary to analyze not only unpredictable but also anticipated oil price shocks.

In particular, we try to answer the following questions: i) Does the endogenous monetary policy response to an oil price hike amplify or dampen the destabilizing effects of this shock? ii) What is the optimal monetary policy response to unanticipated and anticipated oil price shocks? iii) What are the dynamic effects of unanticipated compared to anticipated oil shocks? iv) What is the performance of optimal simple rules relative to the unrestricted optimal rule under commitment? v) What are the properties of an optimal simple interest rate rule?

We seek to answer these questions by employing a calibrated stylized New Keynesian model of a small open economy which is dependent upon raw materials imports (like crude oil).

Our paper is related to different strands of the literature. The first strand deals with the macroeconomic effects of oil price shocks. Bhandari and Turnovsky (1984) analyze anticipated and unanticipated as well as permanent and temporary oil price increases in a traditional open economy framework. Kim and Loungani (1992), Rotemberg and Woodford (1996) and Finn (2000) analyze the effects of oil price shocks in dynamic general equilibrium models of closed economies. Backus and Crucini (2000) consider an open-economy real business cycle model to study the effects of oil on the economy. The latter studies are based on the assumption of completely flexible prices. Hence, there is no role for monetary policy.

A second strand of the literature studies the interaction between oil price shocks and monetary policy. Among the numerous empirical studies see, for example, Hamilton (1983), Hamilton and Herrera (2004), Bernanke et al. (1997, 2004) or Barsky and Kilian (2002). Theoretical contributions are for example, Leduc and Sill (2004), Carlstrom and Fuerst (2006), and Blanchard and Galí (2007). They all consider a model of a closed economy and thus rule out the potentially important impacts of changes in the nominal exchange rate and the terms of trade. Leduc and Sill (2004) and Carlstrom and Fuerst (2006) attempt to isolate the impacts of an oil price shock from the impacts of the endogenous response of monetary policy to this oil price hike. In doing so, they try to shed some light on the question whether monetary policy amplifies or dampens the destabilizing effects of oil price shocks. Carlstrom and Fuerst (2006) challenge the empirical work by Bernanke et al. (1997, 2004) by showing that anticipation effects actually matter for the analysis of the interplay between oil price shocks and monetary policy. We follow this line of thought and analyze the consequences of anticipated oil price shocks under several monetary policy responses.

A further strand of the literature related to our paper deals with optimal monetary policy in open economies. In particular, our paper contributes to the ongoing discussion about the structure of simple interest rate rules for open economies. Ball (1999), Svensson (2000), Taylor (2001), Leitemo and Söderström (2005), and Adolfson (2007) find no or only a limited role for the inclusion of an exchange rate term in non-optimized or optimized simple interest rate rules. In contrast, Wollmershäuser (2006) finds a substantial welfare improvement by adding an exchange rate term.

The performance of simple monetary policy rules in the presence of oil price shocks is studied by Kamps and Pierdzioch (2002) and De Fiori et al. (2006). The latter is the only paper we are aware of, that, as we do too, studies optimal simple monetary policy rules in the presence of oil price shocks. Whereas De Fiori et al. (2006) consider a three-country framework and study simple optimized Taylor-type rules without a direct exchange rate term, we analyze a broader range of possible interest rate policy rules in a model of a small open economy. Moreover, they only deal with unpredictable oil shocks. This paper, however, deals with unanticipated as well as anticipated oil price increases.

From a methodological point of view our paper applies a method proposed by Wohltmann and Winkler (2007), which generalizes the work of Söderlind (1999) by allowing the analysis of optimal monetary policy in the presence of anticipated shocks.

Our main results are the following: Standard calibrated interest rate rules amplify the welfare loss compared to neutral monetary policies, namely a money growth peg or an interest rate peg. The optimal unrestricted policy under precommitment, by contrast, dampens the welfare loss of the oil price shock compared to a neutral monetary policy. Optimized simple monetary policy rules are able to replicate the outcome under the optimal unrestricted rule. This holds for rules which contain both backward-looking elements and an exchange rate term. In case of anticipated shocks an optimal simple rule should, in addition, contains forward-looking elements. Thus, our findings are in favor of an open-economy Taylor rule. Furthermore, we show that anticipated oil shocks lead to higher welfare losses than unanticipated shocks irrespective of the monetary policy response.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 derives the optimal monetary policy under commitment and discusses the dynamic effects of anticipated as well as unanticipated oil price shocks under this policy regime. Section 4 discusses non-optimized and optimized simple monetary policy rules. Section 5 summarizes and concludes.

2 The Model

We consider a stylized New Keynesian model of a small open economy which is dependent upon imported intermediate inputs (like raw materials).[1] We assume forward-looking agents on the foreign exchange market and partially forward-looking behavior on the part of consumers and price setters. The building blocks of our rational expectations model are a hybrid IS equation, a hybrid Phillips curve, where the rate of change of the domestic price of imported intermediate goods enters the inflation equation, the uncovered interest parity condition and a money demand equation. All variables, except for interest rates, are in logs.

The goods market equilibrium, or open economy IS curve, takes the following form:

$$\begin{aligned} q_t = {} & \phi_{y_b} y_{t-1} + \phi_{y_f} \operatorname{E}_t y_{t+1} + \phi_i (i_t - \operatorname{E}_t \pi^c_{t+1}) \\ & + \phi_m (m_t - p^c_t) - \phi_y y_t + \phi_{y^*} y^*_t - \phi_\tau \tau_t. \end{aligned} \tag{1}$$

q denotes domestic final output while y is domestic real income or gross domestic product (GDP). The difference between q and y results from imports of intermediate inputs. i is the nominal interest rate which serves as the operating target of monetary policy. p is the domestic price of domestic output, while $p^c = \alpha p + (1-\alpha)(p^* + e)$, $0 < \alpha < 1$, denotes the consumer price index (CPI) and π^c_{t+1} the CPI inflation rate between period t and $t+1$. The variable e stands for the nominal exchange rate, while $\tau = p - (p^* + e)$ are the terms of trade which are the inverse of the real exchange rate. p^* and y^* denote the foreign price and foreign income respectively. E is the expectations operator where rational expectations are assumed.

Domestic final output q_t depends on past and expected future income, the real interest rate $i_t - \operatorname{E}_t \pi^c_{t+1}$ and net exports of final goods which are a function of domestic and foreign income and price competitiveness (terms of trade). The IS curve reflects the behavior of rational, intertemporally optimizing consumers as well as the assumption of habit formation in consumption.[2] Moreover, we assume that the demand for goods depends directly on real money balances $m_t - p^c_t$, where the nominal money stock is deflated by the consumer price index to allow for the fact that in open economies money is also used for the purchase of imported goods.[3]

Money market equilibrium is given by a standard LM curve:

$$m_t - p^c_t = l_q q_t - l_i i_t. \tag{2}$$

Money demand is assumed to depend on real output rather than on real income which is considered as a more appropriate measure of the volume of transactions.

[1] Similar models are used by, for example, van Aarle et al. (2004), Svensson (2000) or Moons et al. (2007).

[2] For a detailed derivation of a microfounded IS curve with habit formation in consumption see, for example, McCallum and Nelson (1999).

[3] The presence of the real money stock in the IS curve reflects the implicit assumption that the utility function of the representative household is non-separable.

The nominal exchange rate is modeled as a forward-looking and expectations determined asset price. It adjusts in such a manner that the uncovered interest parity (UIP) condition holds throughout:

$$\mathrm{E}_t\, e_{t+1} - e_t = i_t - i_t^*. \tag{3}$$

The UIP condition implies that the domestic interest rate may only deviate from the exogenously given foreign interest rate i^* by the rationally expected depreciation rate between period t and $t+1$.

The difference between domestic production of final goods and national income or gross domestic product is described by the following equation:

$$q_t = y_t + \psi(p_{in,t}^* + e_t - p_t). \tag{4}$$

p_{in}^* denotes the foreign price of imported *in*termediate goods (like raw materials). Such goods are used in the domestic production process. Imports of intermediate inputs depend on the respective real factor price. We assume that imported inputs are denominated in terms of foreign currency so that the domestic real factor price $p_{in}^* + e - p$ depends on the nominal exchange rate.[4]

The dynamics of inflation are given by a hybrid Phillips curve which contains both forward- and backward-looking price setting behavior:[5]

$$\pi_t = \gamma_{\pi_b}\pi_{t-1} + \gamma_{\pi_f}\,\mathrm{E}_t\,\pi_{t+1} + \gamma_q(q_t - \overline{q}) + \gamma_{dp_{in}}(\Delta p_{in,t}^* + \Delta e_t). \tag{5}$$

Domestic inflation between period $t-1$ and t depends on past and expected future inflation, the current value of the output gap $q_t - \overline{q}$ and the inflation of imported intermediate inputs $\Delta(p_{in,t}^* + e_t)$. It is assumed that the pass-trough of exchange rate changes on domestic inflation is instantaneous. Since the domestic economy is assumed to be small relative to the rest of the world, the foreign input price p_{in}^* is exogenously given.

In the following we will discuss the dynamic effects of *anticipated* input price shocks following a stationary $AR(1)$ process $p_{in,t}^* = \beta^* p_{in,t-1}^* + \kappa_t$, where $0 \leq \beta^* < 1$ and κ_t is a one-unit price shock. We assume that at time $t = 0$ the public and the policy maker anticipate a one-unit shock in the foreign price of imported intermediate goods to take effect at some future date $T > 0$. This implies $\kappa_t = 1$ for $t = T$ and $\kappa_t = 0$ for $t \neq T$. If the initial value of p_{in}^* is normalized to zero ($p_{in,0}^* = 0$), then $p_{in,t}^* = 0$ for $0 \leq t < T$ and $p_{in,t}^* = \beta^{*\,t-T}$ for $t \geq T$. For example, we can imagine that in $t = 0$ the OPEC credibly announces a temporary price increase in crude oil to occur at the future date $T > 0$. Although commodity shocks are generally of permanent nature, we will only discuss temporary input price increases ($\beta^* < 1$). In case of permanent price shocks it is impossible to distinguish how much of the persistence of endogenous variables results from the persistence of the shock

[4]The constant ψ can be derived from a profit maximizing approach with a CES production technology which allows for factor substitution between labor and raw materials imports (Bhandari and Turnovsky, 1984).

[5]This assumption is in line with empirical evidence provided by, for example, Galí and Gertler (1999) or Galí et al. (2001, 2005).

itself and how much is intrinsic to the model. In case of temporary price shocks no steady state effects occur so that the domestic variables return to their initial steady state values which are normalized to zero.

We will further assume that a one-unit increase in the foreign nominal price of the imported input is accompanied by a less than proportional increase in the price of the imported final good p^*, i.e., $p^* = (1 - \mu^*)p^*_{in}$ with $0 < \mu^* \leq 1$. Then the nominal price shock also leads to a change in the real foreign price of imported intermediate inputs, $p^*_{in} - p^* = \mu^* p^*_{in}$.

In case of anticipated price shocks the one-unit price shock κ_t is not white noise, but known to the public before the shock actually occurs. The adjustment dynamics therefore involve two phases: the time span between the anticipation and implementation of the input price increase (anticipation phase) and the time span after the realization of the shock. A further implication of anticipated shocks is that rational expectations are equivalent to perfect foresight. We can therefore omit the expectations operator E in (1), (3) and (5).

The dynamics of the New Keynesian model (1) - (5) can be represented in state space form:

$$Bk_{t+1} = Ck_t + \chi i_t + \varepsilon \kappa_{t+1}, \tag{6}$$

where the state vector

$$k_t = \begin{pmatrix} w_t \\ v_t \end{pmatrix} \tag{7}$$

can be partitioned into a vector of predetermined variables

$$w_t = (p^*_{in,t}, y_{t-1}, \pi_{t-1}, \tau_{t-1}, p^*_{in,t-1})' \tag{8}$$

and a vector of forward-looking variables

$$v_t = (y_t, \pi_t, \tau_t)'. \tag{9}$$

The nominal interest rate is the policy instrument. The 8×8 matrices B and C are defined in the mathematical appendix; B is non-singular while $\det C = 0$. χ and ε are 8×1 unit vectors.

3 Optimal Monetary Policy under Commitment

The optimal monetary policy response to anticipated input price shocks follows from minimizing an intertemporal loss function given the state space representation of the model. We assume the following quadratic loss criterion reflecting the objective of flexible domestic inflation targeting (Svensson, 2000):

$$J_t = E_t \sum_{i=0}^{\infty} \lambda^i \{d_1 \pi^2_{t+i} + d_2 (y_{t+i} - \overline{y})^2 + d_3 \tau^2_{t+i} + d_4 (i_{t+i} - i^*)^2\}, \tag{10}$$

where $0 < \lambda \leq 1$ is the central bank's discount factor and d_1, d_2, d_3 and d_4 are nonnegative parameters measuring the weights on stabilizing inflation, income, terms of trade and interest rate movements. Normalizing the steady state value $\overline{y}$ and the foreign interest rate i^* to zero the objective function of the policy maker can be rewritten as

$$\min_{i_t} E_t \sum_{i=0}^{\infty} \lambda^i \{k'_{t+i} D k_{t+i} + i'_{t+i} R i_{t+i}\} \quad \text{s.t. (6)}, \tag{11}$$

where $D = \mathrm{diag}(0,0,0,0,0,d_2,d_1,d_3)$ and $R = (d_4)$. The central bank optimizes the loss function under precommitment at the beginning of the planning period. It adopts this optimal monetary policy and sticks to it during the entire planning period. We assume that the beginning of the central bank's planning period coincides with the date of anticipation of the input price shock, $t = 0$. Since the total loss over the anticipation interval is non-negligible, it is reasonable to start the optimization at the date of anticipation ($t = 0$) rather than the date of implementation ($t = T$). The loss function of the central bank is then given by

$$J_0 = \sum_{i=0}^{\infty} \lambda^i \{k'_i D k_i + i'_i R i_i\}. \tag{12}$$

In the following we will solve the optimization problem (11) using the methods outlined in Levine (1988), Söderlind (1999), Klein (2000), and Wohltmann and Winkler (2007).

From the Lagrangian

$$L_0 = \sum_{t=0}^{\infty} \lambda^t [k'_t D k_t + i'_t R i_t + 2\varrho'_{t+1}(C k_t + \chi i_t + \varepsilon \kappa_{t+1} - B k_{t+1})] \tag{13}$$

with the Lagrange multiplier ϱ_{t+1} we get the first-order conditions with respect to the new costate vector $p_{t+1} = \lambda^{-1}\varrho_{t+1}$, the state vector k_t and the control variable i_t:

$$\begin{pmatrix} B & 0 & 0 \\ 0 & 0 & \lambda C' \\ 0 & 0 & -\lambda\chi' \end{pmatrix} \begin{pmatrix} k_{t+1} \\ i_{t+1} \\ p_{t+1} \end{pmatrix} = \begin{pmatrix} C & \chi & 0 \\ -D & 0 & B' \\ 0 & R & 0 \end{pmatrix} \begin{pmatrix} k_t \\ i_t \\ p_t \end{pmatrix} + \begin{pmatrix} \varepsilon \\ 0 \\ 0 \end{pmatrix} \kappa_{t+1}. \tag{14}$$

(14) consists of 17 equations in the unknowns k, i and p. To solve (14), expand the state and costate vector k and p as (w', v') and (p'_w, p'_v) respectively and reorder the columns by placing the predetermined costate vector p_v after the predetermined state vector w. The reordered first-order conditions can be written as

$$F \begin{pmatrix} \widetilde{w}_{t+1} \\ \widetilde{v}_{t+1} \end{pmatrix} = G \begin{pmatrix} \widetilde{w}_t \\ \widetilde{v}_t \end{pmatrix} + \begin{pmatrix} \varepsilon \\ 0 \\ 0 \end{pmatrix} \kappa_{t+1}, \tag{15}$$

where F and G are 17×17 matrices resulting from the reordering of (14), and $\widetilde{w}$ and $\widetilde{v}$ are the vectors

$$\widetilde{w} = \begin{pmatrix} w \\ p_v \end{pmatrix}, \quad \widetilde{v} = \begin{pmatrix} v \\ i \\ p_w \end{pmatrix}. \tag{16}$$

$\widetilde{w}$ is 8×1 and contains the 'backward-looking' or predetermined variables (with the initial value $\widetilde{w}_0' = (w_0', p_{v,0}') = (w_0', 0'))$ while the 9×1 vector $\widetilde{v}$ consists of 'forward-looking' variables.

The matrices F and G are singular. To solve (15) we apply the complex generalized Schur decomposition (Klein, 2000). The decomposition gives square matrices of complex numbers Q, Z, S and T, where Q and Z are unitary ($Q\overline{Q}' = \overline{Q}'Q = Z\overline{Z}' = \overline{Z}'Z = I$), S and T are upper triangular such that

$$F = \overline{Q}' S \overline{Z}', \quad G = \overline{Q}' T \overline{Z}'. \tag{17}$$

$\overline{Z}'$ is non-singular and denotes the transpose of $\overline{Z}$ which is the complex conjugate of Z. $\overline{Q}'$ is the transpose of the complex conjugate of Q. Premultiply both sides of (15) with Q and define the auxiliary variables

$$\begin{pmatrix} \widetilde{z} \\ \widetilde{x} \end{pmatrix} = \overline{Z}' \begin{pmatrix} \widetilde{w} \\ \widetilde{v} \end{pmatrix}, \tag{18}$$

where $\widetilde{z}$ is an 8×1 vector and $\widetilde{x}$ is 9×1. Partition the triangular matrices S and T conformably with $\widetilde{z}$ and $\widetilde{x}$ and define

$$Q \begin{pmatrix} \varepsilon \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}. \tag{19}$$

We then obtain the equivalent system

$$\begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} \widetilde{z}_{t+1} \\ \widetilde{x}_{t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} \widetilde{z}_t \\ \widetilde{x}_t \end{pmatrix} + \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \kappa_{t+1}, \tag{20}$$

where the square matrices S_{11} and T_{22} are invertible while S_{22} is singular. The lower block of the triangular system (20) contains the unstable generalized eigenvalues and must be solved forward. Since

$$\widetilde{x}_{t+s} = T_{22}^{-1} S_{22} \widetilde{x}_{t+s+1} - T_{22}^{-1} q_2 \kappa_{t+s+1} \quad (s = 0, 1, 2, \ldots), \tag{21}$$

where the eigenvalues of $T_{22}^{-1} S_{22}$ are stable, we get the unique stable solution

$$\widetilde{x}_t = -\sum_{s=0}^{\infty} (T_{22}^{-1} S_{22})^s T_{22}^{-1} q_2 \kappa_{t+s+1} = \begin{cases} -(T_{22}^{-1} S_{22})^{T-1-t} T_{22}^{-1} q_2 & \text{for} \quad 0 \le t < T \\ 0 & \text{for} \quad t \ge T. \end{cases} \tag{22}$$

The last equation follows from the definition of the unit price shock κ_t. During the anticipation phase the auxiliary vector $\widetilde{x}_t$ is a function of the anticipated future input price shock while for $t \ge T$ it is equal to zero.

The upper block of the decomposed system (20) contains the stable generalized eigenvalues and can be solved backward. Since

$$\widetilde{z}_{t+1} = S_{11}^{-1} T_{11} \widetilde{z}_t + S_{11}^{-1} (T_{12} \widetilde{x}_t - S_{12} \widetilde{x}_{t+1}) + S_{11}^{-1} q_1 \kappa_{t+1}, \tag{23}$$

the general solution is given by

$$\begin{aligned}
\widetilde{z}_t &= (S_{11}^{-1}T_{11})^t K + \sum_{s=0}^{t-1}(S_{11}^{-1}T_{11})^{t-s-1}S_{11}^{-1}(T_{12}\widetilde{x}_s - S_{12}\widetilde{x}_{s+1} + q_1\kappa_{s+1}) \\
&= \begin{cases} (S_{11}^{-1}T_{11})^t K + \sum_{s=0}^{t-1}(S_{11}^{-1}T_{11})^{t-s-1}S_{11}^{-1}(T_{12}\widetilde{x}_s - S_{12}\widetilde{x}_{s+1}) & \text{for} \quad 0 \le t < T \\ (S_{11}^{-1}T_{11})^t K + \sum_{s=0}^{T-1}(S_{11}^{-1}T_{11})^{t-s-1}S_{11}^{-1}(T_{12}\widetilde{x}_s - S_{12}\widetilde{x}_{s+1}) & \\ + (S_{11}^{-1}T_{11})^{t-T}S_{11}^{-1}q_1 & \text{for} \quad t \ge T, \end{cases}
\end{aligned} \tag{24}$$

where K is an arbitrary vector of constants and $\widetilde{x}$ given by (22). The constant K can be uniquely determined using the initial condition of the predetermined vector $\widetilde{x}$. Premultiply both sides of equation (18) with Z and partition the matrix Z conformably with the auxiliary variables $\widetilde{z}$ and $\widetilde{x}$. We then obtain

$$\begin{pmatrix} \widetilde{w} \\ \widetilde{v} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \widetilde{z} \\ \widetilde{x} \end{pmatrix}. \tag{25}$$

Since $\widetilde{z}_0 = K$, $\widetilde{w}_0 = 0$ (in case $T > 0$), and

$$\widetilde{x}_0 = -(T_{22}^{-1}S_{22})^{T-1}T_{22}^{-1}q_2, \tag{26}$$

we get from (25)

$$K = Z_{11}^{-1}\widetilde{w}_0 - Z_{11}^{-1}Z_{12}\widetilde{x}_0 = Z_{11}^{-1}Z_{12}(T_{22}^{-1}S_{22})^{T-1}T_{22}^{-1}q_2 \quad \text{if} \quad T > 0. \tag{27}$$

In the special case $T = 0$ (unanticipated oil price shocks) we have $\widetilde{x}_t = 0$ for all t and

$$\widetilde{z}_t = (S_{11}^{-1}T_{11})^t K + (S_{11}^{-1}T_{11})^t S_{11}^{-1}q_1 \quad \text{for} \quad t \ge 0 \tag{28}$$

implying $\widetilde{z}_0 = K + S_{11}^{-1}q_1$. If $T = 0$, the initial value of $\widetilde{w}$ is given by

$$\widetilde{w}_0' = (w_0', p_{v,0}') = (1,0,0,0,0,0,0,0)'. \tag{29}$$

From (25) we obtain $\widetilde{w}_0 = Z_{11}\widetilde{z}_0$ so that

$$K = Z_{11}^{-1}\widetilde{w}_0 - S_{11}^{-1}q_1 \quad \text{if} \quad T = 0. \tag{30}$$

In order to get a unique solution for $\widetilde{z}$ we must assume that the square matrix Z_{11} is invertible (Klein, 2000). The unique stable time path of the original state vector $(\widetilde{w}', \widetilde{v}')$ then follows from equation (25). Assuming the invertibility of Z_{11} equation (25) also implies the following relationship between the state vectors $\widetilde{v}$ an $\widetilde{w}$:

$$\widetilde{v} = Z_{21}Z_{11}^{-1}\widetilde{w} + (Z_{22} - Z_{21}Z_{11}^{-1}Z_{12})\widetilde{x}. \tag{31}$$

Let $N = Z_{21}Z_{11}^{-1}$, $\hat{Z} = (Z_{22} - Z_{21}Z_{11}^{-1}Z_{12})$ and partition the matrices $N = (n_{ij})_{1\le i\le 9, 1\le j\le 8}$ and $\hat{Z} = (\hat{z}_{ij})_{1\le i,j\le 9}$ conformably with the components of $\widetilde{v}' = (v', i, p_w')$, i.e.

$$N = \begin{pmatrix} N_{11} & & N_{12} \\ n_{41} & \cdots & n_{48} \\ N_{21} & & N_{22} \end{pmatrix}, \qquad \hat{Z} = \begin{pmatrix} & \hat{Z}_1 & \\ \hat{z}_{41} & \cdots & \hat{z}_{49} \\ & \hat{Z}_2 & \end{pmatrix}, \tag{32}$$

where the matrices N_{11}, N_{12}, N_{21}, N_{22}, $\hat{Z}_1$ and $\hat{Z}_2$ are 3×5, 3×3, 5×5, 5×3, 3×9 and 5×9 respectively.[6] Equation (31) can be written as

$$v = N_{11}w + N_{12}p_v + \hat{Z}_1\widetilde{x}, \tag{33}$$
$$i = (n_{41}, \ldots, n_{45})w + (n_{46}, \ldots, n_{48})p_v + (\hat{z}_{41}, \ldots, \hat{z}_{49})\widetilde{x}, \tag{34}$$
$$p_w = N_{21}w + N_{22}p_v + \hat{Z}_2\widetilde{x}. \tag{35}$$

Assume the invertibility of the square matrix N_{12} and solve equation (33) for the predetermined costate vector p_v. Inserting this expression into (34) yields the following expression for the optimal unrestricted control rule under commitment

$$\begin{aligned} i_t = {} & (n_{46}, \ldots, n_{48})N_{12}^{-1}v_t + \left[(n_{41}, \ldots, n_{45}) - (n_{46}, \ldots, n_{48})N_{12}^{-1}N_{11}\right] w_t \\ & + \left[(\hat{z}_{41}, \ldots, \hat{z}_{49}) - (n_{46}, \ldots, n_{48})N_{12}^{-1}\hat{Z}_1\right] \widetilde{x}_t \qquad (t = 0, 1, 2, \ldots), \end{aligned} \tag{36}$$

where the auxiliary vector $\widetilde{x}_t$ is defined in (22). The optimal policy rule can be expressed as a linear feedback on the current state vector $k = (w', v')'$ and the auxiliary vector $\widetilde{x}$. In the special case of unanticipated input price shocks ($T = 0$) stability requires $\widetilde{x}_t = 0$ for all t. In this case i_t only depends on k_t. In particular i_t is a linear function of the current and *lagged* values of the forward-looking variables (i.e., π_t, π_{t-1}, y_t, y_{t-1}, τ_t, τ_{t-1}). It is well known that the optimal unrestricted rule may also be expressed as a feedback on the current value of the vector w of predetermined state variables plus a discounted linear combination of past values of w (Levine, 1988).

Figure 1 illustrates the impulse response functions that represent the dynamic adjustments resulting from the input price shock in the case of the optimal monetary policy rule under commitment. Underlying the simulations is a set of baseline model parameters (see Table 1) that appear plausible from empirical studies.[7]

Table 1: Baseline parameters

ψ	α	β^*	ϕ_{y_b}	ϕ_{y_f}	ϕ_i	ϕ_τ	ϕ_m	ϕ_y	ϕ_{y^*}	γ_{π_b}	γ_{π_f}	γ_q	$\gamma_{dp_{in}}$	l_q	l_i	μ^*
0.175	0.75	0.8	0.4	0.6	0.3	0.25	0.03	0.25	0.25	0.32	0.56	0.1	0.2	1	1	0.95

It is assumed that the parameters showing the forward-looking behavior of consumers and price setters are greater than the corresponding 'backward-looking' parameters. We consider both an unanticipated and anticipated input price shock which takes place at time $T = 2$. In the unanticipated case we get short-term stagflationary results and thereafter hump-shaped adjustments of domestic income and inflation. In the case of an anticipated commodity price increase domestic income rises on impact. The temporary increase in domestic GDP can be traced back to a simultaneous decrease in the terms of trade τ

[6] If the unitary matrix $\overline{Z}'$ is partitioned via $\overline{Z}' = \begin{pmatrix} \overline{Z}'_{11} & \overline{Z}'_{12} \\ \overline{Z}'_{21} & \overline{Z}'_{22} \end{pmatrix}$ then the identity $Z \cdot \overline{Z}' = I_{17\times 17}$ implies $\hat{Z} \cdot \overline{Z}'_{22} = I_{9\times 9}$ so that $\hat{Z} = \overline{Z}'^{-1}_{22}$.

[7] See, for example, Moons et al. (2007) and the references therein.

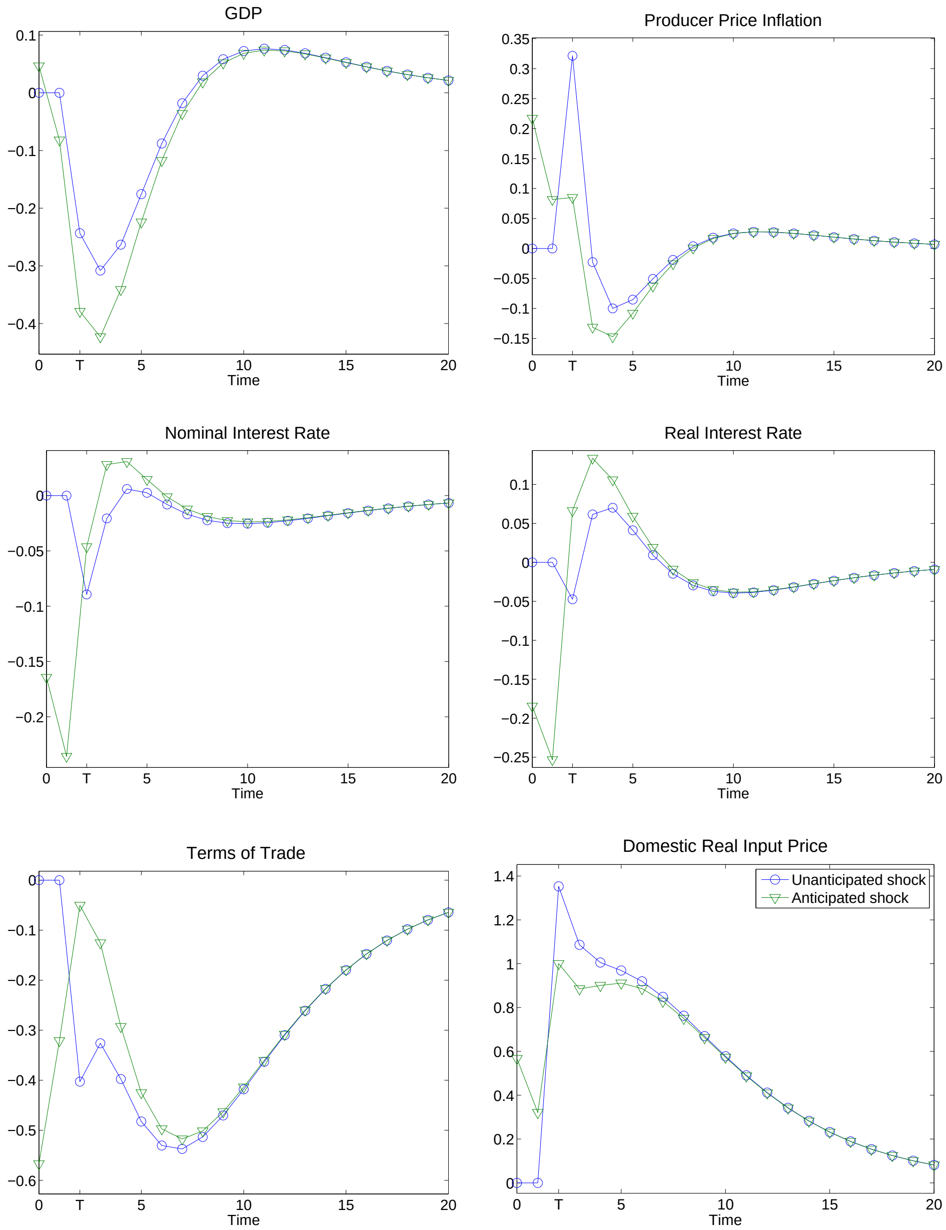

Figure 1: Economy's response to an increase in the price of imported inputs. Solid lines with *triangles* are responses to an *anticipated* shock taking place in period $T = 2$; solid lines with *circles* are responses to an *unanticipated* shock taking place in period $T = 2$. Note that in this case the variables remain at their steady state values (normalized to zero) until $T = 2$.

and the real interest rate and a slight increase in the domestic real commodity price $(p^*_{in} + e - p)$.[8] At the time of implementation of the input price increase

[8]The identity equation $p^*_{in} + e - p = (p^*_{in} - p^*) - \tau$ implies that during the anticipation phase the development of $p^*_{in} + e - p$ is equivalent to the adjustment of the domestic real exchange rate $-\tau$.

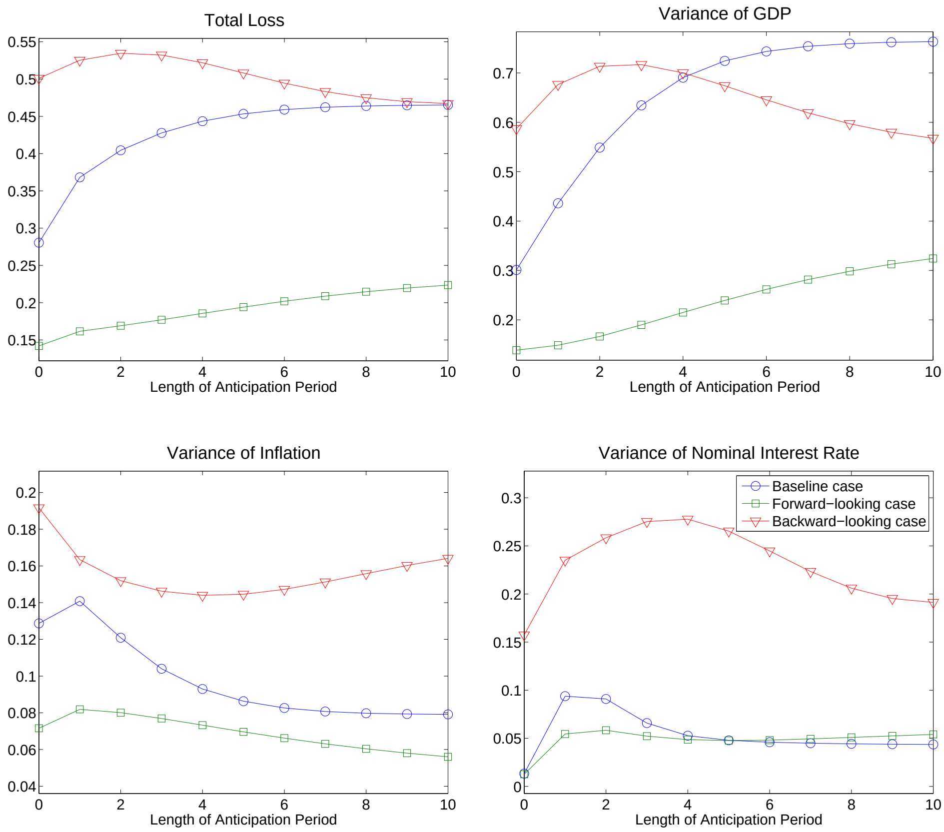

Figure 2: Total loss and variances for different lengths of anticipation period. Solid lines with *circles* show the baseline case; solid lines with *squares* show the more forward-looking case $\phi_{y_b} = 0.05$, $\phi_{y_f} = 0.95$, $\gamma_{\pi_b} = 0.04$, $\gamma_{\pi_f} = 0.76$; solid lines with *triangles* show the more backward-looking case $\phi_{y_b} = 0.95$, $\phi_{y_f} = 0.05$, $\gamma_{\pi_b} = 0.76$, $\gamma_{\pi_f} = 0.04$.

the fall in domestic income is stronger than in the unanticipated case which can be explained by the rise in the real interest rate and the terms of trade. On the other hand, the producer price inflation is weaker than in the case of an unanticipated commodity price shock. In any case the domestic nominal interest rate falls on impact although the domestic real commodity price lies above its initial steady state level during the whole course of adjustment.

Comparing the total loss J_0 in the unanticipated and anticipated case (Figure 2) we get the surprising result that – given the baseline calibration of our New Keynesian model – J_0 is an *increasing* function in the length of the anticipation phase (T).

The total loss in case of anticipated input price shocks is always greater than the total loss resulting from an unanticipated commodity price increase.[9] In Figure 2 the development of the intertemporal quadratic loss function J_0 is based on the following parameter set representing the objective of flexible

[9]The determination of the value of the loss function is presented in the mathematical appendix.

domestic inflation targeting:

$$d_1 = 1 > d_2 = 0.5 > d_4 = 0.1 > d_3 = 0.$$

In a setting of perfect capital mobility where the uncovered interest parity condition (3) holds, interest rate stability and stability of the real exchange rate are intrinsically linked. We therefore assume $d_3 = 0$. Flexible inflation targeting implies that inflation is the most important objective of the central bank but that it is also concerned about output and interest rate stability (Svensson, 2000).

To explain the *loss puzzle* consider the volatility or total variance of the target variables y, π and i (Figure 2). Obviously, the variance of GDP (VAR(y)) dominates the variance of the inflation rate and the nominal interest rate. Moreover, VAR(y) is an increasing function in the length of the anticipation phase T. This can be traced back to the strong GDP contraction at the date of implementation (T) which is *not* independent of T.[10] Instead, the income contraction *increases* with rising T, since the opposing effects of the contractionary input price shock, i.e. the fall in the domestic terms of trade and the real interest rate, get weaker the longer the time span between the anticipation and realization of the commodity price increase.

Similar results hold if the degree of forward-looking behavior is increased. It is well-known that the volatility of output and inflation decreases if the part of forward-looking consumers and price setters increases. Therefore, the total loss in the more forward-looking case runs below the J_0 curve in the baseline scenario, but it also increases in T (Figure 2). In the more backward-looking case, J_0 lies above the loss function in the baseline calibration. Its development is now hump-shaped. However, as long as T is not too large (i.e. $T \leq 5$), J_0 again increases if T becomes larger.

4 Simple Monetary Policy Rules

In this section, we evaluate the performance of several monetary policy rules in the presence of both unanticipated and anticipated oil price shocks. In particular, we study the performance of simple rules relative to the optimal unrestricted policy under commitment. The optimal unrestricted control rule (36) can not be implemented as an instrument rule for two reasons. First, it leads to an indeterminacy problem with respect to the original system (6) since the number of unstable eigenvalues would be smaller than the number of forward-looking state variables (Blanchard and Kahn, 1980). Second, the rule is rather complicated because it depends on all forward- and backward-looking state variables including the exogenous shock variable. We therefore analyze calibrated and optimized simple monetary policy rules which guarantee saddle path stability of the original system (6). In doing so, we search for simple rules that are good approximations of the fully optimal policy.

[10] The opposite result, i.e., the *in*dependence of the output jump from T can be found in the literature to traditional Dornbusch-type models (Turnovsky, 1986; 2000).

Table 2: Coefficients of monetary policy rules in case of unanticipated ($T = 0$) and anticipated ($T = 2$) commodity price shocks

Rule	T	π_{t+1}	π_t	π_{t-1}	y_{t+1}	y_t	y_{t-1}	τ_{t+1}	τ_t	τ_{t-1}	$p^*_{in,t}$	$p^*_{in,t-1}$
Com	0/2	-	-0.82	0.80	-	-19.67	8.28	-	-5.80	0.50	-6.95	-0.48
Non-optimized rules												
Mpeg	0/2	-	2.00	-	-	1.00	-1.00	-	-1.43	-1.43	0.12	-0.12
Ipeg	0/2	-	1.01	-	-	-	-	-	-0.99	0.99	-0.05	0.05
TR	0/2	-	1.50	-	-	0.50	-	-	-	-	-	-
TRS	0/2	-	1.10	-	-	0.10	-	-	-0.80	0.80	-0.04	0.04
Optimized rules												
R1	0	-	1.91	-0.25	-	0.55	0.02	-	-0.88	1.23	-0.93	1.23
	2	-	1.69	-0.31	-	1.74	-1.29	-	1.07	-0.41	0.30	0.44
R2	0	-	-0.59	2.11	-	2.93	-1.89	-	-2.02	2.09	-	-
	2	-	-0.17	0.99	-	1.18	-1.27	-	0.10	0.01	-	-
R3	0	-	2.62	1.36	-	2.77	-3.00	-	-	-	-	-
	2	-	0.53	2.64	-	2.66	-3.00	-	-	-	-	-
R4	0	-	0.87	-	-	-0.37	-	-	0.04	-	-	-
	2	-	3.00	-	-	-0.62	-	-	-0.28	-	-	-
TRopt	0	-	1.22	-	-	-0.38	-	-	-	-	-	-
	2	-	3.00	-	-	-0.56	-	-	-	-	-	-
TRSopt	0	-	1.28	-	-	0.01	-	-	-0.86	0.86	-0.04	0.04
	2	-	1.02	-	-	0.03	-	-	-0.99	0.99	-0.05	0.05
SL	0	-	3.00	-	-	2.47	-2.47	-	-	-	-	-
	2	-	3.00	-	-	2.57	-2.57	-	-	-	-	-
R5	0	-	-	-	-	-	-	-	-	-	-	-
	2	0.05	0.12	0.53	1.73	-1.77	-0.09	0.68	-0.45	-0.05	-	-

Notes: The rows display interest rate rules which depend linearly on the variables in the columns. The first row presents the coefficients of the unrestricted optimal policy rule (36) with respect to the whole vector of state variables k_t (cf. equation (7)). The coefficients in this case and in case of the non-optimized rules are based on our calibration and are therefore identical for $T = 0$ and $T = 2$. The rows in the lower part of the table display the set of optimized rules for both the case of unanticipated shocks $T = 0$ and anticipated shocks $T = 2$.

Non-optimized simple rules

We analyze four types of simple non-optimized monetary policy rules discussed in the literature, namely a money growth peg (Mpeg), an interest rate peg (Ipeg), a standard Taylor rule (TR), and a Taylor rule with smoothing (TRS).

Under a money growth peg, the growth rate of the money supply is kept constant by the central bank ($\Delta m_t = 0$). Combining the condition $\Delta m_t = 0$, the money demand equation (4) (which is assumed to be stable), the interest parity condition (3) and the definition of the consumer price index we can represent a money growth peg in terms of an interest rate rule. Mpeg is then a linear function of π_t, y_t, y_{t-1}, τ_t, τ_{t-1}, $p^*_{in,t}$, and $p^*_{in,t-1}$, where the corresponding coefficients are presented in the row of Table 2 belonging to the rule Mpeg.

Under an interest rate peg, the central bank targets the nominal interest rate. To achieve a constant interest rate without an indeterminacy problem we specify an interest rate peg as follows:

$$i_t = \rho i_{t-1} + (1-\rho) v_1 \pi_t \tag{37}$$

with $\rho = 0.999$ and $v_1 = 1.001$ implying $\Delta i_t \approx 0$ (Collard and Dellas, 2005). By using the interest parity condition (3) we get the rule presented in Table 2. We follow Leduc and Sill (2004) and Carlstrom and Fuerst (2006) and define both a money growth peg and an interest peg as neutral monetary policies.

Furthermore, we consider two types of Taylor-type interest rate rules. TR is a rule originally proposed by Taylor (1993) with the standard coefficients 1.5 for inflation and 0.5 for GDP. TRS is an instrument rule with smoothing of the form

$$i_t = \rho i_{t-1} + (1-\rho)(v_1 \pi_t + v_2 y_t) \tag{38}$$

with $\rho = 0.8$, $v_1 = 1.5$ and $v_2 = 0.5$ (Clarida et al., 1998; 2000). By using the interest parity condition (3) we get the rule presented in Table 2.

Table 3 shows the absolute and relative loss resulting from unanticipated ($T = 0$) and anticipated shocks ($T = 2$) under the monetary policy rules discussed above. The relative loss expresses the loss in percent of the loss under the optimal unrestricted monetary policy under commitment (Com). Two results should be emphasized. First, the standard Taylor rule (TR) as well as the Taylor rule with smoothing (TRS) perform worse than the neutral monetary policies (Mpeg, Ipeg). This result holds for unanticipated as well as for anticipated oil price hikes. Second, a Taylor rule with smoothing performs considerably better than a Taylor rule without smoothing.

The rationale behind these results is the following: The neutral monetary policies as well as the Taylor rule with smoothing exhibit one of the main characteristics of the optimal unrestricted monetary policy under commitment, namely its history-dependence (see equation (36) as well as Levine, 1988; Woodford, 1999). Both a money growth peg, an interest rate peg and the Taylor rule with smoothing respond to lagged variables, in particular y_{t-1} and τ_{t-1} respectively (see Table 2). This conclusion offers an alternative perception of the discussion about active versus passive monetary policy rules in the presence of oil price shocks and questions the appropriateness of characterizing money or interest rate pegs as neutral monetary policies.[11]

Optimized simple rules

We now consider a set of possible simple interest rate rules and minimize the objective function (10) of the central bank with respect to the coefficients of the given structure of the rule. The question arises whether a *restricted* optimal monetary policy can lead to the same or nearly the same welfare loss as the optimal *unrestricted* policy (where in both cases commitment is assumed).

The rules we consider are presented in Table 2.[12] The optimized rule R1, as well as the optimal unrestricted rule, depends on the whole vector of state variables k_t. By subtracting from this rule the actual and lagged price of oil,

[11] See, for example, Leduc and Sill (2004) or Carlstrom and Fuerst (2006).

[12] We have done numerical optimizations for a much larger set of possible interest rate rules. Qualitatively, the results are similar to those reported and are available upon request.

Table 3: Performance of monetary policy rules in case of unanticipated and anticipated commodity price shocks

	Unanticipated shock		*Anticipated shock*	
Rule	Absolute Loss	Relative Loss	Absolute Loss	Relative Loss
Com	0.2805	100.00	0.4044	100.00
Non-optimized rules				
Mpeg	0.3421	121.94	0.4889	120.89
Ipeg	0.3792	135.17	0.5087	125.79
TR	2.5935	924.48	3.5352	874.15
TRS	0.5210	185.72	0.6946	171.75
Optimized rules				
R1	0.2805	100.00	0.4050	100.14
R2	0.2811	100.20	0.4204	103.95
R3	0.3041	108.41	0.4547	112.43
R4	0.3914	139.52	0.7549	186.66
TRopt	0.4303	153.38	0.8197	202.69
TRSopt	0.3446	122.85	0.4956	122.55
SL	0.3337	118.93	0.5829	144.13
R5	-	-	0.4052	100.19

Note: The relative loss is the percentage of the loss from the simple rule relative to the loss from the optimal unrestricted monetary policy (Com).

$p^*_{in,t}$ and $p^*_{in,t-1}$ respectively, we get the rule R2. R3 is a history-dependent closed economy interest rate rule implying that it responds only to the actual and lagged values of domestic inflation and GDP. The interest rate rule R4, however, reacts to the actual but not the lagged values of inflation, GDP and the terms of trade τ. TRopt and TRSopt are optimized standard Taylor rules with and without smoothing respectively. SL is a so called speed limit policy which reacts to actual inflation and the growth rate of GDP $\Delta y_t = y_t - y_{t-1}$ (Walsh, 2003; Stracca, 2007). R5 is an open economy rule which is both forward- and backward-looking.

Table 2 displays the optimized parameter values of the rules both for the case of an unanticipated rise in the price of oil ($T = 0$) and for the case of an anticipated oil price hike ($T = 2$).[13] The corresponding values of the absolute and relative loss are shown in Table 3.

Three results are worth mentioning. First, there exists optimized simple rules which are able to replicate the outcome under the optimal unrestricted rule (in particular R1 and R2 in case $T = 0$ and R1 and R5 in case $T = 2$). These rules must exhibit a similar structure as the unrestricted policy rule. For *un*anticipated shocks the rule R1 has exactly the same structure as the optimal unrestricted rule (36) and attains virtually the same level of welfare. In case of unanticipated oil price increases the simplified rule R2 (which does not react directly to the shock itself) performs only 0.2% worse than the best simple or the optimal unrestricted rule. Therefore, this more simple rule is superior to the rule R1. In the case of *an*ticipated shocks this result does not

[13]We restrict the coefficients to lie between -3 and 3 (Schmitt-Grohé and Uribe, 2007).

hold. As emphasized above, the unrestricted optimal control rule (36) in case of anticipated shocks is a linear feedback rule on the current state vector k_t *and* the auxiliary vector $\widetilde{x}_t$. Since the latter can be approximated by the forward-looking elements $\mathrm{E}_t\, \pi_{t+1}$, $\mathrm{E}_t\, y_{t+1}$ and $\mathrm{E}_t\, \tau_{t+1}$, the forward-looking rule R5 leads to the second-best results and is superior to R1 for the same reason as explained above. We conclude that anticipated shocks provide a theoretical justification for forward-looking simple monetary policy rules.

The next two results follow directly from the rationale about the structure of optimal simple rules discussed above. First, welfare-enhancing interest rate rules should react to an exchange rate term.[14] This result can be demonstrated by comparing the rules R2 and R3 as well as R4 and TRopt. By adding the actual terms of trade to the Taylor rule TRopt or by adding the actual and lagged terms of trade to the rule R3 the loss decreases for both anticipated and unanticipated shocks. Second, welfare enhancing interest rate rules should be history-dependent, a point already emphasized in the context of non-optimized simple rules. An interest rule which reacts not only to actual inflation and GDP but also to their lagged values (compare TRopt with R3) performs remarkably better. The same holds if we compare the optimized Taylor rule (TRopt) with the history-dependent speed limit policy SL or the optimized Taylor rule with and without smoothing (TRopt and TRSopt respectively).

A final but important issue that should be mentioned is the somewhat puzzling result that anticipated shocks lead to a higher welfare loss than unanticipated shocks irrespective of the monetary policy response. This loss puzzle has already been discussed in the context of the optimal unrestricted policy rule. An implication of this result is that it would be socially optimal to disregard the information about a future oil price hike. However, a rationale agent will use the knowledge about future interest rate and exchange rate changes to make risk-free profits. Thus, the no-arbitrage condition reflected in the uncovered interest parity condition (3) leads necessarily to changes in the non-predetermined variables at the date of anticipation.

5 Summary and Conclusion

This paper evaluates the welfare effects of several monetary policy rules in the presence of anticipated and unanticipated oil price shocks. The analysis shows that calibrated Taylor-type interest rate rules amplify the welfare loss of an increase in the price of oil compared to neutral monetary policy rules. By contrast, the unrestricted optimal monetary policy under commitment dampens the welfare loss of the oil price shock compared to neutral monetary policy rules. Optimal simple rules are welfare-enhancing if they have a similar structure as the optimal unrestricted policy under commitment, namely history-dependence and the inclusion of an exchange rate term. In case of anticipated shocks the optimal unrestricted policy rule is also forward-looking. Optimal simple rules which are very good approximations of the optimal unrestricted

[14] This result is in line with the study of Wollmershäuser (2006). For an opposite result, see Adolfson (2007) or Leitemo and Söderström (2005).

rule should therefore also contain forward-looking elements. These conclusions offer an alternative perception of the discussion about Taylor rules versus neutral monetary policy. We show that the structure of a money growth peg as well as an interest rate peg is more similar to the optimal policy under commitment than to the policy rule proposed by Taylor (1993). A further somewhat puzzling result is that anticipated oil shocks lead to higher welfare losses than unanticipated shocks irrespective of the monetary policy response.

Of course, there are some limitations to our analysis. In future research it would be desirable i) to employ a fully microfounded model of an oil-dependent economy, ii) to relax the assumption of an exogenously given oil price, iii) to extend the analysis to the case of large open economies and to iv) estimate the model parameters.

Mathematical Appendix

State Space Representation

The matrices B and C and the vectors χ and ε of the state-space representation (6) are given by

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ b_{61} & 0 & 0 & 0 & 0 & \phi_{y_f} & -\phi_m l_i & b_{68} \\ \mu^* - 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{\pi_f} & 0 \end{pmatrix} \tag{A1}$$

with

$$b_{61} = (\phi_i + \phi_m l_i)(1 - \mu^*)\,, \qquad b_{68} = \phi_i \alpha + \phi_m l_i,$$

$$C = \begin{pmatrix} \beta^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{61} & -\phi_{y_b} & 0 & 0 & 0 & c_{66} & 0 & c_{68} \\ \mu^* - 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ c_{81} & 0 & -\gamma_{\pi_b} & -\gamma_{dp_{in}} & \gamma_{dp_{in}}\,\mu^* & -\gamma_q & 1 - \gamma_{dp_{in}} & c_{88} \end{pmatrix} \tag{A2}$$

with

$$c_{61} = (1 - \phi_m l_q)\psi\mu^* + (\phi_i + \phi_m l_i)(1 - \mu^*), \qquad c_{66} = 1 + \phi_y - \phi_m l_q,$$
$$c_{68} = \phi_i \alpha + \phi_m l_i + \phi_\tau - (1 - \phi_m l_q)\psi, \qquad c_{81} = -(\gamma_q \psi + \gamma_{dp_{in}})\mu^*,$$
$$c_{88} = \gamma_{dp_{in}} + \gamma_q \psi,$$

$$\chi = (0,0,0,0,0,0,1,0)', \tag{A3}$$
$$\varepsilon = (1,0,0,0,0,0,0,0)'. \tag{A4}$$

The first five state equations define the predetermined state variables (8). The sixth state equation is a combination of the model equations (1), (2), (3) and (4); the seventh state equation results from the uncovered interest parity condition (3), while the last state equation follows from the Phillips curve (5). Without loss of generality the exogenous variables i_t^* and y_t^* as well as the initial steady state values of the endogenous variables are set equal to zero.

If the time span between the anticipation and the implementation of the foreign price shock is positive ($T > 0$), the initial value of the vector of backward-looking variables w is zero ($w_0 = 0$). In the special case $T = 0$ w_0 is equal to the unit vector $(1,0,0,0,0)'$.

Total Loss under Commitment
To determine the minimum value of the loss function (10) first consider the case $T = 0$. Let $\widetilde{D}$ be the diagonal matrix

$$\widetilde{D} = \mathrm{diag}(d_2, d_1, d_3, d_4, 0, 0, 0, 0, 0). \tag{A5}$$

Since $\widetilde{v} = N\widetilde{w}$ ($N = Z_{21}Z_{11}^{-1}$), we get

$$J_t = \mathrm{E}_t \sum_{i=0}^{\infty} \lambda^i \widetilde{v}'_{t+i} \widetilde{D} \widetilde{v}'_{t+i} = \mathrm{E}_t \sum_{i=0}^{\infty} \lambda^i \widetilde{w}_{t+1} N' \widetilde{D} N \widetilde{w}'_{t+i}. \tag{A6}$$

From (25), (28) and (30) we obtain

$$\widetilde{w}_t = Z_{11}\widetilde{z}_t = Z_{11}(S_{11}^{-1}T_{11})^t Z_{11}^{-1}\widetilde{w}_0 = \left(Z_{11}(S_{11}^{-1}T_{11})Z_{11}^{-1}\right)^t \widetilde{w}_0 = \Gamma^t \widetilde{w}_0, \tag{A7}$$

where

$$\Gamma = Z_{11}(S_{11}^{-1}T_{11})Z_{11}^{-1}. \tag{A8}$$

Define

$$\varphi_t = \Gamma^t \widetilde{w}_0 \tag{A9}$$

and

$$P = N'\widetilde{D}N. \tag{A10}$$

Then the policy maker's welfare loss at time t can be rewritten as

$$\begin{aligned} J_t &= \sum_{i=0}^{\infty} \lambda^i \widetilde{w}'_{t+i} P \widetilde{w}'_{t+i} = \sum_{i=0}^{\infty} \lambda^i (\Gamma^{t+i}\widetilde{w}_0)' P (\Gamma^{t+i}\widetilde{w}_0) \\ &= (\Gamma^t \widetilde{w}_0)' \left(\sum_{i=0}^{\infty} \lambda^i \Gamma^{i'} P \Gamma^i \right) \Gamma^t \widetilde{w}_0 = \varphi'_t V \varphi_t, \end{aligned} \tag{A11}$$

where V is the geometric sum of matrices

$$V = \sum_{i=0}^{\infty} \lambda^i \Gamma^{i'} P \Gamma^i. \tag{A12}$$

Since the matrix $S_{11}^{-1} T_{11}$ is stable by construction, we have

$$\lim_{i\to\infty} \lambda^i \Gamma^{i'} P \Gamma^i = 0 \qquad \text{for all } 0 < \lambda \leq 1. \tag{A13}$$

The definition of V implies

$$\begin{aligned} V &= P + \sum_{i=1}^{\infty} \lambda^i \Gamma^{i'} P \Gamma^i = P + \sum_{i=0}^{\infty} \lambda^{i+1} \Gamma^{i+1'} P \Gamma^{i+1} \\ &= P + \lambda \Gamma' \left(\sum_{i=0}^{\infty} \lambda^i \Gamma^{i'} P \Gamma^i \right) \Gamma = P + \lambda \Gamma' V \Gamma. \end{aligned} \tag{A14}$$

Since $P = N' \widetilde{D} N$, the matrix $V = (v_{ij})_{1 \leq i,j \leq 8}$ satisfies the matrix equation

$$V = N' \widetilde{D} N + \lambda \Gamma' V \Gamma \tag{A15}$$

with the solution

$$\operatorname{vec}(V) = [I - \lambda \Gamma' \otimes \Gamma']^{-1} \cdot (N' \otimes N') \operatorname{vec}(\widetilde{D}), \tag{A16}$$

where $\operatorname{vec}(V)$ denotes the vector of stacked column vectors of V and $\otimes$ denotes the Kronecker product of matrices (Klein, 2000; Rudebusch and Svensson, 1999).

The optimal unrestricted policy under commitment yields a loss given by (Currie and Levine, 1993)

$$J_0 = \varphi_0' V \varphi_0 = \widetilde{w}_0 V \widetilde{w}_0 = \operatorname{trace}(V \widetilde{w}_0 \widetilde{w}_0') = v_{11} \quad \text{if } T = 0. \tag{A17}$$

Next consider the case $T > 0$. Partition the loss function J_t via

$$J_t = J_t^{(1)} + J_t^{(2)} = \mathrm{E}_t \sum_{i=0}^{T-1} \lambda^i \widetilde{v}_{t+i}' \widetilde{D} \widetilde{v}_{t+i} + \mathrm{E}_t \sum_{i=T}^{\infty} \lambda^i \widetilde{v}_{t+i}' \widetilde{D} \widetilde{v}_{t+i}. \tag{A18}$$

We will derive a formula for $J_t^{(2)}$ which is similar to (A11). For $t \geq T$ we have

$$\begin{aligned} \widetilde{w}_t = Z_{11} \widetilde{z}_t &= -Z_{11} (S_{11}^{-1} T_{11})^t Z_{11}^{-1} Z_{12} \widetilde{x}_0 \\ &\quad + Z_{11} \sum_{s=0}^{T-1} (S_{11}^{-1} T_{11})^{t-s-1} S_{11}^{-1} (T_{12} \widetilde{x}_s - S_{12} \widetilde{x}_{s+1}) + Z_{11} (S_{11}^{-1} T_{11})^{t-T} S_{11}^{-1} q_1 \\ &= Z_{11} (S_{11}^{-1} T_{11})^{t-T} \Bigg(S_{11}^{-1} q_1 - (S_{11}^{-1} T_{11})^T Z_{11}^{-1} Z_{12} \widetilde{x}_0 \\ &\quad + \sum_{s=0}^{T-1} (S_{11}^{-1} T_{11})^{T-s-1} S_{11}^{-1} (T_{12} \widetilde{x}_s - S_{12} \widetilde{x}_{s+1}) \Bigg) \\ &= Z_{11} M^{t-T} \widetilde{K}, \end{aligned} \tag{A19}$$

where

$$M = S_{11}^{-1} T_{11} \tag{A20}$$

and

$$\widetilde{K} = S_{11}^{-1} q_1 - M^T Z_{11}^{-1} Z_{12} \widetilde{x}_0 + \sum_{s=0}^{T-1} M^{T-s-1} S_{11}^{-1} (T_{12} \widetilde{x}_s - S_{12} \widetilde{x}_{s+1}). \tag{A21}$$

Then

$$\begin{aligned} J_t^{(2)} &= \sum_{i=T}^{\infty} \lambda^i \widetilde{w}'_{i+t} P \widetilde{w}_{i+t} = \sum_{i=T}^{\infty} \lambda^i (Z_{11} M^{i+t-T} \widetilde{K})' P (Z_{11} M^{i+t-T} \widetilde{K}) \\ &= (M^t \widetilde{K})' \lambda^T \left(\sum_{i=T}^{\infty} \lambda^{i-T} (Z_{11} M^{i-T})' P (Z_{11} M^{i-T}) \right) (M^t \widetilde{K}) \\ &= \lambda^T \widetilde{\varphi}'_t \widetilde{V} \widetilde{\varphi}_t, \end{aligned} \tag{A22}$$

where

$$\widetilde{\varphi}_t = M^t \widetilde{K} \tag{A23}$$

and

$$\widetilde{V} = \sum_{i=T}^{\infty} \lambda^{i-T} (Z_{11} M^{i-T})' P (Z_{11} M^{i-T}). \tag{A24}$$

Since $M = S_{11}^{-1} T_{11}$ is a stable matrix, $\widetilde{V}$ is well-defined. We get the matrix equation

$$\begin{aligned} \widetilde{V} &= Z'_{11} P Z_{11} + \sum_{i=T+1}^{\infty} \lambda^{i-T} (Z_{11} M^{i-T})' P (Z_{11} M^{i-T}) \\ &= Z'_{11} P Z_{11} + \sum_{i=T}^{\infty} \lambda^{i+1-T} (Z_{11} M^{i+1-T})' P (Z_{11} M^{i+1-T}) \\ &= Z'_{11} P Z_{11} + \lambda M' \widetilde{V} M. \end{aligned} \tag{A25}$$

Since

$$Z'_{11} P Z_{11} = Z'_{11} N' \widetilde{D} N Z_{11} = Z'_{11} Z_{11}^{-1\,\prime} Z'_{21} \widetilde{D} Z_{21} Z_{11}^{-1} Z_{11} = Z'_{21} \widetilde{D} Z_{21}, \tag{A26}$$

the matrix $\widetilde{V}$ satisfies the equation

$$\widetilde{V} = Z'_{21} \widetilde{D} Z_{21} + \lambda M' \widetilde{V} M \tag{A27}$$

with the solution

$$\mathrm{vec}(\widetilde{V}) = [I - \lambda M' \otimes M']^{-1} \cdot (Z'_{21} \otimes Z'_{21}) \mathrm{vec}(\widetilde{D}). \tag{A28}$$

(A22) and (A23) imply

$$J_0^{(2)} = \lambda^T \widetilde{\varphi}_0' \widetilde{V} \widetilde{\varphi}_0 = \lambda^T \widetilde{K}' \widetilde{V} \widetilde{K} = \lambda^T \operatorname{trace}(\widetilde{V} \widetilde{K} \widetilde{K}') \tag{A29}$$

with $\widetilde{K}$ given by (A21), where

$$\widetilde{x}_s = -(T_{22}^{-1} S_{22})^{T-1-s} T_{22}^{-1} q_2 \quad \text{for } s < T. \tag{A30}$$

Next consider the finite sum $J_0^{(1)}$. Since

$$\widetilde{v}_i = Z_{21} \widetilde{z}_i + Z_{22} \widetilde{x}_i \quad \text{for } i < T, \tag{A31}$$

we obtain

$$\begin{aligned} J_0^{(1)} &= \sum_{i=0}^{T-1} \lambda^i \widetilde{v}_i' \widetilde{D} \widetilde{v}_i = \sum_{i=0}^{T-1} \lambda^i (Z_{21} \widetilde{z}_i + Z_{22} \widetilde{x}_i)' \widetilde{D} (Z_{21} \widetilde{z}_i + Z_{22} \widetilde{x}_i) \\ &= \sum_{i=0}^{T-1} \lambda^i \widetilde{z}_i' (Z_{21}' \widetilde{D} Z_{21}) \widetilde{z}_i + \sum_{i=0}^{T-1} \lambda^i \widetilde{x}_i' (Z_{22}' \widetilde{D} Z_{22}) \widetilde{x}_i + 2 \sum_{i=0}^{T-1} \lambda^i \widetilde{z}_i' (Z_{21}' \widetilde{D} Z_{22}) \widetilde{x}_i. \end{aligned} \tag{A32}$$

In case $T > 0$ the optimal unrestricted policy under commitment yields a total loss given by $J_0^{(1)} + J_0^{(2)}$ where $J_0^{(1)}$ and $J_0^{(2)}$ can be determined with the help of (A32) and (A29).

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