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Monetary Policy Dynamics in Large Oil-Dependent Economies

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Monetary Policy Dynamics in Large Oil-Dependent Economies

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Abstract

The paper analyzes the impacts of anticipated and unanticipated monetary policies on two large open economies that are dependent upon raw materials imports from a small third country. The analysis is based on asymmetric behavior on the supply side of both economies and an endogenous commodity pricing equation of Phillips' curve type. It is shown that an increase in the growth rate of domestic money supply is not neutral in the long run but induces contractionary output effects in both economies. The paper also discusses the impacts of monetary policy rules that either reduce the inflationary or contractionary output effects of commodity price shocks.

JEL classification: E63, F42, Q43
Keywords: Monetary Policy, Oil Price Shocks, International Policy Coordination

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1 Introduction

The paper analyzes the dynamic effects of anticipated and unanticipated monetary policies on two large open economies that are dependent upon raw materials imports (like crude oil) from a small third country. We can identify the small commodity-exporting country with the OPEC nations and the domestic and the foreign economy with the European Monetary Union and the USA respectively. It is assumed that the dependency upon oil imports is stronger for the domestic than the foreign economy and that oil imports are priced in terms of the foreign currency (dollars). This implies an asymmetric macroeconomic structure of the two economies. We assume sluggish price and wage adjustment on the supply side and a dynamic commodity pricing rule which takes the form of a Phillips curve. We then discuss the impact of monetary expansion in one economy on both the domestic and foreign economy where the analysis is based on asymmetric behavior. We also analyze the dynamic effects of commodity price disturbances and various monetary policy responses that could be employed by the domestic and foreign central bank in an effort to reduce the stagflationary effects of oil price shocks.

It is shown that as a consequence of the endogenization of the rate of change of the raw materials price a rise in the growth rate of domestic money supply is not neutral in the long run but increases the domestic and foreign real commodity price. By contrast, a rise in the growth rate of foreign money supply generally leads to a decline in the real factor prices in the long run. A coordination of domestic and foreign monetary policy is able to neutralize the inflationary effects of commodity price shocks. An international monetary policy coordination can also fix the output vector at its initial pre-disturbance equilibrium level at all times. There exist optimal monetary policies that guarantee perfect output stabilization and simultaneously minimize deviations of the domestic and foreign inflation rate from their respective initial steady state level.

The paper is organized as follows: Section 2 describes the macroeconomic model of two large oil-dependent economies. Section 3 analyzes the dynamic effects both of a unilateral increase in the growth rate of domestic and foreign money supply on the two economies. Section 4 discusses the impacts of commodity price shocks and various monetary policy responses to oil price disturbances. Section 5 summarizes the main results. At the end, the paper includes an extensive mathematical appendix.

2 The Macroeconomic Model

The following model stands in the Mundell/Fleming/Dornbusch/Phillips (MFDP) tradition and extends similar models of Bhandari (1981) and Turnovsky (1986, 2000) to the case of large oil-dependent asymmetric economies. We consider two open economies, each specializing in the production of a distinct final good. Both countries use imported raw materials from a small third country for the production of their respective domestic output.[1] The model is described by the equations (1)–(20):

$$
\begin{aligned}
q &= (a_0 + a_1 y - a_2(i - \dot{p}^c)) + g + (c_0 - c_1 y + c_2 y^* - c_3 \tau) && (1)\\
q^* &= (a_0 + a_1 y^* - a_2(i^* - \dot{p}^{*c})) + g^* - (c_0 - c_1 y + c_2 y^* - c_3 \tau) && (2)\\
\tau &= p - (p^* + e) && (3)\\
m - p^c &= l_0 + l_1 q - l_2 i && (4)\\
m^* - p^{*c} &= l_0 + l_1 q^* - l_2 i^* && (5)\\
i &= i^* + \dot{e} && (6)\\
p^c &= \alpha p + (1-\alpha)(p^* + e) \qquad (0.5 < \alpha < 1) && (7)\\
p^{*c} &= \alpha^* p^* + (1-\alpha^*)(p - e) \qquad (0.5 < \alpha^* < 1) && (8)\\
y &= q - \psi(p_R^* + e - p) - d_0 && (9)\\
y^* &= q^* - \psi^*(p_R^* - p^*) - d_0 && (10)\\
\dot{p} &= \mu \dot{w} + (1-\mu)(\dot{p}_R^* + \dot{e}) \qquad (0 < \mu < 1) && (11)\\
\dot{p}^* &= \mu^* \dot{w}^* + (1-\mu^*)\dot{p}_R^* \qquad (0 < \mu^* < 1) && (12)\\
\dot{w} &= \pi + \delta(q - \overline{q}) && (13)\\
\dot{w}^* &= \pi^* + \delta^*(q^* - \overline{q}^*) && (14)\\
\dot{p}_R^* &= \pi_R^* + \delta_R^*(q + q^* - (\overline{q} + \overline{q}^*)) && (15)\\
\pi &= \beta \dot{m} + (1-\beta)\dot{p}^c \qquad (0 \le \beta \le 1) && (16)\\
\pi^* &= \beta^* \dot{m}^* + (1-\beta^*)\dot{p}^{*c} \qquad (0 \le \beta^* \le 1) && (17)\\
\pi_R^* &= \beta_R^* \dot{m}^* + (1-\beta_R^*)\dot{p}^* \qquad (0 \le \beta_R^* \le 1) && (18)\\
\overline{q} &= f_0 + f_1 \overline{\tau} + f_2 (\overline{p - (p_R^* + e)}) && (19)\\
\overline{q}^* &= f_0^* - f_1^* \overline{\tau} + f_2^* (\overline{p^* - p_R^*}) && (20)
\end{aligned}
$$

where q = real output, y = real income, i = nominal interest rate, $i - \dot{p}^c$ = real interest rate, p^c = consumer price index (CPI), p = price of output, τ = final goods terms of trade, e = nominal exchange rate (domestic currency price of foreign currency), p_R^* = US dollar price of imported raw materials, $p_R^* + e - p$ = domestic real factor price of imported raw materials, $p_R^* - p^*$ = foreign real factor price of imported raw materials, w = nominal wage rate, m = nominal money stock, π = augmentation term in the Phillips curve. All variables – except for the interest rates i and i^* – are expressed in logarithms. Dots denote time derivatives and overbars steady state values. Foreign variables are denoted with an asterisk. We

[1] In what follows we use interchangeably the terms oil imports, raw materials imports and commodity imports.

assume rational expectations with respect to the expected depreciation rate and the inflation rates.

Equations (1) and (2) are IS equations where private absorption (first expression in brackets) depends on national income and the real interest rate and where international trade in final goods (second expression in brackets) depends on domestic income, foreign income and the final goods terms of trade τ (defined in (3)).[2] We assume symmetric private demand functions. The corresponding effects across the two economies are then identical and the coefficients c_1 and c_2 in the bilateral trade balance coincide. Equations (4) and (5) are standard LM equations and reflect money market equilibrium in the economies. We assume symmetric money demand functions. The uncovered interest parity condition (6) describes perfect substitutability of domestic and foreign bonds. Equations (7) and (8) define the domestic and foreign consumer price index. We assume that the proportion of consumption spent on the respective home good (α, α^*) is greater than 0.5 so that residents in both countries have a preference for their own good. The expressions $1 - \alpha$ and $1 - \alpha^*$ can be interpreted as the degree of demand side openness of the respective economy. If we identify the foreign economy with the USA and the domestic economy with the European Monetary Union (EMU) it is reasonable to assume $1 - \alpha^* < 1 - \alpha$ or, equivalently, $\alpha > \alpha^*$. Equations (9) and (10) describe the difference between the respective domestic production and real income or gross national product. The difference results from imports of intermediate goods which in turn depend on the respective real factor price. We assume that raw materials imports (like crude oil) are denominated in terms of the foreign currency (US dollars) so that the domestic real factor price $p_R^* + e - p$ depends on the nominal exchange rate e. The constants ψ and ψ^* can be derived from a microeconomic profit maximizing approach with a CES production technology which allows for factor substitution between labor and raw materials imports. It can be shown that in this case ψ is of the form $(1 - \mu)(1 - \sigma)/\sigma$, where σ is the elasticity of substitution between labor and raw materials imports and where μ measures the share of labor in gross domestic output (Bhandari and Turnvosky (1984)). The constant $1 - \mu$ then measures the share of imported inputs in gross output and can be interpreted as a measure for the supply side openness of the domestic economy. An analogous formula holds for ψ^*, and it is reasonable to assume $\psi > \psi^*$ and $1 - \mu > 1 - \mu^*$, i.e., a stronger oil-dependency and supply side openness for the EMU than for the USA.[3] The equations (11) and

[2]In discrete time *New Neoclassical New Keynesian synthesis* models, private absorption does not only depend positively on current income, but also depends positively on past and/or expected future income (see, for example, Fuhrer and Moore (1995), Clarida, Gali and Gertler (1999), King (2000) or McCallum (2001)). In deterministic continuous time models (like ours) private absorption only depends on the current income.

[3]In addition to the oil *dependency*, i.e. the ratio of net oil imports to GDP, the oil *intensity*, i.e. the ratio of oil consumption to GDP, is greater for the EMU than for the USA (Anderton, di

(12) describe the price adjustment in the domestic and foreign economy respectively and are dynamic versions of mark-up pricing rules. In both countries the rate of change of the price of output is a weighted average of nominal wage inflation and the rate of change of the domestic price of raw materials imports. Equations (13) and (14) describe the dynamics of wage adjustment in form of standard augmented Phillips curves (Buiter and Miller (1982)). The augmentation term is according to (16) and (17) a linear combination of the trend rate and the core rate of inflation, the first given by the growth rate of money stock and the second by the rationally anticipated inflation rate based on the CPI.[4] In (16), the parameter β is a fixed weight between zero and one and can be considered as a measure of wage indexation (van der Ploeg (1990)). In the special case $\beta = 0$ the growth rate of domestic real wages only depends on the output gap $q - \overline{q}$. In the other polar case $\beta = 1$ the wage-Phillips curve describes the adjustment of nominal wages as a positive function of the output gap and the growth rate of money supply. Assuming a low value of the parameter δ, the case $\beta = 1$ is consistent with nominal wage rigidity. In many member countries of the EMU the degree of wage indexation is greater than in the USA (Manasse (1991), OECD (2000)). We therefore assume $1 - \beta > 1 - \beta^*$, i.e., $\beta < \beta^*$. The third Phillips curve (15) describes the adjustment of the second factor price, the price of oil. We assume that the growth rate of the foreign commodity price is a positive function of the world output gap $q + q^* - (\overline{q} + \overline{q}^*)$. The augmentation term π_R^* is a fixed weight linear combination of the growth rate of foreign money supply and the foreign inflation rate. The difference $1 - \beta_R^*$ measures the degree of commodity price indexation to the foreign inflation rate $\dot{p}^*$. Since the commodity price p_R^* is expressed in terms of the foreign currency, a positive inflation rate $\dot{p}^* > 0$ leads to an ongoing reduction in the purchasing power of the oil exporting nation if $\dot{p}_R^*$ is not indexed to $\dot{p}^*$. It is therefore reasonable to assume $1 - \beta_R^* > 0$, i.e., $\beta_R^* < 1$. Moreover, a situation in which the world output lies above its natural level induces a high world wide demand for oil causing an upward pressure on the price of oil. In the following we assume a positive but small demand coefficient δ_R^* and a high value of β_R^* which is smaller than one. The last two equations (19) and (20) describe the long run output supply depending on the final goods terms of trade and the real commodity price. These equations result from the static analogue of the dynamic price and wage equations where complete wage indexation ($\beta = \beta^* = 0$)

Mauro and Moneta (2004)). Note that our model assumption regarding the dependency of the two large open economies on imported raw materials produced only by a third small country leads to an identity of these concepts.

[4]The formulation of the inflation dynamics can also be interpreted as a special type of a continuous time hybrid Phillips curve. In a discrete time model, the hybrid Phillips curve formulation allows inflation to depend on expected future and lagged inflation (see, for example, Gali and Gertler (1999)). Instead of a backward-looking element in the hybrid Phillips curve, we model a dependency of the inflation dynamics on the trend inflation rate given by the growth rate of money supply.

is assumed. In this case the coefficients f_1 and f_2 are given by $f_1 = (1-\alpha)/\delta$ and $f_2 = (1-\mu)/(\mu\delta)$ respectively with $f_2 > f_1$. On the assumption that the domestic economy is stronger oil-dependent than the foreign economy (i.e., $1-\mu > 1-\mu^*$), the parameter f_2 is greater than its foreign analogue f_2^* (provided that the static wage equations $\overline{w} = \overline{p}^c + \delta\overline{q}$ and $\overline{w}^* = \overline{p}^{*c} + \delta^*\overline{q}^*$ have similar coefficients δ and δ^* so that $(1-\mu)\mu^*/((1-\mu^*)\mu) > \delta/\delta^*$ holds).

The dynamic behavior of the whole system (1)–(20) can be described by a three-dimensional dynamic system. Assuming positive growth rates of domestic and foreign money supply, steady state values of the domestic and foreign price level and the nominal exchange rate do not exist. It is therefore convenient to express the dynamics of the system in terms of real liquidity and real competitiveness (Buiter and Miller (1982)). Exploiting the decomposition method by Aoki (1981) and Fukuda (1993) we use the state variables $l^s = (m-p)+(m^*-p^*)$, $l^d = (m-p)-(m^*-p^*)$ and $\tau = p-(p^*+e)$ for the state space representation of the model.[5] The sum and the difference of domestic and foreign real money supply are backward-looking or predetermined variables containing the sluggish price variables p and p^*. By contrast, the terms of trade τ are a forward-looking or jump variable moving discontinuously whenever the nominal exchange rate jumps. Although the adjustment of final goods prices and factor prices is assumed to be sluggish the inflation rates $\dot{p}$ and $\dot{p}^*$ are jump variables as well that can adjust instantaneously. In the following we assume that at time $t = 0$ both economies are in steady state and of equal size. At time $t = 0$ a monetary policy change is anticipated to take place at some future time $T > 0$.

3 Dynamic Effects of Expansionary Monetary Policy

In this chapter we discuss the dynamic effects of a unilateral once-and-for-all increase in the growth rate of money supply ($d\dot{m} > 0, d\dot{m}^* > 0$). We first consider the steady state effects, thereafter the dynamic effects where we distinguish between an unanticipated and an anticipated monetary policy.

3.1 Steady State Effects

The steady state effects of an increase in the domestic and foreign growth rate of money supply are presented in table 1. They result from the steady state condition $\dot{\tau} = \dot{l}^s = \dot{l}^d = 0$ implying classical results with respect to the change of the

[5]The state space representation and the solution time paths of the state variables are provided in the mathematic appendix. As long as the degree of supply side openness is not extremely large, the state form of the dynamic macro model exhibits saddle point stability.

inflation rates at home and abroad and the rate of depreciation $\dot{e}$. Since during the course of adjustment a rise in the growth rate of *domestic* money supply leads to an overshooting of the corresponding inflation rate ($\dot{p} > \dot{m}$), the real money stock $m - p$ declines in the long run implying a steady state rise in the nominal interest rate.[6] In case $\dot{m} > 0$ and given a constant level of the foreign money stock ($\dot{m}^* = 0$) a similar result holds with respect to the foreign real money stock and the foreign nominal interest rate where the interest parity condition (6) implies a positive nominal interest rate differential ($d\bar{i} - d\bar{i}^* = d\bar{\dot{e}} = d\bar{\dot{m}} > 0$).[7] Since the real appreciation rate $\dot{\tau}$ is zero in the long run there is no real steady state interest rate differential. Instead, expansionary domestic monetary policy causes a long run rise in domestic and foreign real interest rate of equal size. This result stands in sharp contrast to traditional MFDP models where monetary policy is neutral in the long run. In our dynamic macro model with an endogenous commodity pricing rule for the rate of change of the price of oil, domestic monetary policy is not neutral.[8] Instead, it has *perverse* real steady state effects. Under the fairly weak condition (cf. Wohltmann and Winkler (2005 a))

$$(1 - a_1 + 2c_1)(f_2 - f_2^*) > (a_1 - 2c_1)(\psi - \psi^*) \tag{21}$$

an increase in the growth rate of domestic money stock causes a long run real appreciation of the domestic currency ($d\overline{\tau} > 0$). Moreover, a steady state rise in the real factor prices $p_R^* - p^*$ and $p_R^* + e - p$ occurs. The reason is that during the course of adjustment the world output gap $q^w - \overline{q}^w$ ($q^w = q + q^*$) is always positive. The commodity pricing rule (15) which is equivalent to the equation

$$\dot{p}_R^* - \dot{p}^* = \beta_R^*(\dot{m}^* - \dot{p}^*) + \delta_R^*(q^w - \overline{q}^w) \tag{22}$$

then implies a positive growth rate of the foreign real factor price $p_R^* - p^*$ (although there is a weak positive foreign inflation rate $\dot{p}^*$ after the implementation of the domestic monetary shock inducing an opposing effect on $\dot{p}_R^* - \dot{p}^*$). Compared to the steady state change of the terms of trade τ, the rise in the real commodity price $p_R^* - p^*$ is strong. According to the relationship

$$p_R^* + e - p = -\tau + p_R^* - p^* \tag{23}$$

[6] This is a well known result of MFDP models with rational expectations and sluggish price and wage adjustment.

[7] On the assumption $\dot{m}^* = 0$ a steady state value of the foreign price level p^* exists so that $d(\overline{m^* - p^*}) = -d\overline{p}^*$ holds.

[8] In our model, neutrality of domestic monetary policy only holds in the special case $\delta_R^* = 0$, i.e., if the world output gap is not an argument of the commodity pricing rule.

the domestic real factor price must also increase in the long run, where the steady state rise in $p_R^* + e - p$ is smaller than the increase in $\overline{p_R^* - p^*}$ if $d\overline{\tau}/d\dot{m} > 0$ holds. The long run rise in the domestic and foreign real commodity price leads to the striking result that an *increase* in the growth rate of domestic money supply causes a steady state *contraction* of real output and income in both economies.[9] Since we have assumed that the domestic economy is stronger oil-dependent than the foreign economy the domestic output contraction is generally stronger than the decline in foreign output. This result holds although the steady state rise in the domestic terms of trade τ has a positive impact on long run domestic output $\overline{q}$ while its impact on $\overline{q}^*$ is negative (cf. equations (19) and (20)). However, since the rise in $\overline{\tau}$ is weak compared with the steady state increase in the real factor prices $p_R^* - p^*$ and $p_R^* + e - p$, the output contraction occurs in both economies. The assumption $f_2 > f_2^*$ then generally leads to a stronger decline in $\overline{q}$ than $\overline{q}^*$. A further remarkable result is that the magnitude of the output contraction depends on the length T of the time interval between the anticipation and the implementation of the monetary policy shock. The shorter this time span, the stronger are the steady state effects of monetary policy. This implies that an unanticipated increase in the growth rate of domestic money supply has stronger contractionary steady state output effects than an anticipated monetary expansion. To explain this, consider the domestic and foreign price and wage equations (11), (12), (13) and (14) which can be combined and transformed into the aggregate supply functions

$$q = \overline{q} + \left(\frac{1}{\delta}(1-\beta)(1-\alpha) + \frac{1-\mu}{\mu}\right)\dot{\tau} - \frac{\beta}{\delta}(\dot{m} - \dot{p}) - \frac{1-\mu}{\delta\mu}(\dot{p}_R^* - \dot{p}^*) \quad (24)$$

$$q^* = \overline{q}^* - \frac{1}{\delta^*}(1-\beta^*)(1-\alpha^*)\dot{\tau} - \frac{\beta^*}{\delta^*}(\dot{m}^* - \dot{p}^*) - \frac{1-\mu^*}{\delta^*\mu^*}(\dot{p}_R^* - \dot{p}^*) \quad (25)$$

Domestic output is a positive function of the domestic real appreciation rate $\dot{\tau}$ while q^* is a negative function of $\dot{\tau}$. Since the supply side of both economies is asymmetric, aggregate or real world output q^w also depends on $\dot{\tau}$.[10] We have assumed that oil imports are completely invoiced in terms of the foreign currency and that the degree of wage indexation is greater in the domestic than in the foreign economy. On these assumptions the world output gap $q^w - \overline{q}^w$ is a positive function of $\dot{\tau}$. Inserting this equation into equation (22) for the growth rate of the foreign real commodity price it follows by integration that the real factor price $p_R^* - p^*$ is a positive function of the terms of trade τ. Since $p_R^* - p^*$ adjusts sluggishly by assumption while τ jumps at the date of anticipation by the amount $\tau(0+) - \overline{\tau}_0$, the foreign real commodity price

[9] A similar result with respect to domestic real output (but not with respect to domestic private consumption) also holds in models of the New Open Economy Macroeconomics. See Obstfeld and Rogoff (1995, 1996).

[10] In the special case $\beta = \beta^*$, $\alpha = \alpha^*$, $\delta = \delta^*$ and $\mu = \mu^* = 1$ world output is independent of $\dot{\tau}$ and equals $\overline{q}^w$ for all t.

must be a negative function of the initial jump of τ. This holds during the whole course of adjustment so that the steady state change of $p_R^* - p^*$ depends negatively on the initial terms of trade difference $\tau(0+) - \overline{\tau}_0$. Now it is a well known result that the initial jump of τ increases the shorter the anticipation phase $(0, T)$.[11] Since an expansionary domestic monetary policy leads on impact to a nominal and real depreciation of the domestic currency ($\tau(0+) - \overline{\tau}_0 < 0$), the steady state rise in $p_R^* - p^*$ and the output contraction in both economies must be stronger in case $T = 0$ than in case $T > 0$.

An analogous result holds for *foreign* monetary policy, i.e., the steady state effects of an unanticipated increase in the growth rate of foreign money supply ($d\dot{m}^* > 0$) are stronger than an anticipated foreign monetary expansion. An interesting result is that the long run real output, income and relative price effects of foreign monetary policy are usually opposite in sign than the corresponding steady state effects of domestic monetary policy (cf. table 1). In contrast to domestic monetary policy *expansionary foreign* monetary policy generally causes a long run *fall* in the real commodity prices $p_R^* - p^*$ and $p_R^* + e - p$ implying a steady state rise in domestic and foreign output and income. This result holds as long as the coefficient β_R^* in equation (22) for the growth rate of the real factor price $p_R^* - p^*$ is not too small relative to the second coefficient δ_R^*. In this case the overshooting of the foreign inflation rate ($\dot{p}^* > \dot{m}^*$) which occurs during the course of adjustment induces a negative growth rate of the foreign real commodity price $p_R^* - p^*$ that can not be reversed by a positive world output gap.[12] This implies a steady state fall in the real factor price $p_R^* - p^*$ inducing a long run rise in foreign and domestic output. Although there is in general a long run real appreciation of the foreign currency (provided condition (21) is met) and – as a consequence – a weaker fall in the domestic than the foreign real commodity price, the steady state rise in domestic output is stronger than the foreign output expansion. This is a consequence of our assumption $f_2 > f_2^*$ in the long run supply functions (19) and (20). Foreign monetary policy is inefficient in the long run only if the polar case $\beta_R^* = 0 = \delta_R^*$ is considered. Equation (22) then implies $\dot{p}_R^* - \dot{p}^* = 0$ so that the foreign real commodity price is constant over time. In this special case the steady state values of all relative prices remain unchanged so that foreign monetary policy is neutral in the long run. As in the case of domestic monetary policy, foreign monetary policy can also be connected with perverse steady state output effects. While a rise in the domestic monetary growth rate $\dot{m}$ always induces a steady state fall in $\overline{q}$ and $\overline{q}^*$ (provided $\delta_R^* > 0$ holds), an analogous foreign monetary expansion only leads to a long run output contraction if δ_R^* is positive and the coefficient β_R^* in (22) very small, in particular equal to zero.

[11] See, for example, Turnovsky (2000).

[12] Note that in case of domestic monetary policy ($d\dot{m}^* > 0$) the growth rate $\dot{p}_R^* - \dot{p}^*$ is always positive. This result even holds if β_R^* is large and δ_R^* very small.

$d\dot{m} > 0$, $\dot{m}^* = 0$	$d\dot{m}^* > 0$, $\dot{m} = 0$
$d\overline{\dot{p}} = d\overline{\dot{p}^c} = d\overline{\dot{w}} = d\dot{m} > 0$	$d\overline{\dot{p}^*} = d\overline{\dot{p}^{*c}} = d\overline{\dot{w}^*} = d\dot{m}^* > 0$
$d\overline{\dot{p}^*} = d\overline{\dot{p}^{*c}} = d\overline{\dot{w}^*} = d\dot{p}^*_R = 0$	$d\overline{\dot{p}} = d\overline{\dot{p}^c} = d\overline{\dot{w}} = 0$
$d\overline{\dot{e}} = d\dot{m} > 0$	$d\overline{\dot{e}} = -d\dot{m}^* < 0$
$d(\overline{m - p}) < 0$	$d(\overline{m - p}) > 0$
$d(\overline{m^* - p^*}) < 0$	$d(\overline{m^* - p^*}) < 0$
$d\overline{p}^* > 0$	$d\overline{p} < 0$
$d(\overline{i - \dot{p}^c}) = d(\overline{i^* - \dot{p}^{*c}}) > 0$	$d(\overline{i - \dot{p}^c}) = d(\overline{i^* - \dot{p}^{*c}}) < 0$
$d\overline{i} > d\overline{i}^* > 0$	$d\overline{i}^* > 0 > d\overline{i}$
$d\overline{\tau} > 0$	$d\overline{\tau} < 0$
$d(\overline{p^*_R + e - p}) > 0$	$d(\overline{p^*_R + e - p}) < 0$
$d(\overline{p^*_R - p^*}) > 0$	$d(\overline{p^*_R - p^*}) < 0$
$d\overline{q} < 0$	$d\overline{q} > 0$
$d\overline{q}^* < 0$	$d\overline{q}^* > 0$
$d\overline{y} < 0$	$d\overline{y} > 0$
$d\overline{y}^* < 0$	$d\overline{y}^* > 0$

Table 1: Long run effects of domestic and foreign monetary policy

3.2 Dynamic Effects of Unanticipated Monetary Policy

We first consider an *unanticipated* unit increase in the growth rate of *domestic* money supply, with the foreign money supply held constant; that is $d\dot{m} = 1, d\dot{m}^* = 0$. The dynamic effects are illustrated in figures 1 to 4 (solid lines), where the assumption is made that $\delta^*_R > 0$. In that case domestic monetary policy is non-neutral in the long run. Figure 1 (a) shows the response of the domestic terms of trade τ which is similar to the adjustment of τ in MFD overshooting models. On impact, the domestic monetary expansion induces a real depreciation of the domestic currency. Thereafter, a process of real appreciation towards the new steady state $\overline{\tau}_1$ takes place. On the assumption (21) $\overline{\tau}_1$ is greater than the initial steady state $\overline{\tau}_0$. An analogous response results for the domestic and foreign real interest rate (figure 1 (b) and 1 (c)) where the new steady states $(\overline{i - \dot{p}^c})_1$ and $(\overline{i^* - \dot{p}^{*c}})_1$ coincide and are larger than the corresponding initial steady states. The development of the domestic and foreign real commodity price is illustrated in figure 2. On impact, the domestic real factor price $p^*_R + e - p$ overshoots its long run response where the jump in $t = 0$ corresponds with the initial jump of τ. Thereafter, $p^*_R + e - p$ converges from above to its new steady state $(\overline{p^*_R + e - p})_1$ which is greater than the initial value $(\overline{p^*_R + e - p})_0$. The foreign real factor price $p^*_R - p^*$ behaves sluggishly at time $t = 0$ and increases gradually during the course of adjustment (figure 2 (b)). The permanent rise in $p^*_R - p^*$ is a consequence of the commodity pricing rule (15) which induces a growth rate of the nominal factor price p_R^* greater than the foreign inflation rate $\dot{p}^*$ for all $0 < t < \infty$. Therefore, the growth rate of the foreign real factor price, i.e., $\dot{p}_R^* - \dot{p}^*$, is positive throughout the whole adjustment process (figure 2 (c)) implying a monotonous rise in $p_R^* - p^*$ for $t > 0$. Figure 3 illustrates the domestic and foreign output response. The short term and medium term development of domestic output q is familiar from traditional MFDP models.[13] On impact, an output expansion takes place which is a consequence of the initial real depreciation of the domestic currency and the fall in the real interest rate. Although the initial rise in the domestic real commodity price induces – in isolation – a reduction in national income and private absorption, this contractionary effect on aggregate demand is dominated by the expansionary effects caused by the fall in the terms of trade and the real interest rate. After its initial rise domestic output begins to fall, which is a consequence of the process of real appreciation and the increase in the domestic real interest rate after the impact phase. Since the new steady state values of τ, $i - \dot{p}^c$ and $p_R^* + e - p$ are larger than the corresponding initial values, a long run reduction in aggregate demand and real output takes place. A similar development results for foreign output q^* (figure 3 (b)). On impact, there is a real appreciation of the foreign currency and a decrease in the foreign real interest rate so that the net effect on aggregate demand is ambiguous. As illustrated in figure 1, the terms of trade effect dominates the opposing interest rate effect implying a slight fall in q^* on impact. Thereafter, the decrease in foreign output is magnified which is a consequence of the gradual rise in the foreign real factor price $p_R^* - p^*$ and the interest rate $i^* - \dot{p}^{*c}$ inducing a decline in private absorption. On the other hand, the process of real depreciation of the foreign currency after the initial response improves net exports of final goods while the domestic output and income contraction causes – in isolation – a deterioration of the foreign trade balance with respect to the domestic economy. Since the overall effect on foreign aggregate demand is unambiguously negative, the initial foreign output contraction is amplified during the course of further adjustment. In the long run foreign output lies below its initial steady state level. However, the steady state output contraction is slightly weaker than the long run fall in domestic output. This implies that the output differential $q - q^*$, which is positive in the short run (i.e. $q > q^*$ for sufficiently small $t > 0$), changes sign during the course of adjustment (figure 3 (c)). On the other hand, the world output gap $q^w - \overline{q}^w$, i.e. the difference between aggregate output and the new steady state level of world output, is always positive (figure 3 (d)).

Figure 4 illustrates the response of the inflation rates $\dot{p}$, $\dot{p}^*$ and $\dot{p}_R^*$.[14] The domestic inflation rate is characterized by an overshooting ($\dot{p} > \dot{m} = 1$ for $0 < t < \infty$) and

[13] See, for example, Turnovsky (1986) and Clausen and Wohltmann (2005).

[14] The development of $\dot{p}^c$ and $\dot{w}$ is similar to the response of $\dot{p}$ and need not be discussed here. The same holds for the foreign inflation rate $\dot{p}^{*c}$ and $\dot{w}^*$ which behave similar like $\dot{p}^*$.

converges from above towards the new level of the growth rate of domestic money supply. On the other hand, the foreign inflation rates $\dot{p}^*$ and $\dot{p}^*_R$, which are smaller than the domestic inflation rate $\dot{p}$, converge from above towards their initial level $\overline{\dot{p}^*_0} = (\overline{\dot{p}^*_R})_0 = \dot{m}^* = 0$. Since $\dot{p}^*$ is positive for $0 < t < \infty$ the foreign price level has increased permanently ($d\overline{p}^* > 0$) so that expansionary domestic monetary policy induces permanent stagflation abroad.

Figures 1 to 4 also contain the response of the endogenous variables in case of a unilateral *foreign* monetary shock, that is, $d\dot{m}^* = 1$, $d\dot{m} = 0$ (dashed lines). The adjustment processes are based on the assumption that the coefficient β^*_R in the commodity pricing rule (15) is sufficiently large compared to δ^*_R. In that case the domestic and foreign steady state output increase while the steady state values of the real factor prices decrease. The dynamic response of the real commodity prices $p^*_R - p^*$ and $p^*_R + e - p$ is opposite to the corresponding development of the factor prices in case $d\dot{m} > 0$ (figure 1). Foreign output behaves similar to the adjustment of domestic output in case $d\dot{m} > 0$. After its initial increase, which is a result of the initial real depreciation of the foreign currency, foreign output falls towards its new steady state level which is greater than the initial steady state of q^*. This holds since the new steady state values of τ, $p^*_R - p^*$ and $i^* - \dot{p}^{*c}$ lie below their corresponding initial values. The development of domestic output is opposite to the corresponding response of foreign output q^* in case of the domestic monetary policy $d\dot{m} > 0$. On impact, a domestic output expansion takes place, although the domestic currency appreciates in the short run and the domestic real interest rate rises. On the other hand, the short term decrease in the real factor price $p^*_R + e - p$ causes a rise in domestic private absorption, while the foreign output expansion leads to an increase in domestic final goods exports. After the impact phase, domestic output increases further, which is a consequence of the continuous reduction in the domestic terms of trade and the domestic real interest rate after their initial jump.

The steady state decline in the foreign real factor price $p^*_R - p^*$ results from an overshooting of the foreign inflation rate ($\dot{p}^* > \dot{m}^* = 1$) which is larger than the positive growth rate of the commodity price ($\dot{p}^* > \dot{p}^*_R$).[15] Since the aggregate steady state output increases ($d\overline{q} + d\overline{q}^* > 0$) the world output gap is small so that the rate of change of the commodity price is basically determined by the weighted sum of $\dot{m}^*$ and $\dot{p}^*$. By contrast, in the case of domestic monetary policy, the foreign inflation rate is small and the world output gap large so that the response of $\dot{p}^*_R$ is similar to the development of the world output gap. An interesting result concerning the domestic inflation rate $\dot{p}$ is that the foreign monetary policy $d\dot{m}^* > 0$ causes temporary deflation in the domestic economy. $\dot{p}$ falls on impact and runs below

[15] Note that this result does not hold if β^*_R is very small (in particular equal to zero) and δ^*_R sufficiently large. In this case $\dot{p}^*_R$ increases on impact by a large amount and is greater than $\dot{p}^*$ so that the real factor price $p^*_R - p^*$ rises and q^* and q fall in the long run.

its initial steady state level $\overline{p}_0 = 0$ for all $0 < t < \infty$ (figure 4). The economic explanation is the negative domestic output gap ($q - \overline{q}_1 < 0$) which leads to a negative rate of change of the domestic wage rate ($\dot{w} < 0$). This in turn causes a negative inflation rate $\dot{p}$ so that expansionary *foreign* monetary policy is connected with a *disinflationary boom* in the domestic economy.[16] By contrast, an increase in the growth rate of *domestic* money supply causes an *inflationary recession* in the foreign economy.

Parameter	Value	Description
a_1	0.8	Marginal propensity to consume
$c_1 = c_2$	0.2	Income elasticity of international trade in final goods
c_3	0.3	Terms of trade elasticity of international trade in final goods
a_2	0.8	Semi-interest elasticity of private absorption
$1-\alpha$	0.25	Degree of demand side openness of the domestic economy
$1-\alpha^*$	0.2	Degree of demand side openness of the foreign economy
l_1	1.0	Income elasticity of money demand
l_2	1.5	Semi-interest elasticity of money demand
$1-\mu$	0.3	Degree of supply side openness of the domestic economy
$1-\mu^*$	0.2	Degree of supply side openness of the foreign economy
$1-\beta$	0.8	Degree of wage indexation in the domestic economy
$1-\beta^*$	0.4	Degree of wage indexation in the foreign economy
$\delta = \delta^*$	0.25	Output gap elasticity of wage inflation
σ	0.3	Elasticity of substitution between labor and oil
$1-\beta_R^*$	0.2	Degree of commodity price indexation to foreign inflation
δ_R^*	0.1	World output gap elasticity of commodity price inflation
λ_V	0.85	Weight of long run monetary change in the loss function V_3

Table 2: Parameter values

[16] Note that this result does not hold if β_R^* is zero and $\delta_R^* > 0$. In this special case q and $\dot{p}$ on impact rise by a large amount and fall strongly thereafter, where $\overline{q}_1$ lies below its initial pre-disturbance level.

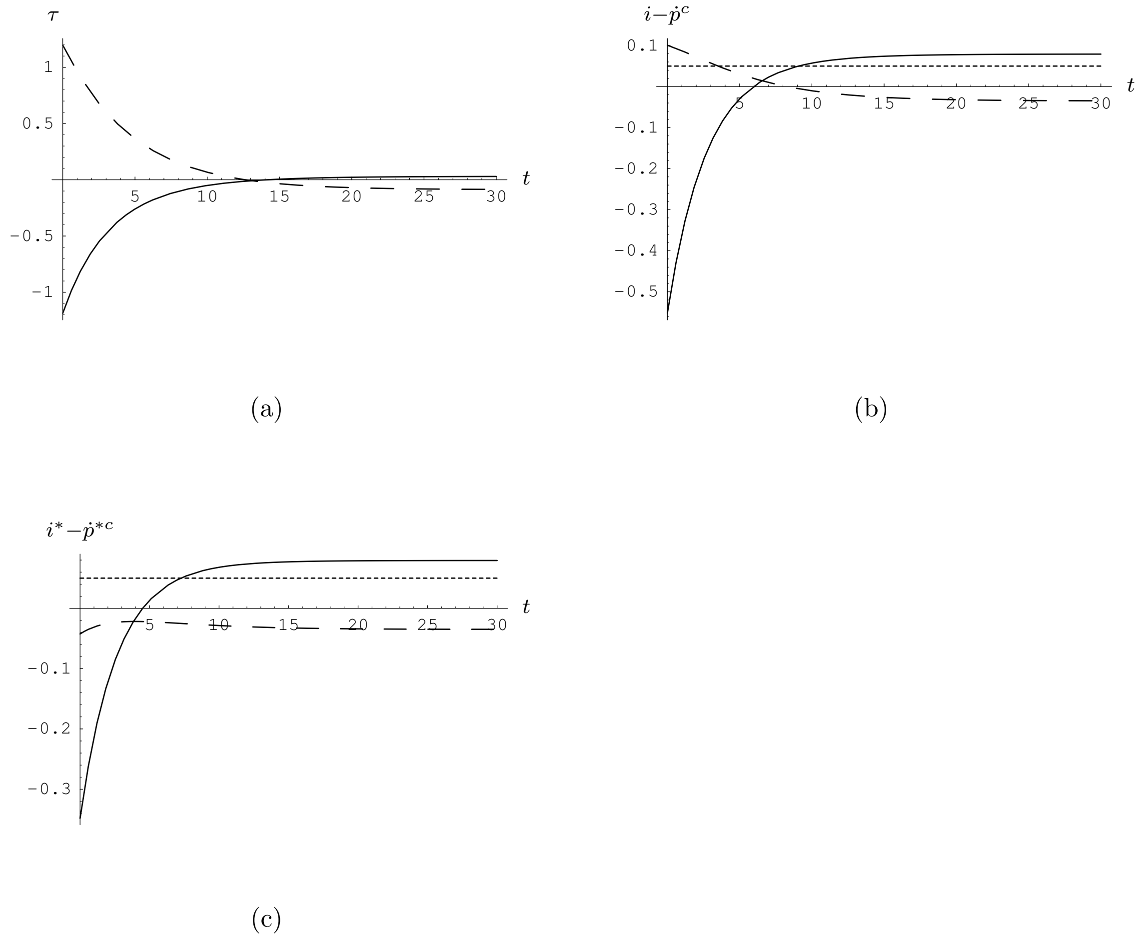


Figure 1: Responses of terms of trade τ, domestic real interest rate $i - \dot{p}^c$ and foreign real interest rate $i^* - \dot{p}^{*c}$ to an unanticipated domestic (**solid lines**) and to an unanticipated foreign (**dashed lines**) permanent expansionary monetary policy shock; initial steady states: **dotted lines**

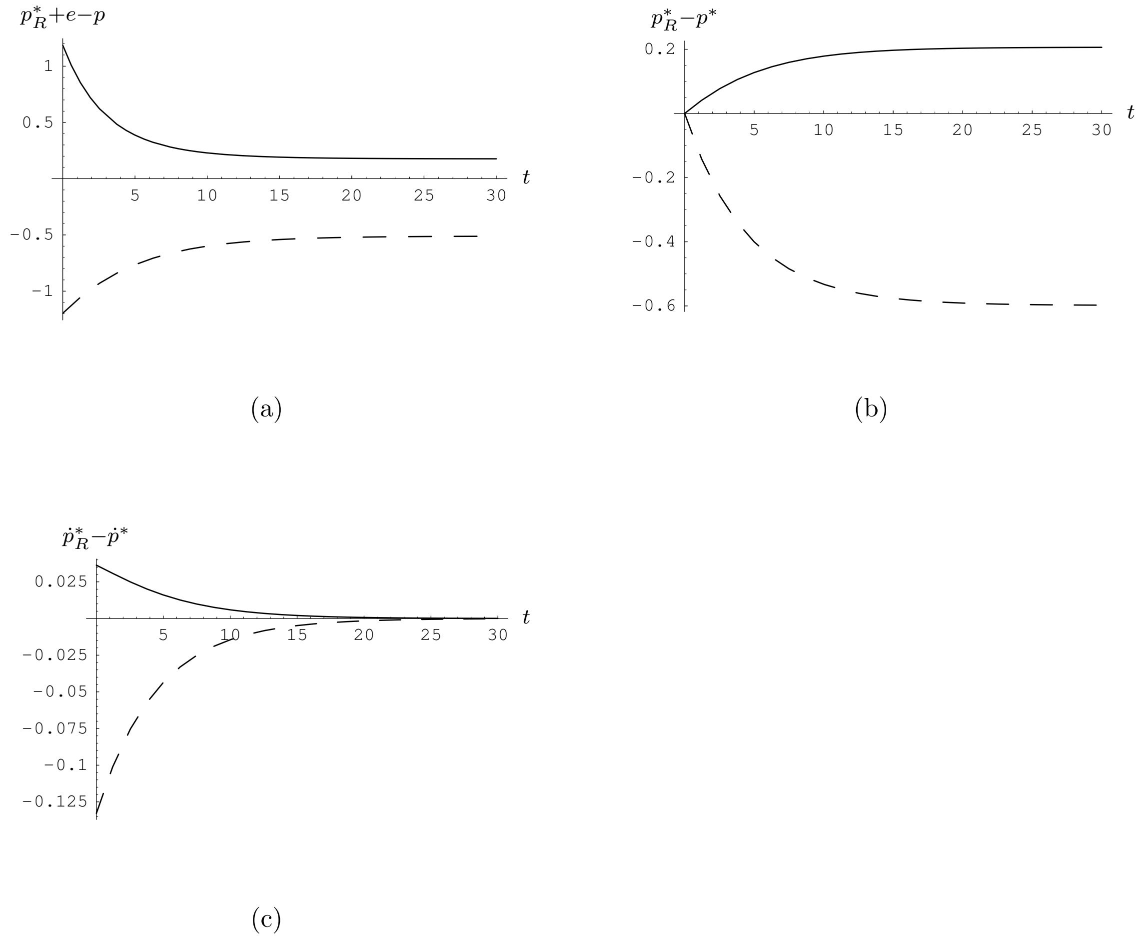


Figure 2: Responses of domestic real commodity price $p_R^* + e - p$, foreign real commodity price $p_R^* - p^*$ and growth rate of foreign real commodity price $\dot{p}_R^* - \dot{p}^*$ to an unanticipated domestic (**solid lines**) and to an unanticipated foreign (**dashed lines**) permanent expansionary monetary policy shock

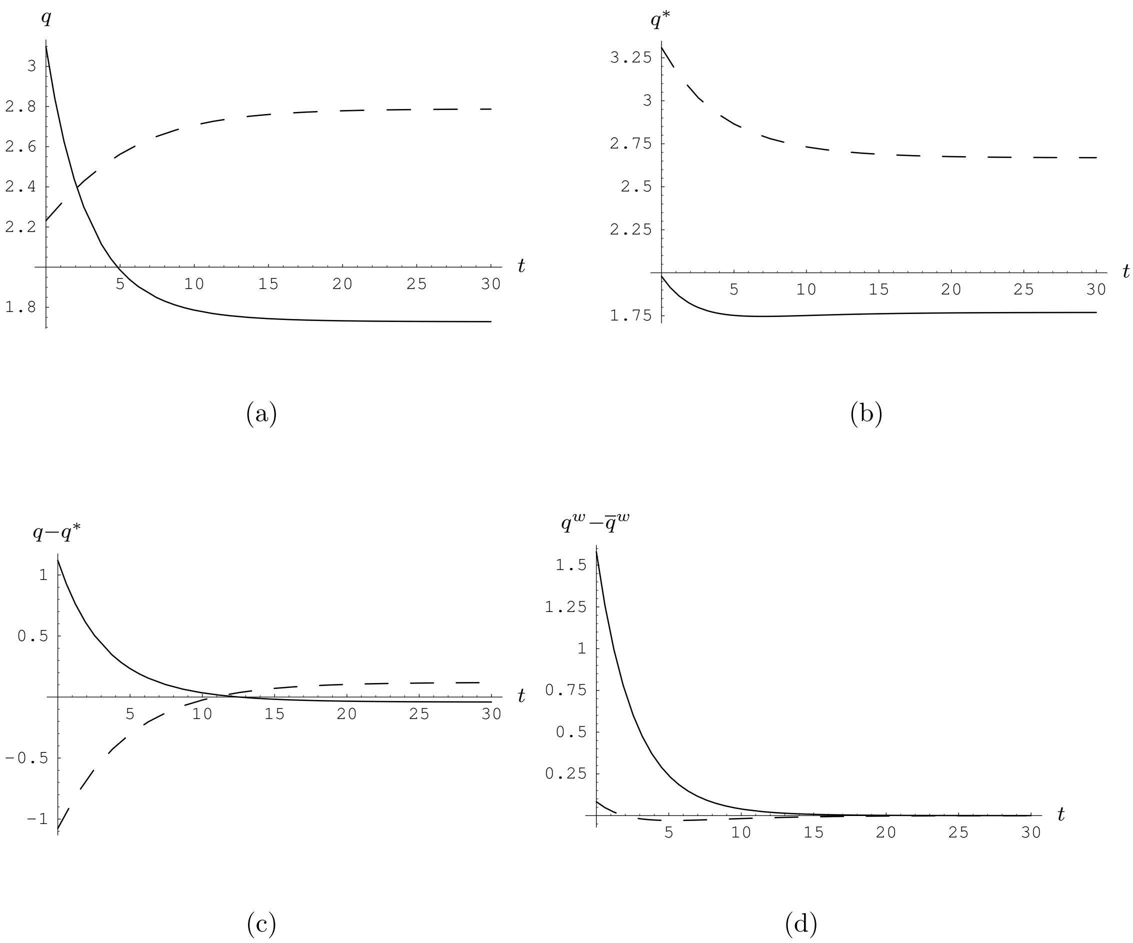


Figure 3: Responses of domestic output q, foreign output q^*, output differential $q - q^*$ and world output gap $q^w - \overline{q}^w$ to an unanticipated domestic (**solid lines**) and to an unanticipated foreign (**dashed lines**) permanent expansionary monetary policy shock

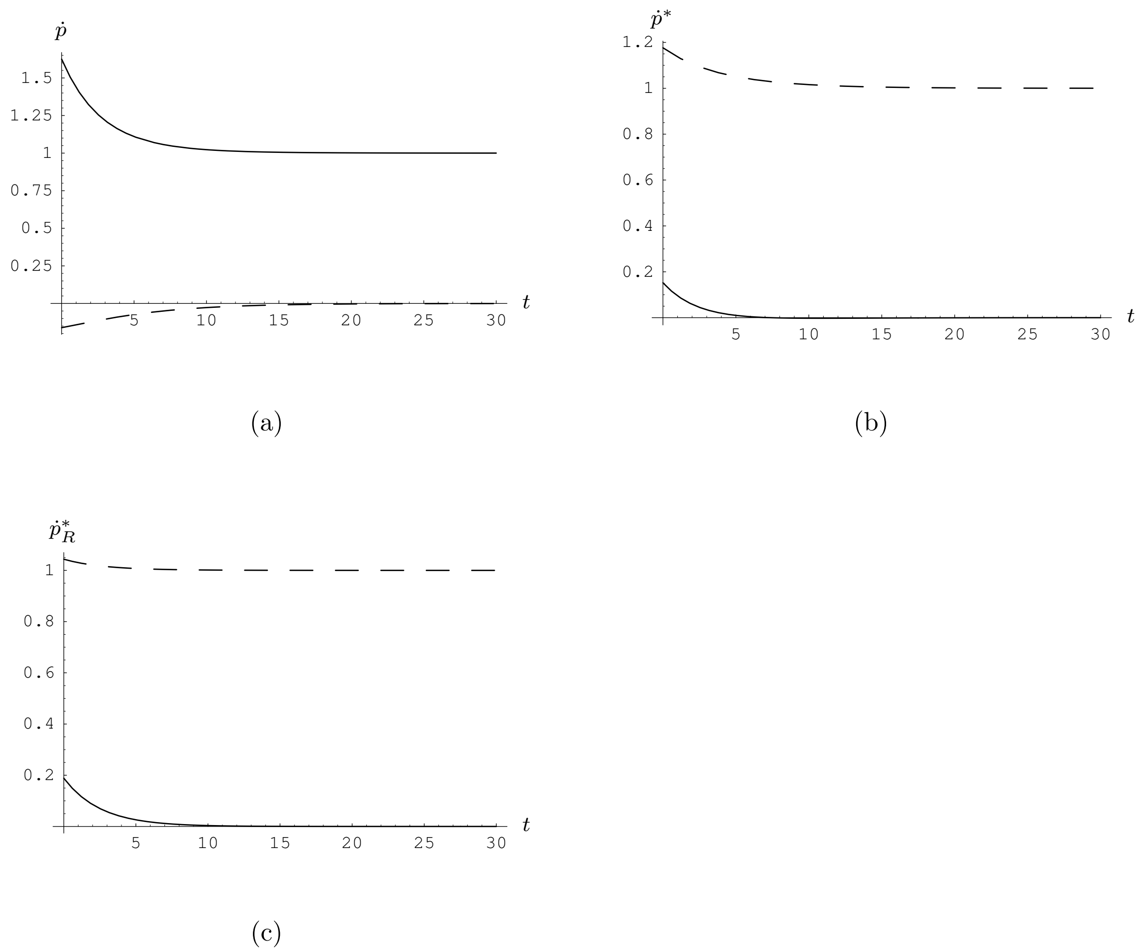


Figure 4: Responses of domestic inflation rate $\dot{p}$, foreign inflation rate $\dot{p}^*$ and growth rate nominal commodity price $\dot{p}_R^*$ to an unanticipated domestic (**solid lines**) and to an unanticipated foreign (**dashed lines**) permanent expansionary monetary policy shock

3.3 Dynamic Effects of Anticipated Monetary Policy

We now consider the behavior of the domestic and foreign economy in response to a unilateral monetary expansion which is credibly announced at time $t = 0$ to take place at some future time $T > 0$. We restrict the discussion to the response of the relevant variables during the anticipation phase $0 < t < T$, i.e., before the monetary shock actually occurs. The behavior of the economies for $t > T$ is very similar to the case $T = 0$ and need not be discussed once more. Note that the steady state effects in case of an anticipated monetary policy are in absolute terms smaller than in case of an equivalent unanticipated monetary shock.

The dynamic effects of an anticipated unilateral increase in the growth rate of domestic or foreign money supply are illustrated in figures 5 to 8. The solid lines show the response in case of domestic monetary policy ($d\dot{m} = 1$, $d\dot{m}^* = 0$), the dashed lines illustrate the dynamic behavior of the system if the growth rate of foreign money supply is increased ($d\dot{m}^* = 1$, $d\dot{m} = 0$). We first consider the dynamic effects of an anticipated rise in $\dot{m}$. As in the case of an unanticipated monetary policy the terms of trade τ fall on impact where the real depreciation of the domestic currency is now smaller than in case $T = 0$. If the time span between the anticipation and the implementation of the domestic monetary policy is sufficiently large the initial depreciation is slightly magnified until the end of the anticipation period. For $t > T$ the development of τ is equivalent to the case $T = 0$. The domestic and foreign real interest rate fall on impact and lie below their initial steady state level throughout the whole anticipation period $0 < t < T$. At the date of implementation of the expansionary monetary policy both interest rates decrease once more. For $t > T$ the behavior of the real interest rates is similar to the corresponding response in case $T = 0$. The foreign real factor price $p_R^* - p^*$, which is illustrated in figure 6 (b), continuously increases which already occurs before the implementation of the monetary policy. The reason is a deflationary process in the foreign economy throughout the time interval $0 < t < T$, while the rate of change of the commodity price is positive on impact. The growth rate $\dot{p}_R^* - \dot{p}^*$ of the foreign real factor price is therefore positive on impact and remains greater zero during the whole course of adjustment. The domestic real commodity price $p_R^* + e - p$ jumps on impact by the same amount as τ (but in the opposite direction) and increases further until time T (figure 6 (a)). Although the domestic inflation rate $\dot{p}$ is positive and increasing during the time span $0 < t < T$, and the rate of change of the commodity price ($\dot{p}_R^*$) is small, the depreciation rate $\dot{e}$ is so large that it induces a positive growth rate of the domestic real factor price $p_R^* + e - p$ for $0 < t < T$.[17] After

[17] Since $\dot{e}$ is positive and increasing over the interval $(0, T)$ this leads to rising inflation rates $\dot{p}$, $\dot{p}^c$ and $\dot{w}$ in the domestic economy and to deflation in the foreign economy. In contrast to domestic inflation, foreign deflation is weak since the depreciation rate $\dot{e}$ is not a direct argument of the foreign pricing rule (12).

the realization of the monetary shock at time T, $p_R^* + e - p$ begins to fall towards its new steady state level. As in the case $T = 0$, domestic output q increases on impact and falls thereafter until time T (figure 7 (a)). The behavior of q after the impact phase is a consequence of the rise in the domestic real commodity price and the domestic real interest rate during this time span. Moreover, foreign output declines on impact and thereafter until time T leading to a reduction in domestic exports of final goods. At the time of implementation of the expansionary monetary policy, both output variables increase (due to the fall in the real interest rates at time T). Thereafter, q and q^* behave as in the case $T = 0$, i.e. fall towards their new steady state levels.

The anticipation effects of an anticipated increase in the growth rate of *foreign* money supply with respect to the domestic terms of trade, the real factor prices, the real interest rates, the output variables and the inflation rates are opposite to the corresponding effects of an expansionary domestic monetary policy. The fall in the foreign real commodity price $p_R^* - p^*$ results from the positive foreign inflation rate $\dot{p}^*$ and the initial decrease in the rate of change of the commodity price, $\dot{p}_R^*$. Since the anticipation of a future rise in $\dot{m}^*$ is connected with a process of nominal depreciation of the foreign currency ($\dot{e} < 0$) this induces a deflationary process in the domestic economy and a reduction in the real factor price $p_R^* + e - p$ during the anticipation period. The domestic output q falls on impact which is a consequence of the rise in the domestic terms of trade and the domestic real interest rate. The decline in the real factor price $p_R^* + e - p$, the dampening of the initial rise in the interest rate $i - \dot{p}^c$ and the increase in foreign output cause a weak domestic output expansion after the impact phase. During the whole time span $0 < t < T$ domestic output lies below its initial steady state level which can be explained with the strong increase in τ. By contrast, foreign output q^* lies above its pre-disturbance equilibrium level $\overline{q}_0^*$, where the initial expansion is magnified until time T. The implementation of the foreign monetary policy shock generates positive jumps of the output variables which are a consequence of the discontinuous fall in the real interest rates at time T. Thereafter, foreign output converges from above to its new steady state level, whereas for $t > T$ the development of domestic output is generally not monotonic, but hump-shaped.[18] After a sharp fall, induced by the rise in the domestic real factor price and an overshooting of the domestic real interest rate, domestic output rises again and converges from below to its new steady state level.

[18] An exception is the case that β_R^* is sufficiently large and $\delta_R^* = 0$. The other polar case $\beta_R^* = 0$ and δ_R^* sufficiently large leads again to a hump-shaped development of q. This case causes extremely high jumps of q and q^* at time T, and thereafter a very strong contractionary process ending in lower steady state levels of $\overline{q}$ and $\overline{q}^*$ as initially.

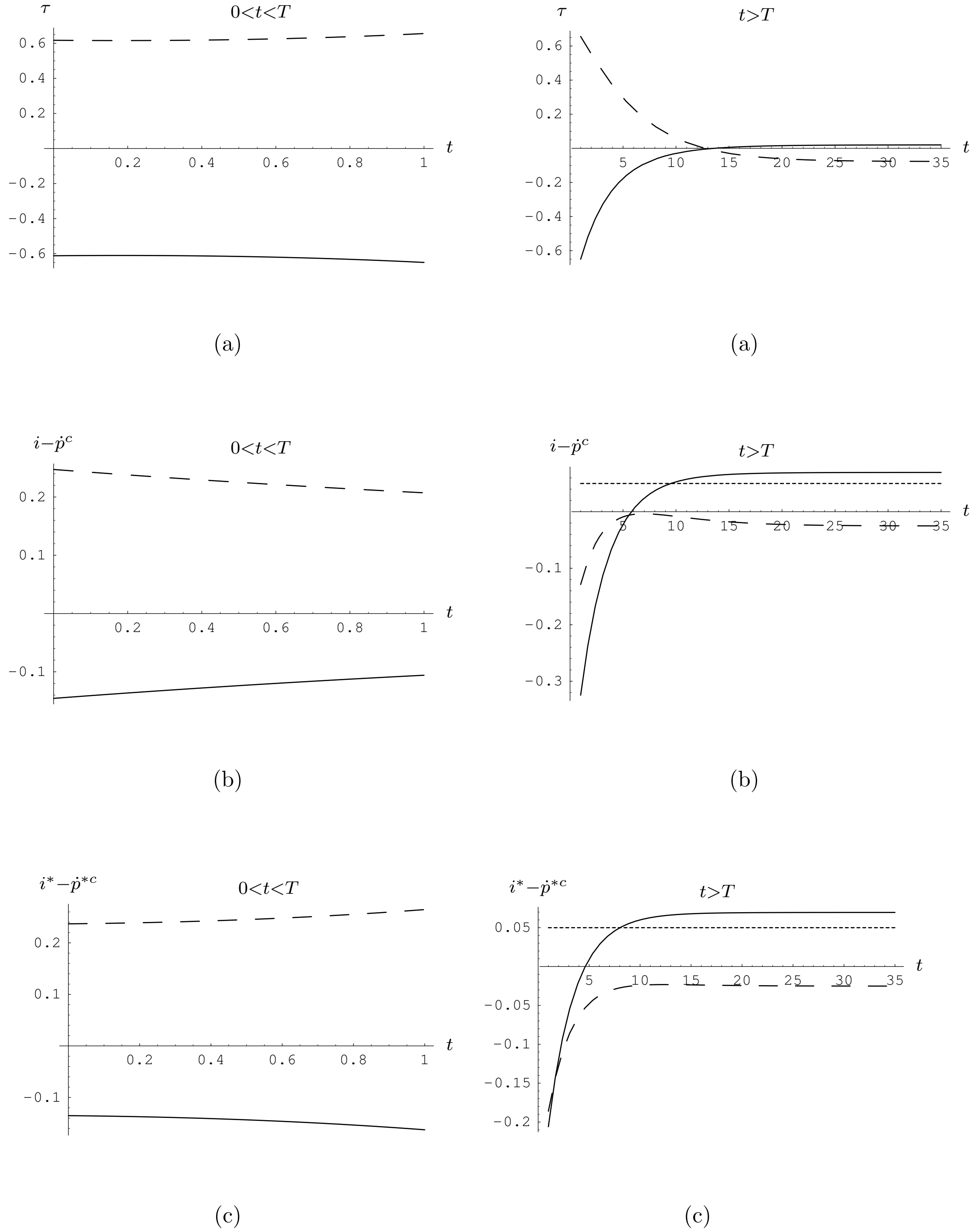
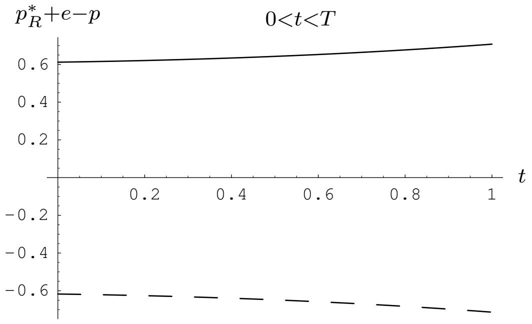
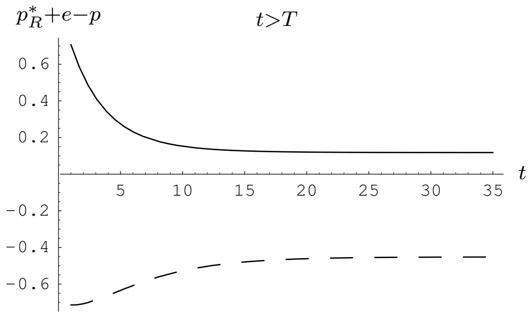


Figure 5: Responses of terms of trade τ, domestic real interest rate $i-\dot{p}^c$ and foreign real interest rate $i^*-\dot{p}^{*c}$ to an anticipated domestic (**solid lines**) and to an anticipated foreign (**dashed lines**) permanent expansionary monetary policy shock during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**); initial steady states: **dotted lines**



(a)



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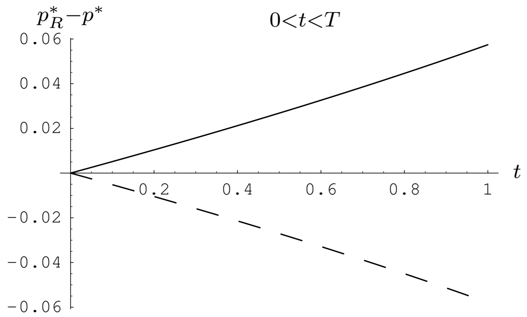

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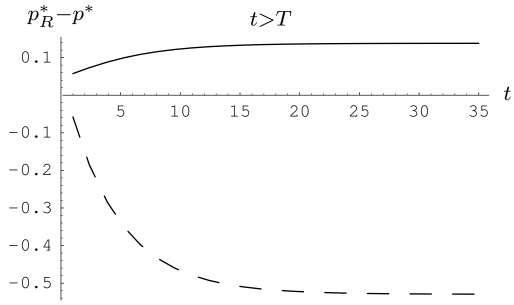

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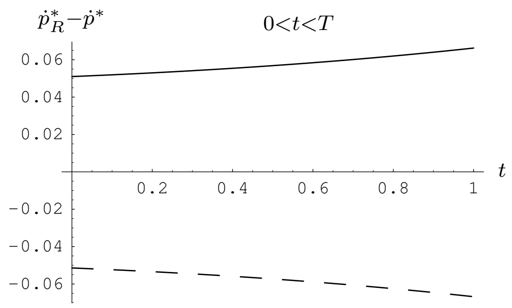

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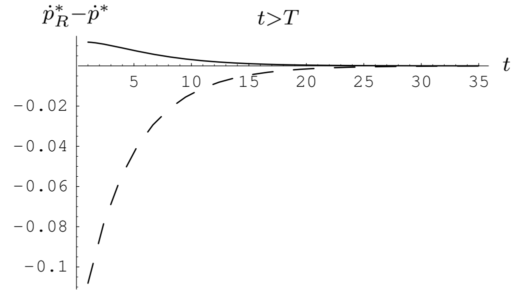


(c)

Figure 6: Responses of domestic real commodity price $p_R^* + e - p$, foreign real commodity price $p_R^* - p^*$ and growth rate of foreign real commodity price $\dot{p}_R^* - \dot{p}^*$ to an anticipated domestic (**solid lines**) and to an anticipated foreign (**dashed lines**) permanent expansionary monetary policy shock during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

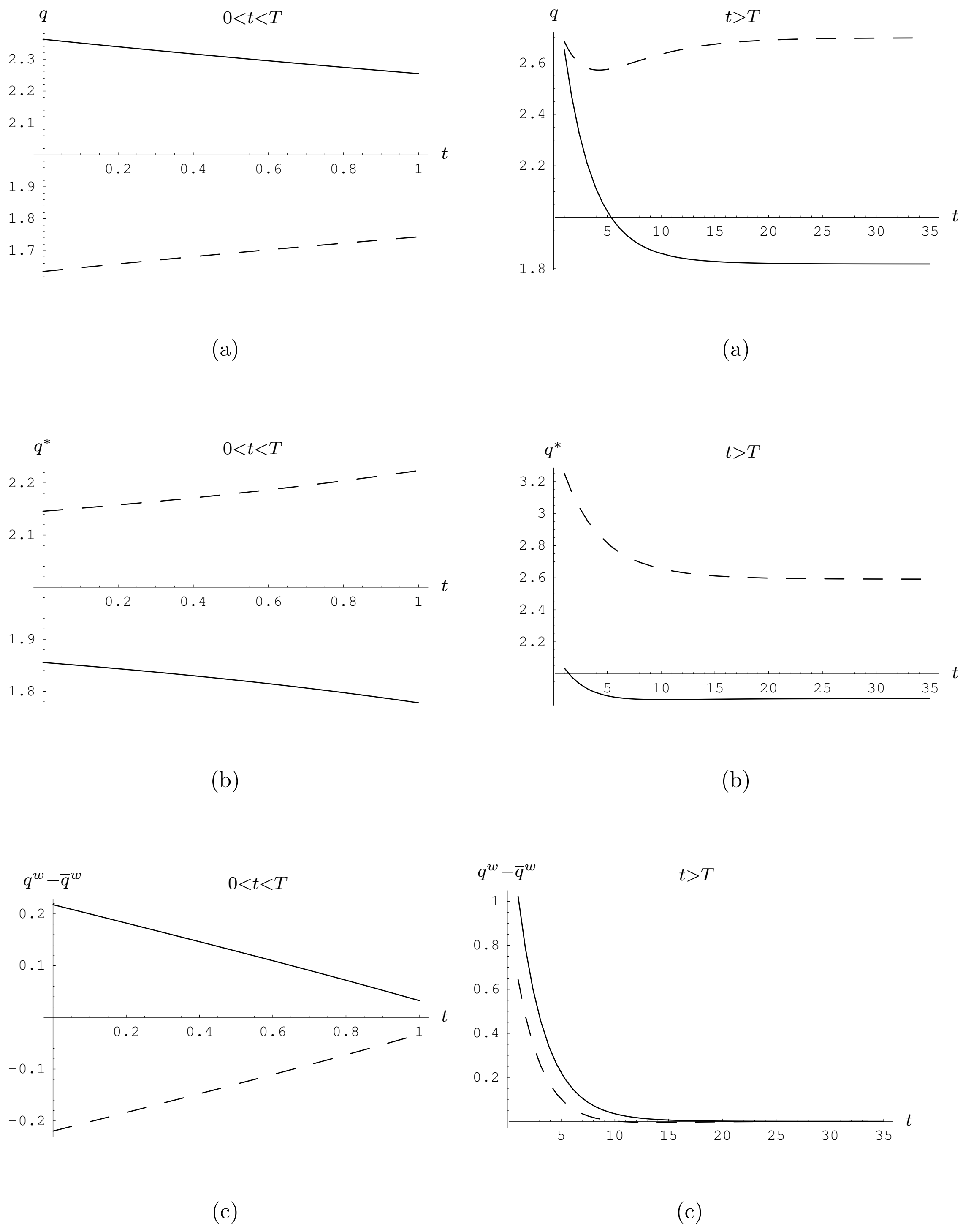


Figure 7: Responses of domestic output q, foreign output q^* and world output gap $q^w - \overline{q}^w$ to an anticipated domestic (**solid lines**) and to an anticipated foreign (**dashed lines**) permanent expansionary monetary policy shock during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

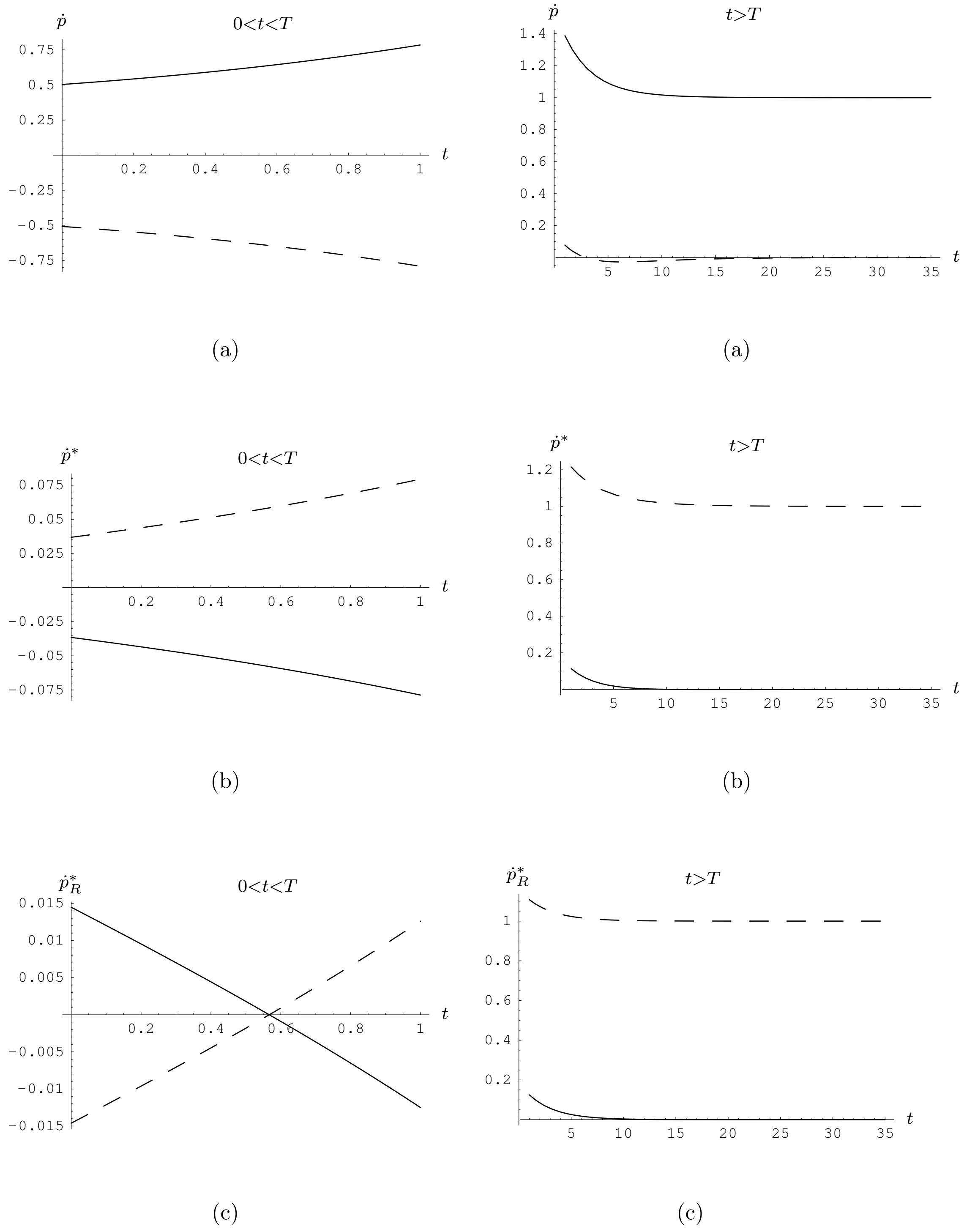


Figure 8: Responses of domestic inflation rate $\dot{p}$, foreign inflation rate $\dot{p}^*$ and growth rate of nominal commodity price $\dot{p}_R^*$ to an anticipated domestic (**solid lines**) and to an anticipated foreign (**dashed lines**) permanent expansionary monetary policy shock during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

4 Commodity Price Shocks and Monetary Policy Response

In this section we discuss at first briefly the dynamic effects of an exogenous commodity price shock (like an anticipated increase in the oil price). We then analyze the consequences of monetary policy reactions that could be employed in an effort to reduce the potentially disruptive effects of oil price shocks.

4.1 Dynamic Effects of Oil Price Shocks

It is well known that a commodity price increase ($dp_R^* > 0$) causes long run stagflation for oil-dependent economies (that is, $d\overline{q} < 0$, $d\overline{p} > 0$, $d\overline{q}^* < 0$, $d\overline{p}^* > 0$) and balance of trade problems.[19] The strength of the output contraction and the price increase depends on the degree of oil-dependency (i.e., the values of the coefficients ψ, $1 - \mu$ and f_2 in the model equations (9), (11) and (19)). On the assumption $\psi > \psi^*$, $\mu < \mu^*$ and $f_2 > f_2^*$ the stagflationary effects of a commodity price shock are stronger for the domestic than for the foreign economy. It can be shown that the contractionary steady state effects of oil price shocks can be reduced for both economies if oil imports are denominated in terms of the domestic rather than the foreign currency (Wohltmann and Winkler (2005 a)).

The dynamic effects of an anticipated increase in the price of oil ($dp_R^* > 0$) are illustrated in figures 9 to 13. The solid lines show the behavior of the domestic and foreign economy in the polar case $\beta_R^* = 1$, $\delta_R^* = 0$, the dashed lines illustrate the response of the system if $\beta_R^* < 1$ and $\delta_R^* > 0$ (but sufficiently small). Assuming constant values of the domestic and foreign money stock ($\dot{m} = \dot{m}^* = 0$) the case $\beta_R^* = 1$, $\delta_R^* = 0$ is equivalent to a zero growth rate of the commodity price ($\dot{p}_R^* = 0$), while in the case $\beta_R^* < 1$, $\delta_R^* > 0$ the rate of change $\dot{p}_R^*$ is a positive function of the foreign inflation rate and the world output gap.[20] The anticipation effects of a unit increase in the commodity price p_R^* are in qualitative terms identical to the corresponding impacts of a unit rise in the growth rate of domestic money supply ($d\dot{m} = 1$). In quantitative terms, the anticipation effects of the price shock $dp_R^* = 1$ are smaller (in particular the initial jumps of τ, q, q^* and the interest rates). On the other hand, the steady state effects with respect to real output and relative prices are considerably larger than in case $d\dot{m} = 1$. The reason is that at time T the real factor prices $p_R^* + e - p$ and $p_R^* - p^*$ behave discontinuously and increase by the amount $dp_R^* = 1$ while they behave sluggishly in case of a monetary shock. Since the commodity price shock is by assumption of permanent nature, the steady state

[19] Bhandari (1981), Bhandari and Turnovsky (1984), Wohltmann and Winkler (2005 a).

[20] The case $\dot{p}_R^* = 0$ is discussed at length in Wohltmann and Winkler (2005 a).

increase in the real factor prices is stronger than in case $d\dot{m} = 1$. The figures (9) to (13) illustrate that the steady state effects of the oil price shock also depend on the coefficients β_R^* and δ_R^* in the commodity pricing rule (15). The stagflationary as well as the relative price effects in absolute terms increase if β_R^* decreases and/or δ_R^* increases. Since the world output gap $q^w - \overline{q}^w$ and the foreign inflation rate $\dot{p}^*$ are positive for $t > T$, a decreasing β_R^* and/or an increasing δ_R^* lead to a rising positive growth rate of the commodity price at time T inducing higher inflation rates in T and larger steady state levels of the real factor prices and the domestic and foreign price levels. Moreover, in this case the fall in the output variables and the inflation rates in the time span after the realization of the oil price shock is much stronger. A further aspect is that a falling β_R^* and/or a rising δ_R^* increases the dependency of the rate of change of the commodity price on the jump variables $\dot{p}^*$ and $q^w - \overline{q}^w$ so that the volatility of the system in the time span after the realization of the oil price shock is magnified. The change of the output variables and the inflation rates is now much higher, whereas the time path of the terms of trade τ is characterized by a delayed overshooting which increases with falling β_R^* and/or increasing δ_R^*.

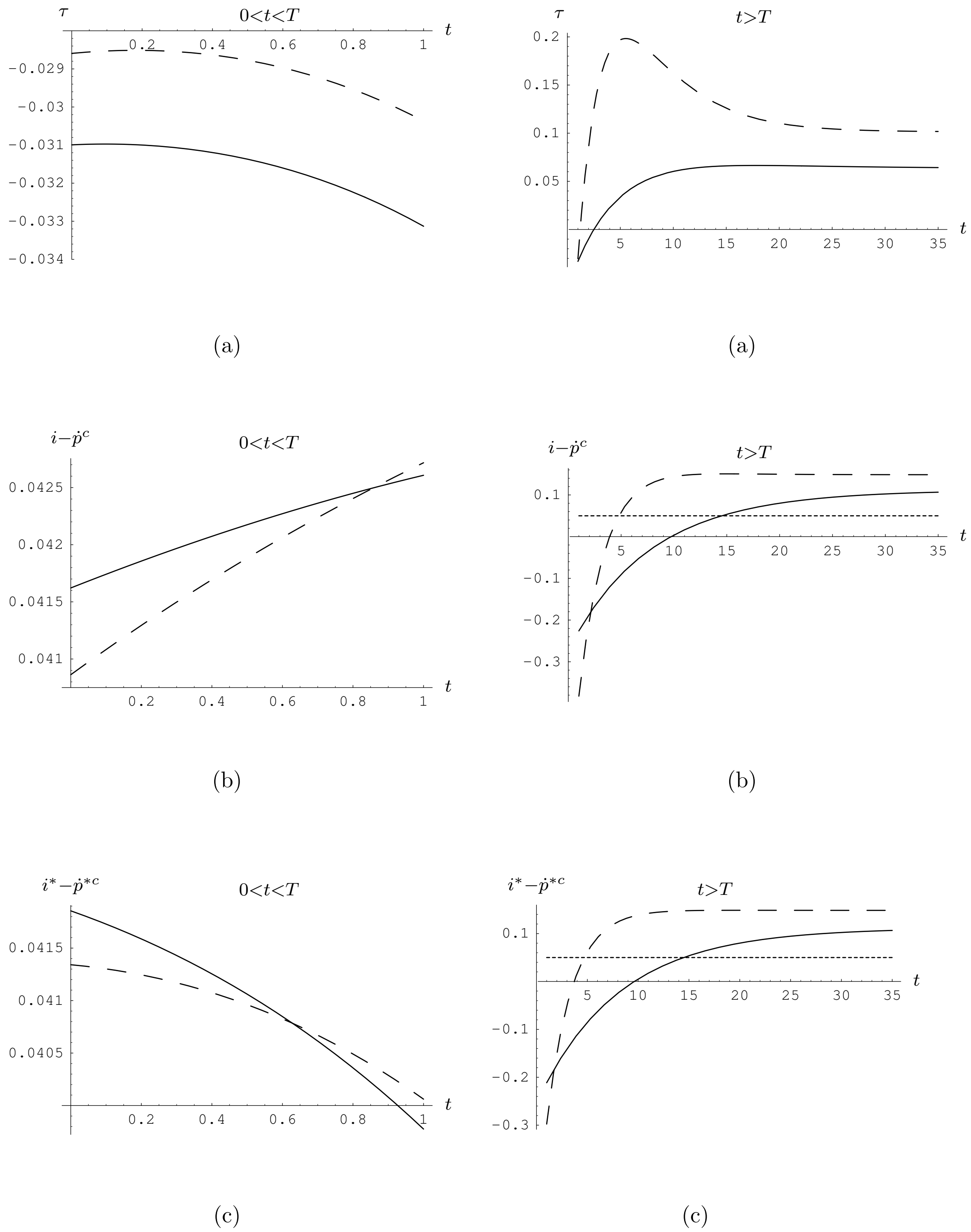
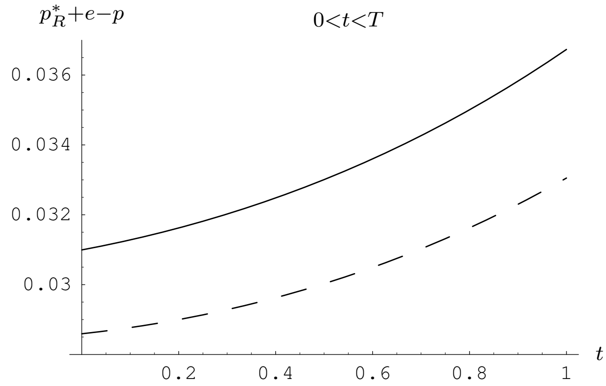
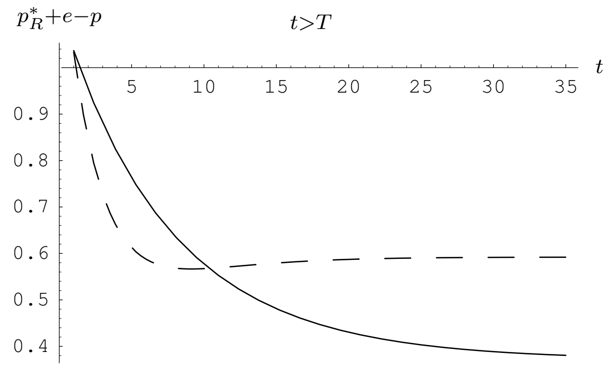


Figure 9: Responses of terms of trade τ, domestic real interest rate $i - \dot{p}^c$ and foreign real interest rate $i^* - \dot{p}^{*c}$ to an anticipated permanent commodity price shock in the case $\beta_R^* = 1$, $\delta_R^* = 0$ (**solid lines**) and in the case $\beta_R^* < 1$, $\delta_R^* > 0$ (**dashed lines**) during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**); initial steady states: **dotted lines**



(a)



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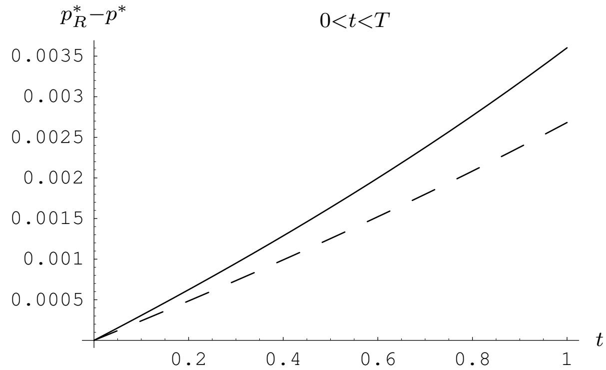

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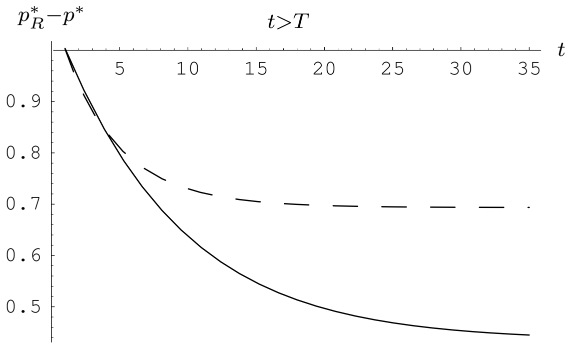

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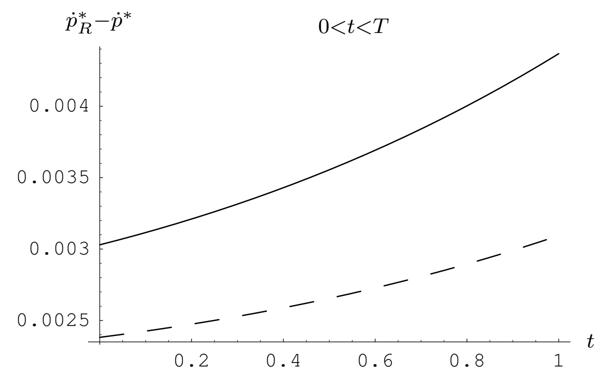
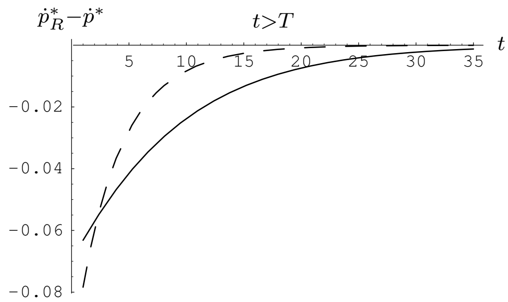
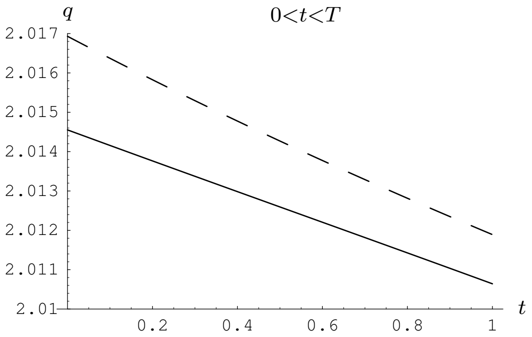


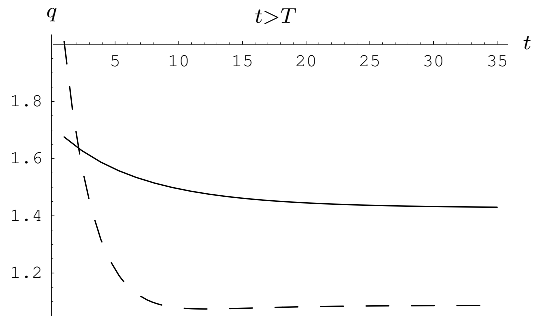
(c)



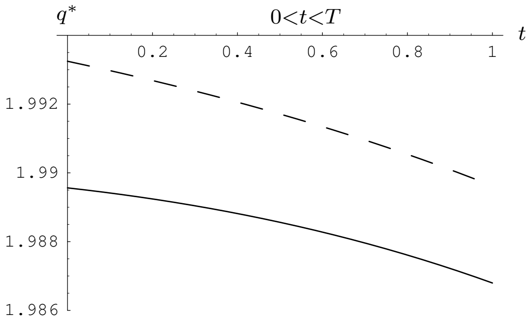
(c)

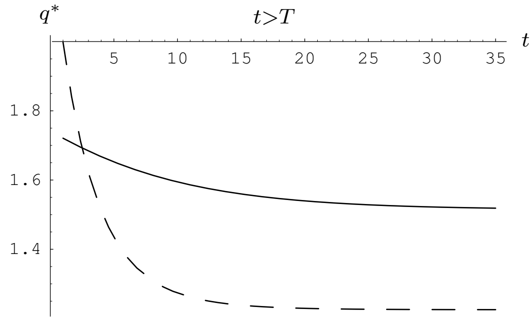
Figure 10: Responses of domestic real commodity price p_R^*+e-p, foreign real commodity price $p_R^* - p^*$ and growth rate of foreign real commodity price $\dot{p}_R^* - \dot{p}^*$ to an anticipated permanent commodity price shock in the case $\beta_R^* = 1$, $\delta_R^* = 0$ (**solid lines**) and in the case $\beta_R^* < 1$, $\delta_R^* > 0$ (**dashed lines**) during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)





(a) (a)





(b) (b)

Figure 11: Responses of domestic and foreign output q and q^* to an anticipated permanent commodity price shock in the case $\beta_R^* = 1$, $\delta_R^* = 0$ (**solid lines**) and in the case $\beta_R^* < 1$, $\delta_R^* > 0$ (**dashed lines**) during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

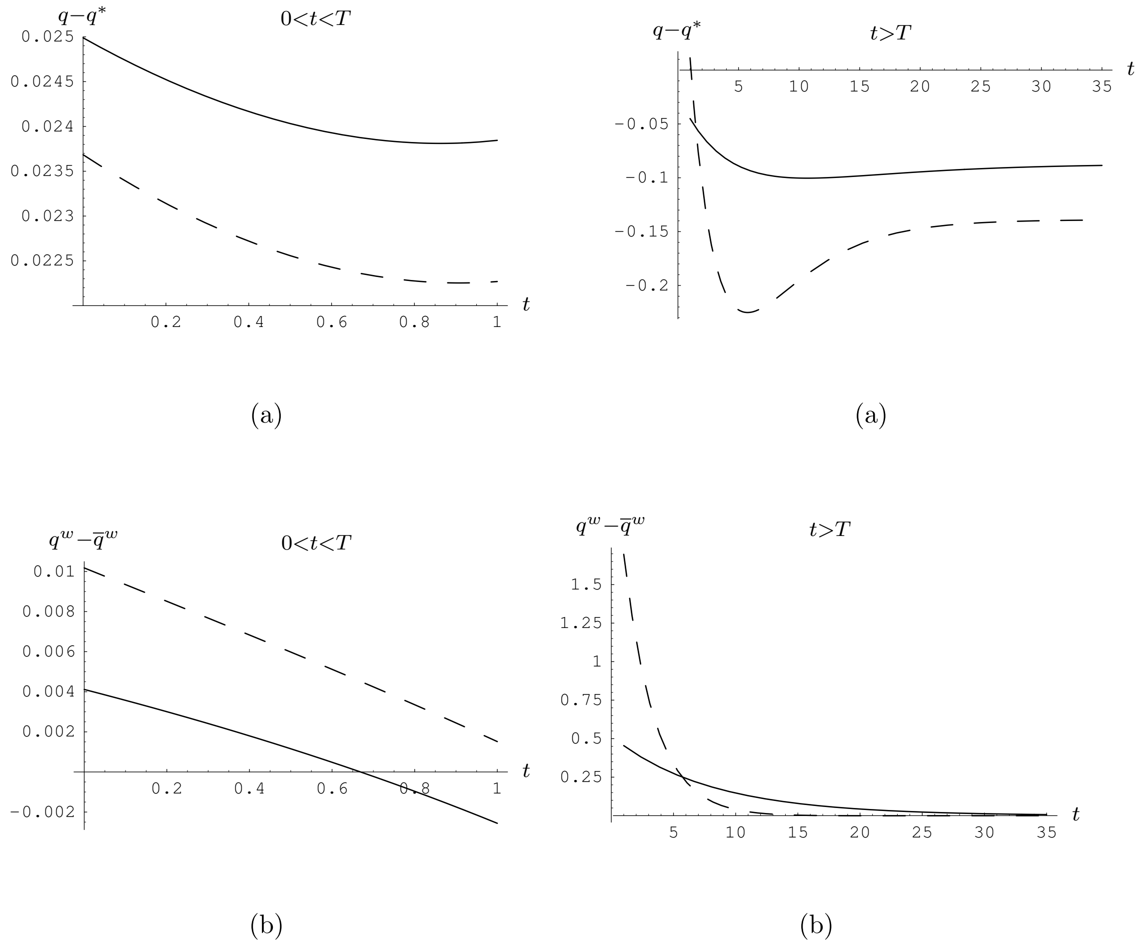


Figure 12: Responses of output differential $q - q^*$ and world output gap $q^w - \overline{q}^w$ to an anticipated permanent commodity price shock in the case $\beta_R^* = 1$, $\delta_R^* = 0$ (**solid lines**) and in the case $\beta_R^* < 1$, $\delta_R^* > 0$ (**dashed lines**) during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

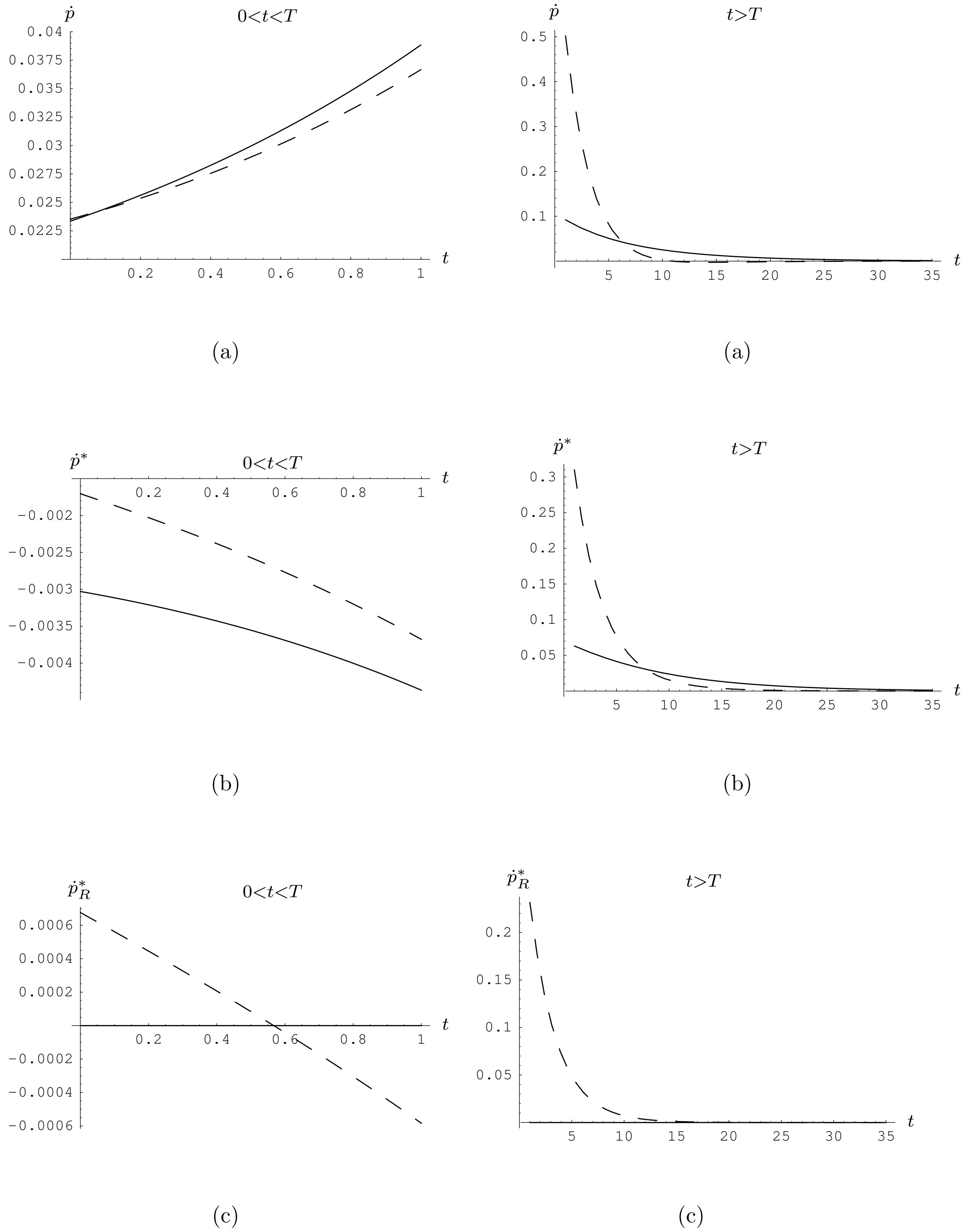


Figure 13: Responses of domestic inflation rate $\dot{p}$, foreign inflation rate $\dot{p}^*$ and growth rate of nominal commodity price $\dot{p}_R^*$ to an anticipated permanent commodity price shock in the case $\beta_R^* = 1$, $\delta_R^* = 0$ (**solid lines**) and in the case $\beta_R^* < 1$, $\delta_R^* > 0$ (**dashed lines**) during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

4.2 Stabilization of the Inflation Rates

Since a once-and-for-all increase in the commodity price p_R^* is connected with temporary inflation which may be substantial if the growth rate of p_R^* follows a rule similar to a wage Phillips equation, the question arises whether a coordination of domestic and foreign monetary policy is able to stabilize the inflation rates at their initial pre-disturbance level. In the following we analyze the dynamic effects of monetary policy rules that are employed by the central bank in an effort to fix the consumer inflation rates $\dot{p}^c$ and $\dot{p}^{*c}$ at their respective initial steady state level $\overline{\dot{p}^c}_0 = \overline{\dot{p}^{*c}}_0 = 0$ at all times. This inflationary target leads to the decision rules

$$\dot{m} = (1-\alpha)\dot{\tau} + \frac{1}{2}\dot{l}^s + \frac{1}{2}\dot{l}^d \tag{26}$$

$$\dot{m}^* = -(1-\alpha^*)\dot{\tau}^* + \frac{1}{2}\dot{l}^s - \frac{1}{2}\dot{l}^d \tag{27}$$

(Wohltmann and Winkler (2005 b)). Since the commodity price shock $dp_R^* > 0$ causes temporary inflation in both economies during the time span $T < t < \infty$, the growth rates of domestic and foreign money stock resulting from the policy rules (26) and (27) are negative over this time interval (figures 14 (a) and 15 (a)). Domestic monetary policy is stronger contractionary than foreign monetary policy immediately after the oil price shock (i.e., $\dot{m}(T+) < \dot{m}^*(T+) < 0$). This holds since in the short run after the realization of an isolated commodity price shock the inflation rate based on the consumer price index is higher in the domestic than in the foreign economy ($\dot{p}^c(T+) > \dot{p}^{*c}(T+)$). Fixing the consumer inflation rates at their respective pre-disturbance steady state level at all times requires the implementation of the policy rules (26) and (27) at the time of anticipation of the materials price shock, i.e., at time $t = 0$. An isolated and anticipated commodity price shock causes domestic inflation and foreign deflation during the anticipation phase $0 < t < T$ (figure 13 (a) and 13 (b)). To avoid this, the monetary policy rules must already be implemented at time $t = 0$. Figure 14 (a) illustrates that the growth rate of domestic money supply is positive during the time span $0 < t < T$. This holds since the anticipation of a strong contractionary monetary policy in response to the realization of an oil price shock induces deflation over the time interval $0 < t < T$. The disinflation is stronger than the weak inflationary effects caused by the anticipation of a future commodity price shock so that the overall effect on the domestic consumer inflation rate $\dot{p}^c$ is negative. To remove disinflation an expansionary domestic monetary policy over the time interval $0 < t < T$ is necessary. By contrast, foreign monetary policy is both contractionary for $t < T$ and $t > T$, that is, before and after the occurrence of the oil price shock. The growth rate of foreign money supply is also negative for $t < T$, since the anticipation of a relatively strong contractionary domestic monetary policy which is realized immediately after the occurrence of the commodity price shock, causes inflation in the foreign economy over the time interval $0 < t < T$. These inflationary effects are stronger than the overall disinflationary effects induced by the anticipation of the oil price shock and the contractionary foreign monetary policy at time T. Therefore, $\dot{m}^*$ must also be negative over the time interval $0 < t < T$.[21] The implementation of the monetary policy rules (26) and (27) at time $t = 0$ induces a complete fixing of the consumer inflation rates at their respective initial steady state level $\overline{\dot{p}^c}_0 = \overline{\dot{p}^{*c}}_0 = 0$ and reduces substantially the growth rates of the domestic and foreign price level and the factor prices ($\dot{p}$, $\dot{p}^*$, $\dot{w}$, $\dot{w}^*$, $\dot{p}^*_R$) (figures 19 and 20). Moreover, the permanent rise in the domestic and foreign price level induced by the realization of the oil price shock is substantially weakened (figure 21). The implementation of the monetary policy rules causes a long run fall in the domestic and foreign money stock ($d\overline{m} < 0$, $d\overline{m}^* < 0$) inducing a strong reduction in the steady state rise in the domestic and foreign price variables.

Although inflation targeting requires contractionary monetary policies in the sense of negative monetary growth rates $\dot{m}$ and $\dot{m}^*$ for $T < t < \infty$ and a permanent fall in the domestic and foreign money stock, this does not lead to a magnification of the steady state output contraction of an isolated oil price shock. On the contrary, the output contraction is slightly reduced which is a consequence of the long run output expansion caused by a contractionary domestic monetary policy. These effects are marginally larger than the contractionary output effects of the foreign monetary policy.[22] Moreover, inflation targeting also has output stabilizing effects, i.e. weakens the dynamic output effects of an isolated oil price shock (figure 18). With inflation targeting the deviation of domestic and foreign output from its respective initial steady state level during the anticipation phase is smaller than in case of a passive monetary policy. Furthermore, the output volatility after the realization of the commodity price increase is reduced.

[21] Note that in the case of weak anticipation effects of the domestic monetary policy rule the growth rate of foreign money supply is positive for $0 < t < T$, while $\dot{m}$ is negative during this time span. This special case is possible if the commodity pricing rule is characterized by a very high value of β^*_R (in particular $\beta^*_R = 1$) and a small value of δ^*_R. The inflationary effects of an isolated oil price increase are then relatively small.

[22] The monetary policy rules (26) and (27) imply $d\overline{m} < d\overline{m}^* < 0$. Although the long run output multipliers of domestic monetary policy are smaller (in absolute terms) than the corresponding multipliers of foreign monetary policy, the overall output effect is nearly zero since in the long run the domestic money stock is stronger reduced than the foreign money stock.

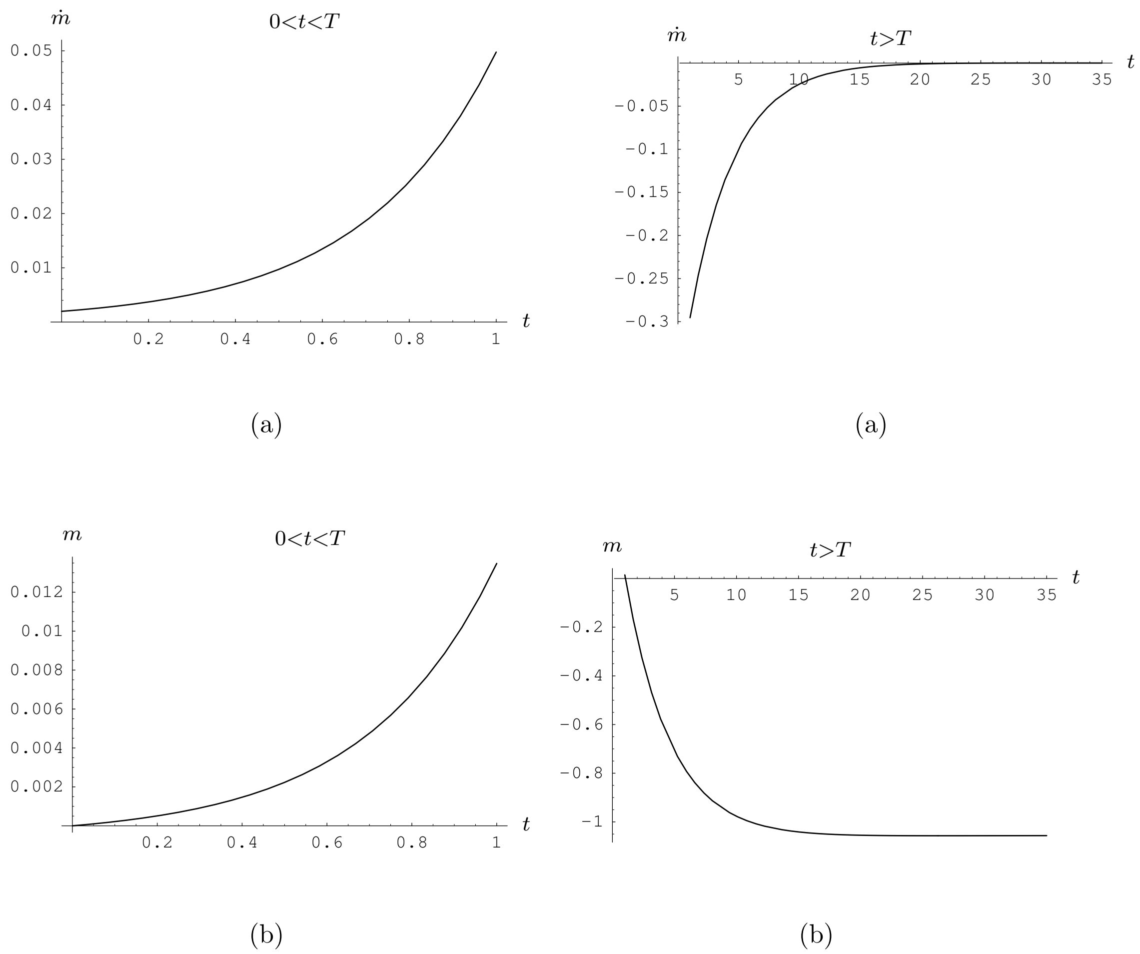


Figure 14: Growth rate of domestic money supply $\dot{m}$ needed to achieve the inflation target $\dot{p}^c = 0 = \dot{p}^{*c}$ due to an anticipated commodity price shock and the resulting development of domestic money stock m during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

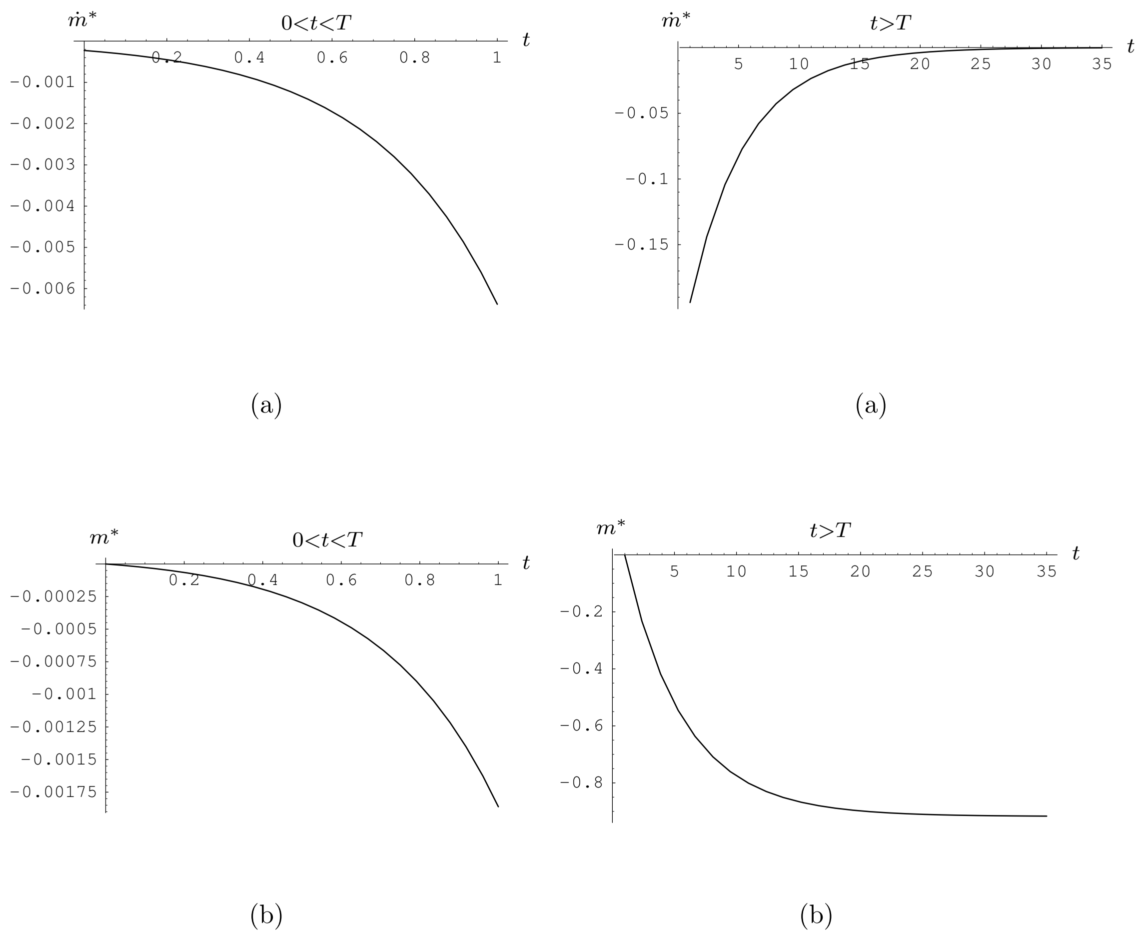


Figure 15: Growth rate of foreign money supply $\dot{m}^*$ needed to achieve the inflation target $\dot{p}^c = 0 = \dot{p}^{*c}$ due to an anticipated commodity price shock and the resulting development of foreign money stock m^* during the anticipation phase $0 < t < T$ (**left**) and during the implementation phase $t > T$ (**right**)

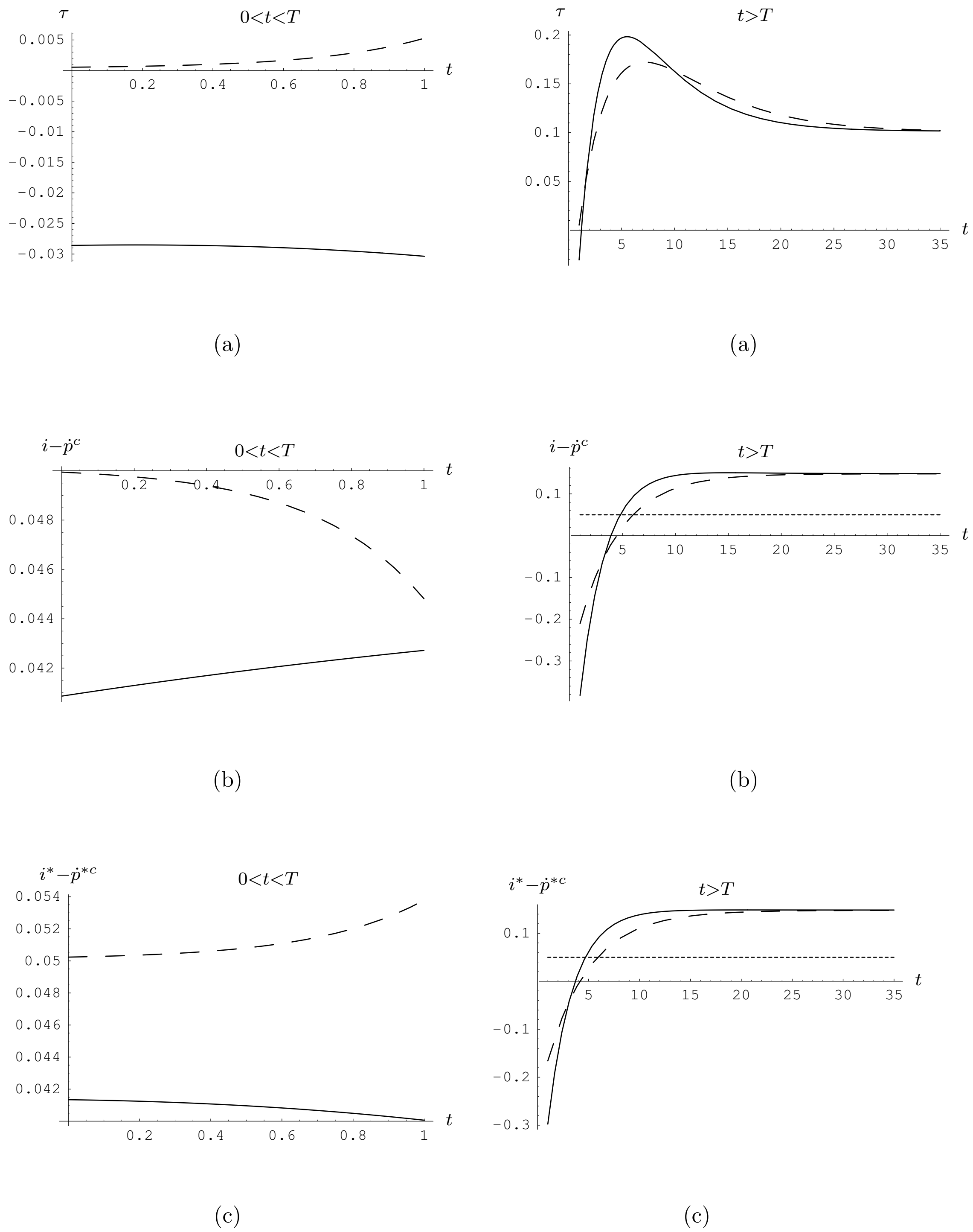


Figure 16: Responses of terms of trade τ, domestic real interest rate $i - \dot{p}^c$ and foreign real interest rate $i^* - \dot{p}^{*c}$ to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated oil price shock with an inflation targeting monetary policy (**dashed lines**); initial steady states: **dotted lines**

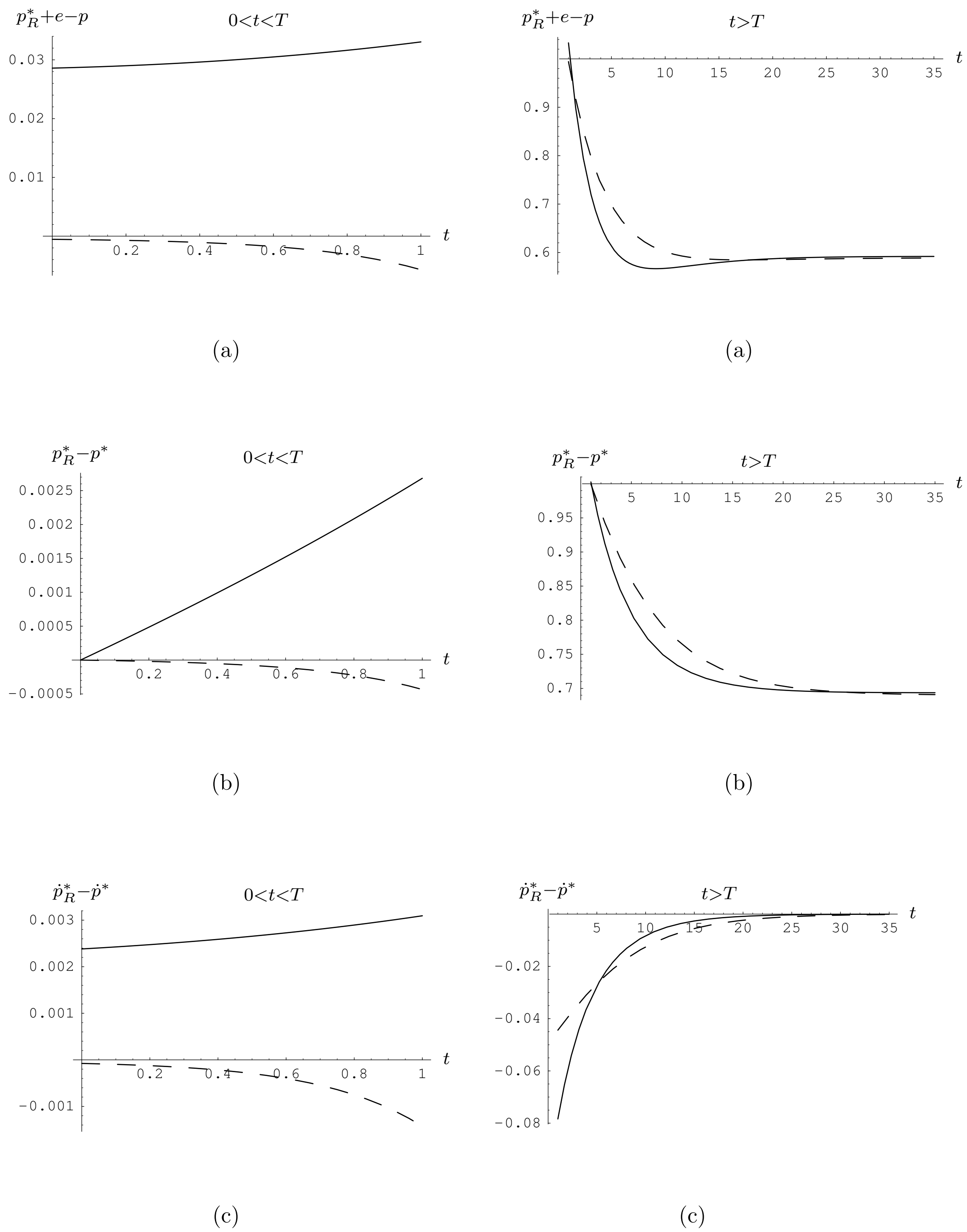


Figure 17: Responses of domestic real commodity price p_R^*+e-p, foreign real commodity price $p_R^* - p^*$ and growth rate of foreign real commodity price $\dot{p}_R^* - \dot{p}^*$ to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated oil price shock with an inflation targeting monetary policy (**dashed lines**)

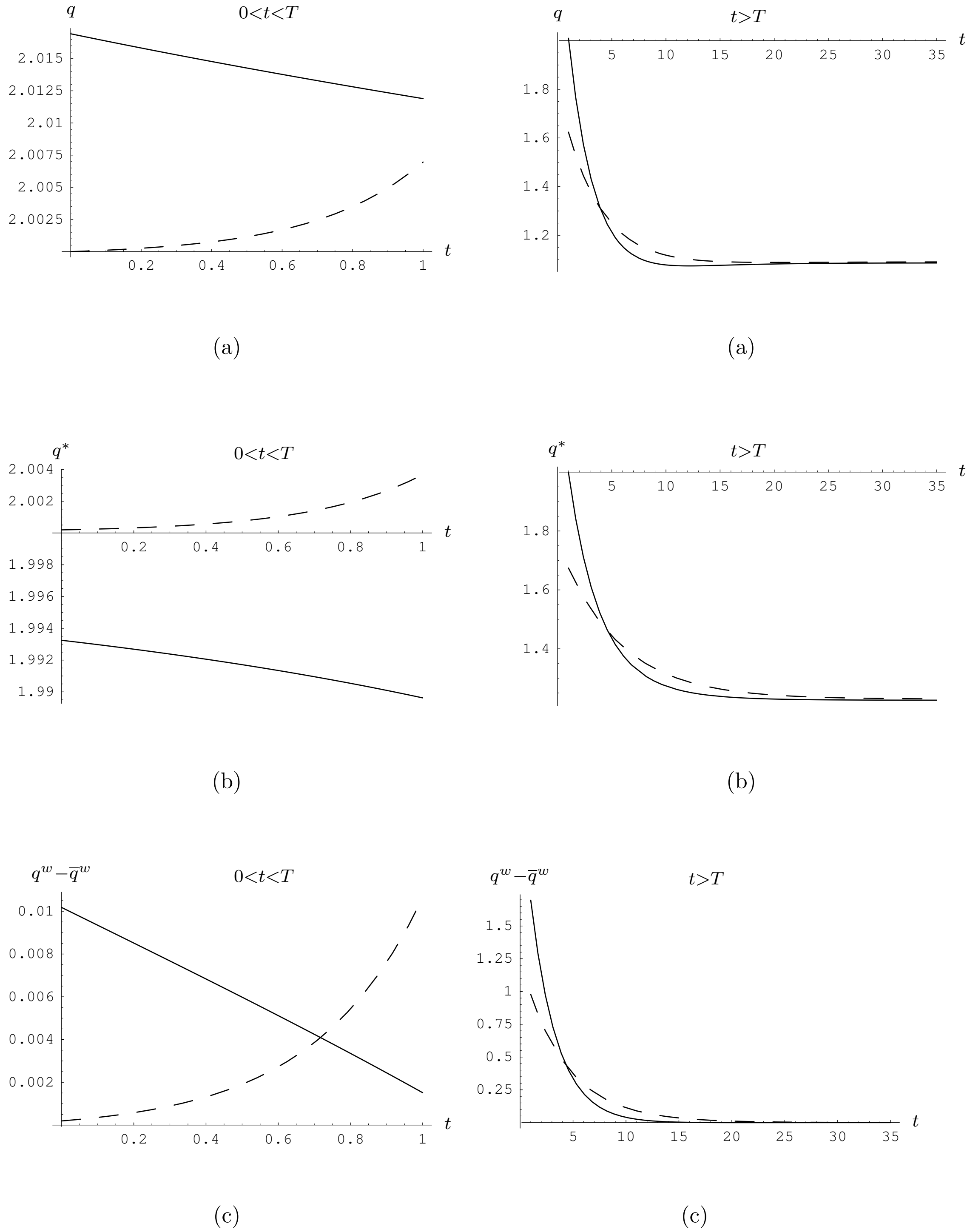


Figure 18: Responses of domestic output q, foreign output q^* and output differential $q-q^*$ to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated oil price shock with an inflation targeting monetary policy (**dashed lines**)

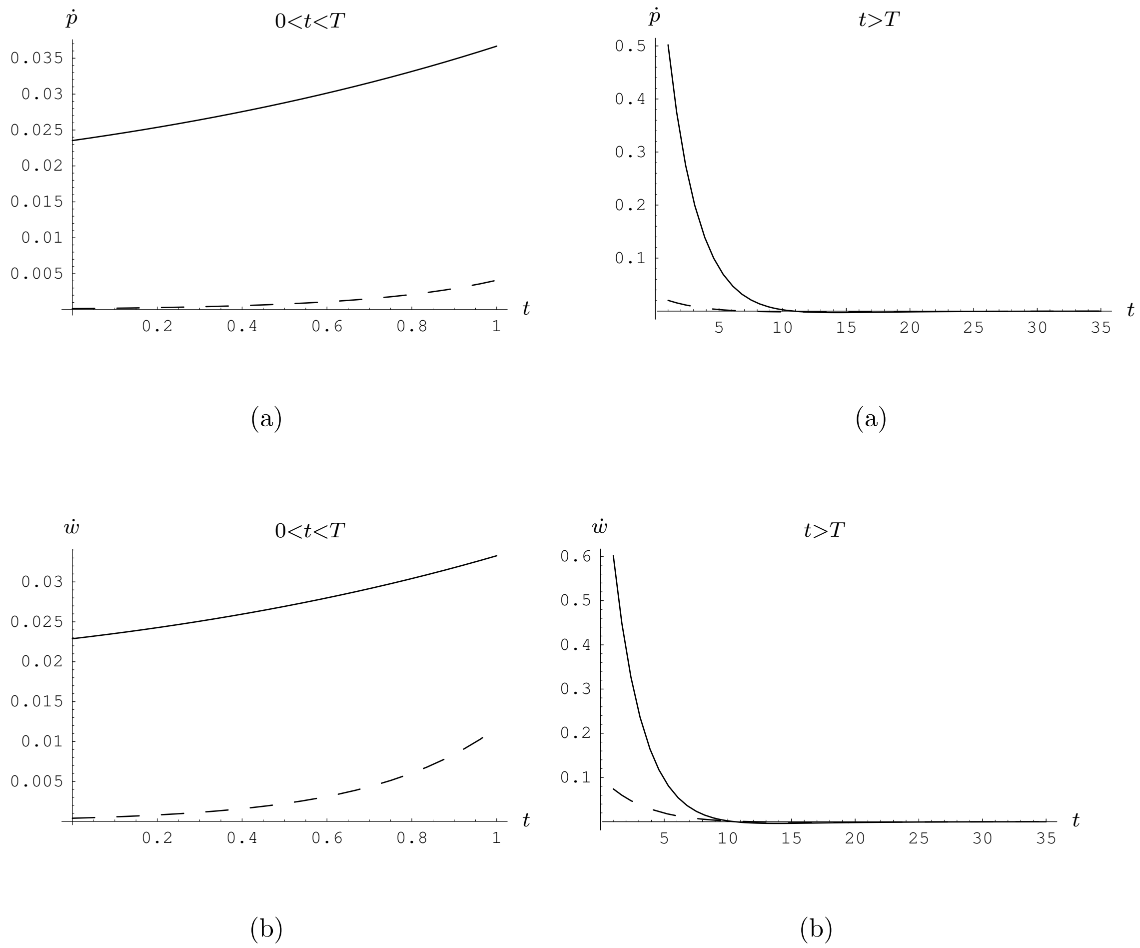


Figure 19: Responses of domestic price and wage inflation rate $\dot{p}$ and $\dot{w}$ to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated commodity price shock with an inflation targeting monetary policy (**dashed lines**)

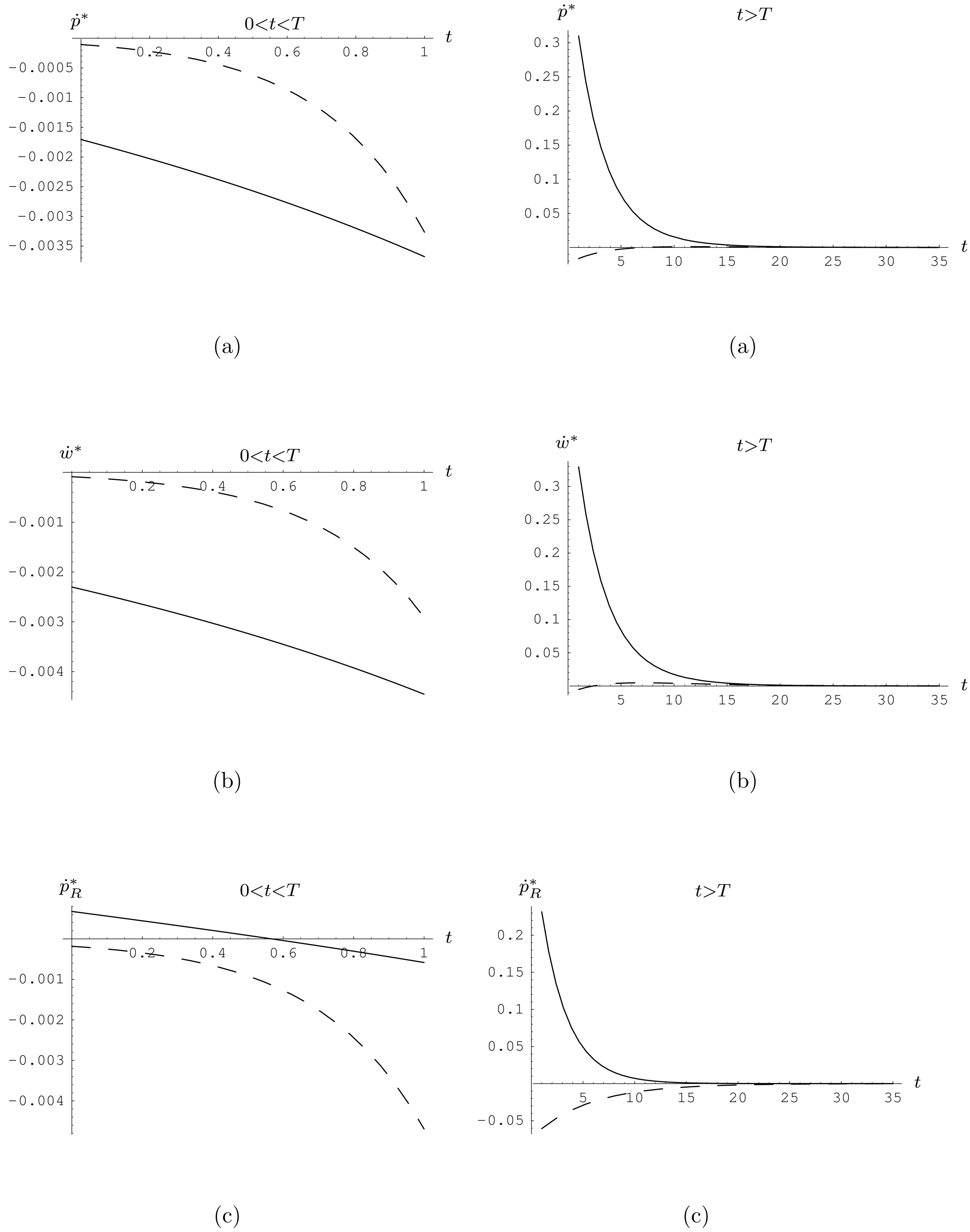
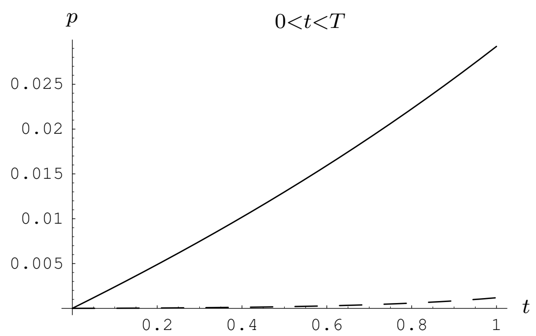
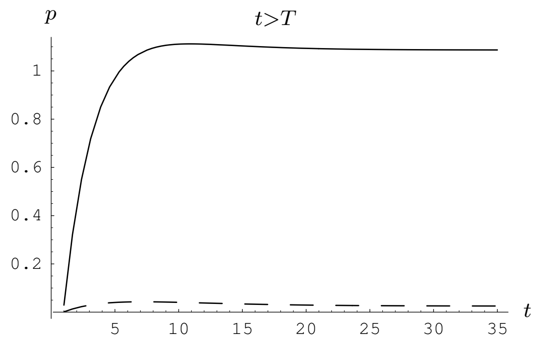


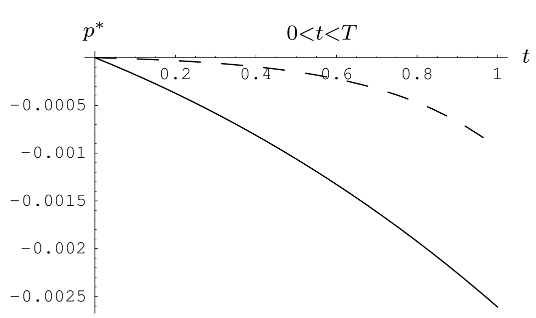
Figure 20: Responses of foreign price and wage inflation rate $\dot{p}^*$ and $\dot{w}^*$ and growth rate of nominal commodity price $\dot{p}_R^*$ to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated commodity price shock with an inflation targeting monetary policy (**dashed lines**)



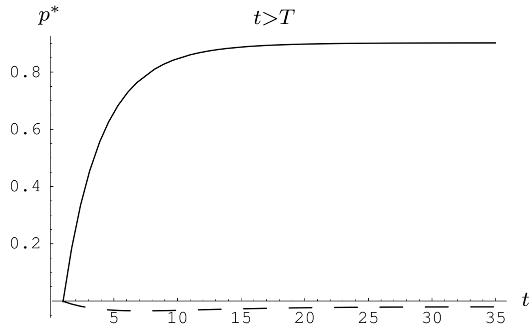
(a)



(a)



(b)



(b)

Figure 21: Responses of domestic and foreign price level p and p^* to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated commodity price shock with an inflation targeting monetary policy (**dashed lines**)

4.3 Output Stabilization

A commodity price shock of the form $dp_R^* > 0$ causes a permanent rise in the real factor prices $p_R^* - p^*$ and $p_R^* + e - p$ and a permanent decline in real output and income in the domestic and foreign economy. The question therefore arises whether monetary policy is able to fix the steady state output variables $\overline{q}$ and $\overline{q}^*$ at their respective pre-disturbance level $\overline{q}_0$ and $\overline{q}_0^*$ respectively. It can be shown that the relative multipliers of domestic and foreign monetary policy as well as the corresponding multipliers of a commodity price shock with respect to real output, income and relative prices coincide:[23]

$$\begin{aligned} \frac{d\overline{x}_i/d\dot{m}}{d\overline{x}_j/d\dot{m}} &= \frac{d\overline{x}_i/d\dot{m}^*}{d\overline{x}_j/d\dot{m}^*} = \frac{d\overline{x}_i/dp_R^*}{d\overline{x}_j/dp_R^*} \\ \text{for } x_i, x_j &\in \{q, q^*, y, y^*, \tau, p_R^* - p^*, p_R^* + e - p, i - \dot{p}^c, i^* - \dot{p}^{*c}\} \end{aligned} \tag{28}$$

This results holds since

$$\begin{aligned} \frac{d\overline{\tau}/d\dot{m}}{d(\overline{p_R^* - p^*})/d\dot{m}} &= \frac{d\overline{\tau}/d\dot{m}^*}{d(\overline{p_R^* - p^*})/d\dot{m}^*} = \frac{d\overline{\tau}/dp_R^*}{d(\overline{p_R^* - p^*})/dp_R^*} = \\ &\frac{\lambda(f_2 - f_2^*) - (a_1 - 2c_1)(\psi - \psi^*)}{\lambda(f_1 + f_1^* + f_2) + 2c_3 - (a_1 - 2c_1)\psi} \quad (\lambda = 1 - a_1 + 2c_1 > 0) \end{aligned} \tag{29}$$

and $d\overline{x}_i$ is representable as linear combination of $d\overline{\tau}$ and $d(\overline{p_R^* - p^*})$, where $x_i \in \{q, q^*, y, y^*, p_R^* + e - p, i - \dot{p}^c, i^* - \dot{p}^{*c}\}$. An implication of the equality of corresponding relative multipliers is that the steady state output vector $(\overline{q}, \overline{q}^*)'$ is not statically controllable with the help of the monetary policy instruments. The corresponding steady state multiplier matrix has not full rank, i.e.,

$$rank \begin{pmatrix} d\overline{q}/d\dot{m} & d\overline{q}/d\dot{m}^* \\ d\overline{q}^*/d\dot{m} & d\overline{q}^*/d\dot{m}^* \end{pmatrix} = 1 < 2 \tag{30}$$

The policy variables $\dot{m}$ and $\dot{m}^*$ are therefore linearly dependent with respect to the target vector $(\overline{q}, \overline{q}^*)'$ so that the number of independent target variables exceeds the number of linearly independent control variables.[24] Nevertheless, monetary policy is able to compensate the steady state output contraction of a commodity price shock, i.e., to stabilize the steady state output vector at its initial (pre-disturbance) level. The equality of the relative multipliers (28) implies that the two-dimensional target $d\overline{q} = d\overline{q}^* = 0$ is attainable by means of only *one* monetary policy instrument, i.e., either by means of a contractionary domestic monetary policy or an expansionary

[23]The proof is presented in the mathematical appendix of this paper.

[24]Cf. Tinbergen (1952) and Preston and Pagan (1982) for the concept of static controllability.

foreign monetary policy:

$$d\dot{m} = -\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}} \cdot dp_R^* = -\frac{\partial\overline{q}^*/\partial p_R^*}{\partial\overline{q}^*/\partial\dot{m}} \cdot dp_R^* < 0\,, \qquad d\dot{m}^* = 0 \tag{31}$$

or

$$d\dot{m}^* = -\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}^*} \cdot dp_R^* = -\frac{\partial\overline{q}^*/\partial p_R^*}{\partial\overline{q}^*/\partial\dot{m}^*} \cdot dp_R^* > 0\,, \quad d\dot{m} = 0 \tag{32}$$

In both cases the targets $d\overline{q} = 0$ and $d\overline{q}^* = 0$ are simultaneously met so that no steady state change of the output variables occurs. The monetary policy rules (31) and (32) also imply $d\overline{\tau} = 0 = d(\overline{p_R^* - p^*}) = d(\overline{p_R^* + e - p})$, i.e., the constancy of the steady state values of the terms of trade and the real factor prices. The stabilization of domestic and foreign steady state output with the help of domestic monetary policy requires a reduction in the growth rate of domestic money supply, since the policy $d\dot{m} < 0$ has *positive* output effects in the long run. Alternatively, the target $d\overline{q} = 0 = d\overline{q}^*$ can be realized with the help of an expansionary foreign monetary policy ($d\dot{m}^* > 0$) since this policy induces a permanent rise in domestic and foreign output. A drawback of the policy rules (31) and (32) is that they either lead to strong deflation in the domestic economy, while the foreign inflation rate remains unchanged in the long run, or to high inflation in the foreign economy leaving the domestic steady state inflation rate constant. This disadvantage can be relaxed by an international policy coordination, i.e., a simultaneous implementation of a contractionary domestic and an expansionary foreign monetary policy:

$$d\dot{m} = -(1-\gamma^*)\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}} \cdot dp_R^* \tag{33}$$

$$d\dot{m}^* = -\gamma^*\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}^*} \cdot dp_R^* \tag{34}$$

with $0 < \gamma^* < 1$.[25] Compared to the polar case $\gamma^* = 0$ the foreign inflation rate is now weaker while the domestic deflation rate is weaker than in case $\gamma^* = 1$. Figures 22 and 23 illustrate the behavior of the domestic and foreign inflation rate and the output variables if γ^* is set equal to

$$\gamma^* = \frac{\partial\overline{q}/\partial\dot{m}^*}{\dfrac{\partial\overline{q}}{\partial\dot{m}^*} - \dfrac{\partial\overline{q}}{\partial\dot{m}}} \tag{35}$$

[25] Note that steady state output stabilization holds for *any* value of $\gamma^* \in \mathbb{R}$. If $\gamma^* < 0$, $d\dot{m}$ and $d\dot{m}^*$ are both negative, while they are both positive if $\gamma^* > 1$.

implying a change of the growth rate of domestic and foreign money supply of equal size, but in opposite direction ($d\dot{m} = -d\dot{m}^*$).[26] The dynamic response of the output variables to an anticipated commodity price shock is characterized by a temporary deviation from their respective initial steady state level. In the long run they return to their initial values.

The question arises whether monetary policy is able to fix the output variables at their respective pre-disturbance steady state level $\overline{q}_0$ and $\overline{q}_0^*$ at *all* times, i.e., to prevent both permanent and temporary deviations from $\overline{q}_0$ and $\overline{q}_0^*$ respectively. To answer this question it is useful to transform the dynamic macro model into an economic policy decision model consisting of dynamic state equations and a set of static equations for the output vector $(q, q^*)'$.[27] In the mathematical appendix it is shown that the system (1) to (20) can be reduced to the following set of equations:

$$\mathbf{B}\dot{\mathbf{x}} = \mathbf{C}(\mathbf{x} - \overline{\mathbf{x}}) + \mathbf{K}(\mathbf{z} - \overline{\mathbf{z}}) \tag{36}$$

$$\mathbf{v} - \overline{\mathbf{v}} = \mathbf{D}\dot{\mathbf{x}} \tag{37}$$

where $\mathbf{x} = (l^s, \tau, l^d)'$ is the state vector, $\mathbf{z} = (\dot{m}, \dot{m}^*)'$ the vector of policy instruments, $\mathbf{v} = (q, q^*)'$ the target vector and $\mathbf{B}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{D}$ are matrices of appropriate size. Since the system matrix $\mathbf{B}$ is invertible, the state equations (36) can be solved for $\dot{\mathbf{x}}$ and then inserted into the target equations leading to the static output equations

$$\mathbf{v} - \overline{\mathbf{v}} = \mathbf{D}\mathbf{B}^{-1}\mathbf{C}(\mathbf{x} - \overline{\mathbf{x}}) + \mathbf{D}\mathbf{B}^{-1}\mathbf{K}(\mathbf{z} - \overline{\mathbf{z}}) \tag{38}$$

Generally, the impact multiplier matrix $\mathbf{D}\mathbf{B}^{-1}\mathbf{K}$ has full rank 2. The target vector $\mathbf{v}$ is therefore perfectly or globally path controllable with the help of the control vector $\mathbf{z}$.[28] Fixing the output vector $\mathbf{v}$ at its initial steady state level $\mathbf{v} = \overline{\mathbf{v}}_0$ at all times is possible with the help of the monetary policy rule

$$\mathbf{z} = \overline{\mathbf{z}}_0 - \mathbf{S}(\mathbf{x} - \overline{\mathbf{x}}_0) \qquad \text{where} \quad \mathbf{S} = (\mathbf{D}\mathbf{B}^{-1}\mathbf{K})^{-1}\mathbf{D}\mathbf{B}^{-1}\mathbf{C} \tag{39}$$

The output stabilization condition

$$\mathbf{v} - \overline{\mathbf{v}} = \mathbf{D}\dot{\mathbf{x}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{40}$$

causes a reduction in the number of linearly independent state variables. The dynamics of the stabilized system can be completely represented by *one* dynamic state

[26] Of course, the choice of the policy combination ($d\dot{m}$, $d\dot{m}^*$) may be the result of an optimization approach with a common loss function of the form $V = \widetilde{\alpha}(d\dot{m})^2 + \widetilde{\beta}(d\dot{m}^*)^2$ ($\widetilde{\alpha}, \widetilde{\beta} > 0$). See below, equation (42).

[27] Cf. Preston and Pagan (1982).

[28] Cf. Aoki (1975), Preston and Pagan (1982) and Wohltmann (1985).

equation in the state variable l^d. Since the corresponding eigenvalue is positive in general, the new steady state resulting from the exogenous price shock $dp_R^* > 0$ and the policy rule (39), is completely unstable. If the real liquidity variables l^d and l^s are still considered as predetermined variables there is no nonexplosive solution of the stabilized state equations.[29] A unique convergent solution time path for the state variable l^d only exists if l^d is interpreted as a jump variable that jumps at the date of anticipation. Pegging the output vector $\mathbf{v}$ at its initial pre-disturbance level at all times requires price flexibility so that the price variables p and p^* are no longer treated as predetermined. This requires a sufficiently flexible money wage what appears acceptable since the augmentation term in the wage Phillips curves (13) and (14) contains the actual rate of CPI inflation which is perfectly foreseen by the public.[30].

Figures 24 and 25 illustrate the dynamic behavior of the domestic and foreign growth rate of money supply according to the policy rule (39) and the resulting development of the inflation rates, the terms of trade and the foreign real factor price. Saddlepoint stability of the stabilized system requires that the new steady state is already attained at time T and that the system remains there thereafter. It can be shown that the steady state change of the monetary policy vector $\mathbf{z}$ resulting from the rule (39), is not uniquely determined.[31] There exists a continuum of policy combinations (a line in $d\overline{\dot{m}}/d\overline{\dot{m}^*}$ – space) such that each of the corresponding time paths of the control vector $\mathbf{z}$ satisfying the rule (39), completely offsets the dynamic output effects of the oil price shock. The feasible policy combinations $(d\overline{\dot{m}}, d\overline{\dot{m}^*})$ are lying on a straight line of the form

$$d\overline{\dot{m}} = a + b \cdot d\overline{\dot{m}^*} \qquad (a < 0, b > 0) \tag{41}$$

with

$$\begin{aligned} a &= -\frac{2(1+\kappa\delta_R^*)\delta}{l_2(\beta_R^*\delta + 2\delta_R^*\beta)}\ , \quad \kappa = \frac{1-\mu}{\delta\mu} + \frac{1-\mu^*}{\delta^*\mu^*} \\ b &= \frac{\beta_R^*\delta^* - 2\delta_R^*\beta^*}{\beta_R^*\delta + 2\delta_R^*\beta} \end{aligned}$$

(cf. figure 26). The adjustment time paths in figure 24 result from monetary policy time paths for which $d\overline{\dot{m}} = d\overline{\dot{m}^*} = a/(1-b) > 0$ holds. Such a policy combination causes a long run fall in the domestic and foreign real money stock of exactly equal size so that $d\bar{l}^d = 0$ holds and no adjustment dynamics occur. The terms of trade τ

[29] Cf. Turnovsky (2000), p. 147.

[30] Cf. Buiter (1984).

[31] Note that a similar result holds in the case of steady state output stabilization, i.e. $d\bar{q} = d\bar{q}^* = 0$. Cf. equations (33) and (34).

remain unchanged over time ($\tau = \overline{\tau}_0$) while the variables $\dot{m}$, $\dot{m}^*$, $\dot{p}$, $\dot{p}^*$ and $p_R^* - p^*$ jump on impact and remain constant until time T. At that time they jump into their new steady state levels and stay there afterwards. Since $0 < d\overline{\dot{m}} = d\overline{\dot{m}^*}$ holds, perfect output stabilization is connected with long run inflation in both economies. The long run inflation rates of the symmetric monetary policy $d\overline{\dot{m}} = d\overline{\dot{m}^*}$ can be reduced if the choice of a feasible policy combination ($d\overline{\dot{m}}$, $d\overline{\dot{m}^*}$) is the result of an optimal international policy coordination in which the minimization of the deviation of the growth rate of money supply from its respective initial pre-disturbance level is aspired. Each of figures 25 (a) to (25 (f) contain three time paths resulting from an *optimal* and asymmetric choice of the policy combination ($d\overline{\dot{m}}$, $d\overline{\dot{m}^*}$). The *solid* lines result from a monetary policy combination which minimizes the loss function

$$V_1 = \frac{1}{2}(d\overline{\dot{m}})^2 + \frac{1}{2}(d\overline{\dot{m}^*})^2 \tag{42}$$

i.e., which tries to stabilize the *long run* inflation and monetary growth rates at their respective pre-disturbance steady state level. In this case $d\overline{\dot{m}} < 0 < d\overline{\dot{m}^*}$ holds so that perfect output stabilization is connected with long run deflation in the domestic and small long run inflation in the foreign economy. The foreign long run inflation rate $\overline{\dot{p}^*}$ is now smaller than in the case $d\overline{l}^d = 0$. In the short run, the domestic inflation rate as well as the domestic growth rate of money supply undershoot their respective new steady state level by a large amount. Moreover, this policy combination also leads on impact to a strong undershooting of the domestic terms of trade τ, while the immediate fall in the foreign real factor price is smaller than in case $d\overline{l}^d = 0$. The foreign inflation rate increases on impact and rises further until time T. The increase in $\dot{p}^*$ over the interval $0 < t < T$ is strong which is a consequence of the international inflationary effects of contractionary domestic monetary policy (cf. figure 8). At time T the inflation rates jump into their new steady state levels where the jumps are stronger than case in $d\overline{l}^d = 0$. The *dashed* lines underly a policy combination ($d\overline{\dot{m}}, d\overline{\dot{m}^*}$) which results from the minimization of the alternative loss function

$$V_2 = \frac{1}{2}\big(\dot{m}(0+) - \overline{\dot{m}}_0\big)^2 + \frac{1}{2}\big(\dot{m}^*(0+) - \overline{\dot{m}}_0^*\big)^2 \tag{43}$$

In this case perfect output stabilization is realized with the aid of a time path of the policy vector for which its initial jump is minimized. Compared to the first optimization approach (solid lines), the short run undershooting of $\dot{m}$ and $\dot{p}$ is now considerably smaller while the long run value of $\dot{m}$ and $\dot{p}$ changes sign, i.e. is positive, but smaller than in case $d\overline{l}^d = 0$. The initial and long term response of the foreign inflation rate is slightly stronger than in the first optimization approach. However, the volatility of $\dot{p}^*$ over the interval $0 < t < T$ is now considerably reduced.[32] The volatility of the inflation rates over the interval $0 < t < T$ is also small if the choice of the policy combination $(d\overline{\dot{m}}, d\overline{\dot{m}^*})$ results from the minimization of a loss function which is a linear combination of V_1 and V_2:

$$V_3 = \lambda_V V_1 + (1 - \lambda_V) V_2 \qquad 0 \leq \lambda_V \leq 1 \tag{44}$$

The weight λ_V measures the importance of the long run deviation of the growth rate of money supply from its initial value in the loss function V_3. The long run inflation rates (*dotted lines*) are now smaller than in case V_2 (*dashed lines*) but larger than in the first optimization approach (*solid lines*). The mixed loss function V_3 also generates initial responses of $\dot{p}$, $\dot{p}^*$, τ and $p_R^* - p^*$ which lie between the corresponding values in the polar cases V_1 and V_2. In any of the three optimization approaches the long run foreign inflation rate $\overline{\dot{p}^*}$ is *greater* than the corresponding domestic rate $\overline{\dot{p}}$. The policy trade-off curve in figure 26 illustrates that perfect output stabilization is also possible with asymmetric monetary policy combinations for which $d\overline{\dot{m}} > d\overline{\dot{m}^*}$ and thus $d\overline{\dot{p}} > d\overline{\dot{p}^*}$ holds. But this would induce high and non acceptable long run inflation rates in both economies.

[32] Note that nearly the same time paths of the variables $\dot{m}$, $\dot{m}^*$, $\dot{p}$, $\dot{p}^*$, τ and $p_R^* - p^*$ result if the loss function V_2 is replaced by $\widetilde{V}_2 = \int_0^T \left(\frac{1}{2}(\dot{m} - \overline{\dot{m}}_0)^2 + \frac{1}{2}(\dot{m}^* - \overline{\dot{m}}_0^*)^2\right) dt$.

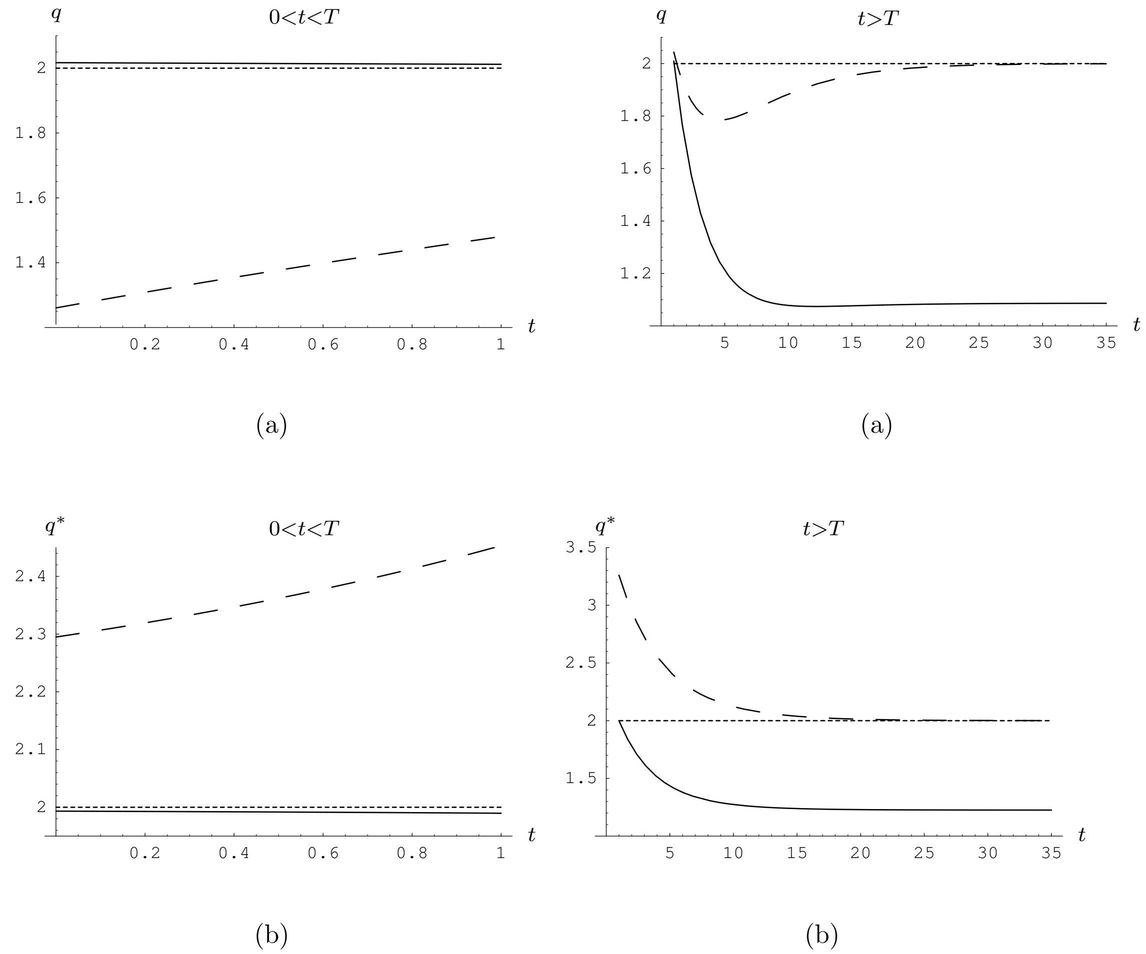


Figure 22: Responses of domestic and foreign output q and q^* to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated commodity price shock with an output stabilizing monetary policy (**dashed lines**); initial steady states: **dotted lines**

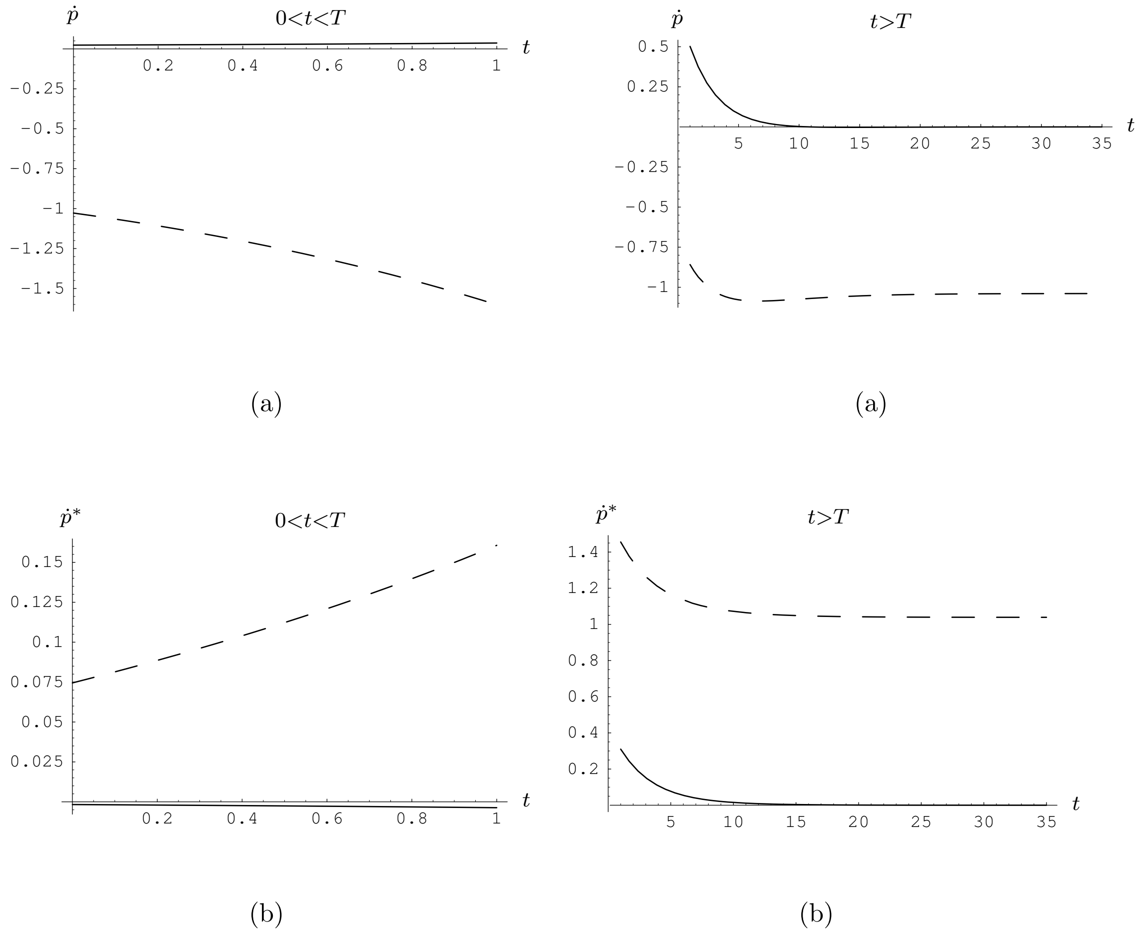
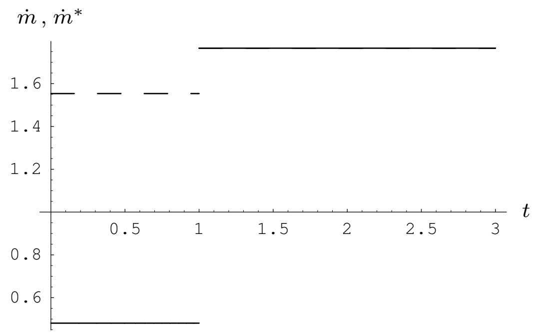
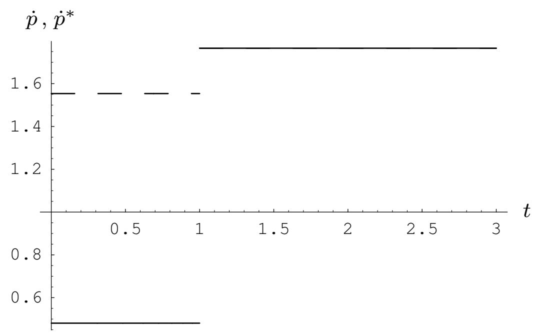


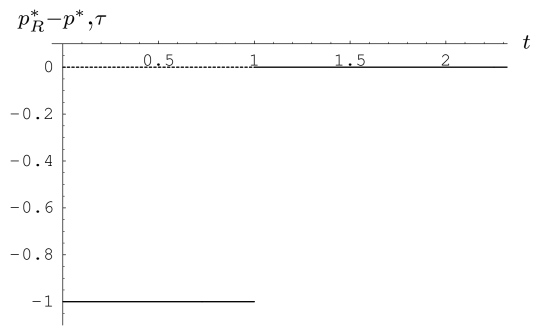
Figure 23: Responses of domestic and foreign price inflation rate $\dot{p}$ and $\dot{p}^*$ to an anticipated commodity price shock with passive monetary policy (**solid lines**) and to an anticipated oil price shock with an output stabilizing monetary policy (**dashed lines**)



(a)



(b)



(c)

Figure 24: Output stabilizing growth rates of domestic (**solid lines**) and foreign money supply (**dashed lines**) in case $d\bar{l}^d = 0$ and the resulting responses of domestic inflation rate $\dot{p}$ (**solid lines**), foreign inflation rate $\dot{p}^*$ (**dashed lines**), foreign real factor price $p_R^* - p^*$ and the terms of trade τ (**dotted lines**)

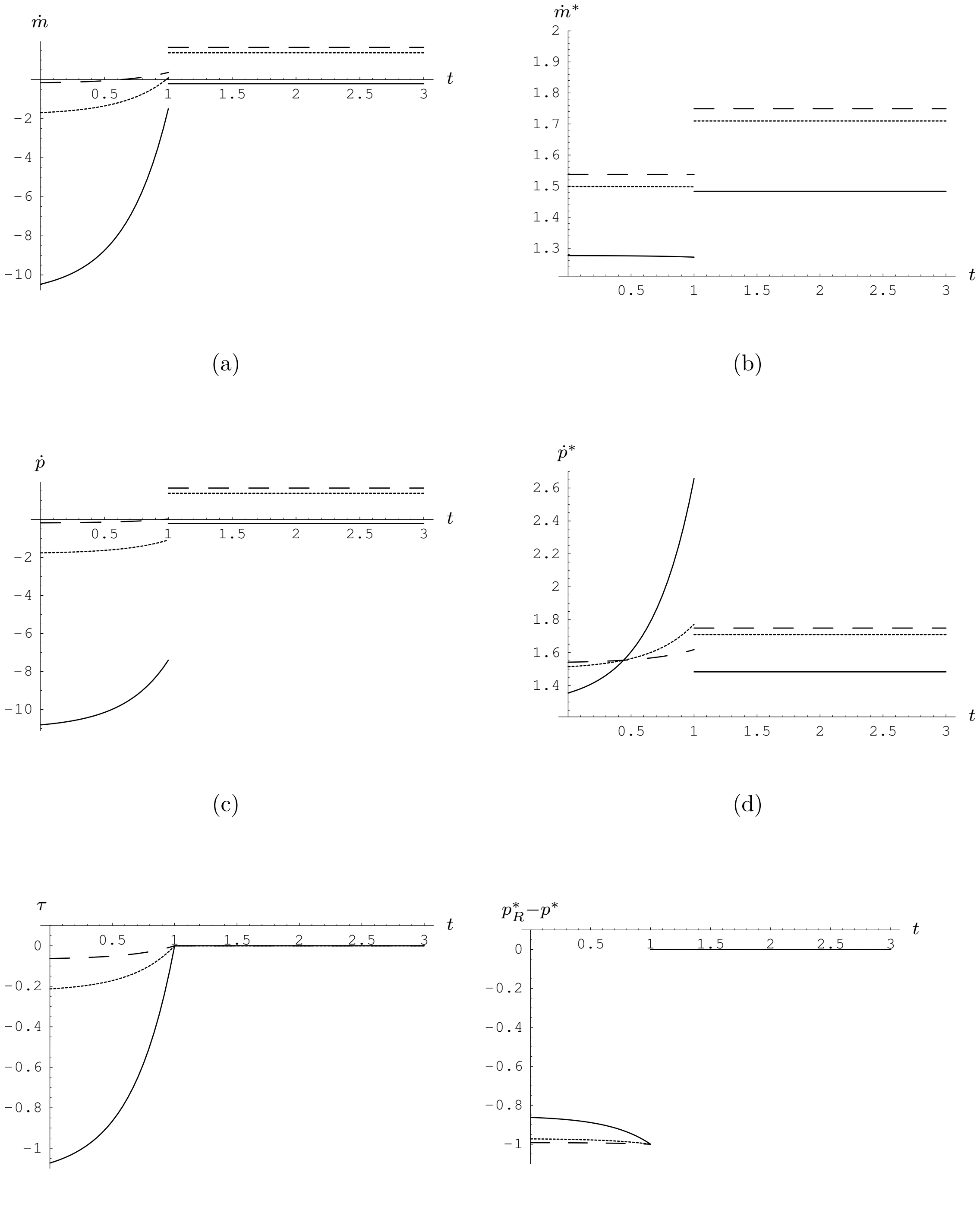


Figure 25: Optimal output stabilizing domestic and foreign monetary policies ($\dot{m}|_{V_1}$, $\dot{m}^*|_{V_1}$) (**solid lines**), ($\dot{m}|_{V_2}$, $\dot{m}^*|_{V_2}$) (**dashed lines**), ($\dot{m}|_{V_3}$, $\dot{m}^*|_{V_3}$) (**dotted lines**) and the respective responses of domestic inflation rate $\dot{p}$, foreign inflation rate $\dot{p}^*$, foreign real factor price $p_R^* - p^*$ and the terms of trade τ

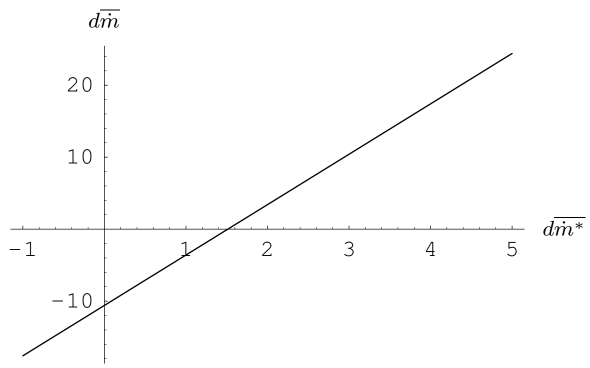


Figure 26: Perfect output stabilizing monetary policy trade-off

5 Summary of the Main Results

This paper has analyzed a macrodynamic model of two large open economies which are dependent upon commodity imports from a small third country. It is assumed that the domestic economy (the European Monetary Union) is stronger dependent on raw materials imports than the foreign economy (the USA) and that commodity imports are priced in dollars. The main results of the analysis may be summarized as follows:

(a) A rise in the growth rate of domestic money supply is not neutral in the long run but induces a decline in the domestic and foreign real output. Since the growth rate of the commodity price is coupled with the foreign inflation rate and the world output gap, a domestic monetary expansion yields a permanent increase in the domestic and foreign real commodity price. Only in the special case that the commodity price does not follow a rule, but is exogenously given, domestic monetary policy is neutral in the long run. By contrast, a rise in the growth rate of foreign money supply generally leads to a steady state decline in the real factor prices and an output expansion in both economies.

(b) The properties of the dynamic time paths following an anticipated or unanticipated monetary expansion have been discussed in the text and the details need not be repeated here. A consequence of the asymmetric supply side structure of the two economies is that the domestic and foreign economies are not affected in precisely offsetting ways during the anticipation phase. Moreover, the dynamic response of the system to a domestic monetary shock is not symmetric to the corresponding response of an equivalent foreign monetary shock.

(c) A commodity price shock causes temporary inflation and a permanent output contraction in both economies. Due to the assumption that the domestic economy is stronger dependent on raw materials imports, the stagflationary effects are stronger for the domestic than for the foreign economy. A magnification of the stagflationary effects occurs if the commodity price development is stronger coupled with the foreign inflation rate and/or the world output gap.

(d) The inflationary effects caused by commodity price shocks can be removed by a coordination of domestic and foreign monetary policy. The policy rules imply a temporary decline in the domestic and foreign growth rate of money supply after the realization of the raw materials price shock. Such an international monetary policy coordination has negligible steady state output effects so that the long run output contraction resulting from an isolated commodity price shock is not amplified.

(e) Active monetary policy as response to oil price shocks is also able to fix simultaneously the domestic and foreign steady state output level at its respective pre-disturbance value. This two-dimensional target is achievable although the relative output multipliers of domestic and foreign monetary policy are identical. Fixing the steady state output vector at its initial value requires a once-and-for-all decrease in the growth rate of domestic money supply and/or an increase in the growth rate of foreign money supply. Such a policy is connected with domestic deflation and/or foreign inflation. These price effects are of moderate size if the change of the domestic growth rate of money supply equals in absolute terms the change of the corresponding foreign monetary growth rate.

(f) A remarkable result is that an international monetary policy coordination is able to fix the output vector at its initial equilibrium level at all times. Since the output vector is impact controllable there exist policy rules for the domestic and foreign growth rate of money supply that ensure the target path controllability of the output vector.

(g) Perfect output stabilization may result in a saddlepoint instability problem if the domestic and foreign price level are considered as predetermined variables. However, with price flexibility, there exist convergent time paths of the stabilized system. Since the long run change of the growth rate of money supply is not uniquely determined, fixing the output vector at its initial level at all times is possible with an infinite number of time paths of the monetary policy control vector. It is shown that perfect output stabilization is attainable both without and with adjustment dynamics. In the first case monetary policy is symmetric in the long run and generates relatively high long run inflation rates. In the latter case the monetary policy combination may be the result of optimization approaches where deviations from the initial value of the control vector are minimized. In this case output stabilization is associated with smaller long run inflation but may lead to strong overshooting phenomena in the short run.

Mathematical Appendix

State Space Representation

Using the decomposition method of Aoki (1981) the dynamics of the multi-country model can be represented with the help of three state variables,

$$\begin{aligned} l^s &= (m-p)+(m^*-p^*) && \text{(A1)} \\ l^d &= (m-p)-(m^*-p^*) && \text{(A2)} \\ \tau &= p-(p^*+e) && \text{(A3)} \end{aligned}$$

where the real liquidity variables (l^s, l^d) are predetermined and real competitiveness (τ) is a jump variable (cf. Wohltmann and Winkler (2005a) for details).

In the following we use the abbreviations

$$\begin{aligned} \lambda &= 1-a_1+2c_1 && \text{(A4)} \\ \kappa_1 &= \frac{1}{\delta}\left((1-\beta)(1-\alpha)+\frac{1-\mu}{\mu}\right)+\frac{1}{\delta^*}(1-\beta^*)(1-\alpha^*) && \text{(A5)} \\ \kappa_2 &= \frac{1-\mu}{\delta\mu}-\frac{1-\mu^*}{\delta^*\mu^*} && \text{(A6)} \\ \kappa_3 &= \frac{1}{\delta}\left((1-\beta)(1-\alpha)+\frac{1-\mu}{\mu}\right)-\frac{1}{\delta^*}(1-\beta^*)(1-\alpha^*) && \text{(A7)} \\ \kappa_4 &= \frac{1-\mu}{\delta\mu}+\frac{1-\mu^*}{\delta^*\mu^*} && \text{(A8)} \\ \phi_1 &= \frac{1}{1+\kappa_4\delta_R^*}\left(\beta_R^*+\delta_R^*\left(\frac{\beta}{\delta}-\frac{\beta^*}{\delta^*}\right)\right) && \text{(A9)} \\ \phi_2 &= \frac{1}{1+\kappa_4\delta_R^*}\left(\beta_R^*-\delta_R^*\left(\frac{\beta}{\delta}+\frac{\beta^*}{\delta^*}\right)\right) && \text{(A10)} \\ \Lambda &= \frac{\delta_R^*\kappa_3}{1+\kappa_4\delta_R^*} && \text{(A11)} \end{aligned}$$

The model equations (1) – (20) can be transformed into two subsystems, a difference and an aggregate system. The difference system consists of the difference of corresponding equations of the domestic and foreign economy (like the difference of the IS equations (1) and (2)) and has the following structure:

$$\begin{aligned}
\lambda(q-q^*) &= 2c_0+g-g^*-a_2 i_r^d-(2c_3-(a_1-2c_1)\psi)\tau \\
&\quad -(a_1-2c_1)(\psi-\psi^*)(p_R^*-p^*) && \text{(A12)}\\
y-y^* &= q-q^*+\psi\tau-(\psi-\psi^*)(p_R^*-p^*) && \text{(A13)}\\
l^d &= (\alpha+\alpha^*-2)\tau+l_1(q-q^*)+l_2\dot{\tau}+l_2\dot{l}^d-l_2(\dot{m}-\dot{m}^*) && \text{(A14)}\\
q-q^* &= \overline{q}-\overline{q}^*+\kappa_1\dot{\tau}-\frac{1}{2}\left(\frac{\beta}{\delta}-\frac{\beta^*}{\delta^*}\right)\dot{l}^s-\frac{1}{2}\left(\frac{\beta}{\delta}+\frac{\beta^*}{\delta^*}\right)\dot{l}^d \\
&\quad -\kappa_2(\dot{p}_R^*-\dot{p}^*) && \text{(A15)}\\
\overline{q}-\overline{q}^* &= f_0-f_0^*+(f_1+f_1^*+f_2)\overline{\tau}-(f_2-f_2^*)(\overline{p_R^*-p^*}) && \text{(A16)}
\end{aligned}$$

where

$$\begin{aligned}
i_r^d &= (i-\dot{p}^c)-(i^*-\dot{p}^{*c})=(1-(\alpha+\alpha^*))\dot{\tau} && \text{(A17)}\\
\dot{p}_R^*-\dot{p}^* &= \beta_R^*(\dot{m}^*-\dot{p}^*)+\delta_R^*\big(q+q^*-(\overline{q}+\overline{q}^*)\big) && \text{(A18)}\\
&= \frac{1}{2}\beta_R^*\dot{l}^s-\frac{1}{2}\beta_R^*\dot{l}^d+\delta_R^*\big(q+q^*-(\overline{q}+\overline{q}^*)\big)
\end{aligned}$$

Similarly, the aggregate system consists of the equations

$$\begin{aligned}
(1-a_1)(q+q^*) &= 2a_0-2d_0a_1-a_2 i_r^s+g+g^*+a_1\psi\tau \\
&\quad -a_1(\psi+\psi^*)(p_R^*-p^*) && \text{(A19)}\\
y+y^* &= q+q^*+\psi\tau-(\psi+\psi^*)(p_R^*-p^*)-2d_0 && \text{(A20)}\\
l^s &= (\alpha-\alpha^*)\tau+2l_0+l_1(q+q^*)-l_2(2i^*+\dot{e}) && \text{(A21)}\\
q+q^* &= \overline{q}+\overline{q}^*+\kappa_3\dot{\tau}-\left(\frac{\beta}{2\delta}+\frac{\beta^*}{2\delta^*}\right)\dot{l}^s-\left(\frac{\beta}{2\delta}-\frac{\beta^*}{2\delta^*}\right)\dot{l}^d \\
&\quad -\kappa_4(\dot{p}_R^*-\dot{p}^*) && \text{(A22)}\\
\overline{q}+\overline{q}^* &= f_0+f_0^*+(f_1+f_2-f_1^*)\overline{\tau}-(f_2+f_2^*)(\overline{p_R^*-p^*}) && \text{(A23)}
\end{aligned}$$

where

$$\begin{aligned}
i_r^s &= (i-\dot{p}^c)+(i^*-\dot{p}^{*c})=2i^*+\dot{e}-(\dot{p}^c+\dot{p}^{*c}) && \text{(A24)}\\
\dot{p}^c+\dot{p}^{*c} &= (\alpha-\alpha^*)\dot{\tau}-\dot{l}^s+\dot{m}+\dot{m}^* && \text{(A25)}
\end{aligned}$$

Combining the equations (A18) and (A22) for the growth rate of foreign real factor price and aggregate real output yields

$$q+q^*-\overline{q}+\overline{q}^*= \tag{A26}$$
$$\frac{1}{1+\kappa_4\delta_R^*}\left(\kappa_3\dot{\tau}-\frac{1}{2}\left(\beta_R^*\kappa_4+\frac{\beta}{\delta}+\frac{\beta^*}{\delta^*}\right)\dot{l}^s+\frac{1}{2}\left(\beta_R^*\kappa_4-\frac{\beta}{\delta}+\frac{\beta^*}{\delta^*}\right)\dot{l}^d\right)$$

$$\dot{p}_R^* - \dot{p}^* = \Lambda\dot{\tau} + \frac{1}{2}\phi_2 \dot{l}^s - \frac{1}{2}\phi_1 \dot{l}^d \tag{A27}$$

It follows by integration that the foreign real factor price level $p_R^* - p^*$ is given by

$$\begin{aligned} p_R^* - p^* &= (\overline{p_R^* - p^*})_0 + \Lambda\big(\tau - \tau(0+)\big) + \frac{1}{2}\phi_2(l^s - \overline{l}_0^s) - \frac{1}{2}\phi_1(l^d - \overline{l}_0^d) \\ &= (\overline{p_R^* - p^*})_0 + \Lambda(\tau - \overline{\tau}_0) + \Lambda\big(\overline{\tau}_0 - \tau(0+)\big) \\ &\quad + \frac{1}{2}\phi_2(l^s - \overline{l}_0^s) - \frac{1}{2}\phi_1(l^d - \overline{l}_0^d) \qquad \text{for } 0 \leq t < T \end{aligned} \tag{A28}$$

$$\begin{aligned} p_R^* - p^* &= (\overline{p_R^* - p^*})_0 + dp_R^* + \Lambda(\tau - \overline{\tau}_0) + \Lambda\big(\overline{\tau}_0 - \tau(0+)\big) \\ &\quad + \frac{1}{2}\phi_2(l^s - \overline{l}_0^s) - \frac{1}{2}\phi_1(l^d - \overline{l}_0^d) \qquad \text{for } t > T \end{aligned} \tag{A29}$$

where we have assumed that $p_R^* - p^*$ behaves sluggishly at the time of anticipation, $t = 0$:

$$\lim_{\substack{t\to 0\\ t>0}}(p_R^* - p^*) = (\overline{p_R^* - p^*})_0 + \Lambda \cdot \lim_{\substack{t\to 0\\ t>0}} \big(\tau - \tau(0+)\big) = (\overline{p_R^* - p^*})_0 \tag{A30}$$

Note that $\tau(0+)$ denotes the right-hand side limit of τ if t converges to zero from the right. If a once-and-for-all increase in the oil price takes places at time T, the real factor price $p_R^* - p^*$ jumps by the amount dp_R^* in T:

$$(p_R^* - p^*)(T+) - (p_R^* - p^*)(T-) = dp_R^* \tag{A31}$$

Thereafter, $p_R^* - p^*$ converges to a new steady state $(\overline{p_R^* - p^*})_1$ given by

$$\begin{aligned} (\overline{p_R^* - p^*})_1 &= (\overline{p_R^* - p^*})_0 + dp_R^* + \Lambda(\overline{\tau}_1 - \overline{\tau}_0) + \Lambda(\overline{\tau}_0 - \tau(0+)) \\ &\quad + \frac{1}{2}\phi_2(\overline{l}_1^s - \overline{l}_0^s) - \frac{1}{2}\phi_1(\overline{l}_1^d - \overline{l}_0^d) \end{aligned} \tag{A32}$$

and depending on the initial jump of τ, i.e. $\tau(0+) - \overline{\tau}_0$. Inserting the formulas for $q - q^*$ and $p_R^* - p^*$ into the difference of the IS and LM equations yields the following differential equations in deviational form:

$$\begin{aligned} &\Big(\lambda\kappa_1 + a_2(1 - (\alpha + \alpha^*))\Big)\dot{\tau} - \frac{1}{2}\lambda\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right)\dot{l}^s - \frac{1}{2}\lambda\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right)\dot{l}^d \\ &- \lambda\kappa_2\left(\Lambda\dot{\tau} + \frac{1}{2}\phi_2\dot{l}^s - \frac{1}{2}\phi_1\dot{l}^d\right) = -\big(2c_3 - (a_1 - 2c_1)\psi\big)(\tau - \overline{\tau}) \\ &- (a_1 - 2c_1)(\psi - \psi^*)\left(\Lambda(\tau - \overline{\tau}) + \frac{1}{2}\phi_2(l^s - \overline{l}^s) - \frac{1}{2}\phi_1(l^d - \overline{l}^d)\right) \end{aligned} \tag{A33}$$

$$
\begin{aligned}
&(l_1\kappa_1 + l_2)\dot{\tau} - \frac{1}{2}l_1\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right)\dot{l}^s + \left(l_2 - \frac{1}{2}l_1\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right)\right)\dot{l}^d \\
&- l_1\kappa_2\left(\Lambda\dot{\tau} + \frac{1}{2}\phi_2\dot{l}^s - \frac{1}{2}\phi_1\dot{l}^d\right) \;=\; (l^d - \overline{l}^d) + \big(2 - (\alpha + \alpha^*)\big)(\tau - \overline{\tau})
\end{aligned} \tag{A34}
$$

Combining the aggregate IS and LM equation (A19) and (A21) and inserting the aggregate supply function (A22) the dynamics of the aggregate system can be represented by

$$
\begin{aligned}
&\big(a_2 l_1 + l_2(1-a_1)\big)\kappa_3\dot{\tau} - \frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right)\dot{l}^s \\
&- \frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right)\dot{l}^d - a_2 l_2\big((\alpha - \alpha^*)\dot{\tau} - \dot{l}^s + \dot{m} + \dot{m}^*\big) \\
&- \big(a_2 l_1 + l_2(1-a_1)\big)\kappa_4\left(\Lambda\dot{\tau} + \frac{1}{2}\phi_2\dot{l}^s - \frac{1}{2}\phi_1\dot{l}^d\right) \\
&+ \big(a_2 l_1 + l_2(1-a_1)\big)(\overline{q} + \overline{q}^*) \;=\; a_2 l^s + \big(l_2 a_1\psi - a_2(\alpha - \alpha^*)\big)\tau + l_2(g + g^*) \\
&\qquad - l_2 a_1(\psi + \psi^*)(p_R^* - p^*) \\
&\qquad + 2a_0 l_2 - 2l_0 a_2 - 2d_0 a_1 l_2
\end{aligned} \tag{A35}
$$

Inserting the formula (A29) for the foreign real factor price the dynamics of the aggregate system can be rewritten in deviational form as follows:

$$
\begin{aligned}
&\Big(\big(a_2 l_1 + l_2(1-a_1)\big)\kappa_3 - a_2 l_2(\alpha - \alpha^*)\Big)\dot{\tau} \\
&+ \left(a_2 l_2 - \frac{1}{2}(a_2 l_1 + l_2(1-a_1))\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right)\right)\dot{l}^s \\
&- \frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right)\dot{l}^d \\
&- \big(a_2 l_1 + l_2(1-a_1)\big)\kappa_4\left(\Lambda\dot{\tau} + \frac{1}{2}\phi_2\dot{l}^s - \frac{1}{2}\phi_1\dot{l}^d\right) \;= \\
&\qquad a_2(l^s - \overline{l}^s) + \big(l_2 a_1\psi - a_2(\alpha - \alpha^*)\big)(\tau - \overline{\tau}) \\
&\qquad - l_2 a_1(\psi + \psi^*)\left(\Lambda(\tau - \overline{\tau}) + \frac{1}{2}\phi_2(l^s - \overline{l}^s) - \frac{1}{2}\phi_1(l^d - \overline{l}^d)\right)
\end{aligned} \tag{A36}
$$

The dynamic equations (A33), (A34) and (A36) have the following matrix representation:

$$
\mathbf{B}\begin{pmatrix} \dot{l}^s \\ \dot{\tau} \\ \dot{l}^d \end{pmatrix} \;=\; \mathbf{C}\begin{pmatrix} l^s - \overline{l}^s \\ \tau - \overline{\tau} \\ l^d - \overline{l}^d \end{pmatrix} \tag{A37}
$$

where $\mathbf{B} = (b_{ij})_{1\leq i,j,\leq 3}$ and $\mathbf{C} = (c_{ij})_{1\leq i,j,\leq 3}$ are given by

$$
\begin{aligned}
b_{11} &= a_2 l_2 - \frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right) \\
&\quad - \frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\kappa_4 \phi_2 && \text{(A38)} \\
b_{12} &= \big(a_2 l_1 + l_2(1-a_1)\big)\kappa_3 - a_2 l_2(\alpha - \alpha^*) - \big(a_2 l_1 + l_2(1-a_1)\big)\kappa_4 \Lambda && \text{(A39)} \\
b_{13} &= -\frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right) + \frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\kappa_4 \phi_1 && \text{(A40)} \\
b_{21} &= -\frac{1}{2}\lambda\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right) - \frac{1}{2}\lambda\kappa_2\phi_2 && \text{(A41)} \\
b_{22} &= \lambda\kappa_1 + a_2\big(1 - (\alpha + \alpha^*)\big) - \lambda\kappa_2\Lambda && \text{(A42)} \\
b_{23} &= -\frac{1}{2}\lambda\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right) + \frac{1}{2}\lambda\kappa_2\phi_1 && \text{(A43)} \\
b_{31} &= -\frac{1}{2}l_1\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*}\right) - \frac{1}{2}l_1\kappa_2\phi_2 && \text{(A44)} \\
b_{32} &= l_1\kappa_1 + l_2 - l_1\kappa_2\Lambda && \text{(A45)} \\
b_{33} &= l_2 - \frac{1}{2}l_1\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right) + \frac{1}{2}l_1\kappa_2\phi_1 && \text{(A46)} \\
c_{11} &= a_2 - \frac{1}{2}l_2 a_1(\psi + \psi^*)\phi_2 && \text{(A47)} \\
c_{12} &= l_2 a_1\psi - a_2(\alpha - \alpha^*) - l_2 a_1(\psi + \psi^*)\Lambda && \text{(A48)} \\
c_{13} &= \frac{1}{2}l_2 a_1(\psi + \psi^*)\phi_1 && \text{(A49)} \\
c_{21} &= -\frac{1}{2}(a_1 - 2c_1)(\psi - \psi^*)\phi_2 && \text{(A50)} \\
c_{22} &= -\big(2c_3 - (a_1 - 2c_1)\psi\big) - (a_1 - 2c_1)(\psi - \psi^*)\Lambda && \text{(A51)} \\
c_{23} &= \frac{1}{2}(a_1 - 2c_1)(\psi - \psi^*)\phi_1 && \text{(A52)} \\
c_{31} &= 0 && \text{(A53)} \\
c_{32} &= 2 - (\alpha + \alpha^*) && \text{(A54)} \\
c_{33} &= 1 && \text{(A55)}
\end{aligned}
$$

Solution to Dynamics

The dynamic system (A37) exhibits saddlepoint behavior. The system matrix $\mathbf{G} = \mathbf{B}^{-1}\mathbf{C} = (g_{ij})_{1\leq i,j,\leq 3}$ has two stable eigenvalues (r_0, r_2) and one unstable root (r_1). The corresponding eigenvectors h_0, h_1, h_2 can be represented in the following normalized form:

$$
h_j = \begin{pmatrix} h_{1j} \\ h_{2j} \\ 1 \end{pmatrix} \qquad j = 0, 1, 2 \tag{A56}
$$

where

$$h_{1j} = \frac{1}{\Delta_j}(-g_{13}(g_{22} - r_j) + g_{12}g_{23}) \tag{A57}$$

$$h_{2j} = \frac{1}{\Delta_j}(g_{21}g_{13} - g_{23}(g_{11} - r_j)) \tag{A58}$$

and

$$\Delta_j = (g_{11} - r_j)(g_{22} - r_j) - g_{12}g_{21} \tag{A59}$$

The uniquely determined convergent solution time path of the state vector $(l^s, \tau, l^d)'$ is then given by (cf. Wohltmann and Winkler (2005 a))

$$\begin{pmatrix} l^s \\ \tau \\ l^d \end{pmatrix} = \begin{pmatrix} \bar{l}_0^s \\ \bar{\tau}_0 \\ \bar{l}_0^d \end{pmatrix} + A_0 h_0 e^{r_0 t} + A_1 h_1 e^{r_1 t} + A_2 h_2 e^{r_2 t} \quad \text{for } 0 < t \leq T \tag{A60}$$

$$\begin{pmatrix} l^s \\ \tau \\ l^d \end{pmatrix} = \begin{pmatrix} \bar{l}_1^s \\ \bar{\tau}_1 \\ \bar{l}_1^d \end{pmatrix} + \widetilde{A}_0 h_0 e^{r_0 t} + \widetilde{A}_2 h_2 e^{r_2 t} \quad \text{for } t \geq T \tag{A61}$$

where $e^{r_j t} = \exp(r_j t)$,

$$A_0 = \frac{1}{h_{10} - h_{12}}(-h_{11} + h_{12})A_1 \tag{A62}$$

$$A_2 = \frac{1}{h_{10} - h_{12}}(h_{11} - h_{10})A_1 \tag{A63}$$

$$A_1 = \frac{1}{d}e^{-r_1 T}\Big((h_{22} - h_{20})d\bar{l}^s + (h_{10} - h_{12})d\bar{\tau} + (h_{12}h_{20} - h_{10}h_{22})d\bar{l}^d\Big) \tag{A64}$$

$$\widetilde{A}_0 = A_0 - \frac{1}{d}e^{-r_0 T}\Big((h_{21} - h_{22})d\bar{l}^s + (h_{12} - h_{11})d\bar{\tau} + (h_{11}h_{22} - h_{21}h_{12})d\bar{l}^d\Big) \tag{A65}$$

$$\widetilde{A}_2 = A_2 - \frac{1}{d}e^{-r_2 T}\Big((h_{20} - h_{21})d\bar{l}^s + (h_{11} - h_{10})d\bar{\tau} + (h_{10}h_{21} - h_{11}h_{20})d\bar{l}^d\Big) \tag{A66}$$

and

$$d = h_{10}(h_{21} - h_{22}) + h_{11}(h_{22} - h_{20}) + h_{12}(h_{20} - h_{21}) \tag{A67}$$

Note that the constants A_0, A_1, A_2, $\widetilde{A}_0$, $\widetilde{A}_2$ are linear combinations of the steady state change of the state variables. Therefore they are dependent upon the specific form of the shock occurring at time $T \geq 0$.

Steady State Analysis

The steady state of the dynamic system (A37) is obtained if

$$\overline{\dot{l}^s} = \overline{\dot{\tau}} = \overline{\dot{l}^d} = 0 \tag{A68}$$

holds. This implies according to (A26) and (A27) that the steady state world output coincides with the long run output level $\overline{q} + \overline{q}^*$ and that $(\overline{\dot{p}_R^* - \dot{p}^*})$ is equal to zero. Thus,

$$\begin{aligned} \overline{\dot{p}^*} &= \overline{\dot{p}_R^*} = \overline{\dot{p}^{*c}} = \overline{\dot{m}^*} \\ \overline{\dot{p}} &= \overline{\dot{p}^c} = \overline{\dot{m}} \\ \overline{\dot{e}} &= \overline{\dot{m}} - \overline{\dot{m}^*} \end{aligned} \tag{A69}$$

Since there is no long run real interest rate differential ($\overline{i}_r^d = 0$) the total differential of the steady state system is given by the following equations:

$$\lambda(d\overline{q} - d\overline{q}^*) = \big(2c_3 - (a_1 - 2c_1)\psi\big)d\overline{\tau} - (a_1 - 2c_1)(\psi - \psi^*)d(\overline{p_R^* - p^*}) \tag{A70}$$

$$d\overline{l}^d = (\alpha + \alpha^* - 2)d\overline{\tau} + l_1(d\overline{q} - d\overline{q}^*) - l_2(d\dot{m} - d\dot{m}^*) \tag{A71}$$

$$d\overline{q} - d\overline{q}^* = (f_1 + f_1^* + f_2)d\overline{\tau} - (f_2 - f_2^*)d(\overline{p_R^* - p^*}) \tag{A72}$$

$$\begin{aligned} (a_2 l_1 + l_2(1 - a_1))(d\overline{q} + d\overline{q}^*) &= a_2 l_2(d\dot{m} + d\dot{m}^*) + a_2 d\overline{l}^s \\ &\quad + \big(l_2 a_1 \psi - a_2(\alpha - \alpha^*)\big)d\overline{\tau} \\ &\quad - l_2 a_1(\psi + \psi^*)d(\overline{p_R^* - p^*}) \end{aligned} \tag{A73}$$

$$d\overline{q} + d\overline{q}^* = (f_1 + f_2 - f_1^*)d\overline{\tau} - (f_2 + f_2^*)d(\overline{p_R^* - p^*}) \tag{A74}$$

$$\begin{aligned} d(\overline{p_R^* - p^*}) &= (\overline{p_R^* - p^*})_1 - d(\overline{p_R^* - p^*})_0 \\ &= dp_R^* + \Lambda d\overline{\tau} + \frac{1}{2}\phi_2 d\overline{l}^s - \frac{1}{2}\phi_1 d\overline{l}^d - \Lambda(\tau(0+) - \overline{\tau}_0) \end{aligned} \tag{A75}$$

where the initial jump of τ is defined by

$$\begin{aligned} \tau(0+) - \overline{\tau}_0 &= A_0 h_{20} + A_1 h_{21} + A_2 h_{22} \\ &= \left(h_{20}\frac{h_{12} - h_{11}}{h_{10} - h_{12}} + h_{22}\frac{h_{11} - h_{10}}{h_{10} - h_{12}} + h_{21}\right) A_1 \\ &= \frac{1}{h_{10} - h_{12}} e^{-r_1 T}\Bigg((h_{22} - h_{20})d\overline{l}^s + (h_{10} - h_{12})d\overline{\tau} \\ &\qquad + (h_{12}h_{20} - h_{10}h_{22})d\overline{l}^d\Bigg) \end{aligned} \tag{A76}$$

Since the jump of τ is a negative function of the length T of the anticipation interval $(0, T)$, the steady state change of the real factor price and the other endogenous variables depends on the time span between the anticipation and the realization of the underlying shock.

The steady state change of the state variables τ, l^d and l^s is now given by the following set of equations:

$$\big(\lambda(f_1 + f_1^* + f_2) + (2c_3 - (a_1 - 2c_1)\psi)\big)d\overline{\tau} = \Bigg(\lambda(f_2 - f_2^*) - (a_1 - 2c_1)(\psi - \psi^*)\Bigg)\left(dp_R^* + \Lambda d\overline{\tau} + \frac{1}{2}\phi_2 d\overline{l}^s - \frac{1}{2}\phi_1 d\overline{l}^d - \Lambda(\tau(0+) - \overline{\tau}_0)\right) \tag{A77}$$

$$d\overline{l}^d + \big(2 - \alpha - \alpha^* - l_1(f_1 + f_1^* + f_2)\big)d\overline{\tau} + l_1(f_2 - f_2^*)\Bigg(dp_R^* + \Lambda d\overline{\tau} + \frac{1}{2}\phi_2 d\overline{l}^s - \frac{1}{2}\phi_1 d\overline{l}^d - \Lambda(\tau(0+) - \overline{\tau}_0)\Bigg) = -l_2(d\dot{m} - d\dot{m}^*) \tag{A78}$$

$$\big(l_2 a_1 \psi - a_2(\alpha - \alpha^*) - (a_2 l_1 + l_2(1 - a_1))(f_1 + f_2 - f_1^*)\big)d\overline{\tau} + a_2 d\overline{l}^s - \big(l_2 a_1(\psi + \psi^*) - (a_2 l_1 + l_2(1 - a_1))(f_2 + f_2^*)\big)\Bigg(dp_R^* + \Lambda d\overline{\tau} + \frac{1}{2}\phi_2 d\overline{l}^s - \frac{1}{2}\phi_1 d\overline{l}^d - \Lambda(\tau(0+) - \overline{\tau}_0)\Bigg) = -a_2 l_2(d\dot{m} + d\dot{m}^*) \tag{A79}$$

In matrix representation the steady state system is of the form

$$\mathbf{F}\begin{pmatrix} d\overline{\tau} \\ d\overline{l}^d \\ d\overline{l}^s \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}\big(dp_R^* - \Lambda(\tau(0+) - \overline{\tau}_0)\big) + \begin{pmatrix} 0 & 0 \\ -l_2 & l_2 \\ -a_2 l_2 & -a_2 l_2 \end{pmatrix}\begin{pmatrix} d\dot{m} \\ d\dot{m}^* \end{pmatrix} \tag{A80}$$

where $\mathbf{F} = (f_{ij})_{1 \le i,j \le 3}$ is defined by

$$f_{11} = \lambda(f_1 + f_1^* + f_2) + (2c_3 - (a_1 - 2c_1)\psi) - (\lambda(f_2 - f_2^*) - (a_2 - 2c_1)(\psi - \psi^*))\Lambda \tag{A81}$$

$$f_{12} = \frac{1}{2}(\lambda(f_2 - f_2^*) - (a_1 - 2c_1)(\psi - \psi^*))\phi_1 \tag{A82}$$

$$f_{13} = -\frac{1}{2}(\lambda(f_2 - f_2^*) - (a_1 - 2c_1)(\psi - \psi^*))\phi_2 \tag{A83}$$

$$f_{21} = 2 - \alpha - \alpha^* - l_1(f_1 + f_1^* + f_2) + l_1(f_2 - f_2^*)\Lambda \tag{A84}$$

$$f_{22} = 1 - \frac{1}{2}l_1(f_2 - f_2^*)\phi_1 \tag{A85}$$

$$f_{23} = \frac{1}{2}l_1(f_2 - f_2^*)\phi_2 \tag{A86}$$

$$f_{31} = \big(l_2 a_1 \psi - a_2(\alpha - \alpha^*) - (a_2 l_1 + l_2(1 - a_1))(f_1 + f_2 - f_1^*)\big) - \big(l_2 a_1(\psi + \psi^*) - (a_2 l_1 + l_2(1 - a_1))(f_2 + f_2^*)\big)\Lambda \tag{A87}$$

$$f_{32} = \frac{1}{2}\big(l_2 a_1(\psi+\psi^*) - (a_2 l_1 + l_2(1-a_1))(f_2+f_2^*)\big)\phi_1 \qquad \text{(A88)}$$

$$f_{33} = a_2 - \frac{1}{2}\big(l_2 a_1(\psi+\psi^*) - (a_2 l_1 + l_2(1-a_1))(f_2+f_2^*)\big)\phi_2 \qquad \text{(A89)}$$

and

$$\omega_1 = \lambda(f_2 - f_2^*) - (a_1 - 2c_1)(\psi - \psi^*) \qquad \text{(A90)}$$

$$\omega_2 = -l_1(f_2 - f_2^*) \qquad \text{(A91)}$$

$$\omega_3 = l_2 a_1(\psi+\psi^*) - (a_2 l_1 + l_2(1-a_1))(f_2+f_2^*) \qquad \text{(A92)}$$

Inserting the equation (A76) for the initial jump of τ leads to the final steady state system

$$\widetilde{\mathbf{F}} \begin{pmatrix} d\overline{\tau} \\ d\overline{l}^d \\ d\overline{l}^s \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} dp_R^* + \begin{pmatrix} 0 & 0 \\ -l_2 & l_2 \\ -a_2 l_2 & -a_2 l_2 \end{pmatrix} \begin{pmatrix} d\dot{m} \\ d\dot{m}^* \end{pmatrix} \qquad \text{(A93)}$$

where $\widetilde{\mathbf{F}} = (f_{ij})_{1\le i,j\le 3}$ is defined by

$$\widetilde{f}_{11} = f_{11} + \omega_1 \Lambda e^{-r_1 T} \qquad \text{(A94)}$$

$$\widetilde{f}_{12} = f_{12} + \omega_1 \Lambda \frac{h_{12}h_{20} - h_{10}h_{22}}{h_{10} - h_{12}} e^{-r_1 T} \qquad \text{(A95)}$$

$$\widetilde{f}_{13} = f_{13} + \omega_1 \Lambda \frac{h_{22} - h_{20}}{h_{10} - h_{12}} e^{-r_1 T} \qquad \text{(A96)}$$

$$\widetilde{f}_{21} = f_{21} + \omega_2 \Lambda e^{-r_1 T} \qquad \text{(A97)}$$

$$\widetilde{f}_{22} = f_{22} + \omega_2 \Lambda \frac{h_{12}h_{20} - h_{10}h_{22}}{h_{10} - h_{12}} e^{-r_1 T} \qquad \text{(A98)}$$

$$\widetilde{f}_{23} = f_{23} + \omega_2 \Lambda \frac{h_{22} - h_{20}}{h_{10} - h_{12}} e^{-r_1 T} \qquad \text{(A99)}$$

$$\widetilde{f}_{31} = f_{31} + \omega_3 \Lambda e^{-r_1 T} \qquad \text{(A100)}$$

$$\widetilde{f}_{32} = f_{32} + \omega_3 \Lambda \frac{h_{12}h_{20} - h_{10}h_{22}}{h_{10} - h_{12}} e^{-r_1 T} \qquad \text{(A101)}$$

$$\widetilde{f}_{33} = f_{33} + \omega_3 \Lambda \frac{h_{22} - h_{20}}{h_{10} - h_{12}} e^{-r_1 T} \qquad \text{(A102)}$$

In the following we use the abbreviations

$$\overline{f}_{11} = \lambda(f_1 + f_1^* + f_2) + 2c_3 - (a_1 - 2c_1)\psi \qquad \text{(A103)}$$

$$\overline{f}_{12} = \frac{1}{2}\big(\lambda(f_2 - f_2^*) - (a_1 - 2c_1)(\psi - \psi^*)\big) = \frac{1}{2}\omega_1 \qquad \text{(A104)}$$

$$\overline{f}_{13} = -\overline{f}_{12} \qquad \text{(A105)}$$

$$\overline{f}_{21} = 2 - \alpha - \alpha^* - l_1(f_1 + f_1^* + f_2) \qquad \text{(A106)}$$

$$\overline{f}_{22} = 1 - \overline{f}_{23} \tag{A107}$$
$$\overline{f}_{23} = \frac{1}{2} l_1 (f_2 - f_2^*) = -\frac{1}{2}\omega_2 \tag{A108}$$
$$\overline{f}_{31} = l_2 a_1 \psi - a_2(\alpha - \alpha^*) - (a_2 l_1 + l_2(1-a_1))(f_1 + f_2 - f_1^*) \tag{A109}$$
$$\overline{f}_{32} = \frac{1}{2}\big(l_2 a_1(\psi + \psi^*) - (a_2 l_1 + l_2(1-a_1))(f_2 + f_2^*)\big) = \frac{1}{2}\omega_3 \tag{A110}$$
$$\overline{f}_{33} = a_2 - \overline{f}_{32} \tag{A111}$$

The matrix $\overline{\mathbf{F}} = (\overline{f}_{ij})_{1\leq i,j\leq 3}$ is the steady state matrix in the special case that there is no endogenous commodity pricing rule, i.e., $\dot{p}_R^* = 0$ (cf. Wohltmann and Winkler (2005 a)). The determinant $|\overline{\mathbf{F}}|$ is given by

$$|\overline{\mathbf{F}}| = \overline{f}_{12}\overline{f}_{31} - \overline{f}_{11}\overline{f}_{32} + a_2(\overline{f}_{11}\overline{f}_{22} - \overline{f}_{12}\overline{f}_{21}) \tag{A112}$$

and is always positive. The determinant $|\mathbf{F}|$ of the matrix $\mathbf{F} = (f_{ij})_{1\leq i,j\leq 3}$ can be expressed in the following form:

$$\begin{aligned} |\mathbf{F}| &= a_2\Big(\overline{f}_{11}(1 - (\phi_1 - \phi_2)) - 2\overline{f}_{12}\Lambda + (\overline{f}_{11}\overline{f}_{22} - \overline{f}_{12}\overline{f}_{21})(\phi_1 - \phi_2)\Big) \\ &\quad + \phi_2\big(|\overline{\mathbf{F}}| - a_2\overline{f}_{11}\big) \end{aligned} \tag{A113}$$

where

$$\phi_1 - \phi_2 = \frac{2\beta\Lambda}{\kappa_3\delta}$$

and Λ, ϕ_1 and ϕ_2 are defined by (A9), (A10) and (A11). The first expression of $|\mathbf{F}|$, i.e., $a_2(\dots)$, is in general positive (and typically very small), while the second expression may be negative since $\phi_2 \gtrless 0$. For plausible parameter values $|\mathbf{F}| > a_2\overline{f}_{11}$ so that the condition $\phi_2 > 0$, i.e.

$$\beta_R^* > \delta_R^*\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*}\right) \tag{A114}$$

is sufficient (but not necessary) for $|\mathbf{F}| > 0$.[33] Henceforth, we assume a sufficiently small value of δ_R^* so that $|\mathbf{F}|$ is positive (but smaller than $|\overline{\mathbf{F}}|$). A sufficiently small value of δ_R^* also guarantees $|\widetilde{\mathbf{F}}| > 0$, since the difference between $|\mathbf{F}|$ and $|\widetilde{\mathbf{F}}|$ is very small. The steady state system (A93) has the following solution:

$$\begin{pmatrix} d\overline{\tau} \\ d\overline{l}^d \\ d\overline{l}^s \end{pmatrix} = \frac{1}{|\widetilde{\mathbf{F}}|}\begin{pmatrix} \mu_{11} & -\mu_{21} & \mu_{31} \\ -\mu_{12} & \mu_{22} & -\mu_{32} \\ \mu_{13} & -\mu_{23} & \mu_{33} \end{pmatrix}\begin{pmatrix} 2\overline{f}_{12} & 0 & 0 \\ -2\overline{f}_{23} & -l_2 & l_2 \\ 2\overline{f}_{32} & -a_2 l_2 & -a_2 l_2 \end{pmatrix}\begin{pmatrix} dp_R^* \\ d\dot{m} \\ d\dot{m}^* \end{pmatrix} \tag{A115}$$

[33]Even in the special case $\beta_R^* = 0$ the determinant $|\mathbf{F}|$ is positive if δ_R^* is sufficiently small.

where the cofactors μ_{ij} of the steady state matrix $\widetilde{\mathbf{F}}$ are given by

$$\mu_{11} = \widetilde{f}_{22}\widetilde{f}_{33} - \widetilde{f}_{23}\widetilde{f}_{32} = a_2(1-\overline{f}_{23}\Gamma_1) - \overline{f}_{32}\Gamma_2 \quad \text{(A116)}$$
$$\mu_{12} = \widetilde{f}_{21}\widetilde{f}_{33} - \widetilde{f}_{23}\widetilde{f}_{31} = a_2(\overline{f}_{21} + 2\overline{f}_{23}\Gamma_3) - \Gamma_2(\overline{f}_{21}\overline{f}_{32} + \overline{f}_{23}\overline{f}_{31}) \quad \text{(A117)}$$
$$\mu_{13} = \widetilde{f}_{21}\widetilde{f}_{32} - \widetilde{f}_{22}\widetilde{f}_{31} = (\overline{f}_{21}\overline{f}_{32} + \overline{f}_{23}\overline{f}_{31})\Gamma_1 - \overline{f}_{31} + 2\overline{f}_{32}\Gamma_3 \quad \text{(A118)}$$
$$\mu_{21} = \widetilde{f}_{12}\widetilde{f}_{33} - \widetilde{f}_{13}\widetilde{f}_{32} = a_2\overline{f}_{12}\Gamma_1 \quad \text{(A119)}$$
$$\mu_{22} = \widetilde{f}_{11}\widetilde{f}_{33} - \widetilde{f}_{13}\widetilde{f}_{31} = a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) + \Gamma_2(\overline{f}_{12}\overline{f}_{31} - \overline{f}_{11}\overline{f}_{32}) \quad \text{(A120)}$$
$$\mu_{23} = \widetilde{f}_{11}\widetilde{f}_{32} - \widetilde{f}_{12}\widetilde{f}_{31} = \Gamma_1(\overline{f}_{11}\overline{f}_{32} - \overline{f}_{12}\overline{f}_{31}) \quad \text{(A121)}$$
$$\mu_{31} = \widetilde{f}_{12}\widetilde{f}_{23} - \widetilde{f}_{13}\widetilde{f}_{22} = \overline{f}_{12}\Gamma_2 \quad \text{(A122)}$$
$$\mu_{32} = \widetilde{f}_{11}\widetilde{f}_{23} - \widetilde{f}_{13}\widetilde{f}_{21} = \Gamma_2(\overline{f}_{11}\overline{f}_{23} + \overline{f}_{12}\overline{f}_{21}) \quad \text{(A123)}$$
$$\mu_{33} = \widetilde{f}_{11}\widetilde{f}_{22} - \widetilde{f}_{12}\widetilde{f}_{21} = \overline{f}_{11} - 2\overline{f}_{12}\Gamma_3 - \Gamma_1(\overline{f}_{11}\overline{f}_{23} + \overline{f}_{12}\overline{f}_{21}) \quad \text{(A124)}$$

with

$$\Gamma_1 = \phi_1 + 2\Lambda\frac{h_{12}h_{20} - h_{10}h_{22}}{h_{10} - h_{12}}e^{-r_1T} \quad \text{(A125)}$$
$$\Gamma_2 = \phi_2 - 2\Lambda\frac{h_{22} - h_{20}}{h_{10} - h_{12}}e^{-r_1T} \quad \text{(A126)}$$
$$\Gamma_3 = \Lambda\left(1 - e^{-r_1T}\right) \quad \text{(A127)}$$

Note that the determinant $|\widetilde{\mathbf{F}}|$ has the same structure as $|\mathbf{F}|$ with ϕ_1, ϕ_2 and Λ replaced by Γ_1, Γ_2 and Γ_3 respectively:

$$|\widetilde{\mathbf{F}}| = a_2\Big(\overline{f}_{11}(1-(\Gamma_1-\Gamma_2)) - 2\overline{f}_{12}\Gamma_3 + (\overline{f}_{11}\overline{f}_{22} - \overline{f}_{12}\overline{f}_{21})(\Gamma_1-\Gamma_2)\Big) \quad \text{(A128)}$$
$$+ \Gamma_2\big(|\overline{\mathbf{F}}| - a_2\overline{f}_{11}\big)$$

Note further that $|\widetilde{\mathbf{F}}| = |\overline{\mathbf{F}}|$ if $\beta_R^* = 1$ and $\delta_R^* = 0$. Since

$$\widetilde{f}_{11} = \overline{f}_{11} - 2\overline{f}_{12}\Gamma_3 \quad \text{(A129)}$$
$$\widetilde{f}_{12} = \overline{f}_{12}\Gamma_1 \quad \text{(A130)}$$
$$\widetilde{f}_{13} = -\overline{f}_{12}\Gamma_2 \quad \text{(A131)}$$
$$\widetilde{f}_{21} = \overline{f}_{21} + 2\overline{f}_{23}\Gamma_3 \quad \text{(A132)}$$
$$\widetilde{f}_{22} = 1 - \overline{f}_{23}\Gamma_1 \quad \text{(A133)}$$
$$\widetilde{f}_{23} = \overline{f}_{23}\Gamma_2 \quad \text{(A134)}$$
$$\widetilde{f}_{31} = \overline{f}_{31} - 2\overline{f}_{32}\Gamma_3 \quad \text{(A135)}$$
$$\widetilde{f}_{32} = \overline{f}_{32}\Gamma_1 \quad \text{(A136)}$$
$$\widetilde{f}_{33} = a_2 - \overline{f}_{32}\Gamma_2 \quad \text{(A137)}$$

we get the following steady state multipliers:

$$
\frac{d\overline{\tau}}{dp_R^*} = \frac{2\overline{f}_{12}a_2}{|\widetilde{\mathbf{F}}|} > 0 \quad \Leftrightarrow \quad |\widetilde{\mathbf{F}}| > 0 \text{ and } \overline{f}_{12} > 0 \text{ , i.e.} \tag{A138}
$$

$\lambda(f_2 - f_2^*) > (a_1 - 2c_1)(\psi - \psi^*)$[34]

$$
\frac{d\overline{l}^d}{dp_R^*} = \frac{-2a_2}{|\widetilde{\mathbf{F}}|}\big(\overline{f}_{12}\overline{f}_{21} + \overline{f}_{23}\overline{f}_{11}\big) < 0 \quad \text{in general} \tag{A139}
$$

$$
\frac{d\overline{l}^s}{dp_R^*} = \frac{2}{|\widetilde{\mathbf{F}}|}\big(\overline{f}_{32}\overline{f}_{11} - \overline{f}_{12}\overline{f}_{31}\big) < 0 \quad \text{in general} \tag{A140}
$$

$$
\frac{d\overline{\tau}}{d\dot{m}} = \frac{l_2\overline{f}_{12}a_2}{|\widetilde{\mathbf{F}}|}(\Gamma_1 - \Gamma_2) > 0 \quad \text{in general, where} \tag{A141}
$$

$$
\Gamma_1 - \Gamma_2 = 2\Lambda\left(\frac{\beta}{\kappa_3\delta} + \frac{h_{20}(h_{12}-1) + h_{22}(1-h_{10})}{h_{10}-h_{12}} \cdot e^{-r_1T}\right)
$$

$$
\begin{aligned}
\frac{d\overline{l}^d}{d\dot{m}} &= \frac{l_2}{|\widetilde{\mathbf{F}}|}\Bigg(- a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) + \Gamma_2\bigg(a_2(\overline{f}_{11}\overline{f}_{23} + \overline{f}_{12}\overline{f}_{21}) \\
&\qquad + \overline{f}_{11}\overline{f}_{32} - \overline{f}_{12}\overline{f}_{31}\bigg)\Bigg) \\
&= \frac{l_2}{|\widetilde{\mathbf{F}}|}\Big(a_2\big(\overline{f}_{11}(\Gamma_2 - 1) + 2\overline{f}_{12}\Gamma_3\big) - \Gamma_2|\overline{\mathbf{F}}|\Big) \\
&= \frac{l_2}{|\widetilde{\mathbf{F}}|}\big(- a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) - \Gamma_2(|\overline{\mathbf{F}}| - a_2\overline{f}_{11})\big) < 0 \quad \text{in general}
\end{aligned} \tag{A142}
$$

$$
\begin{aligned}
\frac{d\overline{l}^s}{d\dot{m}} &= \frac{l_2}{|\widetilde{\mathbf{F}}|}\Bigg(- a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) + \Gamma_1\bigg(a_2(\overline{f}_{11}\overline{f}_{23} + \overline{f}_{12}\overline{f}_{21}) \\
&\qquad + \overline{f}_{11}\overline{f}_{32} - \overline{f}_{12}\overline{f}_{31}\bigg)\Bigg) \\
&= \frac{l_2}{|\widetilde{\mathbf{F}}|}\Big(a_2\big(\overline{f}_{11}(\Gamma_1 - 1) + 2\overline{f}_{12}\Gamma_3\big) - \Gamma_1|\overline{\mathbf{F}}|\Big) \\
&= \frac{l_2}{|\widetilde{\mathbf{F}}|}\big(- a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) - \Gamma_1(|\overline{\mathbf{F}}| - a_2\overline{f}_{11})\big) < 0 \quad \text{in general}
\end{aligned} \tag{A143}
$$

$$
\frac{d\overline{\tau}}{d\dot{m}^*} = -\frac{l_2\overline{f}_{12}a_2}{|\widetilde{\mathbf{F}}|}(\Gamma_1 + \Gamma_2) \quad \text{where} \tag{A144}
$$

$$
\Gamma_1 + \Gamma_2 = \phi_1 + \phi_2 + 2\Lambda\frac{h_{20}(1+h_{12}) - h_{22}(1+h_{10})}{h_{10}-h_{12}} \cdot e^{-r_1T}
$$

$$
\phi_1 + \phi_2 = \frac{2}{1+\kappa_4\delta_R^*}\left(\beta_R^* - \frac{\delta_R^*\beta^*}{\delta^*}\right)
$$

$$
\begin{aligned}
\frac{d\overline{l}^d}{d\dot{m}^*} &= \frac{l_2}{|\widetilde{\mathbf{F}}|}\Bigg(a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) + \Gamma_2\bigg(a_2(\overline{f}_{11}\overline{f}_{23} + \overline{f}_{12}\overline{f}_{21}) \\
&\qquad + \overline{f}_{12}\overline{f}_{31} - \overline{f}_{11}\overline{f}_{32}\bigg)\Bigg)
\end{aligned} \tag{A145}
$$

[33]See Wohltmann and Winkler (2005 a) for the discussion of this condition.

$$\frac{d\bar{l}^s}{d\dot{m}^*} = \frac{l_2}{|\widetilde{\mathbf{F}}|}\Bigg(- a_2(\overline{f}_{11} - 2\overline{f}_{12}\Gamma_3) + \Gamma_1\Bigg(a_2(\overline{f}_{11}\overline{f}_{23} + \overline{f}_{12}\overline{f}_{21}) + \overline{f}_{12}\overline{f}_{31} - \overline{f}_{11}\overline{f}_{32}\Bigg)\Bigg) \quad \text{(A146)}$$

The multipliers with respect to domestic and foreign real money stock can be obtained by determining the arithmetic mean of $d\bar{l}^s$ and $d\bar{l}^d$. We can rewrite the formula (A75) for the steady state change of the foreign real factor price in the following form:

$$\begin{aligned} d(\overline{p_R^* - p^*}) &= dp_R^* + \Lambda(1 - e^{-r_1T})d\overline{\tau} + \frac{1}{2}\left(\phi_2 - 2\Lambda\frac{h_{22} - h_{20}}{h_{10} - h_{12}}e^{-r_1T}\right)d\bar{l}^s \\ &\quad - \frac{1}{2}\left(\phi_1 + 2\Lambda\frac{h_{12}h_{20} - h_{10}h_{22}}{h_{10} - h_{12}}e^{-r_1T}\right)d\bar{l}^d \\ &= dp_R^* + \Gamma_3 d\overline{\tau} + \frac{1}{2}\Gamma_2 d\bar{l}^s - \frac{1}{2}\Gamma_1 d\bar{l}^d \end{aligned} \quad \text{(A147)}$$

This implies

$$\frac{d(\overline{p_R^* - p^*})}{dp_R^*} = 1 + \frac{1}{|\widetilde{\mathbf{F}}|}\Bigg(2\overline{f}_{12}a_2\Gamma_3 + \Gamma_2(\overline{f}_{32}\overline{f}_{11} - \overline{f}_{12}\overline{f}_{31}) + a_2\Gamma_1(\overline{f}_{12}\overline{f}_{21} + \overline{f}_{23}\overline{f}_{11})\Bigg) = \frac{a_2\overline{f}_{11}}{|\widetilde{\mathbf{F}}|} > 0 \quad \text{(A148)}$$

$$\begin{aligned} \frac{d(\overline{p_R^* - p^*})}{d\dot{m}} &= \Gamma_3\frac{d\overline{\tau}}{d\dot{m}} + \frac{1}{2}\Gamma_2\frac{d\bar{l}^s}{d\dot{m}} - \frac{1}{2}\Gamma_1\frac{d\bar{l}^d}{d\dot{m}} \\ &= \frac{1}{2}\frac{l_2a_2\overline{f}_{11}}{|\widetilde{\mathbf{F}}|}\big(\Gamma_1 - \Gamma_2\big) > 0 \end{aligned} \quad \text{(A149)}$$

$$\frac{d(\overline{p_R^* - p^*})}{d\dot{m}^*} = -\frac{1}{2}\frac{l_2a_2\overline{f}_{11}}{|\widetilde{\mathbf{F}}|}\big(\Gamma_1 + \Gamma_2\big) \quad \text{(A150)}$$

An expansionary domestic monetary policy is connected with a steady state rise in the foreign real factor price and generally leads to a small increase in the domestic terms of trade $\overline{\tau}$. Therefore, an increase in the growth rate of domestic money stock causes a steady state *decline* in domestic and foreign real output and income! By contrast, expansionary foreign monetary policy induces generally a fall in $(\overline{p_R^* - p^*})$ and $\overline{\tau}$ so that the long run change of q, q^*, y and y^* is positive. This is the case if the constant $\Gamma_1 + \Gamma_2$ is positive. The formula for $\Gamma_1 + \Gamma_2$ shows that $\Gamma_1 + \Gamma_2$ may also be negative, in particular, if $\phi_1 + \phi_2$ is negative, leading to a rise in $(\overline{p_R^* - p^*})$ and an output contraction in both economies. This will be the case if the commodity pricing rule (15) is characterized by a small value of β_R^* ($\beta_R^* = 0$ in particular) and a sufficiently large value of δ_R^*. Note that all steady state multipliers of a once-and-for-all commodity price increase are (in absolute terms) increased if the growth rate of the materials price $\dot{p}_R^*$ is not exogenously given (as in Wohltmann and Winkler

(2005 a)) but is endogenized according to the pricing rule (15). In this case the only difference between corresponding steady state multipliers is that the determinant $|\overline{\mathbf{F}}|$ has to be replaced by $|\widetilde{\mathbf{F}}|$ which is smaller than $|\overline{\mathbf{F}}|$. This implies that the steady state stagflationary effects of an oil price shock are magnified if $\dot{p}_R^*$ is coupled with the foreign inflation rate $\dot{p}^*$ and the world output gap $q + q^* - (\overline{q} + \overline{q}^*)$.

The monetary policy multipliers as well as the multiplier of a commodity price shock with respect to $\overline{\tau}$ and $(\overline{p_R^* - p^*})$ imply

$$\frac{d\overline{\tau}/d\dot{m}}{d(\overline{p_R^* - p^*})/d\dot{m}} = \frac{2\overline{f}_{12}}{\overline{f}_{11}} = \frac{d\overline{\tau}/d\dot{m}^*}{d(\overline{p_R^* - p^*})/d\dot{m}^*} = \frac{d\overline{\tau}/dp_R^*}{d(\overline{p_R^* - p^*})/dp_R^*} \quad \text{(A151)}$$

i.e., the equality of the relative multipliers of domestic monetary policy, foreign monetary policy and a materials price shock with respect to $(\overline{\tau}, \overline{p_R^* - p^*})$.[35] Since

$$\begin{aligned}
d\overline{q} &= (f_1 + f_2)d\overline{\tau} - f_2 d(\overline{p_R^* - p^*}) && \text{(A152)}\\
d\overline{q}^* &= -f_1^* d\overline{\tau} - f_2^* d(\overline{p_R^* - p^*}) && \text{(A153)}\\
d(\overline{p_R^* + e - p}) &= -d\overline{\tau} + d(\overline{p_R^* - p^*}) && \text{(A154)}\\
d\overline{y} &= d\overline{q} - \psi d(\overline{p_R^* + e - p}) && \text{(A155)}\\
d\overline{y}^* &= d\overline{q}^* - \psi^* d(\overline{p_R^* - p^*}) && \text{(A156)}\\
d\overline{i}_r^s &= \frac{1}{a_2}\Big(-(1-a_1)(d\overline{q} + d\overline{q}^*) + a_1\psi d\overline{\tau} && \text{(A157)}\\
&\qquad - a_1(\psi + \psi^*)d(\overline{p_R^* - p^*})\Big)
\end{aligned}$$

the steady state change of any variable $x_i \in \{q, q^*, p_R^* + e - p, y, y^*, i_r^s\}$ is representable as a linear combination of $d\overline{\tau}$ and $d(\overline{p_R^* - p^*})$, i.e.,

$$d\overline{x}_i = \alpha_{i1} d\overline{\tau} + \alpha_{i2} d(\overline{p_R^* - p^*}) \quad \text{(A158)}$$

We then have

$$\frac{d\overline{x}_i}{d\dot{m}} = \frac{l_2 a_2}{|\widetilde{\mathbf{F}}|}\left(\alpha_{i1}\overline{f}_{12} + \frac{1}{2}\alpha_{i2}\overline{f}_{11}\right)(\Gamma_1 - \Gamma_2) \quad \text{(A159)}$$

$$\begin{aligned}
\frac{d\overline{x}_i}{d\dot{m}^*} &= -\frac{l_2 a_2}{|\widetilde{\mathbf{F}}|}\left(\alpha_{i1}\overline{f}_{12} + \frac{1}{2}\alpha_{i2}\overline{f}_{11}\right)(\Gamma_1 + \Gamma_2) && \text{(A160)}\\
\frac{d\overline{x}_i}{dp_R^*} &= \frac{2a_2}{|\widetilde{\mathbf{F}}|}\left(\alpha_{i1}\overline{f}_{12} + \frac{1}{2}\alpha_{i2}\overline{f}_{11}\right) && \text{(A161)}
\end{aligned}$$

[35] Note that the relative multiplier of domestic monetary policy is only defined if $\delta_R^* > 0$ (so that $\Lambda > 0$); otherwise, $d\overline{\tau}/d\dot{m} = d(\overline{p_R^* - p^*})/d\dot{m} = 0$.

so that the relative multipliers

$$\frac{d\overline{x}_i/d\dot{m}}{d\overline{x}_j/d\dot{m}} \;=\; \frac{\alpha_{i1}\overline{f}_{12}+\frac{1}{2}\alpha_{i2}\overline{f}_{11}}{\alpha_{j1}\overline{f}_{12}+\frac{1}{2}\alpha_{j2}\overline{f}_{11}} \;=\; \frac{d\overline{x}_i/d\dot{m}^*}{d\overline{x}_j/d\dot{m}^*} \;=\; \frac{d\overline{x}_i/dp_R^*}{d\overline{x}_j/dp_R^*} \tag{A162}$$

are identical for any $\overline{x}_i, \overline{x}_j \in \{\overline{\tau}, \overline{p_R^*-p^*}, \overline{q}, \overline{q}^*, \overline{p_R^*+e-p}, \overline{y}, \overline{y}^*, \overline{i}_r^s\}$.

Stabilization of the Steady State Output

An implication of the equality of the relative multipliers (A162) is that the economic policy target to fix simultaneously the steady state output variables $\overline{q}$ and $\overline{q}^*$ at their respective initial steady state level ($d\overline{q} = 0 = d\overline{q}^*$) is attainable with the help of only *one* policy instrument. The target

$$d\overline{q} \;=\; \frac{\partial\overline{q}}{\partial p_R^*}dp_R^* + \frac{\partial\overline{q}}{\partial\dot{m}}d\dot{m} + \frac{\partial\overline{q}}{\partial\dot{m}^*}d\dot{m}^* \;=\; 0 \tag{A163}$$

implies

$$d\dot{m} \;=\; -\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}}\cdot dp_R^* \;=\; \frac{-2}{l_2(\Gamma_1-\Gamma_2)}\cdot dp_R^* < 0 \tag{A164}$$

(if $d\dot{m}^* = 0$) and

$$d\dot{m}^* \;=\; -\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}^*}\cdot dp_R^* \;=\; \frac{2}{l_2(\Gamma_1+\Gamma_2)}\cdot dp_R^* > 0 \tag{A165}$$

(if $d\dot{m} = 0$) while the target

$$d\overline{q}^* \;=\; \frac{\partial\overline{q}^*}{\partial p_R^*}dp_R^* + \frac{\partial\overline{q}^*}{\partial\dot{m}}d\dot{m} + \frac{\partial\overline{q}^*}{\partial\dot{m}^*}d\dot{m}^* \;=\; 0 \tag{A166}$$

leads to the decision rules

$$d\dot{m} \;=\; -\frac{\partial\overline{q}^*/\partial p_R^*}{\partial\overline{q}^*/\partial\dot{m}}\cdot dp_R^* \;=\; -\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}}\cdot dp_R^* \tag{A167}$$

(in case $d\dot{m}^* = 0$) and

$$d\dot{m}^* \;=\; -\frac{\partial\overline{q}^*/\partial p_R^*}{\partial\overline{q}^*/\partial\dot{m}^*}\cdot dp_R^* \;=\; -\frac{\partial\overline{q}/\partial p_R^*}{\partial\overline{q}/\partial\dot{m}^*}\cdot dp_R^* \tag{A168}$$

(if $d\dot{m} = 0$) which are equivalent to (A164) and (A165) respectively. Equality of (A164) and (A167) and (A165) and (A168) respectively is given since the relative output multipliers are identical. The target $d\overline{q} = d\overline{q}^* = 0$ is also achievable with the help of the policy combination

$$d\dot{m}^* \;=\; -\gamma^* \frac{\partial \overline{q}/\partial p_R^*}{\partial \overline{q}/\partial \dot{m}^*} \cdot dp_R^* \;=\; \frac{2\gamma^*}{l_2(\Gamma_1 + \Gamma_2)} \cdot dp_R^* \tag{A169}$$

$$d\dot{m} \;=\; -(1-\gamma^*) \frac{\partial \overline{q}/\partial p_R^*}{\partial \overline{q}/\partial \dot{m}} \cdot dp_R^* \;=\; -\frac{2(1-\gamma^*)}{l_2(\Gamma_1 - \Gamma_2)} \cdot dp_R^* \tag{A170}$$

with $0 < \gamma^* < 1$. If $\gamma^* < 1$, the inflationary effects caused by $d\dot{m}^* > 0$ are smaller than in the polar case $\gamma^* = 1$. The same holds for the deflationary effects caused by $d\dot{m} < 0$ in case $\gamma^* > 0$ compared to $\gamma^* = 0$. The constant γ^* can be determined if the condition $d\dot{m} = -d\dot{m}^*$ is required to hold. Then

$$\gamma^* = \frac{\partial \overline{q}/\partial \dot{m}^*}{\dfrac{\partial \overline{q}}{\partial \dot{m}^*} - \dfrac{\partial \overline{q}}{\partial \dot{m}}} \tag{A171}$$

and

$$d\dot{m}^* = -\frac{\partial \overline{q}/\partial p_R^*}{\dfrac{\partial \overline{q}}{\partial \dot{m}^*} - \dfrac{\partial \overline{q}}{\partial \dot{m}}} \cdot dp_R^* \;=\; -d\dot{m} \;=\; \frac{1}{l_2\Gamma_1} dp_R^* \tag{A172}$$

Note that the output target $d\overline{q} = 0 = d\overline{q}^*$ is attainable by any policy combination $(d\dot{m}, d\dot{m}^*)$ satisfying the linear relationship

$$\begin{aligned} d\dot{m} \;&=\; -\frac{\partial \overline{q}/\partial p_R^*}{\partial \overline{q}/\partial \dot{m}} dp_R^* - \frac{\partial \overline{q}/\partial \dot{m}^*}{\partial \overline{q}/\partial \dot{m}} d\dot{m}^* \\ &=\; -\frac{2}{l_2(\Gamma_1 - \Gamma_2)} dp_R^* + \frac{\Gamma_1 + \Gamma_2}{\Gamma_1 - \Gamma_2} d\dot{m}^* \end{aligned} \tag{A173}$$

This follows either from equation (A163) or from (A169), (A170) by solving (A169) for γ^* and then inserting this expression into (A170). In general, $\Gamma_1 + \Gamma_2 > \Gamma_1 - \Gamma_2 > 0$ holds; the policy trade-off curve (A173) has therefore a positive slope in $(d\dot{m}^*/d\dot{m})$-space.

Inflation Rate Stabilization

The stabilization of the consumer inflation rates $\dot{p}^c$ and $\dot{p}^{*c}$ with the aid of suitable monetary policy rules leads to the following decision rules: Since

$$\begin{aligned} \dot{p}^c \;&=\; (\dot{p}^c - \dot{p}) + (\dot{p} - \dot{m}) + \dot{m} \\ &=\; -(1-\alpha)\dot{\tau} - \frac{1}{2}\dot{l}^s - \frac{1}{2}\dot{l}^d + \dot{m} \end{aligned} \tag{A174}$$

the target $\dot{p}^c = 0$ is attainable if

$$\dot{m} \;=\; (1-\alpha)\dot{\tau} + \frac{1}{2}\dot{l}^s + \frac{1}{2}\dot{l}^d \tag{A175}$$

Similarly, the target $\dot{p}^{*c} = 0$ implies

$$\dot{m}^* = -(1-\alpha^*)\dot{\tau} + \frac{1}{2}\dot{l}^s - \frac{1}{2}\dot{l}^d \tag{A176}$$

Then

$$\dot{m} - \dot{m}^* = (2-\alpha-\alpha^*)\dot{\tau} + \dot{l}^d \tag{A177}$$

and

$$\dot{m} + \dot{m}^* = (\alpha^*-\alpha)\dot{\tau} + \dot{l}^s \tag{A178}$$

The endogenization of the growth rate of domestic and foreign money supply causes a change in the system matrix $\mathbf{B}$ of the state space representation (A37). In the case of simultaneous inflation rate stabilization the matrix $\mathbf{B}$ has to be replaced by $\mathbf{B}' = (b_{ij})_{1\leq i,j\leq 3}$ where

$$b'_{11} = b_{11} - a_2 l_2 = -\frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*} + \kappa_4\phi_2\right) \tag{A179}$$

$$b'_{12} = b_{12} + a_2 l_2(\alpha-\alpha^*) = \big(a_2 l_1 + l_2(1-a_1)\big)\big(\kappa_3 - \kappa_4\Lambda\big) \tag{A180}$$

$$b'_{13} = b_{13} = -\frac{1}{2}\big(a_2 l_1 + l_2(1-a_1)\big)\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*} - \kappa_4\phi_1\right) \tag{A181}$$

$$b'_{21} = b_{21} = -\frac{1}{2}\lambda\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*} + \kappa_2\phi_2\right) \tag{A182}$$

$$b'_{22} = b_{22} = \lambda(\kappa_1 - \kappa_2\Lambda) + a_2(1-(\alpha+\alpha^*)) \tag{A183}$$

$$b'_{23} = b_{23} = -\frac{1}{2}\lambda\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*} - \kappa_2\phi_1\right) \tag{A184}$$

$$b'_{31} = b_{31} = -\frac{1}{2}l_1\left(\frac{\beta}{\delta} - \frac{\beta^*}{\delta^*} + \kappa_2\phi_2\right) \tag{A185}$$

$$b'_{32} = b_{32} - l_2(2-\alpha-\alpha^*) = l_1(\kappa_1 - \kappa_2\Lambda) + l_2(-1+\alpha+\alpha^*) \tag{A186}$$

$$b'_{33} = b_{33} - l_2 = -\frac{1}{2}l_1\left(\frac{\beta}{\delta} + \frac{\beta^*}{\delta^*} - \kappa_2\phi_1\right) \tag{A187}$$

Note that the matrix $\mathbf{C}$ in (A37) remains unchanged. The monetary policy rules (A175) and (A176) imply that the level of domestic and foreign money supply is given by

$$m = \overline{m}_0 + (1-\alpha)(\tau - \tau(0+)) + \frac{1}{2}(l^s - \overline{l}^s_0) + \frac{1}{2}(l^d - \overline{l}^d_0) \tag{A188}$$

$$m^* = \overline{m}^*_0 - (1-\alpha^*)(\tau - \tau(0+)) + \frac{1}{2}(l^s - \overline{l}^s_0) - \frac{1}{2}(l^d - \overline{l}^d_0) \tag{A189}$$

The formulas for m and m^* result on the condition that m and m^* are predetermined variables which are continuous at time $t = 0$. Although $\overline{\dot{m}} = \overline{\dot{m}^*} = 0$ in the long run, the policy rules for $\dot{m}$ and $\dot{m}^*$ are connected with a permanent change of domestic and foreign money stock:

$$
\begin{aligned}
d\overline{m} &= (1-\alpha)d\overline{\tau} + \frac{1}{2}d\overline{l}^s + \frac{1}{2}d\overline{l}^d - (1-\alpha)(\tau(0+) - \overline{\tau}_0) \qquad \text{(A190)} \\
&= (1-\alpha)\left(1-e^{-r_1T}\right)d\overline{\tau} + \left(\frac{1}{2} - (1-\alpha)\frac{h_{22}-h_{20}}{h_{10}-h_{12}}e^{-r_1T}\right)d\overline{l}^s \\
&\quad + \left(\frac{1}{2} - (1-\alpha)\frac{h_{12}h_{20}-h_{10}h_{22}}{h_{10}-h_{12}}e^{-r_1T}\right)d\overline{l}^d
\end{aligned}
$$

$$
\begin{aligned}
d\overline{m}^* &= -(1-\alpha^*)d\overline{\tau} + \frac{1}{2}d\overline{l}^s - \frac{1}{2}d\overline{l}^d + (1-\alpha^*)(\tau(0+) - \overline{\tau}_0) \qquad \text{(A191)} \\
&= -(1-\alpha^*)\left(1-e^{-r_1T}\right)d\overline{\tau} + \left(\frac{1}{2} + (1-\alpha^*)\frac{h_{22}-h_{20}}{h_{10}-h_{12}}e^{-r_1T}\right)d\overline{l}^s \\
&\quad - \left(\frac{1}{2} - (1-\alpha^*)\frac{h_{12}h_{20}-h_{10}h_{22}}{h_{10}-h_{12}}e^{-r_1T}\right)d\overline{l}^d
\end{aligned}
$$

In the case of the commodity price shock $dp_R^* > 0$ the steady state change of $\overline{m}$ and $\overline{m}^*$ is negative so that the permanent price increase induced by the oil price shock is reduced. Note that the steady state effects of the oil price shock on the state vector $(\tau, l^d, l^s)'$ change in the case of inflation rate stabilization. Although the elements f_{ij} of the matrix $\mathbf{F}$ in (A80) remain unchanged there is a quantitative change in the elements of $\widetilde{\mathbf{F}}$ in (A93). The reason is that the stabilized system causes adjustments in the eigenvalues and eigenvectors leading to a change of the initial jump of τ.

Perfect Stabilization of the Output Time Path

Instead of inflation rate stabilization, monetary policy is also able to fix the output vector $(q, q^*)'$ at its initial steady state level $(\overline{q}_0, \overline{q}_0^*)'$ at all times, i.e. $q = \overline{q}_0$, $q^* = \overline{q}_0^*$ for all $t \geq 0$. The following output equations can be derived from the equations of the aggregate and the difference system:

$$
\begin{pmatrix} q - \overline{q} \\ q^* - \overline{q}^* \end{pmatrix} = \mathbf{D} \begin{pmatrix} \dot{l}^s \\ \dot{\tau} \\ \dot{l}^d \end{pmatrix} \qquad \text{(A192)}
$$

where $\mathbf{D} = (d_{ij})_{\substack{1\leq i\leq 2\\ 1\leq j\leq 3}}$ is given by

$$d_{11} = -\frac{1}{2\delta}\left(\beta + \frac{1-\mu}{\mu}\phi_2\right) \tag{A193}$$

$$d_{12} = \frac{1}{\delta}\left((1-\beta)(1-\alpha) + \frac{1-\mu}{\mu}(\delta - \Lambda)\right) \tag{A194}$$

$$d_{13} = -\frac{1}{2\delta}\left(\beta - \frac{1-\mu}{\mu}\phi_1\right) \tag{A195}$$

$$d_{21} = -\frac{1}{2\delta^*}\left(\beta^* + \frac{1-\mu^*}{\mu^*}\phi_2\right) \tag{A196}$$

$$d_{22} = -\frac{1}{\delta^*}\left((1-\beta^*)(1-\alpha^*) + \frac{1-\mu^*}{\mu^*}\right) \tag{A197}$$

$$d_{23} = \frac{1}{2\delta^*}\left(\beta^* + \frac{1-\mu^*}{\mu^*}\phi_1\right) \tag{A198}$$

The state equations can be represented in the form (cf. (A37))

$$\mathbf{B}\begin{pmatrix} \dot{l}^s \\ \dot{\tau} \\ \dot{l}^d \end{pmatrix} = \mathbf{C}\begin{pmatrix} l^s - \overline{l}^s \\ \tau - \overline{\tau} \\ l^d - \overline{l}^d \end{pmatrix} + \mathbf{K}\begin{pmatrix} \dot{m} - \overline{\dot{m}} \\ \dot{m}^* - \overline{\dot{m}^*} \end{pmatrix} \tag{A199}$$

where $\mathbf{K} = (k_{ij})_{\substack{1\leq i\leq 3\\ 1\leq j\leq 2}}$ is given by

$$k_{11} = a_2 l_2 \tag{A200}$$

$$k_{12} = a_2 l_2 \tag{A201}$$

$$k_{21} = 0 \tag{A202}$$

$$k_{22} = 0 \tag{A203}$$

$$k_{31} = l_2 \tag{A204}$$

$$k_{32} = -l_2 \tag{A205}$$

Since the system matrix $\mathbf{B}$ is invertible, the output equations can be rewritten as

$$\begin{pmatrix} q - \overline{q} \\ q^* - \overline{q}^* \end{pmatrix} = \mathbf{D}\mathbf{B}^{-1}\mathbf{C}\begin{pmatrix} l^s - \overline{l}^s \\ \tau - \overline{\tau} \\ l^d - \overline{l}^d \end{pmatrix} + \mathbf{D}\mathbf{B}^{-1}\mathbf{K}\begin{pmatrix} \dot{m} - \overline{\dot{m}} \\ \dot{m}^* - \overline{\dot{m}^*} \end{pmatrix} \tag{A206}$$

Note that $rank\ \mathbf{D} = rank\ \mathbf{K} = 2$ and in general $|\mathbf{D}\mathbf{B}^{-1}\mathbf{K}| \neq 0$ holds. The target vector $(q, q^*)'$ is therefore target path controllable with the help of the policy vector $(\dot{m}, \dot{m}^*)'$. The invertibility of the impact multiplier matrix $\mathbf{D}\mathbf{B}^{-1}\mathbf{K}$ implies the monetary policy decision rules

$$\begin{pmatrix} \dot{m} - \overline{\dot{m}} \\ \dot{m}^* - \overline{\dot{m}^*} \end{pmatrix} = -(\mathbf{DB^{-1}K})^{-1}\mathbf{DB^{-1}C} \begin{pmatrix} l^s - \overline{l}^s \\ \tau - \overline{\tau} \\ l^d - \overline{l}^d \end{pmatrix} \tag{A207}$$

which yield $q = \overline{q}$, $q^* = \overline{q}^*$ for all $t \geq 0$. If we use the abbreviation

$$\mathbf{S} = (\mathbf{DB^{-1}K})^{-1}\mathbf{DB^{-1}C} = (s_{ij})_{\substack{1\leq i\leq 2 \\ 1\leq j\leq 3}} \tag{A208}$$

the stabilized state equations are given by

$$\mathbf{B} \begin{pmatrix} \dot{l}^s \\ \dot{\tau} \\ \dot{l}^d \end{pmatrix} = \widetilde{\mathbf{C}} \begin{pmatrix} l^s - \overline{l}^s \\ \tau - \overline{\tau} \\ l^d - \overline{l}^d \end{pmatrix} \tag{A209}$$

where $\widetilde{\mathbf{C}} = \mathbf{C} - \mathbf{KS} = (\widetilde{c}_{ij})_{1\leq i,j,\leq 3}$ is defined by

$$\widetilde{c}_{11} = c_{11} - a_2 l_2 (s_{11} + s_{21}) \tag{A210}$$

$$\widetilde{c}_{12} = c_{12} - a_2 l_2 (s_{12} + s_{22}) \tag{A211}$$

$$\widetilde{c}_{13} = c_{13} - a_2 l_2 (s_{13} + s_{23}) \tag{A212}$$

$$\widetilde{c}_{21} = c_{21} \tag{A213}$$

$$\widetilde{c}_{22} = c_{22} \tag{A214}$$

$$\widetilde{c}_{23} = c_{23} \tag{A215}$$

$$\widetilde{c}_{31} = c_{31} - l_2 (s_{11} - s_{21}) \tag{A216}$$

$$\widetilde{c}_{32} = c_{32} - l_2 (s_{12} - s_{22}) \tag{A217}$$

$$\widetilde{c}_{33} = c_{33} - l_2 (s_{13} - s_{23}) \tag{A218}$$

Note that $|\widetilde{\mathbf{C}}| = |\mathbf{B^{-1}}\widetilde{\mathbf{C}}| = 0$ holds since the output equations (A191) and the policy target $q = \overline{q}$, $q^* = \overline{q}^*$ imply that there is only *one* linearly independent state variable. From (A191) we get the equation

$$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \begin{pmatrix} \dot{l}^s \\ \dot{\tau} \end{pmatrix} = - \begin{pmatrix} d_{13} \\ d_{23} \end{pmatrix} \dot{l}^d \tag{A219}$$

and therefore

$$\dot{l}^s = -\eta_1 \dot{l}^d \tag{A220}$$

$$\dot{\tau} = -\eta_2 \dot{l}^d \tag{A221}$$

where

$$\eta_1 = \frac{d_{22}d_{13} - d_{12}d_{23}}{d_{11}d_{22} - d_{12}d_{21}} \tag{A222}$$

$$\eta_2 = \frac{-d_{21}d_{13} + d_{11}d_{23}}{d_{11}d_{22} - d_{12}d_{21}} \tag{A223}$$

The stabilized state equation (A209) then yields the following dynamic equation for l^d:

$$(-b_{31}\eta_1 - b_{32}\eta_2 + b_{33})\dot{l}^d = (-\widetilde{c}_{31}\eta_1 - \widetilde{c}_{32}\eta_2 + \widetilde{c}_{33})(l^d - \bar{l}^d) \tag{A224}$$

The corresponding eigenvalue

$$\widetilde{r}_2 = \frac{\widetilde{c}_{31}\eta_1 + \widetilde{c}_{32}\eta_2 - \widetilde{c}_{33}}{b_{31}\eta_1 + b_{32}\eta_2 - b_{33}} = -(\widetilde{g}_{31}\eta_1 + \widetilde{g}_{32}\eta_2 - \widetilde{g}_{33}) \tag{A225}$$

with $(\widetilde{g}_{ij})_{1\leq i,j,\leq 3} = \mathbf{B^{-1}\widetilde{C}}$ is ambiguous in sign.[36] Several numerical simulations clarify that $\widetilde{r}_2$ is positive if the semi-interest elasticity of private absorption is sufficiently large. It is therefore reasonable to assume $\widetilde{r}_2 > 0$. This implies that the new long run equilibrium resulting from the permanent oil price shock $dp_R^* > 0$ and the monetary policy rules (A207) is completely unstable. A unique convergent solution time path of the state variable l^d only exists if l^d is not predetermined but a jump variable that jumps whenever τ jumps. This is a natural assumption, since (A221) implies by integration that there is a linear relationship between τ and l^d. The assumption is also justified from an economic point of view, since perfect stabilization of the output time path along the initial steady state level requires an immediate change of the growth rate of domestic and foreign money supply at the date of anticipation of the oil price shock inducing inflationary or deflationary processes without delay. In case $\widetilde{r}_2 > 0$ the unique convergent solution time path of (A224) is given by

$$l^d = \bar{l}_0^d + d\bar{l}^d e^{\widetilde{r}_2(t-T)} \qquad \text{for } 0 < t \leq T \tag{A226}$$

$$l^d = \bar{l}_1^d \qquad \text{for } t > T \tag{A227}$$

where $d\bar{l}^d = \bar{l}_1^d - \bar{l}_0^d$. After its initial jump, which occurs if $d\bar{l}^d \neq 0$, the state variable l^d converges monotonically to its new steady state level $\bar{l}_1^d$ which is reached at time T. Thereafter, it must stay there, otherwise no bounded solution time path of l^d would exist. The same must hold for the other endogenous variables of the stabilized system. Assuming $\tau = \bar{\tau}_1$ and $l^s = \bar{l}_1^s$ for $t \geq T$ the time paths of τ and l^s over the

[36] Note that the eigenvalue $\widetilde{r}_2$ does not change if the dynamics of the stabilized system is expressed in terms of τ instead of l^d.

anticipation period $0 < t < T$ are given by

$$\tau = \overline{\tau}_1 - \eta_2(l^d - \overline{l}_1^d) = \overline{\tau}_0 + d\overline{\tau} + \eta_2 d\overline{l}^d \left(1 - e^{\widetilde{r}_2(t-T)}\right) \tag{A228}$$

$$l^s = \overline{l}_0^s + d\overline{l}^s + \eta_1 d\overline{l}^d \left(1 - e^{\widetilde{r}_2(t-T)}\right) \tag{A229}$$

with $d\overline{\tau} = 0$. Inserting the solution time paths of the state variables into (A206) yields the time path of the growth rate of domestic and foreign money supply.

$$\begin{aligned} \dot{m} &= \overline{\dot{m}}_0 - \left(s_{11}(l^s - \overline{l}_0^s) + s_{12}(\tau - \overline{\tau}_0) + s_{13}(l^d - \overline{l}_0^d)\right) \qquad \text{(A230)} \\ &= \overline{\dot{m}}_0 - \Big(s_{11}\big(d\overline{l}^s + \eta_1 d\overline{l}^d(1 - e^{\widetilde{r}_2(t-T)})\big) \\ &\quad + s_{12}\big(d\overline{\tau} + \eta_2 d\overline{l}^d(1 - e^{\widetilde{r}_2(t-T)})\big) \\ &\quad + s_{13} d\overline{l}^d e^{\widetilde{r}_2(t-T)} \Big) \qquad \text{for } 0 < t < T \\ \dot{m} &= \overline{\dot{m}}_1 = \overline{\dot{m}}_0 + d\overline{\dot{m}} \qquad \text{for } t \geq T \qquad \text{(A231)} \end{aligned}$$

$$\begin{aligned} \dot{m}^* &= \overline{\dot{m}_0^*} - \Big(s_{21}\big(d\overline{l}^s + \eta_1 d\overline{l}^d(1 - e^{\widetilde{r}_2(t-T)})\big) \qquad \text{(A232)} \\ &\quad + s_{22}\big(d\overline{\tau} + \eta_2 d\overline{l}^d(1 - e^{\widetilde{r}_2(t-T)})\big) \\ &\quad + s_{23} d\overline{l}^d e^{\widetilde{r}_2(t-T)} \Big) \qquad \text{for } 0 < t < T \\ \dot{m}^* &= \overline{\dot{m}_1^*} = \overline{\dot{m}_0^*} + d\overline{\dot{m}^*} \qquad \text{for } t \geq T \qquad \text{(A233)} \end{aligned}$$

where the steady state change of $\dot{m}$ and $\dot{m}^*$ results from (A207), (A208) and the long run stabilization condition $d\overline{q} = d\overline{q}^* = d\overline{\tau} = d(\overline{p_R^* - p^*}) = 0$:

$$\begin{pmatrix} d\overline{\dot{m}} \\ d\overline{\dot{m}^*} \end{pmatrix} = -\overline{\mathbf{S}} \begin{pmatrix} d\overline{l}^s \\ d\overline{l}^d \end{pmatrix} \tag{A234}$$

where

$$\overline{\mathbf{S}} = \overline{\mathbf{K}}^{-1}\overline{\mathbf{C}} = \begin{pmatrix} k_{11} & k_{12} \\ k_{31} & k_{32} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \overline{c}_{11} & \overline{c}_{13} \\ c_{31} & c_{32} \end{pmatrix} \tag{A235}$$

with (cf. (A47), (A49))

$$\overline{c}_{11} = a_2 = c_{11} + \frac{1}{2} l_2 a_1(\psi + \psi^*)\phi_1 \tag{A236}$$

$$\overline{c}_{13} = 0 = c_{13} - \frac{1}{2} l_2 a_1(\psi + \psi^*)\phi_1 \tag{A237}$$

and k_{11}, k_{12}, k_{31}, k_{32}, c_{31}, c_{33} given by (A200), (A201), (A204), (A205), (A53) and (A55). We then have

$$\overline{\mathbf{S}} = \frac{1}{2a_2l_2}\begin{pmatrix} 1 & a_2 \\ 1 & -a_2 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2l_2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{A238}$$

implying

$$d\overline{\dot{m}} = -\frac{1}{2l_2}(d\bar{l}^s + d\bar{l}^d) = -\frac{1}{l_2}d(\overline{m-p}) \tag{A239}$$

$$d\overline{\dot{m}^*} = -\frac{1}{2l_2}(d\bar{l}^s - d\bar{l}^d) = -\frac{1}{l_2}d(\overline{m^*-p^*}) \tag{A240}$$

If the foreign real factor price $p_R^* - p^*$ is viewed as jump variable that jumps at the date of anticipation, its bounded solution time path is given by

$$\begin{aligned} p_R^* - p^* &= (\overline{p_R^* - p^*})_1 - dp_R^* + \Lambda(\tau - \overline{\tau}_1) + \frac{1}{2}\phi_2(l^s - \bar{l}_1^s) - \frac{1}{2}\phi_1(l^d - \bar{l}_1^d) \qquad \text{(A241)} \\ &= (\overline{p_R^* - p^*})_1 - dp_R^* - \left(\Lambda\eta_2 + \frac{1}{2}\phi_2\eta_1 + \frac{1}{2}\phi_1\right)(l^d - \bar{l}_1^d) \\ &= (\overline{p_R^* - p^*})_1 - dp_R^* \\ &\quad - \left(\Lambda\eta_2 + \frac{1}{2}\phi_2\eta_1 + \frac{1}{2}\phi_1\right) d\bar{l}^d\left(1 - e^{\tilde{r}_2(t-T)}\right) \qquad \text{for } 0 < t < T \\ p_R^* - p^* &= (\overline{p_R^* - p^*})_1 = (\overline{p_R^* - p^*})_0 + d(\overline{p_R^* - p^*}) \qquad \text{for } t \geq T \qquad \text{(A242)} \end{aligned}$$

where steady state output stabilization implies $d(\overline{p_R^* - p^*}) = 0$. At time T, $p_R^* - p^*$ jumps by the amount dp_R^* back into its initial steady state level and remains there thereafter. If $p_R^* - p^*$ is not predetermined at time $t = 0$, the steady state change of $p_R^* - p^*$ is independent of the initial jump of τ, i.e., $\tau(0+) - \overline{\tau}_0$ (cf. (A75)).

The solution time paths for l^s, l^d, τ, $\dot{m}$, $\dot{m}^*$ and $p_R^* - p^*$ depend on the steady state change of l^s and l^d. To determine $d\bar{l}^s$ and $d\bar{l}^d$, consider the policy target $q = \overline{q}_0$, $q^* = \overline{q}_0^*$ (for all t) implying $d\overline{q} = d\overline{q}^* = 0$ and therefore $d\overline{\tau} = d(\overline{p_R^* - p^*}) = 0$. Equation (A75) then implies the relation

$$\phi_1 d\bar{l}^d - \phi_2 d\bar{l}^s = 2dp_R^* \tag{A243}$$

which is equivalent to (cf. A80):

$$f_{12}d\bar{l}^d + f_{13}d\bar{l}^s = \omega_1 dp_R^* \tag{A244}$$

since $f_{12} = \frac{1}{2}\omega_1\phi_1$ and $f_{12} = -\frac{1}{2}\omega_2\phi_2$. Further steady state equations of the stabilized system result from (A71) and (A73):

$$d\bar{l}^d = -l_2(d\bar{\dot{m}} - d\overline{\dot{m}^*}) \quad \text{(A245)}$$

$$d\bar{l}^s = -l_2(d\bar{\dot{m}} + d\overline{\dot{m}^*}) \quad \text{(A246)}$$

Obviously, equations (A245) and (A246) are equivalent to (A239), (A240). They can also be derived from (A78), (A79).

Inserting (A245), (A246) into (A243) yields the relation between $d\bar{\dot{m}}$ and $d\overline{\dot{m}^*}$ which must hold in order to realize the output target $q = \bar{q}_0$ and $q^* = \bar{q}_0^*$ simultaneously:[37]

$$d\bar{\dot{m}} = -\frac{2}{l_2(\phi_1 - \phi_2)}dp_R^* + \frac{\phi_1 + \phi_2}{\phi_1 - \phi_2}d\overline{\dot{m}^*} \quad \text{(A247)}$$

The policy trade-off (A247) is illustrated in figure 26. In the special case $d\bar{l}^d = 0$, i.e.

$$d\bar{\dot{m}} = d\overline{\dot{m}^*} = \frac{1}{l_2\phi_2}dp_R^* \qquad (d\bar{l}^d = 0) \quad \text{(A248)}$$

no adjustment dynamics occur. The state variable l^d and τ remain constant over time ($l^d = \bar{l}_0^d$, $\tau = \bar{\tau}_0$ for $t \geq 0$) while the variable l^s immediately jumps into its new steady state level $\bar{l}_1^s = \bar{l}_0^s - 2/\phi_2 dp_R^*$ and stays there afterwards. According to (A241) the real factor price $p_R^* - p^*$ on impact jumps by the amount $-dp_R^* = -1$ and jumps by the same amount back into its initial steady state level at time T. The policy variables $\dot{m}$ and $\dot{m}^*$ also jump on impact and at time T, where the jump in T is given by

$$\begin{aligned}
\dot{m}(T+) - \dot{m}(T-) &= \bar{\dot{m}}_0 + d\bar{\dot{m}} + s_{11}d\bar{l}^s + s_{13}d\bar{l}^d \quad \text{(A249)}\\
&= \bar{\dot{m}}_0 - \frac{1}{2l_2}(d\bar{l}^s + d\bar{l}^d) + s_{11}d\bar{l}^s + s_{13}d\bar{l}^d\\
&= \bar{\dot{m}}_0 - \left(\frac{1}{2l_2} - s_{11}\right)d\bar{l}^s - \left(\frac{1}{2l_2} - s_{13}\right)d\bar{l}^d
\end{aligned}$$

$$\dot{m}^*(T+) - \dot{m}^*(T-) = \overline{\dot{m}^*}_0 - \left(\frac{1}{2l_2} - s_{21}\right)d\bar{l}^s + \left(\frac{1}{2l_2} + s_{23}\right)d\bar{l}^d \quad \text{(A250)}$$

where $d\bar{l}^s = -2/\phi_2 dp_R^*$ in the special case $d\bar{l}^d = 0$.

The policy combination (A248) is connected with long run inflation in both economies. An alternative solution with a smaller inflation rate in the foreign economy and with long run deflation in the domestic economy results from the optimiza-

[37] Note that the policy trade-off (A247) is equivalent to (A173) if Γ_i $(i = 1, 2)$ is replaced by ϕ_i in (A173).

tion problem

$$V_1 = \left(\frac{1}{2}(d\bar{\dot{m}})^2 + \frac{1}{2}(d\bar{\dot{m}}^*)^2\right) \quad \rightarrow \quad \min_{d\bar{\dot{m}}\,,\,d\bar{\dot{m}}^*} \tag{A251}$$

subject to the condition (A247). Inserting equation (A247) into the loss function (A251) and then differentiating with respect to $d\bar{\dot{m}}^*$ yields the following optimal solution

$$d\bar{\dot{m}}\big|_{V_1} = -\frac{\phi_1 - \phi_2}{l_2(\phi_1^2 + \phi_2^2)} < 0 \tag{A252}$$

$$d\bar{\dot{m}}^*\big|_{V_1} = \frac{\phi_1 + \phi_2}{l_2(\phi_1^2 + \phi_2^2)} > 0 \tag{A253}$$

A drawback of the optimization problem (A251) is that it only considers *long run* deviations of the domestic and foreign growth rate of money stock from its respective initial steady state level, i.e., short term deviations are neglected. On impact, the fall in the domestic growth rate of money supply may be very strong yielding a sharp deflationary process in the domestic economy. An international policy coordination that minimizes both short term and long term deviations of the growth rate of money supply from its respective initial steady state level results from the optimization problem

$$\widetilde{V}_2 = \int_0^T \left(\frac{1}{2}(\dot{m} - \bar{\dot{m}}_0)^2 + \frac{1}{2}(\dot{m}^* - \bar{\dot{m}}_0^*)^2\right) dt \quad \rightarrow \quad \min_{d\bar{\dot{m}}\,,\,d\bar{\dot{m}}^*} \tag{A254}$$

subject to the conditions (A230), (A232), (A245) – (A247). With the abbreviations

$$a = -\frac{2}{l_2(\phi_1 - \phi_2)} \tag{A255}$$

$$b = \frac{\phi_1 + \phi_2}{\phi_1 - \phi_2} \tag{A256}$$

$$\alpha_1 = (s_{11} + s_{11}\eta_1 + s_{12}\eta_2)al_2 \tag{A257}$$

$$\alpha_2 = (s_{11}(1 + b) - (s_{11}\eta_1 + s_{12}\eta_2)(1 - b))l_2 \tag{A258}$$

$$\beta_1 = -al_2(s_{11}\eta_1 + s_{12}\eta_2 - s_{13}) \tag{A259}$$

$$\beta_2 = (s_{11}\eta_1 + s_{12}\eta_2 - s_{13})(1 - b)l_2 \tag{A260}$$

$$\alpha_1^* = (s_{21} + s_{21}\eta_1 + s_{22}\eta_2)al_2 \tag{A261}$$

$$\alpha_2^* = (s_{21}(1 + b) - (s_{21}\eta_1 + s_{22}\eta_2)(1 - b))l_2 \tag{A262}$$

$$\beta_1^* = -al_2(s_{21}\eta_1 + s_{22}\eta_2 - s_{23}) \tag{A263}$$

$$\beta_2^* = (s_{21}\eta_1 + s_{22}\eta_2 - s_{23})(1 - b)l_2 \tag{A264}$$

the solution time paths for $\dot{m}$ and $\dot{m}^*$ over the time interval $(0, T)$ can be written as follows

$$\dot{m} - \overline{\dot{m}}_0 = \alpha_1 + \alpha_2 d\overline{\dot{m}^*} + \beta_1 e^{\tilde{r}_2(t-T)} + \beta_2 e^{\tilde{r}_2(t-T)} d\overline{\dot{m}^*} \quad \text{(A265)}$$

$$\dot{m}^* - \overline{\dot{m}^*}_0 = \alpha_1^* + \alpha_2^* d\overline{\dot{m}^*} + \beta_1^* e^{\tilde{r}_2(t-T)} + \beta_2^* e^{\tilde{r}_2(t-T)} d\overline{\dot{m}^*} \quad \text{(A266)}$$

where we have assumed $dp_R^* = 1$. The loss function (A254) is then given by the expression

$$\begin{aligned} \widetilde{V}_2 &= \frac{1}{2}\int_0^T \left(\left[f_1(t) + f_2(t) d\overline{\dot{m}^*} \right]^2 + \left[g_1(t) + g_2(t) d\overline{\dot{m}^*} \right]^2 \right) dt \quad \text{(A267)} \\ &= \frac{1}{2}\int_0^T (f_1^2(t) + g_1^2(t)) dt + d\overline{\dot{m}^*} \int_0^T (f_1(t) f_2(t) + g_1(t) g_2(t)) dt \\ &\quad + \frac{1}{2}(d\overline{\dot{m}^*})^2 \int_0^T \left(f_2^2(t) + g_2^2(t) \right) dt \end{aligned}$$

where

$$f_1(t) = \alpha_1 + \beta_1 e^{\tilde{r}_2(t-T)} \quad \text{(A268)}$$

$$f_2(t) = \alpha_2 + \beta_2 e^{\tilde{r}_2(t-T)} \quad \text{(A269)}$$

$$g_1(t) = \alpha_1^* + \beta_1^* e^{\tilde{r}_2(t-T)} \quad \text{(A270)}$$

$$g_2(t) = \alpha_2^* + \beta_2^* e^{\tilde{r}_2(t-T)} \quad \text{(A271)}$$

Differentiation with respect to $d\overline{\dot{m}^*}$ yields the optimization condition

$$\frac{d\widetilde{V}_2}{d\overline{\dot{m}^*}} = \int_0^T (f_1 f_2 + g_1 g_2) dt + d\overline{\dot{m}^*} \int_0^T \left(f_2^2 + g_2^2 \right) dt = 0 \quad \text{(A272)}$$

The optimal value of $d\overline{\dot{m}^*}$ is then given by

$$d\overline{\dot{m}^*}\big|_{\widetilde{V}_2} = -\frac{\int_0^T (f_1 f_2 + g_1 g_2) dt}{\int_0^T (f_2^2 + g_2^2) dt} \quad \text{(A273)}$$

where

$$\begin{aligned} \int_0^T (f_1 f_2 + g_1 g_2) dt &= (\alpha_1 \alpha_2 + \alpha_1^* \alpha_2^*) T \quad \text{(A274)} \\ &\quad + (\alpha_1 \beta_2 + \alpha_2 \beta_1 + \alpha_1^* \beta_2^* + \alpha_2^* \beta_1^*) \frac{1}{\tilde{r}_2} \left(1 - e^{-\tilde{r}_2 T} \right) \\ &\quad + (\beta_1 \beta_2 + \beta_1^* \beta_2^*) \frac{1}{2\tilde{r}_2} \left(1 - e^{-2\tilde{r}_2 T} \right) \end{aligned}$$

and

$$\int_0^T (f_2^2 + g_2^2)dt = (\alpha_2^2 + \alpha_2^{*2})T + 2(\alpha_2\beta_2 + \alpha_2^*\beta_2^*)\frac{1}{\widetilde{r}_2}\big(1 - e^{-\widetilde{r}_2 T}\big) + (\beta_2^2 + \beta_2^{*2})\frac{1}{2\widetilde{r}_2}\big(1 - e^{-2\widetilde{r}_2 T}\big) \tag{A275}$$

The optimal solution for $d\dot{m}$ follows from the policy trade-off (A247).

The choice of the policy combination $(d\overline{\dot{m}}, d\overline{\dot{m}^*})$ which satisfies the trade-off (A247) can also result from the minimization of the initial jump of $\dot{m}$ and $\dot{m}^*$:

$$V_2 = \frac{1}{2}\big(\dot{m}(0+) - \overline{\dot{m}}_0\big)^2 + \frac{1}{2}\big(\dot{m}^*(0+) - \overline{\dot{m}^*}_0\big)^2 \quad \rightarrow \quad \min_{d\overline{\dot{m}}\,,\, d\overline{\dot{m}^*}} \tag{A276}$$

subject to the conditions (A230), (A232), (A245) – (A247). In this case the optimal policy choice is given by

$$d\overline{\dot{m}^*}\big|_{V_2} = -\frac{f_1(0)f_2(0) + g_1(0)g_2(0)}{f_2^2(0) + g_2^2(0)} \tag{A277}$$

$$d\overline{\dot{m}}\big|_{V_2} = a + b \cdot d\overline{\dot{m}^*}\big|_{V_2} \tag{A278}$$

where $f_j(0)$ and $g_j(0)$ $(j = 1, 2)$ are the initial values of the functions f_j and g_j defined in (A268) – (A271). The optimal choice of $(d\overline{\dot{m}}, d\overline{\dot{m}^*})$ may also be the result of the minimization of a loss function V_3 which is a combination of the functions V_1 and V_2 with a weight $0 \leq \lambda_V \leq 1$:

$$V_3 = \lambda_V V_1 + (1 - \lambda_V)V_2 \quad \rightarrow \quad \min_{d\overline{\dot{m}}\,,\, d\overline{\dot{m}^*}} \tag{A279}$$

In this case an international monetary policy coordination simultaneously minimizes the initial and steady state deviation of the growth rate of money supply from its initial value. The result of the minimization of V_3 is given by

$$d\overline{\dot{m}^*}\big|_{V_3} = -\frac{\lambda_V ab + (1 - \lambda_V)\big(f_1(0)f_2(0) + g_1(0)g_2(0)\big)}{\lambda_V(1 + b^2) + (1 - \lambda_V)\big(f_2^2(0) + g_2^2(0)\big)} \tag{A280}$$

$$d\overline{\dot{m}}\big|_{V_3} = a + b \cdot d\overline{\dot{m}^*}\big|_{V_3} \tag{A281}$$

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