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# On the Non-Optimality of Information: An Analysis of the Welfare Effects of Anticipated Shocks in the New Keynesian Model

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**Abstract**

This paper compares the welfare effects of anticipated and unanticipated cost-push shocks in the canonical New Keynesian model with optimal monetary policy. We find that, for empirically plausible degrees of nominal rigidity, the anticipation of a future cost-push shock leads to a higher welfare loss than an unanticipated shock. A welfare gain from the anticipation of a future cost shock may only occur if prices are sufficiently flexible. We analytically show that this surprising result holds although unanticipated shocks lead to higher negative impact effects on welfare than anticipated shocks.

*JEL classification*: E31, E32, E52
*Keywords*: Anticipated Shocks, Optimal Monetary Policy, Sticky Prices, Welfare Analysis

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# 1 Introduction

Does the *anticipation* of future shocks has a stabilizing effect on the economy and thus reduces the welfare loss compared to *unanticipated* shocks? In this paper, we seek to answer this question by comparing the welfare effects of unanticipated and anticipated cost-push shocks in the canonical New Keynesian model with a monetary authority which minimizes a standard loss function that weights the volatility of inflation and the output gap. In particular, we analytically solve for dynamics and welfare in case of optimal monetary policy under timeless perspective commitment and discretion. We distinguish the usual case of unanticipated cost-push shocks and the case of future cost-push shocks that are known in advance.

Since the real business cycle revolution of Kydland and Prescott (1982) and his successors, unanticipated random disturbances are considered as the main driving force in explaining business cycles. New Keynesians add nominal rigidities to the real business cycle framework to study the role of monetary policy in aggregate fluctuations but maintain the assumption of unpredictable random shocks (see, e.g., the textbooks of Walsh (2003), Woodford (2003), or Galí (2008)). An exception is the stream of literature that analyzes anticipated disinflations going back to Ball (1994) who shows that a simple variant of the New Keynesian model predicts a boom in response to an anticipated disinflation. However, the literature on the optimal design of monetary policy usually considers unanticipated shocks (see, e.g. Clarida, Galí, and Gertler (1999), Svensson (1999), King, Khan, and Wolman (2000), or Woodford (2003)).

Recently, a number of macroeconometric studies emphasize the role of anticipated shocks as sources of macroeconomic fluctuations. Beaudry and Portier (2006) find that more than half of business cycle fluctuations are caused by news about future technological opportunities. Davis (2007) and Fujiwara, Hirose, and Shintani (2008) analyze the importance of anticipated shocks in large scale DSGE models closely related to the model of Christiano, Eichenbaum, and Evans (2005) and report that these disturbances are important components of aggregate fluctuations. Schmitt-Grohé and Uribe (2008) conduct a Bayesian estimation of a real-business cycle model and find that anticipated shocks are the most important source of aggregate fluctuations. In particular, they report that anticipated shocks explain two thirds of the volatility in consumption, output, investment, and employment.

Theoretical studies on the role of anticipations for business cycle fluctuations include Beaudry and Portier (2004, 2007), Beaudry, Collard, and Portier (2006), Jaimovich and Rebelo (2006, 2008), Den Haan and Kaltenbrunner (2007), or Christiano, Ilut, Motto, and Rostagno (2008).

However, none of these studies considers the welfare effects of the anticipation of future shocks. In this study, we derive a solution of welfare as a function of the time span between the anticipation and the realization of the shock which enables us to discover the dependency of welfare on the length of the anticipation period. Furthermore, we contribute to the literature by systematically investigating the role of nominal rigidities for the welfare impacts of anticipations.

To the best of our knowledge, Wohltmann and Winkler (2008) and Winkler (2008) are the only studies that compare the welfare effects of anticipated and unanticipated shocks. They both analyze energy price shocks under different monetary policy regimes including optimal monetary policy. However, these studies rely on numerical simulations and do not, as we do, investigate the role of nominal rigidities.

The main results of this paper are the following. For empirically plausible degrees of nominal rigidity, the anticipation of a future cost-push shock leads to a higher welfare loss than an analogous unanticipated shock. A welfare gain from the anticipation of a future cost shock may only occur if prices are sufficiently flexible. This result is consistent with the findings of Schmitt-Grohé and Uribe (2008) who show that the anticipation of future shocks has a stabilizing effect on an economy without nominal rigidities. We point out that precisely the degree of nominal rigidity play an important role for the evaluation of the welfare effects of anticipations.

Our results are driven by two opposing effects. On the one hand, we obtain the well-known result that the anticipation of a future shock dampens its impact effect. On the other hand, we show that anticipation of future cost-push shocks enhances the persistence of output and inflation and thus enhances the welfare loss. This persistence effect, in turn, is amplified by the degree of price stickiness.

Nevertheless, at a first glance, our findings seem to be puzzling since it suggests that the information about the occurrence of future shocks is in general welfare-reducing. But then the question arises, why rational agents do not ignore the knowledge about future disturbances. In the remainder of this paper, we will seek to shed more light on this question.

Our paper is organized as follows. Section 2 presents the canonical New Keynesian model and its solution under the policy regimes timeless perspective commitment and discretion. In section 3, we report and discuss our main findings. Furthermore, we provide analytical proofs and, for the sake of illustration, numerical simulations. Section 4 concludes. The paper includes an extensive mathematical appendix.

## 2 The Framework

The canonical New Keynesian model serves as analytical framework. It consists of an optimizing IS-type relationship of the form

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \qquad (\sigma \geq 1) \tag{1}$$

and a price adjustment equation of Calvo-Rotemberg type, often referred to as New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + k_t \qquad (0 < \beta < 1, \kappa > 0) \tag{2}$$

$x_t$ denotes the output gap, $\pi_t$ is inflation, and $i_t$ is the nominal interest rate. $E_t$ is the expectations operator conditional on information up to date $t$. $\beta$ is the discount factor and $1/\sigma$ denotes the intertemporal elasticity of substitution. It is well-known that under the assumptions of Calvo (1983) price setting, a constant returns to scale production function with labor as single input, and perfect labor markets, the slope parameter $\kappa$ is given by $\kappa = (\eta+\sigma)\frac{(1-\omega)(1-\beta\omega)}{\omega}$, where $\eta$ is the inverse of the labor supply elasticity.[1] Obviously, $\kappa$ is negatively correlated with the degree of price rigidity $\omega$. According to the Calvo price adjustment mechanism, a fraction $1-\omega$ of firms can adjust their price in period $t$. Simultaneously, $\omega$ is the probability that a single price which is reoptimized in period $t$, also holds in the next period $t+1$. The Calvo parameter $\omega$ is therefore a measure of the degree of price rigidity on the goods markets.

In the NKPC, $k_t$ represents a temporary cost-push shock that is assumed to be autoregressive of order one with AR parameter $\varphi \in [0, 1)$ and a one-unit cost shock $\varepsilon_t$

$$k_t = \varphi k_{t-1} + \varepsilon_t \qquad (t \geq T > 0) \tag{3}$$

Since we consider anticipated cost-push shocks, the one-unit cost shock $\varepsilon_t$ is not white noise, but known to the public before the shock actually occurs.[2] Assume that at time $t = 0$ the public anticipates the cost-push shock to take place at some future time $T > 0$. Then,

$$\varepsilon_t = \begin{cases} 1 & \text{for } t = T > 0 \\ 0 & \text{for } t \neq T \end{cases} \tag{4}$$

The adjustment dynamics induced by anticipated shocks involve two phases, the time span between the anticipation and the realization of the shock ($0 \leq t < T$) and the time span after the implementation of the shock ($T \leq t \leq \infty$). The lead time $T$ up to the realization of the shock is equal to the length of the anticipation phase $0 \leq t < T$. An implication of our definition of anticipated shocks is that rational expectations are equivalent to perfect foresight so that we can omit the expectations operator.

The policy maker's objective at the time of anticipation $t = 0$ is to minimize the intertemporal loss function

$$V = E_0 \sum_{t=0}^{\infty} \beta^t (\alpha_1 \pi_t^2 + \alpha_2 x_t^2) \qquad (\alpha_1 > \alpha_2 > 0,\ 0 < \beta \leq 1) \tag{5}$$

which reflects the objective of flexible inflation targeting (see, e.g., Svensson (1999)). Rotemberg and Woodford (1999) and Woodord (2003) show that, under certain conditions, a quadratic loss function in inflation and the output gap is the correct approximation to the representative agent's utility function.

The first-order conditions of the policy problem under timeless perspective precommitment monetary policy as well as under discretion are well known and

[1] See, e.g., Walsh (2003) for a derivation of the NKPC under Calvo pricing.

[2] Schmitt-Grohé and Uribe (2007) study the impacts of anticipated cost shocks on the pass-through to prices.

need not to be derived here (see, for example, Walsh (2003)). Under the optimal timeless perspective precommitment policy, inflation satisfies the targeting rule

$$\pi_t = -\frac{\alpha_2}{\alpha_1 \kappa}(x_t - x_{t-1}) \tag{6}$$

while the output gap is described by the second-order difference equation

$$\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) x_t - x_{t-1} - \beta E_t x_{t+1} = -\frac{\alpha_1 \kappa}{\alpha_2} k_t \tag{7}$$

where the expectational operator can be omitted in case of anticipated shocks.

To solve the difference equation for $x_t$, write equation (7) as

$$\begin{pmatrix} x_{t+1} \\ w_{t+1} \end{pmatrix} = C \begin{pmatrix} x_t \\ w_t \end{pmatrix} + \begin{pmatrix} \frac{\alpha_1 \kappa}{\alpha_2 \beta} \\ 0 \end{pmatrix} k_t \tag{8}$$

where $w_t = x_{t-1}$ and

$$C = \begin{pmatrix} \frac{1}{\beta}\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) & -\frac{1}{\beta} \\ 1 & 0 \end{pmatrix} \tag{9}$$

The auxiliary variable $w_t$ is backward-looking (with the initial value $w_0 = 0$, while the output gap $x_t$ is forward-looking. The system matrix $C$ has two real eigenvalues $r_1$ and $r_2$ with $r_1 > 1 > r_2 > 0$ so that the Blanchard and Kahn (1980) saddlepath stability condition is satisfied. A detailed derivation of our results is provided in the mathematical appendix.

The solution for the output gap over the anticipation phase is given by

$$x_t = -\frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} \frac{\alpha_1 \kappa}{\alpha_2 \beta} r_1^{-T} (r_1^{t+1} - r_2^{t+1}) \quad \text{for} \quad t < T \tag{10}$$

with the initial values

$$x_0 = -\frac{1}{r_1 - \varphi} \frac{\alpha_1 \kappa}{\alpha_2 \beta} r_1^{-T}, \qquad x_{-1} = 0 \tag{11}$$

while the solution for $t \geq T$ is defined by

$$\begin{aligned} x_t =& \frac{\alpha_1 \kappa}{\alpha_2 \beta} \frac{1}{(r_1 - \varphi)(r_2 - \varphi)} \cdot \\ & \cdot \left[ \varphi^{t+1-T} - \frac{(r_1 - \varphi) r_2^{-T} - (r_2 - \varphi) r_1^{-T}}{r_1 - r_2} r_2^{t+1} \right] \text{ for } t \geq T \end{aligned} \tag{12}$$

In the limiting case of unanticipated shocks ($T = 0$), the term in brackets in equation (12) simplifies to $\varphi^{t+1} - r_2^{t+1}$. Note that the solution formula (10) also holds in the shock period $t = T$.

Using (6), the solution time path of the inflation rate follows

$$\pi_t = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} r_1^{-T} \left[ (r_1 - 1) r_1^t - (r_2 - 1) r_2^t \right] \quad \text{for } t \leq T \tag{13}$$

with the initial value

$$\pi_0 = \frac{1}{\beta}\frac{1}{r_1-\varphi} r_1^{-T} \tag{14}$$

and

$$\begin{aligned}\pi_t =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi}\cdot \\ &\cdot\left[(1-\varphi)\varphi^{t-T} - \frac{(r_1-\varphi)r_2^{-T}-(r_2-\varphi)r_1^{-T}}{r_1-r_2}(1-r_2)r_2^t\right] \text{ for } t\geq T\end{aligned} \tag{15}$$

In the limiting case $T = 0$, the term in brackets simplifies to $(1-\varphi)\varphi^t - (1-r_2)r_2^t$.

To determine the welfare loss under the optimal precommitment policy, write the loss function $V$ as $V_1 + V_2$, where

$$V_1 = E_0 \sum_{t=0}^{T-1} \beta^t \left(\alpha_1 \pi_t^2 + \alpha_2 x_t^2\right) \tag{16}$$

is the loss in the anticipation period and

$$V_2 = E_0 \sum_{t=T}^{\infty} \beta^t \left(\alpha_1 \pi_t^2 + \alpha_2 x_t^2\right) \tag{17}$$

is the loss caused by the realization of the shock.

By inserting the solution for $x_t$ and $\pi_t$, the loss $V_1$ can be rewritten as

$$V_1 = \alpha_1 \lambda^2 r_1^{-2T}\left(r_1^T - r_2^T\right)\left(\frac{r_1-1}{r_2^T} + \frac{1-r_2}{r_1^T}\right) \tag{18}$$

where

$$\lambda = \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} \tag{19}$$

Accordingly, the loss $V_2$ can be rewritten as

$$V_2 = \frac{\alpha_1\beta^T}{\beta^2(r_1-\varphi)^2}\left\{\frac{\left(r_2^T - r_1^T\right)^2(1-r_2)}{(r_1-r_2)^2 r_1^{2T}} + \frac{r_1}{r_1r_2-\varphi^2}\right\} \tag{20}$$

The total loss $V$ is then simply given by $V = V_1 + V_2$.

Under the policy regime discretion (D), the central bank is unable to make a commitment to future policies. Now private expectations are given for the central bank and the reduced form of the first-order conditions reads as

$$\pi_t = -\frac{\alpha_2}{\alpha_1\kappa} x_t \tag{21}$$

$$E_t x_{t+1} = \frac{1}{\beta}\left[1 + \frac{\alpha_1\kappa^2}{\alpha_2}\right] x_t + \frac{\alpha_1\kappa}{\alpha_2\beta} k_t \tag{22}$$

with $E_t x_{t+1} = x_{t+1}$ in case of anticipated shocks. The difference equation in $x_t$ has the unstable eigenvalue

$$r_D = \frac{1}{\beta}\left[1 + \frac{\alpha_1\kappa^2}{\alpha_2}\right] = \frac{1}{\alpha_2\beta}\left[\alpha_2 + \alpha_1\kappa^2\right] > 1 \tag{23}$$

and the forward solution

$$x_t = -\sum_{s=0}^{\infty} r_D^{-s}\frac{1}{r_D}\frac{\alpha_1\kappa}{\alpha_2\beta}k_{t+s} \tag{24}$$

Since

$$k_{t+s} = \begin{cases} \varphi^{t+s-T} & \text{for } t+s \geq T \\ 0 & \text{for } t+s < T \end{cases} \tag{25}$$

we obtain for $t \geq T$

$$x_t = -\frac{\alpha_1\kappa}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}\varphi^{t-T} \tag{26}$$

and for $t < T$

$$x_t = -\frac{\alpha_1\kappa}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}r_D^{t-T} \tag{27}$$

Due to $r_D^{t-T} = 1$ for $t = T$, the solution formula (27) also holds in the shock period $t = T$. For $t = 0$ we obtain

$$x_0 = -\frac{\alpha_1\kappa}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}r_D^{-T} \tag{28}$$

so that the the size of the initial jump of $x_t$ decreases with increasing $T$.

For the inflation rate $\pi_t$ we obtain the solution time path

$$\pi_t = \begin{cases} \dfrac{\alpha_2}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}r_D^{t-T} & \text{if} \quad 0 \leq t \leq T \\[2ex] \dfrac{\alpha_2}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}\varphi^{t-T} & \text{if} \quad t \geq T \end{cases} \tag{29}$$

Note that the limiting case $\varphi = 0$ implies $\pi_t = x_t = 0$ for $t > T$.

It is well-known that the loss under discretion ($V_D$) is greater than the total loss under the optimal precommitment policy. By inserting the solution time paths for $\pi_t$ and $x_t$ in the loss function, we obtain

$$\begin{aligned} V_D &= V_1^D + V_2^D \\ &= \sum_{t=0}^{T-1}\beta^t\left[\frac{\alpha_2^2}{\alpha_1\kappa^2} + \alpha_2\right]x_t^2 + \sum_{t=T}^{\infty}\beta^t\left[\frac{\alpha_2^2}{\alpha_1\kappa^2} + \alpha_2\right]x_t^2 \\ &= \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\left(\frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} + \frac{\beta^T}{1 - \beta\varphi^2}\right) \\ &= \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{1}{1 - \beta r_D^2}\left(r_D^{-2T} - \frac{\beta(r_D^2 - \varphi^2)}{1 - \beta\varphi^2}\beta^T\right) \end{aligned} \tag{30}$$

where

$$\frac{1}{1 - \beta r_D^2} = \frac{\alpha_2^2\beta}{\alpha_2^2\beta - (\alpha_2 + \alpha_1\kappa^2)^2} < 0 \tag{31}$$

## 3 Main Results

In this section, we compare the welfare loss induced by anticipated shocks ($T > 0$) to the corresponding loss if the same deterministic shock is not anticipated in advance ($T = 0$). In particular, we investigate the properties of the welfare loss $V$ considered as function of the lead time $T$.

Since the size of the initial jumps of the forward-looking variables $x_t$ and $\pi_t$ are negatively correlated with the lead time $T$, we can conjecture that the loss function $V = V(T)$ is a decreasing function in $T$. In the following, we will demonstrate that this conjecture is false in general. It is only true, if the degree of price flexibility is very high.

Our main results can be summarized in the form of four propositions.

**Proposition 1.** *Without discounting (i.e. $\beta = 1$) the welfare loss induced by an anticipated cost-push shock is greater than the corresponding loss in case of an unanticipated shock. This result is independent of the length of the lead time $T$ and the degree of price rigidity $\omega$:*

$$\text{If } \beta = 1, \quad \text{then } V(0) < V(T) \quad \text{for all} \quad T > 0 \quad \text{and all} \quad \omega > 0\,. \tag{32}$$

A similar result holds with discounting ($\beta < 1$) provided the degree of price rigidity $\omega$ is sufficiently high and the time span between anticipation and realization of the shock is not too large.

**Proposition 2.** *If $\beta$ is less than unity and the degree of price flexibility $1 - \omega$ low, there exists a positive upper bound $T_c^*$ for the lead time $T$, positively depending on $\omega$, such that*

$$V(0) < V(T) \quad \text{for all} \quad 0 < T < T_c^*\,. \tag{33}$$

**Proposition 3.** *If the degree of price flexibility is very high (i.e. $\omega$ very small) then $T_c^* = 0$ so that*

$$V(T) < V(0) \quad \text{for all} \quad T > 0\,. \tag{34}$$

*Only in this case (which seems empirically not very realistic), the welfare loss under anticipated cost-push shocks is always smaller than under unanticipated shocks.*

**Proposition 4.** *The propositions 1, 2, and 3 hold under the optimal monetary policy regimes timeless perspective commitment and discretion. They also hold under (optimal) simple rules of Taylor-type.*

**Sketch of Proof of Propositions 1, 2, and 3.** Consider the partial loss function $V_1$ (given by (18)) as function of $T$ (the time span between the anticipation and realization of the cost-push shock).

The function $V_1 = V_1(T)$ has the following properties:

$$V_1(0) = 0, \qquad \lim_{T\to\infty} V_1(T) = \begin{cases} 0 & \text{for } \beta < 1 \\ \overline{V}_1 > 0 & \text{for } \beta = 1 \end{cases} \tag{35}$$

where

$$\overline{V}_1 = \frac{\alpha_1(r_1-1)}{(r_1-\varphi)^2(r_1-r_2)^2} \tag{36}$$

The derivative of $V_1$ with respect to $T$, i. e.

$$\begin{aligned}\frac{dV_1}{dT} = \alpha_1\lambda^2 \Big\{ & 2\ln r_1 \cdot r_1^{-2T}[r_1+r_2-2] - (r_1-1)\ln(r_1 r_2)\cdot(r_1 r_2)^{-T} \\ & - (1-r_2)\ln\left(\frac{r_2}{r_1^3}\right)\cdot\left(\frac{r_2}{r_1^3}\right)^T \Big\}\end{aligned} \tag{37}$$

is positive at time $T=0$:

$$\frac{dV_1}{dT}\Big|_{T=0} = \alpha_1 \frac{1}{\beta^2}\frac{1}{(r_1-\varphi)^2}\frac{1}{r_1-r_2}[\ln r_1 - \ln r_2] > 0 \tag{38}$$

Therefore, $V_1(T)$ starts to rise with increasing $T$ (although the size of the initial jumps of $x_t$ and $\pi_t$ is decreasing in $T$). For $\beta < 1$, the limit value $\lim_{T\to\infty} V_1(T)$ is equal to zero. Therefore, $V_1(T)$ must decrease if $T$ is sufficiently large.

The loss function $V_2 = V_2(T)$ (given by (20)) has the following properties:

$$V_2(0) = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\frac{r_1}{r_1 r_2-\varphi^2} > 0 \tag{39}$$

$$\lim_{T\to\infty} V_2(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \overline{V}_2 > V_2(0)\Big|_{\beta=1} = \dfrac{\alpha_1 r_1}{(r_1-\varphi)^2(1-\varphi^2)} & \text{if } \beta = 1 \end{cases} \tag{40}$$

where

$$\overline{V}_2 = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1-r_2}{(r_1-r_2)^2} + \frac{r_1}{1-\varphi^2}\right\} \tag{41}$$

The first derivative of $V_2$ with respect to $T$

$$\begin{aligned}\frac{dV_2}{dT} = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\beta^T \Bigg\{ & \frac{r_1}{r_1 r_2-\varphi^2}\ln\beta \\ & + \frac{1-r_2}{(r_1-r_2)^2}\left[(\ln r_2 - 3\ln r_1)\left(\frac{r_2}{r_1}\right)^{2T} + 4\ln r_1\left(\frac{r_2}{r_1}\right)^T + \ln\beta\right]\Bigg\}\end{aligned} \tag{42}$$

implies for $\beta < 1$ and $T=0$

$$\frac{dV_2}{dT}\Big|_{T=0} = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\frac{r_1}{r_1 r_2-\varphi^2}\ln\beta < 0 \tag{43}$$

since $\beta = 1/(r_1 r_2)$. For $\beta < 1$, the derivative $dV_2/dT$ is also negative if $T$ is sufficiently large. In the limiting case $\beta = 1$, the loss function $V_2(T)$ is an increasing function in $T$ with a limit value $\overline{V}_2 > V_2(0)$.

We can now investigate the development of the total loss $V = V_1 + V_2$.

In the limiting case $\beta = 1$, the total loss $V(T)$ is an overall increasing function in $T$ with $V(0) = V_2(0) > 0$ and

$$\lim_{T\to\infty} V(T) = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1}{r_1-r_2}+\frac{r_1}{1-\varphi^2}\right\} > V_2(0)\Big|_{\beta=1} > 0 \tag{44}$$

If $\beta = 1$, we can write $V(T)$ as $V_1(T) + V_2(T)$, where

$$V_1(T) = \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2}\left[(r_1-1)+(2-r_1-r_2)r_1^{-2T}-(1-r_2)\left(\frac{r_2}{r_1^3}\right)^T\right] \tag{45}$$

$$V_2(T) = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1-r_2}{(r_1-r_2)^2}\left[1-\left(\frac{r_2}{r_1}\right)^T\right]^2+\frac{r_1}{1-\varphi^2}\right\} \tag{46}$$

Then

$$\begin{aligned}\frac{dV_1}{dT} = \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2}&\Bigg\{2[r_1+r_2-2]\ln r_1 \\ &+[3\ln r_1-\ln r_2](1-r_2)\left(\frac{r_2}{r_1}\right)^T\Bigg\}r_1^{-2T} > 0 \quad \text{for all} \quad T \geq 0\end{aligned} \tag{47}$$

(due to $r_1 + r_2 = \text{tr } C > 2$ and $\ln r_2 < 0$) and

$$\begin{aligned}\frac{dV_2}{dT} &= \frac{\alpha_1}{(r_1-\varphi)^2}\frac{1-r_2}{(r_1-r_2)^2}\left\{-2\left(1-\left(\frac{r_2}{r_1}\right)^T\right)\ln\left(\frac{r_2}{r_1}\right)\right\}\left(\frac{r_2}{r_1}\right)^T \\ &\underset{(=)}{>} 0 \qquad \text{if} \qquad T \underset{(=)}{>} 0\end{aligned} \tag{48}$$

(because $0 < r_2 < 1 < r_1$). Therefore, $dV/dT > 0$ for all $T \geq 0$ so that $V$ is a monotonically increasing function in $T$. This result holds independently of the degree of price rigidity $\omega$.

For $\beta < 1$, $V(0) = V_2(0) > 0$ (with $V_2(0)$ defined in (39)) and $\lim_{T\to\infty} V(T) = 0$. For small values of $\omega$, i.e. a high degree of price flexibility, the total loss $V$ is a decreasing function in $T$ implying $V(T) < V(0)$ for all $T > 0$. With high price flexibility, the welfare loss under anticipated shocks is smaller than under unanticipated shocks.

For the derivative $dV/dT$ at time $T = 0$ we get

$$\begin{aligned}\frac{dV}{dT}\Big|_{T=0} = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}&\Bigg\{\left[\frac{1}{r_1-r_2}-\frac{r_1}{r_1r_2-\varphi^2}\right]\ln r_1 \\ &-\left[\frac{1}{r_1-r_2}+\frac{r_1}{r_1r_2-\varphi^2}\right]\ln r_2\Bigg\}\end{aligned} \tag{49}$$

Then

$$\frac{dV}{dT}\Big|_{T=0} > 0 \Leftrightarrow 2\left(\frac{1}{\beta}-\varphi^2\right)\ln r_1 + \left(r_1^2-\varphi^2\right)\ln\beta > 0 \tag{50}$$

A rising $\omega$ induces a fall in the unstable eigenvalue $r_1$ since $d\kappa/d\omega < 0$. Since the fall in $r_1^2$ is stronger than the decrease in $\ln r_1$, and $1/\beta - \varphi^2 > 0$, inequality (50) is fulfilled if the degree of price rigidity $\omega$ is sufficiently large. In this case $V(T)$ starts to rise and due to $\lim_{T\to\infty} V(T) = 0$ its development must be hump-shaped implying the existence of an upper bound $T_c^* > 0$ such that $V(T) > V(0) > 0$ for all $T < T_c^*$.

The value of the upper bound $T_c^*$ is the positive solution of the equation $V(T) = V(0)$, where $V(0) = V_2(0)$ is given by (39). This leads to the equation

$$1 - \left(\frac{r_2}{r_1}\right)^T = \left[(r_1 r_2)^T - 1\right] \frac{r_1(r_1 - r_2)}{r_1 r_2 - \varphi^2} \tag{51}$$

Equation (51) can be written as

$$\beta^T r_1^{2T}\left[\beta r_1^2\left(1 - \frac{1}{\beta^T}\right) + \frac{1}{\beta^T} - \beta\varphi^2\right] = 1 - \beta\varphi^2 \quad \Leftrightarrow \tag{52}$$

$$r_1^{2T}\left[\beta^{T+1}\left(r_1^2 - \varphi^2\right) + \left(1 - \beta r_1^2\right)\right] = 1 - \beta\varphi^2 \tag{53}$$

so that $T_c^*$ is also the positive solution of (52) and (53). The value of $T_c^*$ is dependent on $\omega$ and $\beta$. A rising $\omega$ (a higher degree of price rigidity) decreases the unstable eigenvalue $r_1$ so that the left-hand side of equation (52) is decreased while the right-hand side remains unchanged. Since $\beta^T r_1^{2T} = (r_1/r_2)^T$ is increasing in $T$, equation (52) implies that the solution value $T_c^*$ must increase if $\omega$ rises. Conversely, a higher degree of price flexibility induces a fall in $T_c^*$. For sufficiently small values of $\omega$, the only solution of (53) is $T_c^* = 0$ (so that $V(T) < V(0)$ for all $T > 0$). If a positive solution $T_c^*$ of (53) exists, then it is also an increasing function in the discount factor $\beta$ with $T_c^* = \infty$ if $\beta = 1$. □

**Sketch of Proof of Proposition 4.** Consider $V_D$ (given by (30)) as function in $T$. Then

$$V_D(0) = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{1}{1-\beta\varphi^2} > 0 \tag{54}$$

and

$$\lim_{T\to\infty} V_D(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\left(\frac{1}{r_D^2-1} + \frac{1}{1-\varphi^2}\right) > V_D(0) > 0 & \text{if } \beta = 1 \end{cases} \tag{55}$$

The partial loss function

$$V_2^D(T) = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{\beta^T}{1-\beta\varphi^2} \tag{56}$$

has the properties

$$V_2^D(0) = V_D(0) \tag{57}$$

$$\lim_{T\to\infty} V_2^D(T) = 0 \quad \text{if} \quad \beta < 1 \tag{58}$$

$$\frac{dV_2^D}{dT} = (\ln\beta)V_2^D(T) < 0 \quad \text{if} \quad \beta < 1 \quad \text{for all} \quad 0 \le T < \infty \tag{59}$$

For $\beta = 1$, the function $V_2^D(T)$ is constant (independent of $T$).

The partial loss function $V_1^D(T)$ given by

$$V_1^D(T) = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} \tag{60}$$

has similar properties as the corresponding function $V_1(T)$ under the policy regime timeless perspective commitment:

$$V_1^D(0) = 0 \tag{61}$$

$$\lim_{T\to\infty} V_1^D(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \dfrac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\dfrac{1}{r_D^2 - 1} > 0 & \text{if } \beta = 1 \end{cases} \tag{62}$$

The first derivative with respect to $T$

$$\frac{dV_1^D(T)}{dT} = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{1}{1 - \beta r_D^2}\left[-2(\ln r_D)r_D^{-2T} - (\ln\beta)\beta^T\right] \tag{63}$$

is positive at time $T = 0$, since $1 - \beta r_D^2 < 0$ and $-2\ln r_D - \ln\beta < 0$ due to $r_D > 1 \ge \beta$.

In case $\beta < 1$, the development of $V_1^D(T)$ is hump-shaped with the maximum value at time $T_d^*$ which is the solution of the equation

$$2(\ln r_D)r_D^{-2T} + (\ln\beta)\beta^T = 0 \tag{64}$$

Equation (64) is equivalent to

$$-\frac{2\ln r_D}{\ln\beta} = (\beta r_D^2)^T \tag{65}$$

with the solution

$$T_d^* = \frac{\ln\left[-\dfrac{2\ln r_D}{\ln\beta}\right]}{\ln(\beta r_D^2)} > 0 \tag{66}$$

The total loss function $V_D(T) = V_1^D(T) + V_2^D(T)$ has a similar development as the corresponding function $V(T)$ under timeless perspective commitment. In the limiting case $\beta = 1$ it is overall increasing. For $\beta < 1$ it is hump-shaped, if the degree of price flexibility is not too large, while it is monotonically decreasing in $T$ if the value of $\omega$ is small. For small values of $\omega$ the derivative of $V_D$ at time $T = 0$ is negative, while it is positive if $\omega$ is sufficiently large. For the sake of brevity, the proof for the case of simple (optimal) Taylor rules is presented in the mathematical appendix. □

The propositions 1 to 3 follow from two opposing effects on the welfare loss which change in opposite directions with increasing lead time $T$. On the one hand, the size of the initial jumps of the forward-looking variables $x_t$ and $\pi_t$ taking place at the time of anticipation, is inversely related to the time span between anticipation and realization of the cost-push shock. The longer the lead time $T$, the smaller is the response of output and inflation on impact so that the contribution of this *anticipation effect* to the welfare loss $V$ *decreases* with increasing $T$. On the other hand, the *persistence effect* of the cost-push shock on the target variables $x_t$ and $\pi_t$ is *increasing* in $T$. Thereby, persistence is measured as the total variation of a variable over time, i.e. its intertemporal deviation from the respective initial steady state. For example, the persistence of the price level $p_t$ is given by $\sum_{t=0}^{\infty} |p_t - \overline{p}_0|$ where the initial steady state can be normalized to zero. In the appendix, we derive the persistence of $p_t, x_t$, and $\pi_t$ under the optimal monetary policy regimes commitment and discretion and show that persistence is smaller in case of unanticipated shocks than in case of anticipated shocks.

For the sake of illustration, we numerically simulated our solutions by using a standard calibration. The time unit is one quarter. The discount rate is equal to $\beta = 0.99$ implying an annual steady state real interest rate of approximately 4 percent. The inverse of the intertemporal elasticity of substitution, $\sigma$, is set to $\sigma = 2$. We set $\eta = 1$ implying a quadratic disutility of labor. The Calvo parameter $\omega$ is either set to 0.25 implying an average duration of price contracts of four months or to 0.75 implying an average duration of price contracts of one year. The weights in the loss function are set to $\alpha_1 = 1$ and $\alpha_2 = 0.5$ reflecting the objective of flexible inflation targeting. Finally, we assume the cost-push shock to be persistent and choose $\varphi$ equal to 0.5.

Figure 1 depicts impulse response functions of inflation, output gap, and price level in case of low ($\omega = 0.25$, left column) and high ($\omega = 0.75$, right column) price rigidity under the optimal monetary policy with timeless perspective commitment. Solid lines with triangles denote responses to a cost-push shock that unexpectedly emerges in period $t = 0$, solid lines with circles denote responses to a cost-push shock whose realization in period $T = 2$ is anticipated in period $t = 0$.

We firstly consider the empirically plausible case of *high price rigidity.* In case of an unanticipated cost-shock, both the price level and inflation rise whereas output falls in response to the realization of the increase in the costs of production.[3] Subsequently, all variables converge in a hump-shaped fashion to their respective steady state values.

Anticipated cost shocks have two effects, namely the anticipation effect which reflects the change in $x_t$, $\pi_t$, and $p_t$ in response to the anticipation of a future change in costs, and the realization effect which occurs when the anticipated change in costs actually takes place. Under the optimal monetary policy with commitment, output starts to decline and prices begin to increase in response to the anticipation of a future rise in the costs of production. Both

[3] We could think about this cost-push shock as an exogenous rise in wage mark-ups (see, for example, Galí (2008)).

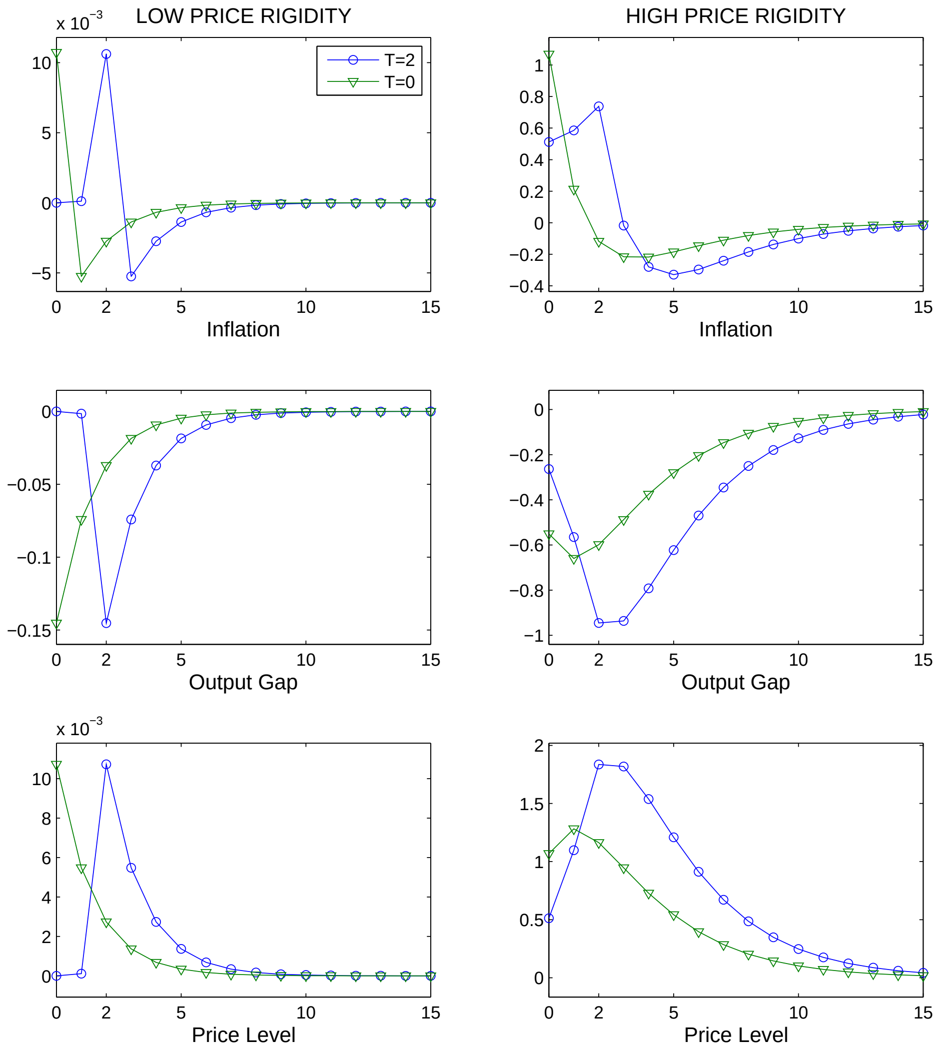


**Figure 1**: Impulse response functions under optimal policy with timeless perspective commitment.

Notes: Solid lines with triangles denote responses to an unanticipated cost-push shock, solid lines with circles denote responses to an anticipated cost-push shock. In case of low price rigidity, the Calvo parameter $\omega$ is set to 0.25; in case of high price rigidity, $\omega$ is set to 0.75.

variables respond in a hump-shaped fashion peaking at the date of realization. The increase in prices causes inflation to jump at the time of anticipation, peaking at the date of realization and then returning in a hump-shaped fashion to its initial steady state level.

In case of *low price rigidity*, an unanticipated cost shock causes an immediate rise in prices and an immediate drop in output. Subsequently, both variables converge monotonically to their initial steady state levels. After the initial jump, inflation falls sharply and converges from below to its pre-shock level. The announcement of a future rise in costs has negligible anticipation effects when prices are highly flexible. The reason is that the price setting problem of firms becomes more of an atemporal (static) nature when the Calvo parameter $\omega$ decreases. In this case firms know that, with a high probability, they will be able to raise their price when the anticipated shock actually materializes in period $T$. Thus, output and prices change only slightly in response to an announcement or anticipation of future cost-push shocks.

Regardless of the degree of price rigidity, Figure 1 illustrates that the initial jumps of inflation, output gap and price level are greater in case of unanticipated ($T = 0$) than in case of anticipated shocks ($T = 2$). On the other hand, anticipated shocks amplify the persistence of $p_t, x_t$, and $\pi_t$ compared to unanticipated shocks.[4]

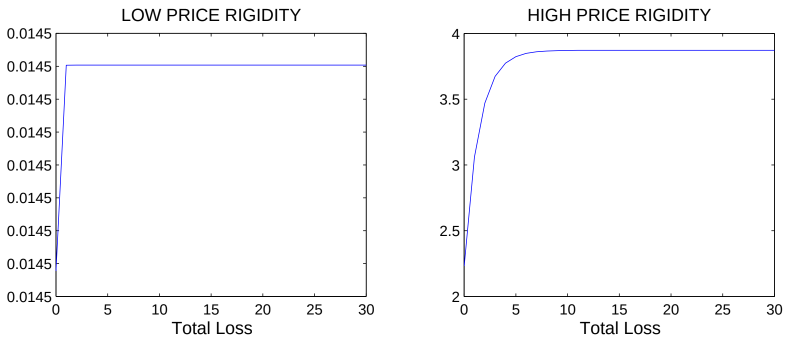


**Figure 2**: Welfare loss for different lengths of anticipation period under optimal timeless perspective commitment policy in case $\beta = 1$.

Figure 2 illustrates the welfare loss $V = V(T)$ in case $\beta = 1$. Without time discounting in the intertemporal loss function, the persistence effect always dominates the anticipation effect so that proposition 1 holds. In Figure 2, the total loss $V = V(T)$ is overall increasing in $T$ if $\beta = 1$.

If future deviations of the state variables from their initial steady state levels are discounted, the contribution of the initial jumps of output and inflation for the determination of the total loss becomes more important. The same holds for increasing degree of price flexibility $1-\omega$, since the persistence of prices, output and inflation is a decreasing function of $1-\omega$. If the degree of price flexibility is high, the value of the total loss is almost completely determined by the size of the initial jumps of $x_t$ and $\pi_t$ which in turn is inversely proportional to the lead time $T$. With a sufficiently high degree of price flexibility, the total loss under unanticipated cost-push shocks is greater than the loss under anticipated shocks so that proposition 3 holds. This result is also illustrated in Figure 3, where $V(T)$ is a monotonically decreasing function in the lead time $T$ if the degree of price rigidity $\omega$ is very small.

[4]This result also holds in the special case $\varphi = 0$, i.e. if the shock exhibits no serial correlation. It is well-known that even in this case the optimal precommitment policy introduces inertia in the impulse response functions.

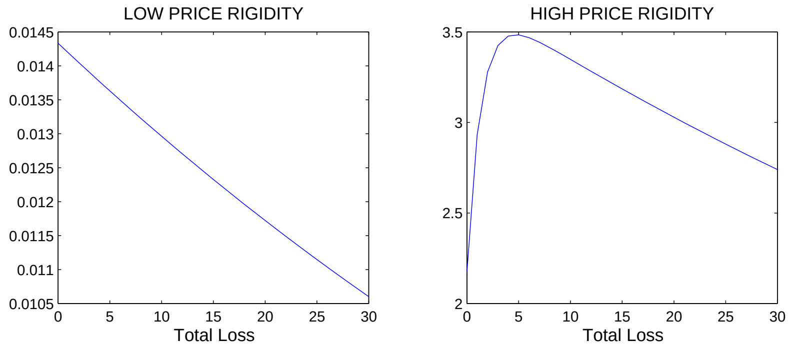


**Figure 3**: Welfare loss for different lengths of anticipation period under optimal timeless perspective commitment policy in case $\beta = 0.99$.

From an empirical point of view, the parameter $\omega$ is not that small so that the development of the impulse response functions displays inertia or strong serial correlation. Then, if the time span between the anticipation and the implementation of the cost-push shock is not too long, the persistence effect dominates and the value of the total loss $V(T)$ is greater than $V(0)$. This is illustrated in Figure 3, where the development of the loss function $V(T)$ is hump-shaped and monotonically increasing for small values of $T$.

Propositions 1 to 3 are independent of the chosen optimal monetary policy regime. They hold under timeless perspective commitment as well as under discretion (see Figure 4 and 5 for a numerical visualization). They also hold under simple monetary policy rules (such as Taylor-type rules or money growth peg).

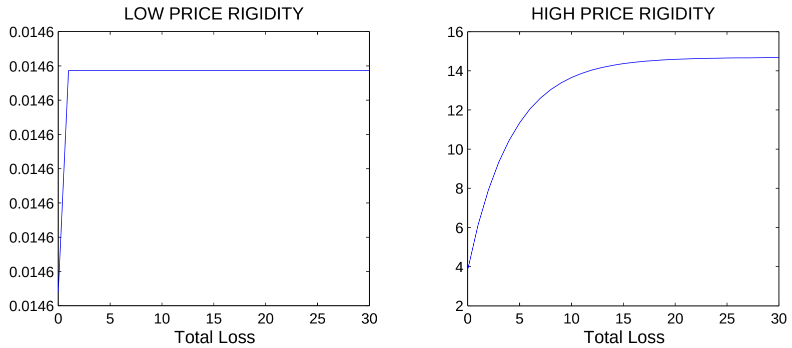


**Figure 4**: Welfare loss for different lengths of anticipation period under the optimal discretionary policy in case $\beta = 1$.

In order to check whether the welfare-reducing effects of anticipations hold for empirically plausible degrees of nominal rigidity, we compute the critical anticipation values $T_c^*$ (commitment) and $T_d^*$ (discretion). Table 1 depicts the values of $T_c^*$ and $T_d^*$ for a persistent ($\varphi = 0.5$) and a one-off cost-push shock ($\varphi = 0$).

Table 1 shows that the anticipation of cost-push shocks dampens the welfare loss induced by such shocks only for empirically unrealistic degrees of nominal

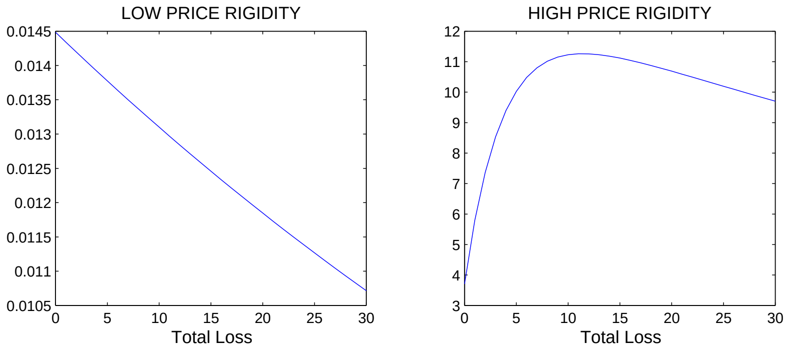


**Figure 5**: Welfare loss for different lengths of anticipation period under the optimal discretionary policy in case $\beta = 0.99$.

rigidity. For the widely applied values of $\omega = 0.75$ or $\omega = 0.66$, the anticipation period or lead time $T$ must be extremely large to obtain a welfare gain from anticipation. Under commitment and a value $\omega = 0.75$, the loss under an anticipated shock is smaller than the loss under an unanticipated shock of same size when the shock is anticipated to take place in $T_c^* = 54$ (for $\varphi = 0.5$) or $T_c^* = 66$ (for $\varphi = 0$) quarters. Even larger values are obtained under optimal discretionary policy. A Calvo parameter of 0.5 represents the lower bound in the range of values that are reported in the literature. In this case and under the monetary policy regime commitment, the anticipation of future cost shocks has a welfare-enhancing effect if the lead time is larger or equal to two quarters for persistent and three quarters for one-off shocks, respectively. Under discretionary monetary policy, these critical values are three and four quarters.

Our simulations illustrate that for a wide range of empirically realistic degrees of nominal rigidities (i.e., $\omega \geq 0.5$) in conjunction with a plausible length of the anticipation period, the welfare loss of anticipated cost shocks exceeds the welfare loss of unanticipated cost shocks.

**Table 1**: Values of the critical lead time $T_c^*$ and $T_d^*$

| | *Degree of price rigidity* $\omega$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Monetary policy | 0.75 | 0.66 | 0.60 | 0.55 | 0.50 | 0.45 | 0.40 | 0.25 |
| *With* $\varphi = 0.5$ | | | | | | | | |
| Commitment | 53.09 | 19.82 | 9.00 | 4.23 | 1.82 | 0.69 | 0.16 | 0 |
| Discretion | 125.90 | 40.41 | 15.61 | 6.37 | 2.42 | 0 | 0 | 0 |
| *With* $\varphi = 0$ | | | | | | | | |
| Commitment | 65.78 | 25.57 | 11.79 | 5.59 | 2.41 | 0.95 | 0.28 | 0 |
| Discretion | 146.99 | 50.77 | 20.25 | 8.38 | 3.20 | 0 | 0 | 0 |

Note: For an anticipation period $0 < T < T_i^*$ it is true that $V|_T > V|_{T=0}$, for $T > T_i^*$ it is true that $V|_T < V|_{T=0}$ where $i = c, d$.

## 4 Conclusion

In this paper we investigate the welfare effects resulting from the anticipation of future shocks. In particular, we analyze the welfare loss for different lengths of the time span between the anticipation and the realization of cost-push shocks. This includes the widely applied case of unanticipated cost-push shocks. Our analysis is based on the canonical New Keynesian model with optimal monetary policy.

We emphasize the role of nominal rigidities for the welfare effects of anticipations. We show that for empirically plausible degrees of nominal rigidity, anticipated cost shocks entail higher welfare losses than unexpected cost shocks. The anticipation of a future cost-push shock dampens the volatility of output and inflation only if prices are highly flexible. These results hold independently of the monetary policy regime (timeless perspective commitment, discretion, (optimal) simple rules).

Our results imply that the knowledge about the realization of future cost shocks is in general welfare-reducing. The question remains why rational agents do not simply ignore this information. However, this would be inconsistent with the profit-maximizing behavior of individual firms and the utility-maximizing behavior of individual households on which our model is based. The firm's optimality condition in fact calls for an increase in prices in response to the anticipation of a future rise in costs. By simply ignoring this information, the firm would make a loss.

Hence, our results reveal a contradiction between the optimal behavior of individuals and the optimum from a social point of view.

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## Mathematical Appendix

### Solution time paths under the optimal timeless perspective precommitment policy

It is well-known that under the optimal timeless perspective precommitment policy inflation and the output gap satisfy

$$\pi_t = -\frac{\alpha_2}{\alpha_1 \kappa}(x_t - x_{t-1}) \tag{1}$$

and

$$\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) x_t - x_{t-1} - \beta E_t x_{t+1} = -\frac{\alpha_1 \kappa}{\alpha_2} k_t \tag{2}$$

where the expectational operator can be omitted in case of anticipated shocks. To solve the difference equation for $x_t$ write equation (2) as

$$\begin{pmatrix} x_{t+1} \\ w_{t+1} \end{pmatrix} = C \begin{pmatrix} x_t \\ w_t \end{pmatrix} + \begin{pmatrix} \frac{\alpha_1 \kappa}{\alpha_2 \beta} \\ 0 \end{pmatrix} k_t \tag{3}$$

where $w_t = x_{t-1}$ and

$$C = \begin{pmatrix} \frac{1}{\beta}\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) & -\frac{1}{\beta} \\ 1 & 0 \end{pmatrix} \tag{4}$$

The auxiliary variable $w_t$ is backward-looking (with the initial value $w_0 = 0$) while the output gap $x_t$ is forward-looking. The system matrix $C$ has two real eigenvalues $r_1$ and $r_2$ with $r_1 > 1 > r_2 > 0$ so that the Blanchard/Kahn (1980) saddlepath stability condition is satisfied. The eigenvalues are given by

$$r_{1,2} = \frac{1}{2}\mathrm{tr}\, C \pm \sqrt{\frac{1}{2}(\mathrm{tr}\, C)^2 - |C|} \tag{5}$$

with

$$\mathrm{tr}\, C = \frac{1}{\beta}\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) = r_1 + r_2 \,, \qquad |C| = \frac{1}{\beta} = r_1 r_2 \tag{6}$$

We can transfer system (3) into Jordan-canonical form using the similarity transformation

$$C = H \cdot \Lambda \cdot H^{-1} \tag{7}$$

where $\Lambda = \mathrm{diag}(r_1, r_2)$ is a diagonal matrix whose diagonal elements are the characteristic roots of $C$ and

$$H = \begin{pmatrix} r_1 & r_2 \\ 1 & 1 \end{pmatrix} \tag{8}$$

is a matrix of linearly independent eigenvectors of $C$. Define auxiliary variables $v_t$ and $z_t$ by

$$H^{-1}\begin{pmatrix}x_t\\ w_t\end{pmatrix} = \begin{pmatrix}v_t\\ z_t\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}x_t\\ w_t\end{pmatrix} = H\begin{pmatrix}v_t\\ z_t\end{pmatrix} \tag{9}$$

Premultiplying equation (3) with $H^{-1}$ yields the Jordan-canonical system

$$\begin{pmatrix}v_{t+1}\\ z_{t+1}\end{pmatrix} = \begin{pmatrix}r_1 & 0\\ 0 & r_2\end{pmatrix}\begin{pmatrix}v_t\\ z_t\end{pmatrix} + \frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}\begin{pmatrix}1\\ -1\end{pmatrix}k_t \tag{10}$$

The difference equation in $v_t$ contains the unstable eigenvalue $r_1$ and has the unique stable forward solution

$$v_t = -\sum_{s=0}^{\infty} r_1^{-s}\frac{1}{r_1}\frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}k_{t+s} \tag{11}$$

Since the cost-push shock $k_{t+s}$ is a AR(1) variable with

$$k_{t+s} = \begin{cases}\varphi^{t+s-T} & \text{for} \quad t+s \geq T\\ 0 & \text{for} \quad t+s < T\end{cases} \tag{12}$$

we obtain

$$v_t = \begin{cases}-\dfrac{1}{r_1-\varphi}\dfrac{1}{r_1 - r_2}\dfrac{\alpha_1\kappa}{\alpha_2\beta}\varphi^{t-T} & \text{for} \quad t \geq T\\ -\dfrac{1}{r_1-\varphi}\dfrac{1}{r_1 - r_2}\dfrac{\alpha_1\kappa}{\alpha_2\beta}r_1^{t-T} & \text{for} \quad t \leq T\end{cases} \tag{13}$$

with the initial value

$$v_0 = -\frac{1}{r_1-\varphi}\frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}r_1^{-T} < 0 \tag{14}$$

The difference equation in $z_t$ has the general backward solution

$$z_t = r_2^t K - \sum_{s=0}^{t-1} r_2^{t-s-1}\frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}k_s \tag{15}$$

where the constant $K$ follows from the initial condition

$$z_0 = K = w_0 - v_0 = -v_0 \tag{16}$$

Since $k_s = 0$ for $s < T$ we obtain

$$z_t = r_2^t K = \frac{1}{r_1-\varphi}\frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}r_1^{-T}r_2^t \quad \text{for } t \leq T \tag{17}$$

and

$$\begin{aligned} z_t &= r_2^t K - \frac{1}{r_2-\varphi}\frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}\left[r_2^{t-T} - \varphi^{t-T}\right] \\ &= \frac{1}{r_1 - r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}\left[\left(\frac{r_1^{-T}}{r_1-\varphi} - \frac{r_2^{-T}}{r_2-\varphi}\right)r_2^t + \frac{\varphi^{-T}}{r_2-\varphi}\varphi^t\right] \quad \text{for} \quad t \geq T \end{aligned} \tag{18}$$

The solution for the output gap $x_t = r_1 v_t + r_2 z_t$ is then given by

$$x_t = -\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2}\frac{\alpha_1\kappa}{\alpha_2\beta}r_1^{-T}(r_1^{t+1}-r_2^{t+1}) \quad \text{for} \quad t \le T \tag{19}$$

with the initial values

$$x_0 = -\frac{1}{r_1-\varphi}\frac{\alpha_1\kappa}{\alpha_2\beta}r_1^{-T}, \qquad x_{-1} = 0 \tag{20}$$

and

$$\begin{aligned} x_t =& \frac{\alpha_1\kappa}{\alpha_2\beta}\frac{1}{(r_1-\varphi)(r_2-\varphi)} \cdot \\ &\cdot \left[\varphi^{t+1-T} - \frac{(r_1-\varphi)r_2^{-T}-(r_2-\varphi)r_1^{-T}}{r_1-r_2}r_2^{t+1}\right] \text{ for } t \ge T \end{aligned} \tag{21}$$

The solution formula (21) also contains the limiting case $T = 0$, i.e., if the cost-push shock is not anticipated. The term in brackets then simplifies to $\varphi^{t+1} - r_2^{t+1}$.

Using (1), the solution time path of the inflation rate follows:

$$\pi_t = \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2}r_1^{-T}\left[(r_1-1)r_1^t - (r_2-1)r_2^t\right] \text{ for } t \le T \tag{22}$$

with the initial value

$$\pi_0 = \frac{1}{\beta}\frac{1}{r_1-\varphi}r_1^{-T} \tag{23}$$

and

$$\begin{aligned} \pi_t =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi} \cdot \\ &\cdot \left[(1-\varphi)\varphi^{t-T} - \frac{(r_1-\varphi)r_2^{-T}-(r_2-\varphi)r_1^{-T}}{r_1-r_2}(1-r_2)r_2^t\right] \text{ for } t \ge T \end{aligned} \tag{24}$$

In the special case $T = 0$ the term in brackets simplifies to $(1-\varphi)\varphi^t - (1-r_2)r_2^t$.

The solution time path of the price level $p_t$ can be derived from the solution of $\pi_t$ due to

$$p_t = \sum_{k=0}^{t} \pi_k \tag{25}$$

We then obtain for or $t \le T$:

$$\begin{aligned} p_t =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2}r_1^{-T}\sum_{k=0}^{t}\left[(r_1-1)r_1^k - (r_2-1)r_2^k\right] \\ =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2}r_1^{-T}\left[(r_1-1)\frac{1-r_1^{t+1}}{1-r_1} - (r_2-1)\frac{1-r_2^{t+1}}{1-r_2}\right] \\ =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2}r_1^{-T}\left[r_1^{t+1} - r_2^{t+1}\right] \end{aligned} \tag{26}$$

and for $t \geq T$

$$
\begin{aligned}
p_t &= \sum_{k=0}^{T-1} \pi_k + \sum_{k=T}^{t} \pi_k \qquad (27)\\
&= \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T}\left[r_1^T - r_2^T\right] + \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi}\cdot\\
&\cdot \sum_{k=T}^{t}\left\{(1-\varphi)\varphi^{k-T} - \frac{(r_1-\varphi)r_2^{-T} - (r_2-\varphi)r_1^{-T}}{r_1-r_2}(1-r_2)r_2^k\right\}\\
&= \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T}\left[r_1^T - r_2^T\right] + \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi}\cdot\\
&\cdot \left[-(1-\varphi)\varphi^{-T}\frac{\varphi^{t+1}-\varphi^T}{1-\varphi} + \frac{(r_1-\varphi)r_2^{-T} - (r_2-\varphi)r_1^{-T}}{r_1-r_2}(1-r_2)\frac{r_2^{t+1}-r_2^T}{1-r_2}\right]\\
&= \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T}\left[r_1^T - r_2^T\right] + \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi}\cdot\\
&\cdot \left[1-\varphi^{t+1-T} + \frac{(r_1-\varphi)r_2^{-T} - (r_2-\varphi)r_1^{-T}}{r_1-r_2}\left(r_2^{t+1}-r_2^T\right)\right]\\
&= \frac{1}{\beta}\frac{1}{r_2-\varphi}\frac{1}{r_1-r_2} r_2^{t+1-T} - \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T} r_2^{t+1} - \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi}\varphi^{t+1-T}
\end{aligned}
$$

Obviously,

$$\lim_{t\to\infty} p_t = 0 \quad \text{for all} \quad T \geq 0 \qquad (28)$$

and

$$p_0 = \frac{1}{\beta}\frac{1}{r_1-\varphi} r_1^{-T} = \pi_0 > 0 \qquad (29)$$

so that the size of the initial jump in $p$ is inversely proportional to the lead time $T$.

Similar results hold for the state variables $x_t$ and $\pi_t$. Since

$$\sum_{k=0}^{t}(x_k - x_{k-1}) = x_t \qquad (30)$$

equation (1) implies

$$p_t = \sum_{k=0}^{t} \pi_k = -\frac{\alpha_2}{\alpha_1 \kappa}\sum_{k=0}^{t}(x_k - x_{k-1}) = -\frac{\alpha_2}{\alpha_1\kappa} x_t \qquad (31)$$

so that $p_t > 0$ if and only if $x_t < 0$. The optimal policy under timeless perspective implies $p_t > 0$ for all $0 \leq t < \infty$ so that $x_t < 0$ for all $t < \infty$. We can also show that the *persistence* or *total variation* of $p_t$ is positive correlated with $T$, i.e.

$$\sum_{t=0}^{\infty} p_t\Big|_{T=0} < \sum_{t=0}^{\infty} p_t\Big|_{T>0} \quad \text{for all} \quad T > 0 \qquad (32)$$

where the infinite sum $\sum_{t=0}^{\infty} p_t\Big|_{T>0}$ is an increasing function in $T$.

The persistence measure used here is based on the deviation of $p_t$ from its initial steady state level $\overline{p}_0$, where the deviation $|p_t - \overline{p}_0|$ is calculated both for $t < T$ and $t \geq T$. Thereafter the differences $|p_t - \overline{p}_0|$ are summed up. Since $\overline{p}_0 = 0$ and $p_t > 0$ for all $t$ we must determine the infinite sum $\sum_{t=0}^{\infty} p_t$.

Inequality (32) holds although the initial jump of $p_t$ is a negative function in $T$. To prove the inequality note that

$$\begin{aligned} \sum_{t=0}^{\infty} p_t\Big|_{T=0} &= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[\frac{r_2}{1-r_2} - \frac{\varphi}{1-\varphi}\right] \\ &= \frac{1}{\beta(r_1-\varphi)(1-r_2)(1-\varphi)} \end{aligned} \tag{33}$$

$$\sum_{t=0}^{T} p_t\Big|_{T>0} = \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} r_1^{-T}\left[r_1\frac{1-r_1^{T+1}}{1-r_1} - r_2\frac{1-r_2^{T+1}}{1-r_2}\right] \tag{34}$$

and

$$\begin{aligned} \sum_{t=T+1}^{\infty} p_t\Big|_{T>0} = &\frac{1}{\beta(r_2-\varphi)(r_1-r_2)} r_2^{1-T}\frac{r_2^{T+1}}{1-r_2} \\ &- \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} r_1^{-T} r_2 \frac{r_2^{T+1}}{1-r_2} \\ &- \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)} \varphi^{1-T}\frac{\varphi^{T+1}}{1-\varphi} \end{aligned} \tag{35}$$

so that

$$\begin{aligned} \sum_{t=0}^{\infty} p_t\Big|_{T>0} &= \frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\left[\frac{r_1^{1-T}}{1-r_1} - \frac{r_1^2}{1-r_1} - \frac{r_2 r_1^{-T}}{1-r_2}\right. \\ &\quad \left. + \frac{r_1^{-T} r_2^{T+2}}{1-r_2} - \frac{r_1^{-T} r_2^{T+2}}{1-r_2}\right] \\ &\quad + \frac{1}{\beta(r_2-\varphi)}\left[\frac{1}{r_1-r_2}\frac{r_2^2}{1-r_2} - \frac{1}{r_1-\varphi}\frac{\varphi^2}{1-\varphi}\right] \\ &= \frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\left[\frac{r_1}{1-r_1}\left(r_1^{-T} - r_1\right) - \frac{r_2 r_1^{-T}}{1-r_2}\right] \\ &\quad + \frac{1}{\beta(r_2-\varphi)}\left[\frac{1}{r_1-r_2}\frac{r_2^2}{1-r_2} - \frac{1}{r_1-\varphi}\frac{\varphi^2}{1-\varphi}\right] \end{aligned} \tag{36}$$

Then

$$\sum_{t=0}^{\infty} p_t\Big|_{T>0} > \sum_{t=0}^{\infty} p_t\Big|_{T=0} \quad \Leftrightarrow \tag{37}$$

$$\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\frac{r_1}{1-r_1}\left(r_1^{-T}-r_1\right)-\frac{r_2}{1-r_2}$$

$$\left[\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}r_1^{-T}-\frac{1}{\beta(r_2-\varphi)}\frac{r_2}{r_1-r_2}+\frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\right]$$

$$-\frac{1}{\beta(r_2-\varphi)}\frac{1}{r_1-\varphi}\frac{\varphi}{1-\varphi}[\varphi-1]>0 \quad \Leftrightarrow$$

$$\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\frac{r_1}{1-r_1}\left(r_1^{-T}-r_1\right)$$

$$-\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\frac{r_2}{1-r_2}\left(r_1^{-T}-r_1\right)-\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\frac{r_2}{1-r_2}r_1$$

$$-\frac{r_2}{1-r_2}\left[-\frac{r_1-\varphi}{\beta(r_1-\varphi)(r_2-\varphi)}\frac{r_2}{r_1-r_2}+\frac{r_1-r_2}{\beta(r_1-\varphi)(r_2-\varphi)(r_1-r_2)}\right]$$

$$+\frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\varphi>0 \quad \Leftrightarrow$$

$$\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\left(r_1^{-T}-r_1\right)\left[\frac{r_1}{1-r_1}-\frac{r_2}{1-r_2}\right]$$

$$+\frac{r_2}{1-r_2}\frac{1}{\beta(r_1-\varphi)(r_2-\varphi)(r_1-r_2)}\cdot$$

$$\cdot\left[r_2(r_1-\varphi)-(r_1-r_2)-(r_2-\varphi)r_1\right]+\frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\varphi>0 \quad \Leftrightarrow$$

$$\frac{1}{\beta(r_1-\varphi)(r_1-1)(1-r_2)}\left(r_1-r_1^{-T}\right)+\frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\cdot$$

$$\cdot\left[\varphi+\frac{r_2}{(1-r_2)(r_1-r_2)}\left(r_2(r_1-\varphi)-(r_1-r_2)-(r_2-\varphi)r_1\right)\right]>0 \quad \Leftrightarrow$$

$$\frac{1}{\beta(r_1-\varphi)(r_1-1)(1-r_2)}\left(r_1-r_1^{-T}\right)$$

$$+\frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\frac{(r_1-r_2)(\varphi-r_2)}{(1-r_2)(r_1-r_2)}>0 \quad \Leftrightarrow$$

$$\frac{1}{\beta(r_1-\varphi)(r_1-1)(1-r_2)}\left(r_1-r_1^{-T}\right)-\frac{1}{\beta(r_1-\varphi)(1-r_2)}>0 \quad \Leftrightarrow$$

$$r_1-r_1^{-T}-(r_1-1)>0 \quad \Leftrightarrow$$

$$1-r_1^{-T}>0$$

Since $r_1 > 1$ the last inequality is fulfilled. Note that the total variation of $p_t$, i.e. $\sum_{t=0}^{\infty} p_t\Big|_{T>0}$ is an increasing function in $T$. This follows from equation (36), since the derivative of $\frac{r_1}{1-r_1}r_1^{-T} - \frac{r_2}{1-r_2}r_1^{-T}$ with respect to $T$ is positive.

An implication of inequality (37) is

$$\sum_{t=0}^{\infty} |x_t|\Big|_{T=0} < \sum_{t=0}^{\infty} |x_t|\Big|_{T>0} \tag{38}$$

since

$$|x_t| = \frac{\alpha_1 \kappa}{\alpha_2} p_t \tag{39}$$

The persistence of the output response in case of anticipated cost-push shocks is therefore stronger than in case of unanticipated shocks.

A similar result can be shown for the inflation rate $\pi_t$ if the limiting case $\varphi = 0$ is considered. We then get for $T = 0$

$$\pi_t = \begin{cases} 1 - (1 - r_2) = r_2 & \text{if} \quad t = 0 \\ -(1 - r_2) r_2^t < 0 & \text{if} \quad t > 0 \end{cases} \tag{40}$$

implying

$$\begin{aligned} \sum_{t=0}^{\infty} \pi_t = \pi_0 + \sum_{t=1}^{\infty} \pi_t &= r_2 - (1 - r_2) \sum_{t=1}^{\infty} r_2^t \\ &= r_2 - (1 - r_2) \left[ \frac{1}{1 - r_2} - 1 \right] = r_2 - r_2 = 0 \end{aligned} \tag{41}$$

and

$$\sum_{t=0}^{\infty} |\pi_t|\Big|_{T=\varphi=0} = r_2 + (1 - r_2) \sum_{t=1}^{\infty} r_2^t = 2 r_2 \tag{42}$$

In case $T > 0$ and $\varphi = 0$ we get

- for $t \leq T$:

$$\pi_t = \frac{r_2}{r_1 - r_2} r_1^{-T} \left[ (r_1 - 1) r_1^t - (r_2 - 1) r_2^t \right] > 0 \tag{43}$$

- for $t > T$:

$$\pi_t = -\frac{r_1 r_2^{-T} - r_2 r_1^{-T}}{r_1 - r_2} (1 - r_2) r_2^t < 0 \tag{44}$$

Then

$$\begin{aligned} \sum_{t=0}^{T} \pi_t &= \frac{r_2}{r_1 - r_2} r_1^{-T} \sum_{t=0}^{T} \left[ (r_1 - 1) r_1^t - (r_2 - 1) r_2^t \right] \\ &= \frac{r_2}{r_1 - r_2} r_1^{-T} \left[ (r_1 - 1) \frac{1 - r_1^{T+1}}{1 - r_1} + (1 - r_2) \frac{1 - r_2^{T+1}}{1 - r_2} \right] \\ &= \frac{r_2}{r_1 - r_2} r_1^{-T} \left[ r_1^{T+1} - r_2^{T+1} \right] \end{aligned} \tag{45}$$

and

$$\sum_{t=T+1}^{\infty} \pi_t = -\frac{1-r_2}{r_1-r_2}\left[r_1 r_2^{-T} - r_2 r_1^{-T}\right]\frac{r_2^{T+1}}{1-r_2} \tag{46}$$
$$= -\frac{r_2}{r_1-r_2} r_1^{-T}\left[r_1^{T+1} - r_2^{T+1}\right]$$

so that

$$\sum_{t=0}^{\infty} \pi_t = 0 \tag{47}$$

and

$$\sum_{t=0}^{\infty} |\pi_t|\Big|_{\substack{T>0\\ \varphi=0}} = 2\frac{r_2}{r_1-r_2} r_1^{-T}\left[r_1^{T+1} - r_2^{T+1}\right] \tag{48}$$

Now

$$\begin{aligned} \frac{r_2}{r_1-r_2} r_1^{-T}\left[r_1^{T+1} - r_2^{T+1}\right] > r_2 &\quad\Leftrightarrow \\ r_1^{-T}\left[r_1^{T+1} - r_2^{T+1}\right] > r_1 - r_2 &\quad\Leftrightarrow \\ r_1^{T+1} - r_2^{T+1} > r_1^{T+1} - r_2 r_1^{T} &\quad\Leftrightarrow \\ r_2 r_1^{T} - r_2^{T+1} > 0 &\quad\Leftrightarrow \\ r_2\left[r_1^{T} - r_2^{T}\right] > 0 & \end{aligned} \tag{49}$$

Due to $r_1 > 1 > r_2 > 0$ the last inequality is met so that

$$\sum_{t=0}^{\infty} |\pi_t|\Big|_{T=\varphi=0} < \sum_{t=0}^{\infty} |\pi_t|\Big|_{\substack{T>0\\ \varphi=0}} \tag{50}$$

The case $\varphi > 0$ is more difficult to analyze since $\pi_t$ can take both positive and negative values for $t > T > 0$. If $T = 0$, $\pi_t$ changes sign immediately after the initial jump. Since

$$\pi_t = \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[(1-\varphi)\varphi^t - (1-r_2)r_2^t\right] \quad (\text{if } T=0) \tag{51}$$

we get

$$\pi_0 = \frac{1}{\beta(r_1-\varphi)} > 0 \tag{52}$$

and

$$\begin{aligned} \sum_{t=1}^{\infty} \pi_t\Big|_{T=0} &= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[(1-\varphi)\sum_{t=1}^{\infty}\varphi^t - (1-r_2)\sum_{t=1}^{\infty} r_2^t\right] \\ &= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[(1-\varphi)\left(\frac{1}{1-\varphi} - 1\right) - (1-r_2)\left(\frac{1}{1-r_2} - 1\right)\right] \\ &= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}(\varphi - r_2) = -\frac{1}{\beta(r_1-\varphi)} = -\pi_0 \end{aligned} \tag{53}$$

so that

$$\sum_{t=0}^{\infty} |\pi_t|\Big|_{T=0} = 2\frac{1}{\beta(r_1 - \varphi)} \tag{54}$$

In case $T > 0$ $\pi_t$ is positive for $0 \leq t \leq T$ and we obtain due to (22)

$$\begin{aligned} \sum_{t=0}^{T} \pi_t &= \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} r_1^{-T} \left[ (r_1-1)\frac{1-r_1^{T+1}}{1-r_1} - (r_2-1)\frac{1-r_2^{T+1}}{1-r_2} \right] \\ &= \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} r_1^{-T} \left[ -\left(1-r_1^{T+1}\right) + 1 - r_2^{T+1} \right] \\ &= \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} r_1^{-T} \left[ r_1^{T+1} - r_2^{T+1} \right] \\ &= \frac{r_1}{\beta(r_1-\varphi)(r_1-r_2)} \left[ 1 - \left(\frac{r_2}{r_1}\right)^{T+1} \right] > 0 \end{aligned} \tag{55}$$

(since $r_1 > 1 > r_2 > 0$). If $t > T$, $\pi_t$ is negative for sufficiently large values of $t$. For small values of $t > T$ $\pi_t$ may be positive. Due to

$$\lim_{t\to\infty} p_t = 0 \quad \text{and} \quad p_t = \sum_{k=0}^{t} \pi_k \tag{56}$$

we must have

$$\sum_{t=0}^{\infty} \pi_t = 0 \tag{57}$$

so that

$$\sum_{t=T+1}^{\infty} \pi_t = -\sum_{t=0}^{T} \pi_t < 0 \tag{58}$$

The last equation also follows from (24): With

$$\psi = -\frac{(r_1-\varphi)r_2^{-T} - (r_2-\varphi)r_1^{-T}}{r_1 - r_2} \tag{59}$$

we obtain

$$
\begin{aligned}
\sum_{t=T+1}^{\infty}\pi_t &= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[(1-\varphi)\varphi^{-T}\sum_{t=T+1}^{\infty}\varphi^t+\psi(1-r_2)\sum_{t=T+1}^{\infty}r_2^t\right] \qquad (60)\\
&= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[(1-\varphi)\varphi^{-T}\frac{\varphi^{T+1}}{1-\varphi}+\psi(1-r_2)\frac{r_2^{T+1}}{1-r_2}\right]\\
&= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[\varphi+\psi r_2^{T+1}\right]\\
&= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[\varphi-\frac{(r_1-\varphi)r_2^{-T}-(r_2-\varphi)r_1^{-T}}{r_1-r_2}r_2^{T+1}\right]\\
&= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)}\left[\varphi-\frac{r_1-\varphi}{r_1-r_2}r_2+\frac{r_2-\varphi}{r_1-r_2}r_1^{-T}r_2^{T+1}\right]\\
&= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)(r_1-r_2)}\left[\varphi(r_1-r_2)-r_2(r_1-\varphi)+(r_2-\varphi)r_1^{-T}r_2^{T+1}\right]\\
&= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)(r_1-r_2)}\left[(\varphi-r_2)r_1+(r_2-\varphi)r_1^{-T}r_2^{T+1}\right]\\
&= \frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\left[-r_1+r_1^{-T}r_2^{T+1}\right]\\
&= -\frac{r_1}{\beta(r_1-\varphi)(r_1-r_2)}\left[1-\left(\frac{r_2}{r_1}\right)^{T+1}\right] = -\sum_{t=0}^{T}\pi_t<0
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\sum_{t=0}^{T}\pi_t\Big|_{T>0}-\sum_{t=T+1}^{\infty}\pi_t\Big|_{T>0} = 2\sum_{t=0}^{T}\pi_t\Big|_{T>0} > \sum_{t=0}^{\infty}|\pi_t|\Big|_{T=0} = 2\pi_0\Big|_{T=0} &\quad\Leftrightarrow \qquad (61)\\
\sum_{t=0}^{T}\pi_t\Big|_{T>0} > \pi_0\Big|_{T=0} &\quad\Leftrightarrow\\
\frac{r_1}{\beta(r_1-\varphi)(r_1-r_2)}\left[1-(\frac{r_2}{r_1})^{T+1}\right] > \frac{1}{\beta(r_1-\varphi)} &\quad\Leftrightarrow\\
\frac{r_1}{r_1-r_2}\left[1-\left(\frac{r_2}{r_1}\right)^{T+1}\right] > 1 &\quad\Leftrightarrow\\
r_1\left[1-\left(\frac{r_2}{r_1}\right)^{T+1}\right] > r_1-r_2 &\quad\Leftrightarrow\\
r_2 > r_1\left(\frac{r_2}{r_1}\right)^{T+1} &\quad\Leftrightarrow\\
1 > \left(\frac{r_2}{r_1}\right)^{T} &\quad\Leftrightarrow\\
r_1^T > r_2^T &
\end{aligned}
$$

The last inequality is met due to $r_1 > 1 > r_2 > 0$. Since

$$- \sum_{t=T+1}^{\infty} \pi_t \Big|_{T>0} \leq \sum_{t=T+1}^{\infty} |\pi_t| \Big|_{T>0} \tag{62}$$

the stronger persistence in case of anticipated shocks follows:

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T>0} = \sum_{t=0}^{T} \pi_t + \sum_{t=T+1}^{\infty} |\pi_t| \geq \sum_{t=0}^{T} \pi_t - \sum_{t=T+1}^{\infty} \pi_t > \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} \tag{63}$$

Note that for arbitrary $T > 0$

$$\pi_0 \Big|_{T=0} < \sum_{t=0}^{T} \pi_t \Big|_{T>0} \tag{64}$$

but

$$\pi_t \Big|_{T>0} < \pi_0 \Big|_{T=0} \quad \text{for all} \quad 0 \leq t \leq T \tag{65}$$

In particular

$$\pi_T \Big|_{T>0} < \pi_0 \Big|_{T=0} \tag{66}$$

since

$$\begin{aligned} \pi_t \Big|_{T>0} &= \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} r_1^{-T} \left[ (r_1 - 1) r_1^T - (r_2 - 1) r_2^T \right] \qquad (67) \\ &= \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} \left[ (r_1 - 1) - (r_2 - 1) \left( \frac{r_2}{r_1} \right)^T \right] < \pi_0 \Big|_{T=0} = \frac{1}{\beta(r_1 - \varphi)} \quad \Leftrightarrow \\ & \frac{1}{r_1 - r_2} \left[ (r_1 - 1) - (r_2 - 1) \left( \frac{r_2}{r_1} \right)^T \right] < 1 \quad \Leftrightarrow \\ & (r_1 - 1) - (r_2 - 1) \left( \frac{r_2}{r_1} \right)^T < r_1 - r_2 \quad \Leftrightarrow \\ & (1 - r_2) \left( \frac{r_2}{r_1} \right)^T < 1 - r_2 \quad \Leftrightarrow \\ & \left( \frac{r_2}{r_1} \right)^T < 1 \end{aligned}$$

Since the last equation holds, the value of the inflation rate at the time of implementation of the cost-push shock is smaller in case of anticipated compared to unanticipated shocks.[5]

[5]This result holds under the optimal timeless perspective precommitment policy. Under the policy regime discretion we have (cf. (138))

$$\pi_0 \Big|_{T=0} = \pi_T \Big|_{T>0} = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi}$$

### The loss under the optimal policy

To determine the welfare loss under the optimal precommitment policy, write the loss function $V$ as $V_1 + V_2$, where

$$V_1 = E_0 \sum_{t=0}^{T-1} \beta^t \left(\alpha_1 \pi_t^2 + \alpha_2 x_t^2\right) \tag{68}$$

is the loss resulting from the anticipation of the shock and

$$V_2 = E_0 \sum_{t=T}^{\infty} \beta^t \left(\alpha_1 \pi_t^2 + \alpha_2 x_t^2\right) \tag{69}$$

is the loss following from the realization of the shock. We first calculate the value of the loss function $V_1$. Since the solution time path of the state vector $(\pi_t, x_t)'$ over the anticipation interval can be written as

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = G \begin{pmatrix} r_1^t & 0 \\ 0 & r_2^t \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (t < T) \tag{70}$$

where

$$G = \begin{pmatrix} \frac{(r_1-1)r_1^{-T}}{\beta(r_1-\varphi)(r_1-r_2)} & \frac{-(r_2-1)r_1^{-T}}{\beta(r_1-\varphi)(r_1-r_2)} \\ \frac{-\alpha_1\kappa}{\alpha_2\beta} \frac{r_1^{1-T}}{(r_1-\varphi)(r_1-r_2)} & \frac{\alpha_1\kappa}{\alpha_2\beta} \frac{r_2 r_1^{-T}}{(r_1-\varphi)(r_1-r_2)} \end{pmatrix} \tag{71}$$

we obtain

$$\begin{aligned} V_1 &= \sum_{t=0}^{T-1} \beta^t \begin{pmatrix} \pi_t \\ x_t \end{pmatrix}' \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} \\ &= \sum_{t=0}^{T-1} \beta^t \begin{pmatrix} 1 \\ 1 \end{pmatrix}' \begin{pmatrix} r_1^t & 0 \\ 0 & r_2^t \end{pmatrix} G' \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} G \begin{pmatrix} r_1^t & 0 \\ 0 & r_2^t \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix}' W_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \operatorname{tr}\left( W_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}' \right) \\ &= w_{11}^{(1)} + 2w_{12}^{(1)} + w_{22}^{(1)} \end{aligned} \tag{72}$$

where the symmetric matrix $W_1 = \left(w_{ij}^{(1)}\right)_{1 \le i,j \le 2}$ is defined as the finite sum of matrices

$$W_1 = \sum_{t=0}^{T-1} \beta^t \begin{pmatrix} r_1^t & 0 \\ 0 & r_2^t \end{pmatrix} D \begin{pmatrix} r_1^t & 0 \\ 0 & r_2^t \end{pmatrix} \tag{73}$$

with

$$D = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix} = G' \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} G \tag{74}$$

The elements of the symmetric matrix $D$ are given by

$$d_{11} = \alpha_1 \lambda^2 r_1^{-2T} \left[ (r_1 - 1)^2 + \frac{\alpha_1 \kappa^2}{\alpha_2} r_1^2 \right] \tag{75}$$

$$d_{12} = -\alpha_1 \lambda^2 r_1^{-2T} \left[ (r_1 - 1)(r_2 - 1) + \frac{\alpha_1 \kappa^2}{\alpha_2} r_1 r_2 \right] \tag{76}$$

$$d_{22} = \alpha_1 \lambda^2 r_1^{-2T} \left[ (r_2 - 1)^2 + \frac{\alpha_1 \kappa^2}{\alpha_2} r_2^2 \right] \tag{77}$$

where we have used the abbreviation

$$\lambda = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} \tag{78}$$

According to (6) we have

$$\begin{aligned} (r_1 - 1)(r_2 - 1) + \frac{\alpha_1 \kappa^2}{\alpha_2} r_1 r_2 &= \\ r_1 r_2 \left[ 1 + \frac{\alpha_1 \kappa^2}{\alpha_2} \right] + 1 - (r_1 + r_2) &= \\ \frac{1}{\beta} \left[ 1 + \frac{\alpha_1 \kappa^2}{\alpha_2} \right] + 1 - (r_1 + r_2) &= \\ \operatorname{tr} C - 1 + 1 - (r_1 + r_2) &= 0 \end{aligned} \tag{79}$$

so that

$$w_{11}^{(1)} = d_{11} \sum_{t=0}^{T-1} \beta^t r_1^{2t} = \frac{1 - \beta^T r_1^{2T}}{1 - \beta r_1^2} d_{11} \tag{80}$$

$$w_{12}^{(1)} = d_{12} \sum_{t=0}^{T-1} \beta^t r_1^t r_2^t = 0 \tag{81}$$

$$w_{22}^{(1)} = d_{22} \sum_{t=0}^{T-1} \beta^t r_2^{2t} = \frac{1 - \beta^T r_2^{2T}}{1 - \beta r_2^2} d_{22} \tag{82}$$

Using (6) we get

$$\begin{aligned} (r_1 - 1)^2 + \frac{\alpha_1 \kappa^2}{\alpha_2} r_1^2 &= \\ r_1^2 \left( 1 + \frac{\alpha_1 \kappa^2}{\alpha_2} \right) + 1 - 2r_1 &= \\ \beta \left( r_1^2 [r_1 + r_2 - 1] + \frac{1}{\beta} [1 - 2r_1] \right) &= \\ \frac{1}{r_2} \left( r_1 [r_1 + r_2 - 1] + r_2 [1 - 2r_1] \right) &= \frac{1}{r_2} (r_1 - r_2)(r_1 - 1) \end{aligned} \tag{83}$$

so that

$$w_{11}^{(1)} = \frac{r_2^T - r_1^T}{(r_2 - r_1) r_2^{T-1}} \alpha_1 \lambda^2 r_1^{-2T} \frac{1}{r_2} (r_1 - r_2)(r_1 - 1) \tag{84}$$
$$= \alpha_1 \lambda^2 r_1^{-2T} \frac{r_1^T - r_2^T}{r_2^T} (r_1 - 1)$$

and analogically

$$w_{22}^{(1)} = \alpha_1 \lambda^2 r_1^{-2T} \frac{r_1^T - r_2^T}{r_1^T} (1 - r_2) \tag{85}$$

Then the loss $V_1$ can be written as

$$V_1 = \alpha_1 \lambda^2 r_1^{-2T} \left( r_1^T - r_2^T \right) \left( \frac{r_1 - 1}{r_2^T} + \frac{1 - r_2}{r_1^T} \right) \tag{86}$$

Consider $V_1$ as function in $T$ (the time span between the anticipation and realization of the cost-push shock). The function $V_1(T)$ has the following properties:

$$V_1(0) = 0, \qquad \lim_{T \to \infty} V_1(T) = \begin{cases} 0 & \text{for } \beta < 1 \\ \overline{V}_1 > 0 & \text{for } \beta = 1 \end{cases} \tag{87}$$

where

$$\overline{V}_1 = \frac{\alpha_1 (r_1 - 1)}{(r_1 - \varphi)^2 (r_1 - r_2)^2} \tag{88}$$

The derivative of $V_1$ with respect to $T$, i. e.

$$\frac{dV_1}{dT} = \alpha_1 \lambda^2 \Big\{ 2 \ln r_1 \cdot r_1^{-2T} [r_1 + r_2 - 2] - (r_1 - 1) \ln(r_1 r_2) \cdot (r_1 r_2)^{-T} \tag{89}$$
$$- (1 - r_2) \ln(\frac{r_2}{r_1^3}) \cdot (\frac{r_2}{r_1^3})^T \Big\}$$

is positive at time $T = 0$:

$$\left. \frac{dV_1}{dT} \right|_{T=0} = \alpha_1 \lambda^2 \Big\{ 2 (\ln r_1)(r_1 + r_2 - 2) \tag{90}$$
$$- (r_1 - 1) \ln(r_1 r_2) - (1 - r_2) \ln \left( \frac{r_2}{r_1^3} \right) \Big\}$$
$$= \alpha_1 \lambda^2 (r_1 - r_2) [\ln r_1 - \ln r_2]$$
$$= \alpha_1 \frac{1}{\beta^2} \frac{1}{(r_1 - \varphi)^2} \frac{1}{r_1 - r_2} [\ln r_1 - \ln r_2] > 0$$

Therefore, $V_1(T)$ starts to rise with increasing $T$ (although the size of the initial jumps of $x_t$ and $\pi_t$ is decreasing in $T$). For $\beta < 1$ the limit value is equal to zero, therefore $V_1(T)$ must decrease if $T$ is sufficiently large. Figure 6 illustrates the hump-shaped development of $V_1(T)$ in case $\beta < 1$.

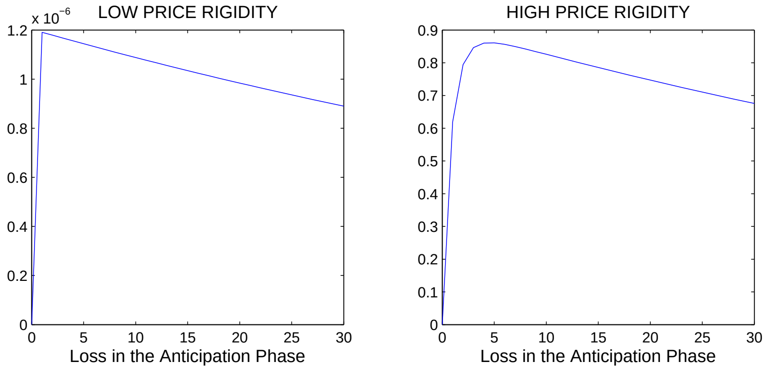


**Figure 6**: Partial loss in the anticipation phase for different lengths of anticipation period under optimal timeless perspective commitment policy in case $\beta = 0.99$.

To calculate the loss function $V_2$, write $(\pi_t, x_t)'$ as

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = F \begin{pmatrix} \varphi^t & 0 \\ 0 & r_2^t \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (t \geq T) \tag{91}$$

where

$$F = \begin{pmatrix} \dfrac{(1-\varphi)\varphi^{-T}}{\beta(r_1-\varphi)(r_2-\varphi)} & \dfrac{\left[(r_2-\varphi)r_1^{-T}-(r_1-\varphi)r_2^{-T}\right](1-r_2)}{\beta(r_1-\varphi)(r_2-\varphi)(r_1-r_2)} \\ \\ \dfrac{\alpha_1\kappa}{\alpha_2\beta}\dfrac{\varphi^{-(T-1)}}{(r_1-\varphi)(r_2-\varphi)} & \dfrac{\alpha_1\kappa}{\alpha_2\beta}\dfrac{\left[(r_2-\varphi)r_1^{-T}-(r_1-\varphi)r_2^{-T}\right]r_2}{\beta(r_1-\varphi)(r_2-\varphi)(r_1-r_2)} \end{pmatrix} \tag{92}$$

Then

$$\begin{aligned} V_2 &= \sum_{t=T}^{\infty} \beta^t \begin{pmatrix} \pi_t \\ x_t \end{pmatrix}' \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} \\ &= \sum_{t=T}^{\infty} \beta^t \begin{pmatrix} 1 \\ 1 \end{pmatrix}' \begin{pmatrix} \varphi^t & 0 \\ 0 & r_2^t \end{pmatrix} F' \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} F \begin{pmatrix} \varphi^t & 0 \\ 0 & r_2^t \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix}' W_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \operatorname{tr}\left( W_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}' \right) \\ &= w_{11}^{(2)} + 2w_{12}^{(2)} + w_{22}^{(2)} \end{aligned} \tag{93}$$

where the symmetric matrix $W_2 = \left(w_{ij}^{(2)}\right)_{1 \leq i,j \leq 2}$ is the geometric sum of matrices

$$W_2 = \sum_{t=T}^{\infty} \beta^t \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^t Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^t \tag{94}$$

with

$$Q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = F' \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} F \tag{95}$$

The elements of the symmetric matrix $Q$ are given by

$$q_{11} = \alpha_1 \delta^2 \varphi^{-2T} \left[ (1-\varphi)^2 + \frac{\alpha_1 \kappa^2}{\alpha_2} \varphi^2 \right] \tag{96}$$

$$q_{12} = \alpha_1 \delta^2 \phi \varphi^{-T} \frac{1}{r_1 - r_2} \left[ (1-r_2)(1-\varphi) + \frac{\alpha_1 \kappa^2}{\alpha_2} r_2 \varphi \right] \tag{97}$$

$$q_{22} = \alpha_1 \delta^2 \phi^2 \frac{1}{(r_1 - r_2)^2} \left[ (1-r_2)^2 + \frac{\alpha_1 \kappa^2}{\alpha_2} r_2^2 \right] \tag{98}$$

with the abbreviations

$$\delta = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_2 - \varphi} \tag{99}$$

and

$$\phi = (r_2 - \varphi) r_1^{-T} - (r_1 - \varphi) r_2^{-T} \tag{100}$$

The definition of $W_2$ implies that the matrix $W_2$ satisfies the matrix equation

$$\begin{aligned} W_2 &= \beta^T \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T + \sum_{t=T+1}^{\infty} \beta^t \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^t Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^t \\ &= \beta^T \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T + \sum_{t=T}^{\infty} \beta^{t+1} \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^{t+1} Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^{t+1} \\ &= \beta^T \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T + \beta \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix} W_2 \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix} \end{aligned} \tag{101}$$

Since

$$\beta^T \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T Q \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix}^T = \begin{pmatrix} \beta^T \varphi^{2T} q_{11} & \beta^T \varphi^T r_2^T q_{12} \\ \beta^T \varphi^T r_2^T q_{21} & \beta^T r_2^{2T} q_{22} \end{pmatrix} \tag{102}$$

and

$$W_2 - \beta \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix} W_2 \begin{pmatrix} \varphi & 0 \\ 0 & r_2 \end{pmatrix} = \begin{pmatrix} (1-\beta\varphi^2) w_{11}^{(2)} & (1-\beta\varphi r_2) w_{12}^{(2)} \\ (1-\beta\varphi r_2) w_{21}^{(2)} & (1-\beta r_2^2) w_{22}^{(2)} \end{pmatrix} \tag{103}$$

we obtain

$$w_{11}^{(2)} = \frac{\beta^T \varphi^{2T}}{1-\beta\varphi^2} q_{11} \tag{104}$$

$$w_{12}^{(2)} = w_{21}^{(2)} = \frac{\beta^T \varphi^T r_2^T}{1-\beta\varphi r_2} q_{12} \tag{105}$$

$$w_{22}^{(2)} = \frac{\beta^T r_2^{2T}}{1-\beta r_2^2} q_{22} \tag{106}$$

Using (6) and the definition of $q_{ij}$ we can write

$$\begin{aligned} w_{11}^{(2)} &= \alpha_1 \delta^2 \frac{\beta^T}{1-\beta\varphi^2} \left[ (1-2\varphi) + \left(1 + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) \varphi^2 \right] \\ &= \alpha_1 \delta^2 \frac{\beta^T}{r_1 r_2 - \varphi^2} \left[ r_1 r_2 (1-2\varphi) + (r_1 + r_2 - 1)\varphi^2 \right] \end{aligned} \tag{107}$$

$$\begin{aligned} w_{12}^{(2)} &= \alpha_1 \delta^2 \phi \frac{\beta^T r_2^T}{1-\beta\varphi r_2} \frac{1}{r_1 - r_2} \left[ (1-r_2)(1-\varphi) + \frac{\alpha_1 \kappa^2}{\alpha_2} r_2 \varphi \right] \\ &= \alpha_1 \delta^2 \phi \frac{\beta^T r_2^T}{r_1 - \varphi} \frac{1}{r_1 - r_2} \left[ r_1 (1-r_2)(1-\varphi) + r_1 r_2 \frac{\alpha_1 \kappa^2}{\alpha_2} \varphi \right] \\ &= \alpha_1 \delta^2 \phi \frac{\beta^T r_2^T}{r_1 - \varphi} \frac{1}{r_1 - r_2} \left[ r_1 (1 - \varphi - r_2) + (r_1 + r_2 - 1)\varphi \right] \\ &= \alpha_1 \delta^2 \phi \beta^T r_2^T \frac{1-r_2}{r_1 - r_2} \end{aligned} \tag{108}$$

$$\begin{aligned} w_{22}^{(2)} &= \alpha_1 \delta^2 \phi^2 \frac{\beta^T r_2^{2T}}{1-\beta r_2^2} \frac{1}{(r_1 - r_2)^2} \left[ (1-r_2)^2 + \frac{\alpha_1 \kappa^2}{\alpha_2} r_2^2 \right] \\ &= \alpha_1 \delta^2 \phi^2 \beta^T r_2^{2T} \frac{r_1}{r_1 - r_2} \frac{1}{(r_1 - r_2)^2} \left[ 1 - 2r_2 + r_2^2 \left( 1 + \frac{\alpha_1 \kappa^2}{\alpha_2} \right) \right] \\ &= \alpha_1 \delta^2 \phi^2 \beta^T r_2^{2T} \frac{1-r_2}{(r_1 - r_2)^2} \end{aligned} \tag{109}$$

Then

$$\begin{aligned} V_2 = \alpha_1 \delta^2 \beta^T \Bigg\{ & \frac{1}{r_1 r_2 - \varphi^2} \left[ r_1 r_2 (1-2\varphi) + (r_1 + r_2 - 1)\varphi^2 \right] \\ & + \frac{2\phi}{r_1 - r_2} r_2^T (1 - r_2) + \frac{\phi^2}{(r_1 - r_2)^2} r_2^{2T} (1 - r_2) \Bigg\} \end{aligned} \tag{110}$$

Since

$$\frac{1}{r_1 r_2 - \varphi^2} \left[ r_1 r_2 (1-2\varphi) + (r_1 + r_2 - 1)\varphi^2 \right] = 1 + \frac{(r_1 + r_2)\varphi - 2 r_1 r_2}{r_1 r_2 - \varphi^2} \varphi \tag{111}$$

and (according to the definition of $\phi$)

$$1 + \frac{1}{r_1 - r_2} \phi r_2^T = \frac{1}{(r_1 - r_2) r_1^T} (r_2 - \varphi)(r_2^T - r_1^T) \tag{112}$$

we can write

$$
\begin{aligned}
V_2 &= \alpha_1 \delta^2 \beta^T \left\{ \left( 1 + \frac{1}{r_1 - r_2} \phi r_2^T \right)^2 (1 - r_2) + r_2 + \frac{(r_1 + r_2)\varphi - 2 r_1 r_2}{r_1 r_2 - \varphi^2} \varphi \right\} \\
&= \alpha_1 \delta^2 \beta^T \left\{ \frac{1}{(r_1 - r_2)^2 r_1^{2T}} (r_2 - \varphi)^2 \left( r_2^T - r_1^T \right)^2 (1 - r_2) + \frac{r_1 (r_2 - \varphi)^2}{r_1 r_2 - \varphi^2} \right\} \\
&= \frac{\alpha_1 \beta^T}{\beta^2 (r_1 - \varphi)^2} \left\{ \frac{\left( r_2^T - r_1^T \right)^2 (1 - r_2)}{(r_1 - r_2)^2 r_1^{2T}} + \frac{r_1}{r_1 r_2 - \varphi^2} \right\}
\end{aligned}
\tag{113}
$$

The loss function $V_2 = V_2(T)$ has the following properties:

$$
V_2(0) = \frac{\alpha_1}{\beta^2 (r_1 - \varphi)^2} \frac{r_1}{r_1 r_2 - \varphi^2} > 0 \tag{114}
$$

$$
\lim_{T \to \infty} V_2(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \overline{V}_2 > V_2(0) \Big|_{\beta = 1} = \dfrac{\alpha_1 r_1}{(r_1 - \varphi)^2 (1 - \varphi^2)} & \text{if } \beta = 1 \end{cases} \tag{115}
$$

where

$$
\overline{V}_2 = \frac{\alpha_1}{(r_1 - \varphi)^2} \left\{ \frac{1 - r_2}{(r_1 - r_2)^2} + \frac{r_1}{1 - \varphi^2} \right\} \tag{116}
$$

The first derivative of $V_2$ with respect to $T$

$$
\begin{aligned}
\frac{dV_2}{dT} = \frac{\alpha_1}{\beta^2 (r_1 - \varphi)^2} \beta^T \Bigg\{ & \frac{r_1}{r_1 r_2 - \varphi^2} \ln \beta \\
& + \frac{1 - r_2}{(r_1 - r_2)^2} \left[ (\ln r_2 - 3 \ln r_1) \left( \frac{r_2}{r_1} \right)^{2T} + 4 \ln r_1 \left( \frac{r_2}{r_1} \right)^T + \ln \beta \right] \Bigg\}
\end{aligned}
\tag{117}
$$

implies for $\beta < 1$

$$
\frac{dV_2}{dT} \Big|_{T=0} = \frac{\alpha_1}{\beta^2 (r_1 - \varphi)^2} \frac{r_1}{r_1 r_2 - \varphi^2} \ln \beta < 0 \tag{118}
$$

(since $\beta = 1/(r_1 r_2)$). For $\beta < 1$, $dV_2/dT$ is also negative if $T$ is sufficiently large. Figure 7 illustrates that the development of $V_2$ is overall decreasing if the value of $\omega$ is sufficiently small (i.e., the degree of price flexibility is high); otherwise it is not monotone, but hump-shaped. For sufficiently large values of $\omega$ the loss function $V_2$ has two extrema (a maximum and a minimum) which can be determined from the first-order condition $dV_2/dT = 0$. The first extremum of $V_2$ cannot be represented graphically since the corresponding value of $T$ is very small. Note that in the limiting case $\beta = 1$ the loss function $V_2(T)$ is an increasing function in $T$ with a limit value $\overline{V}_2 > V_2(0)$.

We can now derive the development of the total loss $V = V_1 + V_2$. We first assume $\beta < 1$. Then $V(0) = V_2(0) > 0$ (with $V_2(0)$ defined in (114)) and

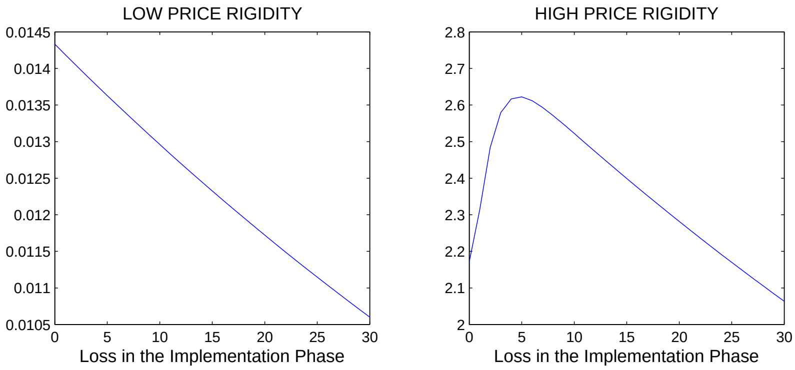


**Figure 7**: Partial loss in the implementation phase for different lengths of anticipation period under optimal timeless perspective commitment policy in case $\beta = 0.99$.

$\lim_{T\to\infty} V(T) = 0$. For small values of $\omega$, i.e. a high degree of price flexibility, the total loss $V$ is a decreasing function in $T$ implying $V(T) < V(0)$ for all $T > 0$. With high price flexibility the welfare loss under anticipated shocks is smaller than under unanticipated shocks. If $\omega$ is small, the persistence of the state variables is weak and the total loss is mainly determined by the size of the initial jumps of $x_t$ and $\pi_t$ which is a decreasing function in $T$. By contrast, for sufficiently large values of $\omega$ (i.e. a high degree of price rigidity) the jump variables $x_t$ and $\pi_t$ display a strong persistence so that the welfare loss starts to rise with increasing lead time $T$. For the derivative $dV/dT$ at time $T = 0$ we get

$$\begin{aligned}
\left.\frac{dV}{dT}\right|_{T=0} &= \left.\frac{dV_1}{dT}\right|_{T=0} + \left.\frac{dV_2}{dT}\right|_{T=0} \qquad (119)\\
&= \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\left\{\frac{1}{r_1-r_2}\left[\ln r_1 - \ln r_2\right] + \frac{r_1}{r_1r_2-\varphi^2}\ln\beta\right\}\\
&= \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\left\{\frac{1}{r_1-r_2}\left[\ln r_1 - \ln r_2\right] - \frac{r_1}{r_1r_2-\varphi^2}\left[\ln r_1 + \ln r_2\right]\right\}\\
&= \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\left\{\left[\frac{1}{r_1-r_2} - \frac{r_1}{r_1r_2-\varphi^2}\right]\ln r_1 - \left[\frac{1}{r_1-r_2} + \frac{r_1}{r_1r_2-\varphi^2}\right]\ln r_2\right\}
\end{aligned}$$

Then

$$\begin{aligned}
\left.\frac{dV}{dT}\right|_{T=0} > 0 \quad &\Leftrightarrow \qquad (120)\\
\left[r_1(2r_2-r_1)-\varphi^2\right]\ln r_1 - \left[r_1^2-\varphi^2\right]\ln r_2 > 0 \quad &\Leftrightarrow\\
2\left(\frac{1}{\beta}-\varphi^2\right)\ln r_1 + \left(r_1^2-\varphi^2\right)\ln\beta > 0 &
\end{aligned}$$

The last equivalence holds since $\ln r_2 = -(\ln r_1 + \ln\beta)$. A rising $\omega$ induces a fall in the unstable eigenvalue $r_1$ since $d\kappa/d\omega < 0$. Since the fall in $r_1^2$ is stronger than the decrease in $\ln r_1$, and $1/\beta - \varphi^2 > 0$, inequality (120) is fulfilled if $\omega$ is sufficiently large. In this case $V(T)$ starts to rise and due to $\lim_{T\to\infty} V(T) = 0$

its development must be hump-shaped implying the existence of an upper bound $T_c^* > 0$ such that $V(T) > V(0)$ for all $T < T_c^*$.

With low price flexibility and a lead time $T$ which is not too long, the welfare loss under anticipated shocks is greater than under unanticipated shocks. The reason is the stronger persistence of $x_t$ and $\pi_t$ in case $T > 0$ (compared to $T = 0$) which dominates the determination of the total loss if $\omega$ is sufficiently large.

The value of the upper bound $T_c^*$ is the positive solution of the equation $V(T) = V(0)$, where $V(0) = V_2(0)$ is given by (114). This leads to the equation

$$\frac{1}{(r_1 - r_2)^2} r_1^{-2T} (r_1^T - r_2^T) \left[ \frac{r_1 - 1}{r_2^T} + \frac{1 - r_2}{r_1^T} + \beta^T (r_1^T - r_2^T)(1 - r_2) \right] = \left(1 - \beta^T\right) \frac{r_1}{r_1 r_2 - \varphi^2} \tag{121}$$

which is equivalent to

$$1 - \left(\frac{r_2}{r_1}\right)^T = \left[(r_1 r_2)^T - 1\right] \frac{r_1(r_1 - r_2)}{r_1 r_2 - \varphi^2} \tag{122}$$

Equation (122) can be written as

$$\begin{aligned}
1 - \frac{1}{\beta^T r_1^{2T}} &= \frac{\left(\frac{1}{\beta}\right)^T - 1}{\frac{1}{\beta} - \varphi^2} \left(r_1^2 - \frac{1}{\beta}\right) \quad \Leftrightarrow \\
\beta^T r_1^{2T} - 1 &= \frac{\left(\frac{1}{\beta}\right)^T - 1}{1 - \beta\varphi^2} \left(\beta r_1^2 - 1\right) \beta^T r_1^{2T} \quad \Leftrightarrow \\
\beta^T r_1^{2T} \left[1 - \frac{\left(\frac{1}{\beta}\right)^T - 1}{1 - \beta\varphi^2} \left(\beta r_1^2 - 1\right)\right] &= 1 \quad \Leftrightarrow \\
\beta^T r_1^{2T} \left[\beta \left(r_1^2 - \varphi^2\right) + \frac{1}{\beta^T}(1 - \beta r_1^2)\right] &= 1 - \beta\varphi^2 \quad \Leftrightarrow \\
\beta^T r_1^{2T} \left[\beta r_1^2 \left(1 - \frac{1}{\beta^T}\right) + \frac{1}{\beta^T} - \beta\varphi^2\right] &= 1 - \beta\varphi^2 \quad \Leftrightarrow \qquad (123) \\
r_1^{2T} \left[\beta^{T+1} \left(r_1^2 - \varphi^2\right) + \left(1 - \beta r_1^2\right)\right] &= 1 - \beta\varphi^2 \qquad (124)
\end{aligned}$$

so that $T_c^*$ is also the positive solution of (123) and (124). The value of $T_c^*$ is dependent on $\omega$ and $\beta$. A rising $\omega$ (a higher degree of price rigidity) decreases the unstable eigenvalue $r_1$ so that the left-hand side of equation (123) is decreased while the right-hand side remains unchanged. Since $\beta^T r_1^{2T} = (r_1/r_2)^T$ is increasing in $T$, equation (123) implies that the solution value $T_c^*$ must increase if $\omega$ rises. Conversely, a higher degree of price flexibility induces a fall in $T_c^*$. For sufficiently small values of $\omega$ the only solution of (124) is $T_c^* = 0$ (so that $V(T) < V(0)$ for all $T > 0$). If a positive solution $T_c^*$ of (124) exists, then it is also an increasing function in the discount factor $\beta$ with $T_c^* = \infty$ if $\beta = 1$.

In the limiting case $\beta = 1$ the total loss $V(T)$ is an overall increasing function in $T$ with $V(0) = V_2(0) > 0$ and

$$\lim_{T\to\infty} V(T) = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1}{r_1-r_2}+\frac{r_1}{1-\varphi^2}\right\} > V_2(0)\Big|_{\beta=1} > 0 \tag{125}$$

If $\beta = 1$, we can write $V(T)$ as $V_1(T) + V_2(T)$, where

$$V_1(T) = \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2} r_1^{-2T}\left(r_1^T - r_2^T\right)\left[(r_1-1)r_2^{-T} + (1-r_2)r_1^{-T}\right] \tag{126}$$

$$= \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2}\left[(r_1-1) + (2-r_1-r_2)r_1^{-2T} - (1-r_2)\left(\frac{r_2}{r_1^3}\right)^T\right]$$

$$V_2(T) = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{\left(r_1^T-r_2^T\right)^2(1-r_2)}{(r_1-r_2)^2 r_1^{2T}} + \frac{r_1}{1-\varphi^2}\right\} \tag{127}$$

$$= \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1-r_2}{(r_1-r_2)^2}\left[1-\left(\frac{r_2}{r_1}\right)^T\right]^2 + \frac{r_1}{1-\varphi^2}\right\}$$

Then

$$\frac{dV_1}{dT} = \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2}\left\{2[r_1+r_2-2]\ln r_1 + [3\ln r_1 - \ln r_2](1-r_2)\left(\frac{r_2}{r_1}\right)^T\right\} r_1^{-2T} > 0 \quad \text{for all} \quad T \geq 0 \tag{128}$$

(due to $r_1 + r_2 = \text{tr } C > 2$ and $\ln r_2 < 0$) and

$$\frac{dV_2}{dT} = \frac{\alpha_1}{(r_1-\varphi)^2}\frac{1-r_2}{(r_1-r_2)^2}\left\{-2\left(1-\left(\frac{r_2}{r_1}\right)^T\right)\ln\left(\frac{r_2}{r_1}\right)\right\}\left(\frac{r_2}{r_1}\right)^T \underset{(=)}{>} 0 \quad \text{if} \quad T \underset{(=)}{>} 0 \tag{129}$$

(because $0 < r_2 < 1 < r_1$). Therefore, $dV/dT > 0$ for all $T \geq 0$ so that $V$ is a monotonically increasing function in $T$. This result holds independently of the degree of price rigidity $\omega$.

## Optimal policy under discretion

Under the policy regime discretion (D), the central bank is unable to make a commitment to future policies. Now private expectations are given for the central bank and the reduced form of the first-order conditions can be written as

$$\pi_t = -\frac{\alpha_2}{\alpha_1\kappa}x_t \tag{130}$$

$$E_t x_{t+1} = \frac{1}{\beta}\left[1+\frac{\alpha_1\kappa^2}{\alpha_2}\right]x_t + \frac{\alpha_1\kappa}{\alpha_2\beta}k_t \tag{131}$$

with $E_t x_{t+1} = x_{t+1}$ in case of anticipated shocks. The difference equation in $x_t$ has the unstable eigenvalue

$$r_D = \frac{1}{\beta}\left[1 + \frac{\alpha_1\kappa^2}{\alpha_2}\right] = \frac{1}{\alpha_2\beta}\left[\alpha_2 + \alpha_1\kappa^2\right] > 1 \tag{132}$$

and the forward solution

$$x_t = -\sum_{s=0}^{\infty} r_D^{-s}\frac{1}{r_D}\frac{\alpha_1\kappa}{\alpha_2\beta}k_{t+s} \tag{133}$$

Since

$$k_{t+s} = \begin{cases} \varphi^{t+s-T} & \text{for } t+s \geq T \\ 0 & \text{for } t+s < T \end{cases} \tag{134}$$

we get for $t \geq T$

$$\begin{aligned} x_t &= -\sum_{s=0}^{\infty} r_D^{-s}\varphi^s\frac{1}{r_D}\frac{\alpha_1\kappa}{\alpha_2\beta}\varphi^{t-T} = -\frac{1}{1-r_D^{-1}\varphi}\frac{1}{r_D}\frac{\alpha_1\kappa}{\alpha_2\beta}\varphi^{t-T} \\ &= -\frac{1}{r_D-\varphi}\frac{\alpha_1\kappa}{\alpha_2\beta}\varphi^{t-T} = -\frac{\alpha_1\kappa}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}\varphi^{t-T} \end{aligned} \tag{135}$$

and for $t < T$

$$\begin{aligned} x_t &= -\sum_{s=T-t}^{\infty} r_D^{-s}\varphi^s\frac{1}{r_D}\frac{\alpha_1\kappa}{\alpha_2\beta}\varphi^{t-T} = -\frac{(r_D^{-1}\varphi)^{T-t}}{1-r_D^{-1}\varphi}\frac{1}{r_D}\frac{\alpha_1\kappa}{\alpha_2\beta}\varphi^{t-T} \\ &= -\frac{(r_D^{-1})^{T-t}}{r_D-\varphi}\frac{\alpha_1\kappa}{\alpha_2\beta} = -\frac{1}{r_D-\varphi}\frac{\alpha_1\kappa}{\alpha_2\beta}r_D^{t-T} = -\frac{\alpha_1\kappa}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}r_D^{t-T} \end{aligned} \tag{136}$$

Since $r_D^{t-T} = 1$ for $t = T$, the solution formula for $x_t$ also holds in the shock period $t = T$. For $t = 0$ we get

$$x_0 = -\frac{\alpha_1\kappa}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}r_D^{-T} \tag{137}$$

so that the the size of the initial jump of $x_t$ decreases with increasing $T$.

For the inflation rate $\pi_t$ we obtain the solution time path

$$\pi_t = \begin{cases} \dfrac{\alpha_2}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}r_D^{t-T} & \text{if} \quad 0 \leq t \leq T \\ \\ \dfrac{\alpha_2}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}\varphi^{t-T} & \text{if} \quad t \geq T \end{cases} \tag{138}$$

Note that the limiting case $\varphi = 0$ implies $\pi_t = x_t = 0$ for $t > T$.

For all $0 \leq \varphi < 1$ the adjustment processes of $x_t$ and $\pi_t$ in case of anticipated cost-push shocks show a stronger persistence than in case $T = 0$. With the abbreviation

$$\tilde{\phi} = \frac{\alpha_2}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi} > 0 \tag{139}$$

we have

$$\sum_{t=0}^{\infty} |\pi_t|\Big|_{T=0} = \tilde{\phi}\sum_{t=0}^{\infty}\varphi^t = \frac{\tilde{\phi}}{1-\varphi} \tag{140}$$

and

$$\begin{aligned}
\sum_{t=0}^{\infty} |\pi_t|\Big|_{T>0} &= \sum_{t=0}^{T-1}\pi_t\Big|_{T>0} + \sum_{t=T}^{\infty}\pi_t\Big|_{T>0} \\
&= \tilde{\phi} r_D^{-T}\sum_{t=0}^{T-1} r_D^t + \tilde{\phi}\varphi^{-T}\sum_{t=T}^{\infty}\varphi^t \\
&= \tilde{\phi} r_D^{-T}\frac{1-r_D^T}{1-r_D} + \tilde{\phi}\varphi^{-T}\frac{\varphi^T}{1-\varphi} \\
&= \tilde{\phi}\frac{1}{1-\varphi} + \tilde{\phi}\frac{1-r_D^{-T}}{r_D-1} > \tilde{\phi}\frac{1}{1-\varphi} = \sum_{t=0}^{\infty}|\pi_t|\Big|_{T=0}
\end{aligned} \tag{141}$$

since $r_D > 1$ and $0 < r_D^{-T} < 1$ if $T > 0$. An analogous result holds for $x_t$.

The policy regime discretion implies

$$\sum_{t=0}^{\infty}\pi_t\Big|_{T=0} = \sum_{t=T}^{\infty}\pi_t\Big|_{T>0} \tag{142}$$

and

$$\sum_{t=0}^{\infty}x_t\Big|_{T=0} = \sum_{t=T}^{\infty}x_t\Big|_{T>0} \tag{143}$$

so that the stronger persistence of $\pi_t$ and $x_t$ in case $T > 0$ is due to the anticipation effects $\sum_{t=0}^{T-1}\pi_t > 0$ and $\sum_{t=0}^{T-1}x_t < 0$.

The solution time path for the price level $p_t$ results from

$$p_t = \sum_{k=0}^{t}\pi_k \tag{144}$$

For $0 \le t \le T$ we get

$$\begin{aligned}
p_t &= \frac{\alpha_2}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}\sum_{k=0}^{t} r_D^{t-T} \\
&= \frac{\alpha_2}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi} r_D^{-T}\frac{1-r_D^{t+1}}{1-r_D}
\end{aligned} \tag{145}$$

and for $t \ge T$

$$\begin{aligned}
p_t &= \sum_{k=0}^{T-1}\pi_k + \sum_{k=T}^{\infty}\pi_k \\
&= \frac{\alpha_2}{\alpha_2+\alpha_1\kappa^2-\alpha_2\beta\varphi}\left[r_D^{-T}\frac{1-r_D^T}{1-r_D} - \varphi^{-T}\frac{\varphi^{t+1}-\varphi^T}{1-\varphi}\right]
\end{aligned} \tag{146}$$

with

$$\lim_{t\to\infty} p_t = \frac{\alpha_2}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}\left[\frac{1 - r_D^{-T}}{r_D - 1} + \frac{1}{1-\varphi}\right] > 0 \tag{147}$$

Note that the limit value of $p_t$ is a positive function in $T$. It is well-known that a temporary cost-push shock yields a permanent rise in the price level under the policy regime discretion. By contrast, under the optimal timeless perspective precommitment policy there is only a temporary rise in the price level.

A further well-known result is that the loss under discretion ($V_D$) is greater than the total loss under the optimal precommitment policy. Inserting the solution time paths for $\pi_t$ and $x_t$ in the loss function we obtain

$$\begin{aligned}
V_D &= \sum_{t=0}^{\infty} \beta^t \left(\alpha_1 \pi_t^2 + \alpha_2 x_t^2\right) &(148)\\
&= \sum_{t=0}^{\infty} \beta^t \left[\alpha_1 \left(\frac{\alpha_2}{\alpha_1\kappa}\right)^2 + \alpha_2\right] x_t^2 = V_1^D + V_2^D\\
&= \sum_{t=0}^{T-1} \beta^t \left[\frac{\alpha_2^2}{\alpha_1\kappa^2} + \alpha_2\right] x_t^2 + \sum_{t=T}^{\infty} \beta^t \left[\frac{\alpha_2^2}{\alpha_1\kappa^2} + \alpha_2\right] x_t^2\\
&= \frac{\alpha_2^2 + \alpha_1\alpha_2\kappa^2}{\alpha_1\kappa^2}\frac{(\alpha_1\kappa)^2}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\left(\sum_{t=0}^{T-1}\beta^t r_D^{2(t-T)} + \sum_{t=T}^{\infty}\beta^t\varphi^{2(t-T)}\right)\\
&= \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\left(r_D^{-2T}\frac{1-(\beta r_D^2)^T}{1-\beta r_D^2} + \varphi^{-2T}\frac{(\beta\varphi^2)^T}{1-\beta\varphi^2}\right)\\
&= \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\left(\frac{r_D^{-2T}-\beta^T}{1-\beta r_D^2} + \frac{\beta^T}{1-\beta\varphi^2}\right)\\
&= \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\frac{1}{1-\beta r_D^2}\left(r_D^{-2T} - \frac{\beta(r_D^2-\varphi^2)}{1-\beta\varphi^2}\beta^T\right)
\end{aligned}$$

where

$$\frac{1}{1-\beta r_D^2} = \frac{\alpha_2^2\beta}{\alpha_2^2\beta - (\alpha_2+\alpha_1\kappa^2)^2} < 0 \tag{149}$$

Consider $V_D$ as function in $T$. Then

$$V_D(0) = \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\frac{1}{1-\beta\varphi^2} > 0 \tag{150}$$

and

$$\lim_{T\to\infty} V_D(T) = \begin{cases} 0 & \text{if } \beta < 1\\ \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\left(\frac{1}{r_D^2-1} + \frac{1}{1-\varphi^2}\right) > V_D(0) > 0 & \text{if } \beta = 1\end{cases} \tag{151}$$

The loss function

$$V_2^D(T) = \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\frac{\beta^T}{1-\beta\varphi^2} \tag{152}$$

has the properties

$$V_2^D(0) = V_D(0) \tag{153}$$

$$\lim_{T\to\infty} V_2^D(T) = 0 \quad \text{if} \quad \beta < 1 \tag{154}$$

$$\frac{dV_2^D}{dT} = (\ln\beta)V_2^D(T) < 0 \quad \text{if} \quad \beta < 1 \quad \text{for all} \quad 0 \le T < \infty \tag{155}$$

For $\beta = 1$ the function $V_2^D(T)$ is constant.

The loss function $V_1^D(T)$ given by

$$V_1^D(T) = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} \tag{156}$$

has similar properties as the corresponding function $V_1(T)$ under the policy regime commitment:

$$V_1^D(0) = 0 \tag{157}$$

$$\lim_{T\to\infty} V_1^D(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \dfrac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\dfrac{1}{r_D^2 - 1} > 0 & \text{if } \beta = 1 \end{cases} \tag{158}$$

The first derivative with respect to $T$

$$\frac{dV_1^D(T)}{dT} = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2}\frac{1}{1 - \beta r_D^2}\left[-2(\ln r_D)r_D^{-2T} - (\ln\beta)\beta^T\right] \tag{159}$$

is positive at time $T = 0$, since $1 - \beta r_D^2 < 0$ and $-2\ln r_D - \ln\beta < 0$ due to $r_D > 1 \ge \beta$.

In case $\beta < 1$ the development of $V_1^D(T)$ is hump-shaped with the maximum value at time $T_d^*$ which is the solution of the equation

$$2(\ln r_D)r_D^{-2T} + (\ln\beta)\beta^T = 0 \tag{160}$$

Equation (160) is equivalent to

$$-\frac{2\ln r_D}{\ln\beta} = (\beta r_D^2)^T \tag{161}$$

with the solution

$$T_d^* = \frac{\ln\left[-\dfrac{2\ln r_D}{\ln\beta}\right]}{\ln(\beta r_D^2)} > 0 \tag{162}$$

The total loss function $V_D(T) = V_1^D(T) + V_2^D(T)$ has a similar development as the corresponding function $V(T)$ under timeless perspective commitment. In the limiting case $\beta = 1$ it is overall increasing while for $\beta < 1$ it is hump-shaped, if the degree of price flexibility is not too large, while it is monotonically decreasing in $T$ if the value of $\omega$ is small. For small values of $\omega$ the derivative of $V_D$ at time $T = 0$ is negative, while it is positive if $\omega$ is sufficiently large.

## Total loss under a simple rule

We can also determine the total loss under an ad hoc Taylor rule

$$i_t = \delta_\pi \pi_t + \delta_x x_t \tag{163}$$

with exogenously given coefficients $\delta_\pi$ and $\delta_x$. It is well-known that under the condition $\delta_\pi > 1$ and $\delta_x \geq 0$ the baseline New Keynesian model satisfies the Blanchard/Kahn (1980) saddlepath condition. The state equations

$$A \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = B \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} k_t \tag{164}$$

with

$$A = \begin{pmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{pmatrix}, \qquad B = \begin{pmatrix} 1 + \frac{\delta_x}{\sigma} & \frac{\delta_\pi}{\sigma} \\ -\kappa & 1 \end{pmatrix} \tag{165}$$

have two unstable eigenvalues belonging to the state matrix $A^{-1}B$. Solving the state equations forward we get with

$$v_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, \quad P = B^{-1}A, \quad q = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{166}$$

the solution time paths in case of anticipated cost-push shocks:

- For $t \geq T$

$$\begin{aligned} v_t &= -\left( \sum_{s=0}^{\infty} \varphi^s P^s \right) B^{-1} q \varphi^{t-T} = -[I_{2\times 2} - \varphi P]^{-1} B^{-1} q \varphi^{t-T} \\ &= -[B - \varphi A]^{-1} q \varphi^{t-T} \end{aligned} \tag{167}$$

- For $t < T$

$$\begin{aligned} v_t &= -\left( \sum_{s=T-t}^{\infty} \varphi^s P^s \right) B^{-1} q \varphi^{t-T} \\ &= -[I_{2\times 2} - \varphi P]^{-1} (\varphi P)^{T-t} B^{-1} q \varphi^{t-T} \\ &= -[I_{2\times 2} - \varphi P]^{-1} P^{T-t} \, B^{-1} q \end{aligned} \tag{168}$$

The solution formula for $t < T$ also holds in $t = T$ since

$$\begin{aligned} v_T &= -[B - \varphi A]^{-1} q \\ &= -[I_{2\times 2} - \varphi P]^{-1} B^{-1} \end{aligned} \tag{169}$$

The total loss under the simple Taylor rule ($V_{STR}$) can be written as

$$V_{STR} = \sum_{t=0}^{\infty} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t = V_1^{STR} + V_2^{STR} \tag{170}$$

where

$$V_1^{STR} = \sum_{t=0}^{T-1} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t \tag{171}$$

and

$$V_2^{STR} = \sum_{t=T}^{\infty} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t \tag{172}$$

Define

$$M = (B - \varphi A)^{-1} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \tag{173}$$

Then

$$\begin{aligned} V_2^{STR} &= \sum_{t=T}^{\infty} \beta^t q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \varphi^{2(t-T)} \\ &= q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \varphi^{-2T} \left( \sum_{t=T}^{\infty} \beta^t \varphi^{2t} \right) \\ &= \frac{(\beta\varphi^2)^T}{1 - \beta\varphi^2} \varphi^{-2T} q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \\ &= \frac{\beta^T}{1 - \beta\varphi^2} \operatorname{tr} \left( M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q q' \right) \end{aligned} \tag{174}$$

where

$$M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M = \begin{pmatrix} \alpha_2 m_{11}^2 + \alpha_1 m_{21}^2 & \alpha_2 m_{11} m_{12} + \alpha_1 m_{21} m_{22} \\ \alpha_2 m_{11} m_{12} + \alpha_1 m_{21} m_{22} & \alpha_2 m_{12}^2 + \alpha_1 m_{22}^2 \end{pmatrix} \tag{175}$$

Since

$$q q' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{176}$$

we obtain

$$\operatorname{tr} \left( M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q q' \right) = \alpha_2 m_{12}^2 + \alpha_1 m_{22}^2 \tag{177}$$

The definition of the matrices $A$ and $B$ implies

$$B - \varphi A = \begin{pmatrix} 1 + \frac{\delta_x}{\sigma} - \varphi & \frac{\delta_\pi}{\sigma} - \frac{\varphi}{\sigma} \\ -\kappa & 1 - \varphi\beta \end{pmatrix} \tag{178}$$

$$\Delta = |B - \varphi A| = \left( 1 + \frac{\delta}{\sigma} - \varphi \right) (1 - \varphi\beta) + \kappa \left( \frac{\delta_\pi}{\sigma} - \frac{\varphi}{\sigma} \right) = \frac{1}{\sigma} b \tag{179}$$

where

$$
\begin{aligned}
b &= (1-\varphi)(1-\varphi\beta)\sigma + \delta_x(1-\varphi\beta) + \kappa(\delta_\pi - \varphi) \\
&> 0 \quad \text{if } \delta_\pi > 1 \text{ and } \delta_x > 0
\end{aligned} \tag{180}
$$

Then

$$
M = (B - \varphi A)^{-1} = \frac{1}{b}\begin{pmatrix} \sigma(1-\varphi\beta) & -(\delta_\pi - \varphi) \\ \sigma\kappa & \sigma(1-\varphi) + \delta_x \end{pmatrix} \tag{181}
$$

so that

$$
m_{12} = -\frac{1}{b}(\delta_\pi - \varphi), \qquad m_{22} = \frac{1}{b}[\sigma(1-\varphi) + \delta_x] \tag{182}
$$

and

$$
V_2^{STR} = \frac{\beta^T}{1-\beta\varphi^2}\frac{1}{b^2}\left[\alpha_2(\delta_\pi - \varphi)^2 + \alpha_1(\sigma(1-\varphi) + \delta_x)^2\right] \tag{183}
$$

The loss function $V_2^{STR} = V_2^{STR}(T)$ hat the same properties as the corresponding function under discretion ($V_2^D(T)$).

To calculate the loss $V_1^{STR}$ set

$$
Q = [I_{2\times 2} - \varphi P]^{-1} \qquad (\text{where } P = B^{-1}A) \tag{184}
$$

and

$$
\tilde{q} = B^{-1}q \tag{185}
$$

Then

$$
v_t = -QP^{T-t}\tilde{q} \qquad \text{for} \quad t \leq T \tag{186}
$$

and

$$
\begin{aligned}
V_1^{STR} &= \tilde{q}'\left(\sum_{t=0}^{T-1}\beta^t(P^{T-t})'Q'\begin{pmatrix}\alpha_2 & 0 \\ 0 & \alpha_1\end{pmatrix}QP^{T-t}\right)\tilde{q} \\
&= \tilde{q}'\left(\sum_{k=1}^{T}\beta^{T-k}(P^k)'Q'\begin{pmatrix}\alpha_2 & 0 \\ 0 & \alpha_1\end{pmatrix}QP^k\right)\tilde{q} \\
&= \beta^T\tilde{q}'\tilde{W}\tilde{q} = \beta^T\operatorname{tr}(\tilde{W}\tilde{q}\tilde{q}')
\end{aligned} \tag{187}
$$

where

$$
\begin{aligned}
\tilde{q}\tilde{q}' &= B^{-1}qq'(B^{-1})' = B^{-1}\begin{pmatrix}0 & 0 \\ 0 & 1\end{pmatrix}(B^{-1})' \\
&= \frac{1}{(\sigma + \delta_x + \kappa\delta_\pi)^2}\begin{pmatrix}\delta_\pi^2 & -\delta_\pi(\sigma+\delta_x) \\ -\delta_\pi(\sigma+\delta_x) & (\sigma+\delta_x)^2\end{pmatrix}
\end{aligned} \tag{188}
$$

and

$$\tilde{W} = \sum_{k=1}^{T} \beta^{-k} (P^k)' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^k \tag{189}$$

$\tilde{W}$ satisfies the following matrix equation: Let

$$\tilde{D} = Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q \tag{190}$$

Then the definition of $\tilde{W}$ implies

$$\begin{aligned} \tilde{W} &= \beta^{-1} P' \tilde{D} P + \sum_{k=2}^{T} \beta^{-k} (P^k)' \tilde{D} P^k \\ &= \beta^{-1} P' \tilde{D} P + \sum_{k=1}^{T-1} \beta^{-(k+1)} (P^{k+1})' \tilde{D} P^{k+1} \\ &= \beta^{-1} P' \tilde{D} P + \beta^{-1} P' \left( \sum_{k=1}^{T-1} \beta^{-k} (P^k)' \tilde{D} P^k \right) P \\ &= \beta^{-1} P' \tilde{D} P + \beta^{-1} P' \left( \sum_{k=1}^{T} \beta^{-k} (P^k)' \tilde{D} P^k \right) P - \beta^{-1} P' \beta^{-T} (P^T)' \tilde{D} P^T P \\ &= \beta^{-1} P' \tilde{D} P - \beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1} + \beta^{-1} P' \tilde{W} P \end{aligned} \tag{191}$$

or in compact representation

$$\tilde{W} = \tilde{H} + \beta^{-1} P' \tilde{W} P \tag{192}$$

where

$$\tilde{H} = \beta^{-1} P' \tilde{D} P - \beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1} \tag{193}$$

To solve for $\tilde{W}$ use the vectorization of a matrix and the Kronecker product of matrices. Since

$$\mathrm{vec}\,(\beta^{-1} P' \tilde{W} P) = [\beta^{-1} P' \otimes P'] \mathrm{vec}\,\tilde{W} \tag{194}$$

we obtain

$$\mathrm{vec}\,\tilde{W} - [\beta^{-1} P' \otimes P'] \mathrm{vec}\,\tilde{W} = \mathrm{vec}\,\tilde{H} \tag{195}$$

with the solution

$$\mathrm{vec}\,\tilde{W} = [I_{4\times 4} - \beta^{-1} P' \otimes P']^{-1} \mathrm{vec}\,\tilde{H} \tag{196}$$

where

$$\begin{aligned} \mathrm{vec}\,\tilde{H} &= \mathrm{vec}\,(\beta^{-1} P' \tilde{D} P) - \mathrm{vec}\,(\beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1}) \\ &= \Big( [\beta^{-1} P' \otimes P'] - [\beta^{-(T+1)} (P^{T+1})' \otimes (P^{T+1})'] \Big) \mathrm{vec}\,\tilde{D} \end{aligned} \tag{197}$$

and

$$\begin{aligned}\operatorname{vec}\tilde{D} &= Q' \otimes Q' \operatorname{vec}\begin{pmatrix}\alpha_2 & 0\\ 0 & \alpha_1\end{pmatrix} \\ &= ([I_{2\times 2} - \varphi P]^{-1})' \otimes ([I_{2\times 2} - \varphi P]^{-1})' \begin{pmatrix}\alpha_2\\ 0\\ 0\\ \alpha_1\end{pmatrix}\end{aligned} \tag{198}$$

Note that $\operatorname{vec}\tilde{D}$ equals $\operatorname{vec}\begin{pmatrix}\alpha_2 & 0\\ 0 & \alpha_1\end{pmatrix}$ in the special case $\varphi = 0$. The development of $V_1^{STR}$ as function in $T$ is analogous to the loss function $V_1^D(T)$. Therefore, the total loss function $V^{STR}(T) = V_1^{STR}(T) + V_2^{STR}(T)$ has the same properties as the total loss under discretion.