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Anticipated Raw Materials Price Shocks and Monetary Policy Response - A New Keynesian Approach

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Anticipated Raw Materials Price Shocks and Monetary Policy Response – A New Keynesian Approach

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Abstract

The paper analyzes the dynamic effects of anticipated raw materials price increases for small open oil-dependent economies and investigates the consequences of several monetary policy rules in response to commodity price shocks. Based on a calibrated New Keynesian open economy model the analysis shows that anticipated increases in the price of oil will involve oil-dependent economies both in temporary inflation and deflation as well as in output expansion and contraction. Compared to an interest rate Taylor rule a money growth rule is more appropriate to reduce the volatility of the CPI inflation rate whereas just the opposite holds for stabilizing the output gap.

JEL classification: E32, E52, F41, Q43
Keywords: Oil price shocks, Monetary Policy, Open Economy

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1 Introduction

The purpose of this paper is to analyze the dynamic effects of *anticipated* raw materials price increases for small open economies and to discuss the impacts of possible monetary policy responses to such price shocks.

The relevance of this paper follows from the strong fluctuations in commodity prices, especially the substantial increase in oil prices during the past decades. It could be seen that raw materials price increases can be of temporary as well as of permanent nature and that they generally are anticipated shocks.[1]

The analysis of the intertemporal effects of anticipated raw materials price shocks is based on a calibrated New Keynesian open economy model with a hybrid IS- and Phillips curve equation.[2] It can be shown that there are output expansion and moderate inflation before the occurrence of the raw materials price shock, whereas typical stagflationary effects only appear directly after raw materials price increases. In the course of the adjustment, deflationary phases and further output increases are possible.

In the second part of this paper it is analyzed in how far monetary policy rules can contribute to a simultaneous stabilization of the cyclical development of output and inflation rate caused by anticipated commodity price shocks. Are interest rate rules of the type originally proposed by Taylor (1993) able to reduce the increased volatility of output and inflation simultaneously or are money growth rules rather able to do so? It will be shown that the inflation rate volatility can clearly be reduced by targeting the money stock growth, whereas in case of temporary input price shocks Taylor-type interest rate rules lead to an *increase* in total inflation rate variance. Irrespective of the type of the input price shock money growth rules are always linked to a lower total variance of the inflation rate than interest rate rules of the Taylor type are, whereas exactly the opposite applies for the output variance. In contrast, interest rate rules of the Taylor type are compared to money growth rules more appropriate in order to stabilize output variance. Both rules are nevertheless accompanied by a strong increase in nominal interest rate volatility. This applies especially to interest rate rules without interest rate smoothing or if perfect stabilization of inflation rates is aimed at.

To our best knowledge, in economic literature there are no articles based on *open economy New Keynesian* models yet that deal with the dynamic effects of *anticipated* raw materials price shocks and the effects of *alternative* interest rate rules and money growth rules, which are used by the central bank to simultaneously stabilize inflation and output. Kim and Loungani (1992), Rotemberg and Woodford (1996) and Finn (2000) analyze the effects of oil price shocks in

[1] Bhandari and Turnovsky (1984) emphasize that most of the oil price increases in the 70's and early 80's were anticipated. They analyze anticipated and unanticipated as well as permanent and temporary raw materials price increases in a traditional open economy framework. More recently, Schubert and Turnovsky (2006) consider anticipated fiscal policy changes in a representative agent economy with investment adjustment costs. They also provide an overview of economic literature that deals with anticipated shocks.

[2] Numerous empirical studies analyze the consequences of oil price shocks and the interplay between oil price shocks and monetary policy responses. See, for example, Hamilton and Herrera (2004) and the references therein.

dynamic general equilibrium models of closed economies. Backus and Crucini (2000) consider an open-economy real business cycle model to study the effects of oil on the economy. All these studies are based on the assumption of completely flexible prices. Hence, there is no role for monetary policy.

Leduc and Sill (2004) as well as Carlstrom and Fuerst (2006) include nominal rigidities in dynamic general equilibrium models of oil-dependent economies to study the interaction between oil price shocks and monetary policy. However, by considering a closed economy they rule out the potentially important impacts of changes in the nominal exchange rate and the terms of trade. The authors of both studies, as we do too, attempt to isolate the impacts of an oil price shock from the impacts of the endogenous response of monetary policy to this oil price hike. In doing so Carlstrom and Fuerst (2006) challenge the empirical work by Bernanke et al. (1997, 2004) by showing that anticipation effects actually matter for the analysis of the interplay between oil price shocks and monetary policy. We follow this line of thought and now analyze the effects of *anticipated* oil price shocks under several monetary policy responses.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 deals with the dynamic effects of raw materials price increases under a neutral monetary policy. Section 4 discusses the impacts of monetary policy rules. Section 5 discusses the possibility of a perfect stabilization of the consumer price inflation. Section 6 compares the monetary policy rules and draws from this economic policy conclusions. At the end, the paper includes an extensive mathematical appendix.

2 The Model

We consider a stylized hybrid New Keynesian model of a small open economy which is dependent upon raw materials imports (like crude oil).[3] The building blocks of our rational expectations model are a hybrid IS and a hybrid Phillips curve, the uncovered interest parity condition and a money demand equation. All variables – except for the interest rates – are in logs. Therefore, the positive model parameters can be interpreted as elasticities or semi-elasticities.

The equilibrium of the goods market can be represented by the following IS curve:

$$\begin{aligned} q_t = {} & a_1\big(\Phi y_{t-1} + (1-\Phi)\,\mathrm{E}_t\, y_{t+1}\big) - a_2(i_t - \mathrm{E}_t\,\Delta p^c_{t+1}) \\ & + a_3(m_t - p^c_t) + g_t - b_1 y_t + b_2 y^*_t - b_3\tau_t + b_0 \end{aligned} \tag{1}$$

q denotes real output, y real income, i the nominal interest rate, m_t the nominal money stock, g real government expenditure and p the domestic price of domestic output. p^* and y^* denotes the foreign price and foreign income respectively. $\tau = p - (p^* + e)$ are the terms of trade, where e is the nominal exchange rate defined as the domestic currency price of foreign currency. $p^c_t = \alpha p_t + (1-\alpha)(p^*_t + e_t)$ is the consumer price index (CPI) where $1-\alpha$ can

[3]Similar models are used by e.g. van Aarle et al. (2004) to study monetary and fiscal policy in the European Monetary Union or by Svensson (2000) to analyze inflation targeting in a small open economy.

be interpreted as the degree of demand side openness. Δp^c_{t+1} denotes the CPI inflation rate between period t and $t+1$. b_0 is a constant and E the expectations operator where rational expectations are assumed.

Domestic output q_t depends on past and expected future income, the real interest rate $i_t - \mathrm{E}_t\, \Delta p^c_{t+1}$, real government expenditure and the aggregate trade balance, where the latter depends on income developments and on the terms of trade. Our IS curve reflects the behavior of rational, intertemporally optimizing consumers as well as the assumption of habit formation in consumption.[4] Moreover, we assume that the demand of goods depends directly on real money balances $m_t - p^c_t$ where the nominal money stock is deflated by the consumer price index to allow for the fact that in open economies money is also used for the purchase of imported goods.[5].

Money market equilibrium is given by a standard LM curve:

$$m_t - p^c_t = l_0 + l_1 q_t - l_2 i_t \tag{2}$$

Money demand is assumed to depend on real output rather than on real income which is considered as a more appropriate measure of the volume of transactions.

We assume perfect substitutability of domestic and foreign bonds and perfect capital mobility, so that the uncovered interest parity condition holds:

$$i_t = i^*_t + \mathrm{E}_t\, \Delta e_{t+1} \tag{3}$$

The domestic interest rate may only deviate from the foreign interest rate i^* by the rationally expected depreciation rate between period t and $t+1$ ($\mathrm{E}_t\, \Delta e_{t+1}$).

The difference between the respective domestic production and real income or gross national product is described by the following equation:

$$q_t = y_t + \psi(p^*_{R,t} + e_t - p_t) + d_0 \tag{4}$$

p^*_R denotes the foreign nominal price of raw materials imports and d_0 a constant. The difference between q and y results from imports of intermediate goods which in turn depend on the respective real factor price. We assume that raw materials imports (like crude oil) are denominated in terms of the foreign currency (US dollars) so that the domestic real factor price $p^*_R + e - p$ depends on the nominal exchange rate e.[6]

The dynamics of inflation are given by a hybrid Phillips curve:

$$\Delta p_t = \mu\big(\omega \Delta p^c_{t-1} + (1-\omega)\,\mathrm{E}_t\, \Delta p^c_{t+1}\big) + \mu\delta(q_t - \overline{q}) + (1-\mu)(\Delta p^*_{R,t} + \Delta e_t) \tag{5}$$

[4]For a detailed derivation of a microfounded IS curve with habit formation in consumption see, for example, McCallum and Nelson (1999).

[5]The presence of the real money stock in the IS curve reflects the implicit assumption that the utility function of the representative household is non-separable.

[6]The constant ψ can be derived from a profit maximizing approach with a CES production technology which allows for factor substitution between labor and raw materials imports. It can be shown that in this case ψ is of the form $(1-\mu)(1-\sigma)/\mu$, where σ is the elasticity of substitution between labor and raw materials imports and where μ measures the share of labor in gross domestic output (Bhandari and Turnvosky (1984)). The constant $1-\mu$ then measures the share of imported inputs in gross output and can be interpreted as a measure for the supply side openness of the domestic economy.

Inflation between period $t-1$ and t depends on past and expected future CPI inflation, the output gap $q-\overline{q}$ and the inflation of raw materials imports $\Delta p^*_{R,t} + \Delta e_t$ between period $t-1$ and t. In the special case $\omega = 0$ we obtain a traditional backward-looking Phillips curve, in the other special case $\omega = 1$ we obtain the forward-looking New Keynesian Phillips curve. We assume that ω lies between 0 and 1 to allow for both backward and forward-looking price setting behavior.[7]

In the long run, assuming labor market equilibrium where labor demand is a negative function of the producer and labor supply a positive function of the consumer real wage rate and, in addition, assuming a perfectly elastic raw materials supply, output supply depends positively on the final goods terms of trade and negatively on the domestic real oil price:[8]

$$\overline{q} = f_0 + f_1 \overline{\tau} - f_2(\overline{p^*_R + e - p}) \tag{6}$$

where f_0 is a constant.

Since the economy is assumed to be small relative to the rest of the world, the foreign variables y^*, i^*, p^* and p^*_R are exogenously given.

3 Dynamic Effects of Anticipated Raw Materials Price Increases

In what follows we use the terms raw materials imports, oil imports and commodity imports interchangeably. We assume that at time $t = 0$ the public anticipates a one-unit price shock in raw materials imports to take effect at some future time $T > 0$. For example, we can assume that in $t = 0$ the OPEC credibly announces a permanent or temporary price increase in crude oil to occur at the future date $T > 0$. In what follows we will discuss the dynamic effects of such commodity price shocks. In particular the anticipation effects of announced oil price increases are analyzed. In this chapter we ask what effect an oil price increase has on the economy if monetary policy is neutral or passive. We follow Leduc and Sill (2004) and define monetary policy as neutral, if the money stock is held constant by the central bank ($\Delta m_t = 0$).[9] In the next chapter we will discuss the impacts of various monetary policy rules in response to anticipated raw materials price shocks. We assume that the foreign nominal price of raw materials imports p^*_R follows the autoregressive AR(1) process

$$p^*_{R,t} = \beta_R \cdot p^*_{R,t-1} + \kappa_t, \quad 0 \leq \beta_R \leq 1 \tag{7}$$

[7]This assumption is in line with empirical evidence provided by e.g. Galí and Gertler (1999) or Galí et al. (2001, 2005).

[8]A more detailed theoretical derivation of the role of the terms of trade in aggregate supply is given in Devereux and Purvis (1990). The supply equation (6) can also be derived by assuming long run static price and wage equations of the form $\overline{p} = \mu\overline{w} + (1-\mu)(p^*_R + \overline{e})$, $\overline{w} = \overline{p}^c + \delta\overline{q}$. In this case the parameters f_1, f_2 are of the form $f_1 = (1-\alpha)/\delta$, $f_2 = (1-\mu)/(\mu\delta)$ where $f_2 > f_1$.

[9]Carlstrom and Fuerst (2006) use the same definition of neutral monetary policy. They also analyze alternative definitions of neutral monetary policy, namely an interest rate peg and a so called "Wicksellian" interest rate policy.

where κ_t is the one-unit price shock

$$\kappa_t = \begin{cases} 1 & \text{for } t = T > 0 \\ 0 & \text{for } t \neq T \end{cases} \tag{8}$$

If the initial value of p_R^* is normalized to zero ($p_{R,0}^* = 0$) then

$$p_{R,t}^* = \begin{cases} 0 & \text{for } 0 < t < T \\ \beta_R^{t-T} & \text{for } t \geq T \end{cases} \tag{9}$$

We assume further that a one-unit increase in the foreign nominal price of the imported input is accompanied by a less than equivalent increase in the price of the imported final good p^*:

$$p_t^* = (1 - \mu^*) p_{R,t}^*, \qquad 0 < \mu^* < 1 \tag{10}$$

Then the nominal price shock represents a change in the real foreign price of imported raw materials:

$$p_{R,t}^* - p_t^* = \mu^* p_{R,t}^* = \begin{cases} 0 & \text{for } 0 \leq t \leq T \\ \mu^* \beta_R^{t-T} & \text{for } t \geq T \end{cases} \tag{11}$$

In case $\beta_R < 1$ the increase in the real foreign input price is transitory whereas it is of permanent nature if $\beta_R = 1$. In the following we will discuss both types of input price disturbances. In case of *anticipated* price shocks the adjustment dynamics involve two phases: the phase before and after the occurrence of the commodity price increase. Figure 1 illustrates the response of the domestic economy to an anticipated temporary and an anticipated permanent oil price increase. It is assumed that the time span between the anticipation and the implementation of the rise in p_R^* consists of two periods ($T = 2$). The simulations are based on a typical parameter set represented in table 1.

The initial steady state value of each endogenous variable is normalized to zero. Each figure contains simultaneously the adjustment process of a domestic variable in case $\beta_R = 0.8$ (temporary price shock) and $\beta_R = 1$ (permanent price shock). In case of transitory commodity price increases no steady state effects occur for the domestic economy so that the domestic variables return to their initial steady state values.

Temporary raw materials price shocks

The dynamic effects of anticipated raw materials price shocks in case $\beta_R < 1$ can be summarized as follows (see also the overview in table 2):

During the anticipation phase (periods $t = 0$ and $t = 1$) there is a moderate increase in real output and national income, which is accompanied by a slight increase in the inflation rates Δp and Δp^c. The temporary increase in output is traced to a short-term decrease in the real interest rate with simultaneous increase in the terms of trade τ. Stagflation in the sense of a decrease in output with a simultaneous increase in inflation does not take place until the period of the commodity price increase $t = T$. During the periods after the shock realization there is a strong decrease in inflation rates, which results from the drop of the output gap $q - \overline{q}$, so that even deflation ($\Delta p_t < \Delta\overline{p}_0 = 0$, $\Delta p_t^c < \Delta\overline{p}_0^c = 0$) occurs in the medium term of the adjustment. The output strongly decreases first for $t = T$ and immediately thereafter, which can be explained by increasing real interest rates, but increases again during the medium phase of adjustment (due to real interest rate decreases and a real depreciation process). As well as during the initial phase of adjustment, an overshooting of output over the initial steady state value $\overline{q}_0$ occurs, so that altogether we have a cyclical, hump-shaped development for q.

The nominal exchange rate e also runs hump-shaped and except from the impact phase above its initial level. The result during the shock period is a strong increase in the exchange rate, which is reinforced in the following periods. This delayed overshooting corresponds to strong increases in price level and in price index in T and $T+1$. A gradual nominal appreciation process then follows, which is linked to strong decreases in the price level p and the price index p^c, so that the nominal appreciation process corresponds to a real devaluation process in the medium term of the adjustment.

The development of the nominal interest rate i_t follows the development of the nominal depreciation rate Δe_{t+1}. During the initial phase of adjustment increases in nominal interest rates, which reach their maximum in $t = T$, take place; then a strong decrease in interest rates follows, so that an undershooting arises during the medium phase of adjustment as it was already the case with the inflation rates Δp and Δp^c. A mostly parallel development of the real interest rate $i_t - \Delta p_{t+1}^c$ corresponds to it, which is basically only during the anticipation phase exactly opposite.

The development of real commodity imports im_R can be determined with the help of the equation

$$im_{R,t} = q_t + (1-\sigma)(p_{R,t}^* + e_t - p_t) \tag{12}$$

where $p_R^* + e - p$ is the domestic real commodity price and σ the elasticity of substitution between the factors labor and commodity.[10]

The domestic real input price $p_R^* + e - p$ increases strongly during the shock period $T = 2$ and remains at a high level during the following periods (the decline towards its initial steady state value only takes place in the long run). In case of a low elasticity of substitution σ it is accompanied by a strong increase in commodity imports. The development of output that makes commodity imports decrease immediately after the shock period T, is opposite to it. Since in the longer run an increase in output at a high level of the real commodity price takes place, there is once more an increase in real commodity imports when $t > T$ is sufficiently great; then temporarily im_R is even above the level of the shock period T. During the entire adjustment after the realization of the oil price shock, im_R is bigger than its initial steady state value.

[10] Cf. Bhandari and Turnovsky (1984) and Bhandari (1981). In Bhandari (1981) σ is set to zero.

Permanent raw materials price shocks

Next consider the case $\beta_R = 1$ (permanent anticipated raw materials price shock). This case is characterized by a permanent rise in the nominal exchange rate, strong permanent price and price index effects and a long run reduction in real output and national income. However, the interest rates i_t and $i_t - \Delta p^c_{t+1}$ as well as the inflation rates π_t and π^c_t return to their initial values in the long run. Here the inflation effects during period T are more strongly developed than in case of a temporary commodity price shock. This is due to the fact that the output gap $q - \overline{q}$ *increases* in T, since the decrease in the steady state output is bigger than the output contraction in T. In the same way, the output contraction after the shock period is due to a strong real appreciation process more strongly developed than in case of $\beta_R < 1$. Furthermore, the expansion process during the anticipation phase turns out to be stronger than at temporary commodity price shocks, which can be attributed to a stronger decrease in real interest rates during this period. On the other hand the real appreciation process is also more strongly developed, where - under the parameter combination used - τ lies also in the long term above its initial steady state value.

The permanent increase in commodity prices causes a permanent increase in domestic real commodity prices, which isolated seen permanently increases real commodity import. On the other hand, the long-term output contraction results in a permanent decrease in im_R, so that the net effect is ambiguous. Due to the chosen parameter values the output effect dominates the opposed real factor price effect, so that im_R runs parallel to the output development for $t > T$.

Remarks

Until now we have analyzed the effects that follow anticipated commodity price increases. It was assumed that an increase in nominal input price p^*_R is at the same time accompanied by an increase in real foreign input price $p^*_R - p^*$. In the following we deal with the *borderline* case that the commodity price shock is *unanticipated* and then treat the case of a *pure nominal input price shock* (i.e. $\mu^* = 0$ resp. $p^*_{R,t} - p^*_t = 0$ for all t).

In case of an *unanticipated* oil price increase, the anticipation phase is omitted, so that there is an *immediate* output contraction, which can be traced back to an increase in real interest rate and terms of trade in $t = T$. The connected inflation effects in $t = T$ are now stronger compared to the case of anticipation and are again weakened after the realization of the unanticipated price shock (figures 2 and 3). Furthermore, an immediate depreciation of the domestic currency takes place. Qualitatively, for $t > T$ we obtain the same development of the endogenous variables as for the anticipated oil price increase. The tables 4 to 7 show the volatility of y, Δp^c and i (measured by the total variance) in case of anticipated and unanticipated shocks. When passing from the unanticipated into the anticipated case, a clear increase in variance can be observed in each case, which can be explained by the hybrid character of the supply equation and the demand equation as well as by the increase in y and Δp^c during the anticipation phase. The increase in volatility is clearly diminished and in case of the CPI inflation rate even reduced, when passing into a purely forward-looking model.

The case of $\mu^* = 0$ (*constant* foreign real input price) provides stronger positive output and income effects during the anticipation phase than the case discussed until now, i.e., $\mu^* > 0$. This can be explained by a stronger decrease in real interest rates during the periods $t = 0$ and $t = 1$ (figures 4 and 5). The short-term real appreciation process that takes place during the anticipation period, is not continued in the shock period $T = 2$, so that the output contraction in T is weaker than in case of $\mu^* > 0$. In case of a permanent input price shock ($\beta_R = 1$) the output level always runs above the time path of q in case of a real commodity price shock ($\mu^* > 0$). Especially the long-term output contraction turns out substantially weaker, which can be traced back to a much less increase in domestic real commodity price $p^*_R + e - p$ on the supply side and to a less increase in terms of trade τ on the demand side. Due to a permanent nominal appreciation in case of $\beta_R = 1$ the real factor price $p^*_R + e - p$ may even decrease in the long run, so that a permanent output contraction does not need to occur either. Furthermore, the case of a permanent input price shock ($\beta_R = 1$) shows that a purely nominal foreign price shock ($\mu^* = 0$) is in the short run accompanied by strong price decreases and in the long and medium term only by slight price increases, so that overall only weak stagflationary effects occur. Nevertheless there are now stronger inflationary effects during the shock period T both at permanent and at temporary commodity price shocks.

4 The Impacts of Monetary Policy Rules

In the last chapter we have shown that anticipated and unanticipated increases in the price of oil or other raw materials import goods will involve oil-dependent economies in temporary inflation and output contraction, the precise degree of severity of these effects depending upon the reaction of the price level of imported final goods. This section investigates the consequences of two types of monetary policy rules that could be employed by the domestic central bank in an effort to reduce the potentially disruptive effects of oil-price shocks.

On the one hand we examine an interest rate rule with interest rate smoothing of the Taylor type[11], i.e.

$$i_t = \beta i_{t-1} + (1-\beta)\big(\bar{i} + v_1(\Delta p^c_t - \Delta\bar{p}^c) + v_2(q_t - \bar{q})\big) \qquad (13)$$

on the other hand we discuss an analogous monetary policy rule for the growth rate of money stock of the type

$$\Delta m_t = \Delta\overline{m} - \widetilde{v}_1(\Delta p^c_t - \Delta\bar{p}^c) - \widetilde{v}_2(q_t - \bar{q}) \qquad (14)$$

Here, more importance is attached to the stabilization of inflation rates than to the stabilization of output ($v_1 > 1 > v_2 > 0$, $\widetilde{v}_1 > 1 > \widetilde{v}_2 > 0$). In the

[11] In economic literature numerous versions of monetary Taylor rules for closed and open economies are discussed. Those may also be of the forward-looking-type proposed by Clarida, Galí and Gertler (2000) and – referring to open economies – be explicitly dependent upon the real or nominal exchange rate. See e.g. Ball (1999) or Taylor (2001).

first case the central bank pursues an interest rate targeting, in the second case a monetary base targeting. By the assumption $v_1 > 1$ ($\widetilde{v}_1 > 1$) the Taylor principle is presumed, according to which an increase in inflation rate leads isolated seen to a more than proportionately high increase (resp. decrease) in nominal interest rate (in nominal growth in money supply), so that the real interest rate decreases resp. the real growth in money supply declines.

In case of a *permanent anticipated* commodity price shock ($\beta_R = 1, T = 2$) the *Taylor rule* with interest rate smoothing ($\beta = 0.8$) ensures that the inflationary effects during the anticipation phase and the shock period are slightly reduced (see figure 7 and table 3); this diminution becomes particularly clear when the interest rate smoothing is *omitted* ($\beta = 0$). The deflationary process that results in the course of the adjustment is also less developed so that in the medium and long term the inflation rates lie above the inflation rate in case of passive monetary policy ($\Delta m_t = 0$). The Taylor rule also ensures a stronger short-term decrease in nominal interest rate and makes this variable increase more subsequent to the shock period T than in case of passive monetary policy. Furthermore, there is a stronger output contraction process subsequent to the commodity price increase in T (see table 3). This becomes particularly clear when interest rate smoothing is not pursued ($\beta = 0$).

The case $\beta_R < 1$ (*temporary* commodity price shock) clarifies that the Taylor rule may also be accompanied by *stronger* inflationary effects during the anticipation phase and the shock period T (figure 6). Again this becomes clear when the simple Taylor rule ($\beta = 0$) is existent. Furthermore, the deflationary process subsequent to the input price shock in T is now more strongly developed, so that the stabilized inflation rate runs below the one in case of non-stabilization in the longer run. In addition, in case of the Taylor rule, a clear increase in volatility of the nominal interest rate is shown again.

If the Taylor rule (13) is substituted by the money growth rule (14), we obtain qualitatively the same developments for the endogenous variables (figures 8 and 9). In case of the money growth rule the output expansion, which results during the anticipation phase, is a little stronger developed than in case of the interest rate rule (with smoothing). In case of *temporary* commodity price shocks it is distinguishable that there is a stronger output contraction subsequent to the oil price shock in case of the money growth rule than in case of the interest rate rule with interest rate smoothing ($\beta = 0.8$). When omitting interest rate smoothing ($\beta = 0$) the output contraction is slightly smaller in case of the money growth rule, since the interest rate rule is linked to a stronger contraction process in the case of $\beta = 0$ than in the case of $\beta = 0.8$.

In case of a *temporary* commodity price shock the inflationary effects are weaker during the initial periods under the money growth rule than under the interest rate rule (with and without smoothing). The deflationary process subsequent to the raw materials price shock is also clearly less distinct in case of the money growth rule than in case of the interest rate rule. Likewise, the money growth rule ensures weaker inflationary effects in case of *permanent* commodity price shocks than the interest rate rule. In particular, the money growth rule is able to clearly diminish the strong increase in inflation during the shock period T, whereas this does not work in case of the interest rate targeting.

This implies that the volatility of the CPI inflation rate under the money growth rule is considerably weaker than under the interest rate rule (with and without smoothing). Note also that the strongest volatility in interest rate occurs in case of the simple interest rate rule, since it does not provide any smoothing of the interest rate.

5 Perfect Stabilization of the CPI Inflation Rate

The previous chapter has shown, that an interest rate rule depending on inflation and output gap $\Delta p_t^c - \Delta\overline{p}^c$ resp. $q_t - \overline{q}$ is *not* able, to clearly reduce the inflationary effects that result from temporary or permanent commodity price shocks. On the contrary even intensifying effects may occur in case of this type of interest rate targeting. As it can be shown, an equivalent statement is valid for forward-looking interest rate rules (see tables 8 and 9). The question arises whether a *perfect* stabilization of the CPI inflation rate is possible with the help of another type of interest rate rules. Indeed, a perfect stabilization of π^c can be achieved at the initial level, if the domestic interest rate is not attached to the inflation rate and the output gap any more, but to the real depreciation rate $-\Delta\tau_{t+1}$:

$$i_t = i_t^* - \Delta p_{t+1}^* - \alpha(\tau_{t+1} - \tau_t) \tag{15}$$

This interest rate rule is in close relationship to the uncovered interest parity condition $i_t = i_t^* + \Delta e_{t+1}$ whereby in small open economies interest rate targeting is equivalent to the targeting of the nominal depreciation rate Δe_{t+1}. When choosing the goal of pure inflation targeting this is equivalent to the targeting of the real depreciation rate.

In case of *temporary* commodity price shocks there are stronger expansive output effects during the anticipation phase than when using Taylor interest rate rules with and without interest rate smoothing. This can be traced back to high short-term decreases in real interest rate. In case of perfect inflation targeting ($\Delta\overline{p}^c = 0$ for all t) those are identical to equal decreases in nominal interest rate. During the shock period large increases in interest rate and terms of trade occur, therefore there is a sharper output contraction (figures 10 and 11) than in case of a Taylor rule with smoothing. In case of *permanent* commodity price shocks the positive output effects of the anticipation phase are in case of perfect inflation targeting reinforced compared to the Taylor rules. The contraction process that appears in consequence of the commodity price shock, is nearly identical to the development of output in case of a Taylor rule with smoothing.

It has to be considered that by perfect stabilization of the CPI inflation rate the domestic inflation rate Δp_t is stabilized as well. Because of $\Delta p_t^c = \Delta\overline{p}^c = 0$ and $\Delta p_t - \Delta p_t^c = (1-\alpha)\Delta\tau_t$, Δp_t develops parallel to the real appreciation rate $\Delta\tau_t$. In contrast, nominal and real interest rate run opposite to $\Delta\tau_{t+1}$.

Remark

If the interest rule (15) that is linked to a perfect stabilization of the CPI inflation rate, is substituted by a money growth rule of the type (14), where the weight $\widetilde{v}_1$ of the inflation gap is chosen to be very large, (e.g. $\widetilde{v}_1 = 100$), we obtain almost the same time paths for the endogenous variables. From the money market equation and the money growth rule the equation

$$\Delta p_t^c = -\frac{l_1}{\widetilde{v}_1 + 1}\Delta q_t - \frac{\widetilde{v}_2}{\widetilde{v}_1 + 1} q_t + \frac{l_2}{\widetilde{v}_1 + 1}\Delta i_t \tag{16}$$

results for Δp^c. For large values of $\widetilde{v}_1$ Δp^c is - apart from the starting phase of adjustment - identical to its initial value $\Delta \overline{p}^c = 0$.

6 Comparison of Monetary Policy Rules and Conclusion

Comparing the volatility of the variables Δp^c, y and i under different monetary policy rules relatively to the volatility of those variables in case of a passive monetary policy, it becomes clear that all rules lead to a strong increase in interest rate variance; the increase when applying the money growth rule is almost identical to the increase in case of the baseline Taylor rule (table 8 and table 11). The money growth rule causes a strong decrease in the total variance of the CPI inflation rate, whereas the Taylor rules – except for the special case of pure inflation targeting – are normally accompanied by intensifying effects. In contrast, in case of the Taylor rule with and without smoothing the total output variance is smaller than in case of the money growth rule. Here the output volatility can even be below the volatility in case of a passive monetary policy. Altogether seen a Taylor rule of the type (13) is more appropriate to output stabilization than to stabilization of inflation rates. For a money growth rule of the type (14) the exact opposite applies. The volatility in inflation rate caused by commodity price shocks can be clearly reduced by a money growth rule, whereas a strong decrease in output variance is realized by a Taylor-type interest rate rule. However, both rules increase the volatility in interest rates to the same degree. These results both hold if we only look at the period after the occurrence of the oil price increase (table 9 and 12) and if we abandon the assumption of anticipated oil price increases and instead consider an unanticipated shock (table 10 and 13).

As it has been shown, an interest rate rule is also able to drastically reduce the variance in the inflation rate; however this requires a policy of pure inflation targeting which is expressed by an interest rate rule that is not of the type (13) any more, but depends on the real depreciation rate. This interest rate rule is yet accompanied by a strong increase in output variance if the input price shock is anticipated by the public. Note that just the opposite holds in case of an unanticipated permanent increase in commodity prices (table 13).

The economic policy conclusion which can be drawn from the theoretical analysis of the effects of anticipated commodity price shocks and the appropriate monetary policy reaction is from our point of view evident: A simultaneous reduction of the increased volatility of output and inflation rates that is generated by anticipated commodity price shocks can be obtained neither by an interest rate rule of the Taylor type nor by an analogous money growth rule.

If the goal of stabilizing inflation rate dominates the goal of stabilizing output, targeting the growth rate of money stock is more appropriate than interest rate targeting of the Taylor type. If in contrast the goal of stabilizing output is dominant, the opposite applies. Relating to the European Monetary Union, which attaches more importance to the goal of price stability, this means the European Central Bank should not renounce the money growth targeting.

Tables and Figures

Table 1: Baseline parameters

a_1	0.8	a_2	0.8	a_3	0.03
b_1	0.2	b_2	0.2	b_3	0.3
b_0	0	α	0.75	Φ	0.8
l_1	1	l_2	1.5	l_0	0
ψ	0.233	d_0	0	β	0.8
v_1	1.5	v_2	0.5	$\widetilde{v}_1$	1.5
$\widetilde{v}_2$	0.5	ω	0.5	δ	0.2
μ	0.75	μ^*	0.95	f_0	0
f_1	1.25	f_2	1.67	$\Delta\overline{m}$	0
$\overline{g}$	0	$\overline{i}^*$	0	$\overline{y}^*$	0

Table 2: Qualitative effects of an anticipated increase in the price of raw materials under a passive monetary policy

Variables	Case	$t < T$	$t = T$	$t > T$
q, y	1	moderate increase	sharp fall	further, temporary fall, thereafter rise with overshooting
	2	stronger rise than in 1	sharp fall	further, permanent fall with undershooting
Δp, Δp^c	1	moderate rise	sharp rise	strong fall in $T+1$, thereafter cyclical development
	2	moderate rise	sharp rise	strong fall in $T+1$, thereafter monotonous development
e	1	small rise	further rise	further increase, monotonous fall for sufficiently large $t > T$
	2	fall	rise	further permanent rise
p, p^c	1	moderate increase	sharp increase	further rise in $T + 1$, thereafter fall with undershooting
	2	moderate decrease	sharp increase	further permanent rise with overshooting
i	1	rise	further rise	temporary fall with undershooting, thereafter increase
	2	fall on impact	rise	monotonous fall

Note: Case 1 denotes a temporary oil price shock, case 2 denotes a permanent oil price shock.

Table 3: Qualitative effects of an anticipated oil price shock in case of the baseline Taylor rule in comparison to the passive monetary policy case

Variables	Case	$t < T$	$t = T$	$t > T$
q, y	1	stronger rise	weaker fall	nearly the same development
	2	slightly stronger rise	slightly stronger fall	output and income contraction stronger
Δp, Δp^c	1	stronger rise	slightly stronger rise	longer deflationary process, no overshooting
	2	slightly weaker rise	sharp, but weaker rise	deflationary process weaker and more persistent
i	1	stronger rise on impact	nearly the same rise	stronger undershooting
	2	stronger fall on impact	weaker rise	delayed overshooting

Note: Case 1 denotes a temporary oil price shock, case 2 denotes a permanent oil price shock.

Table 4: Variances in case of a *temporary* raw materials price shock under a *passive monetary policy*

	VAR(Δp^c)	VAR(y)	VAR(i)
Unanticipated	0.1135	1.165	0.0441
	(68.37%)	(87.07%)	(59.92%)
Anticipated	0.1660	1.338	0.0736
	(100%)	(100%)	(100%)
	VAR(Δp^c)$\|_{t\geq T}$	VAR(y)$\|_{t\geq T}$	VAR(i)$\|_{t\geq T}$
Unanticipated	0.1135	1.165	0.0441
	(74.72%)	(88.02%)	(78.05%)
Anticipated	0.1519	1.3235	0.0565
	(100%)	(100%)	(100%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of an anticipated raw materials price shock. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{\infty}(x_t - \overline{x})^2$ and $\text{VAR}(x)|_{t\geq T} = \sum_{t=T}^{\infty}(x_t - \overline{x})^2$.

Table 5: Variances in case of a *temporary* raw materials price shock under a passive monetary policy in a *purely forward-looking model*

	VAR(Δp^c)	VAR(y)	VAR(i)
Unanticipated	0.0441	0.4996	0.0021
	(124.23%)	(80.91%)	(56.76%)
Anticipated	0.0355	0.6175	0.0037
	(100%)	(100%)	(100%)
	VAR(Δp^c)$\|_{t\geq T}$	VAR(y)$\|_{t\geq T}$	VAR(i)$\|_{t\geq T}$
Unanticipated	0.0441	0.4996	0.0021
	(130.47%)	(94.8%)	(105%)
Anticipated	0.0338	0.527	0.002
	(100%)	(100%)	(100%)

Notes: To obtain a purely forward-looking model we set $\omega = \Phi = 0$. Numbers in parentheses are the ratio of the variance relative to the variance in case of an anticipated raw materials price shock. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{\infty}(x_t - \overline{x})^2$ and $\text{VAR}(x)|_{t\geq T} = \sum_{t=T}^{\infty}(x_t - \overline{x})^2$.

Table 6: Variances in case of a *permanent* raw materials price shock under a *passive monetary policy*

	VAR(Δp^c)	VAR(y)	VAR(i)
Unanticipated	0.1292 (55.88%)	0.3435 (25.67%)	0.0199 (55.71%)
Anticipated	0.2312 (100%)	0.615 (100%)	0.0616 (100%)
	VAR(Δp^c)$\vert_{t\geq T}$	VAR(y)$\vert_{t\geq T}$	VAR(i)$\vert_{t\geq T}$
Unanticipated	0.1292 (58.02%)	0.3435 (25.95%)	0.0199 (72.57%)
Anticipated	0.2227 (100%)	0.531 (100%)	0.0554 (100%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of an anticipated raw materials price shock. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{T-1}(x_t - \overline{x}_0)^2 + \sum_{t=T}^{\infty}(x_t - \overline{x}_1)^2$ and $\text{VAR}(x)|_{t\geq T} = \sum_{t=T}^{\infty}(x_t - \overline{x}_1)^2$.

Table 7: Variances in case of a *permanent* raw materials price shock under a passive monetary policy in a *purely forward-looking model*

	VAR(Δp^c)	VAR(y)	VAR(i)
Unanticipated	0.0698 (132.2%)	0.0169 (15.82%)	0.00007 (7.78%)
Anticipated	0.0528 (100%)	0.1068 (100%)	0.0009 (100%)
	VAR(Δp^c)$\vert_{t\geq T}$	VAR(y)$\vert_{t\geq T}$	VAR(i)$\vert_{t\geq T}$
Unanticipated	0.0698 (145.11%)	0.0169 (84.5%)	0.00007 (8750%)
Anticipated	0.0481 (100%)	0.02 (100%)	0.000008 (100%)

Notes: To obtain a purely forward-looking model we set $\omega = \Phi = 0$. Numbers in parentheses are the ratio of the variance relative to the variance in case of an anticipated raw materials price shock. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{T-1}(x_t - \overline{x}_0)^2 + \sum_{t=T}^{\infty}(x_t - \overline{x}_1)^2$ and $\text{VAR}(x)|_{t\geq T} = \sum_{t=T}^{\infty}(x_t - \overline{x}_1)^2$.

Table 8: Variances in case of an *anticipated temporary* raw materials price shock under *alternative monetary policy responses*

	VAR(Δp^c)	VAR(y)	VAR(i)
Passive policy	0.166 (100%)	1.3338 (100%)	0.0736 (100%)
Baseline Taylor rule	0.2894 (174.34%)	1.2907 (96.77%)	0.2419 (328.67%)
Taylor rule w/o smoothing	1.0929 (658.37%)	1.4227 (106.67%)	1.4752 (2004.35%)
Forward-look. Taylor rule	0.4304 (259.28%)	1.2092 (90.66%)	0.374 (508.15%)
Money growth rule	0.117 (70.48%)	1.5639 (117.25%)	0.2184 (296.74%)
Inflation Targeting	0 (0%)	1.7388 (133.7%)	0.322 (437.5%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of a raw materials price shock under passive monetary policy. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{\infty}(x_t - \overline{x})^2$.

Table 9: Variances after the occurrence of the *anticipated temporary* oil price increase under *alternative monetary policy responses*

	$\text{VAR}(\Delta p^c)\|_{t\geq T}$	$\text{VAR}(y)\|_{t\geq T}$	$\text{VAR}(i)\|_{t\geq T}$
Passive policy	0.1519 (100%)	1.3235 (100%)	0.0565 (100%)
Baseline Taylor rule	0.2582 (169.98%)	1.2765 (96.45%)	0.2185 (386.73%)
Taylor rule w/o smoothing	0.9986 (657.41%)	1.4195 (107.25%)	1.2564 (2223.72%)
Forward-look. Taylor rule	0.3752 (247%)	1.2027 (90.87%)	0.295 (522.12%)
Money growth rule	0.1126 (74.13%)	1.5486 (117.01%)	0.209 (369.91%)
Inflation Targeting	0 (0%)	1.6895 (127.65%)	0.1536 (271.86%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of a raw materials price shock under passive monetary policy. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=T}^{\infty}(x_t - \overline{x})^2$.

Table 10: Variances in case of an *unanticipated temporary* raw materials price shock under *alternative monetary policy responses*

	VAR(Δp^c)	VAR(y)	VAR(i)
Passive policy	0.1135 (100%)	1.165 (100%)	0.0441 (100%)
Baseline Taylor rule	0.2459 (216.65%)	1.1525 (98.93%)	0.223 (505.67%)
Taylor rule w/o smoothing	0.7604 (669.96%)	1.0812 (92.81%)	0.9807 (2223.8%)
Forward-look. Taylor rule	0.3386 (298.33%)	1.0537 (90.45%)	0.3013 (683.22%)
Money growth rule	0.0856 (75.42%)	1.3124 (112.65%)	0.1569 (355.78%)
Inflation Targeting	0 (0%)	1.8358 (157.58%)	0.1457 (330.39%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of a raw materials price shock under passive monetary policy. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{\infty}(x_t - \overline{x})^2$.

Table 11: Variances in case of an *anticipated permanent* raw materials price shock under *alternative monetary policy responses*

	VAR(Δp^c)	VAR(y)	VAR(i)
Passive policy	0.2312 (100%)	0.6150 (100%)	0.0616 (100%)
Baseline Taylor rule	0.2192 (94.81%)	0.6749 (109.74%)	0.173 (280.84%)
Taylor rule w/o smoothing	0.6695 (289.58%)	0.477 (77.56%)	1.0283 (1669.32%)
Forward-look. Taylor rule	0.3092 (133.74%)	0.6114 (99.41%)	0.2321 (376.79%)
Money growth rule	0.0833 (36.03%)	0.7027 (114.26%)	0.1759 (285.55%)
Inflation Targeting	0 (0%)	1.0257 (166.78%)	0.3966 (643.83%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of a raw materials price shock under passive monetary policy. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{T-1}(x_t - \overline{x}_0)^2 + \sum_{t=T}^{\infty}(x_t - \overline{x}_1)^2$.

Table 12: Variances after the occurrence of the *anticipated permanent* oil price increase under *alternative monetary policy responses*

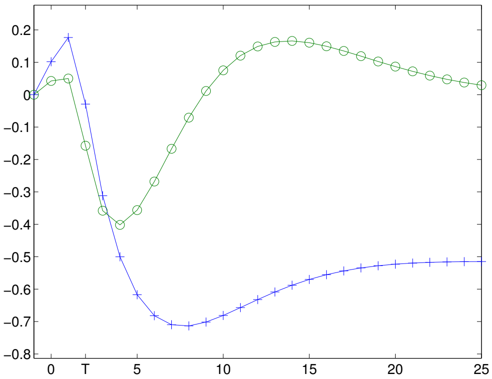
	$\text{VAR}(\Delta p^c)\|_{t\geq T}$	$\text{VAR}(y)\|_{t\geq T}$	$\text{VAR}(i)\|_{t\geq T}$
Passive policy	0.2227 (100%)	0.531 (100%)	0.0554 (100%)
Baseline Taylor rule	0.2152 (96.63%)	0.5678 (106.93%)	0.1597 (288.27%)
Taylor rule w/o smoothing	0.5344 (239.96%)	0.3599 (67.78%)	0.8016 (1446.9%)
Forward-look. Taylor rule	0.3036 (136.33%)	0.5247 (98.81%)	0.2192 (395.67%)
Money growth rule	0.0701 (31.48%)	0.5629 (106.01%)	0.0721 (130.14%)
Inflation Targeting	0 (0%)	0.7476 (140.79%)	0.0252 (45.49%)

Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of a raw materials price shock under passive monetary policy. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=T}^{\infty}(x_t - \overline{x}_1)^2$.

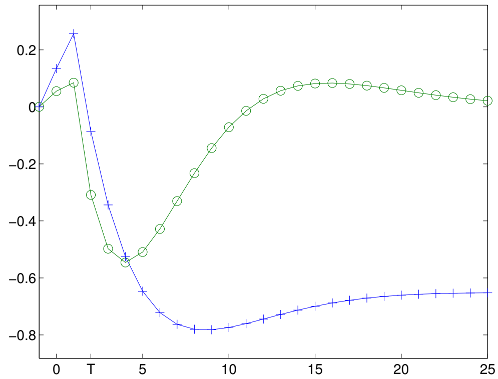
Table 13: Variances in case of an *unanticipated permanent* raw materials price shock under *alternative monetary policy responses*

	$\text{VAR}(\Delta p^c)$	$\text{VAR}(y)$	$\text{VAR}(i)$
Passive policy	0.1292 (100%)	0.3435 (100%)	0.0199 (100%)
Baseline Taylor rule	0.0848 (65.63%)	0.3495 (101.75%)	0.0354 (177.89%)
Taylor rule w/o smoothing	0.1164 (90.09%)	0.1647 (47.95%)	0.1814 (911.56%)
Forward-look. Taylor rule	0.1161 (89.86%)	0.3611 (105.12%)	0.0468 (235.18%)
Money growth rule	0.0315 (24.38%)	0.2982 (86.81%)	0.0162 (81.41%)
Inflation Targeting	0 (0%)	0.2415 (70.31%)	0.0073 (36.68%)

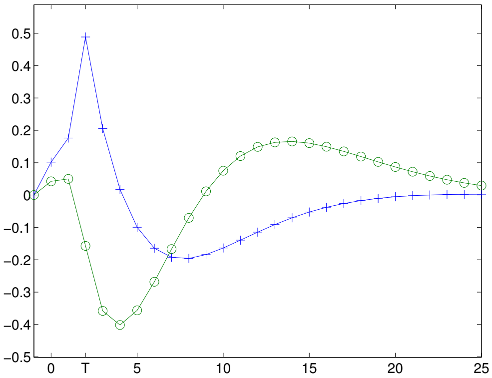
Notes: Numbers in parentheses are the ratio of the variance relative to the variance in case of a raw materials price shock under passive monetary policy. The variances for $x \in \{\Delta p^c, y, i\}$ are calculated as follows: $\text{VAR}(x) = \sum_{t=0}^{\infty}(x_t - \overline{x}_1)^2$.



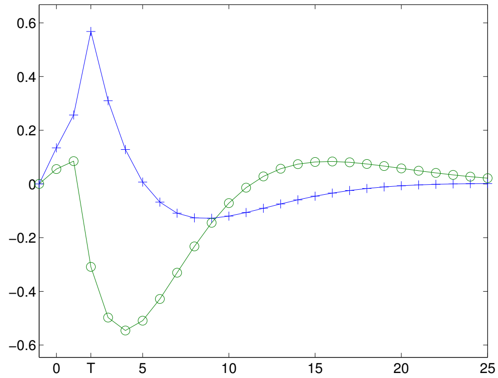
(a) Output



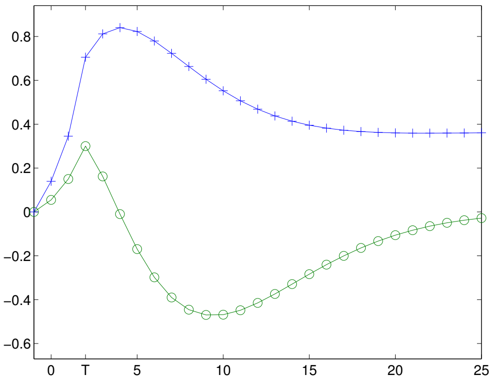
(b) Income



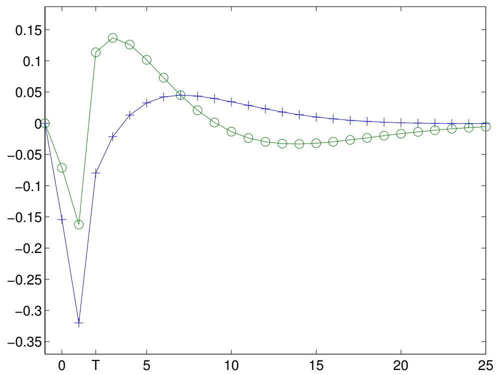
(c) Output Gap



(d) Income Gap

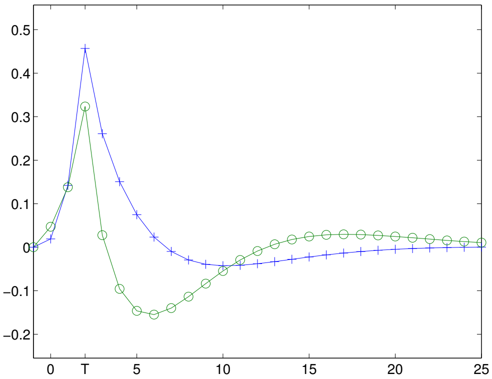


(e) Terms of Trade

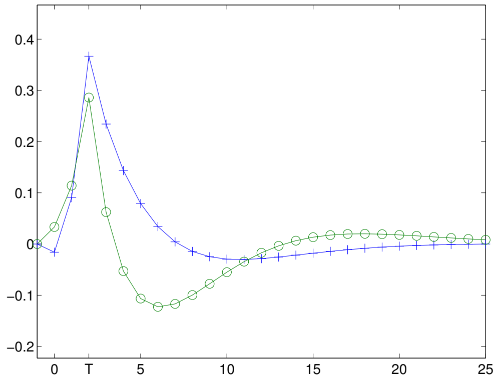


(f) Real Interest Rate

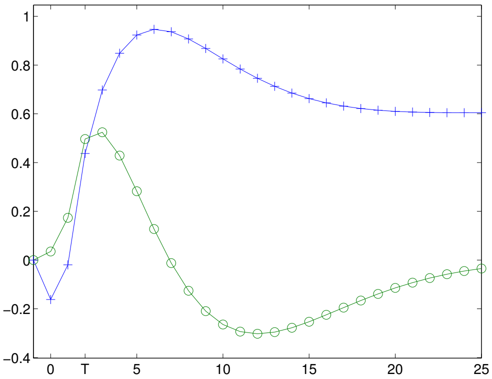
Figure 1: Economy's responses to an anticipated oil price increase taking place in period T. Solid lines with *circles* are responses to a *temporary* oil price increase; solid lines with *plus signs* are responses to a *permanent* oil price increase.



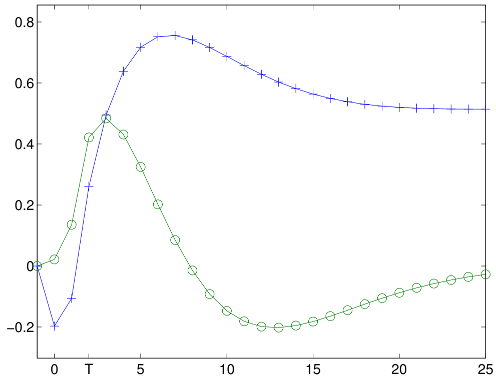
(g) Inflation



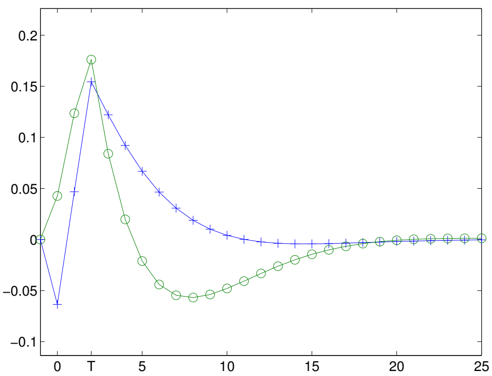
(h) CPI Inflation



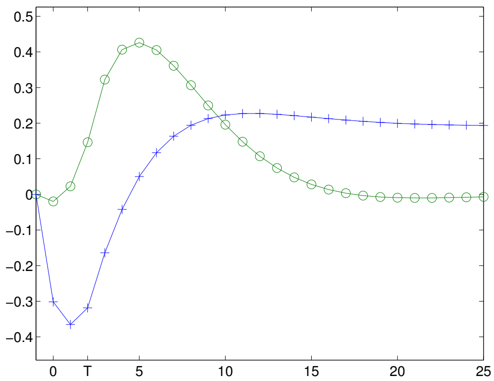
(i) Price



(j) Consumer Price Index

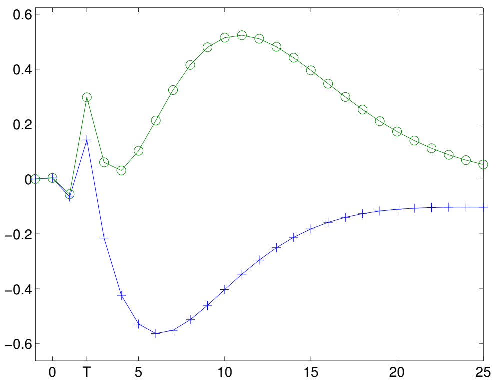


(k) Nominal Interest Rate

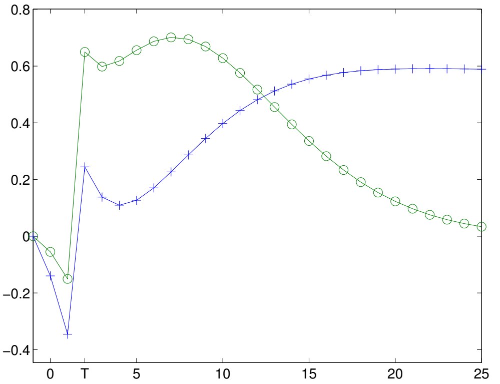


(l) Nominal Exchange Rate

Figure 1: — Continued

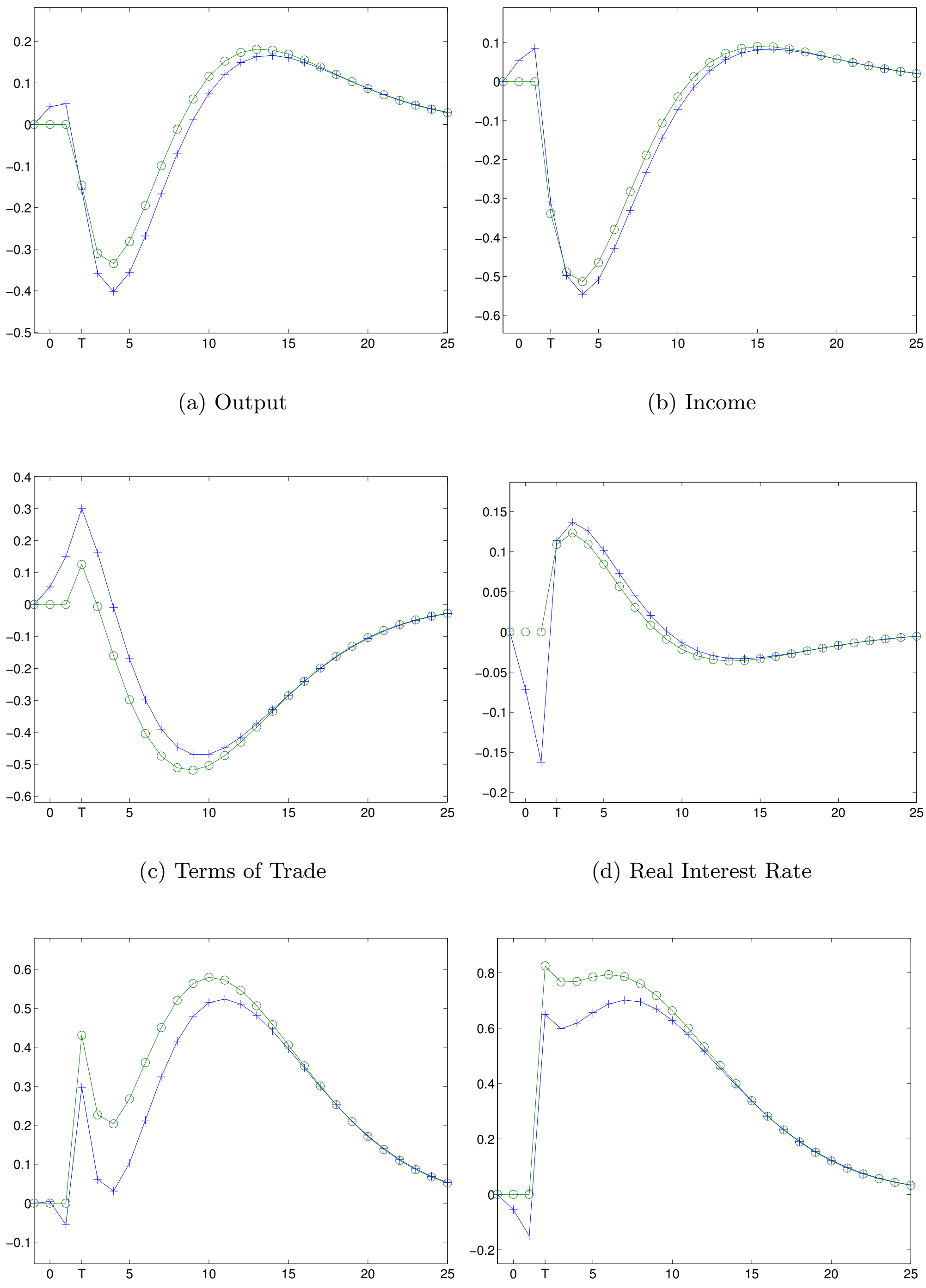


(m) Real Oil Imports



(n) Domestic Real Oil Price

Figure 1: — Continued

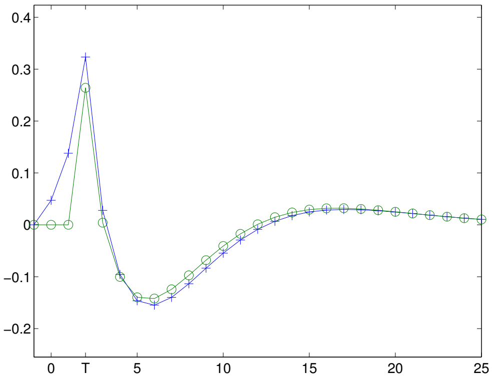


(a) Output (b) Income

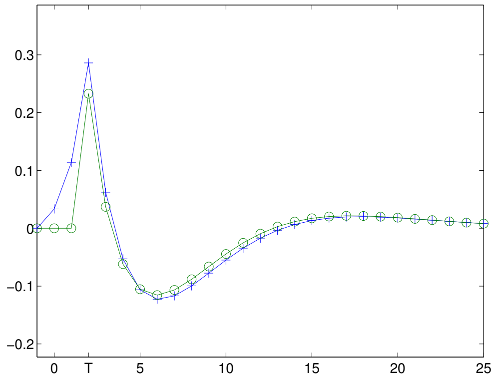
(c) Terms of Trade (d) Real Interest Rate

(e) Real Oil Imports (f) Domestic Real Oil Price

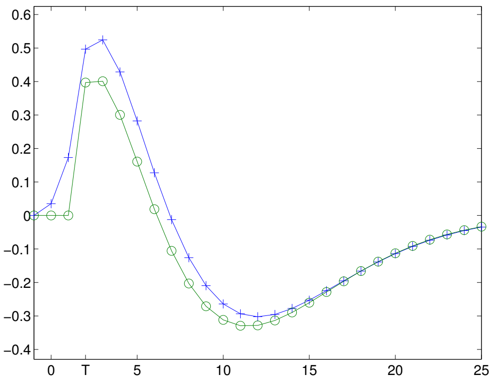
Figure 2: Economy's responses to a *temporary* oil price increase. Solid lines with *circles* are responses to an *unanticipated* oil price increase taking place in period T; solid lines with *plus signs* are responses to an *anticipated* oil price increase taking place in period T.



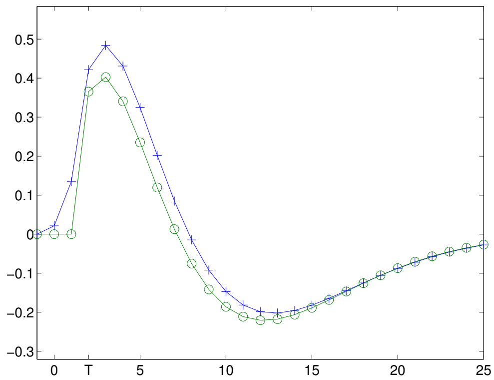
(g) Inflation



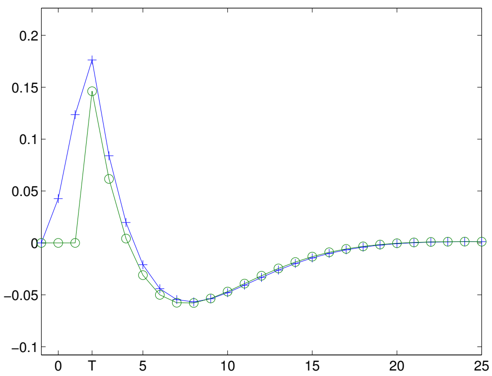
(h) CPI Inflation



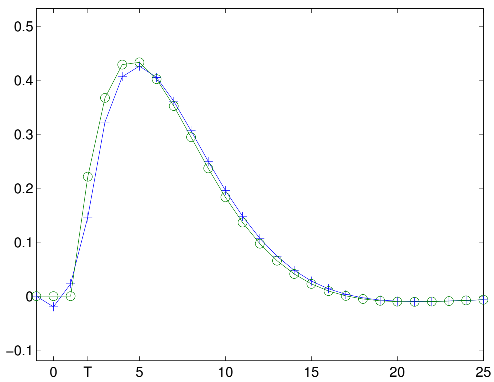
(i) Price



(j) Consumer Price Index

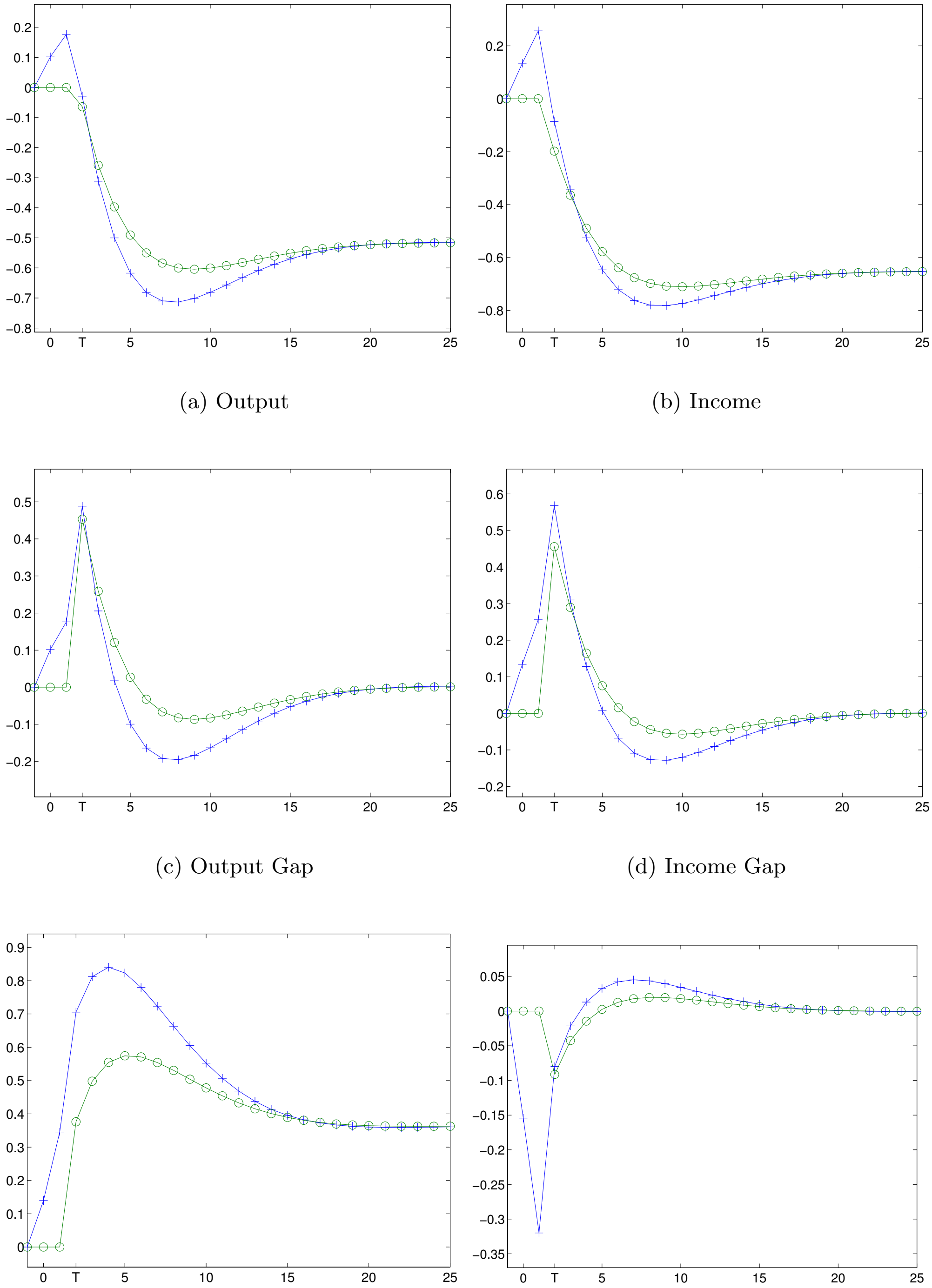


(k) Nominal Interest Rate



(l) Nominal Exchange Rate

Figure 2: — Continued



(a) Output

(b) Income

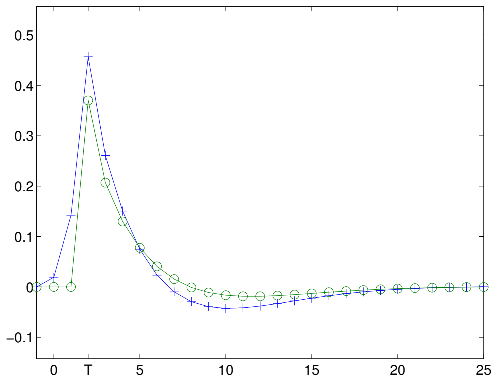
(c) Output Gap

(d) Income Gap

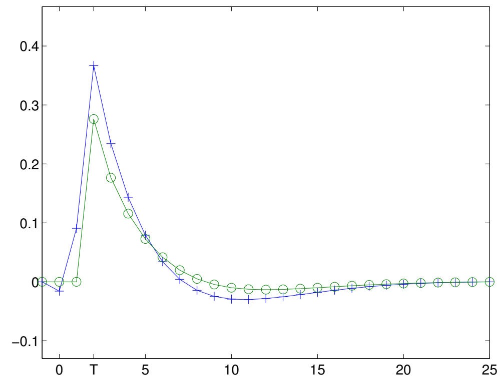
(e) Terms of Trade

(f) Real Interest Rate

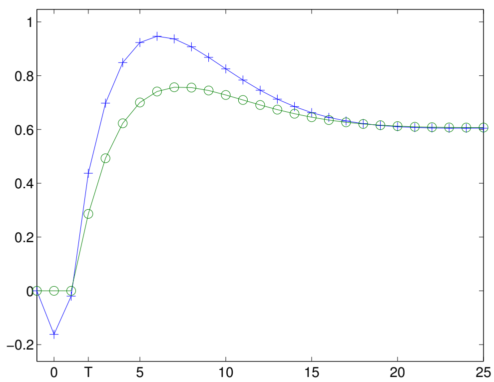
Figure 3: Economy's responses to a *permanent* oil price increase. Solid lines with *circles* are responses to an *unanticipated* oil price increase taking place in period T; solid lines with *plus signs* are responses to an *anticipated* oil price increase taking place in period T.



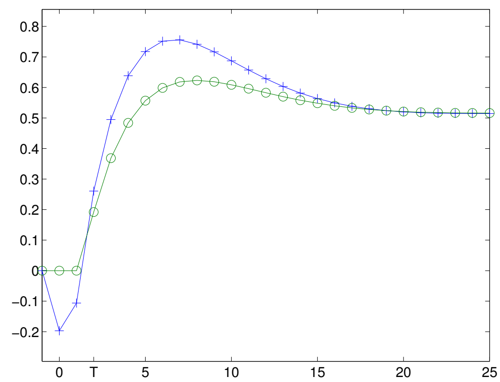
(g) Inflation



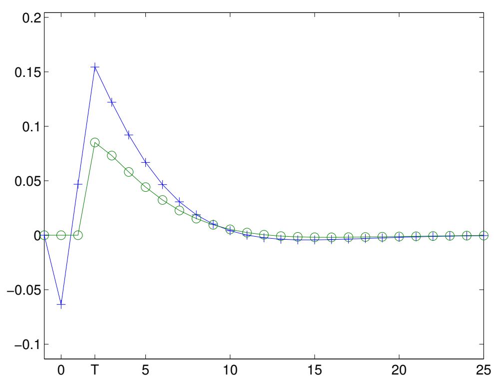
(h) CPI Inflation



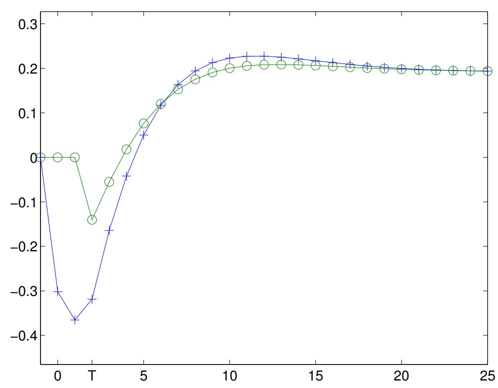
(i) Price



(j) Consumer Price Index

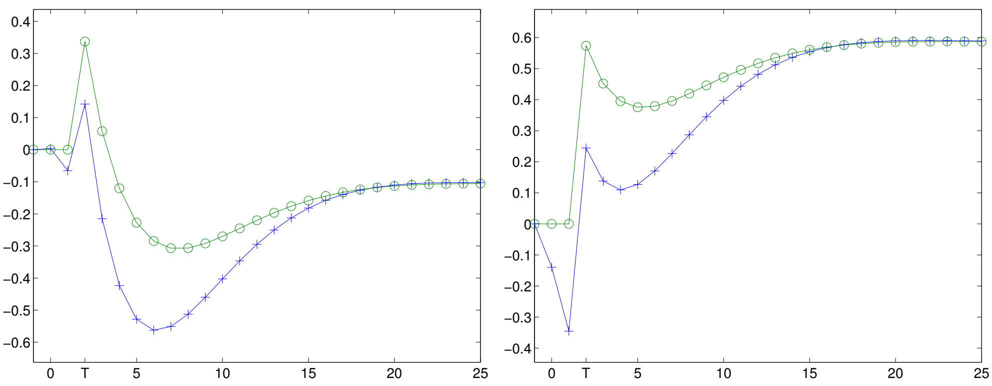


(k) Nominal Interest Rate



(l) Nominal Exchange Rate

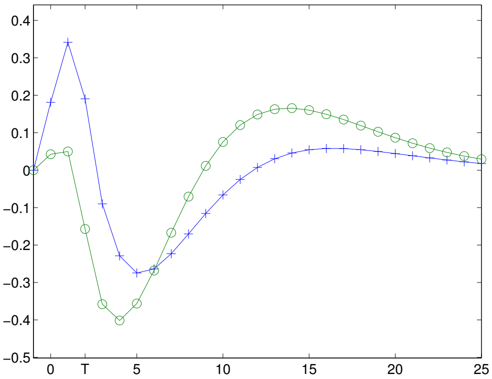
Figure 3: — Continued



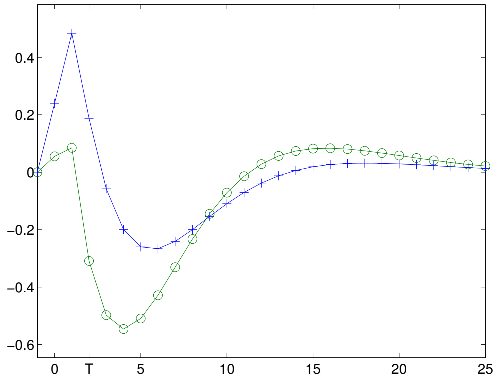
(m) Real Oil Imports

(n) Domestic Real Oil Price

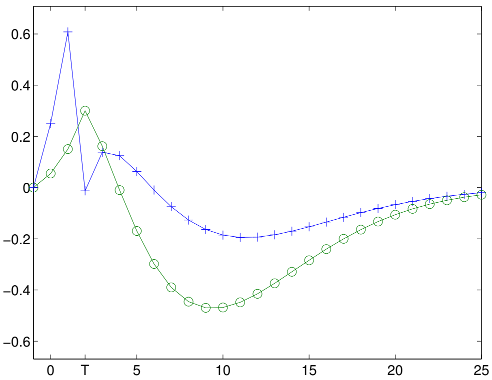
Figure 3: — Continued



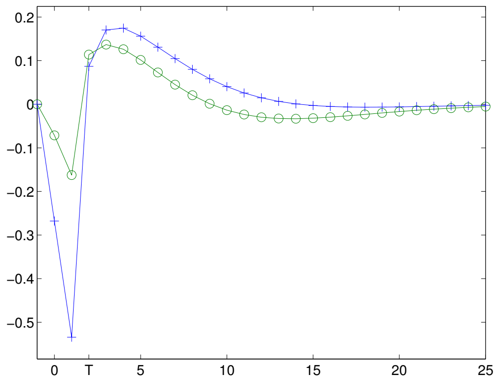
(a) Output



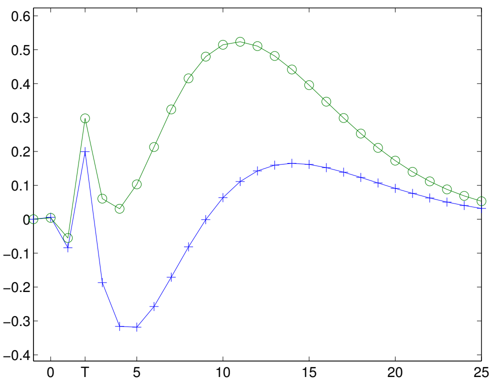
(b) Income



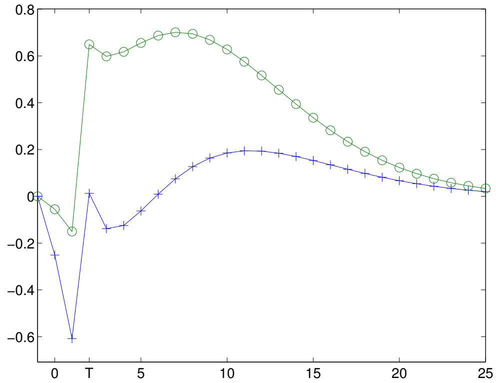
(c) Terms of Trade



(d) Real Interest Rate

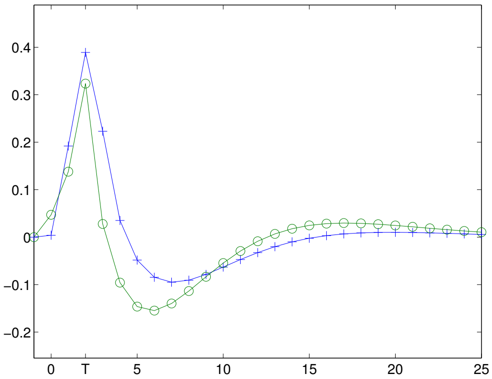


(e) Real Oil Imports

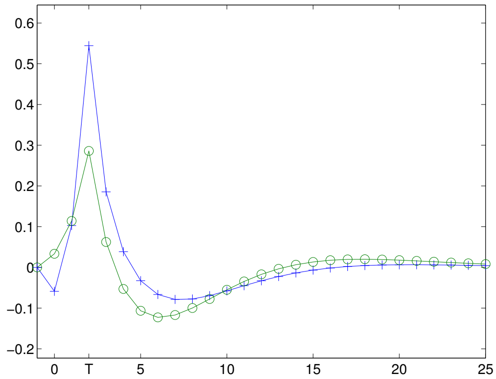


(f) Domestic Real Oil Price

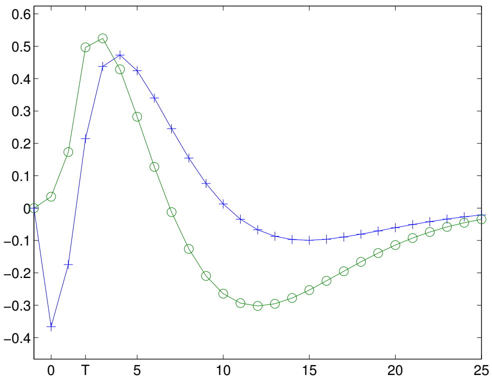
Figure 4: Economy's responses to an anticipated *temporary* oil price increase. Solid lines with *circles* are *baseline model* responses; solid lines with *plus signs* are responses in case of a *constant foreign real oil price* ($\mu^* = 0$).



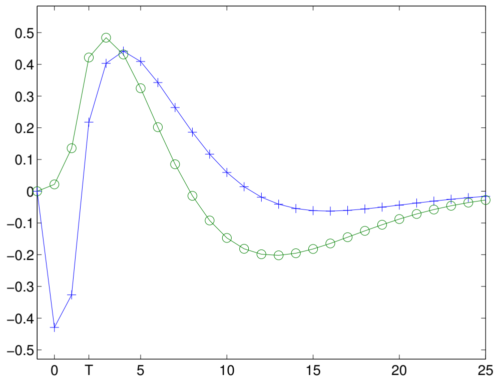
(g) Inflation



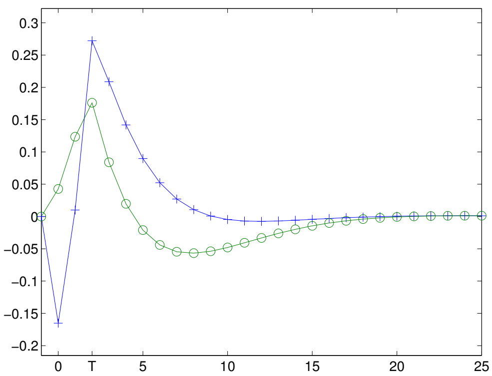
(h) CPI Inflation



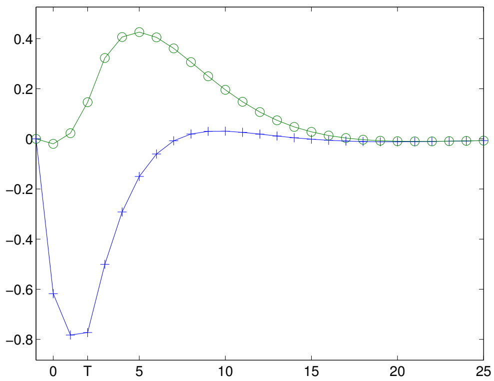
(i) Price



(j) Consumer Price Index

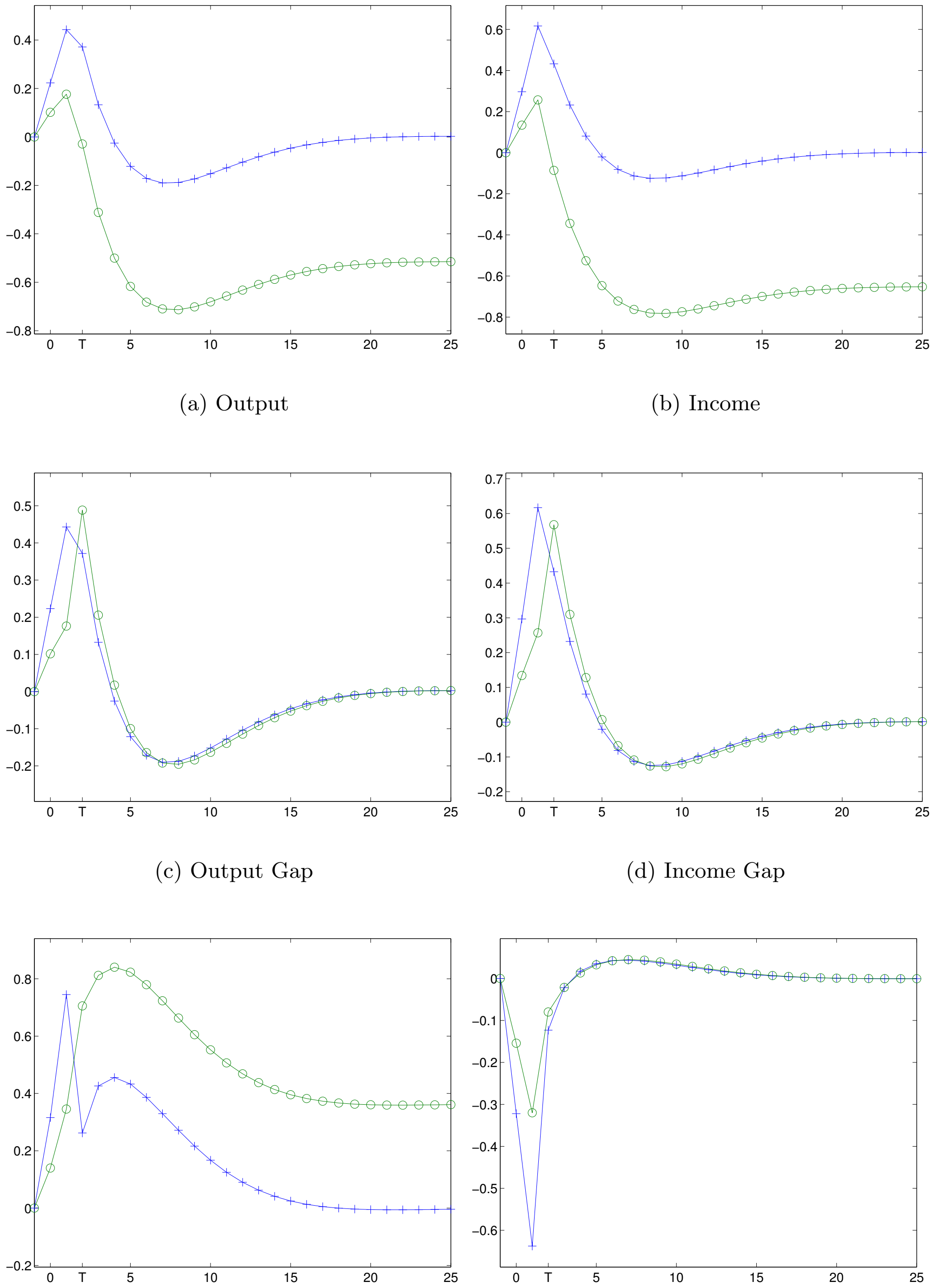


(k) Nominal Interest Rate



(l) Nominal Exchange Rate

Figure 4: — Continued

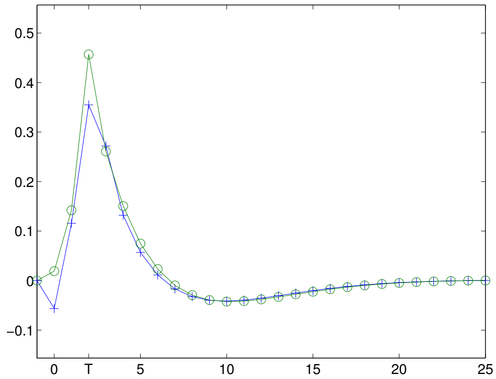


(a) Output (b) Income

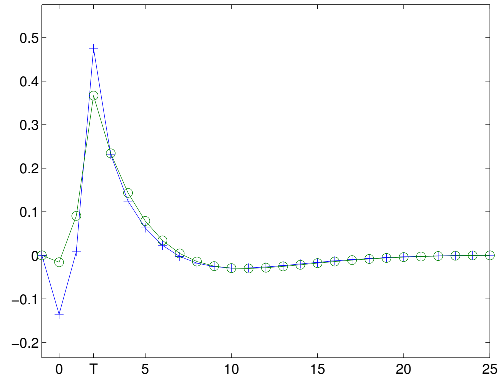
(c) Output Gap (d) Income Gap

(e) Terms of Trade (f) Real Interest Rate

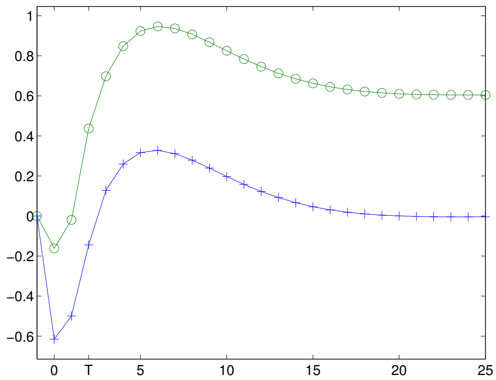
Figure 5: Economy's responses to an anticipated *permanent* oil price increase. Solid lines with *circles* are *baseline model* responses; solid lines with *plus signs* are responses in case of a *constant foreign real oil price* ($\mu^* = 0$).



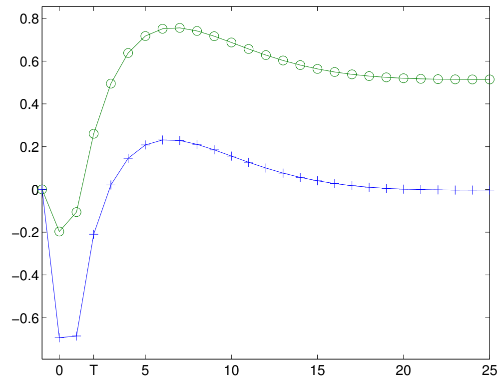
(g) Inflation



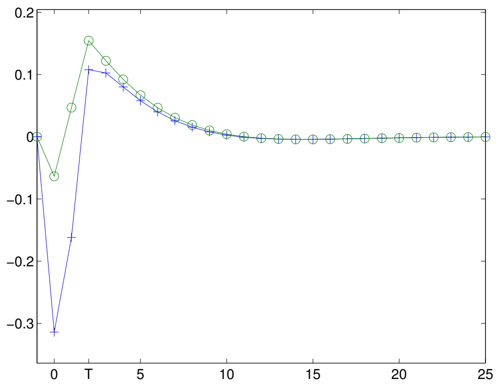
(h) CPI Inflation



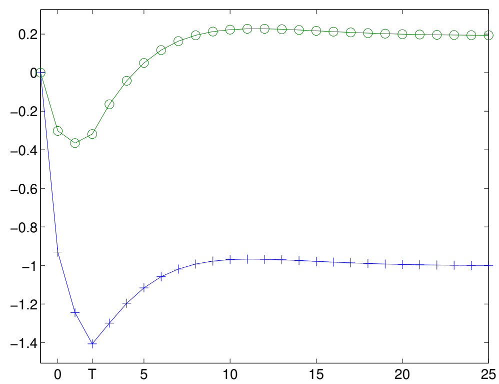
(i) Price



(j) Consumer Price Index

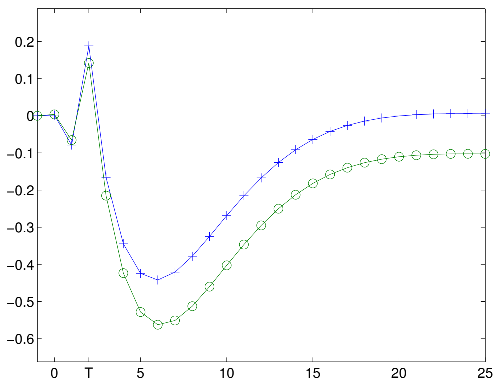


(k) Nominal Interest Rate

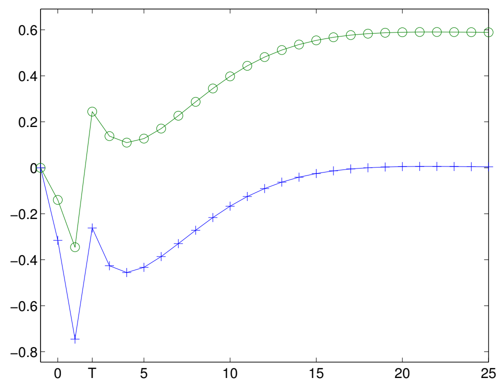


(l) Nominal Exchange Rate

Figure 5: — Continued

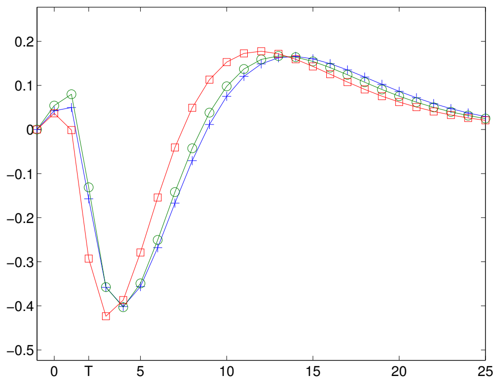


(m) Real Oil Imports

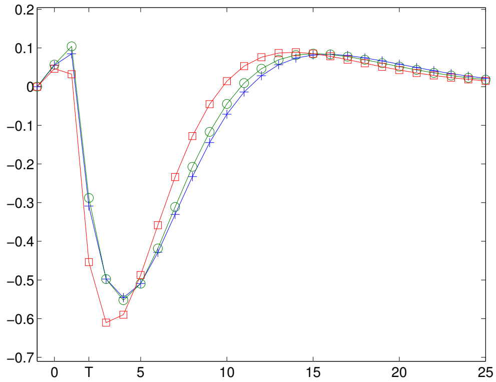


(n) Domestic Real Oil Price

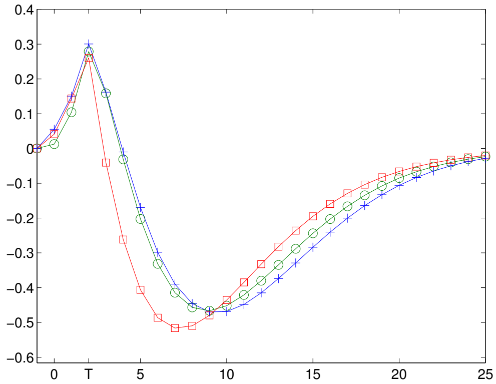
Figure 5: — Continued



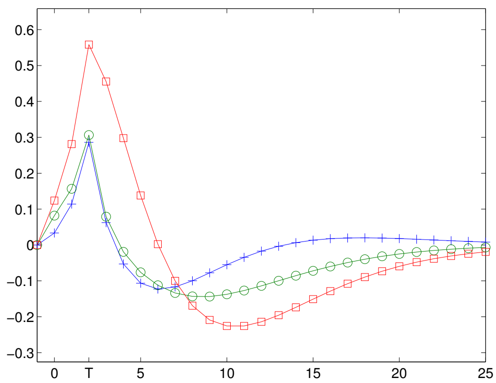
(a) Output



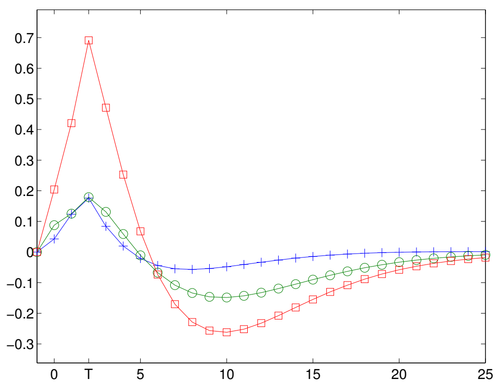
(b) Income Gap



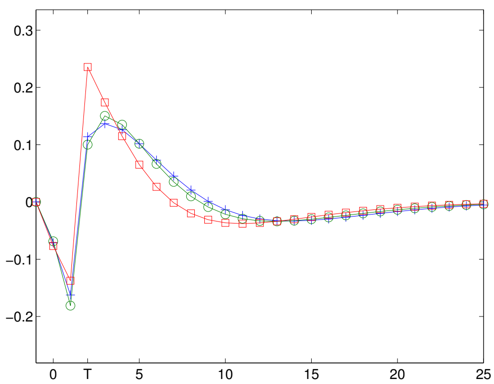
(c) Terms of Trade



(d) CPI Inflation

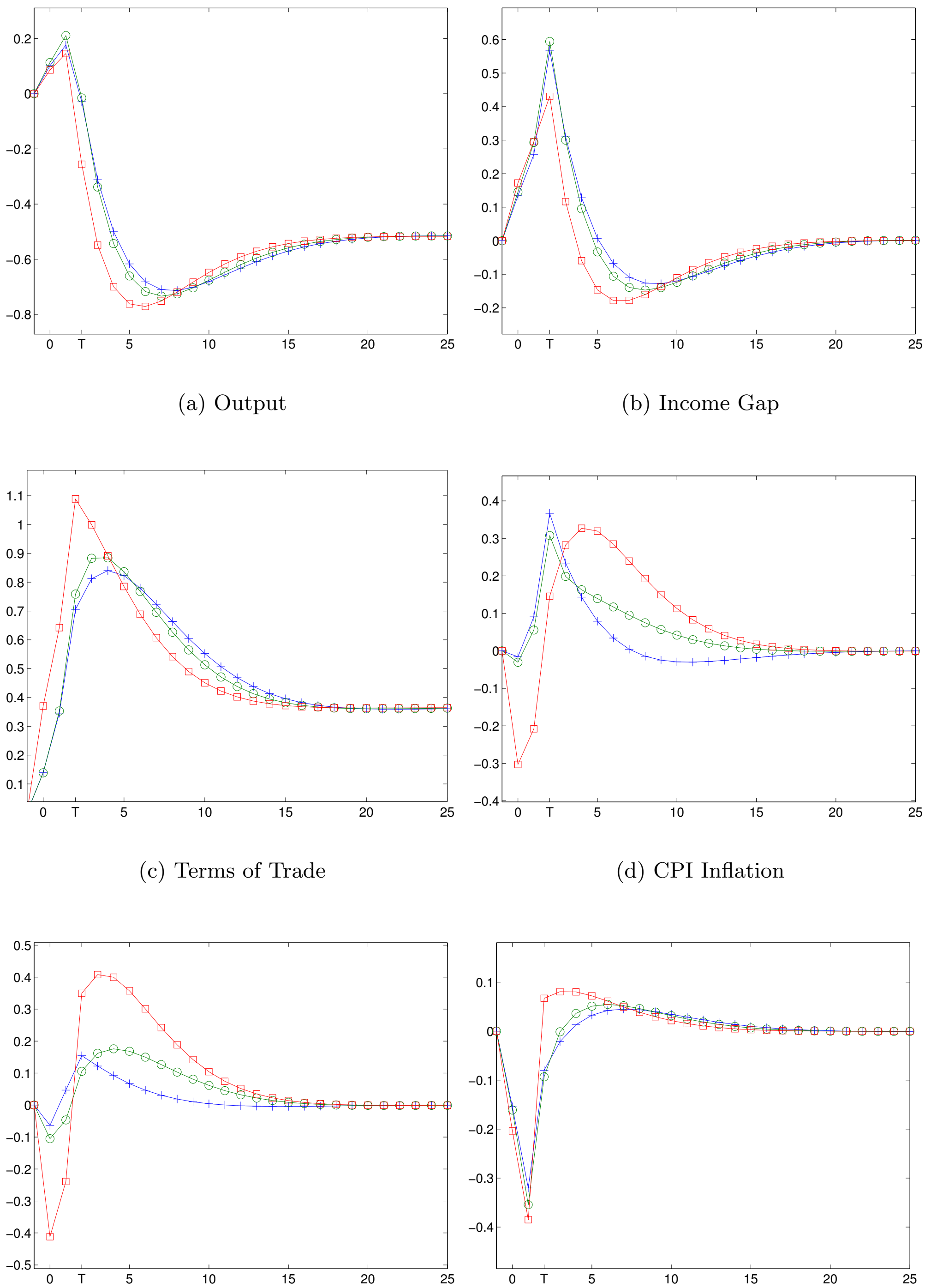


(e) Nominal Interest Rate



(f) Real Interest Rate

Figure 6: Economy's responses to an anticipated *temporary* oil price increase. Solid lines with *plus signs* are responses under a *passive monetary policy* ($\Delta m_t = 0$); solid lines with *circles* are responses under a *Taylor rule with interest rate smoothing* ($\beta = 0.8$); solid lines with *squares* are responses under a *Taylor rule without interest rate smoothing* ($\beta = 0$).



(a) Output

(b) Income Gap

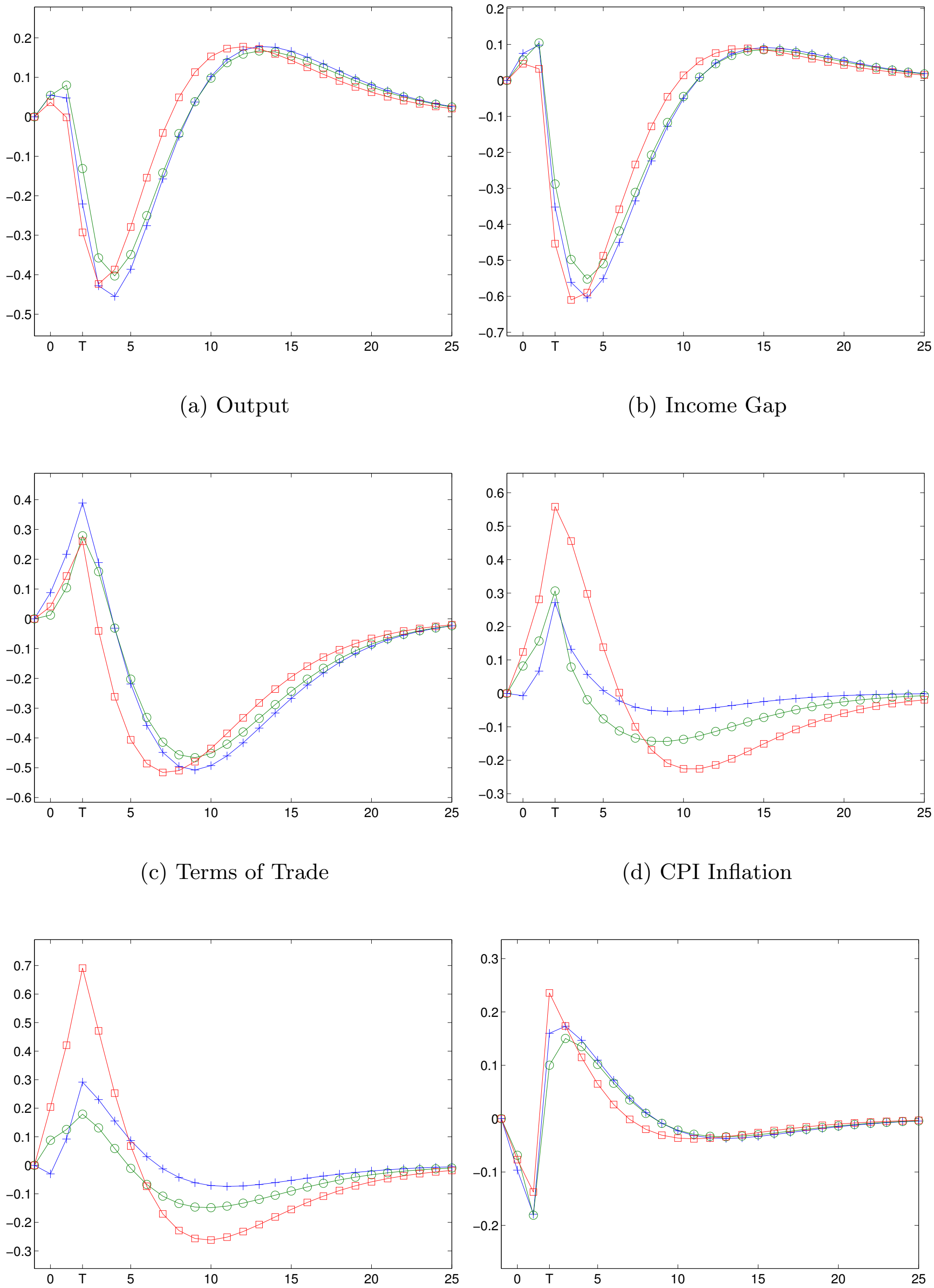
(c) Terms of Trade

(d) CPI Inflation

(e) Nominal Interest Rate

(f) Real Interest Rate

Figure 7: Economy's responses to an anticipated *permanent* oil price increase. Solid lines with *plus signs* are responses under a *passive monetary policy* ($\Delta m_t = 0$); solid lines with *circles* are responses under a *Taylor rule with interest rate smoothing* ($\beta = 0.8$); solid lines with *squares* are responses under a *Taylor rule without interest rate smoothing* ($\beta = 0$).

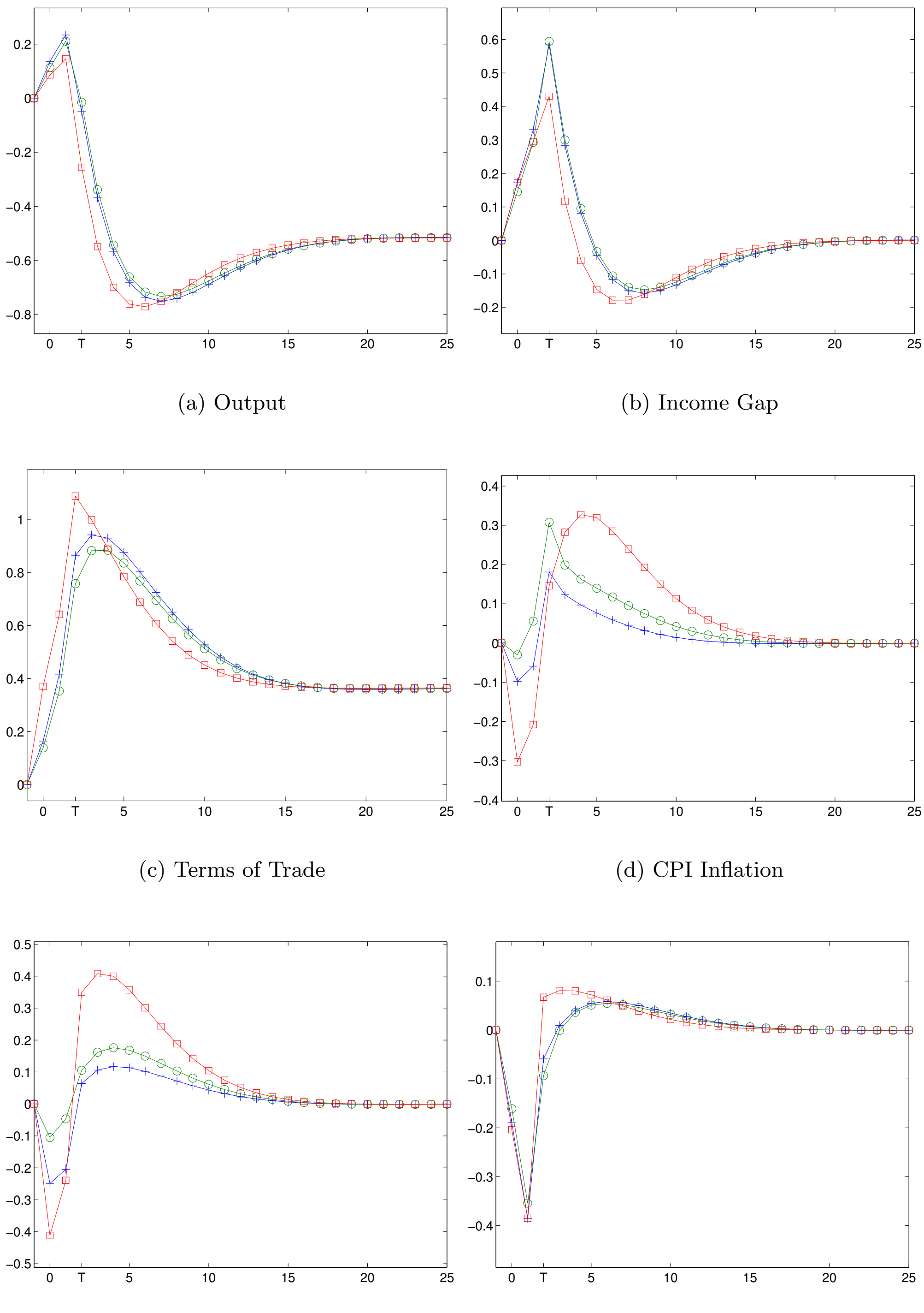


(a) Output (b) Income Gap

(c) Terms of Trade (d) CPI Inflation

(e) Nominal Interest Rate (f) Real Interest Rate

Figure 8: Economy's responses to an anticipated *temporary* oil price increase. Solid lines with *plus signs* are responses under a *money growth rule*; solid lines with *circles* are responses under a *Taylor rule with interest rate smoothing* ($\beta = 0.8$); solid lines with *squares* are responses under a *Taylor rule without interest rate smoothing* ($\beta = 0$).



(a) Output

(b) Income Gap

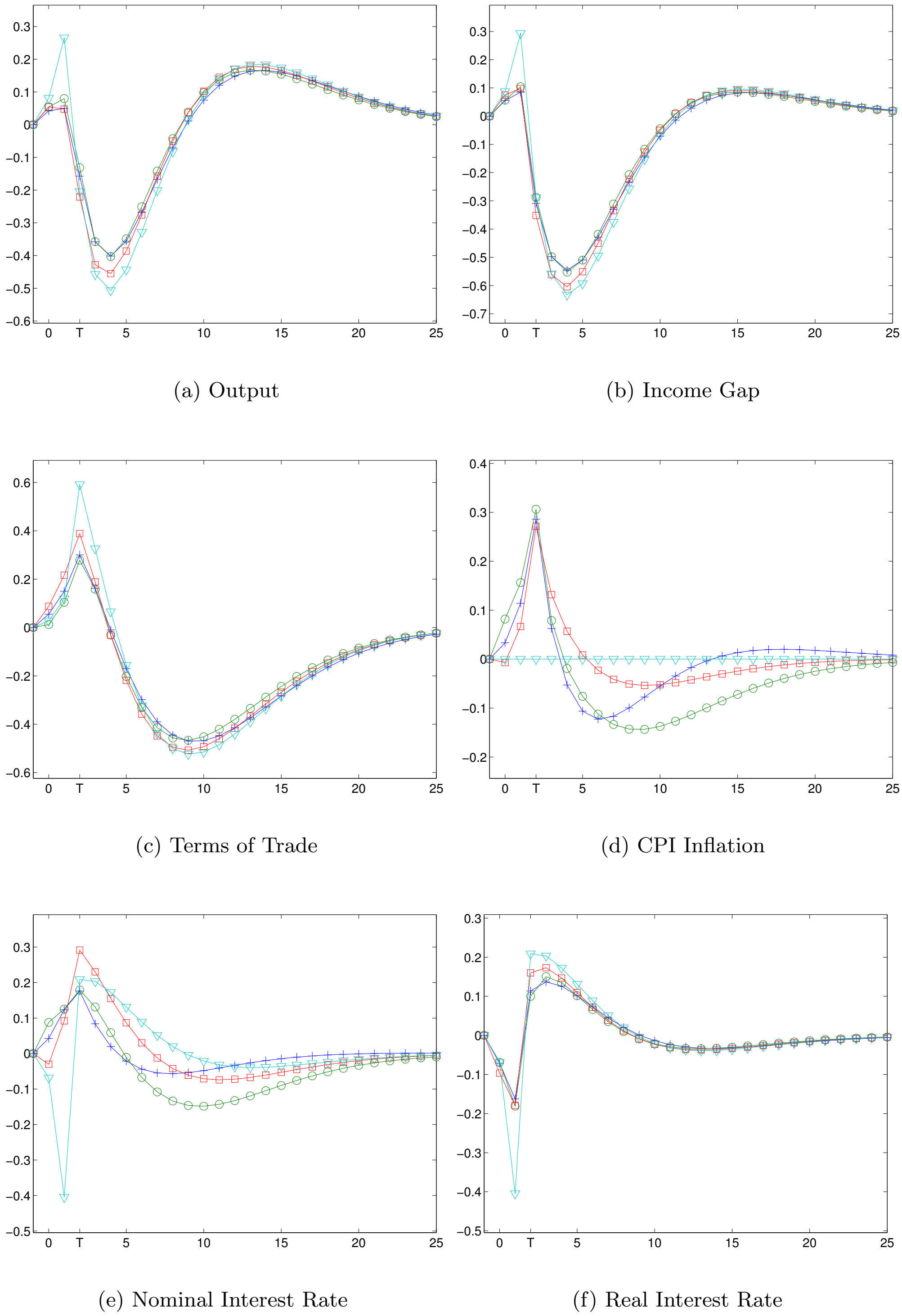
(c) Terms of Trade

(d) CPI Inflation

(e) Nominal Interest Rate

(f) Real Interest Rate

Figure 9: Economy's responses to an anticipated *permanent* oil price increase. Solid lines with *plus signs* are responses under a *money growth rule*; solid lines with *circles* are responses under a *Taylor rule with interest rate smoothing* ($\beta = 0.8$); solid lines with *squares* are responses under a *Taylor rule without interest rate smoothing* ($\beta = 0$).



(a) Output

(b) Income Gap

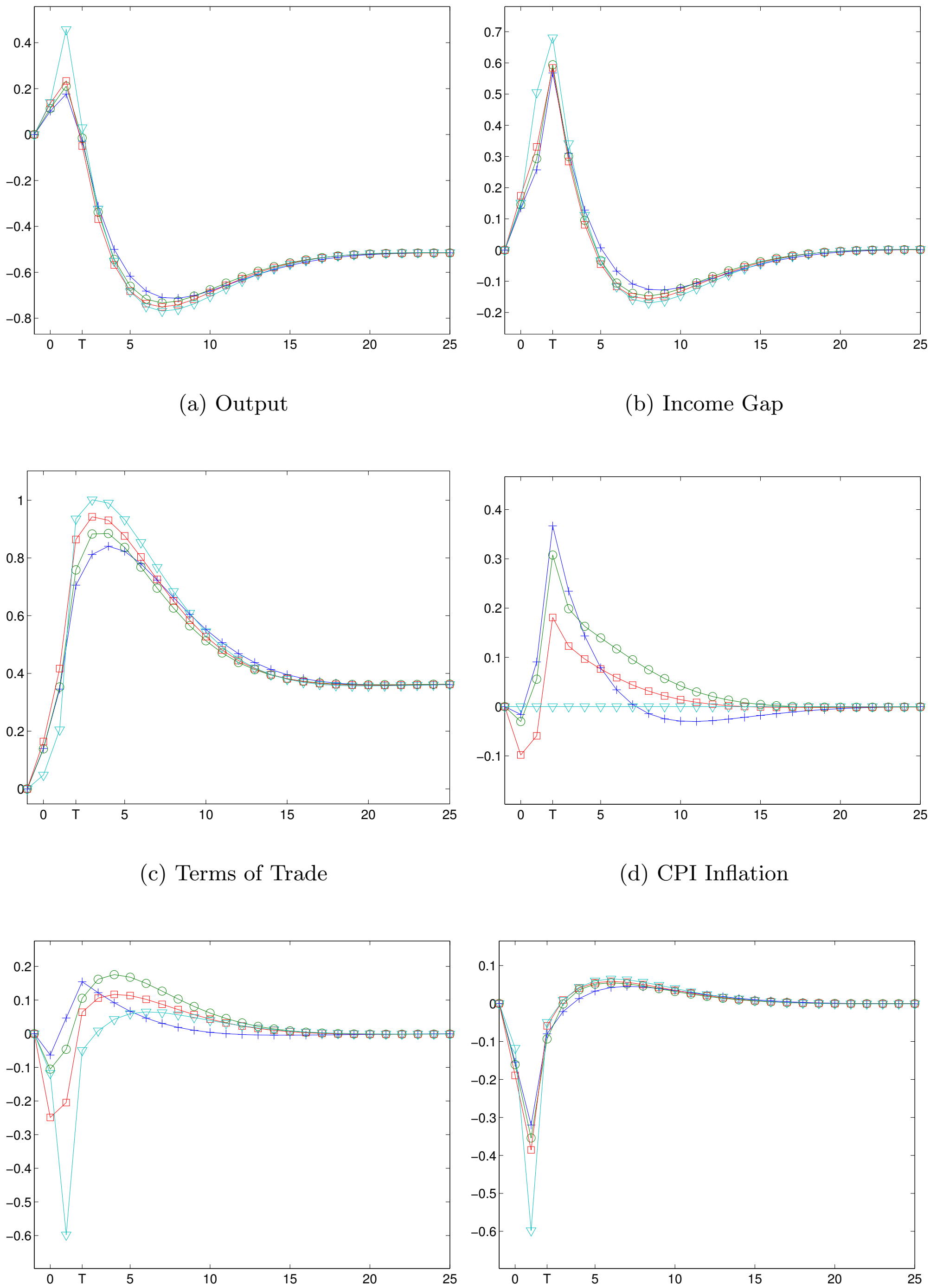
(c) Terms of Trade

(d) CPI Inflation

(e) Nominal Interest Rate

(f) Real Interest Rate

Figure 10: Economy's responses to an anticipated *temporary* oil price increase. Solid lines with *plus signs* are responses under a *passive monetary policy*; solid lines with *circles* are responses under a *Taylor rule with interest rate smoothing*; solid lines with *squares* are responses under a *money growth rule*; solid lines with *triangles* are responses under *inflation targeting*.



(a) Output

(b) Income Gap

(c) Terms of Trade

(d) CPI Inflation

(e) Nominal Interest Rate

(f) Real Interest Rate

Figure 11: Economy's responses to an anticipated *permanent* oil price increase. Solid lines with *plus signs* are responses under a *passive monetary policy*; solid lines with *circles* are responses under a *Taylor rule with interest rate smoothing*; solid lines with *squares* are responses under a *money growth rule*; solid lines with *triangles* are responses under *inflation targeting*.

Mathematical Appendix

The dynamics of the New Keynesian model of a small open economy can be represented by the following set of equations:

$$\begin{aligned}a_1(1-\Phi)y_{t+1} + (a_2+a_3l_2)\alpha\tau_{t+1} - a_3l_2\pi^c_{t+1} = \\ (1+b_1-a_3l_1)y_t - a_1\Phi y_{t-1} + \big((a_2+a_3l_2)\alpha + b_3 - (1-a_3l_1)\psi\big)\tau_t \\ + (1-a_3l_1)\psi(p^*_{R,t}-p^*_t) + (a_2+a_3l_2)(i^*_t - \pi^*_{t+1}) - b_2y^*_t - g_t \\ + (1-a_3l_1)d_0 - a_3l_0 - b_0\end{aligned} \tag{A1}$$

$$\begin{aligned}-\alpha\tau_{t+1} + \pi^c_{t+1} = (1-\beta)v_2y_t - \big((1+\beta)\alpha + (1-\beta)v_2\psi\big)\tau_t \\ + \big(\beta + (1-\beta)v_1\big)\pi^c_t + \alpha\beta\tau_{t-1} + (1-\beta)v_2\psi(p^*_{R,t}-p^*_t) \\ - (i^*_t - \pi^*_{t+1}) + \beta(i^*_{t-1} - \pi^*_t) + (1-\beta)\bar{i} \\ - (1-\beta)v_2(\bar{q}-d_0) - (1-\beta)v_1\bar{\pi}^c\end{aligned} \tag{A2}$$

$$\begin{aligned}\mu(1-\omega)\pi^c_{t+1} = -\mu\delta y_t + (1-\mu\alpha+\mu\delta\psi)\tau_t + \mu\pi^c_t - (1-\mu\alpha)\tau_{t-1} \\ - \mu\omega\pi^c_{t-1} - \mu\delta\psi(p^*_{R,t}-p^*_t) - (1-\mu)(\pi^*_{R,t}-\pi^*_t) + \mu\delta(\bar{q}-d_0)\end{aligned} \tag{A3}$$

where $\pi^c_t = \Delta p^c_t = p^c_t - p^c_{t-1}$, $\pi^*_t = \Delta p^*_t = p^*_t - p^*_{t-1}$, and $\pi^*_{R,t} = \Delta p^*_{R,t} = p^*_{R,t} - p^*_{R,t-1}$ denotes the rate of change of the domestic price index, the foreign price level, and the raw materials price level respectively. Equation (A1) is the combination of the model equations (1), (2), (4) and the real interest rate equation

$$i_t - \pi^c_{t+1} = i^*_t - \alpha\Delta\tau_{t+1} - \pi^*_{t+1} \tag{A4}$$

Equation (A2) results from the Taylor rule (13) where we have substituted the nominal interest rate by equation (A4). The last equation (A3) is the Phillips curve equation (5). Let $\mathbf{v}_t = (y_t, \tau_t, \pi^c_t)'$ be the vector of current and $\mathbf{w}_t = \mathbf{v}_{t-1}$ the vector of lagged state variables. The matrix representation of the implicit state equations (A1) to (A3) is then given by

$$\mathbf{B}\begin{pmatrix}\mathbf{v}_{t+1}\\ \mathbf{w}_{t+1}\end{pmatrix} = \mathbf{C}\begin{pmatrix}\mathbf{v}_t\\ \mathbf{w}_t\end{pmatrix} + \mathbf{k}_t \tag{A5}$$

where the triangular matrix $\mathbf{B} = (b_{ij})_{1\le i,j\le 6}$ and the matrix $\mathbf{C} = (c_{ij})_{1\le i,j\le 6}$ are defined by[12]

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & 0\\ 0 & b_{22} & b_{23} & 0 & 0 & 0\\ 0 & 0 & b_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & b_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & b_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & b_{66}\end{pmatrix} \tag{A6}$$

[12] In the case of the forward-looking Taylor rule

$$i_t = \beta i_{t-1} + (1-\beta)\big(\bar{i} + v_1(\pi^c_{t+1} - \bar{\pi}^c) + v_2(q_t - \bar{q})\big)$$

the element b_{23} of the matrix $\mathbf{B}$ has to be replaced by $b^\star_{23} = 1 - (1-\beta)v_1$, while the element c_{23} of $\mathbf{C}$ must be replaced by $c^\star_{23} = \beta$.

with

$$\begin{array}{lll} b_{11} = a_1(1-\Phi) & b_{12} = (a_2 + a_3 l_2)\alpha & b_{13} = -a_3 l_2 \\ b_{22} = -\alpha & b_{23} = 1 & b_{33} = \mu(1-\omega) \\ b_{44} = 1 & b_{55} = 1 & b_{66} = 1 \end{array}$$

and

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & 0 & c_{14} & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & c_{25} & 0 \\ c_{31} & c_{32} & c_{33} & 0 & c_{35} & c_{36} \\ c_{41} & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{63} & 0 & 0 & 0 \end{pmatrix} \tag{A7}$$

with

$$\begin{array}{lll} c_{11} = 1 + b_1 - a_3 l_1 & c_{12} = (a_2 + a_3 l_2)\alpha + b_3 - (1 - a_3 l_1)\psi & c_{14} = -a_1\Phi \\ c_{21} = (1-\beta)v_2 & c_{22} = -\big((1+\beta)\alpha + (1-\beta)v_2\psi\big) & c_{23} = \beta + (1-\beta)v_1 \\ c_{25} = \alpha\beta & c_{31} = -\mu\delta & c_{32} = 1 - \mu\alpha + \mu\delta\psi \\ c_{33} = \mu & c_{35} = -(1-\mu\alpha) & c_{36} = -\mu\omega \\ c_{41} = 1 & c_{52} = 1 & c_{63} = 1 \end{array}$$

The input vector $\mathbf{k}_t = (k_{1t}, k_{2t}, k_{3t}, 0, 0, 0)'$ contains the exogenous or forcing functions

$$\begin{aligned} k_{1t} &= (1 - a_3 l_1)\psi(p^*_{R,t} - p^*_t) + (a_2 + a_3 l_2)(i^*_t - \pi^*_{t+1}) \\ &\quad - b_2 y^*_t - g_t + (1 - a_3 l_1)d_0 - a_3 l_0 - b_0 \end{aligned} \tag{A8}$$

$$\begin{aligned} k_{2t} &= (1-\beta)v_2\psi(p^*_{R,t} - p^*_t) + \pi^*_{t+1} - \beta\pi^*_t - i^*_t + \beta i^*_{t-1} \\ &\quad + (1-\beta)\bar{i} - (1-\beta)v_2(\bar{q} - d_0) - (1-\beta)v_1\bar{\pi}^c \end{aligned} \tag{A9}$$

$$k_{3t} = -\mu\delta\psi(p^*_{R,t} - p^*_t) - (1-\mu)(\pi^*_{R,t} - \pi^*_t) + \mu\delta(\bar{q} - d_0) \tag{A10}$$

We analyze the dynamic effects of anticipated raw materials price shocks which take the form

$$p^*_{R,t} = \beta_R \cdot p^*_{R,t-1} + \kappa_t \quad , \quad 0 \le \beta_R \le 1 \tag{A11}$$

with the impulse function

$$\kappa_t = \begin{cases} 1 & \text{for } t = T > 0 \\ 0 & \text{for } t \neq T \end{cases} \tag{A12}$$

On the assumption $p^*_{R,0} = 0$ for the initial value of p^*_R, the solution of the difference equation (A11) is given by

$$p^*_{R,t} = \begin{cases} 0 & \text{for } t < T \\ \beta_R^{t-T} & \text{for } t \ge T \end{cases} \tag{A13}$$

Then

$$\pi^*_{R,t} = p^*_{R,t} - p^*_{R,t-1} = \begin{cases} 0 & \text{for } t < T \\ p^*_{R,T} = 1 & \text{for } t = T \\ (\beta_R - 1)\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A14}$$

We assume that an oil price shock of the form (A11) influences the foreign price level p^*, but leaves the foreign income and the foreign interest rate unchanged:

$$p^*_t = (1 - \mu^*)p^*_{R,t} \qquad 0 < \mu^* < 1 \tag{A15}$$

$$i^*_t = \overline{i}^* \,, \qquad y^*_t = \overline{y}^* \tag{A16}$$

Then

$$\pi^*_t = (1 - \mu^*)\pi^*_{R,t} = \begin{cases} 0 & \text{for } t < T \\ (1 - \mu^*) & \text{for } t = T \\ (1 - \mu^*)(\beta_R - 1)\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A17}$$

implying

$$p^*_{R,t} - p^*_t = \mu^* p^*_{R,t} = \begin{cases} 0 & \text{for } t < T \\ \mu^* \beta_R^{t-T} & \text{for } t \geq T \end{cases} \tag{A18}$$

and

$$\pi^*_{R,t} - \pi^*_t = \mu^* \pi^*_{R,t} = \begin{cases} 0 & \text{for } t < T \\ \mu^* & \text{for } t = T \\ \mu^*(\beta_R - 1)\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A19}$$

In the case of a permanent raw materials price shock, i.e. $\beta_R = 1$, equations (A17) to (A19) simplify to

$$\pi^*_t = \begin{cases} 1 - \mu^* & \text{for } t = T \\ 0 & \text{for } t \neq T \end{cases} \qquad \text{if } \beta_R = 1 \tag{A20}$$

$$p^*_{R,t} - p^*_t = \begin{cases} 0 & \text{for } t < T \\ \mu^* & \text{for } t \geq T \end{cases} \qquad \text{if } \beta_R = 1 \tag{A21}$$

$$\pi^*_{R,t} - \pi^*_t = \begin{cases} \mu^* & \text{for } t = T \\ 0 & \text{for } t \neq T \end{cases} \qquad \text{if } \beta_R = 1 \tag{A22}$$

The input functions k_{1t}, k_{2t} and k_{3t} can be rewritten with the help of the steady state equations

$$\overline{q} = a_1\overline{y} - a_2\overline{i}^* + a_3(l_0 + l_1\overline{q} - l_2\overline{i}^*) + \overline{g} - b_1\overline{y} + b_2\overline{y}^* - b_3\overline{\tau} + b_0 \tag{A23}$$

$$\overline{y} = \overline{q} + \psi\overline{\tau} - \psi(\overline{p^*_R - p^*}) - d_0 \tag{A24}$$

$$\overline{q} = (f_1 + f_2)\overline{\tau} - f_2(\overline{p^*_R - p^*}) + f_0 \tag{A25}$$

(A23) is the steady state version of the IS equation (1) where we have used that in the long run $y_{t+1} = y_t = y_{t-1} = \overline{y}$, $\tau_{t+1} = \tau_t = \tau_{t-1} = \overline{\tau}$ and $\pi^c_{t+1} = \pi^c_t = \pi^c_{t-1} = \overline{\pi}^c = 0$ holds (implying $\Delta\overline{e} = 0$). (A24) is the steady state relationship between national income and output while (A25) is a reformulation of the long run supply function (6). Equations (A23) and (A24) imply

$$
\begin{aligned}
&(1-a_1+b_1-a_3l_1)\overline{y}+(b_3-(1-a_3l_1)\psi)\overline{\tau}= && \text{(A26)}\\
&\quad -(a_2+a_3l_2)\overline{i}^*+b_2\overline{y}^*+\overline{g}-(1-a_3l_1)\psi(\overline{p_R^*-p^*})\\
&\quad -(1-a_3l_1)d_0+a_3l_0+b_0\\
&\overline{y}-\psi\overline{\tau}+\psi(\overline{p_R^*-p^*})=\overline{q}-d_0 && \text{(A27)}
\end{aligned}
$$

Using (A26) and (A27) and the long run interest parity condition $\overline{i}=\overline{i}^*$ the forcing functions (A8) to (A10) can be written as

$$
\begin{aligned}
k_{1t}&=(1-a_3l_1)\psi\big[(p_{R,t}^*-p_t^*)-(\overline{p_R^*-p^*})\big]-(a_2+a_3l_2)\pi_{t+1}^* && \text{(A28)}\\
&\quad-(1-a_1+b_1-a_3l_1)\overline{y}-(b_3-(1-a_3l_1)\psi)\overline{\tau}\\
k_{2t}&=(1-\beta)v_2\psi\big[(p_{R,t}^*-p_t^*)-(\overline{p_R^*-p^*})\big]+\pi_{t+1}^*-\beta\pi_t^* && \text{(A29)}\\
&\quad-(1-\beta)v_2(\overline{y}-\psi\overline{\tau})\\
k_{3t}&=-\mu\delta\psi\big[(p_{R,t}^*-p_t^*)-(\overline{p_R^*-p^*})\big]-(1-\mu)(\pi_{R,t}^*-\pi_t^*) && \text{(A30)}\\
&\quad+\mu\delta(\overline{y}-\psi\overline{\tau})
\end{aligned}
$$

The foreign real raw materials price $p_R^*-p^*$ is exogenously given for the domestic small open economy. If $\overline{\mathbf{v}}_0$ denotes the initial and $\overline{\mathbf{v}}_1$ the new steady state level of the state vector $\mathbf{v}$ after the occurrence of an oil price shock, $\overline{\mathbf{v}}_1$ differs from $\overline{\mathbf{v}}_0$ only if $\beta_R=1$, i.e, if a permanent shock takes place. In this case

$$
d(\overline{p_R^*-p^*})=(\overline{p_R^*-p^*})_1-(\overline{p_R^*-p^*})_0=\mu^* \qquad \text{(A31)}
$$

and the steady state change of y and τ follows from the steady state equations (A23) to (A25). The long run multipliers $\partial\overline{y}/\partial(\overline{p_R^*-p^*})$ and $\partial\overline{\tau}/\partial(\overline{p_R^*-p^*})$ result from the set of equations

$$
\begin{pmatrix} 1-a_1+b_1-a_3l_1 & b_3-(1-a_3l_1)\psi \\ 1 & -(f_1+f_2+\psi) \end{pmatrix}\begin{pmatrix} d\overline{y} \\ d\overline{\tau} \end{pmatrix}=-\begin{pmatrix} (1-a_3l_1)\psi \\ f_2+\psi \end{pmatrix} d(\overline{p_R^*-p^*}) \qquad \text{(A32)}
$$

Using the abbreviation

$$
\Delta=-(1-a_1+b_1-a_3l_1)(f_1+f_2+\psi)-(b_3-(1-a_3l_1)\psi) \qquad \text{(A33)}
$$

the multipliers are given by

$$
\begin{aligned}
\frac{\partial\overline{y}}{\partial(\overline{p_R^*-p^*})}&=\frac{1}{\Delta}\big[(f_1+f_2+\psi)(1-a_3l_1)\psi && \text{(A34)}\\
&\qquad+(b_3-(1-a_3l_1)\psi)(f_2+\psi)\big]\\
\frac{\partial\overline{\tau}}{\partial(\overline{p_R^*-p^*})}&=\frac{1}{\Delta}\big((1-a_3l_1)\psi-(1-a_1+b_1-a_3l_1)(f_2+\psi)\big) && \text{(A35)}
\end{aligned}
$$

On the assumption

$$
b_3-(1-a_3l_1)\psi>0 \qquad \text{(A36)}
$$

the long run IS curve (A26) has a negative slope in $\overline{y}/\overline{\tau}$-space implying that a permanent real appreciation of the domestic currency causes isolated seen a long run decline in aggregate demand and national income. Then the multiplier $\partial\overline{y}/\partial(\overline{p_R^* - p^*})$ is unambiguously negative. The long run response of the terms of trade to a permanent rise in the foreign real factor price $p_R^* - p^*$ is not uniquely determined. Assuming (A36), a necessary and sufficient condition for a rise (fall) in $\overline{\tau}$ is given by

$$\frac{\partial\overline{\tau}}{\partial(\overline{p_R^* - p^*})} > (<)0 \quad \Leftrightarrow \quad (a_1 - b_1)\psi < (>)(1 - a_1 + b_1 - a_3 l_1) f_2 \tag{A37}$$

implying a stronger (weaker) increase in the domestic than the foreign price level (the latter expressed in units of the domestic currency).

The input functions k_{1t}, k_{2t} and k_{3t} can be written in the form

$$k_{jt} = \overline{d}_j + \phi_{jt} \qquad j = 1, 2, 3 \tag{A38}$$

where

$$\overline{d}_1 = -(1 - a_1 + b_1 - a_3 l_1)\overline{y} - (b_3 - (1 - a_3 l_1)\psi)\overline{\tau} \tag{A39}$$
$$\overline{d}_2 = -(1 - \beta) v_2 (\overline{y} - \psi\overline{\tau}) \tag{A40}$$
$$\overline{d}_3 = \mu\delta(\overline{y} - \psi\overline{\tau}) \tag{A41}$$

and

$$\phi_{1t} = (1 - a_3 l_1)\psi\big((p_{R,t}^* - p_t^*) - (\overline{p_R^* - p^*})\big) - (a_2 + a_3 l_2)\pi_{t+1}^* \tag{A42}$$
$$\phi_{2t} = (1 - \beta) v_2 \psi\big((p_{R,t}^* - p_t^*) - (\overline{p_R^* - p^*})\big) + \pi_{t+1}^* - \beta\pi_t^* \tag{A43}$$
$$\phi_{3t} = -\mu\delta\psi\big((p_{R,t}^* - p_t^*) - (\overline{p_R^* - p^*})\big) - (1 - \mu)(\pi_{R,t}^* - \pi_t^*) \tag{A44}$$

For $t < T$ the steady state values $\overline{y}$, $\overline{\tau}$ and $(\overline{p_R^* - p^*})$ are equal to their initial values $\overline{y}_0$, $\overline{\tau}_0$ and $(\overline{p_R^* - p^*})_0$ while for $t \geq T$ they coincide with their new steady state values $\overline{y}_1$, $\overline{\tau}_1$ and $(\overline{p_R^* - p^*})_1$. Therefore,

$$\overline{d}_j = \begin{cases} \overline{d}_{j0} & \text{for } t < T \\ \overline{d}_{j1} & \text{for } t \geq T \end{cases} \qquad j = 1, 2, 3 \tag{A45}$$

According to (A13) to (A22) the functions ϕ_{1t}, ϕ_{2t} and ϕ_{3t} take the following form:

- In case $\beta_R = 1$ (permanent raw materials price shock)

$$\phi_{1t} = \begin{cases} 0 & \text{for } t < T - 1 \\ -(a_2 + a_3 l_2)\pi_{t+1}^* = -(a_2 + a_3 l_2)(1 - \mu^*) & \text{for } t = T - 1 \\ 0 & \text{for } t \geq T \end{cases} \tag{A46}$$

$$\phi_{2t} = \begin{cases} 0 & \text{for } t < T-1 \\ \pi^*_{t+1} = 1-\mu^* & \text{for } t = T-1 \\ -\beta\pi^*_t = -\beta(1-\mu^*) & \text{for } t = T \\ 0 & \text{for } t > T \end{cases} \tag{A47}$$

$$\phi_{3t} = \begin{cases} 0 & \text{for } t < T-1 \\ -(1-\mu)(\pi^*_{R,t} - \pi^*_t) = -(1-\mu)\mu^* & \text{for } t = T \\ 0 & \text{for } t > T \end{cases} \tag{A48}$$

- In case $\beta_R < 1$ (temporary shock) we get

$$\phi_{1t} = \begin{cases} 0 & \text{for } t < T-1 \\ -(a_2 + a_3 l_2)(1-\mu^*) & \text{for } t = T-1 \\ \big[(1-a_3 l_1)\psi\mu^* & \\ \quad -(a_2 + a_3 l_2)(1-\mu^*)(\beta_R - 1)\big]\beta_R^{t-T} & \text{for } t \geq T \end{cases} \tag{A49}$$

$$\phi_{2t} = \begin{cases} 0 & \text{for } t < T-1 \\ 1-\mu^* & \text{for } t = T-1 \\ (1-\beta)v_2\psi\mu^* + (1-\mu^*)(\beta_R - 1) - \beta(1-\mu^*) & \text{for } t = T \\ \big[(1-\beta)v_2\psi\mu^* + (1-\mu^*)(\beta_R - 1)\big]\beta_R^{t-T} & \\ \quad -\beta(1-\mu^*)(\beta_R - 1)\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A50}$$

$$\phi_{3t} = \begin{cases} 0 & \text{for } t < T \\ -\mu\delta\psi\mu^* - (1-\mu)\mu^* & \text{for } t = T \\ -\mu\delta\psi\mu^*\beta_R^{t-T} - (1-\mu)\mu^*(\beta_R - 1)\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A51}$$

The state equations (A5) can be rewritten as follows:

$$\begin{pmatrix} \mathbf{v}_{t+1} \\ \mathbf{w}_{t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \mathbf{v}_t \\ \mathbf{w}_t \end{pmatrix} + \mathbf{B}^{-1}(\overline{\mathbf{d}} + \boldsymbol{\phi}_t) \tag{A52}$$

where

$$\mathbf{A} = \mathbf{B}^{-1}\mathbf{C} \tag{A53}$$

$$\overline{\mathbf{d}} = (\overline{d}_1, \overline{d}_2, \overline{d}_3, 0, 0, 0)' \tag{A54}$$

$$\boldsymbol{\phi}_t = (\phi_{1t}, \phi_{2t}, \phi_{3t}, 0, 0, 0)' \tag{A55}$$

The inverse matrix $\mathbf{B}^{-1}$ has the structure

$$\mathbf{B}^{-1} = \begin{pmatrix} \widetilde{b}_{11} & \widetilde{b}_{12} & \widetilde{b}_{13} & 0 & 0 & 0 \\ \widetilde{b}_{21} & \widetilde{b}_{22} & \widetilde{b}_{23} & 0 & 0 & 0 \\ \widetilde{b}_{31} & \widetilde{b}_{32} & \widetilde{b}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{A56}$$

where

$$\widetilde{b}_{11} = -\frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega) \qquad \widetilde{b}_{12} = -\frac{1}{|\mathbf{B}|}(a_2 + a_3 l_2)\alpha\mu(1-\omega) \tag{A57}$$
$$\widetilde{b}_{13} = \frac{1}{|\mathbf{B}|}a_2\alpha \qquad \widetilde{b}_{21} = 0$$
$$\widetilde{b}_{22} = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\mu(1-\omega) \qquad \widetilde{b}_{23} = -\frac{1}{|\mathbf{B}|}a_1(1-\Phi)$$
$$\widetilde{b}_{31} = \widetilde{b}_{32} = 0 \qquad \widetilde{b}_{33} = -\frac{1}{|\mathbf{B}|}a_1(1-\Phi)\alpha$$

and

$$|\mathbf{B}| = \det\mathbf{B} = -a_1(1-\Phi)\alpha\mu(1-\omega) < 0 \tag{A58}$$

Then

$$\mathbf{B}^{-1}\overline{\mathbf{d}} = \left(\sum_{j=1}^{3}\widetilde{b}_{1j}\overline{d}_j, \sum_{j=1}^{3}\widetilde{b}_{2j}\overline{d}_j, \sum_{j=1}^{3}\widetilde{b}_{3j}\overline{d}_j, 0, 0, 0\right)' \tag{A59}$$

and $\mathbf{B}^{-1}\boldsymbol{\phi}_t$ is given by

$$\mathbf{B}^{-1}\boldsymbol{\phi}_t = \left(\sum_{j=1}^{3}\widetilde{b}_{1j}\phi_{jt}, \sum_{j=1}^{3}\widetilde{b}_{2j}\phi_{jt}, \sum_{j=1}^{3}\widetilde{b}_{3j}\phi_{jt}, 0, 0, 0\right)' \tag{A60}$$

In case $\beta_R = 1$ (permanent shock) (A60) takes the form

$$\mathbf{B}^{-1}\boldsymbol{\phi}_t = \begin{cases} 0 & \text{for } t < T-1 \\ (0, \widetilde{\alpha}_2, 0, 0, 0, 0)' & \text{for } t = T-1 \\ (\alpha_1, \alpha_2, \alpha_3, 0, 0, 0)' & \text{for } t = T \\ 0 & \text{for } t > T \end{cases} \tag{A61}$$

where

$$\widetilde{\alpha}_2 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\mu(1-\omega)(1-\mu^*) \tag{A62}$$
$$\alpha_1 = \frac{1}{|\mathbf{B}|}\big[(a_2 + a_3 l_2)\alpha\mu(1-\omega)\beta(1-\mu^*) - a_2\alpha(1-\mu)\mu^*\big] \tag{A63}$$
$$\alpha_2 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\big[-\mu(1-\omega)\beta(1-\mu^*) + (1-\mu)\mu^*\big] \tag{A64}$$
$$\alpha_3 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\alpha(1-\mu)\mu^* \tag{A65}$$

If $\beta_R < 1$ (temporary shock), $\mathbf{B}^{-1}\boldsymbol{\phi}_t$ is given by

$$\mathbf{B}^{-1}\boldsymbol{\phi}_t = \begin{cases} 0 & \text{for } t < T-1 \\ (0, \widetilde{\alpha}_2, 0, 0, 0, 0)' & \text{for } t = T-1 \\ (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, 0, 0, 0)' & \text{for } t = T \\ (\overline{\phi}_1, \overline{\phi}_2, \overline{\phi}_3, 0, 0, 0)'\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A66}$$

where $\widetilde{\alpha}_2$ is defined in (A62) and

$$\hat{\alpha}_1 = \frac{1}{|\mathbf{B}|}\Big[\big(- \alpha\mu(1-\omega)\psi\big((1-a_3l_1) + (a_2+a_3l_2)(1-\beta)v_2\big) \tag{A67}$$

$$- a_2\alpha(\mu\delta\psi + 1 - \mu)\big)\mu^* + \alpha\mu(1-\omega)(a_2+a_3l_2)\beta(1-\mu^*)\Big]$$

$$\hat{\alpha}_2 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\Big[\big(\mu(1-\omega)(1-\beta)v_2\psi + \mu\delta\psi + 1 - \mu\big)\mu^* \tag{A68}$$

$$+ \mu(1-\omega)(1-\mu^*)(\beta_R - 1 - \beta)\Big]$$

$$\hat{\alpha}_3 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\alpha(\mu\delta\psi + 1 - \mu)\mu^* \tag{A69}$$

$$\overline{\phi}_1 = -\frac{1}{|\mathbf{B}|}\Big[\big(\alpha\mu(1-\omega)\big[1 - a_3l_1 + (a_2+a_3l_2)(1-\beta)v_2\big]\psi\beta_R \tag{A70}$$

$$+ a_2\alpha\big(\mu\delta\psi\beta_R + (1-\mu)(\beta_R - 1)\big)\big)\mu^*$$

$$- \alpha\mu(1-\omega)(a_2+a_3l_2)\beta(1-\mu^*)(\beta_R - 1)\Big]$$

$$\overline{\phi}_2 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\Big[\big(\mu(1-\omega)(1-\beta)v_2\psi\beta_R + \mu\delta\psi\beta_R \tag{A71}$$

$$+ (1-\mu)(\beta_R - 1)\big)\mu^* + \mu(1-\omega)(1-\mu^*)(\beta_R - 1)(\beta_R - \beta)\Big]$$

$$\overline{\phi}_3 = \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\alpha\big[\mu\delta\psi\beta_R + (1-\mu)(\beta_R - 1)\big]\mu^* \tag{A72}$$

Solution to Dynamics

The dynamical system (A52) can be solved by transforming it into canonical form using the Jordan decomposition of the system matrix $\mathbf{A} = \mathbf{B}^{-1}\mathbf{C}$. $\mathbf{A}$ has six different eigenvalues $r_1, \ldots, r_6$, where r_1, r_2 and r_3 are unstable (i.e., $|r_j| > 1$ for $j = 1, 2, 3$) and r_4, r_5 and r_6 are stable characteristic roots (i.e., $|r_i| < 1$ for $i = 4, 5, 6$). Since the vector $\mathbf{v}$ of state variables only consists of non-predetermined variables, the number of unstable characteristic roots coincides with the number of jump variables so that the system (A52) has the saddlepath property. The system matrix $\mathbf{A}$ can be diagonalized by the similarity transformation

$$\mathbf{A} = \mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^{-1} = \mathbf{H}\boldsymbol{\Lambda}\mathbf{G} \qquad (\mathbf{G} = \mathbf{H}^{-1}) \tag{A73}$$

where $\mathbf{H} = (h_1, \ldots, h_6)$ consists of the linear-independent (right-) eigenvectors of $\mathbf{A}$ and $\mathbf{G} = (g_{ij})_{1 \le i,j \le 6}$ denotes the inverse of $\mathbf{H}$. $\boldsymbol{\Lambda}$ is a diagonal matrix whose diagonal elements are the eigenvalues of $\mathbf{A}$. Partition the matrices $\mathbf{H}$, $\boldsymbol{\Lambda}$ and $\mathbf{G}$ conformably with the state vectors $\mathbf{v}$ and $\mathbf{w}$,

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{pmatrix}, \qquad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_1 & 0 \\ 0 & \boldsymbol{\Lambda}_2 \end{pmatrix}, \qquad \mathbf{G} = \begin{pmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{pmatrix} \tag{A74}$$

and premultiplying both sides of the state equations (A52) with $\mathbf{G}$. This yields

$$\begin{pmatrix}\mathbf{G}_{11} & \mathbf{G}_{12}\\ \mathbf{G}_{21} & \mathbf{G}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{v}_{t+1}\\ \mathbf{w}_{t+1}\end{pmatrix} = \begin{pmatrix}\mathbf{\Lambda}_1 & 0\\ 0 & \mathbf{\Lambda}_2\end{pmatrix}\begin{pmatrix}\mathbf{G}_{11} & \mathbf{G}_{12}\\ \mathbf{G}_{21} & \mathbf{G}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{v}_{t}\\ \mathbf{w}_{t}\end{pmatrix} + \begin{pmatrix}\mathbf{G}_{11} & \mathbf{G}_{12}\\ \mathbf{G}_{21} & \mathbf{G}_{22}\end{pmatrix}\left[\mathbf{B}^{-1}\overline{\mathbf{d}} + \mathbf{B}^{-1}\boldsymbol{\phi}_t\right] \tag{A75}$$

Using the transformation

$$\begin{pmatrix}\mathbf{x}\\ \mathbf{z}\end{pmatrix} = \begin{pmatrix}\mathbf{G}_{11} & \mathbf{G}_{12}\\ \mathbf{G}_{21} & \mathbf{G}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{v}\\ \mathbf{w}\end{pmatrix} \tag{A76}$$

the canonical form of the state equations (A52) is given by

$$\mathbf{x}_{t+1} = \mathbf{\Lambda}_1\mathbf{x}_t + (\mathbf{G}_{11},\mathbf{G}_{12})\left[\mathbf{B}^{-1}\overline{\mathbf{d}} + \mathbf{B}^{-1}\boldsymbol{\phi}_t\right] \tag{A77}$$

$$\mathbf{z}_{t+1} = \mathbf{\Lambda}_2\mathbf{z}_t + (\mathbf{G}_{21},\mathbf{G}_{22})\left[\mathbf{B}^{-1}\overline{\mathbf{d}} + \mathbf{B}^{-1}\boldsymbol{\phi}_t\right] \tag{A78}$$

If $\{\mathbf{x}_t\}_{t=0,1,\ldots,T-1,T,T+1,\ldots}$ and $\{\mathbf{z}_t\}_{t=0,1,\ldots,T-1,T,T+1,\ldots}$ is a solution of (A77) and (A78) respectively, the solution of the original state variables $\mathbf{v}$ and $\mathbf{w}$ can be obtained by using the inverse transformation

$$\begin{pmatrix}\mathbf{v}\\ \mathbf{w}\end{pmatrix} = \begin{pmatrix}\mathbf{H}_{11} & \mathbf{H}_{12}\\ \mathbf{H}_{21} & \mathbf{H}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{x}\\ \mathbf{z}\end{pmatrix} \tag{A79}$$

Note that the steady state of the canonical system (A77), (A78) is given by

$$\begin{pmatrix}\overline{\mathbf{x}}\\ \overline{\mathbf{z}}\end{pmatrix} = \begin{pmatrix}(\mathbf{I}_3-\mathbf{\Lambda}_1)^{-1} & 0\\ 0 & (\mathbf{I}_3-\mathbf{\Lambda}_2)^{-1}\end{pmatrix}\mathbf{G}\mathbf{B}^{-1}\overline{\mathbf{d}} \tag{A80}$$

where $\mathbf{I}_3$ is the 3×3 identity matrix and $\overline{\mathbf{d}}$ defined in (A54), (A45), (A39), (A40), (A41). According to (A79) and (A52) the steady state of the original state vector is then given by

$$\begin{pmatrix}\overline{\mathbf{v}}\\ \overline{\mathbf{w}}\end{pmatrix} = \mathbf{H}\begin{pmatrix}(\mathbf{I}_3-\mathbf{\Lambda}_1)^{-1} & 0\\ 0 & (\mathbf{I}_3-\mathbf{\Lambda}_2)^{-1}\end{pmatrix}\mathbf{G}\mathbf{B}^{-1}\overline{\mathbf{d}} = (\mathbf{I}_6-\mathbf{A})^{-1}\mathbf{B}^{-1}\overline{\mathbf{d}} \tag{A81}$$

since

$$\begin{aligned}(\mathbf{I}_6-\mathbf{A})^{-1} &= (\mathbf{I}_6-\mathbf{H}\mathbf{\Lambda}\mathbf{G})^{-1} = [\mathbf{H}(\mathbf{I}_6-\mathbf{\Lambda})\mathbf{G}]^{-1}\\ &= \mathbf{G}^{-1}(\mathbf{I}_6-\mathbf{\Lambda})^{-1}\mathbf{H}^{-1} = \mathbf{H}\begin{pmatrix}(\mathbf{I}_3-\mathbf{\Lambda}_1)^{-1} & 0\\ 0 & (\mathbf{I}_3-\mathbf{\Lambda}_2)^{-1}\end{pmatrix}\mathbf{G}\end{aligned} \tag{A82}$$

Solution in case $\beta_R < 1$
We first consider the solution of the transformed system (A77), (A78) in case of temporary raw materials price shocks. In this case $\overline{\mathbf{d}} = \overline{\mathbf{d}}_0 = \overline{\mathbf{d}}_1$ implying $\overline{\mathbf{x}} = \overline{\mathbf{x}}_0 = \overline{\mathbf{x}}_1$ and $\overline{\mathbf{z}} = \overline{\mathbf{z}}_0 = \overline{\mathbf{z}}_1$ so that (A77), (A78) is equivalent to

$$\mathbf{x}_{t+1} - \overline{\mathbf{x}} = \mathbf{\Lambda}_1(\mathbf{x}_t - \overline{\mathbf{x}}) + (\mathbf{G}_{11},\mathbf{G}_{12})\mathbf{B}^{-1}\boldsymbol{\phi}_t \tag{A83}$$

$$\mathbf{z}_{t+1} - \overline{\mathbf{z}} = \mathbf{\Lambda}_2(\mathbf{z}_t - \overline{\mathbf{z}}) + (\mathbf{G}_{21},\mathbf{G}_{22})\mathbf{B}^{-1}\boldsymbol{\phi}_t \tag{A84}$$

Let $\boldsymbol{\theta}_{1t}$ and $\boldsymbol{\theta}_{2t}$ be the input functions

$$\boldsymbol{\theta}_{1t} = (\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\boldsymbol{\phi}_t \tag{A85}$$

$$\boldsymbol{\theta}_{2t} = (\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\boldsymbol{\phi}_t \tag{A86}$$

According to (A66) $\boldsymbol{\theta}_{1t}$ and $\boldsymbol{\theta}_{2t}$ have the following structure:

$$\boldsymbol{\theta}_{1t} = \begin{cases} 0 & \text{for } t < T-1 \\ (\widetilde{g}_1, \widetilde{g}_2, \widetilde{g}_3)' & \text{for } t = T-1 \\ (\hat{e}_1, \hat{e}_2, \hat{e}_3)' & \text{for } t = T \\ (\overline{\varphi}_1, \overline{\varphi}_2, \overline{\varphi}_3)'\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A87}$$

$$\boldsymbol{\theta}_{2t} = \begin{cases} 0 & \text{for } t < T-1 \\ (\widetilde{g}_4, \widetilde{g}_5, \widetilde{g}_6)' & \text{for } t = T-1 \\ (\hat{e}_4, \hat{e}_5, \hat{e}_6)' & \text{for } t = T \\ (\overline{\varphi}_4, \overline{\varphi}_5, \overline{\varphi}_6)'\beta_R^{t-T-1} & \text{for } t > T \end{cases} \tag{A88}$$

where

$$\widetilde{g}_j = g_{j2}\widetilde{\alpha}_2 \qquad (j = 1, \ldots, 6) \tag{A89}$$

$$\hat{e}_j = g_{j1}\hat{\alpha}_1 + g_{j2}\hat{\alpha}_2 + g_{j3}\hat{\alpha}_3 \qquad (j = 1, \ldots, 6) \tag{A90}$$

$$\overline{\varphi}_j = g_{j1}\overline{\phi}_1 + g_{j2}\overline{\phi}_2 + g_{j3}\overline{\phi}_3 \qquad (j = 1, \ldots, 6) \tag{A91}$$

Let us first consider the subsystem (A83) where the system matrix $\boldsymbol{\Lambda}_1$ only consists of unstable eigenvalues. We are interested in the convergent solution time path so that the transversality condition

$$\lim_{t\to\infty}(\mathbf{x}_t - \overline{\mathbf{x}}) = 0 \tag{A92}$$

must hold. We start with the general solution of (A83). We can choose either the backward-looking or the forward-looking solution and then apply the stability condition (A92). The general backward-looking solution of (A83) is given by

$$\mathbf{x}_t - \overline{\mathbf{x}} = \boldsymbol{\Lambda}_1^t\mathbf{K}_1 + \sum_{s=0}^{t-1}\boldsymbol{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} = \boldsymbol{\Lambda}_1^t\mathbf{K}_1 + \sum_{s=T-1}^{t-1}\boldsymbol{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} \tag{A93}$$

where $\mathbf{K}_1$ is an arbitrary three-dimensional vector of constants and

$$\sum_{s=T-1}^{t-1}\boldsymbol{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} = 0 \qquad \text{for } t-1 < T-1\text{, i.e. } t < T \tag{A94}$$

Therefore

$$\mathbf{x}_t = \begin{cases} \overline{\mathbf{x}}_0 + \boldsymbol{\Lambda}_1^t\mathbf{K}_1 & \text{for } t < T \\ \overline{\mathbf{x}}_0 + \boldsymbol{\Lambda}_1^t\mathbf{K}_1 + \sum_{s=T-1}^{t-1}\boldsymbol{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} & \\ = \overline{\mathbf{x}}_0 + \boldsymbol{\Lambda}_1^t\left(\mathbf{K}_1 + \sum_{s=T-1}^{t-1}\boldsymbol{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s}\right) & \text{for } t \geq T \end{cases} \tag{A95}$$

The transversality condition (A92) is satisfied only if

$$\mathbf{K}_1 + \sum_{s=T-1}^{\infty} \mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s} = 0 \tag{A96}$$

implying

$$\mathbf{K}_1 = -\sum_{s=T-1}^{\infty} \mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s} \tag{A97}$$

Thus

$$\mathbf{x}_t = \overline{\mathbf{x}}_0 - \mathbf{\Lambda}_1^t \sum_{s=t}^{\infty} \mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s} = \overline{\mathbf{x}}_0 - \sum_{s=t}^{\infty} \mathbf{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} \qquad \text{for } t \geq T \tag{A98}$$

The same convergent solution can be obtained with the help of the general forward-looking solution

$$\begin{aligned} \mathbf{x}_t - \overline{\mathbf{x}} &= \mathbf{\Lambda}_1^t \widetilde{\mathbf{K}}_1 - \sum_{s=t}^{\infty} \mathbf{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} \qquad\qquad (\text{A99}) \\ &= \begin{cases} \mathbf{\Lambda}_1^t \widetilde{\mathbf{K}}_1 - \sum_{s=T-1}^{\infty} \mathbf{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} & \text{for } t < T-1 \\ \mathbf{\Lambda}_1^t \widetilde{\mathbf{K}}_1 - \sum_{s=t}^{\infty} \mathbf{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} & \text{for } t \geq T-1 \end{cases} \\ &= \begin{cases} \mathbf{\Lambda}_1^t (\widetilde{\mathbf{K}}_1 + \mathbf{K}_1) & \text{for } t < T-1 \\ \mathbf{\Lambda}_1^t (\widetilde{\mathbf{K}}_1 - \sum_{s=t}^{\infty} \mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s}) & \text{for } t \geq T-1 \end{cases} \end{aligned}$$

where the constant $\widetilde{\mathbf{K}}_1$ is arbitrary and $\mathbf{K}_1$ defined by (A97). Equation (A99) will converge only if

$$\widetilde{\mathbf{K}}_1 = 0 \tag{A100}$$

implying the equivalence of the uniquely determined convergent forward- and backward-looking solution. The equivalence also holds at time $t = T-1$, since (A95) and (A97) imply

$$\mathbf{x}_{T-1} - \overline{\mathbf{x}}_0 = \mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 = -\mathbf{\Lambda}_1^{T-1} \sum_{s=T-1}^{\infty} \mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s} = -\sum_{s=T-1}^{\infty} \mathbf{\Lambda}_1^{T-s-2}\boldsymbol{\theta}_{1s} \tag{A101}$$

where the last expression is the convergent forward-looking solution at time $T-1$. Using the definition of the input function $\boldsymbol{\theta}_{1t}$ the constant $\mathbf{K}_1$ can be written as

$$\mathbf{K}_1 = -\mathbf{\Lambda}_1^{-T} \left[\begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \widetilde{g}_3 \end{pmatrix} + \mathbf{\Lambda}_1^{-1} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} + \mathbf{\Lambda}_1^{-1} \begin{pmatrix} \overline{\varphi}_1/(r_1 - \beta_R) \\ \overline{\varphi}_2/(r_2 - \beta_R) \\ \overline{\varphi}_3/(r_3 - \beta_R) \end{pmatrix} \right] \tag{A102}$$

The last expression follows from the fact that for $j = 1, 2, 3$

$$\begin{aligned} -\sum_{s=T+1}^{\infty} r_j^{-s-1}\overline{\varphi}_j \beta_R^{s-T-1} &= -\overline{\varphi}_j \frac{1}{r_j} \frac{1}{\beta_R^{T+1}} \sum_{s=T+1}^{\infty} \left(\frac{\beta_R}{r_j}\right)^s \qquad\qquad (\text{A103}) \\ &= -\overline{\varphi}_j \frac{1}{r_j} \frac{1}{\beta_R^{T+1}} \frac{\left(\frac{\beta_R}{r_j}\right)^{T+1}}{1 - \frac{\beta_R}{r_j}} = -\overline{\varphi}_j \frac{1}{r_j - \beta_R} r_j^{-(T+1)} \end{aligned}$$

holds.[13] This implies

$$-\sum_{s=T+1}^{\infty}\boldsymbol{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s} = -\boldsymbol{\Lambda}_1^{-T-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} \tag{A104}$$

(A95), (A98), (A101) and (A102) imply that the unique convergent solution time path of the vector $\mathbf{x}$ can be expressed in the following form:

- For $t \geq T+1$:

$$\mathbf{x}_t = \overline{\mathbf{x}}_0 - \sum_{s=t}^{\infty}\boldsymbol{\Lambda}_1^{t-s-1}\boldsymbol{\theta}_{1s} = \overline{\mathbf{x}}_0 - \begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\beta_R^{t-(T+1)} \tag{A105}$$

since

$$\begin{aligned}\sum_{s=t}^{\infty} r_j^{t-s-1}\overline{\varphi}_j\beta_R^{s-T-1} &= r_j^{t-1}\overline{\varphi}_j\frac{1}{\beta_R^{T+1}}\sum_{s=t}^{\infty}\left(\frac{\beta_R}{r_j}\right)^s \qquad (A106)\\ &= r_j^{t-1}\overline{\varphi}_j\frac{1}{\beta_R^{T+1}}\frac{\left(\frac{\beta_R}{r_j}\right)^t}{r_j-\beta_R}r_j\\ &= \overline{\varphi}_j\frac{1}{r_j-\beta_R}\beta_R^{t-(T+1)} \qquad (j=1,2,3)\end{aligned}$$

- For $t = T$:[14]

$$\begin{aligned}\mathbf{x}_T &= \overline{\mathbf{x}}_0 - \sum_{s=T}^{\infty}\boldsymbol{\Lambda}_1^{T-s-1}\boldsymbol{\theta}_{1s} = \overline{\mathbf{x}}_0 - \boldsymbol{\Lambda}_1^{-1}\boldsymbol{\theta}_{1T} - \sum_{s=T+1}^{\infty}\boldsymbol{\Lambda}_1^{T-s-1}\boldsymbol{\theta}_{1s} \qquad (A107)\\ &= \overline{\mathbf{x}}_0 - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\end{aligned}$$

since

$$\begin{aligned}\sum_{s=T+1}^{\infty} r_j^{T-s-1}\overline{\varphi}_j\beta_R^{s-T-1} &= r_j^{T}\sum_{s=T+1}^{\infty} r_j^{-s-1}\overline{\varphi}_j\beta_R^{s-T-1} \qquad (A108)\\ &= \overline{\varphi}_j\frac{1}{r_j-\beta_R}r_j^{-1} \qquad \text{(cf. (A103)).}\end{aligned}$$

[13]Note that

$$\sum_{s=t}^{\infty} x^s = \frac{x^t}{1-x} \qquad \text{for } |x|<1$$

$(x = \beta_R/r_j,\ t = T+1)$.

[14]Note that (A107) also follows from (A83) for $t = T$ by substituting (A105) for $\mathbf{x}_{T+1} - \overline{\mathbf{x}}$: Since

$$\mathbf{x}_{T+1} - \overline{\mathbf{x}}_0 = \boldsymbol{\Lambda}_1(\mathbf{x}_T - \overline{\mathbf{x}}_0) + \boldsymbol{\theta}_T\ ,$$

we get

$$\mathbf{x}_T - \overline{\mathbf{x}}_0 = \boldsymbol{\Lambda}_1^{-1}(\mathbf{x}_{T+1} - \overline{\mathbf{x}}_0) - \boldsymbol{\Lambda}_1^{-1}\boldsymbol{\theta}_T = -\boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix}$$

- For $t = T - 1$:[15]

$$\mathbf{x}_{T-1} = \overline{\mathbf{x}}_0 - \sum_{s=T-1}^{\infty} \mathbf{\Lambda}_1^{T-s-2}\boldsymbol{\theta}_{1s} \tag{A109}$$

$$= \overline{\mathbf{x}}_0 - \mathbf{\Lambda}_1^{-1}\boldsymbol{\theta}_{1T-1} - \mathbf{\Lambda}_1^{-2}\boldsymbol{\theta}_{1T} - \mathbf{\Lambda}_1^{T-1}\sum_{s=T+1}^{\infty}\mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s}$$

$$= \overline{\mathbf{x}}_0 - \mathbf{\Lambda}_1^{-1}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} - \mathbf{\Lambda}_1^{-2}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} - \mathbf{\Lambda}_1^{-2}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}$$

since

$$\mathbf{\Lambda}_1^{T-1}\sum_{s=T+1}^{\infty}\mathbf{\Lambda}_1^{-s-1}\boldsymbol{\theta}_{1s} = \mathbf{\Lambda}_1^{T-1}\mathbf{\Lambda}_1^{-T-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} \tag{A110}$$

$$= \mathbf{\Lambda}_1^{-2}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} \qquad \text{(cf. (A104))}.$$

- For $t < T - 1$:

$$\mathbf{x}_t = \overline{\mathbf{x}}_0 + \mathbf{\Lambda}_1^t\mathbf{K}_1 = \overline{\mathbf{x}}_0 - \mathbf{\Lambda}_1^{t-T}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} - \mathbf{\Lambda}_1^{t-T-1}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} - \mathbf{\Lambda}_1^{t-T-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} \tag{A111}$$

Next consider the second subsystem (A84) where the diagonal matrix $\mathbf{\Lambda}_2$ only consists of stable eigenvalues. The general backward-looking solution is given by

$$\mathbf{z}_t - \overline{\mathbf{z}} = \mathbf{\Lambda}_2^t\mathbf{K}_2 + \sum_{s=0}^{t-1}\mathbf{\Lambda}_2^{t-s-1}\boldsymbol{\theta}_{2s} = \mathbf{\Lambda}_2^t\mathbf{K}_2 + \sum_{s=T-1}^{t-1}\mathbf{\Lambda}_2^{t-s-1}\boldsymbol{\theta}_{2s} \tag{A112}$$

$$= \begin{cases}\mathbf{\Lambda}_2^t\mathbf{K}_2 & \text{for } t-1 < T-1, \text{ i.e., } t < T\\ \mathbf{\Lambda}_2^t\mathbf{K}_2 + \sum_{s=T-1}^{t-1}\mathbf{\Lambda}_2^{t-s-1}\boldsymbol{\theta}_{2s} & \text{for } t \geq T\end{cases}$$

with arbitrary constant $\mathbf{K}_2$. The definition of the forcing function $\boldsymbol{\theta}_{2s}$ (cf. (A88)) implies

- for $t = T$:

$$\mathbf{z}_T = \overline{\mathbf{z}}_0 + \mathbf{\Lambda}_2^T\mathbf{K}_2 + \boldsymbol{\theta}_{2T-1} = \overline{\mathbf{z}}_0 + \mathbf{\Lambda}_2^T\mathbf{K}_2 + \begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \tag{A113}$$

[15](A109) also follows from the equation

$$\mathbf{x}_{T-1} - \overline{\mathbf{x}}_0 = \mathbf{\Lambda}_1^{-1}(\mathbf{x}_T - \overline{\mathbf{x}}_0) - \mathbf{\Lambda}_1^{-1}\boldsymbol{\theta}_{T-1}$$

by substituting (A107) for $\mathbf{x}_T - \overline{\mathbf{x}}_0$.

- for $t = T+1$:

$$\mathbf{z}_{T+1} = \overline{\mathbf{z}}_0 + \boldsymbol{\Lambda}_2^{T+1}\mathbf{K}_2 + \sum_{s=T-1}^{T} \boldsymbol{\Lambda}_2^{T-s}\boldsymbol{\theta}_{2s} \tag{A114}$$
$$= \overline{\mathbf{z}}_0 + \boldsymbol{\Lambda}_2^{T+1}\mathbf{K}_2 + \boldsymbol{\Lambda}_2 \begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} + \begin{pmatrix} \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{pmatrix}$$

- for $t > T+1$:

$$\begin{aligned} \mathbf{z}_t &= \overline{\mathbf{z}}_0 + \boldsymbol{\Lambda}_2^{t}\mathbf{K}_2 + \boldsymbol{\Lambda}_2^{t-T}\boldsymbol{\theta}_{2T-1} \\ &\quad + \boldsymbol{\Lambda}_2^{t-T-1}\boldsymbol{\theta}_{2T} + \sum_{s=T+1}^{t-1} \boldsymbol{\Lambda}_2^{t-s-1}\boldsymbol{\theta}_{2s} \\ &= \overline{\mathbf{z}}_0 + \boldsymbol{\Lambda}_2^{t}\mathbf{K}_2 + \boldsymbol{\Lambda}_2^{t-T} \begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} + \boldsymbol{\Lambda}_2^{t-T-1} \begin{pmatrix} \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{pmatrix} \\ &\quad - \begin{pmatrix} \overline{\varphi}_4/(r_4-\beta_R) \\ \overline{\varphi}_5/(r_5-\beta_R) \\ \overline{\varphi}_6/(r_6-\beta_R) \end{pmatrix} \beta_R^{t-T-1} + \boldsymbol{\Lambda}_2^{t-T-1} \begin{pmatrix} \overline{\varphi}_4/(r_4-\beta_R) \\ \overline{\varphi}_5/(r_5-\beta_R) \\ \overline{\varphi}_6/(r_6-\beta_R) \end{pmatrix} \end{aligned} \tag{A115}$$

(A115) holds since[16]

$$\begin{aligned} \sum_{s=T+1}^{t-1} r_j^{t-s-1}\overline{\varphi}_j\beta_R^{s-T-1} &= r_j^{t-1}\overline{\varphi}_j\beta_R^{-T-1} \sum_{s=T+1}^{t-1} \left(\frac{\beta_R}{r_j}\right)^s \\ &= r_j^{t-1}\overline{\varphi}_j\beta_R^{-T-1} \left(\frac{\left(\frac{\beta_R}{r_j}\right)^{T+1} - \left(\frac{\beta_R}{r_j}\right)^{t}}{1-\frac{\beta_R}{r_j}} \right) \\ &= r_j^{t}\overline{\varphi}_j \frac{1}{r_j-\beta_R}\frac{1}{\beta_R^{T+1}} \left(\left(\frac{\beta_R}{r_j}\right)^{T+1} - \left(\frac{\beta_R}{r_j}\right)^{t} \right) \\ &= \overline{\varphi}_j \frac{1}{r_j-\beta_R} \left(r_j^{t-T-1} - \beta_R^{t-T-1} \right) \qquad (j = 4,5,6) \end{aligned} \tag{A116}$$

and therefore

$$\sum_{s=T+1}^{t-1} \boldsymbol{\Lambda}_2^{t-s-1}\boldsymbol{\theta}_{2s} = \boldsymbol{\Lambda}_2^{t-T-1} \begin{pmatrix} \overline{\varphi}_4/(r_4-\beta_R) \\ \overline{\varphi}_5/(r_5-\beta_R) \\ \overline{\varphi}_6/(r_6-\beta_R) \end{pmatrix} - \begin{pmatrix} \overline{\varphi}_4/(r_4-\beta_R) \\ \overline{\varphi}_5/(r_5-\beta_R) \\ \overline{\varphi}_6/(r_6-\beta_R) \end{pmatrix} \beta_R^{t-T-1} \tag{A117}$$

[16] Note that

$$\sum_{s=T+1}^{t-1} x^s = \sum_{s=0}^{t-1} x^s - \sum_{s=0}^{T-1} x^s - x^T = \frac{1-x^t}{1-x} - \frac{1-x^T}{1-x} - x^T = \frac{x^T-x^t}{1-x} - x^T = \frac{x^{T+1}-x^t}{1-x}$$

$(x = \beta_R/r_j)$.

Note that (A115) is equivalent to (A114) in the special case $t = T + 1$.

The solution of the original state vector $\mathbf{v} = (y, \tau, \pi^c)'$ can be obtained using the transformation (A79) and the solution of the canonical system. Since

$$\mathbf{v}_t = \mathbf{H}_{11}\mathbf{x}_t + \mathbf{H}_{12}\mathbf{z}_t \tag{A118}$$

and

$$\overline{\mathbf{v}} = \mathbf{H}_{11}\overline{\mathbf{x}} + \mathbf{H}_{12}\overline{\mathbf{z}} \tag{A119}$$

we get

- for $t > T + 1$:

$$\begin{aligned}\mathbf{v}_t = {} & \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\boldsymbol{\Lambda}_2^t\mathbf{K}_2 + \mathbf{H}_{12}\boldsymbol{\Lambda}_2^{t-T}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \\ & + \mathbf{H}_{12}\boldsymbol{\Lambda}_2^{t-T-1}\left[\begin{pmatrix}\hat{e}_4\\ \hat{e}_5\\ \hat{e}_6\end{pmatrix} + \begin{pmatrix}\overline{\varphi}_4/(r_4-\beta_R)\\ \overline{\varphi}_5/(r_5-\beta_R)\\ \overline{\varphi}_6/(r_6-\beta_R)\end{pmatrix}\right] \\ & - \left[\mathbf{H}_{11}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} + \mathbf{H}_{12}\begin{pmatrix}\overline{\varphi}_4/(r_4-\beta_R)\\ \overline{\varphi}_5/(r_5-\beta_R)\\ \overline{\varphi}_6/(r_6-\beta_R)\end{pmatrix}\right]\beta_R^{t-T-1}\end{aligned} \tag{A120}$$

- for $t = T + 1$:

$$\begin{aligned}\mathbf{v}_{T+1} = {} & \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\boldsymbol{\Lambda}_2^{T+1}\mathbf{K}_2 + \mathbf{H}_{12}\boldsymbol{\Lambda}_2\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \\ & + \mathbf{H}_{12}\begin{pmatrix}\hat{e}_4\\ \hat{e}_5\\ \hat{e}_6\end{pmatrix} - \mathbf{H}_{11}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\end{aligned} \tag{A121}$$

- for $t = T$:

$$\begin{aligned}\mathbf{v}_T = {} & \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\boldsymbol{\Lambda}_2^{T}\mathbf{K}_2 + \mathbf{H}_{12}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \\ & - \mathbf{H}_{11}\boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} - \mathbf{H}_{11}\boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\end{aligned} \tag{A122}$$

- for $t = T - 1$:

$$\begin{aligned}\mathbf{v}_{T-1} = {} & \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\boldsymbol{\Lambda}_2^{T-1}\mathbf{K}_2 - \mathbf{H}_{11}\boldsymbol{\Lambda}_1^{-1}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \\ & - \mathbf{H}_{11}\boldsymbol{\Lambda}_1^{-2}\left[\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} + \begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\right]\end{aligned} \tag{A123}$$

- for $t < T-1$:

$$\mathbf{v}_t = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^t\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\mathbf{K}_2 \tag{A124}$$

where $\mathbf{K}_1$ is defined in (A102). The second constant $\mathbf{K}_2$ can be determined from the initial condition of the vector $\mathbf{w}$ of predetermined state variables:

$$\mathbf{w}(0) = \overline{\mathbf{w}}_0 \tag{A125}$$

(A79), (A111) and (A112) imply

$$\begin{aligned}\mathbf{w}(0) &= \mathbf{H}_{21}\mathbf{x}(0) + \mathbf{H}_{22}\mathbf{z}(0) = \mathbf{H}_{21}(\overline{\mathbf{x}}_0 + \mathbf{K}_1) + \mathbf{H}_{22}(\overline{\mathbf{z}}_0 + \mathbf{K}_2) \\ &= \overline{\mathbf{w}}_0 + \mathbf{H}_{21}\mathbf{K}_1 + \mathbf{H}_{22}\mathbf{K}_2\end{aligned} \tag{A126}$$

so that

$$0 = \mathbf{H}_{21}\mathbf{K}_1 + \mathbf{H}_{22}\mathbf{K}_2 \tag{A127}$$

and therefore

$$\mathbf{K}_2 = -\mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{K}_1 \tag{A128}$$

Note that the solution (A120) also holds in $t = T+1$ since (A120) is equivalent to (A121) in the special case $t = T+1$. (A120) is also equivalent to (A122) if we set $t = T$ in (A120). This equivalence holds since

$$\mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} = -\mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}\begin{pmatrix}\hat{e}_4\\ \hat{e}_5\\ \hat{e}_6\end{pmatrix} \tag{A129}$$

and

$$\begin{aligned}\mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} &= -\mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}\begin{pmatrix}\overline{\varphi}_4/(r_4-\beta_R)\\ \overline{\varphi}_5/(r_5-\beta_R)\\ \overline{\varphi}_6/(r_6-\beta_R)\end{pmatrix} \\ &+ \left[\mathbf{H}_{11}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix} + \mathbf{H}_{12}\begin{pmatrix}\overline{\varphi}_4/(r_4-\beta_R)\\ \overline{\varphi}_5/(r_5-\beta_R)\\ \overline{\varphi}_6/(r_6-\beta_R)\end{pmatrix}\right]\beta_R^{-1}\end{aligned} \tag{A130}$$

To show (A129) and (A130) note that $\mathbf{w}_t = \mathbf{v}_{t-1}$ and

$$\mathbf{w}_t = \mathbf{H}_{21}\mathbf{x}_t + \mathbf{H}_{22}\mathbf{z}_t \tag{A131}$$

According to (A124) we then have

$$\begin{aligned}\mathbf{w}_t &= \overline{\mathbf{v}}_0 + \mathbf{H}_{21}\mathbf{\Lambda}_1^t\mathbf{K}_1 + \mathbf{H}_{22}\mathbf{\Lambda}_2^t\mathbf{K}_2 = \\ \mathbf{v}_{t-1} &= \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{t-1}\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{t-1}\mathbf{K}_2 \qquad \text{for } t < T-1\end{aligned} \tag{A132}$$

implying

$$\mathbf{H}_{21}\mathbf{\Lambda}_1 = \mathbf{H}_{11}, \qquad \mathbf{H}_{22}\mathbf{\Lambda}_2 = \mathbf{H}_{12} \tag{A133}$$

or

$$\mathbf{H}_{21} = \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}, \qquad \mathbf{H}_{22} = \mathbf{H}_{12}\mathbf{\Lambda}_2^{-1} \tag{A134}$$

The identity

$$\begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_3 & 0 \\ 0 & \mathbf{I}_3 \end{pmatrix} \tag{A135}$$

implies

$$\mathbf{H}_{21}\mathbf{G}_{11} + \mathbf{H}_{22}\mathbf{G}_{21} = 0 \tag{A136}$$

or

$$\mathbf{G}_{21}\mathbf{G}_{11}^{-1} = -\mathbf{H}_{22}^{-1}\mathbf{H}_{21} \tag{A137}$$

According to (A90)

$$\begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} = \mathbf{G}_{11} \begin{pmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \end{pmatrix} \tag{A138}$$

$$\begin{pmatrix} \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{pmatrix} = \mathbf{G}_{21} \begin{pmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \end{pmatrix} \tag{A139}$$

implying

$$\begin{pmatrix} \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{pmatrix} = \mathbf{G}_{21}\mathbf{G}_{11}^{-1} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} \tag{A140}$$

Premultiplying (A140) with $\mathbf{\Lambda}_2^{-1}$ and using (A137) and (A134) yields

$$\begin{aligned} \mathbf{\Lambda}_2^{-1} \begin{pmatrix} \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{pmatrix} &= \mathbf{\Lambda}_2^{-1}\mathbf{G}_{21}\mathbf{G}_{11}^{-1} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} = -\mathbf{\Lambda}_2^{-1}\mathbf{H}_{22}^{-1}\mathbf{H}_{21} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} \\ &= -\mathbf{\Lambda}_2^{-1}(\mathbf{\Lambda}_2\mathbf{H}_{12}^{-1})(\mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}) \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} = -\mathbf{H}_{12}^{-1}\mathbf{H}_{11}\mathbf{\Lambda}_1^{-1} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{pmatrix} \end{aligned} \tag{A141}$$

Premultiplying this equation with $-\mathbf{H}_{12}$ yields (A129). To show (A130) note that

$$-\mathbf{H}_{12}\mathbf{\Lambda}_2^{-1} \begin{pmatrix} \overline{\varphi}_4 \\ \overline{\varphi}_5 \\ \overline{\varphi}_6 \end{pmatrix} = \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1} \begin{pmatrix} \overline{\varphi}_1 \\ \overline{\varphi}_2 \\ \overline{\varphi}_3 \end{pmatrix} \tag{A142}$$

The proof is similar to the proof of (A129) since by definition

$$\begin{pmatrix} \overline{\varphi}_1 \\ \overline{\varphi}_2 \\ \overline{\varphi}_3 \end{pmatrix} = \mathbf{G}_{11} \begin{pmatrix} \overline{\phi}_1 \\ \overline{\phi}_2 \\ \overline{\phi}_3 \end{pmatrix}, \qquad \begin{pmatrix} \overline{\varphi}_4 \\ \overline{\varphi}_5 \\ \overline{\varphi}_6 \end{pmatrix} = \mathbf{G}_{21} \begin{pmatrix} \overline{\phi}_1 \\ \overline{\phi}_2 \\ \overline{\phi}_3 \end{pmatrix} \tag{A143}$$

and therefore

$$\begin{pmatrix}\overline{\varphi}_4\\ \overline{\varphi}_5\\ \overline{\varphi}_6\end{pmatrix} = \mathbf{G}_{21}\mathbf{G}_{11}^{-1}\begin{pmatrix}\overline{\varphi}_1\\ \overline{\varphi}_2\\ \overline{\varphi}_3\end{pmatrix} \tag{A144}$$

According to (A141) this implies

$$\mathbf{\Lambda}_2^{-1}\begin{pmatrix}\overline{\varphi}_4\\ \overline{\varphi}_5\\ \overline{\varphi}_6\end{pmatrix} = -\mathbf{H}_{12}^{-1}\mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1\\ \overline{\varphi}_2\\ \overline{\varphi}_3\end{pmatrix} \tag{A145}$$

so that (A142) holds. Denote the elements of the modal matrix $\mathbf{H}$ by h_{ij} $(1 \leq i, j \leq 6)$. Using (A142) the right-hand-side of (A130) can be written as follows:

$$\begin{aligned}
&-\sum_{j=4}^{6}\frac{1}{r_j}\frac{\overline{\varphi}_j}{r_j-\beta_R}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix} + \sum_{j=1}^{6}\frac{\overline{\varphi}_j}{(r_j-\beta_R)\beta_R}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix} \qquad \text{(A146)}\\
&= -\sum_{j=4}^{6}\frac{\overline{\varphi}_j}{r_j-\beta_R}\left(\frac{1}{r_j}-\frac{1}{\beta_R}\right)\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix} + \sum_{j=1}^{3}\frac{\overline{\varphi}_j}{(r_j-\beta_R)\beta_R}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix}\\
&= \sum_{j=4}^{6}\frac{\overline{\varphi}_j}{r_j\beta_R}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix} + \sum_{j=1}^{3}\frac{\overline{\varphi}_j}{(r_j-\beta_R)\beta_R}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix}\\
&= -\frac{1}{\beta_R}\sum_{j=1}^{3}\frac{\overline{\varphi}_j}{r_j}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix} + \frac{1}{\beta_R}\sum_{j=1}^{3}\frac{\overline{\varphi}_j}{r_j-\beta_R}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix}\\
&= \sum_{j=1}^{3}\frac{\overline{\varphi}_j}{\beta_R}\left(\frac{1}{r_j-\beta_R}-\frac{1}{r_j}\right)\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix}\\
&= \sum_{j=1}^{3}\frac{\overline{\varphi}_j}{r_j(r_j-\beta_R)}\begin{pmatrix}h_{1j}\\ h_{2j}\\ h_{3j}\end{pmatrix} = \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}
\end{aligned}$$

Therefore, (A130) holds so that the solution formula (A120) is also valid in period $t = T$. Similar to (A129) and (A142)

$$\mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} = -\mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \tag{A147}$$

holds. The solution formula in $T-1$ (i.e., (A123)) can then also be written in the following form:

$$\begin{aligned}
\mathbf{v}_{T-1} = \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\mathbf{K}_2 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \qquad \text{(A148)}\\
-\mathbf{H}_{11}\mathbf{\Lambda}_1^{-2}\left[\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} + \begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\right]
\end{aligned}$$

Now it is obvious that (A123) also follows from (A120) and (A121). But (A123) can also be written in the form (A124), i.e.

$$\mathbf{v}_{T-1} = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\mathbf{K}_2 \tag{A149}$$

since

$$\begin{aligned}\mathbf{v}_{T-1} &= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\mathbf{K}_2 \\ &\quad - \mathbf{H}_{11}\mathbf{\Lambda}_1^{T-1}\mathbf{\Lambda}_1^{-T}\left[\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} + \mathbf{\Lambda}_1^{-1}\begin{pmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{pmatrix} + \mathbf{\Lambda}_1^{-1}\begin{pmatrix}\overline{\varphi}_1/(r_1-\beta_R)\\ \overline{\varphi}_2/(r_2-\beta_R)\\ \overline{\varphi}_3/(r_3-\beta_R)\end{pmatrix}\right] \\ &= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\mathbf{K}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T-1}\mathbf{K}_1\end{aligned} \tag{A150}$$

according to the definition of $\mathbf{K}_1$ (cf. (A102)). Note that an analogous formula does *not* hold in $t = T$ since

$$\mathbf{H}_{12}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} = \mathbf{H}_{12}\mathbf{G}_{21}\mathbf{G}_{11}^{-1}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \neq -\mathbf{H}_{11}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \tag{A151}$$

(A135) implies

$$\mathbf{H}_{11}\mathbf{G}_{11} + \mathbf{H}_{12}\mathbf{G}_{21} = \mathbf{I}_3 \tag{A152}$$

so that

$$\mathbf{H}_{12}\mathbf{G}_{21}\mathbf{G}_{11}^{-1} = \mathbf{G}_{11}^{-1} - \mathbf{H}_{11} \tag{A153}$$

Therefore, (A122) is equivalent to

$$\begin{aligned}\mathbf{v}_T &= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T}\mathbf{K}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T}\mathbf{K}_1 + \mathbf{H}_{12}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} + \mathbf{H}_{11}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \\ &= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T}\mathbf{K}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T}\mathbf{K}_1 + [\mathbf{H}_{12}\mathbf{G}_{21}\mathbf{G}_{11}^{-1} + \mathbf{H}_{11}]\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \\ &= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T}\mathbf{K}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T}\mathbf{K}_1 + [\mathbf{H}_{12}\mathbf{G}_{21}\mathbf{G}_{11}^{-1} + \mathbf{H}_{11}]\mathbf{G}_{11}\begin{pmatrix}0\\ \widetilde{\alpha}_2\\ 0\end{pmatrix} \\ &= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T}\mathbf{K}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T}\mathbf{K}_1 + [\mathbf{H}_{12}\mathbf{G}_{21} + \mathbf{H}_{11}\mathbf{G}_{11}]\begin{pmatrix}0\\ \widetilde{\alpha}_2\\ 0\end{pmatrix}\end{aligned} \tag{A154}$$

$$= \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T}\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T}\mathbf{K}_2 + \begin{pmatrix}0\\ \widetilde{\alpha}_2\\ 0\end{pmatrix} \tag{A155}$$

In summary, the solution time path of the jump vector $\mathbf{v}$ in response to an anticipated temporary raw materials price shock may be represented as follows:

- For $t < T$:

$$\mathbf{v}_t = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^t\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\mathbf{K}_2 \tag{A156}$$

- For $t = T$:

$$\mathbf{v}_T = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^T\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\mathbf{K}_2 + \begin{pmatrix} 0 \\ \widetilde{\alpha}_2 \\ 0 \end{pmatrix} \tag{A157}$$

- For $t > T$:

$$\begin{aligned} \mathbf{v}_t = {} & \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\mathbf{K}_2 \\ & + \mathbf{H}_{12}\mathbf{\Lambda}_2^{t-T}\left[\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} + \mathbf{\Lambda}_2^{-1}\left[\begin{pmatrix} \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{pmatrix} + \begin{pmatrix} \overline{\varphi}_4/(r_4-\beta_R) \\ \overline{\varphi}_5/(r_5-\beta_R) \\ \overline{\varphi}_6/(r_6-\beta_R) \end{pmatrix}\right]\right] \\ & - (\mathbf{H}_{11}, \mathbf{H}_{12})\begin{pmatrix} \overline{\varphi}_1/(r_1-\beta_R) \\ \vdots \\ \overline{\varphi}_6/(r_6-\beta_R) \end{pmatrix}\beta_R^{t-T-1} \end{aligned} \tag{A158}$$

where (A158) also holds in $t = T$ and $\mathbf{K}_1$ and $\mathbf{K}_2$ are defined by (A102) and (A128) respectively.

Solution in case $\beta_R = 1$
In case $\beta_R = 1$ (permanent raw materials price shock) the canonical form of the state equations (A52) is given by (cf. (A77), (A78))

$$\mathbf{x}_{t+1} = \mathbf{\Lambda}_1\mathbf{x}_t + (\mathbf{G}_{11}, \mathbf{G}_{12})[\mathbf{B}^{-1}\overline{\mathbf{d}} + \mathbf{B}^{-1}\boldsymbol{\phi}_t] \tag{A159}$$

$$\mathbf{z}_{t+1} = \mathbf{\Lambda}_2\mathbf{z}_t + (\mathbf{G}_{21}, \mathbf{G}_{22})[\mathbf{B}^{-1}\overline{\mathbf{d}} + \mathbf{B}^{-1}\boldsymbol{\phi}_t] \tag{A160}$$

where

$$\mathbf{B}^{-1}\overline{\mathbf{d}} = \begin{cases} \mathbf{B}^{-1}\overline{\mathbf{d}}_0 & \text{for } t < T \\ \mathbf{B}^{-1}\overline{\mathbf{d}}_1 & \text{for } t \geq T \end{cases} \tag{A161}$$

and

$$\mathbf{B}^{-1}\boldsymbol{\phi}_t = \begin{cases} 0 & \text{for } t < T-1 \text{ and } t > T \\ (0, \widetilde{\alpha}_2, 0, 0, 0, 0)' & \text{for } t = T-1 \\ (\alpha_1, \alpha_2, \alpha_3, 0, 0, 0)' & \text{for } t = T \end{cases} \tag{A162}$$

(cf. (A45), (A59), ... , (A65)). Let

$$\mathbf{G}\begin{pmatrix} 0 \\ \widetilde{\alpha}_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} g_{12}\widetilde{\alpha}_2 \\ g_{22}\widetilde{\alpha}_2 \\ \vdots \\ g_{62}\widetilde{\alpha}_2 \end{pmatrix} = \begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \vdots \\ \widetilde{g}_6 \end{pmatrix} \tag{A163}$$

(cf. (A89)) and

$$\mathbf{G}\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ 0\\ 0\\ 0\end{pmatrix} = \begin{pmatrix} g_{11}\alpha_1 + g_{12}\alpha_2 + g_{13}\alpha_3 \\ g_{21}\alpha_1 + g_{22}\alpha_2 + g_{23}\alpha_3 \\ \vdots \\ g_{61}\alpha_1 + g_{62}\alpha_2 + g_{63}\alpha_3 \end{pmatrix} = \begin{pmatrix} e_1\\ e_2\\ \vdots \\ e_6\end{pmatrix} \tag{A164}$$

We first consider the solution of the canonical system for $t < T-1$ and $t > T$ and then develop the solution of the transformed state vectors in the periods $t = T$ and $t = T-1$. For $t < T-1$ the system (A159), (A160) is equivalent to

$$\mathbf{x}_{t+1} - \overline{\mathbf{x}}_0 = \mathbf{\Lambda}_1(\mathbf{x}_t - \overline{\mathbf{x}}_0) \tag{A165}$$

$$\mathbf{z}_{t+1} - \overline{\mathbf{z}}_0 = \mathbf{\Lambda}_2(\mathbf{z}_t - \overline{\mathbf{z}}_0) \tag{A166}$$

where the steady state vectors $\overline{\mathbf{x}}_0$ and $\overline{\mathbf{z}}_0$ are defined in (A80). For $t > T$ (A159), (A160) is equivalent to

$$\mathbf{x}_{t+1} - \overline{\mathbf{x}}_1 = \mathbf{\Lambda}_1(\mathbf{x}_t - \overline{\mathbf{x}}_1) \tag{A167}$$

$$\mathbf{z}_{t+1} - \overline{\mathbf{z}}_1 = \mathbf{\Lambda}_2(\mathbf{z}_t - \overline{\mathbf{z}}_1) \tag{A168}$$

where $\overline{\mathbf{x}}_1$ and $\overline{\mathbf{z}}_1$ are the new steady states after the occurrence of the permanent price shock. The general solution of the homogeneous system (A165), (A166) for $t < T-1$ is given by

$$\mathbf{x}_t - \overline{\mathbf{x}}_0 = \mathbf{\Lambda}_1^t \mathbf{K}_1 \tag{A169}$$

$$\mathbf{z}_t - \overline{\mathbf{z}}_0 = \mathbf{\Lambda}_2^t \mathbf{K}_2 \tag{A170}$$

where the constants $\mathbf{K}_1$ and $\mathbf{K}_2$ are arbitrary. For $t > T$ the system (A167), (A168) has the general solution

$$\mathbf{x}_t - \overline{\mathbf{x}}_1 = \mathbf{\Lambda}_1^t \widetilde{\mathbf{K}}_1 \tag{A171}$$

$$\mathbf{z}_t - \overline{\mathbf{z}}_1 = \mathbf{\Lambda}_2^t \widetilde{\mathbf{K}}_2 \tag{A172}$$

with arbitrary constants $\widetilde{\mathbf{K}}_1$ and $\widetilde{\mathbf{K}}_2$. The transversality condition (cf. (A92))

$$\lim_{t\to\infty}(\mathbf{x}_t - \overline{\mathbf{x}}_1) = 0 \tag{A173}$$

requires

$$\widetilde{\mathbf{K}}_1 = 0 \tag{A174}$$

and therefore

$$\mathbf{x}_t = \overline{\mathbf{x}}_1 \qquad \text{for } t > T \tag{A175}$$

Next consider the period T in which the foreign price shock is realized. Equation (A159) then implies

$$\mathbf{x}_{T+1} = \mathbf{\Lambda}_1\mathbf{x}_T + (\mathbf{G}_{11}, \mathbf{G}_{12})[\mathbf{B}^{-1}\overline{\mathbf{d}}_1 + \mathbf{B}^{-1}\boldsymbol{\phi}_T] \tag{A176}$$

or equivalently

$$\mathbf{x}_T = \boldsymbol{\Lambda}_1^{-1}\mathbf{x}_{T+1} - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})[\mathbf{B}^{-1}\overline{\mathbf{d}}_1 + \mathbf{B}^{-1}\boldsymbol{\phi}_T] \tag{A177}$$
$$= \boldsymbol{\Lambda}_1^{-1}\overline{\mathbf{x}}_1 - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

according to (A161), (A164) and (A175). Since $\overline{\mathbf{x}}_1$ is given by

$$\overline{\mathbf{x}}_1 = (\mathbf{I}_3 - \boldsymbol{\Lambda}_1)^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 \tag{A178}$$

(cf. (A80)) the first two expressions on the right-hand side of (A177) can be summarized as follows:

$$\boldsymbol{\Lambda}_1^{-1}\overline{\mathbf{x}}_1 - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 = \tag{A179}$$
$$\boldsymbol{\Lambda}_1^{-1}\big[(\mathbf{I}_3 - \boldsymbol{\Lambda}_1)^{-1} - \mathbf{I}_3\big](\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 =$$
$$\boldsymbol{\Lambda}_1^{-1}\big[\mathbf{I}_3 - (\mathbf{I}_3 - \boldsymbol{\Lambda}_1)\big](\mathbf{I}_3 - \boldsymbol{\Lambda}_1)^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 = \boldsymbol{\Lambda}_1^{-1}\boldsymbol{\Lambda}_1\overline{\mathbf{x}}_1 = \overline{\mathbf{x}}_1$$

Therefore

$$\mathbf{x}_T = \overline{\mathbf{x}}_1 - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \tag{A180}$$

Next consider the period $t = T - 1$. There are two possibilities to determine the solution of the vector $\mathbf{x}$ in this period: the forward- and the backward-looking solution. The forward-looking solution is given by

$$\mathbf{x}_{T-1} = \boldsymbol{\Lambda}_1^{-1}\mathbf{x}_T - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})[\mathbf{B}^{-1}\overline{\mathbf{d}}_0 + \mathbf{B}^{-1}\boldsymbol{\phi}_{T-1}] \tag{A181}$$
$$= \boldsymbol{\Lambda}_1^{-1}\left[\overline{\mathbf{x}}_1 - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}\right] - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \widetilde{g}_3 \end{pmatrix}$$
$$= \boldsymbol{\Lambda}_1^{-1}\overline{\mathbf{x}}_1 - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \widetilde{g}_3 \end{pmatrix} - \boldsymbol{\Lambda}_1^{-2}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

Since

$$\boldsymbol{\Lambda}_1^{-1}\overline{\mathbf{x}}_1 - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 = \boldsymbol{\Lambda}_1^{-1}(\overline{\mathbf{x}}_1 - \overline{\mathbf{x}}_0) \tag{A182}$$
$$+ \boldsymbol{\Lambda}_1^{-1}(\mathbf{I}_3 - \boldsymbol{\Lambda}_1)^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 - \boldsymbol{\Lambda}_1^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0$$
$$= \boldsymbol{\Lambda}_1^{-1}d\overline{\mathbf{x}} + \boldsymbol{\Lambda}_1^{-1}\big[(\mathbf{I}_3 - \boldsymbol{\Lambda}_1)^{-1} - \mathbf{I}_3\big](\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0$$
$$= \boldsymbol{\Lambda}_1^{-1}d\overline{\mathbf{x}} + \boldsymbol{\Lambda}_1^{-1}\big[\mathbf{I}_3 - (\mathbf{I}_3 - \boldsymbol{\Lambda}_1)\big](\mathbf{I}_3 - \boldsymbol{\Lambda}_1)^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0$$
$$= \boldsymbol{\Lambda}_1^{-1}d\overline{\mathbf{x}} + \overline{\mathbf{x}}_0$$

(A181) is equivalent to

$$\mathbf{x}_{T-1} = \overline{\mathbf{x}}_0 + \boldsymbol{\Lambda}_1^{-1}d\overline{\mathbf{x}} - \boldsymbol{\Lambda}_1^{-1}\begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \widetilde{g}_3 \end{pmatrix} - \boldsymbol{\Lambda}_1^{-2}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \tag{A183}$$

The backward-looking solution for $\mathbf{x}$ in period $T-1$ is given by (cf. (A169))

$$\begin{aligned}\mathbf{x}_{T-1} &= \mathbf{\Lambda}_1\mathbf{x}_{T-2} + (\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\ &= \mathbf{\Lambda}_1(\mathbf{\Lambda}_1^{T-2}\mathbf{K}_1 + \overline{\mathbf{x}}_0) + (\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\ &= \mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \mathbf{\Lambda}_1(\mathbf{I}_3 - \mathbf{\Lambda}_1)^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 + (\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\ &= \mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \big[\mathbf{\Lambda}_1(\mathbf{I}_3 - \mathbf{\Lambda}_1)^{-1} + \mathbf{I}_3\big](\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\ &= \mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \big[\mathbf{\Lambda}_1 + (\mathbf{I}_3 - \mathbf{\Lambda}_1)\big](\mathbf{I}_3 - \mathbf{\Lambda}_1)^{-1}(\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\ &= \mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \overline{\mathbf{x}}_0\end{aligned} \tag{A184}$$

Since the forward-looking and backward-looking solution must be equivalent, equality of (A183) and (A184) yields a condition for the determination of the constant K_1:

$$\mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \overline{\mathbf{x}}_0 = \overline{\mathbf{x}}_0 + \mathbf{\Lambda}_1^{-1}d\overline{\mathbf{x}} - \mathbf{\Lambda}_1^{-1}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} - \mathbf{\Lambda}_1^{-2}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} \tag{A185}$$

Solving for $\mathbf{K}_1$ yields the expression

$$\mathbf{K}_1 = \mathbf{\Lambda}_1^{-T}\left[d\overline{\mathbf{x}} - \begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} - \mathbf{\Lambda}_1^{-1}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix}\right] \tag{A186}$$

The solution formula for $\mathbf{K}_1$ may also be obtained if the forward-looking solution in period T, i.e., equation (A180), is compared with the equivalent backward-looking solution in T, the latter given by

$$\begin{aligned}\mathbf{x}_T &= \mathbf{\Lambda}_1\mathbf{x}_{T-1} + (\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 + \begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \\ &= \mathbf{\Lambda}_1(\mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \overline{\mathbf{x}}_0) + (\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 + \begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \\ &= \mathbf{\Lambda}_1^{T}\mathbf{K}_1 + \big[\mathbf{\Lambda}_1(\mathbf{I}_3 - \mathbf{\Lambda}_1)^{-1} + \mathbf{I}_3\big](\mathbf{G}_{11}, \mathbf{G}_{12})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 + \begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} \\ &= \mathbf{\Lambda}_1^{T}\mathbf{K}_1 + \overline{\mathbf{x}}_0 + \begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix}\end{aligned} \tag{A187}$$

Equality of (A187) and (A180) again yields (A186).

The next step is the determination of the solution of the second transformed state vector $\mathbf{z}$ in the periods T and $T-1$. The backward-looking solutions have an analogous structure as the corresponding solutions of the state vector $\mathbf{x}$ (cf.

(A184) and (A187)):

$$\mathbf{z}_{T-1} = \mathbf{\Lambda}_2^{T-1}\mathbf{K}_2 + \overline{\mathbf{z}}_0 \tag{A188}$$

$$\mathbf{z}_T = \mathbf{\Lambda}_2^{T}\mathbf{K}_2 + \overline{\mathbf{z}}_0 + \begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} \tag{A189}$$

where $(\widetilde{g}_4, \widetilde{g}_5, \widetilde{g}_6)'$ is defined in (A163). The corresponding forward-looking solutions are given by (cf. (A172))

$$\begin{aligned}
\mathbf{z}_T &= \mathbf{\Lambda}_2^{-1}\mathbf{z}_{T+1} - \mathbf{\Lambda}_2^{-1}(\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} \qquad \text{(A190)} \\
&= \mathbf{\Lambda}_2^{-1}\big[\mathbf{\Lambda}_2^{T+1}\widetilde{\mathbf{K}}_2 + \overline{\mathbf{z}}_1\big] - \mathbf{\Lambda}_2^{-1}(\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} \\
&= \mathbf{\Lambda}_2^{T}\widetilde{\mathbf{K}}_2 + \mathbf{\Lambda}_2^{-1}\big[(\mathbf{I}_3 - \mathbf{\Lambda}_2)^{-1} - \mathbf{I}_3\big](\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} \\
&= \mathbf{\Lambda}_2^{T}\widetilde{\mathbf{K}}_2 + \mathbf{\Lambda}_2^{-1}\big[\mathbf{I}_3 - (\mathbf{I}_3 - \mathbf{\Lambda}_2)\big](\mathbf{I}_3 - \mathbf{\Lambda}_2)^{-1}(\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_1 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} \\
&= \mathbf{\Lambda}_2^{T}\widetilde{\mathbf{K}}_2 + \overline{\mathbf{z}}_1 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix}
\end{aligned}$$

and (cf. (A183))

$$\begin{aligned}
\mathbf{z}_{T-1} &= \mathbf{\Lambda}_2^{-1}\mathbf{z}_T - \mathbf{\Lambda}_2^{-1}(\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} \qquad \text{(A191)} \\
&= \mathbf{\Lambda}_2^{-1}\left[\mathbf{\Lambda}_2^{T}\widetilde{\mathbf{K}}_2 + \overline{\mathbf{z}}_1 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix}\right] - \mathbf{\Lambda}_2^{-1}(\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} \\
&= \mathbf{\Lambda}_2^{T-1}\widetilde{\mathbf{K}}_2 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} - \mathbf{\Lambda}_2^{-2}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} + \mathbf{\Lambda}_2^{-1}(\overline{\mathbf{z}}_1 - \overline{\mathbf{z}}_0) \\
&\quad + \mathbf{\Lambda}_2^{-1}\big[(\mathbf{I}_3 - \mathbf{\Lambda}_2)^{-1} - \mathbf{I}_3\big](\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\
&= \mathbf{\Lambda}_2^{T-1}\widetilde{\mathbf{K}}_2 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} - \mathbf{\Lambda}_2^{-2}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} + \mathbf{\Lambda}_2^{-1}d\overline{\mathbf{z}} \\
&\quad + \mathbf{\Lambda}_2^{-1}\big[\mathbf{I}_3 - (\mathbf{I}_3 - \mathbf{\Lambda}_2)\big](\mathbf{I}_3 - \mathbf{\Lambda}_2)^{-1}(\mathbf{G}_{21}, \mathbf{G}_{22})\mathbf{B}^{-1}\overline{\mathbf{d}}_0 \\
&= \mathbf{\Lambda}_2^{T-1}\widetilde{\mathbf{K}}_2 - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} - \mathbf{\Lambda}_2^{-2}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} + \mathbf{\Lambda}_2^{-1}d\overline{\mathbf{z}} + \overline{\mathbf{z}}_0
\end{aligned}$$

The solution formulas (A188) and (A191) as well as (A189) and (A190) are equivalent. In both cases equality yields the condition

$$\mathbf{K}_2 - \widetilde{\mathbf{K}}_2 = \mathbf{\Lambda}_2^{-T}\left[d\overline{\mathbf{z}} - \begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix}\right] \tag{A192}$$

where – as before – $\mathbf{K}_2$ follows from the initial condition of the vector of predetermined variables $\mathbf{w}$ (cf. (A125) to (A128)):

$$\mathbf{K}_2 = -\mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{K}_1 \tag{A193}$$

(A192), (A193) and (A186) imply

$$\begin{aligned} \widetilde{\mathbf{K}}_2 &= -\mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{K}_1 - \mathbf{\Lambda}_2^{-T}\left[d\overline{\mathbf{z}} - \begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix}\right] \\ &= -\mathbf{H}_{22}^{-1}\mathbf{H}_{21}\mathbf{\Lambda}_1^{-T}\left[d\overline{\mathbf{x}} - \begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \widetilde{g}_3 \end{pmatrix} - \mathbf{\Lambda}_1^{-1}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}\right] \\ &\quad - \mathbf{\Lambda}_2^{-T}\left[d\overline{\mathbf{z}} - \begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} - \mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix}\right] \end{aligned} \tag{A194}$$

The solution of the original state vector is now given by:

- for $t < T-1$:

$$\mathbf{v}_t = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^t\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\mathbf{K}_2 \tag{A195}$$

- for $t > T$:

$$\mathbf{v}_t = \overline{\mathbf{v}}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\widetilde{\mathbf{K}}_2 \tag{A196}$$

- for $t = T$:

$$\begin{aligned} \mathbf{v}_T &= \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^T\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\mathbf{K}_2 + \mathbf{H}_{11}\begin{pmatrix} \widetilde{g}_1 \\ \widetilde{g}_2 \\ \widetilde{g}_3 \end{pmatrix} + \mathbf{H}_{12}\begin{pmatrix} \widetilde{g}_4 \\ \widetilde{g}_5 \\ \widetilde{g}_6 \end{pmatrix} \\ &= \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^T\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\mathbf{K}_2 + \begin{pmatrix} 0 \\ \widetilde{\alpha}_2 \\ 0 \end{pmatrix} \\ &= \overline{\mathbf{v}}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\widetilde{\mathbf{K}}_2 - \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} - \mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}\begin{pmatrix} e_4 \\ e_5 \\ e_6 \end{pmatrix} \\ &= \overline{\mathbf{v}}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\widetilde{\mathbf{K}}_2 \end{aligned} \tag{A197}$$

- for $t = T-1$;

$$
\begin{aligned}
\mathbf{v}_{T-1} &= \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{T-1}\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\mathbf{K}_2 \\
&= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\widetilde{\mathbf{K}}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}d\overline{\mathbf{x}} + \mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}d\overline{\mathbf{z}} \\
&\quad - \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}\begin{pmatrix}\widetilde{g}_1\\ \widetilde{g}_2\\ \widetilde{g}_3\end{pmatrix} - \mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}\begin{pmatrix}\widetilde{g}_4\\ \widetilde{g}_5\\ \widetilde{g}_6\end{pmatrix} \\
&\quad - \mathbf{H}_{11}\mathbf{\Lambda}_1^{-2}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} - \mathbf{H}_{12}\mathbf{\Lambda}_2^{-2}\begin{pmatrix}e_4\\ e_5\\ e_6\end{pmatrix} \\
&= \overline{\mathbf{v}}_0 + \mathbf{H}_{12}\mathbf{\Lambda}_2^{T-1}\widetilde{\mathbf{K}}_2 + \mathbf{H}_{11}\mathbf{\Lambda}_1^{-1}d\overline{\mathbf{x}} + \mathbf{H}_{12}\mathbf{\Lambda}_2^{-1}d\overline{\mathbf{z}} \\
&\quad - \mathbf{H}_{11}\mathbf{\Lambda}_1^{-2}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} - \mathbf{H}_{12}\mathbf{\Lambda}_2^{-2}\begin{pmatrix}e_4\\ e_5\\ e_6\end{pmatrix}
\end{aligned} \tag{A198}
$$

In compact form, the solution of the jump vector $\mathbf{v}$ in case of an anticipated permanent raw materials price shock is given by:

- for $t > T$:

$$\mathbf{v}_t = \overline{\mathbf{v}}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\widetilde{\mathbf{K}}_2 \tag{A199}$$

- for $t = T$:

$$\mathbf{v}_T = \overline{\mathbf{v}}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\widetilde{\mathbf{K}}_2 = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^T\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^T\mathbf{K}_2 + \begin{pmatrix}0\\ \widetilde{\alpha}_2\\ 0\end{pmatrix} \tag{A200}$$

- for $t < T$:

$$\mathbf{v}_t = \overline{\mathbf{v}}_0 + \mathbf{H}_{11}\mathbf{\Lambda}_1^t\mathbf{K}_1 + \mathbf{H}_{12}\mathbf{\Lambda}_2^t\mathbf{K}_2 \tag{A201}$$

where the constants $\mathbf{K}_1$, $\mathbf{K}_2$ and $\widetilde{\mathbf{K}}_2$ are defined in (A186), (A193) and (A194). Note that the solution formulas in case of a permanent raw materials price shock are similar to the corresponding solution of $\mathbf{v}$ in case of a temporary foreign price shock (cf. (A156) to (A158)), but they are obviously not equivalent.

New Keynesian Model with exogenous money stock

If the interest rate rule (13) is replaced by a monetary policy rule for the growth rate of money supply, the corresponding dynamic state equation results from the first difference of the LM equation

$$
\begin{aligned}
\Delta m_t - \pi_t^c &= l_1(y_t - y_{t-1}) + l_1\psi[(p_{R,t}^* - p_t^*) - (p_{R,t-1}^* - p_{t-1}^*)] \\
&\quad - l_2(\pi_{t+1}^c - \pi_t^c) + l_2\alpha(\tau_{t+1} - \tau_t) - l_2(i_t^* - i_{t-1}^*) \\
&\quad - (l_1\psi + l_2\alpha)(\tau_t - \tau_{t-1}) + l_2(\pi_{t+1}^* - \pi_t^*)
\end{aligned} \tag{A202}
$$

where

$$\Delta m_t = \Delta\overline{m} - \widetilde{v}_1(\pi_t^c - \overline{\pi}^c) - \widetilde{v}_2(q_t - \overline{q}) \tag{A203}$$
$$= \Delta\overline{m} + \widetilde{v}_1\overline{\pi}^c + \widetilde{v}_2(\overline{q} - d_0) - \widetilde{v}_1\pi_t^c - \widetilde{v}_2(y_t - \psi\tau_t + \psi(p_R^* - p_t^*))$$

Inserting the monetary policy rule into (A202) yields the state equation

$$\begin{aligned}
-\alpha\tau_{t+1} + \pi_{t+1}^c = {} & \frac{1}{l_2}(l_1 + \widetilde{v}_2)y_t - \frac{1}{l_2}\big((l_1 + \widetilde{v}_2)\psi + 2l_2\alpha\big)\tau_t \\
& + \frac{1}{l_2}(1 + l_2 + \widetilde{v}_1)\pi_t^c - \frac{l_1}{l_2}y_{t-1} + \frac{1}{l_2}(l_1\psi + l_2\alpha)\tau_{t-1} \\
& + \frac{1}{l_2}(l_1 + \widetilde{v}_2)\psi(p_{R,t}^* - p_t^*) - \frac{l_1\psi}{l_2}(p_{R,t-1}^* - p_{t-1}^*) \\
& - \frac{1}{l_2}\Delta\overline{m} - \frac{\widetilde{v}_1}{l_2}\overline{\pi}^c - \frac{\widetilde{v}_2}{l_2}(\overline{q} - d_0) + \pi_{t+1}^* - \pi_t^* - (i_t^* - i_{t-1}^*)
\end{aligned} \tag{A204}$$

Equation (A204) now replaces the dynamic state equation (A2) while the other state equations (A1) and (A3) remain unchanged. Obviously, the state matrix **B**, defined in (A6), does not change while the second row of the matrix **C** (see (A7)) has to be replaced by

$$\begin{aligned}
c_{21} &= \frac{1}{l_2}(l_1 + \widetilde{v}_2), & c_{22} &= -\frac{1}{l_2}\big((l_1 + \widetilde{v}_2)\psi + 2l_2\alpha\big) \\
c_{23} &= \frac{1}{l_2}(1 + l_2 + \widetilde{v}_1), & c_{24} &= -\frac{l_1}{l_2} \\
c_{25} &= \frac{1}{l_2}(l_1\psi + l_2\alpha), & c_{26} &= 0
\end{aligned} \tag{A205}$$

In the long run the inflation rates π and π^c as well as the nominal depreciation rate Δe are determined by the growth rate of domestic money supply:

$$\Delta\overline{m} = \overline{\pi} = \overline{\pi}^c = \Delta\overline{e} \tag{A206}$$

The input function k_{2t} is then given by (cf. (A9))

$$\begin{aligned}
k_{2t} = {} & \frac{1}{l_2}(l_1 + \widetilde{v}_2)\psi(p_{R,t}^* - p_t^*) - \frac{l_1\psi}{l_2}(p_{R,t-1}^* - p_{t-1}^*) + \pi_{t+1}^* - \pi_t^* \\
& - (i_t^* - i_{t-1}^*) - \frac{1}{l_2}(1 + \widetilde{v}_1)\overline{\pi}^c - \frac{\widetilde{v}_2}{l_2}\big(\overline{y} - \psi\overline{\tau} + \psi(\overline{p_R^* - p^*})\big)
\end{aligned} \tag{A207}$$

Assuming $\Delta\overline{m} = \overline{\pi} = \overline{\pi}^c = 0$ and $i_t^* = i_{t-1}^*$, (A207) can be rewritten as

$$k_{2t} = \overline{d}_2 + \phi_{2t} \tag{A208}$$

where

$$\overline{d}_2 = -\frac{\widetilde{v}_2}{l_2}(\overline{y} - \psi\overline{\tau}) \tag{A209}$$

and

$$\begin{aligned}
\phi_{2t} = {} & \frac{l_1}{l_2}\psi\big[(p_{R,t}^* - p_t^*) - (p_{R,t-1}^* - p_{t-1}^*)\big] \\
& + \frac{\widetilde{v}_2\psi}{l_2}\big[(p_{R,t}^* - p_t^*) - (\overline{p_R^* - p^*})\big] + \pi_{t+1}^* - \pi_t^*
\end{aligned} \tag{A210}$$

Note that on the assumption $\Delta\overline{m} = 0$ the steady state system (A23) to (A27) and the input functions k_{1t} and k_{3t} do not change. According to (A13) to (A22) the function ϕ_{2t} can be written as follows:

- In case $\beta_R = 1$:

$$\phi_{2t} = \begin{cases} 0 & \text{for } t < T-1 \\ 1-\mu^* & \text{for } t = T-1 \\ \frac{l_1}{l_2}\psi\mu^* - (1-\mu^*) & \text{for } t = T \\ 0 & \text{for } t > T \end{cases} \tag{A211}$$

- In case $\beta_R < 1$:

$$\phi_{2t} = \begin{cases} 0 & \text{for } t < T-1 \\ 1-\mu^* & \text{for } t = T-1 \\ \frac{1}{l_2}(l_1+\widetilde{v}_2)\psi\mu^* + (1-\mu^*)(\beta_R - 2) & \text{for } t = T \\ \big[\frac{\psi}{l_2}\mu^*\big(l_1(\beta_R-1)+\widetilde{v}_2\beta_R\big) & \\ \quad +(1-\mu^*)(\beta_R-1)^2\big]\beta_R^{t-T-1} & \text{for } t > T \end{cases}$$

In case $\beta_R = 1$ the input vector $\mathbf{B}^{-1}\boldsymbol{\phi}_t$ has the representation (A61) where $\widetilde{\alpha}_2$ and α_3 are again given by (A62) and (A65) respectively, while α_1 and α_2 are now of the form

$$\begin{aligned} \alpha_1 &= \widetilde{b}_{11}\phi_{1T} + \widetilde{b}_{12}\phi_{2T} + \widetilde{b}_{13}\phi_{3T} \qquad \text{(A212)} \\ &= -\frac{1}{|\mathbf{B}|}(a_2+a_3l_2)\alpha\mu(1-\omega)\left[\frac{l_1}{l_2}\psi\mu^* - (1-\mu^*)\right] - \frac{1}{|\mathbf{B}|}a_2\alpha(1-\mu)\mu^* \\ \alpha_2 &= \widetilde{b}_{22}\phi_{2T} + \widetilde{b}_{23}\phi_{3T} \qquad \text{(A213)} \\ &= \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\left(\mu(1-\omega)\left[\frac{l_1}{l_2}\psi\mu^* - (1-\mu^*)\right] + (1-\mu)\mu^*\right) \end{aligned}$$

In case $\beta_R < 1$ the vector $\mathbf{B}^{-1}\boldsymbol{\phi}_t$ takes the form (A66) where $\widetilde{\alpha}_2$, $\hat{\alpha}_3$ and $\overline{\phi}_3$ do not change (cf. (A62), (A69), (A72)) and the constants $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\overline{\phi}_1$ and $\overline{\phi}_2$ are now given by

$$\begin{aligned} \hat{\alpha}_1 &= \widetilde{b}_{11}\phi_{1T} + \widetilde{b}_{12}\phi_{2T} + \widetilde{b}_{13}\phi_{3T} \qquad \text{(A214)} \\ &= -\frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega)\big[(1-a_3l_1)\psi\mu^* - (a_2+a_3l_2)(1-\mu^*)(\beta_R-1)\big] \\ &\quad - \frac{1}{|\mathbf{B}|}(a_2+a_3l_2)\alpha\mu(1-\omega)\big[\frac{1}{l_2}(l_1+\widetilde{v}_2)\psi\mu^* + (1-\mu^*)(\beta_R-2)\big] \\ &\quad - \frac{1}{|\mathbf{B}|}a_2\alpha[\mu\delta\psi + (1-\mu)]\mu^* \\ &= -\frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega)(1-a_3l_1)\psi\mu^* - \frac{1}{|\mathbf{B}|}a_2\alpha[\mu\delta\psi+(1-\mu)]\mu^* \\ &\quad + \frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega)(a_2+a_3l_2)\big[(1-\mu^*) - \frac{1}{l_2}(l_1+\widetilde{v}_2)\psi\mu^*\big] \end{aligned}$$

$$
\begin{aligned}
\hat{\alpha}_2 &= \widetilde{b}_{22}\phi_{2T} + \widetilde{b}_{23}\phi_{3T} \qquad \text{(A215)}\\
&= \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\bigg(\mu(1-\omega)\big[\frac{1}{l_2}(l_1+\widetilde{v}_2)\psi\mu^* + (1-\mu^*)(\beta_R-2)\big]\\
&\quad + [\mu\delta\psi + (1-\mu)]\mu^*\bigg)
\end{aligned}
$$

$$
\begin{aligned}
\overline{\phi}_1 &= -\frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega)[(1-a_3l_1)\psi\mu^* - (a_2+a_3l_2)(1-\mu^*)(\beta_R-1)]\beta_R \qquad \text{(A216)}\\
&\quad - \frac{1}{|\mathbf{B}|}(a_2+a_3l_2)\alpha\mu(1-\omega)\bigg[\frac{\psi}{l_2}\mu^*(l_1(\beta_R-1)+\widetilde{v}_2\beta_R)\\
&\qquad + (1-\mu^*)(\beta_R-1)^2\bigg] - \frac{1}{|\mathbf{B}|}a_2\alpha[\mu\delta\psi\beta_R + (1-\mu)(\beta_R-1)]\mu^*\\
&= -\frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega)\bigg[(1-a_3l_1)\psi\mu^*\beta_R + (a_2+a_3l_2)\frac{\psi}{l_2}\mu^*(l_1(\beta_R-1)+\widetilde{v}_2\beta_R)\bigg]\\
&\quad + \frac{1}{|\mathbf{B}|}\alpha\mu(1-\omega)(a_2+a_3l_2)(1-\mu^*)(\beta_R-1)\\
&\quad - \frac{1}{|\mathbf{B}|}a_2\alpha[\mu\delta\psi\beta_R + (1-\mu)(\beta_R-1)]\mu^*
\end{aligned}
$$

$$
\begin{aligned}
\overline{\phi}_2 &= \frac{1}{|\mathbf{B}|}a_1(1-\Phi)\bigg[\mu(1-\omega)\bigg(\frac{\psi}{l_2}\mu^*(l_1(\beta_R-1)+\widetilde{v}_2\beta_R) \qquad \text{(A217)}\\
&\quad + (1-\mu^*)(\beta_R-1)^2\bigg) + [\mu\delta\psi\beta_R + (1-\mu)(\beta_R-1)]\mu^*\bigg]
\end{aligned}
$$

Perfect Stabilization of the CPI Inflation Rate

Perfect stabilization of the CPI inflation rate π^c is possible with the help of an interest rate rule that depends on the real appreciation rate $\Delta\tau_{t+1}$. The real interest rate definition and the uncovered interest parity condition yield the equation

$$
i_t - \pi^c_{t+1} = i^*_t - \alpha\Delta\tau_{t+1} - \pi^*_{t+1} \qquad \text{(A218)}
$$

The policy target

$$
\pi^c_{t+1} = \pi^c_t = \overline{\pi}^c = 0 \qquad \text{(A219)}
$$

then implies the interest rate rule

$$
i_t = i^*_t - \pi^*_{t+1} - \alpha(\tau_{t+1} - \tau_t) \qquad \text{(A220)}
$$

which differs from the Taylor-type rules. The dynamics of the stabilized system can be represented by the state equations (A1) and (A3) with $\pi^c_{t+1} = \pi^c_t = 0$:

$$
\begin{aligned}
b_{11}y_{t+1} + b_{12}\tau_{t+1} &= c_{11}y_t + c_{12}\tau_t + c_{14}y_{t-1} + k_{1t} \qquad \text{(A221)}\\
0 &= c_{31}y_t + c_{32}\tau_t + c_{35}\tau_{t-1} + k_{3t} \qquad \text{(A222)}
\end{aligned}
$$

(A222) implies

$$
y_{t+j} = -\frac{1}{c_{31}}(c_{32}\tau_{t+j} + c_{35}\tau_{t+j-1} + k_{3t+j}), \qquad j = -1, 0, 1 \qquad \text{(A223)}
$$

Inserting this equation in (A221) yields the following difference equation of order three in the jump variable τ:

$$\begin{aligned}\gamma_1 \tau_{t+1} = \gamma_2 \tau_t + \gamma_3 \tau_{t-1} + \gamma_4 \tau_{t-2} + b_{11} k_{3t+1} \\ - c_{11} k_{3t} - c_{14} k_{3t-1} + c_{31} k_{1t}\end{aligned} \tag{A224}$$

with

$$\begin{aligned}\gamma_1 &= -b_{11} c_{32} + c_{31} b_{12} \\ \gamma_2 &= b_{11} c_{35} - c_{11} c_{32} + c_{31} c_{12} \\ \gamma_3 &= -(c_{11} c_{35} + c_{14} c_{32}) \\ \gamma_4 &= -c_{14} c_{35}\end{aligned} \tag{A225}$$

Let

$$w_{1t} = \tau_{t-1} \tag{A226}$$

$$w_{2t} = w_{1t-1} = \tau_{t-2} \tag{A227}$$

then the state equation (A224) is equivalent to the system

$$\begin{pmatrix} \tau_{t+1} \\ w_{1t+1} \\ w_{2t+1} \end{pmatrix} = \begin{pmatrix} \gamma_2/\gamma_1 & \gamma_3/\gamma_1 & \gamma_4/\gamma_1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \tau_t \\ w_{1t} \\ w_{2t} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u_t \tag{A228}$$

with the input function

$$u_t = \frac{1}{\gamma_1}(b_{11} k_{3t+1} - c_{11} k_{3t} - c_{14} k_{3t-1} + c_{31} k_{1t}) \tag{A229}$$

The state matrix of the stabilized system (A228) has one unstable and two stable eigenvalues ($|r_1| > 1$, $|r_2| < 1$, $|r_3| < 1$) where r_1 is real and r_2 and r_3 are conjugate complex numbers. The system (A228) therefore exhibits saddle point behavior, since the auxiliary variables w_1 and w_2 are predetermined. The unique convergent solution time path for τ can be obtained by transforming (A228) into Jordan-canonical form given by (cf. (A73) to (A79))

$$\begin{pmatrix} \widetilde{\mathbf{x}}_{t+1} \\ \widetilde{\mathbf{z}}_{t+1} \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{\Lambda}}_1 & 0 \\ 0 & \widetilde{\mathbf{\Lambda}}_2 \end{pmatrix} \begin{pmatrix} \widetilde{\mathbf{x}}_t \\ \widetilde{\mathbf{z}}_t \end{pmatrix} + \begin{pmatrix} \widetilde{\mathbf{G}}_{11} & \widetilde{\mathbf{G}}_{12} \\ \widetilde{\mathbf{G}}_{21} & \widetilde{\mathbf{G}}_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u_t \tag{A230}$$

$$\begin{pmatrix} \widetilde{\mathbf{x}} \\ \widetilde{\mathbf{z}} \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{G}}_{11} & \widetilde{\mathbf{G}}_{12} \\ \widetilde{\mathbf{G}}_{21} & \widetilde{\mathbf{G}}_{22} \end{pmatrix} \begin{pmatrix} \tau \\ \mathbf{w} \end{pmatrix} \tag{A231}$$

$$\begin{pmatrix} \widetilde{\mathbf{G}}_{11} & \widetilde{\mathbf{G}}_{12} \\ \widetilde{\mathbf{G}}_{21} & \widetilde{\mathbf{G}}_{22} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} = \widetilde{\mathbf{G}} = \widetilde{\mathbf{H}}^{-1} \tag{A232}$$

$$\widetilde{\mathbf{H}} = \begin{pmatrix} \widetilde{\mathbf{H}}_{11} & \widetilde{\mathbf{H}}_{12} \\ \widetilde{\mathbf{H}}_{21} & \widetilde{\mathbf{H}}_{22} \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \tag{A233}$$

$$\widetilde{\mathbf{A}} = \widetilde{\mathbf{H}} \widetilde{\mathbf{\Lambda}} \widetilde{\mathbf{G}} \tag{A234}$$

$$\widetilde{\mathbf{\Lambda}} = \begin{pmatrix} \widetilde{\mathbf{\Lambda}}_1 & 0 \\ 0 & \widetilde{\mathbf{\Lambda}}_2 \end{pmatrix} = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{pmatrix} \tag{A235}$$

$$\begin{pmatrix} \tau \\ \mathbf{w} \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{H}}_{11} & \widetilde{\mathbf{H}}_{12} \\ \widetilde{\mathbf{H}}_{21} & \widetilde{\mathbf{H}}_{22} \end{pmatrix} \begin{pmatrix} \widetilde{\mathbf{x}} \\ \widetilde{\mathbf{z}} \end{pmatrix} \tag{A236}$$

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{A237}$$

$\widetilde{\mathbf{H}}$ consists of the (right-) eigenvectors of $\widetilde{\mathbf{A}}$, and $\widetilde{\mathbf{G}}$ denotes its inverse. According to the definition of k_{1t} and k_{3t} (cf. (A28) and (A30)) the input function u_t takes the following form (cf. (A38) to (A51)):

- In case $\beta_R = 1$:

$$u_t = \begin{cases} \overline{u}_0 & \text{for } t < T-1 \\ \overline{u}_0 + m_1 & \text{for } t = T-1 \\ \overline{u}_1 + m_2 & \text{for } t = T \\ \overline{u}_1 + m_3 & \text{for } t = T+1 \\ \overline{u}_1 & \text{for } t > T+1 \end{cases} \tag{A238}$$

where

$$\overline{u} = \frac{1}{\gamma_1}\big((b_{11} - c_{11} - c_{14})\overline{d}_3 + c_{31}\overline{d}_1\big) \tag{A239}$$

$$m_1 = \frac{b_{11}}{\gamma_1}(\overline{d}_{31} - \overline{d}_{30}) - \frac{1}{\gamma_1}\big(b_{11}(1-\mu)\mu^* + c_{31}(a_2 + a_3 l_2)(1-\mu^*)\big) \tag{A240}$$

$$m_2 = \frac{c_{14}}{\gamma_1}(\overline{d}_{31} - \overline{d}_{30}) + \frac{c_{11}}{\gamma_1}(1-\mu)\mu^* \tag{A241}$$

$$m_3 = \frac{c_{14}}{\gamma_1}(1-\mu)\mu^* \tag{A242}$$

$$\overline{d}_3 = \mu\delta(\overline{y} - \psi\overline{\tau}) \tag{A243}$$

$$\overline{d}_1 = -(1 - a_1 + b_1 - a_3 l_1)\overline{y} - (b_3 - (1 - a_3 l_1)\psi)\overline{\tau} \tag{A244}$$

- In case $\beta_R < 1$:

$$u_t = \begin{cases} \overline{u}_0 & \text{for } t < T-1 \\ \overline{u}_0 + \widetilde{m}_1 & \text{for } t = T-1 \\ \overline{u}_0 + \widetilde{m}_2 & \text{for } t = T \\ \overline{u}_0 + \widetilde{m}_3 & \text{for } t = T+1 \\ \overline{u}_0 + \widetilde{m}_4\beta_R^{t-T-1} & \text{for } t > T+1 \end{cases} \tag{A245}$$

where

$$\widetilde{m}_1 = -\frac{1}{\gamma_1}(b_{11}\lambda_2 + c_{31}\lambda_0) \tag{A246}$$

$$\widetilde{m}_2 = \frac{1}{\gamma_1}(-b_{11}\lambda_3 + c_{11}\lambda_2 + c_{31}\lambda_1) \tag{A247}$$

$$\widetilde{m}_3 = \frac{1}{\gamma_1}(-b_{11}\lambda_3\beta_R + c_{11}\lambda_3 + c_{14}\lambda_2 + c_{31}\lambda_1\beta_R) \tag{A248}$$

$$\widetilde{m}_4 = \frac{1}{\gamma_1}(-b_{11}\lambda_3\beta_R + c_{11}\lambda_3 + c_{14}\lambda_3\beta_R^{-1} + c_{31}\lambda_1\beta_R) \tag{A249}$$

$$\lambda_0 = (a_2 + a_3 l_2)(1 - \mu^*) \tag{A250}$$

$$\lambda_1 = (1 - a_3 l_1)\psi\mu^* - (a_2 + a_3 l_2)(1 - \mu^*)(\beta_R - 1) \tag{A251}$$

$$\lambda_2 = (\mu\delta\psi + (1 - \mu))\mu^* \tag{A252}$$

$$\lambda_3 = (\mu\delta\psi\beta_R + (1 - \mu)(\beta_R - 1))\mu^* \tag{A253}$$

We first discuss the canonical system (A230) in case of *permanent* raw materials price shocks ($\beta_R = 1$). For $t < T - 1$ it is equivalent to

$$\widetilde{\mathbf{x}}_{t+1} - \overline{\overline{\mathbf{x}}}_0 = \widetilde{\mathbf{\Lambda}}_1(\widetilde{\mathbf{x}}_t - \overline{\overline{\mathbf{x}}}_0) \qquad \text{for } t < T - 1 \tag{A254}$$

$$\widetilde{\mathbf{z}}_{t+1} - \overline{\overline{\mathbf{z}}}_0 = \widetilde{\mathbf{\Lambda}}_2(\widetilde{\mathbf{z}}_t - \overline{\overline{\mathbf{z}}}_0) \qquad \text{for } t < T - 1 \tag{A255}$$

where the steady-state values $\overline{\overline{\mathbf{x}}}$ and $\overline{\overline{\mathbf{z}}}$ are given by

$$\overline{\overline{\mathbf{x}}} = (\mathbf{I}_1 - \widetilde{\mathbf{\Lambda}}_1)^{-1}\widetilde{\mathbf{G}}_{11}\overline{u} \tag{A256}$$

$$\overline{\overline{\mathbf{z}}} = (\mathbf{I}_2 - \widetilde{\mathbf{\Lambda}}_2)^{-1}\widetilde{\mathbf{G}}_{21}\overline{u} \tag{A257}$$

For $t > T + 1$ it is equivalent to

$$\widetilde{\mathbf{x}}_{t+1} - \overline{\overline{\mathbf{x}}}_1 = \widetilde{\mathbf{\Lambda}}_1(\widetilde{\mathbf{x}}_t - \overline{\overline{\mathbf{x}}}_1) \qquad \text{for } t > T + 1 \tag{A258}$$

$$\widetilde{\mathbf{z}}_{t+1} - \overline{\overline{\mathbf{z}}}_1 = \widetilde{\mathbf{\Lambda}}_2(\widetilde{\mathbf{z}}_t - \overline{\overline{\mathbf{z}}}_1) \qquad \text{for } t > T + 1 \tag{A259}$$

The bounded solution for $t < T - 1$ and $t > T + 1$ is given by (cf. (A169) to (A175))

$$\widetilde{\mathbf{x}}_t = \overline{\overline{\mathbf{x}}}_0 + \widetilde{\mathbf{\Lambda}}_1^t\mathbf{K}_1 \qquad \text{for } t < T - 1 \tag{A260}$$

$$\widetilde{\mathbf{z}}_t = \overline{\overline{\mathbf{z}}}_0 + \widetilde{\mathbf{\Lambda}}_2^t\mathbf{K}_2 \qquad \text{for } t < T - 1 \tag{A261}$$

$$\widetilde{\mathbf{x}}_t = \overline{\overline{\mathbf{x}}}_1 \qquad \text{for } t > T + 1 \tag{A262}$$

$$\widetilde{\mathbf{z}}_t = \overline{\overline{\mathbf{z}}}_1 + \widetilde{\mathbf{\Lambda}}_2^t\widetilde{\mathbf{K}}_2 \qquad \text{for } t > T + 1 \tag{A263}$$

The solution for $t = T + 1$, $t = T$ and $t = T - 1$ can be derived from these solution formulas and the state equations (A230) where u_t is given by (A238) in case $\beta_R = 1$. Since

$$\widetilde{\mathbf{x}}_{T+2} = \widetilde{\mathbf{\Lambda}}_1\widetilde{\mathbf{x}}_{T+1} + \widetilde{\mathbf{G}}_{11}u_{T+1} \tag{A264}$$

we get the forward-looking solution

$$\begin{aligned}\widetilde{\mathbf{x}}_{T+1} &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{x}}_{T+2} - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}u_{T+1} \\ &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}\overline{\widetilde{\mathbf{x}}}_1 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}(\overline{u}_1 + m_3) = \overline{\widetilde{\mathbf{x}}}_1 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_3\end{aligned} \tag{A265}$$

(A265) implies

$$\begin{aligned}\widetilde{\mathbf{x}}_{T} &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{x}}_{T+1} - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}u_{T} \\ &= \widetilde{\boldsymbol{\Lambda}}_1(\overline{\widetilde{\mathbf{x}}}_1 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_3) - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}(\overline{u}_1 + m_2) \\ &= \overline{\widetilde{\mathbf{x}}}_1 - \widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}m_3 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_2\end{aligned} \tag{A266}$$

For $t = T-1$ the backward-looking solution of the variable $\widetilde{\mathbf{x}}$ is given by

$$\begin{aligned}\widetilde{\mathbf{x}}_{T-1} &= \widetilde{\boldsymbol{\Lambda}}_1\widetilde{\mathbf{x}}_{T-2} + \widetilde{\mathbf{G}}_{11}u_{T-2} \\ &= \widetilde{\boldsymbol{\Lambda}}_1(\overline{\widetilde{\mathbf{x}}}_0 + \widetilde{\boldsymbol{\Lambda}}_1^{T-2}\mathbf{K}_1) + \widetilde{\mathbf{G}}_{11}\overline{u}_0 = \overline{\widetilde{\mathbf{x}}}_0 + \widetilde{\boldsymbol{\Lambda}}_1^{T-1}\mathbf{K}_1\end{aligned} \tag{A267}$$

while the forward-looking solution takes the form

$$\begin{aligned}\widetilde{\mathbf{x}}_{T-1} &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{x}}_{T} - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}(\overline{u}_0 + m_1) \\ &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}(\overline{\widetilde{\mathbf{x}}}_1 - \widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}m_3 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_2) \\ &\quad - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\overline{u}_0 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_1 \\ &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}(\overline{\widetilde{\mathbf{x}}}_1 - \overline{\widetilde{\mathbf{x}}}_0) + \widetilde{\boldsymbol{\Lambda}}_1^{-1}(\overline{\widetilde{\mathbf{x}}}_0 - \widetilde{\mathbf{G}}_{11}\overline{u}_0) \\ &\quad - \widetilde{\boldsymbol{\Lambda}}_1^{-3}\widetilde{\mathbf{G}}_{11}m_3 - \widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}m_2 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_1 \\ &= \widetilde{\boldsymbol{\Lambda}}_1^{-1}d\overline{\widetilde{\mathbf{x}}} + \overline{\widetilde{\mathbf{x}}}_0 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}[\widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}m_3 + \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_2 + \widetilde{\mathbf{G}}_{11}m_1]\end{aligned} \tag{A268}$$

Since (A267) and (A268) must be equivalent, the constant $\mathbf{K}_1$ is given by

$$\mathbf{K}_1 = \widetilde{\boldsymbol{\Lambda}}_1^{-T}[d\overline{\widetilde{\mathbf{x}}} - \widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}m_3 - \widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_2 - \widetilde{\mathbf{G}}_{11}m_1] \tag{A269}$$

The corresponding solution formulas for the variable $\widetilde{\mathbf{z}}$ can be derived as follows:

$$\begin{aligned}\widetilde{\mathbf{z}}_{T-1} &= \widetilde{\boldsymbol{\Lambda}}_2\widetilde{\mathbf{z}}_{T-2} + \widetilde{\mathbf{G}}_{21}u_{T-2} \\ &= \widetilde{\boldsymbol{\Lambda}}_2^{T-1}\mathbf{K}_2 + \widetilde{\boldsymbol{\Lambda}}_2\overline{\widetilde{\mathbf{z}}}_0 + \widetilde{\mathbf{G}}_{21}\overline{u}_0 = \overline{\widetilde{\mathbf{z}}}_0 + \widetilde{\boldsymbol{\Lambda}}_2^{T-1}\mathbf{K}_2\end{aligned} \tag{A270}$$

For $t = T$ the backward-looking solution is given by

$$\begin{aligned}\widetilde{\mathbf{z}}_{T} &= \widetilde{\boldsymbol{\Lambda}}_2\widetilde{\mathbf{z}}_{T-1} + \widetilde{\mathbf{G}}_{21}u_{T-1} = \widetilde{\boldsymbol{\Lambda}}_2(\overline{\widetilde{\mathbf{z}}}_0 + \widetilde{\boldsymbol{\Lambda}}_2^{T-1}\mathbf{K}_2) + \widetilde{\mathbf{G}}_{21}(\overline{u}_0 + m_1) \\ &= \overline{\widetilde{\mathbf{z}}}_0 + \widetilde{\mathbf{G}}_{21}m_1 + \widetilde{\boldsymbol{\Lambda}}_2^{T}\mathbf{K}_2\end{aligned} \tag{A271}$$

while for $t = T+1$ it takes the form

$$\begin{aligned}\widetilde{\mathbf{z}}_{T+1} &= \widetilde{\boldsymbol{\Lambda}}_2\widetilde{\mathbf{z}}_{T} + \widetilde{\mathbf{G}}_{21}u_{T} \\ &= \widetilde{\boldsymbol{\Lambda}}_2\overline{\widetilde{\mathbf{z}}}_0 + \widetilde{\boldsymbol{\Lambda}}_2\widetilde{\mathbf{G}}_{21}m_1 + \widetilde{\boldsymbol{\Lambda}}_2^{T+1}\mathbf{K}_2 + \widetilde{\mathbf{G}}_{21}\overline{u}_1 + \widetilde{\mathbf{G}}_{21}m_2 \\ &= \widetilde{\boldsymbol{\Lambda}}_2\overline{\widetilde{\mathbf{z}}}_1 + \widetilde{\mathbf{G}}_{21}\overline{u}_1 - \widetilde{\boldsymbol{\Lambda}}_2(\overline{\widetilde{\mathbf{z}}}_1 - \overline{\widetilde{\mathbf{z}}}_0) + \widetilde{\boldsymbol{\Lambda}}_2^{T+1}\mathbf{K}_2 \\ &\quad + \widetilde{\boldsymbol{\Lambda}}_2\widetilde{\mathbf{G}}_{21}m_1 + \widetilde{\mathbf{G}}_{21}m_2 \\ &= \overline{\widetilde{\mathbf{z}}}_1 - \widetilde{\boldsymbol{\Lambda}}_2 d\overline{\widetilde{\mathbf{z}}} + \widetilde{\boldsymbol{\Lambda}}_2^{T+1}\mathbf{K}_2 + \widetilde{\boldsymbol{\Lambda}}_2\widetilde{\mathbf{G}}_{21}m_1 + \widetilde{\mathbf{G}}_{21}m_2\end{aligned} \tag{A272}$$

An alternative solution representation for $\widetilde{\mathbf{z}}_{T+1}$ follows from the equation

$$\widetilde{\mathbf{z}}_{T+2} = \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{z}}_{T+1} + \widetilde{\mathbf{G}}_{21} u_{T+1} \tag{A273}$$

and (A263):

$$\begin{aligned} \widetilde{\mathbf{z}}_{T+1} &= \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{z}}_{T+2} - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} u_{T+1} \\ &= \widetilde{\mathbf{\Lambda}}_2^{-1} \overline{\widetilde{\mathbf{z}}}_1 + \widetilde{\mathbf{\Lambda}}_2^{T+1} \widetilde{\mathbf{K}}_2 - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} (\overline{u}_1 + m_3) \\ &= \overline{\widetilde{\mathbf{z}}}_1 - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} m_3 + \widetilde{\mathbf{\Lambda}}_2^{T+1} \widetilde{\mathbf{K}}_2 \end{aligned} \tag{A274}$$

Since (A272) and (A274) are equivalent, we get the equation

$$\mathbf{K}_2 - \widetilde{\mathbf{K}}_2 = \widetilde{\mathbf{\Lambda}}_2^{-T} [d\overline{\widetilde{\mathbf{z}}} - \widetilde{\mathbf{G}}_{21} m_1 - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} m_2 - \widetilde{\mathbf{\Lambda}}_2^{-2} \widetilde{\mathbf{G}}_{21} m_3] \tag{A275}$$

Note that condition (A275) also follows from the equality of (A271) and

$$\begin{aligned} \widetilde{\mathbf{z}}_T &= \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{z}}_{T+1} - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} u_T \\ &= \widetilde{\mathbf{\Lambda}}_2^{-1} [\overline{\widetilde{\mathbf{z}}}_1 - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} m_3 + \widetilde{\mathbf{\Lambda}}_2^{T+1} \widetilde{\mathbf{K}}_2] - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} (\overline{u}_1 + m_2) \\ &= \overline{\widetilde{\mathbf{z}}}_1 + \widetilde{\mathbf{\Lambda}}_2^{T} \widetilde{\mathbf{K}}_2 - \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} m_2 - \widetilde{\mathbf{\Lambda}}_2^{-2} \widetilde{\mathbf{G}}_{21} m_3 \end{aligned} \tag{A276}$$

After having derived the solution of the transformed state vector $(\widetilde{\mathbf{x}}, \widetilde{\mathbf{z}})$ we can use (A236) to determine the solution time path of the terms of trade τ in case $\beta_R = 1$. The time path for y_t then follows from equation (A223). For $t \leq T-1$ we get

$$\tau_t = \overline{\tau}_0 + \widetilde{\mathbf{H}}_{11} \widetilde{\mathbf{\Lambda}}_1^t \mathbf{K}_1 + \widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^t \mathbf{K}_2 \qquad \text{for } t \leq T-1 \tag{A277}$$

where $\mathbf{K}_1$ is defined by (A269) and $\mathbf{K}_2$ follows from the initial condition of the predetermined state vector $\mathbf{w}$ (cf. (A193)):

$$\mathbf{K}_2 = -\widetilde{\mathbf{H}}_{22}^{-1} \widetilde{\mathbf{H}}_{21} \mathbf{K}_1 \tag{A278}$$

For $t > T+1$ the bounded solution of the state variable τ is given by

$$\tau_t = \overline{\tau}_1 + \widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^t \widetilde{\mathbf{K}}_2 \qquad \text{for } t > T+1 \tag{A279}$$

where $\widetilde{\mathbf{K}}_2$ follows from (A275) and (A278). For $t = T+1$ we get

$$\begin{aligned} \tau_{T+1} &= \overline{\tau}_1 + \widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^{T+1} \widetilde{\mathbf{K}}_2 - \widetilde{\mathbf{H}}_{11} \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} m_3 - \widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} m_3 \\ &= \overline{\tau}_1 + \widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^{T+1} \widetilde{\mathbf{K}}_2 \end{aligned} \tag{A280}$$

since

$$\widetilde{\mathbf{H}}_{11} \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} = -\widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^{-1} \widetilde{\mathbf{G}}_{21} \tag{A281}$$

Moreover,

$$\widetilde{\mathbf{H}}_{11} \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} = -\widetilde{\mathbf{H}}_{12} \widetilde{\mathbf{\Lambda}}_2^{-2} \widetilde{\mathbf{G}}_{21} \tag{A282}$$

(A281) and (A282) imply that for $t = T$ the solution of τ is given by

$$\begin{aligned}\tau_T &= \bar{\tau}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^T\widetilde{\mathbf{K}}_2 - \widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}m_2 - \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^{-1}\widetilde{\mathbf{G}}_{21}m_2 \\ &\quad - \widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}m_3 - \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21}m_3 \\ &= \bar{\tau}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^T\widetilde{\mathbf{K}}_2 - \underbrace{[\widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11} + \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21}]}_{0}m_3 \\ &= \bar{\tau}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^T\widetilde{\mathbf{K}}_2\end{aligned} \tag{A283}$$

To show (A281) and (A282) note that

$$\begin{pmatrix}\widetilde{\mathbf{H}}_{11} & \widetilde{\mathbf{H}}_{12} \\ \widetilde{\mathbf{H}}_{21} & \widetilde{\mathbf{H}}_{22}\end{pmatrix}\begin{pmatrix}\widetilde{\mathbf{G}}_{11} & \widetilde{\mathbf{G}}_{12} \\ \widetilde{\mathbf{G}}_{21} & \widetilde{\mathbf{G}}_{22}\end{pmatrix} = \begin{pmatrix}\mathbf{I}_1 & 0 \\ 0 & \mathbf{I}_2\end{pmatrix} \tag{A284}$$

implying

$$\widetilde{\mathbf{H}}_{22}\widetilde{\mathbf{G}}_{21} = -\widetilde{\mathbf{H}}_{21}\widetilde{\mathbf{G}}_{11} \tag{A285}$$

or

$$\begin{pmatrix}h_{22} & h_{23} \\ h_{32} & h_{33}\end{pmatrix}\begin{pmatrix}g_{21} \\ g_{31}\end{pmatrix} = -\begin{pmatrix}h_{21} \\ h_{31}\end{pmatrix}g_{11} \tag{A286}$$

Then

$$h_{22}g_{21} + h_{23}g_{31} = -h_{21}g_{11} \tag{A287}$$

$$h_{32}g_{21} + h_{33}g_{31} = -h_{31}g_{11} \tag{A288}$$

For $t < T - 1$ the solution of the predetermined state vector $\mathbf{w}$ is given by

$$\mathbf{w}_t = \begin{pmatrix}w_{1t} \\ w_{2t}\end{pmatrix} = \begin{pmatrix}\bar{\tau}_0 \\ \bar{\tau}_0\end{pmatrix} + \widetilde{\mathbf{H}}_{21}\widetilde{\boldsymbol{\Lambda}}_1^t\mathbf{K}_1 + \widetilde{\mathbf{H}}_{22}\widetilde{\boldsymbol{\Lambda}}_2^t\mathbf{K}_2 \tag{A289}$$

Since $w_{1t} = \tau_{t-1}$ and $w_{2t} = \tau_{t-2}$, the following equations must also hold for $t < T - 1$:

$$w_{1t} = \tau_{t-1} = \bar{\tau}_0 + \widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1^{t-1}\mathbf{K}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^{t-1}\mathbf{K}_2 \tag{A290}$$

$$w_{2t} = \tau_{t-2} = \bar{\tau}_0 + \widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1^{t-2}\mathbf{K}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2^{t-2}\mathbf{K}_2 \tag{A291}$$

Comparing these equations with (A289) yields

$$\begin{aligned}&\widetilde{\mathbf{H}}_{21}\widetilde{\boldsymbol{\Lambda}}_1^2\widetilde{\boldsymbol{\Lambda}}_1^{t-2}\mathbf{K}_1 + \widetilde{\mathbf{H}}_{22}\widetilde{\boldsymbol{\Lambda}}_2^2\widetilde{\boldsymbol{\Lambda}}_2^{t-2}\mathbf{K}_2 = \\ &\quad\begin{pmatrix}\widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1 \\ \widetilde{\mathbf{H}}_{11}\end{pmatrix}\widetilde{\boldsymbol{\Lambda}}_1^{t-2}\mathbf{K}_1 + \begin{pmatrix}\widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2 \\ \widetilde{\mathbf{H}}_{12}\end{pmatrix}\widetilde{\boldsymbol{\Lambda}}_2^{t-2}\mathbf{K}_2\end{aligned} \tag{A292}$$

implying

$$\begin{pmatrix}\widetilde{\mathbf{H}}_{11}\widetilde{\boldsymbol{\Lambda}}_1 \\ \widetilde{\mathbf{H}}_{11}\end{pmatrix} = \widetilde{\mathbf{H}}_{21}\widetilde{\boldsymbol{\Lambda}}_1^2, \qquad \begin{pmatrix}\widetilde{\mathbf{H}}_{12}\widetilde{\boldsymbol{\Lambda}}_2 \\ \widetilde{\mathbf{H}}_{12}\end{pmatrix} = \widetilde{\mathbf{H}}_{22}\widetilde{\boldsymbol{\Lambda}}_2^2 \tag{A293}$$

According to (A233) and (A235) the equations (A293) are equivalent to

$$h_{11}r_1 = h_{21}r_1^2 \tag{A294}$$
$$h_{11} = h_{31}r_1^2 \tag{A295}$$
$$h_{12}r_2 = h_{22}r_2^2 \tag{A296}$$
$$h_{13}r_3 = h_{23}r_3^2 \tag{A297}$$

implying

$$h_{11} = h_{21}r_1 \tag{A298}$$
$$h_{21} = h_{31}r_1 \tag{A299}$$
$$h_{12} = h_{22}r_2 \tag{A300}$$
$$h_{13} = h_{23}r_3 \tag{A301}$$
$$h_{22} = h_{32}r_2 \tag{A302}$$
$$h_{23} = h_{33}r_3 \tag{A303}$$

Then the left-hand side of (A281) is given by

$$\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11} = h_{11}r_1^{-1}g_{11} = h_{21}r_1r_1^{-1}g_{11} = h_{21}g_{11} \tag{A304}$$

Using (A287), (A300) and (A301), the right-hand side of (A281) can be written as

$$\begin{aligned}-\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-1}\widetilde{\mathbf{G}}_{21} &= -(h_{12}, h_{13})\begin{pmatrix} r_2^{-1} & 0 \\ 0 & r_3^{-1}\end{pmatrix}\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} \\ &= -(h_{12}r_2^{-1}, h_{13}r_3^{-1})\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} = -(h_{22}, h_{23})\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} \\ &= -h_{22}g_{21} - h_{23}g_{31} = h_{21}g_{11} = \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\end{aligned} \tag{A305}$$

To show (A282) note that the left-hand side is given by

$$\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11} = h_{11}r_1^{-2}g_{11} = h_{21}r_1(r_1^{-2}g_{11}) = h_{21}r_1^{-1}g_{11} = h_{31}g_{11} \tag{A306}$$

(cf. (A299)). The right-hand side of (A282) is given by

$$\begin{aligned}-\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21} &= -(h_{12}, h_{13})\begin{pmatrix} r_2^{-2} & 0 \\ 0 & r_3^{-2}\end{pmatrix}\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} \\ &= -(h_{12}r_2^{-2}, h_{13}r_3^{-2})\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} \\ &= -(h_{22}r_2^{-1}, h_{23}r_3^{-1})\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} \\ &= -(h_{32}, h_{33})\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix} = -(h_{32}g_{21} + h_{33}g_{31}) = h_{31}g_{11}\end{aligned} \tag{A307}$$

(cf. (A288), (A302), (A303)). Therefore, (A282) holds.

We now analyze the canonical system (A230) in case of *temporary* raw materials price shocks ($\beta_R < 1$). For $t < T-1$ its solution is given by (A260) and (A261). For $t > T+1$ the forward-looking solution of $\widetilde{\mathbf{x}}$ is given by (cf. (A105), (A106))

$$\begin{aligned}\widetilde{\mathbf{x}}_t &= \overline{\overline{\mathbf{x}}} - \sum_{s=t}^{\infty} \widetilde{\mathbf{\Lambda}}_1^{t-s-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_4 \beta_R^{s-T-1} \\ &= \overline{\overline{\mathbf{x}}} - g_{11} \widetilde{m}_4 \sum_{s=t}^{\infty} r_1^{t-s-1} \beta_R^{s-T-1} \\ &= \overline{\overline{\mathbf{x}}} - \frac{g_{11}\widetilde{m}_4}{r_1 - \beta_R} \beta_R^{t-(T+1)} \qquad \text{for } t > T+1\end{aligned} \tag{A308}$$

implying

$$\widetilde{\mathbf{x}}_{T+2} = \overline{\overline{\mathbf{x}}} - \frac{g_{11}\widetilde{m}_4}{r_1 - \beta_R} \beta_R \tag{A309}$$

Since

$$\widetilde{\mathbf{x}}_{T+2} = \widetilde{\mathbf{\Lambda}}_1 \widetilde{\mathbf{x}}_{T+1} + \widetilde{\mathbf{G}}_{11} u_{T+1} \tag{A310}$$

we get

$$\begin{aligned}\widetilde{\mathbf{x}}_{T+1} &= \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{x}}_{T+2} - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} u_{T+1} \\ &= \widetilde{\mathbf{\Lambda}}_1^{-1} \overline{\overline{\mathbf{x}}} - \widetilde{\mathbf{\Lambda}}_1^{-1} \frac{g_{11}\widetilde{m}_4}{r_1 - \beta_R} \beta_R - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} (\overline{u}_0 + \widetilde{m}_3) \\ &= \overline{\overline{\mathbf{x}}} - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_3 - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \frac{\widetilde{m}_4}{r_1 - \beta_R} \beta_R\end{aligned} \tag{A311}$$

The forward-looking solution of $\widetilde{\mathbf{x}}_T$ is then given by

$$\begin{aligned}\widetilde{\mathbf{x}}_T &= \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{x}}_{T+1} - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} u_T \\ &= \widetilde{\mathbf{\Lambda}}_1^{-1} \overline{\overline{\mathbf{x}}} - \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \widetilde{m}_3 - \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \frac{\widetilde{m}_4}{r_1 - \beta_R} \beta_R \\ &\quad - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \overline{u}_0 - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_2 \\ &= \overline{\overline{\mathbf{x}}} - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_2 - \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \widetilde{m}_3 - \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \frac{\widetilde{m}_4}{r_1 - \beta_R} \beta_R\end{aligned} \tag{A312}$$

For $t = T-1$ we get

$$\begin{aligned}\widetilde{\mathbf{x}}_{T-1} &= \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{x}}_T - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} u_{T-1} \\ &= \widetilde{\mathbf{\Lambda}}_1^{-1} \overline{\overline{\mathbf{x}}} - \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \widetilde{m}_2 - \widetilde{\mathbf{\Lambda}}_1^{-3} \widetilde{\mathbf{G}}_{11} \widetilde{m}_3 - \widetilde{\mathbf{\Lambda}}_1^{-3} \widetilde{\mathbf{G}}_{11} \frac{\widetilde{m}_4}{r_1 - \beta_R} \beta_R \\ &\quad - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \overline{u}_0 - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_1 \\ &= \overline{\overline{\mathbf{x}}} - \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_1 - \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \widetilde{m}_2 - \widetilde{\mathbf{\Lambda}}_1^{-3} \widetilde{\mathbf{G}}_{11} \widetilde{m}_3 - \widetilde{\mathbf{\Lambda}}_1^{-3} \widetilde{\mathbf{G}}_{11} \frac{\widetilde{m}_4}{r_1 - \beta_R} \beta_R\end{aligned} \tag{A313}$$

The backward-looking solution at time $t = T - 1$ is given by

$$\widetilde{\mathbf{x}}_{T-1} = \widetilde{\mathbf{\Lambda}}_1 \widetilde{\mathbf{x}}_{T-2} + \widetilde{\mathbf{G}}_{11} u_{T-2} \\ = \widetilde{\mathbf{\Lambda}}_1 \overline{\overline{\mathbf{x}}} + \widetilde{\mathbf{\Lambda}}_1^{T-1} \mathbf{K}_1 + \widetilde{\mathbf{G}}_{11} \overline{u}_0 = \overline{\overline{\mathbf{x}}} + \widetilde{\mathbf{\Lambda}}_1^{T-1} \mathbf{K}_1 \tag{A314}$$

Since (A313) and (A314) are equivalent, the constant $\mathbf{K}_1$ must satisfy the condition

$$\mathbf{K}_1 = -\widetilde{\mathbf{\Lambda}}_1^{-T} \big[\widetilde{\mathbf{G}}_{11} \widetilde{m}_1 + \widetilde{\mathbf{\Lambda}}_1^{-1} \widetilde{\mathbf{G}}_{11} \widetilde{m}_2 \\ + \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \widetilde{m}_3 + \widetilde{\mathbf{\Lambda}}_1^{-2} \widetilde{\mathbf{G}}_{11} \frac{\widetilde{m}_4}{r_1 - \beta_R} \beta_R \big] \tag{A315}$$

The next step is the development of the solution time path of the transformed vector $\widetilde{\mathbf{z}}$. Since

$$\widetilde{\mathbf{z}}_{T-2} = \overline{\overline{\mathbf{z}}} + \widetilde{\mathbf{\Lambda}}_2^{T-2} \mathbf{K}_2 \tag{A316}$$

the backward-looking solution for $t = T - 1$, T, $T + 1$ and $T + 2$ is given by

$$\widetilde{\mathbf{z}}_{T-1} = \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{z}}_{T-2} + \widetilde{\mathbf{G}}_{21} u_{T-2} = \overline{\overline{\mathbf{z}}} + \widetilde{\mathbf{\Lambda}}_2^{T-1} \mathbf{K}_2 \tag{A317}$$

$$\widetilde{\mathbf{z}}_{T} = \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{z}}_{T-1} + \widetilde{\mathbf{G}}_{21} u_{T-1} = \overline{\overline{\mathbf{z}}} + \widetilde{\mathbf{\Lambda}}_2^{T} \mathbf{K}_2 + \widetilde{\mathbf{G}}_{21} \widetilde{m}_1 \tag{A318}$$

$$\widetilde{\mathbf{z}}_{T+1} = \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{z}}_{T} + \widetilde{\mathbf{G}}_{21} u_{T} \\ = \overline{\overline{\mathbf{z}}} + \widetilde{\mathbf{\Lambda}}_2^{T+1} \mathbf{K}_2 + \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{G}}_{21} \widetilde{m}_1 + \widetilde{\mathbf{G}}_{21} \widetilde{m}_2 \tag{A319}$$

$$\widetilde{\mathbf{z}}_{T+2} = \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{z}}_{T+1} + \widetilde{\mathbf{G}}_{21} u_{T+1} \\ = \overline{\overline{\mathbf{z}}} + \widetilde{\mathbf{\Lambda}}_2^{T+2} \mathbf{K}_2 + \widetilde{\mathbf{\Lambda}}_2^{2} \widetilde{\mathbf{G}}_{21} \widetilde{m}_1 + \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{G}}_{21} \widetilde{m}_2 + \widetilde{\mathbf{G}}_{21} \widetilde{m}_3 \tag{A320}$$

Let $\widetilde{\boldsymbol{\theta}}_t$ be the input function of the difference equation (cf. (A230))

$$\widetilde{\mathbf{z}}_{t+1} = \widetilde{\mathbf{\Lambda}}_2 \widetilde{\mathbf{z}}_t + \widetilde{\mathbf{G}}_{21} u_t \tag{A321}$$

Then the definition of u_t in case $\beta_R < 1$ implies

$$\widetilde{\boldsymbol{\theta}}_t = \widetilde{\mathbf{G}}_{21} u_t = \begin{cases} 0 & \text{for } t < T - 1 \\ \widetilde{\mathbf{G}}_{21} \widetilde{m}_1 & \text{for } t = T - 1 \\ \widetilde{\mathbf{G}}_{21} \widetilde{m}_2 & \text{for } t = T \\ \widetilde{\mathbf{G}}_{21} \widetilde{m}_3 & \text{for } t = T + 1 \\ \widetilde{\mathbf{G}}_{21} \widetilde{m}_4 \beta_R^{t-T-1} & \text{for } t > T + 1 \end{cases} \tag{A322}$$

The general solution of (A321) for $t > T+1$ is then given by

$$\begin{aligned}\widetilde{\mathbf{z}}_t &= \overline{\widetilde{\mathbf{z}}} + \widetilde{\boldsymbol{\Lambda}}_2^t \mathbf{K}_2 + \sum_{s=T-1}^{t-1} \widetilde{\boldsymbol{\Lambda}}_2^{t-s-1} \widetilde{\boldsymbol{\theta}}_s \\ &= \overline{\widetilde{\mathbf{z}}} + \widetilde{\boldsymbol{\Lambda}}_2^t \mathbf{K}_2 + \widetilde{\boldsymbol{\Lambda}}_2^{t-T} \widetilde{\boldsymbol{\theta}}_{T-1} + \widetilde{\boldsymbol{\Lambda}}_2^{t-T-1} \widetilde{\boldsymbol{\theta}}_T \\ &\quad + \widetilde{\boldsymbol{\Lambda}}_2^{t-T-2} \widetilde{\boldsymbol{\theta}}_{T+1} + \sum_{s=T+2}^{t-1} \widetilde{\boldsymbol{\Lambda}}_2^{t-s-1} \widetilde{\boldsymbol{\theta}}_s \\ &= \overline{\widetilde{\mathbf{z}}} + \widetilde{\boldsymbol{\Lambda}}_2^t \mathbf{K}_2 + \widetilde{\boldsymbol{\Lambda}}_2^{t-T} \widetilde{\mathbf{G}}_{21} \widetilde{m}_1 + \widetilde{\boldsymbol{\Lambda}}_2^{t-T-1} \widetilde{\mathbf{G}}_{21} \widetilde{m}_2 \\ &\quad + \widetilde{\boldsymbol{\Lambda}}_2^{t-T-2} \widetilde{\mathbf{G}}_{21} \widetilde{m}_3 + \sum_{s=T+2}^{t-1} \widetilde{\boldsymbol{\Lambda}}_2^{t-s-1} \widetilde{\mathbf{G}}_{21} \widetilde{m}_4 \beta_R^{s-T-1}\end{aligned} \tag{A323}$$

Since

$$\sum_{s=T+2}^{t-1} x^s = \sum_{s=0}^{t-1} x^s - \sum_{s=0}^{T+1} x^s = \frac{1-x^t}{1-x} - \frac{1-x^{T+2}}{1-x} = \frac{x^{T+2}-x^t}{1-x} \tag{A324}$$

we get for $j = 2, 3$:

$$\begin{aligned}\sum_{s=T+2}^{t-1} r_j^{t-s-1} g_{j1} \widetilde{m}_4 \beta_R^{s-T-1} &= r_j^{t-1} g_{j1} \widetilde{m}_4 \beta_R^{-T-1} \sum_{s=T+2}^{t-1} \left(\frac{\beta_R}{r_j}\right)^s \\ &= r_j^{t-1} g_{j1} \widetilde{m}_4 \beta_R^{-T-1} \left[\frac{\left(\frac{\beta_R}{r_j}\right)^{T+2} - \left(\frac{\beta_R}{r_j}\right)^t}{1-\frac{\beta_R}{r_j}}\right] \\ &= r_j^t g_{j1} \widetilde{m}_4 \frac{1}{r_j - \beta_R} \frac{1}{\beta_R^{T+1}} \left[\left(\frac{\beta_R}{r_j}\right)^{T+2} - \left(\frac{\beta_R}{r_j}\right)^t\right] \\ &= g_{j1} \widetilde{m}_4 \frac{1}{r_j - \beta_R} \left[r_j^{t-T-2} \beta_R - \beta_R^{t-T-1}\right]\end{aligned} \tag{A325}$$

and therefore

$$\begin{aligned}\sum_{s=T+2}^{t-1} \widetilde{\boldsymbol{\Lambda}}_2^{t-s-1} \widetilde{\mathbf{G}}_{21} \widetilde{m}_4 \beta_R^{s-T-1} &= \widetilde{\boldsymbol{\Lambda}}_2^{t-T-2} \begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R) \end{pmatrix} \beta_R \\ &\quad - \begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R) \end{pmatrix} \beta_R^{t-T-1}\end{aligned} \tag{A326}$$

(A323) then implies

$$\begin{aligned}\widetilde{\mathbf{z}}_t = \overline{\widetilde{\mathbf{z}}} &+ \widetilde{\boldsymbol{\Lambda}}_2^t \mathbf{K}_2 + \widetilde{\boldsymbol{\Lambda}}_2^{t-T} \widetilde{\mathbf{G}}_{21} \widetilde{m}_1 + \widetilde{\boldsymbol{\Lambda}}_2^{t-T-1} \widetilde{\mathbf{G}}_{21} \widetilde{m}_2 \\ &+ \widetilde{\boldsymbol{\Lambda}}_2^{t-T-2} \widetilde{\mathbf{G}}_{21} \widetilde{m}_3 + \widetilde{\boldsymbol{\Lambda}}_2^{t-T-2} \begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R) \end{pmatrix} \beta_R \\ &- \begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R) \end{pmatrix} \beta_R^{t-T-1} \qquad \text{for } t > T+2\end{aligned} \tag{A327}$$

Note that (A327) also holds for $t = T + 2$ (see (A320)).

The convergent solution time path of the state variable $\tau = \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{x}} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{z}}$ in case $\beta_R < 1$ is now given by the following expressions:

- For $t \leq T - 1$:

$$\tau_t = \overline{\tau} + \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^t\mathbf{K}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^t\mathbf{K}_2 \tag{A328}$$

- For $t = T$:

$$\begin{aligned}\tau_t &= \overline{\tau} - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\widetilde{m}_2 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\widetilde{m}_3 \\ &\quad - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\frac{\widetilde{m}_4}{r_1 - \beta_R}\beta_R + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^T\mathbf{K}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 \\ &= \overline{\tau} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^T\mathbf{K}_2 + \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^T\mathbf{K}_1 + \widetilde{m}_1\end{aligned} \tag{A329}$$

  since

$$\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 = (\mathbf{I}_1 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{G}}_{11})\widetilde{m}_1 \tag{A330}$$

  and

$$\begin{aligned}\widetilde{\mathbf{\Lambda}}_1^T\mathbf{K}_1 &= -\widetilde{\mathbf{G}}_{11}\widetilde{m}_1 - \widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\widetilde{m}_2 - \widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\widetilde{m}_3 \\ &\quad - \widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\frac{\widetilde{m}_4}{r_1 - \beta_R}\beta_R\end{aligned} \tag{A331}$$

  (cf. (A284), (A315)).

- For $t = T + 1$:

$$\begin{aligned}\tau_{T+1} &= \overline{\tau} - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\widetilde{m}_3 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\frac{\widetilde{m}_4}{r_1 - \beta_R}\beta_R \\ &\quad + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{T+1}\mathbf{K}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{G}}_{21}\widetilde{m}_2\end{aligned} \tag{A332}$$

- For $t = T + 2$:

$$\begin{aligned}\tau_{T+2} &= \overline{\tau} - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{G}}_{11}\frac{\widetilde{m}_4}{r_1 - \beta_R}\beta_R + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{T+2}\mathbf{K}_2 \\ &\quad + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^2\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2\widetilde{\mathbf{G}}_{21}\widetilde{m}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{G}}_{21}\widetilde{m}_3\end{aligned} \tag{A333}$$

- For $t > T + 2$:

$$\begin{aligned}\tau_t &= \overline{\tau} - \widetilde{\mathbf{H}}_{11}\frac{g_{11}\widetilde{m}_4}{r_1 - \beta_R}\beta_R^{t-(T+1)} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^t\mathbf{K}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{t-T}\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 \\ &\quad + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{t-T-1}\widetilde{\mathbf{G}}_{21}\widetilde{m}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{t-T-2}\widetilde{\mathbf{G}}_{21}\widetilde{m}_3 \\ &\quad + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{t-T-2}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2 - \beta_R) \\ g_{31}\widetilde{m}_4/(r_3 - \beta_R)\end{pmatrix}\beta_R \\ &\quad - \widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2 - \beta_R) \\ g_{31}\widetilde{m}_4/(r_3 - \beta_R)\end{pmatrix}\beta_R^{t-T-1}\end{aligned} \tag{A334}$$

  where $\mathbf{K}_2$ is defined by (A278).

The solution formula (A334) also holds for $t = T + 2$ (see (A333)). Moreover, it also holds for $t = T + 1$ since

$$\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{t-T-2}\widetilde{\mathbf{G}}_{21}\widetilde{m}_3\Big|_{t=T+1} = \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-1}\widetilde{\mathbf{G}}_{21}\widetilde{m}_3 = -\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\widetilde{m}_3 \tag{A335}$$

(according to (A281)) and

$$\begin{aligned} &-\widetilde{\mathbf{H}}_{11}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-1}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R \\ &-\widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix} = -\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}g_{11}\frac{\widetilde{m}_4}{r_1-\beta_R}\beta_R \end{aligned} \tag{A336}$$

To show (A336), divide (A336) by $(-\beta_R)$ yielding the equivalent expression (cf. (A130))

$$\begin{aligned} \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R} &= -\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-1}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix} \\ &+\left[\widetilde{\mathbf{H}}_{11}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R} + \widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\right]\beta_R^{-1} \end{aligned} \tag{A337}$$

Using (A281) the right-hand side of (A337) can be summarized as follows:

$$\begin{aligned} &-\sum_{j=2}^{3}\frac{1}{r_j}\frac{g_{j1}\widetilde{m}_4}{r_j-\beta_R}h_{1j} + \sum_{j=1}^{3}\frac{g_{j1}\widetilde{m}_4}{(r_j-\beta_R)\beta_R}h_{1j} \\ &= -\sum_{j=2}^{3}\frac{g_{j1}\widetilde{m}_4}{r_j-\beta_R}\left[\frac{1}{r_j}-\frac{1}{\beta_R}\right]h_{1j} + \frac{g_{11}\widetilde{m}_4}{(r_1-\beta_R)\beta_R}h_{11} \\ &= \sum_{j=2}^{3}\frac{g_{j1}\widetilde{m}_4}{r_j\beta_R}h_{1j} + \frac{g_{11}\widetilde{m}_4}{(r_1-\beta_R)\beta_R}h_{11} = \frac{\widetilde{m}_4}{\beta_R}\left(\sum_{j=2}^{3}h_{1j}r_j^{-1}g_{j1} + \frac{h_{11}}{(r_1-\beta_R)}g_{11}\right) \\ &= \frac{\widetilde{m}_4}{\beta_R}\left(-h_{11}r_1^{-1}g_{11} + \frac{h_{11}}{(r_1-\beta_R)}g_{11}\right) = \frac{\widetilde{m}_4}{\beta_R}h_{11}g_{11}\left(-\frac{1}{r_1}+\frac{1}{r_1-\beta_R}\right) \\ &= \widetilde{m}_4h_{11}g_{11}\frac{1}{r_1(r_1-\beta_R)} = \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R} \end{aligned} \tag{A338}$$

Therefore, (A334) also holds for $t = T + 1$. In the following we will show that the solution time path (A334) also holds for $t = T$. In case $t = T$ the solution formula (A334) is equivalent to (A329) if and only if

$$\begin{aligned} &\overline{\tau} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{T}\mathbf{K}_2 + (\mathbf{I}_1 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{G}}_{11})\widetilde{m}_1 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\widetilde{m}_2 \\ &-\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\widetilde{m}_3 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\frac{\widetilde{m}_4}{r_1-\beta_R}\beta_R = \\ &\quad \overline{\tau} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{T}\mathbf{K}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-1}\widetilde{\mathbf{G}}_{21}\widetilde{m}_2 + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21}\widetilde{m}_3 \\ &\quad -\widetilde{\mathbf{H}}_{11}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R}\beta_R^{-1} + \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R \\ &\quad -\widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} \end{aligned} \tag{A339}$$

(A339) holds if and only if the following four equations hold simultaneously:

$$(\mathbf{I}_1 - \widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{G}}_{11})\widetilde{m}_1 = \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{G}}_{21}\widetilde{m}_1 \tag{A340}$$

$$\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-1}\widetilde{\mathbf{G}}_{21}\widetilde{m}_2 = -\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-1}\widetilde{\mathbf{G}}_{11}\widetilde{m}_2 \tag{A341}$$

$$\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21}\widetilde{m}_3 = -\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\widetilde{m}_3 \tag{A342}$$

$$\begin{aligned}
-\widetilde{\mathbf{H}}_{11}\widetilde{\mathbf{\Lambda}}_1^{-2}\widetilde{\mathbf{G}}_{11}\frac{\widetilde{m}_4}{r_1-\beta_R}\beta_R &= \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R \\
&- \widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} \\
&- \widetilde{\mathbf{H}}_{11}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R}\beta_R^{-1}
\end{aligned} \tag{A343}$$

(A340) is equivalent to (A330), (A341) follows from the identity (A281), while (A342) follows from (A282). It remains to show equation (A343). (A282) implies that (A343) is equivalent to

$$\begin{aligned}
&\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R - \widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}\widetilde{m}_4/(r_2-\beta_R) \\ g_{31}\widetilde{m}_4/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} \\
&- \widetilde{\mathbf{H}}_{11}\frac{g_{11}\widetilde{m}_4}{r_1-\beta_R}\beta_R^{-1} = \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21}\frac{\widetilde{m}_4}{r_1-\beta_R}\beta_R
\end{aligned} \tag{A344}$$

Dividing by $\widetilde{m}_4$ yields the equation

$$\begin{aligned}
&\widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\begin{pmatrix} g_{21}/(r_2-\beta_R) \\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R - \widetilde{\mathbf{H}}_{12}\widetilde{\mathbf{\Lambda}}_2^{-2}\widetilde{\mathbf{G}}_{21}\frac{1}{r_1-\beta_R}\beta_R \\
&- \widetilde{\mathbf{H}}_{12}\begin{pmatrix} g_{21}/(r_2-\beta_R) \\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} = \widetilde{\mathbf{H}}_{11}\frac{g_{11}}{r_1-\beta_R}\beta_R^{-1}
\end{aligned} \tag{A345}$$

or

$$\begin{aligned}
&(h_{12}, h_{13})\begin{pmatrix} r_2^{-2} & 0 \\ 0 & r_3^{-2}\end{pmatrix}\begin{pmatrix} g_{21}/(r_2-\beta_R) \\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R \\
&- (h_{12}, h_{13})\begin{pmatrix} r_2^{-2} & 0 \\ 0 & r_3^{-2}\end{pmatrix}\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix}\frac{1}{r_1-\beta_R}\beta_R \\
&- (h_{12}, h_{13})\begin{pmatrix} g_{21}/(r_2-\beta_R) \\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} = h_{11}g_{11}\frac{1}{r_1-\beta_R}\beta_R^{-1}
\end{aligned} \tag{A346}$$

Since $h_{12}r_2^{-2} = h_{22}r_2^{-1}$, $h_{13}r_3^{-2} = h_{23}r_3^{-1}$, (A346) is equivalent to

$$\begin{aligned}
&(h_{22}r_2^{-1}, h_{23}r_3^{-1})\begin{pmatrix} g_{21}/(r_2-\beta_R) \\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R \\
&- (h_{22}r_2^{-1}, h_{23}r_3^{-1}\begin{pmatrix} g_{21} \\ g_{31}\end{pmatrix}\frac{1}{r_1-\beta_R}\beta_R \\
&- (h_{12}, h_{13})\begin{pmatrix} g_{21}/(r_2-\beta_R) \\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} = h_{11}g_{11}\frac{1}{r_1-\beta_R}\beta_R^{-1}
\end{aligned} \tag{A347}$$

or

$$(h_{32},h_{33})\begin{pmatrix}g_{21}/(r_2-\beta_R)\\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R-(h_{32},h_{33})\begin{pmatrix}g_{21}\\ g_{31}\end{pmatrix}\frac{1}{r_1-\beta_R}\beta_R= \tag{A348}$$
$$h_{11}g_{11}\frac{1}{r_1-\beta_R}\beta_R^{-1}+(h_{12},h_{13})\begin{pmatrix}g_{21}/(r_2-\beta_R)\\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1}$$

since $h_{22}r_2^{-1}=h_{32}$, $h_{23}r_3^{-1}=h_{33}$. (A348) is equivalent to

$$(h_{31},h_{32},h_{33})\begin{pmatrix}g_{11}/(r_1-\beta_R)\\ g_{21}/(r_2-\beta_R)\\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R \;=(h_{11},h_{12},h_{13})\begin{pmatrix}g_{11}/(r_1-\beta_R)\\ g_{21}/(r_2-\beta_R)\\ g_{31}/(r_3-\beta_R)\end{pmatrix}\beta_R^{-1} \tag{A349}$$

since $-h_{32}g_{21}-h_{33}g_{31}=h_{31}g_{11}$. (A349) is equivalent to

$$\sum_{j=1}^{3}\frac{h_{1j}g_{j1}}{(r_j-\beta_R)\beta_R}=\sum_{j=1}^{3}\frac{h_{1j}g_{j1}}{r_j^2(r_j-\beta_R)}\beta_R \tag{A350}$$

since $h_{3j}=h_{1j}r_j^{-2}$ $(j=1,2,3)$. Equation (A350) holds if and only if

$$\sum_{j=1}^{3}\frac{h_{1j}g_{j1}}{r_j-\beta_R}\left(\frac{\beta_R}{r_j^2}-\frac{1}{\beta_R}\right)=-\sum_{j=1}^{3}h_{1j}g_{j1}\frac{\beta_R+r_j}{\beta_R r_j^2}=0 \tag{A351}$$

or

$$-\sum_{j=1}^{3}h_{3j}g_{j1}\left(1+\frac{r_j}{\beta_R}\right)=0 \tag{A352}$$

(since $h_{1j}/r_j^2=h_{3j}$). (A352) is equivalent to

$$-\sum_{j=1}^{3}h_{3j}g_{j1}=\frac{1}{\beta_R}\sum_{j=1}^{3}h_{3j}r_jg_{j1} \tag{A353}$$

But this equation holds since

$$\sum_{j=1}^{3}h_{3j}g_{j1}=0 \tag{A354}$$

and

$$\sum_{j=1}^{3}h_{3j}r_jg_{j1}=\sum_{j=1}^{3}h_{2j}g_{j1}=0 \tag{A355}$$

according to (A287) and (A288).

The solution time path of τ_t in case $\beta_R<1$ is therefore given by (A328) (for $t\le T-1$), (A329) (for $t=T$) and (A334) (for $t\ge T$).

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