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Competition through Technical Progress

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Abstract

The relationship between technical progress and price competition is a controversial issue in economics. This paper highlights the fact that investment in technical progress is an authentic type of competition which benefits the consumers rather than the industry. This type of competition exists when the potential for technical progress, which can be incorporated by firms through investment, is high enough. Competition is, in fact, made up of two components: A static one which is known as price or quantity competition and a dynamic one, the Technical Progress competition which also contribute to reduce prices and increase quantities for consumers. Consequently, the economic factors that increase a firm's margin do not have to be viewed as the consumers' enemy, but rather as an ally, under specific conditions, because they allow higher investments in new technology by which firms increase their capacities and attract higher demand from consumers. This paper also underlines that, for a mature market, the maximum Consumer Surplus as well as Social Welfare are attained by a constant level of combined competition which is only dependent on the size of the market and the number of firms. The level of combined competition can be defined as the product of the static and the dynamic level of competition. As a consequence, the higher the potential of technical progress is, the lower the level of static competition must be in order to reach the maximum level of Consumer Surplus and Social Welfare.

JEL codes: D21, D43, D92, L13, L51, L96, O12

Keywords: Investment, Competition, Technical Progress, Dynamic Competition

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1 Introduction

Technical progress is commonly considered, among economists, to be one of the main factors that leads to economic growth as Solow (1956). The relationship between technical progress and competition is a controversial issue in economics. Schumpeter (1934, 1950) thought that monopolies or highly concentrated markets were favourable to innovation due to the higher potential for profits as well as their economies of scale which can be used for Research and development. Arrow (1962), on the contrary, illustrated that a firm in competitive industry is more willing to innovate in a new technology than a pure monopoly because the difference of the profits it can earn is higher. A monopoly can increase its profits from the new technology, but at the expense of its profits from the old technology. This is known as the "replacement effect" mentioned by Tirole (1997). Moreover, there is no clear evidence whether market concentration influences innovations more favourably rather than less. Gilbert (2006) indicates that this depends on market structures, particularly entry barriers, the kind of innovation, product innovation or process innovation, and issues such as intellectual property rights.

In general, technological progress is regarded as the sole outcome of R&D. However, investments in technological progress are not investments in R&D alone, but also investments in production capacities which integrate technical progress. For instance, telecommunication networks' operators invest in network capacities and buy new generation facilities in order to improve the service they deliver to their customers. They do not necessarily create the new technology and may not have even been involved in the R&D process that brought about the new technology, yet their investment allows end-users to benefit from them.

R&D is generally regarded as a process that leads to unpredictable stochastic shocks on the industry which are hard to anticipate. This explains why Schumpeterian competition, according to Barney (1986), has resisted application into strategic thinking. This is also why, Industrial Organization models take neither the static effects (prices, product differentiation, market structure,…) nor the dynamic effects (innovation, new products, new technologies,…) into account at the same time. They separate them into different frameworks: On the one hand, the static competition and on the other, the dynamic competition.

This paper will be based on the principle that investments in technical progress provide a means to increase production capacities (network capacities for a network operator). This phenomenon has become more predictable and all competitors have equal access to the new technologies. The aim of the paper is to focus on such a context, in order to study the strategic interactions among competitors which reconcile the static and the dynamic aspects into a single framework.

Over time, production capacities tend to increase exponentially. Koh and Magee (2006, 2008) have studied the evolution of technical progress from the middle of the nineteenth century to the present and focused on Information technologies and Energy sectors. During this period, the technical progress in both sectors has increased almost exponentially. This implies an annual technical progress rate which is relatively steady over time even if it may vary from year to year. The order of magnitude is clearly higher for Information Technologies (about 20 to 30%) than for Energy (about 3 to 6%).

The power of technical progress in the Information Technologies highlights the relevance of the recognition of the dynamic point of view particularly in this sector. This paper is organized as follows:

Section 1 is the present introduction, Section 2 is the theoretical model that studies the strategic interaction from the static and dynamic points of view. It reveals an inverted U relationship between price competition level and Consumer Surplus or Social Welfare like Aghion and al (2002). Price competition encourages firms to invest yet, at the same time, it reduces their investment capabilities. The higher the technical progress rate is, the lower the level of price competition must be to maximize Welfare. As a result, technical progress appears as a type of competition comparable to price competition. The level of technical progress competition is characterized by the technical progress rate. Competition is characterized by two components, on the one hand is the static aspect which is price competition, and on the other hand is the dynamic aspect which is Technical progress competition. Section 3 is a discussion that develops the idea that there is an optimal social level of competition comprised of price competition, number of firms and technical progress competition. An increase in a certain type of competition leads to a proportional decrease of the other kinds of competition in order to maintain the optimal social level. Section 4 is the conclusion and the policy implications of the model.

2 The model

Firms compete in price. They invest in technical progress in order to improve the value of their product. The duopoly case will be considered first, to focus on the strategic interactions then we shall generalize the model to *N* firms.
First let us consider the model from the static point of view.

2.1 Static model of duopoly

The two firms compete in price. A total of *n* consumers are uniformly distributed along a line whose length is normalized to one. Firm 1 is located at the extreme left and Firm 2 at the opposite end. Each consumer desires one unit of a good. We assume that the consumers' values of both firms' products v_1 and v_2 are high enough to assure a complete coverage of the market. A consumer located at a distance *x* from the extreme left incurs a disutility of *hx* if he buys from Firm 1 at price p_1 and $h(1-x)$ if he buys from Firm 2 at price p_2. *h* is the transportation cost. We assume that both firms have an equal marginal cost *c*.
Each firm maximizes its profit, and there is a Nash equilibrium where the market share of firm *i* ; $i, j \in \{1,2\}$, is:

$$\sigma_i = \frac{1}{2} + \frac{v_i - v_j}{6h} \qquad (1)$$

(see annex 1)
With the condition $v_i - v_j \leq 3h$ to remain in a duopoly with $\sigma_i \geq 0$
The price of firm *i*'s good is:

$$p_i = c + h + \frac{v_i - v_j}{3} \qquad (2)$$

And each firm earns:

$$\pi_i = \frac{n}{2h}\left(h + \frac{v_i - v_j}{3}\right)^2 \qquad (3)$$

The consumer surplus *cs* is the difference between consumer utility and price:

$$cs = n\left(\int_0^{\sigma_1} (v_1 - hx - p_1)\,dx + \int_{\sigma_1}^{1} (v_2 - h(1-x) - p_2)\,dx \right)$$

$$cs = n\left(\frac{v_1 + v_2}{2} - c - \frac{5}{4}h + \frac{(v_1 - v_2)^2}{36h} \right) \qquad (4)$$

The Welfare as the sum of Consumer Surplus and both firms' profits:

$$w = n\left(\frac{v_1 + v_2}{2} - c - \frac{1}{4}h + \frac{5(v_1 - v_2)^2}{36h} \right) \qquad (5)$$

Consumer Surplus, as well as Welfare, decreases with the transportation cost *h*.
Notice that if both firms propose an equal consumers' value for their offer, they will earn the same profit. If $v_1 = v_2 = v$ then $\pi_1 = \pi_2 = \frac{nh}{2}$; $p_1 = p_2 = c + h$ and $\sigma_1 = \sigma_2 = \frac{1}{2}$. In such a case, consumer surplus becomes:

$$cs = n(v - c - \frac{5}{4}h) \qquad (6)$$

And Welfare:

$$w = n(v - c - \frac{1}{4}h) \qquad (7)$$

h can also be read into a coefficient of differentiation between the offers of both firms. When $h = 0$, the offers of both firms are perfect substitutes and the more *h* increases, the more the offers of both firms are differentiated. This differentiation allows firms to make higher profits. However, when $v_1 = v_2 = v$, *h* also represents the margin. Competition is all the more fierce the lower *h* is. $\frac{1}{h}$ can be regarded here as a proxy for the intensity of competition.
From this static point of view, we can notice that Consumer Surplus as well as Welfare are both at their maximum level for perfect competition when $h = 0$. Competition causes prices to decrease and Consumer Surplus and Welfare to increase.
The outcome may vary when we take the dynamic point of view into account.

2.2 Dynamic model of duopoly

Let us assume that at time t_0, both firms sell a good that all consumers value to the same degree $v_1 = v_2 = v$. Each firm can reinvest a part of its profit in technical progress in order to improve the consumers' value of its offer. This investment can be a R&D investment or an investment in the production capacities which allows technical progress to integrate the good. For instance, an Internet Service Provider can invest in network capacities by buying new generation devices which improve the bandwidth and the available bit rate of its subscribers.
This investment I_i increases the consumers' value of the product from v_i at time t_0 to $v_i+V(I_i)$ at time t_1.
$V(I_i)$ represents the response of the consumers' value to the investment in technical progress. Let us assume function $V(I_i)$ is increasing and concave and when there is no investment there is no improvement of consumer value. $V(0) = 0$.

At time $t = t_0$, firms invest. What is the amount I_i firm *i* has to invest in order to maximize its profit if firm *j* invest I_j?

At time $t = t_1$, firm *i*'s static profit becomes:

$$\pi_i(t_1, I_i) = \frac{n}{2h}\left(h + \frac{V(I_i) - V(I_j)}{3}\right)^2 \quad (8)$$

The investment I_i at t_0 is worth $I_i(1+\rho)$ at time t_1. ρ is the discount rate.
The dynamic profit of firm *i* is:

$$\pi_i(t_1, I_i) - I_1(1+\rho) = \frac{n}{2h}\left(h + \frac{V(I_i) - V(I_j)}{3}\right)^2 - I_1(1+\rho) \quad (9)$$

Firm *i* tries to maximize its dynamic profit, the first order condition leads to

$$\frac{n}{2h}\frac{2}{3}\frac{dV(I_1)}{dI_1}\left(h + \frac{V(I_1) - V(I_2)}{3}\right) - (1+\rho) = 0 \quad (10)$$

Firms *i* and *j* face the same constraints, therefore they will react similarly, thus $I_i = I_j = I$ and $V(I_i) = V(I_j) = V(I)$. Finally:

$$\frac{dV(I)}{dI} = \frac{3(1+\rho)}{n} \quad (11)$$

Since V(I) is concave, $\frac{dV(I)}{dI}$ is strictly decreasing. Consequently, there are two possibilities according to the value of $\frac{dV(0)}{dI}$ in comparison to $\frac{3(1+\rho)}{n}$. (figure 1)

If $\frac{dV(0)}{dI} < \frac{3(1+\rho)}{n}$ there is no solution to equation 11. The response of the consumers' value to the investment in technical progress is too low. The investment is too costly compared to the consumers' value improvement, firms do not invest in such a case.

If $\frac{dV(0)}{dI} \geq \frac{3(1+\rho)}{n}$ there is a solution to equation 11 and therefore, there is a value of investment I^* that maximizes dynamic profit. The response of the consumers value to the investment in technical progress is strong enough, firms incentive to invest is sufficient and so both firms will invest an amount of I^* in technical progress such that: $\frac{dV(I^*)}{dI} = \frac{3(1+\rho)}{n}$ and we can define $I^*(\rho, n)$ as a function depending on nothing but the discount rate ρ and the number of customers *n*.

2.3 Discussion

We can notice that I^* is independent of *h*, which is linked to the level of price competition. The level of price competition has no influence on the amount I^* firms are encouraged to invest. However, it can have an influence on the capability of firms to have this amount at their disposal.
I^* decreases with ρ and increases with *n* (figure 1).

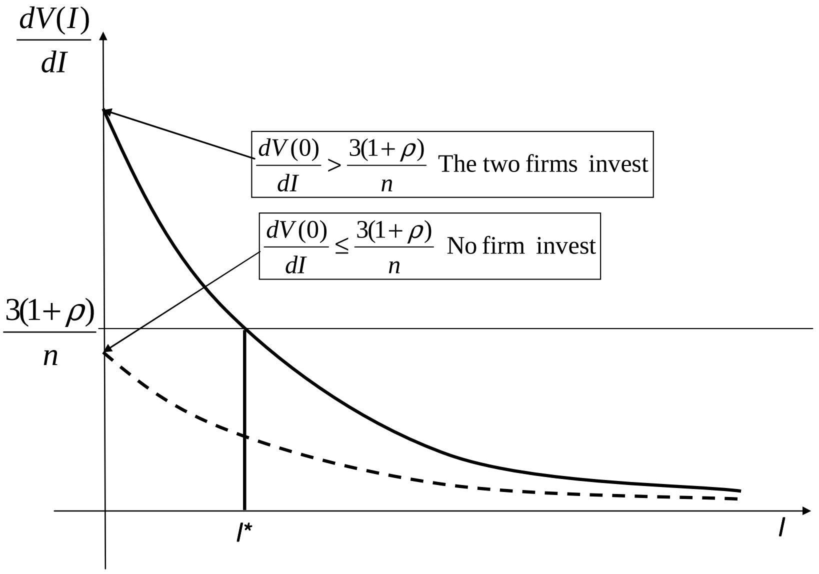


figure1

The discount rate, ρ, represents the valuation of the present time in comparison to the future. The higher ρ is, the more the present is important in comparison to the future, and then the less investment is encouraged. As a result, I^* decreases when ρ increases.
However, the number of customers' *n* encourages investment, because the willingness to pay of each consumer increases thanks to the investment. In figure 1, we notice that the line $\frac{3(1+\rho)}{n}$ increases with ρ and then crosses the $\frac{dV(I)}{dI}$ curve for a lower value of I^*. By contrast, it decreases with *n* and therefore crosses the $\frac{dV(I)}{dI}$ curve for a higher value of I^*.

2.4 Reinvestment rate

Firms try to invest the exact amount I^* in technical progress each time. When they earn a profit higher than I^*, they have no problem doing it, but when their profit is not sufficient, they cannot invest enough unless they resort to an external source of financing. Such sources of financing, in the long run, are costly and not sustainable in this particular model, because firms' profits and financing needs to remain steady. Firms will never be able to pay back such external sources of financing. In this paper, we assume that firms resort only to self-financing.
Let us assume that α is the reinvestment rate such that $\alpha\pi = I^*$
If $\pi \geq I^*$ then $\alpha \leq 1$. Firms do not need to reinvest all their profits.
If $\pi < I^*$, we should obtain $\alpha > 1$, but this is not possible because firms resort only to self-financing which creates the condition $\alpha \leq 1$. In such a case, $\alpha = 1$, and firms reinvest all the profit in technical progress even if it is not sufficient to attain I^*. This investment in technical

progress, which is lower than expected, will slow down the rate of improvement of the goods on the market and therefore restrict the Consumer Surplus growth. (figure 2)

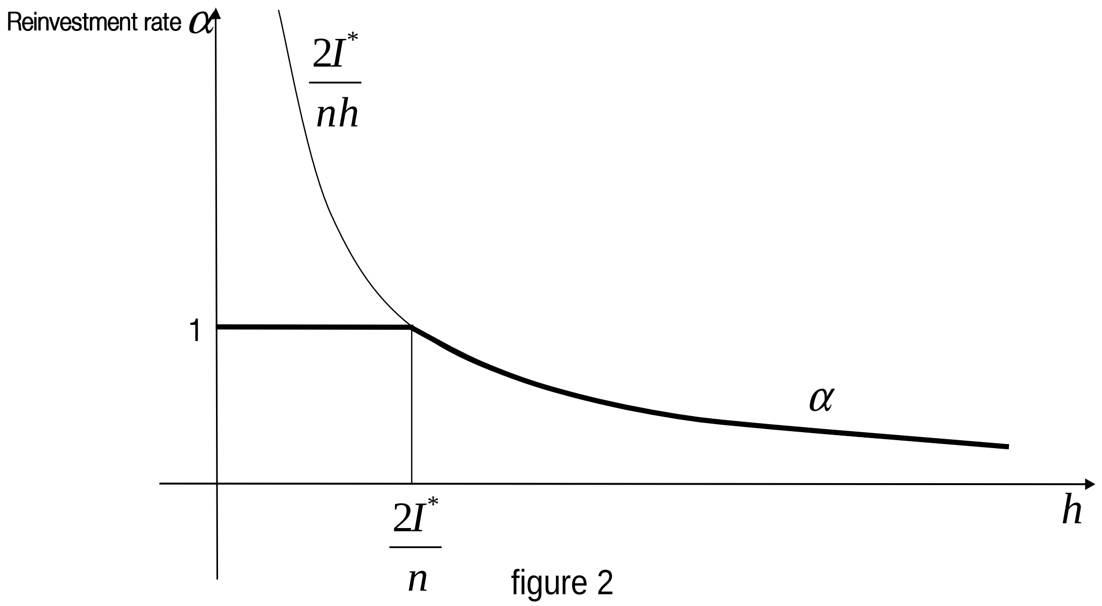


figure 2

Figure 2 represents the reinvestment rate α according to the differentiation parameter h which also represents the margin here. When the margin increases, profits increase as well.

$\pi = \frac{hn}{2}$; If $\pi \geq I^* \Rightarrow h \geq \frac{2I^*}{n}$ in such a case, $\alpha = \frac{2I^*}{nh} \leq 1$ the optimal level of investment in technical progress I^* is reached.

If $\pi < I^* \Rightarrow h < \frac{2I^*}{n}$ in such a case, $\alpha = 1$. The optimal level of investment in technical progress I^* is not reached even if firms reinvest all their profits.

2.5 Prisoner's dilemma

When we consider the global outcome, if technical progress is strong enough to incite firms to invest, both firms will have invested the same amount, and likewise they will have increased the consumers' willingness to pay for their offer. As a result, neither of them has acquired any competitive advantage, therefore, at time t_1, they still earn the same profit as at time t_0, in spite of their financial effort. Whereas the situation is quite unchanged for firms at the end of the game, customers, on the other hand, have gained a lot of Surplus thanks to technical progress. Thus firms' investments have not been advantageous for them, but rather for the customers. In this respect, firms would be better off not investing, however they made the choice to invest, because not investing is very risky in the event that their competitor does. Fear of the competitor urges them to invest. This is the consequence of competition. In fact, investment in technical progress, as a result, is a competitive behaviour. The amount of investment I^* depends on the strength of technical progress. The higher the latter is, the higher I^* is. The level of competition through technical progress is then determined by I^*. The competitive behaviour through technical progress appears in the prisoner's dilemma firms are confronted with.

The table below (table 1) summarizes this dilemma and indicates the gains each firm earns whether it invests or not according to its rival's behaviour.

Firm 2 / Firm 1	Invest	Do not invest
Invest	$-I(1+\rho)$ $-I(1+\rho)$	$n\left(\frac{V(I)^2}{18h}-\frac{V(I)}{3}\right)$ $n\left(\frac{V(I)}{3}+\frac{V(I)^2}{18h}\right)-I(1+\rho)$
Do not invest	$n\left(\frac{V(I)}{3}+\frac{V(I)^2}{18h}\right)-I(1+\rho)$ $n\left(\frac{V(I)^2}{18h}-\frac{V(I)}{3}\right)$	0 0

table 1

The situation mentioned in the table above is a prisoner's dilemma if :

$$n\left(\frac{V(I)}{3}+\frac{V(I)^2}{18h}\right)-I(1+\rho)>0>-I(1+\rho)>n\left(\frac{V(I)^2}{18h}-\frac{V(I)}{3}\right) \qquad (12)$$

Equation 12 leads to $I<\frac{nV(I)}{3(1+\rho)}$ (see annex 2) and by replacing (11) on the right-hand side:

$$\frac{V(I)}{\left(\frac{dV(I)}{dI}\right)}>I \qquad (13)$$

This inequality is checked for all $I>0$, because *V(I)* is concave and $V(0)=0$ (see annex 3). Firms face a prisoner's dilemma since the technical progress is dynamic enough to incite them to invest. That is to say if $\frac{dV(0)}{dI}\geq\frac{3(1+\rho)}{n}$

A prisoner's dilemma can incite firms to cooperate if the game is repeated. This incentive appears when $0>n\left(\frac{V(I)^2}{18h}+\frac{V(I)}{3}\right)-I(1+\rho)+n\left(\frac{V(I)^2}{18h}-\frac{V(I)}{3}\right)$ which means:

$$I>\frac{nV(I)^2}{9(1+\rho)h} \qquad (14)$$

By replacing the right-hand side by equation 11

$$I^*>\frac{V(I^*)^2}{3h\left(\frac{dV(I^*)}{dI}\right)}$$

Equation 14 shows that the incentive to cooperate increases with h and decreases with I^*. Both kinds of competition, Price competition and Technical progress competition tend to reduce incentive to cooperate.

3 Consumer Surplus and Welfare in dynamics

In the dynamic model, investment in technical progress, even if it has no impact on firms' profits it has a great impact on Consumer Surplus and Welfare. Improvement of firms' goods increases consumers' willingness to pay. As both firms' prices remain unchanged, the Consumer Surplus, i.e. the difference between the price and the consumers' willingness to pay, increases.

Social Welfare, defined as the sum of Consumer Surplus and firms' profits, increases too because the firms' profits remain steady, whereas Consumer Surplus increases.

Investment in technical progress increases the consumers' value of firms' products.

Let us assume that at t_0 this value is v_0, at time t_1 this value is upgraded to $v_1 = v_0 + V(I)$. At time t_u, it becomes $v_u = v_0 + uV(I)$ and consumer surplus $cs_u = v_u - c - \frac{5h}{4}$ (equation 6).

The dynamic Consumer Surplus, CS is the discounted sum of static consumer surplus over time and the disounted consumer surplus at time t_u is:

$$\overline{cs_u} = \frac{cs_u}{(1+\rho)^u} = cs_u e^{-\ln(1+\rho)u}$$

therefore

$$CS = n\int_0^\infty \left(v_u - c - \frac{5h}{4}\right) e^{-\ln(1+\rho)u} du \qquad (15)$$

then

$$CS = \frac{n}{\ln(1+\rho)}\left(\frac{V(I)}{\ln(1+\rho)} + \underbrace{v_0 - c - \frac{5h}{4}}_{\text{static consumer surplus}}\right) \text{ with } I = \begin{cases} \frac{hn}{2} & \text{if } h \le \frac{2I^*}{n} \\ I^* & \text{if } h > \frac{2I^*}{n} \end{cases} \qquad (16)$$

(see annex 4)

From this equation, contingent on the level of price competition, the variations of Consumer Surplus can be deduced. There are two possible cases:

When $\pi < I^* \Rightarrow h < \frac{2I^*}{n}$, firms reinvest all their profit then $I = \frac{nh}{2}$. In such a case, I depends on the margin, h.When the level of price competition decreases, h increases and the static consumer surplus decreases. However, at the same time, the investment capabilities of firm, I increases and compensates for the static decrease. As a result, CS increases. (figure 3)

When $\pi \ge I^* \Rightarrow h \ge \frac{2I^*}{n}$, firms can attain the optimal level of investment I^* therefore $I = I^*$. In such a case, I no longer depends on the margin, h. Only the static portion remains dependent on h. Therefore, while h increases, CS decreases. (figure 3)

Likewise, we can calculate the dynamic Welfare as:

$$W = \frac{n}{\ln(1+\rho)}\left(\frac{V(I)}{\ln(1+\rho)} + \underbrace{v_0 - c - \frac{h}{4}}_{static\ welfare}\right) \text{ with } I = \begin{cases} \frac{hn}{2} & \text{if } h \le \frac{2I^*}{n} \\ I^* & \text{if } h > \frac{2I^*}{n} \end{cases} \tag{17}$$

(see annex 4)

CS attains its maximum level for $\min(\frac{2I^*}{n}, h^*_{CS})$ and W for $\min(\frac{2I^*}{n}, h^*_W)$.

With h^*_{CS}, the value of h which maximizes Consumer Surplus when $h = \frac{2I}{n}$

and h^*_W, the value of h which maximizes Welfare when $h = \frac{2I}{n}$

(see figure 3)

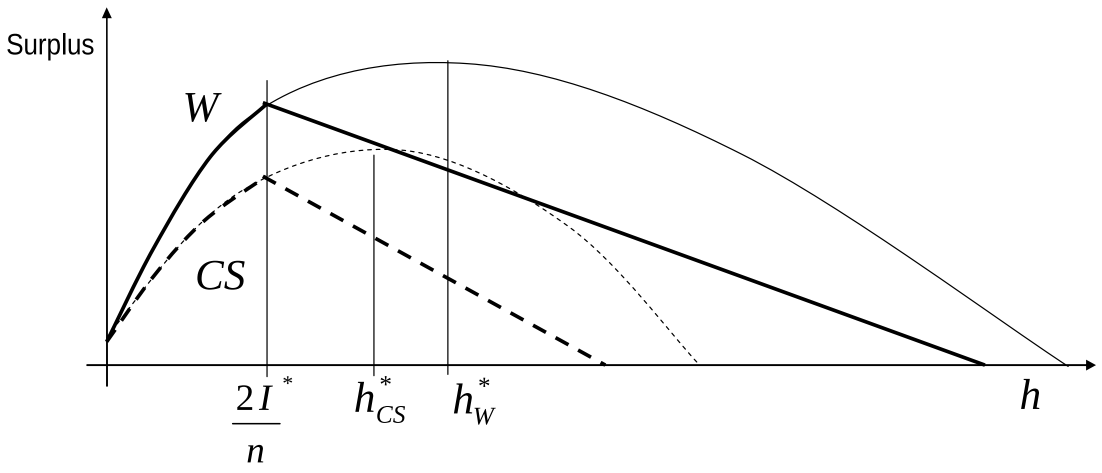


figure 3

We can prove that $\frac{2I^*}{n} < h^*_{CS} < h^*_W$ (see annex 5)

This property highlights that the Maximum Consumer Surplus or Welfare is attained for $h = \frac{2I^*}{n}$, in other words for $I^* = \pi$.

3.1 Law of maximum Welfare conservation

Let us denote $\gamma = \frac{1}{h}$, γ represents the substitutability coefficient between firms which represents the indicator of the level of price competition.

Maximum Consumer Surplus and Welfare are achieved for

$$2\gamma I^* = n \tag{18}$$

γ represents price competition and I^* technical progress competition. Equation 18 highlights that the product of those two kinds of competition is constant and equal to the market's size n divided by 2. Now 2 is the number of firms. We can prove that equation 18 can be generalized to N firms (see annex 6) and becomes:

$$N \gamma I^* = n \qquad (19)$$

Equation 19 means that the product of the two kinds of competition and the number of firms must be equal to the market's size in order to maximize Consumer Surplus as well as Welfare. This also means that the two kinds of competition play a symmetrical and inversely proportional role in Welfare maximization. The higher I^* is, the lower γ must be. If I^* is multiplied by a coefficient, γmust be divided by the same coefficient in order to maintain Consumer Surplus and Welfare at their maximum level.

Eventually, Competition is composed of both types of competition, Price competition can be regarded as the static component of competition and technical progress competition as the dynamic component of competition. The number of firms *N* reinforces competition in both of its components.

The level of competition which leads to the social optimum is proportional to the market's size and inversely proportional to the number of firms. For a given market's size n and a given number of firms *N*. The socially optimal level of competition is constant and not nil. It is equal to the product of its static and its dynamic components γI^*. (figure 4)

I^* depends on the strength of the technical progress, it is an exogenous parameter, upon which policy makers have no hold. However, they have a slight influence over the static parameter of competition γ. That is why the author of this paper believes policy makers should strive to adapt the static component to the dynamic one. For example, the dynamic component of competition is particularly high in the Information technology sector according to the Koh and Magee (works 2006, 2008). This high level of technical progress suggests that the static component of competition should be lowered.

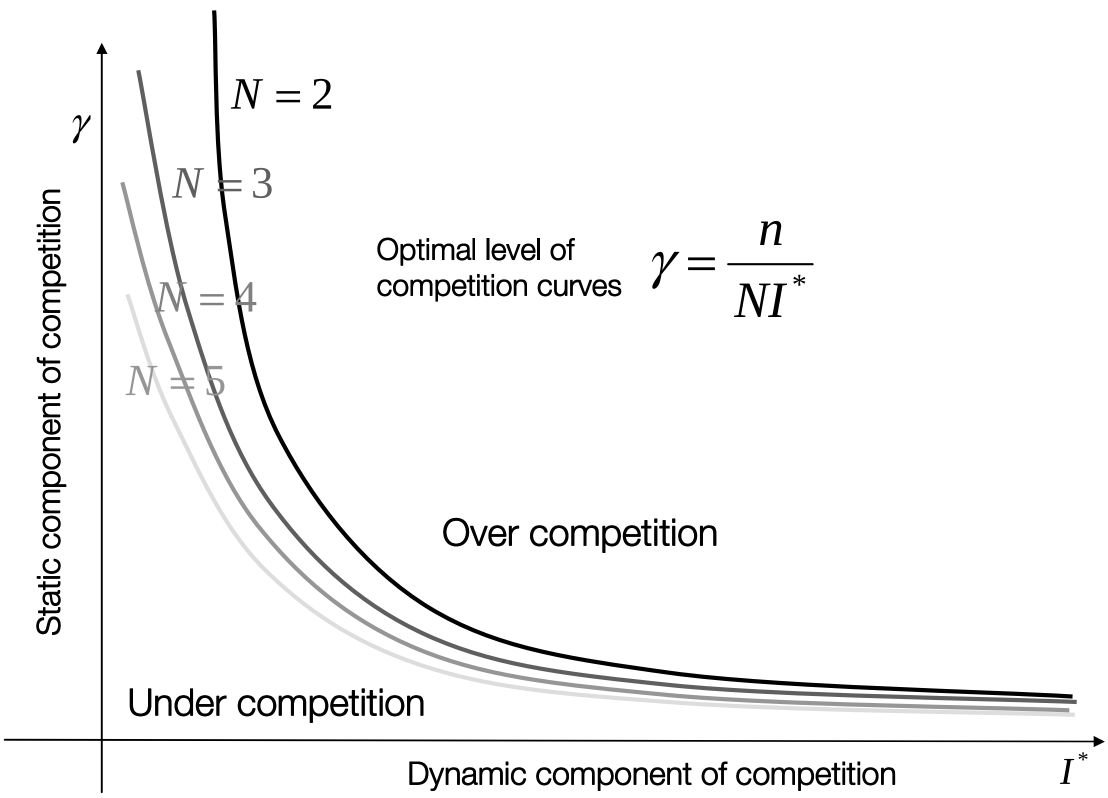


figure 4

In figure 4, competition is represented by its two components which correspond to the two axes: I^* and γ.

When competition falls below the curve $\gamma = \frac{n}{NI^*}$, the optimal level of competition is not attained, because there is not enough competition, this is “under-competition”.
When it is located above the curve, the optimal level of competition is not attained either because there is too much competition, this is the “over-competition”.

When the technical progress is not dynamic enough, firms are not encouraged to invest $I^* = 0$. In such a case, the optimal level of competition is attained when N or γ tend towards infinity, in other words, for a state of perfect competition. The result of the static model is found again. When the dynamic competition is missing, only the static component remains. The corollary is that when the static competition is nil, the dynamic component should tend towards infinity. This also would be a state of perfect competition in its dynamic version. However, this is a theoretical point of view because firms' resources and consequently I^* are both necessarily limited.

Notice that $\forall N; \frac{NI^*}{n} < h^*_{CS} < h^*_W$ can be proven (see annex 7)

3.2 Sustainability of competition

When there is over-competition, $N\gamma I^* > n$, firms cannot sustain the amount of investment I^* in technical progress. They invest as much as they can of a certain amount of money, $I < I^*$ such that $N\gamma I = n$. However, they are encouraged to invest more. This leads to a tension on the parameters N and γ. Firms are incited to lower N, market concentration, or γ, product substitutability, in order to reduce the tension.
In case of under-competition $N\gamma I^* < n$. Firms can invest I^* without reinvesting all their profits. This situation is sustainable even if Consumer Surplus and Welfare are not optimized. There can be an opportunity for new market entry if $(N+1)\gamma I^* \leq n$. However, when the market strays from the social optimum, Consumer Surplus and Welfare decrease more slowly into the side of “under-competition” than into the side of “over-competition”. Indeed, the slope of CS and W is steeper for $h < NI^*$ than for $h > NI^*$

$$\lim_{h \to NI^*-} \frac{\partial CS}{\partial h} > -\lim_{h \to NI^*+} \frac{\partial CS}{\partial h} \text{ and } \lim_{h \to NI^*-} \frac{\partial W}{\partial h} > -\lim_{h \to NI^*+} \frac{\partial W}{\partial h} \text{ (see annex 8)}$$

3.3 Performance unit price (Hedonic price)

The improvement of firms' products at a constant price leads to a price decrease for a constant level of performance.
Let us assume that performance is K_0 at time t_0 and K_u at time t_u . Firms invest an amount of money, I at each stage of technical progress which leads to a technical progress rate, $\tau(I)$ such that:

$$\tau(I) = \frac{K_u - K_{u-1}}{K_{u-1}} \tag{20}$$

Notice that, at the end of the game, the situation of firms is unchanged, even though Consumer Surplus and Welfare has increased. The scenario is repeated each time and firms' investment remain constant, (inflation is not taken into account). The technical progress rate also remains constant. This is consistent with the MIT works of Koh and Magee (2006, 2008). From performance at time t_0, K_0 it can be deduced that the performance at t_u, K_u is:

$$K_u = K_0 (1 + \tau(I))^u$$

Let us assume that consumers' values for firms' products is logarithmically related to their performance. This is a common assumption which has been confirmed by Peter Reichl, Bruno Tuffin and Raimund Schatz (2010). Therefore, it can be deduced that:

$$\ln(K_u) = \ln(K_0) + u \ln(1 + \tau(I))$$

with $v_u = \ln(K_u); v_0 = \ln(K_0)$ and as a result $V(I) = \ln(1 + \tau(I))$

The price of the goods remains constant $p = c + h$ but the price of a performance unit, P_u decreases. The price of a performance unit at time t_u is the price of the good divided by the total performance at t_u.

$$P_u = \frac{p}{K_u} = \frac{c + h}{K_0 (1 + \tau(I))^u} \tag{21}$$

Figure 5 illustrates the evolution of performance price over time according to the level of the two components of competition. For this figure, we assume that $K_0 = 1$.

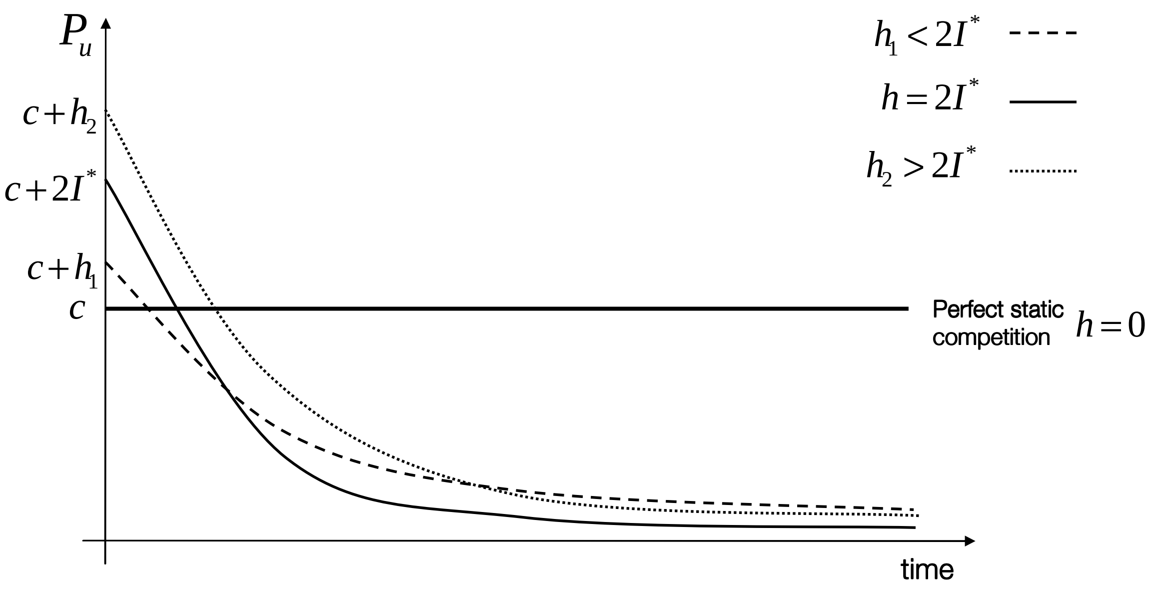
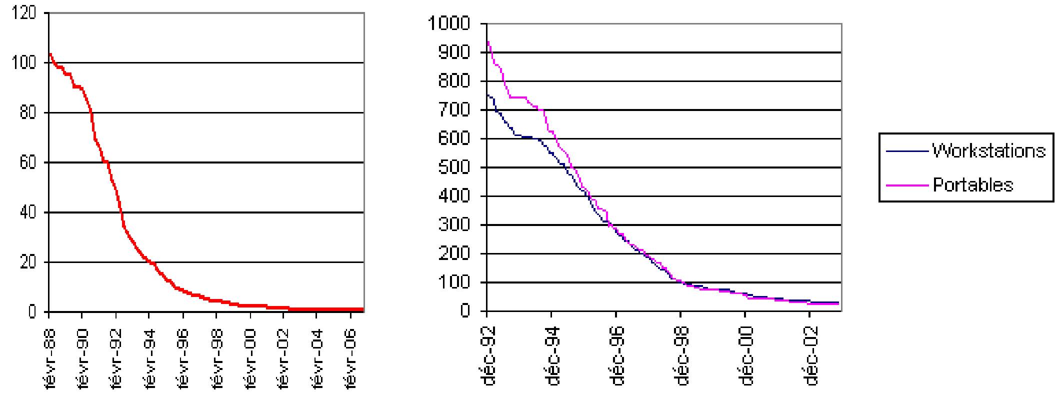


figure 5

Under conditions of perfect static competition, the unit price of performance is lower than under conditions of imperfect static competition at the beginning of the period of time, t_0. If $h = 0$; $P_u = c$. But the unit price of performance remains constant over time because there is no dynamic competition. The imperfect static competition allows the rise of dynamic competition that improves performance. Under imperfect static competition, performance, K_u increases exponentially over time and therefore, the unit price of performance decreases exponentially as well. The fastest and lowest decrease is attained for $h = 2I^*$ under optimal investment, $I = I^*$. For $h < 2I^*$, investment is not optimal and the technical progress rate is lower than expected. Performance increases more slowly and then the unit price of performance decreases more slowly as well. For $h > 2I^*$ the investment is optimal $I = I^*$ but nothing more and static competition is inefficiently lowered. As a consequence, the unit price of performance is higher at the beginning of the period of time and will remain higher over time than when $h = 2I^*$.

Such unit price of performance decrease has been highlighted by Koh and Magee. Three examples, the prices of computer memory, bandwidth and computing performance have dramatically fallen.
Computer memory: Price of Mbits (in US $ 2004), from $420 in 1952 on a magnetic tape, to $8.75 10^{-4}$ in 2004 on an optical disk.
Bandwidth: Price of Kbps/km (in US$ 2004) from $2.136 billion in 1858 by telegraph, to $440.41 in 1951 by TAT_1 (undersea cable) and $3.04 10^{-5} in 2002 by Apollo (undersea cable).
Computing performance: Price of the instruction per second capacity (in US$ 2004), from $1000 in 1951 with UNIVAC_I to $8.5 10^{-9} in 2004 with Athlon 63 3800.

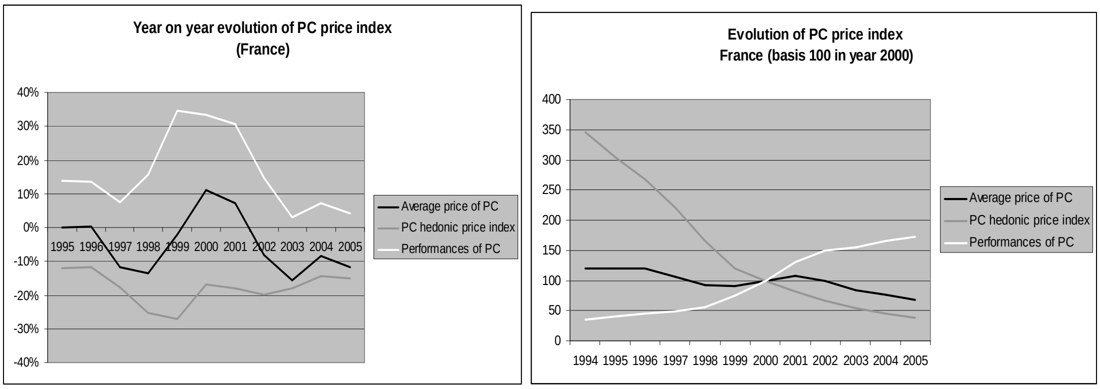
Due to the fact that computer component prices are decreasing, the hedonic price index of computers (of comparable performance) is also decreasing in France as well as in United States and figure 6 illustrates that the evolution of the index is quite similar.



France (Insee) United state (Department of labour)

figure 6

However, even if the hedonic prices of computers have rapidly fallen, the average price of a PC has decreased much more slowly. The graph below (figure 7) illustrates a year by year comparison of the price variation rate of PC (black curve), the hedonic price index (grey curve), and the performance improvement (white curve).

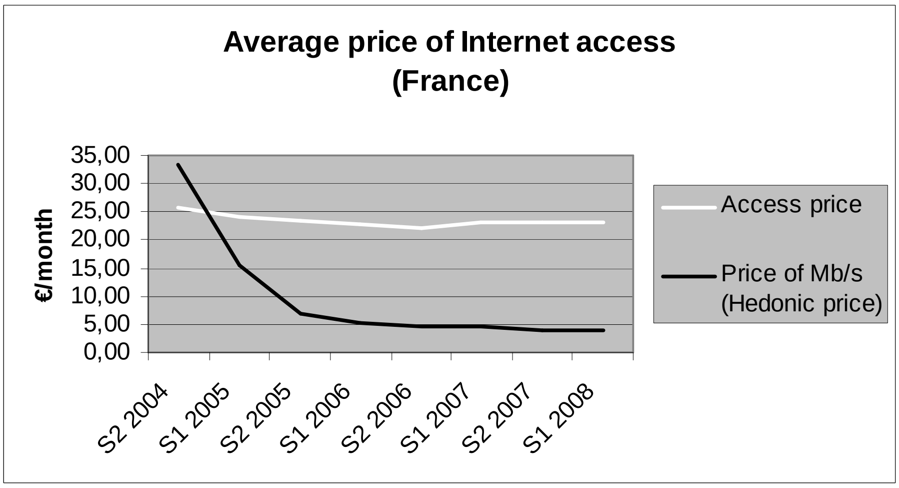


Source Insee Arthaut (2006)

figure 7

Performance evolution is the difference between the evolution of the hedonic price and the evolution of the average price of a PC. The slow decrease of the average price of a PC compared to the stability of the price used in the model above, arise from the growth of the PC market which is not totally mature yet even in the OECD countries such as France. Indeed, the economy of scale allows the PC industry to lower prices.
Furthermore, we can notice the dramatic acceleration of technical progress from 1997 to 2000 which coincides with the Internet bubble. The investments in technical progress were very important during this period. Investments exceeded the financial capacities of the industry which was not sustainable in the long run. After 2000 the rate of technical progress decreased and fell heavily below the pre-bubble level.

The following example, Internet access in France, shows a quasi stable price of internet access with an exponential decrease of the hedonic price index (figure 8).



Source: Author appraisal from ARCEP and Enov Research data
(see annex 9)
figure 8

Internet Service Providers' investments in network capacities in order to implement the technical progress allowed them to propose more and more internet bit rate connectivity to their subscribers.

In the same period of time, the static competition on the market tended to decrease as time went on, as illustrated by the increase of the Herfindahl index for the corresponding period of time. Therefore the dramatic fall in the price of a Mb/s is more due to technical progress rather than price competition. (figure 9)

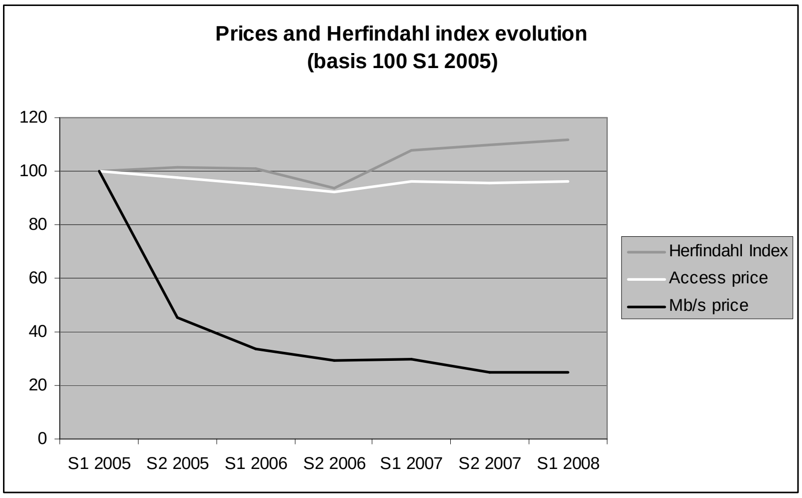


figure 9

Figure 9 highlights a positive correlation, 20%, between the Internet access price and the Herfindahl Index but a negative correlation between the hedonic price (Mb/s price) and the Herfindahl Index, -35%. Static competition appears to have only a limited impact on hedonic prices even if it seems to have a large one on Internet access price. The primary cause of the decrease in the hedonic price clearly stems from technical progress.

3.4 Limits of the model

This model, based on a hotelling competition, presumes that the entire market is covered, this is a mature market. When the market is not totally covered, competition is not the only incentive to invest, the conquest of newcomers is another one. In such a case, if there were increasing economies of scale, marginal costs should decrease as the number of newcomers increases, and prices should decrease, as illustrated in the example of Personal Computers. Moreover, a monopoly also has an incentive to invest because the monopolist’s margin increases with the consumers’ willingness to pay.

Nevertheless, in case of imperfect competition, incentive to invest to gain newcomers tends to vanish when the market approaches maturity. Also, in the end, thanks to the constant improvement of the performance of a good, a market always becomes mature.

4 Conclusion

Technical progress improves the services and goods that firms provide and increases consumers’ willingness to pay. In mature markets, this improvement essentially benefits the customers. Indeed, competition, more precisely, the fear of the competitors, urges firms to invest in technical progress without increasing their margin.

Competition has two components: a static one, known as price competition, and a dynamic one, known as technical progress competition. When technical progress potential goes beyond a certain threshold, the investment in technical progress becomes profitable and the dynamic component of the competition appears. When the technical progress potential is not high enough, investment in technical progress is not profitable and the competition has only its static component.

The intensity of dynamic competition is determined by the amount of investment firms are encouraged to invest. This amount is independent of the level of static competition, it depends solely on the potential technical progress of the industry. This potential is particularly high in the information technologies. For a given number of firms, the maximum level of Consumer Surplus as well as Social Surplus is attained for a constant product of the two kinds of competition. This competition product multiplied by the number of firms is equal to the market size: Equation 19 $N\gamma I^{*}=n$.

When the competition product is higher, the market faces a structure of "over-competition". This over-competition structure is not sustainable in the long run and finally firms invest only the maximum amount they can, but that lowers the rate of technical progress.

When the competition product is lower, the market faces a structure of "under-competition". This under-competition structure is sustainable. It lowers the level of Consumer Surplus a little, but to a lesser extent than "over-competition". Consumer Surplus and Social Welfare are better off if the competition product is a little too low than if it is a little too high.

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5 Annexes

5.1 Annex 1

Utility for a consumer located at x to purchase respectively firm 1's and firm 2's goods is:

$$U_1 = v_1 - p_1 - hx$$
$$U_2 = v_2 - p_2 - h(1-x)$$

The indifferent consumer is located at x such that $U_1 = U_2$

$$x = \frac{1}{2} + \frac{v_1 - v_2}{2h} + \frac{p_2 - p_1}{2h}$$

x is the market share of firm 1 and (1-x) the market share of firm 2

$$\sigma_1 = \frac{1}{2} + \frac{v_1 - v_2}{2h} + \frac{p_2 - p_1}{2h}$$

$$\sigma_2 = \frac{1}{2} + \frac{v_2 - v_1}{2h} + \frac{p_1 - p_2}{2h}$$

As a result firm i's market share for $i, j \in \{1,2\}$ $\sigma_i = \frac{1}{2} + \frac{v_i - v_j}{2h} + \frac{p_j - p_i}{2h}$

Firm i's profit is $\pi_i = (p_i - c)\sigma_i$

The first order condition for the maximization of profit leads to

$$\frac{d\pi_i}{dp_i} = \frac{h + v_i - v_j + p_j - 2p_i + c}{2h} = 0$$

And as a result $p_j = v_j - v_i + 2p_i - c - h$

Same manner from the profit of firm j $\pi_j = (p_j - c)\sigma_j$

We can write $p_i = v_i - v_j + 2p_j - c - h$

Now we replace p_i in the expression of p_j and p_j in the expression of p_i

We obtain: $p_i = \frac{v_i - v_j}{3} + c + h$ and $p_j = \frac{v_j - v_i}{3} + c + h$

As the result we obtain equation (1) $\sigma_i = \frac{1}{2} + \frac{v_i - v_j}{6h}$

5.2 Annex 2

Let us break down the equation 12 into three inequalities

1) $n\left(\frac{V(I)}{3} + \frac{V(I)^2}{18h}\right) - I(1+\rho) > 0$

2) $0 > -I(1+\rho)$

3) $-I(1+\rho) > n\left(\frac{V(I)^2}{18h} - \frac{V(I)}{3}\right)$

The first inequality becomes $n\frac{V(I)}{3}\left(\frac{6h + V(I)}{6h}\right) > I(1+\rho)$

If $I > 0$ the second inequality is checked.

The third inequality becomes $n\frac{V(I)}{3}\left(\frac{6h - V(I)}{6h}\right) > I(1+\rho)$

The sum of inequalities 1) and 3) leads to $I < n\frac{V(I)}{3(1+\rho)}$

5.3 Annex 3

It is known that $V(0) = 0$, $\frac{dV(0)}{dI} \geq 0$ because *V(I)* is increasing, then $\frac{V(0)}{\left(\frac{dV(0)}{dI}\right)} = 0$

Equation $\frac{V(I)}{\left(\frac{dV(I)}{dI}\right)}$ increases faster than *I* ; $\frac{d\left(\frac{V(I)}{\left(\frac{dV(I)}{dI}\right)}\right)}{dI} = 1 - \frac{V(I)\left(\frac{d^2V(I)}{dI^2}\right)}{\left(\frac{dV(I)}{dI}\right)^2}$

$\frac{d^2V(I)}{dI^2} < 0$ because *v(I)* is concave and *V(I) > 0* then $\frac{V(I)\left(\frac{d^2V(I)}{dI^2}\right)}{\left(\frac{dV(I)}{dI}\right)^2} < 0$ and therefore $\frac{d\left(\frac{V(I)}{\left(\frac{dV(I)}{dI}\right)}\right)}{dI} > 1$

and as a result, $\frac{V(I)}{\left(\frac{dV(I)}{dI}\right)} > I$ for *I > 0*

5.4 Annex 4: Calculation of Dynamic Consumer Surplus an Welfare

Dynamic consumer surplus is provided by equation 15:

$$CS = n\int_0^\infty \left(v_u - c - \frac{5h}{4} \right) e^{-\ln(1+\rho)u} du \text{ and } v_u = v_0 + uV(I) \text{ so:}$$

$$CS = n\int_0^\infty \left(v_0 + uV(I) - c - \frac{5h}{4} \right) e^{-\ln(1+\rho)u} du = n\int_0^\infty \left(v_0 - c - \frac{5h}{4} \right) e^{-\ln(1+\rho)u} du + n\int_0^\infty uV(I) e^{-\ln(1+\rho)u} du$$

$$CS = n\left(\frac{4(v_0 - c) - 5h}{4\ln(1+\rho)} + \frac{V(I)}{(\ln(1+\rho))^2} \right)$$

$$CS = \frac{n}{\ln(1+\rho)} \left(\frac{V(I)}{\ln(1+\rho)} + \underbrace{v_0 - c - \frac{5h}{4}}_{\text{static consumer surplus}} \right)$$ This is equation 16.

Likewise, dynamic Welfare can be deduced from static Welfare, equation 7 $w = n(v - c - \frac{1}{4}h)$

$$W = n\int_0^\infty \left(v_u - c - \frac{h}{4} \right) e^{-\ln(1+\rho)u} du \text{ and } v_u = v_0 + uV(I) \text{ so:}$$

$$W = n\int_0^\infty \left(v_0 + uV(I) - c - \frac{h}{4} \right) e^{-\ln(1+\rho)u} du = n\int_0^\infty \left(v_0 - c - \frac{h}{4} \right) e^{-\ln(1+\rho)u} du + n\int_0^\infty uV(I) e^{-\ln(1+\rho)u} du$$

$$W = n\left(\frac{4(v_0 - c) - h}{4\ln(1+\rho)} + \frac{V(I)}{(\ln(1+\rho))^2} \right)$$

$$W = \frac{n}{\ln(1+\rho)} \left(\frac{V(I)}{\ln(1+\rho)} + \underbrace{v_0 - c - \frac{h}{4}}_{\text{static welfare}} \right)$$ This is equation 17.

5.5 *Annex 5: Proof of* $\frac{2I^*}{n} < h^*_{CS} < h^*_W$

$$\frac{\partial CS}{\partial h} = \frac{n}{\ln(1+\rho)} \left(\frac{\left(\frac{dV(I)}{dh}\right)}{\ln(1+\rho)} - \frac{5}{4} \right) \text{ with } \frac{dV(I)}{dh} = \frac{dV(I)}{dI}\frac{dI}{dh} = \frac{n}{2}\frac{dV(I)}{dI}$$

Then $$\frac{\partial CS}{\partial h} = \frac{n}{2\ln(1+\rho)} \left(\frac{n\left(\frac{dV(I)}{dI}\right)}{\ln(1+\rho)} - \frac{5}{2} \right)$$

If $h = h^*_{CS}$ then $\frac{\partial CS}{\partial h} = 0 \Rightarrow \frac{dV\left(\frac{n h^*_{CS}}{2}\right)}{dI} = \frac{5\ln(1+\rho)}{2n}$

We know that $\frac{dV(I^*)}{dI} = \frac{3(1+\rho)}{n}$ equation 11, and $\forall \rho > 0; \frac{1+\rho}{\ln(1+\rho)} \geq e$, then, $\frac{3(1+\rho)}{n} > \frac{e\ln(1+\rho)}{n} > \frac{5\ln(1+\rho)}{2n}$ because $e > \frac{5}{2}$

Then $\frac{dV(I^*)}{dI} > \frac{dV\left(\frac{nh_{CS}^*}{2}\right)}{dI}$. We know that $\frac{dV(I)}{dI}$ is decreasing because *V(I)* is concave, as a consequence: $I^* < \frac{nh_{CS}^*}{2}$ or $\frac{2I^*}{n} < h_{CS}^*$

Likewise, for Welfare: $\frac{\partial W}{\partial h} = 0 \Rightarrow \frac{dV\left(\frac{n\,h_W^*}{2}\right)}{dI} = \frac{\ln(1+\rho)}{2n}$ and $\frac{5\ln(1+\rho)}{2n} > \frac{\ln(1+\rho)}{2n}$. As a result, $\frac{dV\left(\frac{nh_{CS}^*}{2}\right)}{dI} > \frac{dV\left(\frac{nh_W^*}{2}\right)}{dI}$ therefore $\frac{nh_{CS}^*}{2} < \frac{nh_W^*}{2}$

Finally: $\frac{2I^*}{n} < h_{CS}^* < h_W^*$

5.6 Annex 6: Generalization to N firms

Let us consider a *N* firms' model of price competition. Each firm is differentiated from the others "à la Hotelling". Let assume v_1, v_2, ….., v_N the consumers' values for respectively firm 1,2,…,N product.
Whatever $i, j \in \{1,2,..., N\}$ we assume that consumers are regularly shared on lines named *(i,j)* where firm *i* is located at one end and firm *j* at the other. A total of n consumers are uniformly distributed along each *(i,j)*line whose length is normalized to one. A consumer located at point x on the line *(i,j)* incurs a disutility of *hx* if he buys from firm *i* and *h(1-x)* if he buys from firm *j*.
The ratio of consumers which purchase from firm *i* is:

$$\sigma_{ij} = \frac{1}{2} + \frac{v_i - p_i - v_j + p_j}{2h}$$

When there are N firms, there are $\frac{N(N-1)}{2}$ braces *(i,j)* where $i \neq j$. This means there are $\frac{N(N-1)}{2}$ lines and then $\frac{2n}{N(N-1)}$ consumers on each line.
Firm i appears in $(N-1)$ braces.

Demand for firm *i* q_i is:

$$q_i = \frac{2n}{N(N-1)} \sum_{j \neq i} \sigma_{ij}$$

$$q_i = \frac{2n}{N(N-1)}\left(\frac{(N-1)}{2} + \frac{(N-1)v_i - \sum_{j\neq i} v_j}{2h} + \frac{\sum_{j\neq i} p_j - (N-1)p_i}{2h}\right)$$

$$q_i = \frac{n}{N} + \frac{n}{Nh}\left(v_i - p_i + \frac{\sum_{j\neq i} p_j - \sum_{j\neq i} v_j}{(N-1)}\right)$$

Let us assume that each firm incurs the same production cost *c*

Firm *i*'s profit is: $\pi_i = (p_i - c)\,q_i$

At Nash equilibrium: $\frac{\partial \pi_i}{\partial p_i} = 0$, this leads to

$$p_i = \frac{h+c}{2} + \frac{\sum_{j\neq i} p_j + (N-1)v_i - \sum_{j\neq i} v_j}{2(N-1)}$$ and then

$$\sum_{j\neq i} p_j = \frac{(N-1)}{2}(h+c) + \frac{p_i - v_i}{2} + \frac{(N-2)\sum_{j\neq i} p_j + \sum_{j\neq i} v_j}{2(N-1)}$$

$$\sum_{j\neq i} p_j = \frac{(N-1)^2}{N}(h+c) + \frac{(N-1)}{N}(p_i - v_i) + \frac{\sum_{j\neq i} v_j}{N}$$

Therefore $p_i = \frac{(2N-1)(h+c) + p_i + (N-1)v_i - \sum_{j\neq i} v_j}{2N}$

And as a result: $p_i = c + h + \frac{(N-1)v_i - \sum_{j\neq i} v_j}{(2N-1)}$ and $q_i = \frac{n}{N}\left(1 + \frac{(N-1)v_i - \sum_{j\neq i} v_j}{(2N-1)h}\right)$

Finally, $\pi_i = \frac{n}{N\,h}\left(h + \frac{(N-1)v_i - \sum_{j\neq i} v_j}{(2N-1)}\right)^2$

We can check this when *N=2* and equation 3 is obtained again.

If all firms have an equal consumers' valuation for their good $v_1 = v_2 = \cdots = v_N = v$ then:

$$\pi_1 = \pi_2 = \cdots = \pi_N = \pi = \frac{n\,h}{N}$$

The generalization of equation 9 becomes:

$$\pi_i(t_1, I_i) - I_1(1+\rho) = \frac{n}{Nh}\left(h + \frac{(N-1)V(I_1) - \sum_{i\neq j} V(I_j)}{2N-1}\right)^2 - I_1(1+\rho)$$

The first order condition leads to:

$$\frac{2n}{N\,h}\frac{(N-1)}{(2N-1)}\frac{dV(I_1)}{dI_1}\left(h+\frac{(N-1)V(I_1)-\sum_{i\neq j}V(I_j)}{2N-1}\right)-(1+\rho)=0$$

If $I_1 = I_2 = \cdots = I_N = I$ which happens when at time t_0 all firms earn the same profit

then $\frac{dV(I_i)}{dI_i}=\frac{(2N-1)N\,(1+\rho)}{2(N-1)n}$. It can be observed that when *N=2* equation 11 is obtained again.

In such a case, I^* is defined by $\frac{dV(I^*)}{dI}=\frac{(2N-1)N\,(1+\rho)}{2(N-1)n}$ and maximum Consumer Surplus and Welfare are reached when: $I^*=\pi$, this means $N\,\gamma\,I^*=n$ with $\gamma=\frac{1}{h}$

5.7 Annex 7 Generalization of annex 4

Is the equation $\frac{NI^*}{n}<h^*_{CS}<h^*_W$ always true whatever N ?

When $v_1 = v_2 = \cdots = v_N = v$ Consumer Surplus and Welfare do not depend on the number of firms because the number of customers n and prices do not vary according to N.

$$\frac{\partial CS}{\partial h}=\frac{n}{\ln(1+\rho)}\left(\frac{\left(\frac{dV(I)}{dh}\right)}{\ln(1+\rho)}-\frac{5}{4}\right) \text{ with } \frac{dV(I)}{dh}=\frac{dV(I)}{dI}\frac{dI}{dh}=\frac{n}{N}\frac{dV(I)}{dI}$$

Then $\frac{\partial CS}{\partial h}=\frac{n}{N\ln(1+\rho)}\left(\frac{n\left(\frac{dV(I)}{dI}\right)}{\ln(1+\rho)}-\frac{5N}{4}\right)$

If $h=h^*_{CS}$ then $\frac{\partial CS}{\partial h}=0 \Rightarrow \frac{dV\left(\frac{n\,h^*_{CS}}{N}\right)}{dI}=\frac{5N\ln(1+\rho)}{4n}$

It is known that $\frac{dV(I^*)}{dI}=\frac{(2N-1)N\,(1+\rho)}{2(N-1)n}$,

and $\forall\rho>0$, $\frac{(2N-1)N\,(1+\rho)}{2(N-1)n}>\frac{5\ln(1+\rho)N}{4n}$

Then $\frac{dV(I^*)}{dI}>\frac{dV\left(\frac{nh^*_{CS}}{N}\right)}{dI}$. It is known that $\frac{dV(I)}{dI}$ is decreasing because *V(I)* is concave, as a consequence: $I^*<\frac{nh^*_{CS}}{N}$ or $\frac{NI^*}{n}<h^*_{CS}$.

Likewise, for Welfare:

$\frac{\partial W}{\partial h} = 0 \Rightarrow \frac{dV\left(\frac{n\,h_W^*}{N}\right)}{dI} = \frac{N\ln(1+\rho)}{4n}$ and $\frac{(2N-1)N(1+\rho)}{2(N-1)n} > \frac{N\ln(1+\rho)}{4n}$. As a result, $\frac{dV\left(\frac{nh_{CS}^*}{N}\right)}{dI} > \frac{dV\left(\frac{nh_W^*}{N}\right)}{dI}$ therefore $\frac{nh_{CS}^*}{N} < \frac{nh_W^*}{N}$. Finally: $\frac{NI^*}{n} < h_{CS}^* < h_W^*$

5.8 *Annex 8: Proof of* $\lim_{h\to NI^*-}\frac{\partial CS}{\partial h} > -\lim_{h\to NI^*+}\frac{\partial CS}{\partial h}$ *and* $\lim_{h\to 2I^*-}\frac{\partial W}{\partial h} > -\lim_{h\to 2I^*+}\frac{\partial W}{\partial h}$

$$\frac{\partial CS}{\partial h} = \frac{n}{N\ln(1+\rho)}\left(\frac{n\left(\frac{dV(I)}{dI}\right)}{\ln(1+\rho)} - \frac{5N}{4}\right)$$

$$\lim_{h\to NI^*-}\frac{\partial CS}{\partial h} = \frac{n}{N\ln(1+\rho)}\left(\frac{(2N-1)N(1+\rho)}{2(N-1)\ln(1+\rho)} - \frac{5N}{4}\right)$$

$$\lim_{h\to NI^*+}\frac{\partial CS}{\partial h} = \frac{n}{N\ln(1+\rho)}\left(-\frac{5N}{4}\right)$$

Function $\forall\rho > 0; \frac{1+\rho}{\ln(1+\rho)} \geq e$; *e* is the minimum of the function attained for $\rho = e-1$

then $\forall N \geq 2, \frac{(2N-1)(1+\rho)}{2(N-1)\ln(1+\rho)} > e$ and yet it is known that $e > \frac{5}{2}$, therefore

$\frac{(2N-1)N(1+\rho)}{2(N-1)\ln(1+\rho)} - \frac{5N}{4} > \frac{5N}{4}$ and as a result $\lim_{h\to NI^*-}\frac{\partial CS}{\partial h} > -\lim_{h\to NI^*+}\frac{\partial CS}{\partial h}$ which means

that $CS(NI^*)$ decreases faster when *h* is a little too low rather than if it is a little too high.

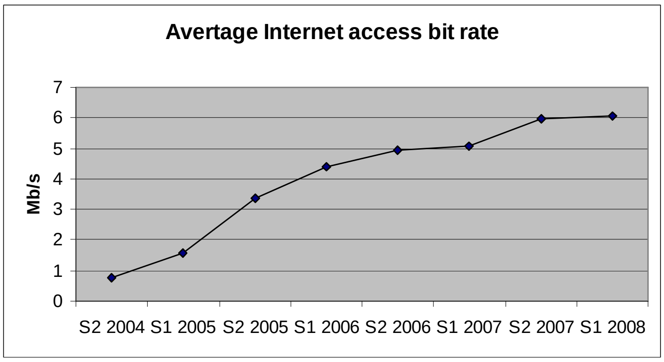
A fortiori $\frac{(2N-1)N(1+\rho)}{2(N-1)\ln(1+\rho)} - \frac{N}{4} > \frac{N}{4}$ and likewise $\lim_{h\to NI^*-}\frac{\partial W}{\partial h} > -\lim_{h\to NI^*+}\frac{\partial W}{\partial h}$

5.9 *Annex 9:*

The chart below indicates the consumers' internet bit rate in France, source (Enov research)

Bit Rate	S2 2004	S1 2005	S2 2005	S1 2006	S2 2006	S1 2007	S2 2007	S1 2008
>8Mbts/s		1%	9%	14%	16%	15%	18%	15%
3-8Mbts/s	2%	8%	13%	15%	17%	17%	20%	21%
2Mbts/s	3%	13%	16%	16%	13%	12%	11%	7%
1Mbts/s	23%	24%	16%	15%	18%	19%	15%	13%
512 Kbts/s and less	60%	37%	27%	21%	16%	11%	8%	7%

This leads to the author's following appraisal of the average internet bit rate (see graph below):



The Herfindahl index has been calculated (by the author) from the market shares (revenues) of the operators given by Enov research.

	S2 2004	S1 2005	S2 2005	S1 2006	S2 2006	S1 2007	S2 2007	S1 2008
Herfindahl index		2407	2447	2426	2255	2597	2644	2689

Whereas, the number of connection and the revenues comes from ARCEP in the following chart:

	S2 2004	S1 2005	S2 2005	S1 2006	S2 2006	S1 2007	S2 2007	S1 2008
Connections (millions)	5,5	7,3	8,5	10,5	11,8	13,7	14,8	16,3
Revenues (€ millions)	425	527	599	722	785	949	1021	1128