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On The Recent Debate on Capital Theory and General Equilibrium

Contribution to the Festschrift in honourof Prof. Bertram Schefold

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Abstract - The paper disputes the negative conclusion of prof. Mandler on the thesis by Garegnani, Schefold, Parrinello that intertemporal general equilibrium theory too is undermined by reswitching and reverse capital deepening. The paper argues that Mandler's conclusion rests upon highly criticisable assumptions that render the equations of intertemporal general equilibrium identical to those of general equilibria without capital goods. The Walrasian treatment of the capital endowment is criticized in Part I on the basis of its insufficient persistence, and of other 'methodological' criticisms that are systematically surveyed. In Part II it is shown through a numerical example that Mandler's claim, that the assumption of a single consumer guarantees uniqueness of intertemporal equilibrium independently of reswitching or reverse capital deepening, rests on the absence of production of capital goods in the last period of the equilibrium; this assumption is thus revealed to be one of the tricks that prevents the existence of capital goods from changing the properties of the equilibrium relative to those of equilibria without capital.

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(comments welcome: please send them to petri@unisi.it)

INTRODUCTION

1. In this paper[1] I try to contribute to the recent debate, whether the deficiencies of traditional neoclassical capital theory highlighted in the Cambridge controversies affect the modern, neo-Walrasian versions of general equilibrium theory that specify the capital endowment of the economy as a given vector. Hoping that what I shall say will help the debate to make some progress however small, I would like this effort of mine to testify my admiration for Professor Bertram Schefold and my gratitude for having learnt so much from him.

The latest stage of the debate on this topic is Professor Michael Mandler's entry "Sraffian economics (new developments)" in the second edition (2008) of *The New Palgrave Dictionary of Economics*. I leave aside here most of the claims of this contribution, many of which are criticisable, e.g. Professor Mandler's insistence on a supposed "Sraffian indeterminacy"; his mistaken identification of Sraffa's analysis with neoclassical full-employment steady states[2]; his acceptance of a mistaken view criticized again and again by the "Sraffians", the identification of the use of the notion of capital as a single factor with the aggregate-production-function models[3].

[1] I thank Profs. Enrico Sergio Levrero, Fabio Ravagnani, and Bertram Schefold for their comments, not all of which were heeded but all of which stimulated rewritings that I hope have improved the readability of the paper although lengthening it.

[2] Since this identification remains implicit, it may be useful to point out that on p. 804 Mandler (2008) writes that "the particular way Sraffa and his followers have spelled out their long-run view of the economy, by requiring that relative prices be constant through time, undermines their 'missing equation' criticism: linear activity models with constant relative prices have determinate factor prices". The last statement in this sentence is false unless one interprets it as referring to neoclassical full-employment steady states where saving propensities are such as to determine uniquely the interest rate ensuring a growth rate equal to the growth rate of the supply of labour. On the difference between long-period and steady-state analyses cf. below, fn. 26??

[3] He thus overlooks the dominance in the neoclassical tradition until at least the 1940s of long-period general equilibria, completely disaggregated but including (and needing) a value

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These mistakes – already present in his previous papers – have been discussed in Garegnani (2005a) and in Fratini and Levrero (2007). I am interested here in his conclusions on my present topic. These conclusions are starkly negative on the import of the recent Sraffian criticisms of general equilibrium theory. Relying on arguments put forth in his 2002 and 2005 papers, Mandler dismisses as wrong the attempts by Garegnani (2000, 2003, 2005a, 2005b), Schefold (1997, 2005a, 2005b) and Parrinello (2005) to argue that reswitching and reverse capital deepening undermine, not only the older long-period formulations of neoclassical equilibrium theory relying on capital treated as a single value factor, but also intertemporal neo-Walrasian general equilibria; his conclusion is that the standard neoclassical view, that multiple and tâtonnement-unstable equilibria can only be due to insufficiently "well-behaved"[4] consumer preferences and in particular to heterogeneity of consumers, is fully correct: "Instability can arise in general equilibrium but it stems from the demand side of the model, not the failure of capital goods to aggregate" (p. 804)[5]; the possibility that of two steady states the one with a higher consumption per unit of labour has a higher rate of interest is dismissed as no cause of problems: "the move to a steady state with higher consumption per worker requires a sacrifice of consumption at some set of dates prior to arrival at the new steady state. The no-free-lunch moral of neoclassical growth theory rears its head" (p. 814)[6].

No doubt a reader who comes across these conclusions without direct knowledge of the contributions Mandler criticizes gets the message of a defeat of the critics. This message, I will try to argue, is misleading. It is true that one or two claims advanced by the critics appear not easily defensible; but the results at the basis of Mandler's conclusions are of very limited relevance for the central question in this

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endowment of capital because the endowments of the several capital goods were treated as variables (as required by the uniform rate of return on supply price characterizing these equilibria) and therefore the equilibrium needed an endowment of capital consisting of a single quantity of variable 'form'. This allows Mandler to avoid discussing the true analytical roles in the neoclassical approach of the conception of capital as a single factor of variable 'form' (Petri 2004, chs. 1, 3), and more generally the shift of neoclassical value theory away from the long-period method. This mistake of Mandler is particularly surprising, considering his presence at the 1999 Siena Summer School (Petri and Hahn 2003) where several interventions insisted on these points.

[4] The "well" in this term is of course an expression of attachment to neoclassical theory rather than a scientific term. The same goes for adjectives such as 'counterintuitive' or 'perverse' used to indicate phenomena (e.g. a demand for a factor that increases when the factor's rental rises) that are disturbing for the validity of neoclassical theory; they should be avoided.

[5] In the *New Palgrave* entry Mandler does not discuss equilibrium uniqueness; I base myself on his views on this issue put forth in Mandler (2002, 2005).

[6] This conclusion of Mandler again relies on taking for granted the full employment of labour, without apparently realizing that this assumption is rejected by the critics precisely on the basis of the implications of their criticisms, so to assume it in order to deny some of the critics' views is illegitimate.

debate, namely, whether the marginalist (or neoclassical, or supply-and-demand) approach to value and distribution is capable of a satisfactory treatment of capital.

The point is that the results Mandler bases himself on rely on assumptions that eliminate the specificities of production with capital goods, and render the equilibrium equations as well as the adjustment process formally identical to those in models without capital goods, where all factors of production are non-produced and there is no interest rate (I will call 'atemporal' the equilibria of these models, but their precise nature needs discussion, cf. §**11**). The assumptions that I particularly intend to discuss are three:

- *First Assumption*: it is legitimate to include given initial endowments of the several capital goods among the data of the intertemporal equilibrium (this obliges one to study stability via instantaneous recontracting processes that exclude the implementation of disequilibrium productions and exchanges);

- *Second Assumption*: the equilibrium is over a finite number of periods after which the economy 'ends', that is, in the last period(s) there is no production of capital goods since these would be useless[7];

- *Third Assumption*: the adjustment toward equilibrium is, to use Mandler's terminology, a 'factor tâtonnement' where for each vector of non-produced factor prices the quantities produced are assumed equal to the quantities demanded at minimum-cost prices (on the basis of incomes that assume that all factor supplies are bought), so disequilibria arise on the sole markets of non-produced factors or goods.

Given these assumptions, as far as I can see the formal equivalence holds (I discuss the issue in §§**11-13**) and the theorems on uniqueness and stability valid for atemporal general equilibria hold for intertemporal equilibria too; thus Mandler, having implicitly made these assumptions, can contend that the additional assumption of a single consumer with a differentiable strictly quasiconcave utility function ensures uniqueness and tâtonnement stability of the intertemporal equilibrium, independently of whether the technology is such as to cause reswitching or reverse capital deepening[8]. Two claims advanced by the critics are thereby questioned:

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[7] The assumption of production in the last period of an exogenously given vector of capital goods is only a special case of this assumption, because these final capital goods are in fact treated like consumption goods demanded in fixed quantities.

[8] There is one formal claim of Mandler concerning comparative statics that I find it difficult to accept but is of secondary importance for the topic of this paper, so its discussion can be confined to a footnote. Mandler (2002 p. 217, 2005 p. 477) argues that, if consumer excess demand satisfies the weak axiom and the utility function of the representative consumer is concave, then in general equilibrium models an increase in the endowment of a factor entails a lower equilibrium rental of that factor. He forgets to add that such a comparative-statics result requires in addition that no good be inferior (Quah 2003). Anyway the possibility, that a greater endowment of a factor be associated with a higher equilibrium rental of that factor owing to some other good being inferior, is due to income effects, so it is not relevant for the issue whether "anti-neoclassical" results can emerge in general equilibrium models owing to the presence of capital goods rather than to income effects. A

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Professor Garegnani's (2000, 2003) claim of possible non-uniqueness of intertemporal equilibria in spite of absence of income effects (I discuss it in §§**11-12**); and Professor Schefold's (1997, 2005a) claim of possible instability of the 'factor tâtonnement' (which he calls Recursive-Samuelson tâtonnement) when the single-consumer intertemporal equilibrium includes a 'perverse' technology switch (I do not discuss this claim because I leave stability issues aside in this paper; I only note, first, that Mandler (2008) does not refute Schefold's (2005b) rejoinder, in particular Schefold's claim that he has demonstrated the possibility of a local instability and that "the loss of local stability is crucial, even if there is global stability all the same, for global stability means little if wages may go to zero in the process of adaptation" (p. 504), and second, that the relevance of the 'factor tâtonnement' is highly doubtful, cf. below, §§ **3** and **9**).

But the critics had also advanced criticisms of the assumptions behind Mandler's contention, and these criticisms Mandler avoids discussing. Thus Garegnani explicitly criticizes the First Assumption in (2000, pp. 410-11; 2003, p. 127; 2005a, pp. 417-28), and his insistence on the need for an explicit consideration of savings-investment adjustments is a criticism of the Third Assumption; Professor Parrinello agrees with Garegnani on the need to reject the Third Assumption to make room for savings-investment markets; and Schefold insists that a 'factor tâtonnement' is only one possible way – and a very unrealistic one, that assumes a 'perfect planner' (2005b, p. 509) – to conceive adjustments on the basis of the first two assumptions, and he proposes as a possible alternative an 'uninformed auctioneer' (2005a, p. 467; 2005b, p. 508 fn. 6) that violates the Third Assumption.

Now, those assumptions are clearly questionable and the implications of dropping them should not have been left undiscussed by Mandler. All one gets from him is: "Perhaps in a more realistic setting [than the 'factor tâtonnement'] the paradoxes of capital theory will turn out to be a distinct source of instability – but the case remains to be made." (Mandler 2008, p. 813) The implicit admission of scarce realism of the 'factor tâtonnement' must be appreciated, but unfortunately neither he nor other neoclassical economists appear interested in exploring the possible consequences of "a more realistic setting" for economies with heterogeneous capital[9].

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different comparative statics issue, more relevant to the debate but still to be explored, is that in intertemporal equilibrium models an increased endowment of labour cannot but mean a simultaneous increase in the endowments of many 'nonproduced' factors, namely labour endowments of several consecutive periods, a phenomenon with no correspondence with what is conceivable for atemporal models, and whose consequences on marginal products can be the most varied.

[9] There is a growing literature on non-tâtonnement adjustments, but with the single exception of F. M. Fisher (1983) none of these papers admits capital goods; and Fisher (as one might have expected on the basis of the considerations I will advance in §**9**) reaches no

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In particular, dropping the fundamental First Assumption would cause an immediate collapse of general equilibrium theory, whose system of equations would become underdetermined. No doubt for this reason, neoclassical economists stubbornly avoid discussing this assumption in spite of the numerous articles and books criticizing it. I repeat the criticisms in Part I of the present paper, which does not advance novel arguments but may be useful nonetheless as a systematic summary that includes a mention of perhaps less widely known criticisms. This part concludes that the problems with the First Assumption suffice to discard the neoclassical or supply-and-demand approach to value and distribution. But in view of the deafness of the neoclassical side on this issue, in Part II I join the attempts to show that the First Assumption does not earn neoclassical economists a defensible theory anyway.

So far these attempts have concentrated on stability but have not produced definitive results analogous, say, to the demonstration of the possibility of reswitching, apart from Schefold's demonstration that one can observe reswitching in an intertemporal equilibrium (but, as mentioned, the implications of this fact remain to be assessed). The stability implications of the tâtonnement based on the 'uninformed auctioneer' are still to be developed. Garegnani has not produced a numerical example proving the possible instability – on which he insists with arguments not without plausibility – of the savings-investment market(s) implicit in intertemporal models independently of 'badly behaved' consumer preferences. Parrinello has proposed a different tâtonnement that again explicitly embodies a savings-investment market but he has not proved that it can generate instability in cases where the standard tâtonnement would be stable.

Anyway there is a general problem with studies of the stability of intertemporal equilibria: the difficulty of producing adjustment processes that grant the First Assumption and complete futures markets and yet preserve a relevance for the understanding of real market economies, a "thorny question" as Schefold puts it (2005b, pp. 511). My considerations in Part I, especially in §§ **3** and **9**, certainly do not reduce the difficulty and may even suggest that very different avenues should be explored, e.g. the application of Gintis's (2007) approach to economies with capital goods.

So I have preferred to concentrate on the implications on *uniqueness* of equilibrium of introducing at least some elements of "a more realistic setting"; and I have chosen to work on an issue so far little discussed, the implications of dropping the Second, rather than the Third, Assumption. In Part II of this paper, after a discussion of some claims by Garegnani, I argue with the help of a numerical example that, without the assumption that in the last period of a finite-horizon intertemporal equilibrium the production of capital goods is either zero or a fixed unexplained vector, the First and the Third Assumptions plus the assumption of a

result supporting the neoclassical approach (cf. Petri 2004, pp. 48-50, 67-71), and has since given up studying the topic.

single "well behaved" consumer do not exclude the possibility of multiple equilibria. So the uniqueness result referred to by Mandler relies on excluding the undeniable existence of investment of *endogenously* determined composition in any period that one may choose to consider 'final' (but that cannot mean that the economy really ends there if one wants the theory not to be ridiculous); the moment such a limitation of the horizon up to which futures markets are assumed to exist is accepted (i.e. the moment the extension of the assumption of complete futures markets to an infinite number of periods is admitted to be nonsensical) and the existence is admitted of an investment component of final output whose composition is not determined in the same way as for consumption goods – a component that would not exist if there were no capital goods –, multiple equilibria can arise independently of consumer heterogeneity. This confirms the role of the Second Assumption as a neutralizer of the specificities of production with heterogeneous capital goods.

PART I

2. An important part of the Sraffa-inspired criticism of modern (neo-Walrasian) general equilibrium theory is totally absent from Mandler's entry: the part that points to what Garegnani (1990, p. 45) has called "methodological difficulties"[10] caused by the First Assumption:

A) insufficient persistence of the equilibrium's data relative to the endowments of capital goods;

B) insufficient factor substitutability;

C) dilemma (due to the need to take into account the changes that equilibrium prices will undergo over time) between on the one hand the intertemporal-equilibrium road, with its totally unrealistic assumptions of either complete futures markets or perfect foresight; and on the other hand the temporary-equilibrium road, with the indefiniteness (plus a Pandora box of other problems) that arises owing to unobservable subjective non-uniform expectations.

But the adjective "methodological", although not easily replaced (I too will use it below), is a potentially misleading one, and it may have contributed to the little attention given so far to these problems by the great majority of mainstream economists. Methodological discussions are dismissed by many scientists as the futile attempt by philosophers to tell scientists how to do science. Quite to the contrary, the deficiencies of neo-Walrasian general equilibrium theory highlighted by these criticisms are no less analytical, and no less grave, than for example the inconsistency of a given value endowment of capital in J. B. Clark or Wicksell.

[10] These difficulties (without the adjective 'methodological') are remembered in a paper by Garegnani (2005a: 422-23, 430-31) which is a direct reply to Mandler (2002), but they have been left unheeded by Mandler in his rejoinder (2005), except for a striking admission on perfect foresight which is commented upon in fn. 21?? below.

Indeed, to assume that the endowments of capital goods do not change during the adjustments toward equilibrium is no less absurd than to assume that, when distribution changes, the value of capital does not change. Fairy-tale institutions as unreal as the auctioneer might well be imagined, capable of maintaining the value of capital unchanged in the face of changes of real wages, for example by imposing by fiat such productions and savings as will quickly re-establish the previous value of the capital endowment after a change in prices; the behaviour of such an institution would have no correspondence with the behaviour of a market economy, but the same is true of the auctioneer. The result is, analogously to when one takes as given the value of capital, that when assuming a given vector of endowments of capital goods one takes something as given in the equilibrium equations, which in fact cannot be treated as given because it is relevantly altered by any realistic process of adjustment[11]; thus the assumption that adjustment operates through an auctioneer-guided tâtonnement hides an indeterminacy of the system of equilibrium equations, analogous to the one Friedrich Lutz (1967, p. 69) admitted for long-period equilibria when he wrote

> the subsistence fund, in the sense of a given value magnitude, cannot be taken as a datum but is itself one of the unknowns, so that the system of these writers [he was referring to Böhm-Bawerk, Wicksell, and other 'Austrian' authors, *F.P.*] lacked one equation for determining the equilibrium.

Allow me to insist on the analytical nature – that risks being obscured by the adjective 'methodological' – of this difficulty. After any change or novelty, firms must find out anew the demand for their products at the several product prices; and the only way is by trial and error, with unwanted accumulations or decumulations of inventories in the meanwhile; adapting production to demand and bringing inventories back to normal takes time, and during this time production goes on and alters the amounts in existence of the several capital goods; hence *it is contradictory* to assume a gravitation of produced quantities toward an equality between supply and demand, and not to assume that this process alters the amounts in existence of capital goods, and alters them in a way different from the one that would be brought about by the instantaneous reaching, at the inital moment, of an intertemporal equilibrium[12].

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[11] The inconsistency of a given value of capital too can be expressed in terms of insufficient persistence, because the value of capital goods is altered by the changes in prices and in quantities going on during disequilibrium adjustments.

[12] Thus consider an economy at date zero. Assume uniqueness of equilibrium in order to see the problem more clearly. The intertemporal equilibrium from date zero onwards is what the theory allows us to determine on the basis of the date-0 data (if we grant complete futures markets, of course). If adjustments are not instantaneous, when this economy reaches date 1 its endowments of capital goods are neither the ones of date zero, nor the ones it

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3. That the adjustment of produced quantities to demand necessarily requires actual trial-and-error productions is highlighted by the absurd role one is obliged to assign to the auctioneer the moment Constant Returns to Scale for industries (i.e. CRS for individual firms and/or free entry[13]) are admitted. In the auctioneer-guided tâtonnement the size and/or number of price-taking firms in an industry becomes infinite the moment positive extraprofits[14] can be made; therefore the auctioneer can only call zero-extraprofit prices, but at these prices the size and/or number of firms in each industry is indeterminate, so, unless the auctioneer becomes a true planner who ensures equality between supply and demand by *imposing* how many firms (and which ones of the potential ones) will be active and also (if there are firm-level CRS) how much each of them shall produce, equilibrium cannot be reached: outputs at equilibrium prices are indeterminate[15]. If on the contrary production is required to

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would have had at date 1 if at date 0 it had instantaneously reached the intertemporal equilibrium corresponding to the date-0 data. For the same reason, the intertemporal equilibrium that might be established at date 1 on the basis of the new date-1 data will not be reached either, so the danger arises of a cumulation of deviations from the original equilibrium path, deviations that can be of any magnitude in the absence of a theory of quantities and employment in disequilibrium. The implications of this fact have been described by Franklin M. Fisher (1983) as follows: "In a real economy, however, trading, as well as production and consumption, goes on out of equilibrium. It follows that, in the course of convergence to equilibrium (assuming that occurs), endowments change. In turn this changes the set of equilibria. Put more succinctly, the set of equilibria is path dependent ... [This path dependence] makes the calculation of equilibria corresponding to the initial state of the system essentially irrelevant. What matters is the equilibrium that the economy will reach from given initial endowments, not the equilibrium that it would have been in, given initial endowments, had prices happened to be just right" (Fisher, 1983, p. 14). Since Fisher explictly mentions production, he is not referring only to reallocations of given total endowments among consumers but also to changes in the endowments themselves. And the problem is worse than one might infer from this quotation: since general equilibrium theory is silent on what happens in non-virtual disequilibrium, "the equilibrium that the economy will reach from given initial endowments" cannot be determined; one cannot even establish whether an equilibrium will be reached at all; the theory tells us nothing at all as to how the economy will behave.

[13] That many general equilibrium theorists may have found it legitimate to assume a given number of decreasing-returns-to-scale firms, even in intertemporal equilibrium models (where the assumption means forbidding the birth of new firms even over time horizons of decades! white-hot nonsense), is truly a scandal, indicative of a disastrous readiness to accept mathematically convenient assumptions without worrying about their economic soundness.

[14] To circumvent the difference in the definition of profits in the classical and in the neoclassical tradition, I use 'extraprofits' to mean profits in the neoclassical sense (i.e. net of interest charges and risk allowance: but risk is neglected in the present paper).

[15] This shows the illegitimacy of the 'factor tâtonnement' as a representation, however idealized, of the working of markets in economies with production, and is therefore a criticism of the Third Assumption. Note that the same need for the auctioneer to act as a planner if outputs are to adjust to demands arises for Schefold's 'uninformed auctioneer' too,

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be actual, as in real economies, then the instantaneous explosion of industry capacity to infinite size cannot happen because firms must get hold of factors in order to produce and therefore their supply is limited by the availability of plants and other specialized factors whose supply takes time to be altered; which is why traditionally the indeterminateness of long-period supply at cost-covering prices was not considered a problem. The point is that if production must be actual then, as Marshall pointed out, supply can be assumed to be along a short-period industry curve conditioned by the availability of plants and other specialized factors; on the contrary in the tâtonnement firms promise to produce on the basis of their *demands* for factors (including fixed plants[16]), which need not be equal to the availability of those factors, so factor availability is no constraint on firms' supply decisions, which are accordingly infinite if there are extraprofits to be made, and are indeterminate if extraprofits are zero. Thus, unless one assumes a planned economy, actual productions are *indispensable* for individual decisions to bring about an adjustment of productions to demands; but the adjustment takes time and alters the amounts in existence of the several capital goods.

Thus the idea of product markets reaching equality between supply and demand not because of a planner's intervention but because of individual decisions requires actual trial-and-error productions, and as a result the system of general equilibrium equations becomes underdetermined because the data relative to the endowments of the several capital goods cannot be treated as data, they must be treated as variables. The equations no longer suffice to determine an equilibrium. The theory is revealed to be as incapable of determining an equilibrium, as the long-period versions without a given value endowment of capital.

4. I turn to the substitutability problem. That there will be too little substitutability among factors if the 'form' of capital is given was implicit in the writings of many traditional marginalist economists, and was explicitly admitted by Hicks in his 1932 *Theory of Wages* with reference to the demand for labour, in a passage quoted or cited so many times by Garegnani and by myself that there is no need to quote it again now. But allow me to quote, with a few minor corrections, my 1991 comment on that passage (*Review of Political Economy* 1991, p. 272):

> The important implication of this line of argument is that short-period neoclassical analyses cannot aim at endogenously determining the real wage as an

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at least in the description supplied in (Schefold 2005a p. 467): without his intervention, first-period intended outputs will not be $c_0^*+q_1^*A_\alpha$.

[16] A firm can demand plants too; in spite of the very-short-period nature of modern general equilibria, there is no justification for assuming that firms can only use the fixed plants they are initially endowed with. They can try to buy fixed plants of other firms; therefore it would be as arbitrary to exclude markets for fixed plants from the tâtonnement, as to exclude markets for lands.

equilibrium real wage. In order to avoid implausible results, the real wage in the short period must be admitted to be essentially sticky; perhaps slowly moving up or down according to the sign of the excess demand for labour, but anyway starting from a level that the analysis must take as exogenously given. Only long-period neoclassical analysis can plausibly try to explain the average level of the real wage in terms of an equilibrium between supply and demand, where the demand for labour is derived from the schedule of the long-period marginal product of labour, a notion based on the conception of capital as a single factor capable of changing 'form'. Short-period analyses cannot do without long-period theory in the marginalist approach.

These considerations apply with even greater force to neo-Walrasian, i.e. *very*-short-period, equilibria; here again I can rely on Hicks who in 1980-1981, in the course of a discussion of the difference between the IS-LM approach and temporary equilibrium theory, admitted that within the single 'week' of his *Value and Capital* temporary equilibria "The actual outputs of products, and probably also the actual input of labour, would be largely predetermined" (1980-81, p. 55). The essentially vertical labour demand curve thus admitted by Hicks undermines any attempt at a very-short-period equilibrium theory of wages (given the little right to presume with sufficient generality a positively sloped and highly elastic labour supply curve). The equilibrium wage might easily be zero or anyway implausibly low, or so high as to reduce other factor rentals to zero or nearly so; and small changes in labour endowment might bring about enormous changes in income distribution, cf. Fig. 1(b). The contradiction of such theoretical predictions with observation would deprive the theory of plausibility.

The neoclassical need for a highly elastic labour demand curve, and therefore for a long-period determination of the labour demand curve, is reinforced by the need to avoid multiple equilibria or the possibility of what can be called *nearly or practically indeterminate equilibria*, in the presence of a 'backward-bending' labour supply curve. The second of these possibilities is almost never mentioned so it deserves a graphical illustration, cf. Fig. 1(a). The labour demand curve must be sufficiently elastic as to render it highly improbable not only that it may cross more than once the backward-bending supply curve but also that, when the two curves cross only once and hence the equilibrium on the labour market is unique, excess demand may remain extremely close to zero in a wide interval around the equilibrium wage, as in Fig. 1(a) where the equilibrium wage is w_2 but excess demand is extremely close to zero between w_1 and w_3. If the latter case were to occur, since the tendency of the wage to change cannot but be the weaker the closer supply and demand are to equality, the supply-and-demand forces tending to alter a disequilibrium wage in the interval (w_1,w_3) would be extremely weak and thus, to all practical effects, a supply-and-demand theory of wages would have to admit an indeterminacy of wages in a wide interval.

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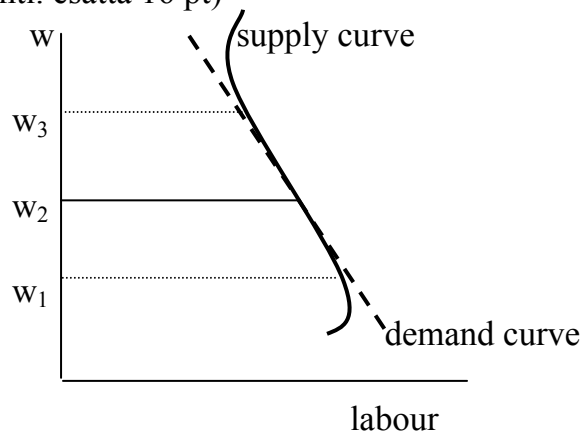

Fig. 1(a)

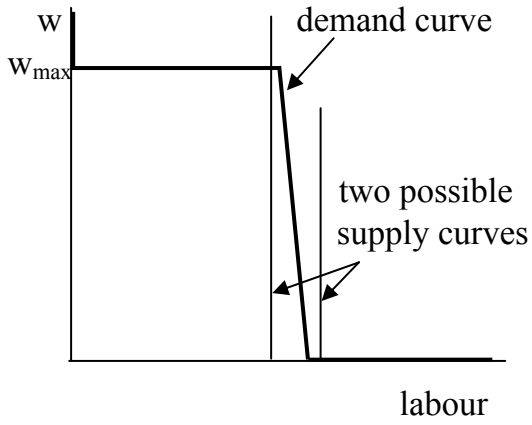

Fig. 1(b)

5. It may be useful – although perhaps a digression – to stress some further implications of treating the quantities of the several capital goods as variables, so as to show what vistas tend to be excluded from the economist's vision when the need for such a treatment of capital goods is neglected.

When one asks what determines the quantities of capital goods, their tendency to adapt to the demand for them at supply prices is the obvious answer. But this adaptation assumes a variability of the quantities produced of them, whose implications are not always fully grasped. The variability of the quantities produced of capital goods is part and parcel of that considerable capacity of production quickly to adjust to demand, that characterizes most markets of produced goods and without which our society would be very different. Without a quick adaptability of the flow supply of capital goods to demand, the adaptability of supply to demand would be non-existent for most produced goods: a flexible supply of flour is indispensable for the supply of bread to adapt to demand without frequent and large changes in the price of bread, changes that would enrage consumers and shake the smooth functioning of society. Competition contributes: firms *want* to be able to guarantee a quick adaptation of production to demand, in order to maintain customer goodwill and not to lose market shares. Thus they maintain some 'slack' in the normal utilization of their available quickly mobilizable resources: spare capacity, inventories, some unused potential supply of labour services. Part of this 'slack' is also ensured by cost minimization: the need to pay higher wages for night shifts induces firms not to use fixed plants 24 hours a day; but increases in the utilization of fixed plant remain possible if demand increases. Inventories (usually neglected in general equilibrium theory) of raw materials and of parts to be assembled ensure the possibility of rapid increases of production; the consequent run down inventories are rapidly reconstituted by the increased production of the industries producing them. Capitalist economies add to this flexibility by maintaining a considerable amount of unemployment; but even employed labour can generally supply an increased amount of labour services if asked to do so. Without this 'slack', the absence of short-period

factor substitutability would make it very difficult to vary production by simply transferring some variable factors, e.g. labour, from one industry or firm to another; the observed generally quick adaptation of supplies to demands would not be possible. (This 'slack' was implicitly conceded as a positive element of capitalist economies in the Western literature that criticized the 'tautness' of planning in the USSR.) The flexibility of production must be expected to be even greater than average in the durable capital goods industries, where firms know that they must stand ready to face strong swings in demand (for the well-known reasons that gave birth to the term 'accelerator').

This generalized 'slack' strongly suggests a view of economic growth and capital accumulation as dependent on the evolution of aggregate demand[17], because it implies that *aggregate* production too can quickly adjust not only to decreases of aggregate demand, but also – within limits rarely approached – to *increases* in aggregate demand, so that it is generally possible, even in economies very close to full employment, to raise at the same time consumption *and* investment, if aggregate demand increases[18]. Hence investment is almost never constrained by savings; capital accumulation will result from the demand for additions to capital stocks due to increases in desired capacity, in turn due to increases of aggregate demand.

What seems to emerge here is that a paradigm, or vision, or theory, is also a mental straitjacket. It can act as blinkers, it can make it more difficult to perceive, or to admit the importance of, certain aspects of reality because it has difficulty in fitting them into its theoretical framework. The dominance of the neoclassical paradigm, with its tendency to an equilibrium based on given and fully utilized factor supplies, has obscured the flexibility of aggregate production in response to changes in aggregate demand, with its radically anti-neoclassical implications for growth theory.

The implications of this 'slack' can also be discussed for wages and the demand for labour, and again they radically undermine the neoclassical 'vision' of the basic forces at work in a capitalist economy. The flexibility of production in response to changes in demand implies that there is no necessary influence, in the short as well as in the long period, of changes in real wages on the demand for labour. For example, let us suppose that the government wants to raise both the level of real wages, and employment. An increase in aggregate demand will be generally capable of attaining both objectives: the adaptability of production to demand in the capital goods industries means that there will be little problem with adopting in new

[17] Cf. e.g. Garegnani and Palumbo, 1988; or Petri, 2003; and the vast literature only partially there mentioned.

[18] Most plants can usually produce much more than 'normal' production, by having recourse to multiple-shift production or full-pace production 24 hours a day. Labour constraints are usually non-existent in the short run because of visible or hidden unemployment and underemployment, and over the longer run there are migrations and structural social adaptations, e.g. changes in the participation of women, that suggest that in the longer run labour supply, like capacity, adapts to demand.

plants[19] the new optimal technical choices associated with the higher real wage, while at the same time increasing the overall level of production and employment. This will be so even when it were the case that a higher wage implied a shift to more capital-intensive techniques and therefore required more savings per unit of labour in new plants (the traditional neoclassical thesis whose general validity has been undermined by reverse capital deepening): the increase in savings will be brought about by the increase in production[20].

6. I come to complete futures markets. Their need arises from the illegitimacy, in this framework, of the assumption that changes in equilibrium relative prices are so slow as to be negligible, the assumption made possible for neoclassical long-period theorists by their treating the capital endowment as a single quantity of variable 'form' and its composition as adapted rather than given, and therefore not needing quick changes (as on the contrary is generally the case with arbitrary given initial proportions among capital goods). Since agents know that relative prices are going to change, their decisions in the initial period cannot be assumed to be based on stationary price expectations, and the theory must determine the future prices on which they base their current decisions. Then the alternatives are either subjective expectations as in temporary equilibria (discussed later) or the simultaneous determination of objective current and future prices through complete futures markets.

Now, complete futures markets do not exist and cannot possibly exist. People not yet born cannot be present to-day to indicate how much labour they will intend to supply and whether they will want apples or pears. Contracts foreseeing all possible contingencies in the future are inconceivable. It is indeed amazing that such an absurd assumption should have gained general acceptance among general equilibrium specialists, to the point of being even extended to assuming complete markets for the *infinite* future (an incredible development to be interpreted mainly as an admission that the assumption of a finitely far date where the economy will end was unacceptable, coupled with an inability or unwillingness to question the root of the trouble – the First Assumption).

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[19] It is only in new plants that optimal technical choices can be adopted. Already existing plants earn quasi-rents, and there is little reason to assume that – as long as they are not shut down earlier than otherwise – normal labour utilization in them will be relevantly affected by a higher real wage, given the little room for changes in production methods. Thus, employed labour will be combined with capital goods adapted to new optimal technical choices only gradually, as existing plants gradually reach the end of their economic life and are replaced by new plants.

[20] In looking for possible dangers created by wage rises for labour employment, one will then consider much more relevant the political considerations stressed in Kalecki's "Political Aspects of Full Employment" or, more recently, in Armstrong, Glyn and Harrison's *Capitalism since 1945*.

Interestingly, there are some indications that when put with their backs against the wall on the plausibility of such an assumption, general equilibrium theorists appear to prefer the interpretation of the intertemporal model as assuming, not complete futures markets, but a sequence of one-period equilibria plus perfect foresight[21]; however, even leaving aside the *plausibility* of the perfect foresight assumption[22], it is known that there are excellent reasons to doubt the possibility of a *logically satisfactory definition* of perfect foresight. For example, if the intertemporal equilibrium is not unique[23], what can perfect foresight mean? And what about perfect foresight of future advances of knowledge, which are by definition unpredictable?

Also, the previous discussion of the need for a planner-auctioneer owing to indeterminacy of outputs under free entry/CRS implies that, absent such a planner-auctioneer, even if (i) the intertemporal equilibrium is unique, (ii) correct equilibrium prices for the initial period are announced and future equilibrium prices and quantities are correctly derived by agents, and (iii) everybody acts as price taker relative to these prices, still equilibrium quantities will *not* be produced except by a totally improbable fluke; thus correct knowledge of future equilibrium prices and equilibrium quantities does not ensure that equilibrium will be realized; then of what can there be perfect foresight?

A less widely noticed problem with perfect foresight is the following: how is one to reconcile the perfect foresight assumption with the admission, implicit in the recourse to the tâtonnement for the study of stability, that the equilibrium must be *found*? The need for a tâtonnement implies that equilibrium prices are not known, so

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[21] Thus Mas-Colell, Whinston and Green in their well-known 1995 textbook admit that the Arrow-Debreu model (i.e. the intertemporal model with complete futures markets) "is hardly realistic" and for this reason take care to "show that Arrow-Debreu equilibria can be reinterpreted by means of trading processes that actually unfold through time", of course under an assumption that "the spot equilibrium prices [of subsequent periods] are correctly anticipated at t=0", an assumption whose realism unfortunately they forget to discuss (Mas-Colell *et al.*, 1995, pp. 694-95). And Arrow has said: "The trouble is that with heterogeneous capital goods almost anything is possible. Now, I do not think this in any way interferes with the consistency of the general equilibrium theory of capital formation based on perfect foresight. You may not like the assumption of perfect foresight; that I can understand. But what I am saying is that there is no logical inconsistency in the perfect foresight model" (Arrow, 1989, p. 155). Mandler too appears to interpret intertemporal equilibrium models as based on perfect foresight, when he writes that "Perfect foresight models are not designed to deliver descriptive accuracy" (2005: 487). This is an interesting admission; the question then inevitably arises, which neoclassical general equilibrium models are designed to deliver "descriptive accuracy"? None, it would seem.

[22] The amount of information and the computing abilities that agents must be assumed to have (outside a situation where nothing is changing) is mind-boggling.

[23] Or if the intertemporal equilibrium would be unique if based on complete futures markets but in the corresponding sequential equilibrium there arises Mandler's indeterminacy.

perfect foresight cannot be interpreted to mean correct foreknowledge of future equilibrium prices before the tâtonnement starts (it would be absurd to assume such a foreknowledge but not the knowledge of the equilibrium prices for current markets); on the other hand, the perfect foresight assumption is made in order to try and surmount the absence of complete futures markets; so, it would seem, one must assume that the tâtonnement operates only in the current-period markets, and perfect foresight must be interpreted to mean that all agents agree on what the prices called on current markets at a stage of the tâtonnement imply for future prices: all agents would have to have the same function determining (and determining correctly!) the vector of expected future prices as a function of the prices called for current markets at each stage of the tâtonnement; but at present no theory exists of what such a function might mean; and it is difficult to imagine a non-arbitrary theory. What seems clear is that present tâtonnement theory[24] would be inapplicable: this reveals the absence of any stability theory for the intertemporal general equilibrium model if complete futures markets are missing – and they are.

7. The absurdity of the complete futures markets assumption and of the hypothesis that the equilibrium is reached by an instantaneous tâtonnement has been stressed in a number of recent articles – no less than seven – by Mark Blaug (1997, 1999, 1999b, 2002, 2002b, 2003, 2003b). These articles are interesting, first of all, because they make it difficult to go on writing, as Mandler does, "While a couple of assertions in Solow growth theory about steady states hinge on whether the economy has a single sector and whether capital aggregates, *the operation of competitive markets does not*." (2008 p. 804, italics added). In these lines Mandler, abandoning earlier cautions, unproblematically identifies "the operation of competitive markets" with how this operation is described in intertemporal general equilibrium theory. Against such an identification, Blaug accuses modern GE theory of preventing

> consideration of all dimensions of competitive rivalry other than price, such as availability, quality of product, quality of delivery, quantity and quality of information about the product, etc.; in short, all aspects of non-price competition *because those take place sequentially in real time*. This is precisely what competition meant to Smith, Ricardo and Marx and, even after Cournot, this is what it meant to Marshall (Blaug 1999b: 266, emphasis added).

Blaug's articles are one more proof of the existence of a widespread unease with modern general equilibrium theory in the profession. But they are also interesting because of their deficiencies. Blaug insists on the need to admit elements of non-price competitive behaviour that "take place sequentially in real time", and

[24] By this I mean the usual tâtonnement for intertemporal exchange economies, and the 'factor tâtonnement' for economies with production.

declares that their being inevitably associated with sequences of decisions in real time is precisely the reason why they find no place in modern competitive price theory; but doesn't price competition (with the entailed adjustments in quantities) take time too? So, the root of the deficiencies of modern competitive price theory that Blaug denounces is the inability of modern general equilibrium theory to admit time-consuming adjustments, which is what obliges the theorist to assume a totally unreal working of competition and of adjustments, and to determine a very-short-period equilibrium where the lack of persistence of relative prices cannot be neglected (whence the assumption of complete futures markets). But then, shouldn't one look for the reasons why modern GE theory suffers from this deficiency, differently from the analyses of earlier authors? However, Blaug is unable to trace the real root of the 'sickness' of which he accuses modern economic theory and in particular the Arrow-Debreu model[25]. He appears unable to see that both the main shortcomings he finds in the Arrow-Debreu model, the fairy-tale tâtonnement and the complete futures markets, were due to the replacement (in neoclassical equilibrium theory) of the traditional specification of the capital endowment as a single quantity of endogenously determined 'form' with a given vector of endowments of capital goods. This blindness makes it possible for Blaug to advocate a return to Marshall, without realizing that we cannot go back to notions of 'centres of gravitation' of market prices and quantities while retaining the marginalist/neoclassical approach to distribution. It was precisely the decision not to abandon that approach in the face of its inability to determine long-period[26] 'centres of gravitation', that prompted the shift to very-

[25] "...modern economics is sick; economics has increasingly become an intellectual game played for its own sake and not for its practical consequences; economists have gradually converted the subject into a sort of Social Mathematics in which analytical rigor as understood in math departments is everything and empirical relevance (as understood in physics departments) is nothing…To paraphrase the title of a popular British musical: 'No Reality, Please. We're Economists' " (Blaug 2002: 36). "The Formalist Revolution made the existence and determinacy of equilibrium the be all and end all of economic analysis...What is little understood about the Formalist Revolution of the 1950's is precisely that the process-conception of equilibrium was so effectively buried in that period that what is now called neoclassical orthodoxy, mainstream economics, consists entirely of static end-state equilibrium theorizing with little attention to the stability of equilibrium" (Blaug 2003: 146). "If we can date the onset of the illness at all, it is the publication in 1954 of a famous paper by ... Kenneth Arrow and Gerard Debreu; it is this paper that marks the beginning of what has since become a cancerous growth in the very center of microeconomics." (Blaug 2002: 36). For a more detailed commentary on these articles by Blaug cf. Petri (2006).

[26] Which does not mean steady-state. Nowadays it seems to be often forgotten that the potential speed of change in the composition of capital is of a higher order of magnitude than the speed of capital accumulation or of changes in population, which is the reason why both classical and traditional neoclassical economists esteemed – and rightly so, it would seem – that it is generally legitimate to neglect the slow effects of accumulation on income distribution when one studies the determinants of normal relative product prices. Therefore Marshall, for example, carefully distinguished long-period analysis, where capital *composition* was assumed to have adjusted but the total capital *endowment* was given, from

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short-period, neo-Walrasian, general equilibrium theory. The root of the problem is the supply-and-demand approach to distribution itself, which comes out to be unable satisfactorily to treat capital in all its versions.

8. Temporary equilibria (without perfect foresight) have not been considered in the debate on which I am focusing. Actually, nowadays there is no reference to temporary general equilibria in advanced neoclassical textbooks on value theory, which suggests a general pessimism on the possibility to obtain useful results in that direction[27]. However, the moment one admits that complete futures markets do not exist and cannot exist and that perfect foresight makes no sense, some kind of temporary-equilibrium analysis appears to be the only avenue left for a supply-and-demand approach to prices and quantities. Unfortunately for this approach, the 'methodological' criticisms still apply; only, the absurdity of complete futures markets is replaced by the given subjective expectations or expectation functions, which not only constitute a second group of data deprived of persistence, but are also unknowable and accordingly render the theory's results indefinite because depending on essentially arbitrary assumptions on expectations and on their evolution. Here I only wish to render this criticism concrete by remembering Garegnani's argument, in his 1978 "Notes on Consumption etc.", that a short-period approach to investment such as one can find in Marshall, if truly severed from reference to the long-period substitution mechanisms based on capital the homogeneous factor, would be unable to reach definite conclusions, because too many indeterminate influences would be present: disproportions between capacity and demand in each industry; age structure of equipment and effects of changes of the interest rate on scrapping; and above all, the state of expectations:

> The attempt to determine the effects on investment of changes in the rate of interest on such indefinite grounds wold seem liable to dissolve into casuistry concerning the influence of these changes on the expectations of entrepreneurs. And this influence would differ from situation to situation, thus making impossible any general and unambiguous conclusions concerning direction and intensity of the effects of interest on investment. (Garegnani 1978, p. 347).

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very-long-period (or secular) equilibria where the amount of capital was endogenously determined at the level inducing zero net savings, and he based his theory of normal prices on the former, where the slow changes of long-period prices over time were neglected. Cf. Petri (2004, pp. 36-38, 72-73, 119).

[27] There has been a series of negative results on the possibility to prove the existence of a temporary equilibrium under sufficiently general assumptions. But no doubt the arbitrariness of the assumptions necessary to render the system of equations definite (e.g. assumptions on how firms take decisions when the share owners have different expectations) has also contributed to the disillusionment with temporary equilibrium theory.

As an example of this indeterminacy of conclusions, consider the possibility that a *decrease* of the rate of interest induces a generalized expectation that the rate of interest will keep decreasing, and causes therefore a postponement of investment decisions i.e. a *decrease* of investment in the short period.

The implication is that only a theory of investment that aims at determining average behaviour over sufficiently long periods, giving time to expectations to be revised in the light of experience, can hope to reach unambiguous, sufficiently general conclusions[28]. Once the traditional conception of capital-labour substitution is abandoned, neoclassical theory is unable to supply such a theory[29].

9. Summing up: only persistent forces, persistently acting through the trial-and-error choices implemented by agents and pushing toward well-defined states, can allow reaching definite conclusions on the tendencies of market economies and can therefore be the basis of a theory of value, distribution, employment and growth.

Modern general equilibrium theory does not satisfy this requirement. It is able to give indications only on the behaviour of fairy-tale economies continuously in equilibrium. The moment one admits that, since adjustments take time, some of the data of the equilibrium equations (the ones relative to the endowments of capital goods) cannot be taken as data, the theory becomes unable to determine the behaviour of the economy.

This radically questions the relevance of any stability result obtained for modern general equilibrium theory. Results on the behaviour of tâtonnement-type adjustments have no implications for the behaviour of real economies. That the tâtonnement is stable does not prove that a real economy would be stable. Conversely, the non-convergence of the tâtonnement does not prove that more realistic adjustments will not converge.

Support for this last statement is provided by Professor Gintis's commendable

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[28] Keynes states this very clearly in a letter to Kalecki, dated 12 April 1937: "I hope you are not right in thinking that my General Theory depends on an assumption that the immediate reaction of a capitalist is of a particular kind. I tried to deal with this on page 271 [? probably 261, *F.P.*], where I assume that the immediate reaction of capitalists is the most unfavourable to my conclusion. I regard behaviour as arrived at by trial and error, and no theory can be regarded as sound which depends on the initial reaction being of a particular kind. One must assume that the initial reaction may be anything in the world, but that the process of trial and error will eventually arrive at the conclusion which one is predicting" (Keynes 1973-79, vol. XII, p. 797).

[29] Cf. Petri 2004, ch. 7, for a criticism of modern neoclassical theories that attempt to prove a negative elasticity of investment vis-à-vis the interest rate without explicitly relying on capital-labour substitution. That the 'Sraffian' criticism of neoclassical capital theory aims at questioning the assumption of full utilization of resources appears to escape Mandler, who never discusses the possibility of a theory of long-period income distribution admitting labour unemployment; his discussion of "Sraffian indeterminacy" assumes the full employment of labour and implicitly attributes such an assumption to Sraffa.

attempt (Gintis 2007) to replace the tâtonnement with something more acceptable, and not only for exchange economies but also for production economies. He proposes a very realistic picture; adjustments are time-consuming and entail exchanges and productions at disequilibrium prices; products in each industry are homogeneous across firms but firms can afford to be price-makers because consumers take time to find out the several firms' prices, so firms modify prices according to sales, and consumers continuously choose between buying (or supplying labour, if that is the decision to be taken) or going on searching for better exchange opportunities; a fundamental role is assigned to imitation of the more successful agents. There is also entry of new firms and exit (bankruptcy) of unsuccessful firms. Through simulations Gintis proves that even in a situation (the one of Scarf's famous exchange-economy example) where the tâtonnement does not converge to (the unique) equilibrium, his imitation-based adjustment converges; and in the production economy for a very wide range of parameters the economy converges over time to a neighbourhood of the neoclassical general equilibrium.

However, his is an a-capitalistic economy[30], essentially without aggregate demand problems[31], and with factor endowments unaffected by disequilibrium (which is what allows him to consider time-consuming adjustments involving disequilibrium transactions and productions); thus, his conclusions that his result 'provides some justification for the importance placed upon the Walrasian model in contemporary economic theory' (p. 1303) and that 'models allowing traders, consumers, workers and firms to imitate successful others lead to an economy with a

[30] Bilancini and Petri (2009), as part of their critical appraisal of Gintis's conclusions, show that the factor that Gintis calls 'capital' is in fact land.

[31] Interestingly, the possibility of consumers temporarily hoarding money implies that aggregate demand can fall short of aggregate supply in Gintis's model in spite of the absence of investment, highlighting the often forgotten fact that the absence of capital goods (and hence of investment) does not guarantee the validity of Say's Law. However, Gintis assumes that in each period consumers spend all the money they have if only they can find the goods they plan to buy and if they are not discouraged by prices that look too high (thus obtaining that hoarding is compensated by dishoarding and the propensity to consume is unitary on average), and he also assumes that loss-making firms are almost entirely re-financed by the monetary authority (and the bankruptcies are roughly compensated by creation of new firms); under these assumptions for a wide range of parameters the occurring of a relevant or persistent insufficiency of aggregate expenditure relative to the value of aggregate production will be highly improbable. Then the tendency toward the full employment of labour, permitted by the decreasing demand curve for labour ensured by the treatment of 'capital' as strictly analogous to land *and fully employed by assumption* (a way to sweep under the carpet some little-noticed stability problems of neoclassical equilibrium arising when all factors are unemployed and one admits that expenditure can only come from the employed factors), encounters no Keynesian aggregate-demand problem. If capital were treated as produced, heterogeneous and of variable 'form', and therefore of changing composition during adjustments, then the decreasing demand curve for labour would lose any firm basis; and the presence of investment expenditure in aggregate demand would make it necessary to have a theory of investment in order to determine labour employment.

reasonable level of stability and efficiency' (p. 1304), are illegitimate; the extension of his approach to an economy with capital goods would require examining the dynamic properties of time-consuming adjustments that *change* the amounts of capital goods; the results could not possibly confirm or disprove the stability of a general equilibrium based on a *given* vector of capital endowments; furthermore, his adjustments ensure some flexibility of production, but in a capitalistic economy any flexibility of production will introduce room for the phenomena mentioned in paragraph **5** above, implying the need for a theory of aggregate demand and for a theory of wages in order to surmount the indeterminacy of effects of changes of real wages on the demand for labour; thus no conclusion about the implications of Gintis's adjustments for capitalistic economies can be reached before one decides on the theory of wages and on the theory of aggregate investment; on these issues the inability of general equilibrium theory to indicate the behaviour of real economies obliges one to turn to non-neoclassical approaches, and then the field is wide open and, for example, multiplier-accelerator instabilities cannot be excluded. What *can* be excluded is traditional neoclassical approaches to investment theory and to the demand for labour, because these approaches rely on the conception of capital-labour substitution undermined by the Cambridge results[32].

Therefore Gintis's interesting exercise shows that if, on the one hand, more realistic adjustments can surmount some of the negative results on tâtonnement stability, on the other hand the moment one tries to conceive of the effects of these time-consuming adjustments in economies with capital goods one realizes that nothing can be concluded until at least a theory of wages and a theory of investment are introduced to render the economy's behaviour determinate – theories that the neoclassical tradition no longer supplies now that the conception of capital as a single factor has come out to be indefensible. And non-neoclassical theories of wages and of investment are far from bringing to conclusions similar to those of the neoclassical approach on the determinants of income distribution, employment and growth.

10. The thoroughly destructive implications for neoclassical theory of these 'methodological' deficiencies of modern general equilibrium theory appear to be still little recognized. Neoclassical theory as applied to the real world, for example neoclassical macroeconomic theory, comes out to have no foundation; the pillars of its applications, namely the traditional neoclassical beliefs in a decreasing labour demand curve and in a decreasing investment function, cannot be defended by referring to modern general equilibrium theory, and recourse to their original basis (the old marginalist theory based on capital the homogeneous factor) is just as

[32] Cf. chapters 7 and 8 of Petri (2004). It would be very interesting to study the implications of Gintis's assumptions on adjustments for economies with heterogeneous capital and with realistic assumptions on wages and on investment decisions. I am quite sure that multiplier-accelerator interactions would indeed develop.

illegitimate; but so far these criticisms have raised no reactions from the (not many) neoclassicals who know something about the Cambridge debates on capital theory, and as a result a majority of economists, the ones who delegate to the 'high-brow' gurus of neoclassical value theory the appraisal of these debates, no doubt remain ignorant of these criticisms. However, the deficiencies pointed out by these criticisms are undeniable; and they are sufficient for a rejection of the supply-and-demand approach to value and distribution.

Therefore the efforts, to show that reswitching and reverse capital deepening are damaging for general equilibrium theory in its modern formulations, can be seen as attempts at overkill, actually superfluous if the aim is to ascertain whether the supply-and-demand approach to value and distribution is defensible. I think that perhaps the purpose of these efforts needs more careful explanation than has been supplied so far, because they can have the side effect of making the neoclassical side somewhat reassured on the relevance of modern general equilibrium theory and on a *limited* relevance of the 'methodological' criticisms: why otherwise, the neoclassical economist might argue, would the "Sraffians" consider it so important to criticize modern general equilibrium theory on its own terrain, accepting complete futures markets and instantaneous tâtonnements?

Why indeed? And why will I too produce an exercise of this type, after stating that actually it is superfluous? My reply is as follows. That an argument be scientifically correct is not always enough for it to be recognized as such; barriers may need surmounting to it being understood or even only heard of. In the present case, considerable barriers existed to a correct understanding of what was at stake in the Cambridge controversies, owing to the confusions at the time on the development of neoclassical theory; there resulted grave misunderstandings that still persist; the confusions were compounded by insufficient clarity or erroneousness of the arguments of some of the critics too, especially Joan Robinson (Petri, 2004, pp. 227-38). The persisting misunderstandings help the resistance to appreciate or even only confront the 'methodological' criticisms: for example it is understandable that there be a reluctance to abandon the First Assumption and, with it, a short-period approach to value and distribution if – owing to an inability to distinguish long-period analysis from steady-state theory – the only alternative is thought to be a steady-state theory necessarily far removed from actual conditions. To this one must add the usual difficulty with escaping long-absorbed ways of thinking (a difficulty compounded at present by the frequent absence of any introduction to non-neoclassical approaches in university economics courses), and the understandable resistance of scientists to entertain the possibility that their lifelong efforts may have been wasted on a wrong approach. For all these reasons, repetition of the 'methodological' criticisms appears to have limited effects: as the old adage goes, "None so deaf as those who will not hear". Considerable time and effort would be required anyway of economists of mainstream formation to surmount the confusions and fully understand the criticisms. The hope is to give some of them the motivation to make the effort, by producing results that accept the First Assumption and nonetheless show that the shift to very-

short-period equilibria does not eliminate the difficulties of the supply-and-demand approach with capital. It may be hoped that these results, if it will be possible to produce them, will show the need to reconsider the whole issue, and will make economists more open to a reflection on the 'methodological' criticisms, to the need to drop the First Assumption, to a reappraisal of the evolution of economic theory, and to the need to explore alternative theories of income distribution, employment and growth.

PART II

11. In this second part of the paper I completely leave aside the issue of stability. I will be only concerned with the uniqueness or multiplicity of solutions to the equilibrium conditions; I concentrate on whether equilibrium can be non-unique owing to the presence of capital goods when the assumptions on consumer behaviour would guarantee uniqueness in the absence of capital goods.

One can usefully enter the issue by discussing Garegnani's claim that his 2000-2003 model can yield multiple equilibria *not attributable to income effects*. The precise meaning of the italicized words is not pointed out by Garegnani, he has only noted in (2005a, pp. 412-13) that his argument is based exclusively on the possible shape of the investment schedule and would still hold if in the second period of his two-periods model there were only a single consumption good. Mandler (2002, 2005) has countered that 'discrete' multiple equilibria in Garegnani's model must be due at least in part to 'badly behaved' consumer choice and more specifically to consumer heterogeneity, because it is a theorem that in that model and in fact in all general equilibrium models of that kind (i.e. with a finite number of commodities and hence of periods) the equilibrium production vector is unique and equilibrium prices form a convex set if consumer excess demands obey the weak axiom of revealed preferences, which, apart from very special examples, implies that there is a representative consumer with strictly convex indifference curves; the existence of the representative consumer does not exclude income effects but it renders them unable to cause (discretely) multiple equilibria (some intuition for this claim will be provided in §**15**). Garegnani has denied the applicability of the theorem to his model, arguing that "the results to which Mandler refers concern production systems where production of capital goods is in effect overlooked, and therefore both demand for them from firms (but ultimately from savers) and supply of them from producing firms, are also overlooked in each period" (2005a, p. 413).

Contrary to the case with his immensely important previous contributions, in this case I, like Schefold (2008), find it impossible to agree with Garegnani. The reason is simple: the equilibrium conditions of his model are formally *identical* to the

equilibrium conditions of an *atemporal acapitalistic* economy[33] where two consumption goods are produced by three different types of labour supplied in fixed amounts, two of which (corresponding to the goods a and b of date zero of Garegnani's model) yield services which can also be used for direct consumption (e.g. domestic labour, massage, etc.), while the third one (corresponding to Garegnani's labour) yields services which are only useful for production. Try writing down the equilibrium equations of this economy, and you will see that they are the same as Garegnani's. (Of course the prices determined in this re-interpretation are not discounted prices.) Garegnani's claim is particularly surprising since production of capital goods is definitely 'overlooked' in his model too: there is no production of capital goods in the model. It is then unclear how he can believe that the model can produce results, on equilibrium uniqueness, different from those of the atemporal reinterpretation for which, it would seem, he would accept the correctness of Mandler's contention.

It is important to stress at this point that I am only concerned with the uniqueness issue. From this point of view, it is furthermore unclear in what sense one might say that, in the intertemporal general equilibrium models where there *is* production of capital goods, supply and demand for produced capital goods are overlooked: the equilibrium equations do specify that there must be equilibrium between supply and demand for each capital good. True, the explicit indication of these conditions might be omitted together with the explicit consideration of

[33] In an *acapitalistic* economy there is no production of goods which are then used to produce other goods; if there is production the sole factors of production are non-produced, i.e. types of labour and types of land. By an *atemporal* economy I mean an economy where variables do not need dating, i.e. an economy whose factor endowments consist only of non-produced factors, whose final product consists only of consumption goods, and where there is no saving and lending and, if significant time intervals exist between application of inputs and production of outputs, the interest rate is zero. An acapitalistic economy need not be atemporal, as shown by intertemporal pure-exchange equilibria. Traditional marginalist analyses of the production-and-exchange economy without capital determined equilibria that were atemporal *and* acapitalistic, with rentals of factor services (labours and lands) and prices of consumption goods being the only prices determined by the equilibrium; however, atemporal equilibria need not be acapitalistic, they can include produced means of production although only through 'Austrian' production processes where intermediate goods are intermediate stages of the production process of a consumption good, with the process started by non-produced factors alone (in Sraffian terminology there is no basic commodity); then pricing the intermediate goods may be possible but is irrelevant to determining the cost of production of the final consumption good which is simply the sum of the payments to the non-produced factors. An atemporal equilibrium of this type can always be rendered acapitalistic, at least in the absence of joint production, by omitting the intermediate goods by treating production as if all production of consumption goods were vertically integrated (cf. § **12**); which was the implicit assumption in the traditional treatment of the production-and-exchange equilibria, since all non-instantaneous production processes of consumption goods can be decomposed into successive stages which (except for the last one) produce intermediate goods to be then utilized in the next stage.

'intermediate' capital goods, via the treatment of the production of 'final' goods as performed by non-produced factors only (that include the initial endowments of capital goods) according to intertemporal production functions of hypothetical vertically integrated firms (see below). In such a case the equilibrium on the markets of 'intermediate' capital goods is taken for granted, as resulting from the cost minimization of the hypothetical vertically integrated firms, and one may well question this assumption of equilibrium on the markets for 'intermediate' capital goods which excludes *by assumption* the possibility of disequilibrium on the savings-investment markets of intermediate periods[34]; but this would seem to have a potential relevance only for the issue of stability (questioning the 'factor tâtonnement', which entails precisely such a treatment of production of intermediate goods). On uniqueness, as long as the intertemporal equilibrium makes the Second Assumption I do not see reasons to deny the reinterpretability of the intertemporal equilibrium conditions as describing an atemporal economy, and therefore the theorems on uniqueness of standard general equilibrium theory do apply. Relative to general atemporal equilibria an intertemporal equilibrium includes some *additional* constraints on technology[35], but this only means that not all atemporal economies can be reinterpreted as intertemporal economies, the converse is always possible; and

[34] Garegnani (2000, 2003) argues that there is a savings-investment market in the initial period too: in his model he calls savings the value of the supply to firms of the not-consumed portion of the endowments of commodities a and b of period zero, and he calls investment the value of the demand for them by firms in order to produce the consumption goods of period 1. Then even with a 'factor tâtonnement' there can be disequilibrium on a savings-investment market, the one of the initial period. However, it must be noted that in (2000, pp. 436-38) as well as in (2005b, p. 495) in order to discuss the stability implications of his analysis Garegnani prefers to consider the savings and investment of period zero as coming out, not of stocks (he defines such a treatment 'misleading'), but of production, by rendering period zero an intermediate period through the introduction of a period t = –1. (Why misleading, Garegnani does not explain, but one can note that if capital goods are not directly consumable – the general case –, the supply to firms of the given initial endowments of capital goods does not indicate renouncing period-zero consumption; an alternative between consuming and investing in period zero would then require considering the possibility of alternative compositions of the period-zero endowment of consumption and capital goods, which would require considering production of period -1; furthermore the little direct substitutability between factors when capital is of given 'form' would obscure the possibility of varying production methods as distribution varies, which is at the heart of the possibility of anti-neoclassical investment behaviour that Garegnani wants to prove.)

[35] Dates on factors and products must respect the fact that it must be impossible to produce a period-t good with period-(t+1) factors; also, at least in the absence of joint production the intertemporal production functions must exhibit weak Leontief separability, reflecting the fact that earlier inputs are used to produce intermediate goods that then are used together with later inputs. The fact that the equilibrium of an atemporal economy can be conceived as a long-period equilibrium with a zero interest rate, with all factors being indestructible like lands and their services being repeatable, while in intertemporal equilibria factor prices are the prices of services of dated factors that last only one period, does not seem to prevent the reinterpretability.

these additional constraints on technology do not alter the applicability of the theorems on uniqueness of atemporal equilibria, because these theorems are valid for very general technologies, of which the intertemporal ones are only special cases. Kehoe's uniqueness theorems based on index theory are perhaps devoid of economic content as argued by Fratini (2008), but as far as I know no fault has been found in the theorem that states that, if consumer excess demand satisfies the weak axiom (e.g. if there is a representative consumer with strictly convex indifference curves), then equilibrium is unique in the quantities produced, and the set of equilibrium relative prices is convex (so equilibrium is unique in relative prices too if in addition consumer excess demand is a one-to-one function of relative prices, e.g. if the representative consumer's indifference curves are differentiable).

12. Some support for my view comes already from the history of economic theory: the atemporal (Walras-Cassel) economy of production and exchange, with no dating of commodities, just factors and consumption goods, was the one whose equilibrium was more rigorously studied by Wald and the starting point of Arrow and Debreu; the intertemporal re-interpretation was attached to the *same* model, with little initial reflection on its legitimacy[36]. This means that when one speaks of the (finite-horizon) intertemporal Arrow-Debreu model, one is in fact speaking of the atemporal model (although satisfying the additional constraints needed for the intertemporal reinterpretability), adding to it simply a *verbal reinterpretation* of the symbols that changes *nothing* in the equilibrium equations[37]. Own-rates of return are *deduced*

[36] The issue needs further study, but it may be noted that there is no mention of the intertemporal reinterpretability of the equilibrium equations in terms of dated commodities in Wald (1951), who is explicit that his model is essentially Cassel's and that in it "the problem of capital formation and of the rate of interest is not treated at all" (p. 379); and Arrow and Debreu (1954) make it clear that Walras's model of production and exchange (*not* Walras's model with 'capitalization'), and Wald, are their starting point. They do mention the reinterpretability of their commodities (finite in number) as dated (p. 266), but without much reflection, it would seem, on the doubtful implications of such a reinterpretation: no mention is made of the need for futures markets or for no savings in the final period. As far as I know it is only with the first of Koopmans's *Three Essays on the State of Economic Science* (1957) that there starts some reflection on the difficulties of the intertemporal reinterpretation, cf. Petri (2004, p. 162).

[37] On this issue of perfect equivalence there is an observation by Garegnani that leaves me perplexed. He derives from his analysis of what can happen in his 2000-2003 model that a difference exists between intertemporal equilibria and atemporal equilibria in that only in intertemporal equilibria the possibility arises that a good (b_0 in his case), which is a free good relative to a *future* good, may have a positive price relative to a *contemporary* good, and therefore may not be "'free' in the generally accepted sense – when a tendency to zero of the price of the commodity in terms of one scarce commodity (b_1 in this case) would entail a tendency to zero of its price in terms of all other scarce commodities (like a_0 here) whether of the same, or of another, date." (2000, p. 418; 2003, p. 146). However, equation (7f) in (2000, p. 403) or (5.7f) in (2003, p. 123) has the consequence that if b_0 is in excess supply so is a_0, thus it is unclear how Garegnani can consider a_0 scarce. It may be added that atemporal

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from relative prices *re-interpreted* as discounted prices, once two goods are re-interpreted as the same physical good but at two different dates. (I leave aside here the further reinterpretability in terms of contingent commodities.)

The above means that, as long as in the last period of the intertemporal equilibrium there is no production of capital goods, there is no need for a *demonstration* of the equivalence of the equilibrium conditions of atemporal and intertemporal equilibria: the equilibrium conditions are necessarily the same because they need no change, all they need is a reinterpretation of the symbols.

For production decisions, the reinterpretability can be carried further; the intertemporal equilibrium conditions can be shown to be equivalent to those of an atemporal equilibrium *without* intermediate goods, i.e. atemporal *and* acapitalistic. One can define a *vertically integrated production method* for a good as one in which the direct produced inputs of the good are replaced by one possible vector of inputs sufficient to produce those inputs, and so on backwards until one reaches the initial period of the equilibrium[38]. In this way one obtains a vertically integrated production method where all intermediate inputs have disappeared, the sole inputs are non-produced inputs of the several periods (by which I mean not produced *inside the equilibrium*: labour and land services of the several periods, and initial capital goods: the latter will be reinterpreted as types of labour or of land in the atemporal reinterpretation). Given the (discounted) rentals of all non-produced inputs, the (discounted) cost of producing the good with that vertically integrated production method is the cost of its inputs, and a hypothetical vertically integrated firm will choose, among all vertically integrated methods to produce that good, the one (or one of the ones) that minimizes the cost; under competition and constant returns to scale, the same result will emerge even when there is no vertically integrated firm, because

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equilibria do not exclude the possibility that two goods have both zero price relative to the numéraire good or to a group of other goods, while having a positive rate of exchange between them. Imagine that in a pure exchange atemporal economy some consumers have endowments of goods A and B and desire also goods C and D, while goods C and D are only held by a second group of consumers who do not care for A nor B; imagine that the excess supply of both goods A and B causes both their prices relative to goods C and D to fall to zero; once the prices of both A and B in terms of C or D have become zero, this means that the consumers of the first group cannot obtain goods C or B, but they still have their endowments of goods A and B (because by assumption the owners of goods C and D do not desire A nor B) and they can exchange them at a perfectly well defined ratio; if these consumers realize that they cannot obtain anything by offering A or B in exchange for C or D, it is conceivable that they resign themselves to exchanging and consuming A and B, so that these goods are fully utilized and have a well-defined exchange rate between them. The same will hold for the exchange rate between C and D, whose owners actually form a separate economy from the owners of A and B. This shows that in atemporal equilibria too, a good A can be non-scarce relative to a group of goods and be scarce relative to another group of goods.

[38] I am implicitly excluding joint production here. I leave the study of the implications of joint production to others more competent than me on this issue.

competition implies that produced inputs are sold at their minimum average cost, and therefore a firm that were to produce its inputs internally would be unable to obtain them at a lower cost than by buying them. If then we consider an atemporal economy where production factors are only the non-produced factors of the intertemporal economy (of course reinterpreted as atemporal inputs) and the available production methods are the vertically integrated methods available to the intertemporal economy, then for each vector of factor rentals (interpreted as discounted in the intertemporal economy, not discounted in the atemporal one) technical choices and minimum average costs of produced goods will be the same mathematically (only the interpretation will be different) as in the intertemporal economy. The equilibrium conditions for the production sector will be then equivalent, even if not formally identical because the intermediate goods will only appear explicitly in the intertemporal economy.

Thus, for example, suppose that in an intertemporal economy unassisted labour and land of time zero produce a circulating capital good of type α at time 1 which, together with labour and land of time 1, produces a circulating capital good of type β at time 2 which, together with labour and land of time 2, produces a consumption good C at time 3; suppose all these processes are characterized by fixed technical coefficients as follows (the symbol * stands for 'together with' and the symbol $\rightarrow$ stands for 'produce'):

$L_\alpha * T_\alpha \rightarrow$ 1 unit of capital good α

$L_\beta * T_\beta * \alpha_\beta \rightarrow$ 1 unit of capital good β

$L_c * T_c * \beta_c \rightarrow$ 1 unit of consumption good C.

L stands for labour, T for land; technical coefficients per unit of product are shown on the left-hand side of the arrows. The subscripts refer to the product, for example α_β is the input of capital good α per unit of output of capital good β. Then we can also say that in order to produce 1 unit of consumption good C one needs L_c and T_c units of labour and land one period earlier, $\beta_c L_\beta$ and $\beta_c T_\beta$ units of labour and land two periods earlier, and $\beta_c \alpha_\beta L_\alpha$ and $\beta_c \alpha_\beta T_\alpha$ units of labour and land three periods earlier. Putting

$\beta_c \alpha_\beta L_\alpha = L_0$,

$\beta_c \alpha_\beta T_\alpha = T_0$,

$\beta_c L_\beta = L_1$,

$\beta_c T_\beta = T_1$,

$L_c = L_2$,

$T_c = T_2$,

one can write the 'vertically integrated' version of the production method of C:

$L_0 * T_0 * L_1 * T_1 * L_2 * T_2 \rightarrow$ 1 unit of consumption good C.

By interpreting these six factors as, for example, three different types of labour and three different types of land one obtains an atemporal and acapitalistic production method formally identical to the vertically integrated intertemporal method.

If the efficient production methods of each output of time t via inputs of time t-1 can be represented via a constant-returns-to-scale production function with smooth

strictly convex isoquants, then one can derive a vertically integrated production function, and hence an atemporal acapitalistic production function, with the same properties. If there are several alternative intertemporal methods available to produce the same final good, each one of them characterized by fixed coefficients, the atemporal equivalent is simply a series of 'activities', generating activity-analysis isoquants that eliminate the inefficient methods.[39]

The above means that in intertemporal economies too there is no need explicitly to consider intermediate goods; one can derive 'vertically integrated' production functions of 'final' goods in terms of non-produced factors alone, formally analogous to those of atemporal acapitalistic economies, by utilizing the vertically integrated production methods. When, in the 'factor tâtonnement', demands for non-produced inputs are derived from the demands for 'final' goods and from given rentals of the non-produced inputs, the thing might be equivalently accomplished through the use of the 'vertically integrated' production functions.

Coming to consumers, under the assumption of no savings in the last period one can reinterpret their intertemporal utility maximization as an atemporal one, the

[39] On this issue there is a statement by Garegnani which I would like to be further clarified, concerning the existence or not of marginal products. He writes (2003: 133; also 2000: 430): "Despite our assumption that all alternative methods of production require the same three factors, there is no assurance that marginal products, even of the discontinuous kind, will exist." And in the attached 2003 endnote 42 he explains: "Marginal products, whether of the discontinuous or the continuous variety, require that the available techniques be susceptible of being ordered so that they can be made to differ by the quantity of only one factor at a time. That, it seems, cannot generally be done when the factors are more than two: weighted averages of the different methods available which could give the above result will not make general economic sense, since it would be an exception when the methods entering such averages could coexist." (2003, p. 164; also 2000, p. 430, fn. 53). The second sentence of this endnote appears to assume that the alternative methods consist of a finite number of fixed-coefficients methods, and thus it appears insufficient to deny the existence of marginal products when production functions are differentiable. But even in the case of a finite number of alternative fixed-coefficients methods, I am unable to understand Garegnani's point. When there are three factors (as in the model Garegnani uses in 2000 and 2003), of course one needs the simultaneous use of at least three alternative methods in order to be able to vary the employment of only one factor at a time in the production of a good. But the construction of discrete marginal products is not different from the one in the two-factors case. For a given employment of the first factor, one can derive output as an increasing function of the quantities employed of the other two, obtaining a kinked concave surface whose level curves are activity-analysis kinked isoquants; if the amount of the second factor is fixed too, one obtains a kinked total productivity curve of the third factor, that yields the marginal product of the factor as a step function of the amount employed. The three rentals of the factors will determine the minimum-cost method or convex set of methods; the *equality* between all three rentals and marginal products will mean that if one draws isoquants in three-dimensional space the isocost plane is tangent to a facet of a kinky isoquant, and this can be considered accidental; but even if an edge or a vertex of the kinky isoquant is optimal, each rental will be included in the discontinuity between two marginal products, in perfect analogy with the case with two factors only.

discounted prices as atemporal prices, and their endowments of dated non-produced factors as endowments of atemporal factors, in an obvious way on which I think I need not dwell. Again, it is only a matter of reinterpreting the symbols; nothing changes in the equations that consumer behaviour is assumed to satisfy.

A complication in the reinterpretation of intertemporal equilibria as atemporal equilibria arises if one assumes, as Schefold (1997, p. 462) does, that in the final period of the finite-horizon intertemporal equilibrium there is production of an exogenously given vector of (capital) goods, that do not enter the representative consumer's utility function. In the atemporal economy the existence of such a component of output can be justified by assuming, for example, that it is a tribute to be paid to a nasty foreign nation in order to avoid being invaded, or that it is a medicine necessary for the representative consumer's survival: Schefold's (1997) proof of existence, uniqueness and optimality of equilibrium then implies that in equilibrium the resources going to produce that vector are such as to minimize the loss of utility of the representative consumer. (If there were several heterogeneous consumers, determinateness of equilibrium would be endangered by the different effects of subtracting the resources necessary for the production of that vector from one consumer rather than from another one, and some arbitrary assumption would be necessary to surmount this problem, but the single-consumer assumption eliminates the difficulty.) Under such a reinterpretation, again there is no need to alter the equilibrium equations, hence no mathematical demonstration of the formal equivalence of the intertemporal and of the atemporal equilibrium conditions is necessary, they are the same by assumption[40].

I conclude that on the uniqueness issue the known theorems on atemporal equilibria are also valid for finite-horizon Arrow-Debreu intertemporal equilibria (that make the Second Assumption).

Therefore if the assumptions on consumer preferences are such as to guarantee uniqueness in Garegnani's 2000 model reinterpreted as an atemporal economy, then there will be uniqueness for the intertemporal original interpretation too. On the uniqueness issue concerning Garegnani's 2000 model, Mandler and Schefold appear to be right and Garegnani wrong.

13. As an aside, since sometimes the theorems on existence, uniqueness and stability of equilibrium are formulated for atemporal *and acapitalistic* economies (i.e. without intermediate goods), it may be opportune to add the following consideration on the applicability of those theorems to intertemporal economies. The

[40] Schefold (2008, p. 174) has recourse to a mathematical demonstration because he has specified the equilibrium conditions for Garegnani's model (p. 134) in a way different from the more usual and general one for atemporal models and he wants to show that the two specifications are equivalent; he might also have simply reinterpreted the former specification without manipulating it at all.

disappearance, in the 'vertically integrated' representation of intertemporal production methods, of the intermediate (i.e. capital) goods produced inside the equilibrium takes it for granted that intermediate goods are produced and utilized in the correct amounts, and thus it prevents the consideration of disequilibria on the markets for these intermediate goods. Thus the 'vertically integrated' representation of production is inappropriate for the study of the *stability* of the markets for intermediate goods (and hence, as already noted, for the study of stability problems arising in savings-investment markets of intermediate periods).

But for the determination *of the equilibria*, the 'vertically integrated' representation would not cause problems. Equilibrium on a market requires either equality of supply and demand, or excess supply and a zero price. The vertically integrated representation assumes equality between supply and demand for the intermediate goods, so if there is equilibrium with such a representation there is also equilibrium with the explicit consideration of the supplies and demands for intermediate goods. Hence the sole danger is that the vertically integrated representation may cause us to miss out some *equilibria* where there isn't equality between supply and demand for some intermediate goods. But in these equilibria for these intermediate goods there can only be excess supply. Then their price i.e. their cost of production must be zero; this must mean that all the factors which are employed in their production have zero rentals, so if we reduce the amounts produced of the intermediate goods in excess supply to just what is demanded, this will only mean an even greater excess supply of those factors, with no change in prices nor on any other markets. Hence in this case we can equivalently formulate the equilibrium as an equilibrium with *equality* between supply and demand for *all* intermediate goods, and with some non-produced factors in excess supply and zero rentals; but then this equilibrium is among those describable through the 'vertically integrated' representation.

The above seems sufficient to conclude that the theorems on existence and uniqueness of atemporal and acapitalistic economies do apply to intertemporal economies (where the three Assumptions hold). With even more reason they will apply to Garegnani's 2000-2003 model, where there are no intermediate goods.

14. But how is the equilibrium-conditions equivalence between intertemporal and atemporal model achieved? By having, in the intertemporal model, not only complete futures markets but also a finite horizon with no production of capital goods in the last period.

(An exogenous vector of zero-utility goods produced in the last period, like vector **f** in Schefold (1997), can be useful for certain analytical purposes but cannot be considered truly to represent production of capital goods: the choice of which capital goods to produce cannot be independent of their prospective profitability, which cannot be independent of what has happened up to then and is therefore endogenous – and in fact Schefold uses **f** essentially in order to prove "that steady states of finite duration may be represented as intertemporal equilibria" (1997, p. 483;

cf. also p. 462), in which case the vector although formally taken as given is actually fixed ex-post as precisely the one required by the steady state –; therefore such a vector, even when assumed to exist, is best considered a special case of rigid demand for consumption goods; so the relevant case to confront is the one with only consumption goods produced in the last period.)

Mandler's dismissal of Schefold's and Garegnani's contentions has been made possible by the fact that Schefold and Garegnani have conceded these assumptions in their formal critical models. I contend that it is because of these assumptions that a "well behaved" consumer side (i.e. satisfying the weak axiom) guarantees uniqueness of the intertemporal equilibrium, or at least convexity of the set of equilibrium prices and uniqueness of productions. In support of this contention I will produce a numerical example of a single-consumer intertemporal equilibrium where in the last period considered there is production of an *endogenously determined* vector of capital goods. Obviously this vector cannot be derived from the demands for the goods that these capital goods will produce (the futures markets for these goods do not exist), it will be determined by considerations that appear plausible and much closer to how actual economies work than complete futures markets over an infinite horizon, or than no production of capital goods in the last period.

I shall construct an intertemporal model over a finite number of periods with a representative consumer, where the economy does not end with the last period, only futures markets end, so in the last period there is production of capital goods; their total value is given by Say's Law plus an assumption on savings; but *which* capital goods will be produced depends on the real wage inside the equilibrium, because this determines the wage expected for the first period beyond the equilibrium's horizon (I assume it is the same real wage) and the prices expected for that same period (I assume they are the minimum-cost-prices associated with the expected wage and the last-period prices of capital goods, and with the choice of technique that maximizes the expected rate of return on investment). Labour supply for the last production cycle inside the equilibrium is assumed to be the result of a choice between leisure and consumption that obeys usually-shaped indifference curves and therefore depends on the wage, and what I show with a numerical example is that as the wage changes the capital goods produced can change such that it is possible that the consumer's choices between consumption and leisure are such that the demand for labour and supply of consumption can coincide with the supply of labour and demand for consumption at more than one wage rate. Thus what I shall produce can be described as a mixed intertemporal-temporary equilibrium model with savings in the last period, identical consumers, and shared expectations on subsequent prices, and I show that simple assumptions suffice to point to the possibility of multiple equilibria. Since both the assumption that the economy ends after a finite number of periods, and the assumption of complete futures markets over the infinite future, are more than faintly ridiculous, I think that any attempt to make general equilibrium theory

somewhat more realistic must accept the introduction of some elements of temporary equilibrium, and then my result shows some implications of such a modification.

The multiplicity of equilibria in the numerical example will concern the choice of the representative consumer between consumption and leisure, an aspect absent from the articles I am discussing where labour supply has been assumed rigid; so it may be useful to illustrate first why the choice between leisure and consumption is unable to produce multiple atemporal equilibria when there is a representative consumer, in spite of the possibility of backward-bending labour supply; this background will make it easier to understand why multiple equilibria connected with the choice between consumption and leisure become possible if some aspects of the model are changed, e.g. if the equilibrium is a long-period one with a given *value* endowment of capital, or if in an otherwise neo-Walrasian intertemporal equilibrium one does not reduce all demand to consumption demand.

15. Consider an 'atemporal' competitive production economy where two factors, labour and land, which can be combined in variable proportions, produce a single consumption good. The many consumers are identical both in preferences and in endowments, and hence a representative consumer exists. Land supply is rigid. The representative consumer spends all her income on the consumption good. The consumption good is the numéraire. The real wage rate w and the real rent rate ρ are tied together by the condition that extraprofits are zero.

The standard marginalist analysis of this economy assumes that the available methods of production form a differentiable production function c=f(N,B) homogeneous of degree 1, where N is labour employment and B is land employment; the real wage rate w and the real rent rate ρ are equal to the respective marginal products, and therefore by the product-exhaustion theorem firms make zero extraprofits. If land is fully employed, then the economy-wide marginal product curve of labour is also its demand curve; it is realistic to assume that marginal products become zero for a sufficiently high factor proportion; then, if labour employment is measured on the abscissa and the real wage on the ordinate, the labour demand curve has initially a horizontal stretch, then decreases, and becomes horizontal (coinciding with the abscissa) to the right of the point where labour's marginal product becomes zero. A backward-bending labour supply curve can have multiple intersections with the labour demand curve, entailing multiple equilibria; but this cannot happen if there is a representative consumer with convex indifference curves, because the PPF (production possibility frontier) is convex and the (Walrasian) budget constraint of the single consumer is tangent to the PPF; hence if there are two equilibria, the budget surface corresponding to each equilibrium passes above the other equilibrium, violating the weak axiom of revealed preferences.

Let me give a graphical confirmation of this statement. I adopt the Walrasian specification of the budget constraint, i.e. that specification which assumes that

consumers count on an income equal to the value of the factors they supply and not to the value of the factors which find purchasers. For the purpose of determining the equilibria (rather than out-of-equilibrium behaviour), this specification does not make any difference relative to the more satisfactory one, because the equilibria are the same. And it is easier to use graphically.

The essential point is that, if there are two factors, but one consumer, this consumer owns them both; therefore her income derives both from wages and from land rent. If we draw the labour total productivity function C=F(N), where C is the production of the consumption good and N is labour employment, the real wage is given by its derivative, but the remainder of the product goes anyway to the consumer as land rent.

In order to consider the consumer choice between leisure λ and corn, we simply re-draw the function C=F(N) from right to left, as C=F(λ_{max}–λ), a concave decreasing curve representing the economy's PPF between leisure (which can be at most λ_{max}, e.g. 24 hours a day) and consumption: then for each value of the real wage the budget constraint is the line *tangent* to this curve, with a slope equal to –w. Thus in Fig. 2 let us suppose that the real wage is such that the (non-equilibrium) budget constraint is as drawn, tangent to the production possibility frontier in E. Let us demonstrate that this is indeed the (Walrasian) budget constraint: the income from supplying labour is indicated by the λ_{max}F line; for example labour income is AG when labour supply is λ_{max}–A; the income going to the fixed land supply is GE, determined by the marginal product of land when labour demand is A[41]; therefore as labour supply increases, at the given factor rentals the consumer's total income varies along the budget constraint as drawn.

41 . The point of tangency of the budget constraint with the production possibility frontier indicates the intended production choice of firms at that wage rate and at the associated land rent rate equal to the marginal product of land when the supply of land is fully utilized (owing to constant returns to scale firms must make zero profits, so land too must be receiving its marginal product); therefore it also indicates the demand for labour, e.g. the segment from A to λ_{max} when the tangency is in E in Fig. 1.

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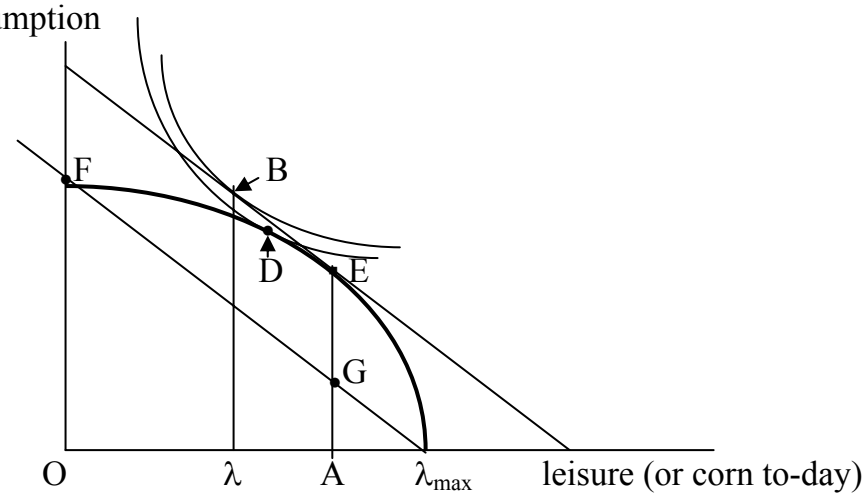

Fig. 2

The budget constraint is therefore such that the consumer choice can *never* be inside the production possibility frontier. Consider the (unique) equilibrium point D in Fig. 2. When the real wage changes from its equilibrium level, unless the PPF has a kink just in D the tangency point moves away from D and the consumer's optimum choice necessarily moves to a higher indifference curve (e.g. to point B) and therefore cannot touch again the concave PPF; and even if the budget constraint still goes through the equilibrium point (because that point is a kink of the PPF), unless the indifference curve through D too has a kink in that point any rotation of the budget constraint will again move the consumer's optimum choice to a higher indifference curve: so the equilibrium allocation is necessarily unique if indifference curves are *strictly* convex and the production possibility frontier is concave([42]), or if indifference curves are convex and the production possibility frontier is *strictly* concave; and the equilibrium relative prices too are unique, except for kinks of both the indifference curve and the production possibility frontier at the equilibrium allocation, or corner solutions.

[42] . Constant returns to scale for the industries suffice to ensure this.

(intl. 19pt)

consumption

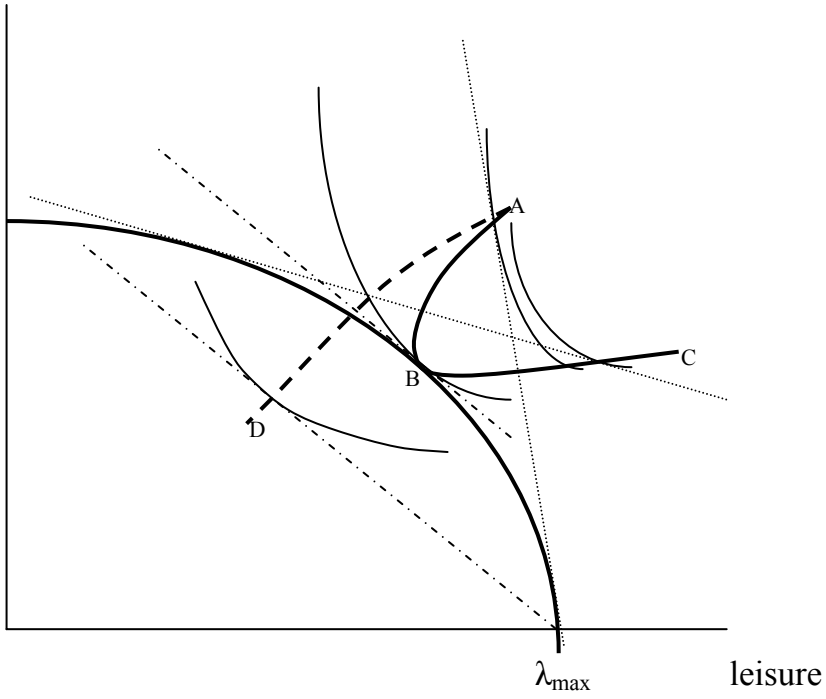

Fig. 3

In order to grasp the difference made by the assumption of a single consumer, I draw in Fig. 3 a possible "offer curve" (the broken curve AD[43]) of the consumer between leisure time and consumption as the real wage changes when she does *not* own the land and her sole endowment is leisure time: in this case the budget line always goes through λ_{max}, and becomes less and less steep as the wage rate decreases; for this case I assume a downward-sloping offer curve, i.e. such that the supply of labour would be backward-bending if the consumer did not own the land, with a possibility of multiple equilibria. The same Figure shows how that offer curve changes when the same consumer, with the same indifference curves, also owns the land: then the budget constraint is always tangent to the PPF, so the offer curve becomes the curve ABC which implies that the unique equilibrium is in B: along the portion of the offer curve from A to B labour demand is less than supply (the supply is to the left of the point where the budget constraint touches the PPF), while along the portion from B to C labour demand is always greater than supply([44]).

[43] As an aside, the term "offer curve" would be better replaced by "choice curve" since the points of the curve describe the consumer's choices (demands), not her "offers" (net supplies) which are only indirectly shown as differences between endowments and demands.

[44] On the contrary, when the offer curve is AD, generated by budget constraints through λ_{max} because leisure is the sole endowment (land belongs to other consumers), the equilibrium points are the points on the offer curve on or below the PPF, vertically aligned with the points on the PPF corresponding to the same wage, and there may well be several of

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Of course there is no reason why indifference curves should be strictly convex; but the multiple equilibria that can arise owing to such a cause are not due to the presence of capital, and therefore we can neglect them. For analogous reasons we can neglect the multiple equilibria that can arise owing to increasing returns and can render the PPF convex.

As far as I can see the above considerations generalize to any atemporal production economy. The same drawings can be interpreted as applying to a stationary economy where land is replaced by corn and the PPF shows the possible alternatives between leisure time and stationary net corn product.

16. *Non-uniqueness with value capital.* When, as in traditional long-period marginalist equilibria, the other factor besides labour is *value* capital, and there is a given endowment of value capital (in terms of the consumption good) while prices are long-period prices then, even with a single consumer, multiple equilibria *are* possible. This is because the relationship, corresponding to the PPF, between leisure and stationary net output of the consumption good is no longer necessarily a concave function with the budget constraint tangent to it and shifting leftwards as w decreases.

This relationship must now be derived as follows. For each real wage, the (stationary) economy chooses a technique which implies a certain value of capital per unit of labour k, hence an employment of labour per unit of value capital equal to its reciprocal, $1/k$, and a certain physical net output of consumption good per unit of labour y. The articles in the Cambridge controversy have analyzed the possible shapes of these functions of the real wage (or of the rate of profit) for a variety of numerical examples. All we need is to derive from these magnitudes how the point (leisure, net output per unit of value capital) changes with the real wage. The net output per unit of value capital is given by $z(w)=y(w)/k(w)$; leisure is given by $\lambda(w) = \lambda_{max} - 1/k(w)$. The locus of points thus obtained in (λ,z) space can be called the pseudo-PPF curve corresponding to a given value endowment of capital of 1 unit. For each w, through the corresponding point of the pseudo-PPF curve one draws a line with slope equal to $-w$, which indicates the budget constraint of the single consumer under a stationariness assumption; if this slope equals the slope of the indifference curve through that point, one has an equilibrium.

But now the points $(\lambda(w),z(w))$ corresponding to the changing value of w need no longer form a concave curve, and the budget constraint will not be tangent to the pseudo-PPF curve, which can have nearly any shape, even crossing itself several times. So, more than one point on this pseudo-PPF curve can exhibit equality

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these. Notice how the elimination of multiple equilibria due to the single consumer does not derive from the abolition of the possibility that the supply of labour be a decreasing function of the real wage: it still is, along most of the AB portion of the ABC offer curve.

between the slopes of the budget constraint and of the indifference curve through that point, i.e. there can be multiple equilibria.

Table 1 reports the values of *1/k* and *y/k* for selected values of *w* for the well-known example developed by Garegnani (1970: Fig. 3, p. 413, and Fig. 7, p.430: for the reader's convenience, Garegnani's Fig. 3 is reproduced here as Fig. 4). These values are appended to the values calculated by Garegnani (1970) in his Table II, p. 429.

Table 1

r	*w*	*y*	*k*	*1/k*	*y/k*
0	0.200	0.200	1.080	0.926	0.185
2.6	0.175	0.192	0.635	1.575	0.302
4.1	0.169	0.183	0.393	2.545	0.466
6.1	0.159	0.175	0.257	3.891	0.681
8.3	0.151	0.167	0.184	5.435	0.908
10.5	0.144	0.159	0.148	6.757	1.074
12.9	0.129	0.152	0.179	5.587	0.849
14.4	0.105	0.159	0.379	2.639	0.420
15.1	0.083	0.167	0.552	1.812	0.303
15.9	0.061	0.175	0.715	1.399	0.245
16.9	0.041	0.183	0.850	1.177	0.215
17.5	0.026	0.192	0.947	1.056	0.203
20.0	0	0.200	1.000	1.000	0.200

[Insert Fig. 4 about here]
[Reproduction of Fig. 3 p. 413 from Garegnani 1970]

The endowment λ_{max} of total leisure time can be fixed arbitrarily, here I assume it is 10 for ease of calculation. The values thus calculated of λ_{max}–*1/k* and of *y/k* are plotted in Fig. 5, and are connected by a curve that reproduces the behaviour of the pseudo-PPF curve, with the corresponding value of *w* derivable from Table 1, and the corresponding budget line shown by the thin straight segment through the point. The resulting pseudo-PPF curve is strikingly different from the convex curve one would obtain with given endowments of physically specified factors. It is apparent that it would be easy to draw families of indifference curves that are tangent to the budget lines in more than one point of the pseudo-PPF curve; three such tangent indifference curves are drawn in Fig. 5. Multiplicity of equilibria is no longer impossible.

[Insert Fig. 5 about here]

The reason why multiple equilibria are possible is that the budget line through

some points of the pseudo-PPF curve passes *below* some other points of the curve, so a convex indifference curve that goes through a point A of that curve and is tangent to the budget line through that point can pass *below* other points of the curve, an impossibility with a concave PPF curve with budget lines tangent to it. Thus in Fig. 5 nothing prevents both the indifference curve through point A and the indifference curve through point B (shown as dotted curves) from being both tangent to the respective budget lines through those two points.

17. The possibility of multiple equilibria thus obtained is due to two differences from the usual formalization of intertemporal equilibria: 1) the value specification of the capital endowment, 2) the presence of an endogenously determined production of capital goods in the last (coinciding here with the first) period considered. Now I will try to show that the second difference is alone sufficient to obtain multiple equilibria.

Since there is production of, and hence demand for, capital goods in the *last* period of the intertemporal equilibrium I will consider, this demand cannot be derived from the demands for the products that those capital goods might produce: the futures markets of those products do not exist. On the other hand, one cannot take this final demand for capital goods as given: it clearly will depend on what investors expect to be profitable in the future, and this cannot be independent of what they have observed to be going on in the markets where equilibrium is being established. It is therefore indispensable to make some assumption as to how the vector of capital goods demanded in the last period, in other words investment, is determined. One can distinguish two issues here; I announce my solutions in intuitive terms first, the precise solutions will become clear when I specify the model.

The first issue is the *total amount* (total value) of gross investment. Since the purpose of the exercise is to prove the possibility of multiple equilibria while remaining as close as possible to neoclassical reasonings, it seems legitimate here to assume Say's Law, that is, investment determined by savings. There remains to determine savings. In keeping with a long tradition in neoclassical theorizing, I will assume that consumers (in fact, the representative consumer) wish to perform no net savings; they want stationariness, so they want the capital stock to remain unaltered. This assumption is not in line with usual contemporary theorizing, that generally assumes a given rate of discount of future consumption; but the latter assumption is not less arbitrary than mine, and anyway I believe that, once the basic problem has been made clear through as simple an example as possible, then modifying the example by introducing a propensity to net savings will not be difficult and will not alter the conclusions. My assumption on the amount of investment is, in conclusion,

that it is the one required by stationariness[45].

The second issue is the *composition* of investment. I will assume that, after an initial period zero where production of capital goods is by labour alone but very inefficient, production can utilize either one or another of two types of capital good, each one of which produces the consumption good and itself (as in Garegnani 1970), and that investment will be in the type of capital good that, on the basis of the given real wage (which is expected to last) and of an expectation of product prices determined by minimum average cost, promises to yield the higher rate of profit in subsequent periods. The assumption that the given real wage is expected to last appears reasonable, in view of the rather small difference between rates of change of real wages and rates of change in labour productivity in actual economies most of the time, and of the fact that in my model I abstract from technical change so the abovementioned "rather small difference" comes to mean a generally very slow change of real wages over time.

18. I come to the model. The equilibrium, established at date 0, includes markets for good of three dates, 0, 1 and 2; these dates mark the beginning of the respective periods. At dates zero and one, demand for the consumption good is zero. At date zero the sole factor is labour supplied in fixed amount, which produces either capital goods of type α or capital goods of type β according to fixed-coefficients methods; capital goods come out at date 1; the stock of capital good of type α available at date 1 will be indicated as $K_{1\alpha}$, the other stock as $K_{1\beta}$. Either kind of capital good, together with labour, is capable of producing itself, or the single consumption good corn, according to a single fixed-coefficients method for each production, specific to the capital good; production takes one period. From date 1 onwards for some reason the production methods adopted in period zero become unfeasible, and one must produce with the methods that use one or the other capital good. Thus apart from period 0 the available production methods are like in the two-techniques case of Garegnani 1970 article. The futures markets for date 2 include the market for corn, that comes out of the production of period 1 in amount C_2, and the

[45] My assumption can be rationalized in the same way as was done by traditional marginalist authors, who considered capital *accumulation* to be sufficiently slow relative to the speed with which the *composition* of capital can adjust, to make it legitimate to assume an adjusted composition while treating the total capital stock as constant (some quotations to such an effect are in Petri, 2004, p. 119). Or it can be rationalized by assuming that from period 1 onwards the myriad identical consumers (cf. the next footnote) last one period and each one bequeathes his capital to his son, an identical one-period consumer for whose utility the father cares but only to the point of desiring that the son starts with as much wealth as the father. Anyway I can see no reason why introducing a propensity to net savings should make it impossible to reach the results I will reach.

markets for capital goods, whose supplies will be indicated as $K_{2\alpha}$ and $K_{2\beta}$; the demands for them constitute investment and I will assume supplies equal to demands; there is no futures market for labour of period 2. Demand for labour in period 1, L_{D1}, depends on how much C_2 is produced and how (it can be produced with either method), and on how much is demanded of $K_{2\alpha}$ and of $K_{2\beta}$; this demand for labour can be compared with the supply of labour, that depends on the consumer's choice between C_2 and leisure in period 1. This choice between leisure and consumption depends uniquely on the real (corn) wage w. The absence of futures markets for labour of date 2 means that the myriad identical consumers[46] cannot determine what will happen to labour employment in period 2; they can only determine their labour supply choices for period 1; their preference for stationariness of the capital stock determines their individually taken saving decisions without any awareness of what this will imply for future labour employment[47].

Let $K_{1\alpha}$ (respectively, $K_{1\beta}$) stand for the amount of capital goods of type α (respectively, of type β) produced by the full employment of labour of date zero. Let us choose units such that $K_{1\alpha}=K_{1\beta}=1$.Then the price of one unit of capital good of date 1 of either type must be the same. Having adopted these units for the capital goods, let the technical coefficients be defined thus (k stands for capital, c for corn or consumption good, l for labour):

$k_{c\alpha} \oplus l_{c\alpha} \rightarrow$ 1 unit of consumption good (with method α)

$k_{k\alpha} \oplus l_{k\alpha} \rightarrow$ 1 unit of capital good of type α

$k_{c\beta} \oplus l_{c\beta} \rightarrow$ 1 unit of consumption good (with method β)

$k_{k\beta} \oplus l_{k\beta} \rightarrow$ 1 unit of capital good of type β

Let P_{it} stand for the *discounted* price of good i of date t, while p_{it} stands for its *undiscounted* price. Corn of date 2 is the numéraire, i.e. $P_{c2}=p_{c2}=1$. The discounted price of corn of date 2 produced with method α is

$$P_{c2\alpha} = 1 = W_2\, l_{c\alpha} + P_{k1\alpha}\, k_{c\alpha}.$$

The undiscounted price, with wages paid in arrears, is

$$p_{c2\alpha} = 1 = w\, l_{c\alpha} + (1+r_{1\alpha})\, p_{k1\alpha}\, k_{c\alpha}.$$

Because corn of date 2 is the numéraire and wages are paid in arrears, it is

[46] The existence of a representative consumer should not make one forget that we are discussing competitive economies, where therefore the representative consumer has no market power; this must be due to the fact that consumers are many, and the representative consumer results e.g. from their being identical in tastes and in endowments.

[47] No doubt this assumption will be found disturbing by economists accustomed to assume perfect foresight or complete futures markets over the infinite future, but realism obliges one to admit that in market economies individual decisions are taken independently of the collective results coming out of them. More analytically, if one assumes a finite decision horizon in an economy, it is inevitable that disequilibria may arise in subsequent periods.

$W_2=w$. Strictly analogous price equations of course hold for corn of date 2 produced with method β. It is unnecessary[48] to distinguish $p_{k1\alpha}$ from $r_{1\alpha}$, or $p_{k\beta}$ from $r_{1\beta}$, because all we need is to determine which capital good is preferred to produce C_2, to such an end what is important is that the minimum-cost method will be preferred, and for that P_{k1} suffices. The two capital goods require the same amount of period-0 labour to be produced so they must have the same price, so for each given P_{k1} and given real wage the preferred method will be the one that yields the smaller sum of labour cost and capital cost; since C_2 is the numéraire its price is 1, and, because of the zero-extraprofit implication of constant returns to scale, whatever part of the value of C_2 that does not go to pay wages must go to pay date-0 labour in the form of price of date-1 capital goods; this means that for a given w the minimum-cost method is the one that allows paying the higher P_{k1}. Minimizing the price of C_2 for a given P_{k1} and a given real wage implies maximizing P_{k1} for a price of C_2 equal to 1 and a given w. The implication is that for the production of C_2, choice of production method will be guided by which method allows paying the higher P_{k1} on the basis of the given w. It is

$P_{k1\alpha}=(1 - w\, l_{c\alpha}) / k_{c\alpha}$

$P_{k1\beta}=(1 - w\, l_{c\beta}) / k_{c\beta}$.

The common discounted price of the capital goods of date 1, that we can indicate simply as P_{k1}, will be the greater of the two. Therefore for $w=0$ the preferred method for the production of C_2 is the one with the smaller coefficient k_c, and if for example this is method α i.e. if $k_{c\alpha}<k_{c\beta}$, then as w rises there can be a switch of method in the production of C_2 only if $l_{c\beta}<l_{c\alpha}$.

If, for a given w, method α is preferred for the production of C_2, this does not mean that only capital good α will be produced by date-0 labour. If investment demand at date 2 is for capital good β, then date-0 labour will produce some of both capital goods, the production of capital good β of date 1 being required for the production of capital good β of date 2.

Can this happen? It depends on what determines the type of capital goods in which there is investment at date 2. I assume that individual producers (who borrow the consumers's savings[49]), when deciding the type of capital in which to invest, are led by their experience to assume that product price is determined by average cost; furthermore, as already said, they assume that the real wage is not going to change; therefore they assume that the undiscounted price of corn of date 3 will be

[48] And actually impossible in this model, where there is no consumption good of date 1, and capital goods have no direct utility and therefore there is no utility-based choice among capital goods of date 1 and 2 that might allow the determination of r_1.

[49] Alternatively one can imagine that capital goods are bought by the consumers themselves, who learn from consultants the capital goods that it is most convenient to purchase.

$p_{c3\alpha} = 1 = w\, l_{c\alpha} + (1 + r_{2\alpha})\, p_{k2\alpha}\, k_{c\alpha}$

if method α is adopted, and

$p_{c3\beta} = 1 = w\, l_{c\beta} + (1 + r_{2\beta})\, p_{k2\beta}\, k_{c\beta}$

if method β is adopted.

Since corn is the numéraire and its undiscounted price is 1, if $p_{k2\alpha}$ and $p_{k2\beta}$ can be determined beforehand then the two rates of return $r_{2\alpha}$ and $r_{2\beta}$ are the sole unknowns in these two equations and can be determined and compared. The previous determination of $p_{k2\alpha}$ and $p_{k2\beta}$ is indeed possible, but there is a complication: they depend on which method is chosen for the production of C_2, because that choice determines P_{k1}. Suppose it is method α; then $P_{k1}=P_{k1\alpha}$ for either capital good of date 1. Let $p_{k2\alpha\alpha}$ stand for the undiscounted cost of production of capital good α of date 2 when C_2 is produced with method α, let $p_{k2\alpha\beta}$ stand for the cost of production of a unit of capital good β of date 2 when C_2 is produced with method α (so the sequence of indices $\alpha\beta$ indicates that the methods preferred for the production of C_2 and successively of C_3 are first α and then β), and analogously for $p_{k2\beta\beta}$ and $p_{k2\beta\alpha}$ (the last one is the cost of production of $K_{2\alpha}$ when $P_{k1}=P_{k1\beta}$). Then the equations for the undiscounted prices of capital goods of date 2 become

$p_{k2\alpha\alpha} = w\, l_{k\alpha} + P_{k1\alpha}\, k_{k\alpha}$,

$p_{k2\alpha\beta} = w\, l_{k\beta} + P_{k1\alpha}\, k_{k\beta}$,

$p_{k2\beta\beta} = w\, l_{k\beta} + P_{k1\beta}\, k_{k\beta}$,

$p_{k2\beta\alpha} = w\, l_{k\alpha} + P_{k1\beta}\, k_{k\alpha}$,

where $P_{k1\alpha}=(1 - w\, l_{c\alpha})/k_{c\alpha}$ is the payment to capital goods of date 1, whichever the type, when C_2 is produced with method α, and analogously for $P_{k1\beta}$; thus, for a given w, one must distinguish four costs of production of corn of date 3, and four expected rates of return if the price of that corn is 1, depending on the sequence of methods. For brevity I shall indicate this sequence as αα, αβ, ββ, βα where the first letter indicates the method used to produce C_2, the second letter indicates the method (or technique[50]) firms intend to use to produce C_3 and therefore also the capital good in which firms invest at date 2. The equations determining these rates of return are

$p_{c3\alpha\alpha} = 1 = w\, l_{c\alpha} + (1 + r_{2\alpha\alpha})\, [w\, l_{k\alpha} + k_{k\alpha}\, (1 - w\, l_{c\alpha})/k_{c\alpha}]\, k_{c\alpha}$

$p_{c3\alpha\beta} = 1 = w\, l_{c\beta} + (1 + r_{2\alpha\beta}))\, [w\, l_{k\beta} + k_{k\beta}\, (1 - w\, l_{c\alpha})/k_{c\alpha}]\, k_{c\beta}$.

$p_{c3\beta\beta} = 1 = w\, l_{c\beta} + (1 + r_{2\beta\beta}))\, [w\, l_{k\beta} + k_{k\beta}\, (1 - w\, l_{c\beta})/k_{c\beta}]\, k_{c\beta}$.

$p_{c3\beta\alpha} = 1 = w\, l_{c\alpha} + (1 + r_{2\beta\alpha}))\, [w\, l_{k\alpha} + k_{k\alpha}\, (1 - w\, l_{c\beta})/k_{c\beta}]\, k_{c\alpha}$.

These equations determine the four possible expected rates of return in the

[50] It is usual to mean by 'method' a direct production process adopted in the production of a certain good, and by 'technique' a set of methods required to produce both a net product and the capital goods directly or indirectly required for that net product. Since production of C_3 with a method implies use of the corresponding capital good which must have been produced in period 1, I could also speak of *technique* producing C_3.

production of corn of date 3. For each level of w, only two of these are relevant, because the level of w selects the method preferred for the production of C_2, thus if this is for example method β, only $r_{2\beta\beta}$ and $r_{2\beta\alpha}$ need be compared in order to ascertain which method will be chosen for the production of C_3 and therefore the capital good in which firms invest at date 2. I assume that investment will be entirely into the type of capital good that yields the higher of the two relevant rates of return. Why entirely? Because as long as w is not expected to change, it is reasonable to assume that the method chosen for the production of C_3 will be presumed by firms to be the one yielding the higher rate of return for the production of C_4 too. This presumption will turn out to be certainly correct if the method chosen for the production of C_3 is different from the one chosen for the production of C_2, because the reason for the change is the tendency of relative prices determined by average costs to converge toward long-period prices if w remains unchanged, a well known result; so the correctness of the change will be confirmed by the subsequent evolution of prices. On the other hand, suppose that method α is preferred for the production of C_2; it is conceivable that method α may be found again the most convenient one for the production of C_3 while further extrapolation of the development of prices would show that C_4 or C_5 should be produced with method β, because the latter is the most convenient technique at the long-period prices connected with the given w. But if one assumed that in such a case date-2 investment will partly consist of capital goods β and partly of capital goods α, one would be assuming something close to perfect foresight by assuming that consumers and firms are able to extrapolate what will happen in periods more and more into the future, and to trust their extrapolations; anyway in the numerical example to be presented this case does not occur.

The above determines the *type* of investment once w is given. As to the *amount* of investment, I assume a desire of consumers for a stationary stock of capital. This easily determines investment in the sequences αα and ββ. The initial stocks of capital, produced by date-0 labour, must be fully utilized in equilibrium. Suppose method α is preferred for C_2 and for C_3. Then only capital good α is produced by date-0 labour, its stock is 1 unit, and its full utilization implies

$$k_{c\alpha}\, C_2 + k_{k\alpha}\, K_{2\alpha} = 1.$$

In this case a stationary physical capital stock can be defined, and it implies $K_{2\alpha}=K_{1\alpha}=1$, so $C_2=(1 - k_{k\alpha}) / k_{c\alpha}$. Let us indicate this amount of C_2, associated with stationariness when method α is preferred for C_2 and for C_3, as $C_{2\alpha\alpha}$:

$$C_{2\alpha\alpha} = (1 - k_{k\alpha}) / k_{c\alpha}.$$

Analogously

$$C_{2\beta\beta} = (1 - k_{k\beta}) / k_{c\beta}.$$

Suppose instead that method α is preferred for C_2 but method (technique) β is preferred for C_3 so date-2 investment consists entirely of capital goods β. Both prices and physical type are changing, and furthermore what subsequent wages,

employment and prices will actually be is not certain. Therefore the meaning to be attributed to "maintaining capital intact" is necessarily arbitrary to an extent. I choose the following. Since a unit of $K_{1\alpha}$ and a unit of $K_{1\beta}$ cost the same, when both types are produced they are perceived by consumers as being the same amount of "real" capital as just one unit of either of them; of the two, one type of capital good comes out on the basis of firms' expectations to be more profitable than the other one for investment, so it seems reasonable to assume that consumers will consider 1 unit of the more profitable capital good to be at least as much "real capital" as the mixture of the two types that they considered to be 1 unit of capital at date 1. So I assume that in this case zero net "real" savings will be interpreted by consumers as meaning that it must be $K_{2\beta}=1$. Then date-1 capital goods must include the amount of capital good β necessary to produce $K_{2\beta}=1$, which is $k_{k\beta}$; therefore the amount of date-1 capital good α is $1 - k_{k\beta}$ and the amount of C_2 that can be produced with it satisfies $k_{c\alpha}\, C_2 = 1 - k_{k\beta}$; let us indicate as $C_{2\alpha\beta}$ this amount of C_2 associated with initial preference for method α and subsequent preference for method β, that is with the methods I will call it *sequence* αβ:

$C_{2\alpha\beta} = (1 - k_{k\beta}) / k_{c\alpha}$.

Analogously

$C_{2\beta\beta} = (1-k_{k\beta}) / k_{c\beta}$

$C_{2\beta\alpha} = (1 - k_{k\alpha}) / k_{c\beta}$.

Now we can determine labour demand in period 1:

$L_{D\alpha\alpha} = l_{c\alpha} (1 - k_{k\alpha}) / k_{c\alpha} + l_{k\alpha}$

$L_{D\alpha\beta} = l_{c\alpha} (1 - k_{k\beta}) / k_{c\alpha} + l_{k\beta}$

$L_{D\beta\beta} = l_{c\beta} (1 - k_{k\beta}) / k_{c\beta} + l_{k\beta}$

$L_{D\beta\alpha} = l_{c\beta} (1 - k_{k\alpha}) / k_{c\beta} + l_{k\alpha}$.

19. The above equations plus the specified conditions of technical choice permit the determination of C_2 and of labour demand in period 1 as functions of the real corn wage, once the eight technical coefficients $k_{c\alpha}$, $l_{c\alpha}$, $k_{k\alpha}$, $l_{k\alpha}$, $k_{c\beta}$, $l_{c\beta}$, $k_{k\beta}$, $l_{k\beta}$ are given. Let us then assume that these coefficients are:

$k_{c\alpha}$	$l_{c\alpha}$	$k_{k\alpha}$	$l_{k\alpha}$	$k_{c\beta}$	$l_{c\beta}$	$k_{k\beta}$	$l_{k\beta}$
1/81	1/9	1/9	1	5/768	13/96	1/8	1

At w=0, method β is the preferred one for the production of C_2 because[51] $P_{k1\beta}=1/k_{c\beta}=153.6>P_{k1\alpha}=1/k_{c\alpha}=81$, and method β remains preferred up to w=6.1525 where α and β allow paying the same P_{k1}.

So for w=6.150 method β is still the preferred one for the production of C_2; but

[51] Calculations have been performed with Maple V Release 3 Student Edition. I can only supply upon request a printout of the Maple file, the file itself will not run on current Windows machines (I run it on an old Windows Millennium Edition computer).

method α is preferred for the production of C_3 because $1+r_{2\beta\alpha}=2.847$ while $1+r_{2\beta\beta}=2.744$. (The preference for the sequence βα persists as long as $6.0583<w<6.1525$.) At this wage, the sequence being βα, it is $C_{2\beta\alpha}=136.53$, $l_{D\beta\alpha}=19.49$. With the sequence ββ it is $C_{2\beta\beta}=134.4$, $l_{D\beta\beta}=19.20$, and assuming any maximum leisure time $\lambda_{max}>20$ one can compare the value of the two leisure-consumption bundles $(\lambda_{max}-l_{D\beta\alpha},\ C_{2\beta\alpha})$ and $(\lambda_{max}-l_{D\beta\beta},\ C_{2\beta\beta})$; the difference between the two values for $w=6.150$ is

$$(C_{2\beta\alpha} - w\cdot l_{D\beta\alpha}) - (C_{2\beta\beta} - w\cdot l_{D\beta\beta}) = 0.3567.$$

If there were equilibrium for this w, the budget line through the equilibrium leisure-consumption bundle (the one associated with the βα sequence) would pass above the leisure-consumption bundle associated with the ββ sequence. This is as neoclassical theory would lead one to expect.

But for w=3 for example[52], method β is preferred both for the production of C_2, and for the production of C_3 because $1+r_{2\beta\beta}=6.33$ while $r_{2\beta\alpha}=4.11$. So the chosen sequence is ββ. (And there is no mistake in this case in the choice to produce only capital good β, because if the real wage remains constant, β will remain the most convenient technique for the production of the consumption good, because at w=3 technique β is the cost-minimizing technique at long-period prices: in long-period technical choice the two techniques reswitch, with α the most profitable one for $w<2$ i.e. $r>6$, and for $w>6$ i.e. $r<2$; the graph of the two $w(r)$ curves is in Fig. 6; it is also seen that when the sequence is βα, technique α is the cost-minimizing one at long-period prices because $w>6$, so in that case too the decision to invest entirely in capital good α is justified.)

[Insert Fig. 6 about here]

However, the value of the leisure-consumption bundle associated with the sequence ββ remains inferior to the value of the leisure-consumption bundle associated with the sequence βα; for $w=3$ it is

$$(C_{2\beta\alpha} - w\cdot l_{D\beta\alpha}) - (C_{2\beta\beta} - w\cdot l_{D\beta\beta}) = 1.267.$$

This means that if there were equilibrium for w=3, the budget line through the chosen leisure-consumption bundle (associated with the ββ sequence) would pass *below* the leisure-consumption bundle associated with the βα sequence. It is then perfectly possible that the indifference curve through the leisure-consumption bundle of sequence βα may have slope -6.150 there, while at the same time the indifference curve through the leisure-consumption bundle of sequence ββ may have slope -3 there, that is, nothing excludes the existence of two equilibria.

[52] In fact, for any w<6.0583 down to w=0.

This is shown in Fig. 7, which has leisure, λ_{max}-l_D, on the abscissa and C_2 on the ordinate; assuming now λ_{max}=24 (but as mentioned, any other number greater than 20 might be chosen), the leisure-consumption bundle associated with sequence βα is (24-19.49, 136.53*)*, the one associated with sequence ββ is (24-19.20, 134.4) and it is to the South-East of the first one. If one of these points is an equilibrium at the corresponding wage rate, the budget line passes through it with slope -w and the convex indifference curve through the point is tangent to the budget line. It is possible to have tangency at both points, because the budget line through the point associated with sequence ββ passes, for an ample range of values of w, below the other point.

[Insert Fig. 7 about here]

Thus it is perfectly possible that this economy has two equilibria. The ultimate reason is that the last period's output includes goods whose demand is not determined in the same way as for consumption goods. The presence of capital goods makes a difference.

Some of the assumptions made in this numerical example may appear rather arbitrary, but I would argue that they are not more arbitrary than currently fashionable assumptions (they are certainly *less* arbitrary than complete futures markets or perfect foresight for the indefinite future, or the end of the economy after a finite number of periods). Anyway I would be surprised if similar results of possible multiplicity of equilibria could not be obtained with different assumptions, as long as there is an endogenously determined production of capital goods in the last period. I would conjecture that along this route it must also be possible to produce multiple equilibria for the case where labour supply is rigid, because it must be possible to find examples where the change in investment composition as income distribution changes leaves demand for labour unchanged.

A final caution: the example relies on technology that is associated with reswitching, but I have not been able to ascertain whether that is a necessary condition for the result. Let me hope that "someone younger and better equipped for the task" may succeed in understanding better the general characteristics of the possibility I have uncovered.

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(rough draft to be replaced by reproduction of Figure from Garegnani 1970)

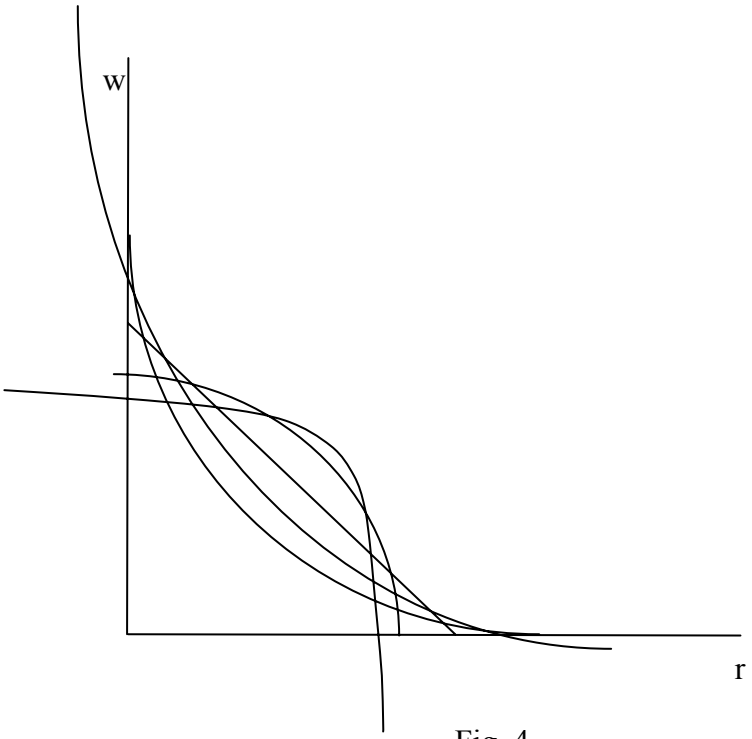

Fig. 4

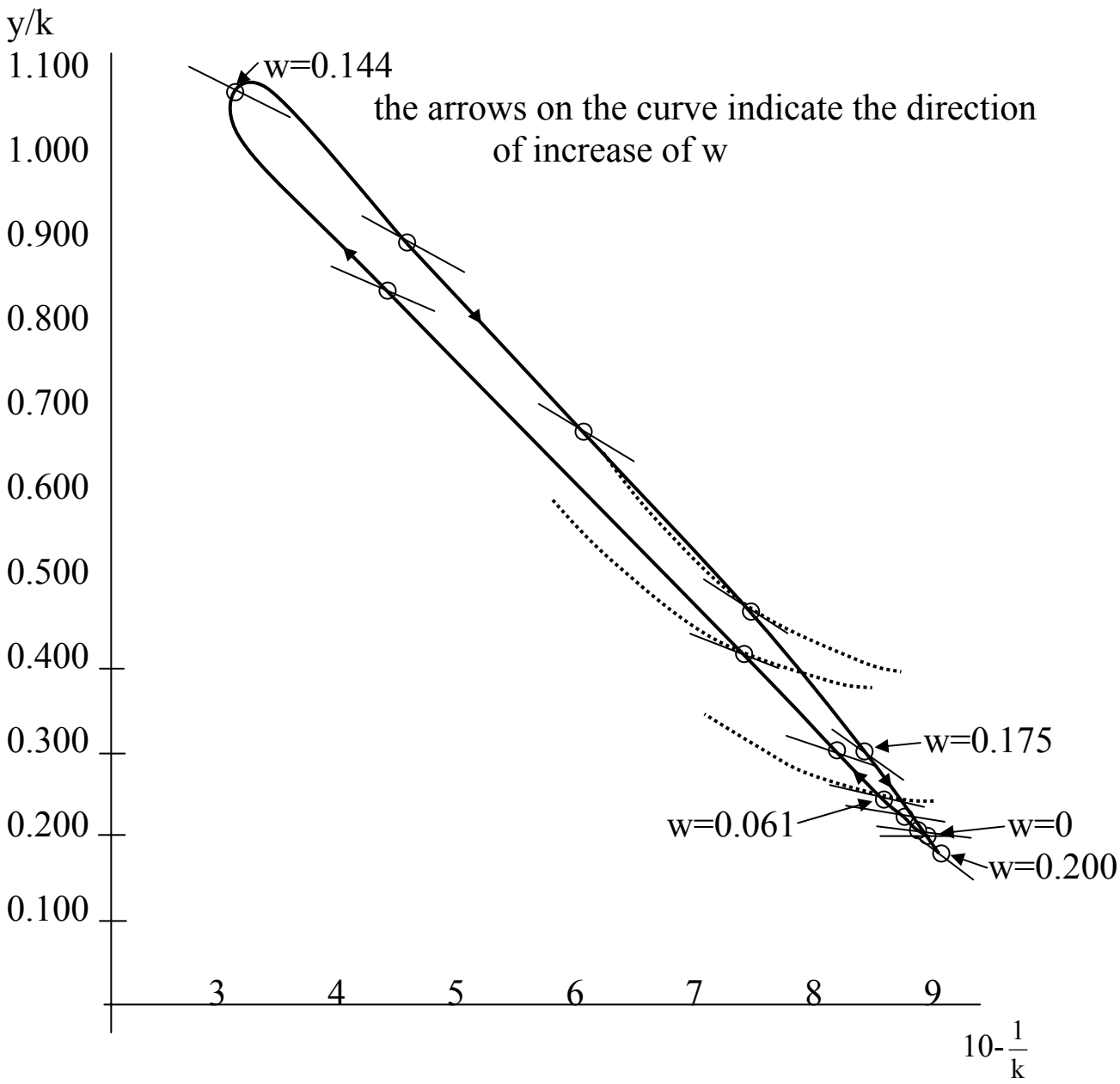

Fig. 5

Behaviour of the leisure-consumption tradeoff as the real wage varies, for the case of given endowment of value capital, long-period prices and technology of Fig. 4. The straight lines are the budget lines corresponding to the various levels of w.

(intl 16 pt)

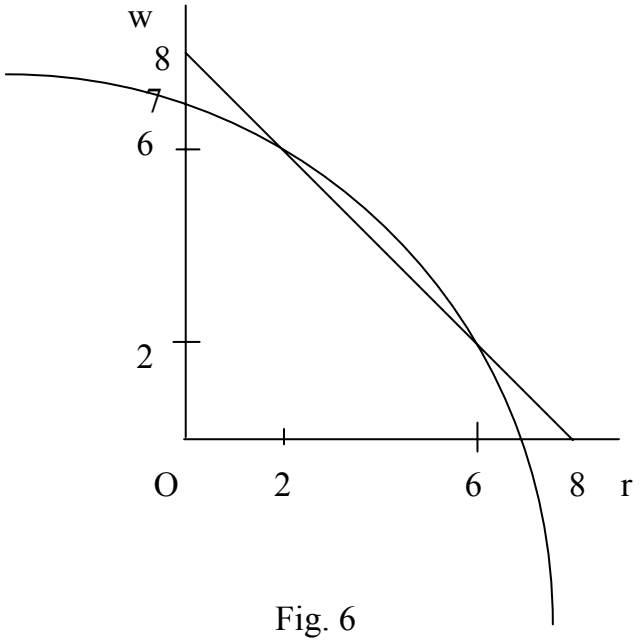

Fig. 6

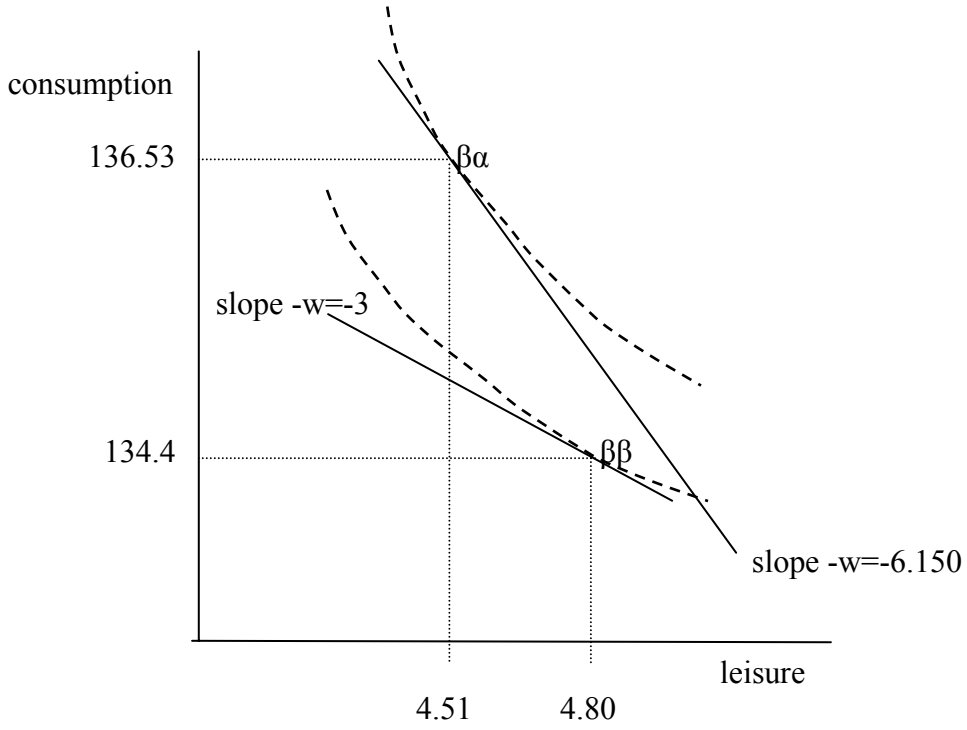
consumption
136.53
βα
slope -w=-3
134.4
ββ
slope -w=-6.150
leisure
4.51
4.80

Fig. 7