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3 • 2004

Martin Ellison – Lucio Sarno –
Jouko Vilmunen
Research Department
20.1.2004

Monetary policy and
learning in an open economy

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The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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Monetary policy and learning in an open economy

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Martin Ellison – Lucio Sarno – Jouko Vilmunen
Research Department

Abstract

In this paper, we examine the incentives for central bank activism and caution in a two-country open-economy model with uncertainty and learning. We find that the presence of a strategic interaction between the home and foreign central banks creates an additional motivation for caution in monetary policy. An activist policy designed to help the learning of the home central bank is suboptimal since it generates a strong reaction from the foreign central bank. As joint learning by the home and foreign central banks is shown to be detrimental to welfare, the optimal policy is cautious.

Key words: activism, learning, monetary policy, open economy

JEL classification numbers: D81, D83, E52, E58, F41

Oppiminen ja rahapolitiikka avoimessa taloudessa

Suomen Pankin keskustelualoitteita 3/2004

Martin Ellison – Lucio Sarno – Jouko Vilmunen
Tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa tarkastellaan optimaalisen rahapolitiikan määräytymistä kahden avoimen talouden tapauksessa, jossa keskuspankin oppiminen liittyy sen pyrkimyksiin päivittää oma käsityksensä inflaation aktiivisuusvaikutusten suuruudesta. Tutkimuksessa osoitetaan, että koti- ja ulkomaisen rahapolitiikan strateginen vuorovaikutus lisää syytä varovaisuuteen rahapolitiikassa. Aktivistinen rahapolitiikka, jonka tarkoituksena on edistää keskuspankin oppimisprosessia, ei ole optimaalista, koska se laukaisee voimakkaan vastareaktion ulkomaisen keskuspankin taholta. Tasapainossa aktivistinen tai kokeileva kotimaan rahapolitiikka ei näin ollen edistä hyvinvointia, mistä puolestaan seuraa, että optimaalinen rahapolitiikka korostaa varovaisuutta.

Avainsanat: rahapolitiikka, oppiminen, avoin talous

JEL-luokittelu: D81, D83, E52, E58, F41

Contents

Abstract	3
1 Introduction	7
2 The model.....	8
2.1 Structure of the open economy	8
2.2 Central bank loss function	9
2.3 Beliefs.....	9
2.4 Learning.....	10
2.5 Equilibrium.....	11
3 Calibration	11
4 Results	12
4.1 Passive learning policy	13
4.2 Active learning policy.....	15
4.3 Sensitivity analysis	18
5 Conclusions	20
References.....	21
Appendix A. Numerical approximation of the active learning policy	23

1 Introduction

The choice between an activist or cautious monetary policy remains a contentious issue between academics and central bankers. Academics such as Bertocchi and Spagat (1993) and Beck and Wieland (2002) argue that monetary policy should be activist to help the central bank in learning the key features of the economy. In contrast, central bankers such as Blinder (1998) and Issing (1999) believe that policy should predominantly be cautious due to uncertainty, following the result of Brainard (1967). As Blinder (1998) puts it, ‘My intuition tells me that [Brainard’s] finding is more general – or at least more wise – in the real world than the [simple] mathematics would suggest’.

In this paper, we attempt to reconcile these conflicting views by showing one way in which the call for caution made by Brainard (1967) is more general than it at first appears. We change the focus of the debate, from the closed economy models used in the existing academic arguments¹, to open economy models with two countries. Our results suggest that optimal monetary policy may actually be even more cautious when learning is taken into account. In an open economy model, learning by the home central bank cannot be considered in isolation since the foreign central bank is also learning at the same time. It is the balance of the costs and benefits of the *joint* learning of the home and foreign central banks that determines whether monetary policy is activist or cautious. We show that the costs of joint learning dominate the benefits so there are no incentives for an activist policy – monetary policy should be cautious.

Our results derive from the strategic interaction between the home and foreign central banks that is central to the open economy model. Existing papers based on closed economy models do not take this strategic interaction into account and so do not fully capture the costs and benefits of an activist policy.²

The structure of the paper is as follows. In Section 2 we describe our two-country open economy model and discuss the roles of uncertainty and learning. The model is calibrated in Section 3. In Section 4 we present our results and show how optimal monetary policy becomes more cautious when learning is taken into account. We perform an extensive sensitivity analysis to check the robustness of our result. The conclusions are presented in Section 5.

¹See, *inter alia*, Basar and Salmon (1990), Bertocchi and Spagat (1993), Balvers and Cosimano (1994), Wieland (1998, 2000b) and Beck and Wieland (2002).

²Similar issues have been explored in the game theory literature. The closed economy model is analogous to a monopolist adjusting its pricing policy to learn the demand conditions it faces (Mirman, Samuelson and Urbano (1993)), whereas the open economy two country model is analogous to two duopolists learning about their common demand conditions (Aghion, Espinosa and Jullien (1993)). The strategic interaction between the two firms in the duopoly case leads to much richer dynamics in optimal pricing behaviour than in monopoly.

2 The model

2.1 Structure of the open economy

Our two-country open economy model is described by the Phillips curve relationships (2.1) and (2.2). Output in the home country, y_t , is determined by the difference between home inflation, π_t , and inflation in the foreign country, π_t^* . Similarly, output in the foreign country, y_t^* , depends on the level of foreign inflation relative to inflation in the home country. To maintain symmetry and maximise the degree of strategic interaction between the two countries, we assume that the Phillips curve parameter β_t is always the same in both countries. z_t and z_t^* are observable i.i.d. output shocks with a variance-covariance matrix Ω . The covariance between z_t and z_t^* reflects the relative importance of symmetric versus asymmetric shocks in the open economy. ε_t and ε_t^* are unobservable i.i.d. output shocks with a variance-covariance matrix Σ . We define the activism of monetary policy in the home country as the degree to which the home central bank reacts to its observable output shock z_t . Equations (2.1) and (2.2) can be considered as a stylised reduced form of a more complicated open economy macroeconomic model in which relative inflation is a driving force behind output fluctuations, for example Canzoneri and Henderson (1988) and Obstfeld and Rogoff (1995).³

$$y_t = \beta_t(\pi_t - \pi_t^*) + z_t + \varepsilon_t \quad (2.1)$$

$$y_t^* = \beta_t(\pi_t^* - \pi_t) + z_t^* + \varepsilon_t^* \quad (2.2)$$

To introduce a role for learning in the model, we assume that the Phillips curve parameter β_t cannot be observed directly by either the home or foreign central bank. For simplicity of the learning process, β_t is restricted to be either high or low (β_H or β_L), with switches occurring according to a two-state Markov process. In other words, the economy switches between periods in which the Phillips curve parameter is high or low. The conditional probabilities of the parameter not switching, ie $\rho_H = P(\beta_{t+1} = \beta_H | \beta_t = \beta_H)$ and $\rho_L = P(\beta_{t+1} = \beta_L | \beta_t = \beta_L)$, are treated as exogenous. Due to the symmetry assumed in the model, the Phillips curve parameter switches identically and simultaneously in both the home and foreign countries.

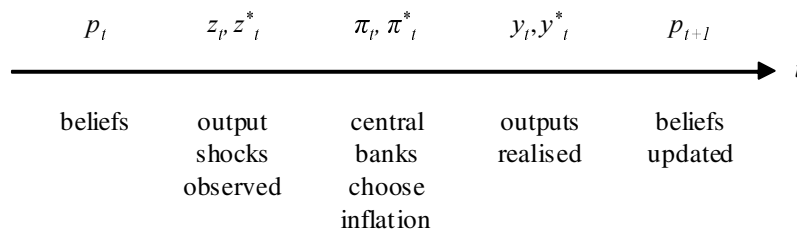


Figure 1: Timing of the model

³Our model is basically a reduced form for the output gap in a model of two large open economies under endogenous real exchange rate fluctuations.

The timing of the model is shown in Figure 1. The observable output shocks are revealed to both the home and foreign central banks at the beginning of the period. The two central banks then set inflation, the instrument of monetary policy.⁴ Activism or caution is reflected in the degree to which the central banks react to the observable output shocks. The final realisations of output are revealed at the end of the period. There is no asymmetric information in the model since each central bank always knows both observable output shocks.

2.2 Central bank loss function

The loss function of each central bank is assumed to be quadratic in the deviations of its own output and inflation from target. We assume that the targets are set consistent with the natural rate of output so they can be normalised to zero and there is no inflation bias. In the terminology of Svensson (1999), each central bank has a flexible inflation target. Equation (2.3) shows the algebraic form of the home central bank loss function, with the parameter χ measuring the relative weight of home inflation and output.

$$\mathcal{L}_t = y_t^2 + \chi\pi_t^2 \tag{2.3}$$

The loss function of the foreign central bank is given analogously in terms of foreign output and inflation by equation (2.4). Both central banks care about inflation and output with the same relative weight.

$$\mathcal{L}_t^* = y_t^{*2} + \chi\pi_t^{*2} \tag{2.4}$$

2.3 Beliefs

The central banks cannot observe the Phillips curve parameter β_t directly and so form beliefs about its current value. The beliefs of the home central bank can be conveniently represented by a single variable, $p_t = P(\beta_t = \beta_H)$, the belief at time t that the Phillips curve parameter is high. If $p_t = 1$ the home central bank is certain that the parameter is high. Conversely, if $p_t = 0$ there is certainty that the parameter is low. The beliefs of the foreign central bank can similarly be summarised as $p_t^* = P(\beta_t = \beta_H)$. Since all information in the model is symmetric, the beliefs of the home and foreign central banks always coincide and $p_t = p_t^*$ for all t .

⁴Allowing the central banks complete control over inflation abstracts from uncertainty in the monetary transmission mechanism. Transmission uncertainty could be included in our model but would not change our main conclusions.

2.4 Learning

Beliefs are not static in the model but are updated as the central banks receive information about the current value of the parameter in the Phillips curve. In Figure 1, initial beliefs p_t are updated to p_{t+1} at the end of the period on the basis of the observable output shocks, central bank inflation choices, and realised outputs. This forms the link between policy activism and learning in the model. In general, an activist policy involves making inflation choices that are more informative, leading to faster learning and quicker updating of beliefs.

The central banks update their beliefs at the end of the period according to whether the outputs realised are more consistent with a high or low parameter in the Phillips curve. Equations (2.5) and (2.6) show the predicted distributions of output, conditional on prior information $\mathcal{I}_t \equiv (z_t, z_t^*, \pi_t, \pi_t^*)$ and the two possible values of the Phillips curve parameter, β_H and β_L . The central banks have to infer which of these two distributions has most likely generated y_t and y_t^* . The role for policy activism and strategic interaction in learning is apparent since it is the relative inflation differential, $\pi_t - \pi_t^*$, that determines how the distributions (2.5) and (2.6) differ and how easy it is for the central bank to learn.

$$\begin{pmatrix} y_t \\ y_t^* \end{pmatrix} \Big|_{\mathcal{I}_t, \beta_t = \beta_H} \sim N \left[\begin{pmatrix} \beta_H(\pi_t - \pi_t^*) + z_t \\ \beta_H(\pi_t^* - \pi_t) + z_t^* \end{pmatrix}; \Sigma \right] \quad (2.5)$$

$$\begin{pmatrix} y_t \\ y_t^* \end{pmatrix} \Big|_{\mathcal{I}_t, \beta_t = \beta_L} \sim N \left[\begin{pmatrix} \beta_L(\pi_t - \pi_t^*) + z_t \\ \beta_L(\pi_t^* - \pi_t) + z_t^* \end{pmatrix}; \Sigma \right] \quad (2.6)$$

A simple application of Bayes rule solves the inference problem of the home central bank. Equation (2.7) shows how initial beliefs of the home central bank, p_t , are updated to p_t^+ at the end of the period on the basis of the new information contained in the observed output shocks, inflation choices and realised outputs. Under such Bayesian learning, p_t^+ depends on the relative probability of observing the outcome $\mathbf{y}_t \equiv (y_t, y_t^*)'$ under high and low values of the Phillips curve parameter.

$$p_t^+ = \frac{p_t P(\mathbf{y}_t | \mathcal{I}_t, \beta_t = \beta_H)}{p_t P(\mathbf{y}_t | \mathcal{I}_t, \beta_t = \beta_H) + (1 - p_t) P(\mathbf{y}_t | \mathcal{I}_t, \beta_t = \beta_L)} \quad (2.7)$$

p_t^+ is the optimal inference for the home central bank of the current value of the Phillips curve parameter, given the observed output shocks, inflation choices and realised outputs. The home central bank is consequently able to make a prediction p_{t+1} of whether the parameter will be high in the next period, taking into account the possibility that the parameter may shift before then. In equation (2.8), the prediction is calculated as a weighted average of the probability of keeping a high value and the probability of switching back from a low to a high value.

$$p_{t+1} = p_t^+ \rho_H + (1 - p_t^+) (1 - \rho_L) \quad (2.8)$$

Equations (2.7) and (2.8), when combined with the conditional distributions (2.5) and (2.6) for \mathbf{y}_t , define a non-linear equation (2.9) for updating the beliefs of the home central bank. Updated beliefs are a function of current beliefs, observed output shocks, relative inflation choices and the realised outputs. $\mathcal{B}(\cdot)$ represents the Bayesian operator modified to take into account Markov-switching effects.

$$p_{t+1} = \mathcal{B}(p_t, z_t, z_t^*, \pi_t - \pi_t^*, y_t, y_t^*) \quad (2.9)$$

The symmetric nature of information in the model means that the beliefs of the foreign central bank, p_t^* , are updated using exactly the same information and Bayesian formula (2.9) as the home central bank. In the model, there is always joint learning and the beliefs of the home and foreign central banks are updated simultaneously and identically. With such joint learning, $dp_t = dp_t^*$ for all t .

2.5 Equilibrium

We assume that the home and foreign central banks play a non-cooperative Nash game, in which each central bank takes the actions of the other central bank as given. Both central banks therefore follow ‘beggar thy neighbour’ policies. In equilibrium, each central bank chooses inflation in its own country, taking inflation in the other country as given.

3 Calibration

The model is calibrated to match monthly data, reflecting the frequency with which monetary policy decisions are made. Table 1 shows our baseline calibration. The first four parameters are taken from the empirical estimation of a similar closed economy model by Ellison and Valla (2001). Whilst not directly applicable to our open economy model, we use their estimates as our baseline calibration and test the sensitivity of our results to alternative calibrations. The calibrated values of $\beta_H = 3$ and $\beta_L = 0.5$ imply that a 0.1 percentage point inflation differential leads to monthly output 0.3% or 0.05% above trend, depending on the Phillips curve parameter. In other words, the effect on output of a given inflation differential is six times higher when β_t is high than when it is low. The process determining the switching of the Phillips curve parameter is calibrated as symmetric, with a persistence parameter of $\rho = 0.975$ corresponding to an average duration between switches of $1/(1 - 0.975) = 40$ months.

Parameter	Value
β_H	3
β_L	0.5
ρ_H	0.975
ρ_L	0.975
Ω	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Σ	$\begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}$
χ	9
δ	0.997

Table 1: Baseline calibrated values

The remaining four parameters in Table 1 cannot be estimated directly from the data. For the baseline calibration, we normalise the variance-covariance matrix of the observable output shocks, Ω , to be the identity matrix. This implies there are no common observable output shocks and the variance of idiosyncratic observable output shocks is the same in both countries. The variance-covariance matrix of the unobservable output shocks, Σ , is calibrated so that the ratio of unobservable to observable output shocks in each country is 0.3. In addition, there is no correlation between the unobservable home and foreign output shocks. χ reflects the relative weight that each central bank places on inflation and output deviations from target. It is calibrated for monthly data to give equal weight to inflation and output deviations at the quarterly frequency. The choice of the discount factor, δ , gives a quarterly discount rate of 1%.

4 Results

To analyse the incentives for activist or cautious monetary policy in the model, we derive the policy of the home central bank under two alternative assumptions about how learning issues are taken into account. With the *passive learning* policy, both central banks learn but neither consciously attempts to influence the speed of learning by adjusting the degree of activism or caution in policy. This policy forms our baseline case since, although each central bank is learning, neither takes into account that current actions affect learning. In contrast, under the *active learning* policy, the home central bank does internalise the consequences of its actions for learning.⁵ From the viewpoint of the home central bank, we refer to this as the optimal policy.⁶

⁵In the active learning case, we assume that the foreign central bank continues to follow a passive learning policy.

⁶The policy is optimal for the home central bank in the absence of a commitment technology by which the central banks can coordinate their actions.

4.1 Passive learning policy

In the passive learning policy, each central bank optimally accounts for current uncertainty but fails to realise that current policy actions also affect expected future losses. Learning is ignored. Since learning is the only source of dynamics in the model, the problem of the home central bank reduces to that of minimising the expected one-period loss function each period, subject to foreign inflation, the observable home output shock and the Phillips curve. The loss minimisation problem of the home central bank is given in (4.1).

$$\begin{aligned} \min_{\pi_t} E_t(\mathcal{L}_t(y_t, \pi_t) | \pi_t^*, z_t, p_t) \\ \text{s.t. } y_t = \beta_t(\pi_t - \pi_t^*) + z_t + \varepsilon_t \end{aligned} \quad (4.1)$$

By substituting the Phillips curve into the loss function, the problem can be written as equation (4.2). The expected one-period loss is a weighted average of the expected losses conditional on the true value of the Phillips curve parameter, with the weights being the beliefs of the home central bank.

$$\min_{\pi_t} \left\{ \begin{aligned} & p_t E_t \left[(\beta_H(\pi_t - \pi_t^*) + z_t + \varepsilon_t)^2 + \chi \pi_t^2 | \pi_t^*, z_t \right] \\ & + (1 - p_t) E_t \left[(\beta_L(\pi_t - \pi_t^*) + z_t + \varepsilon_t)^2 + \chi \pi_t^2 | \pi_t^*, z_t \right] \end{aligned} \right\} \quad (4.2)$$

Solving the first order condition for loss minimisation, the passive learning policy of the home central bank policy is given by equation (4.3).

$$\pi_t = \frac{p_t \beta_H^2 + (1 - p_t) \beta_L^2}{p_t \beta_H^2 + (1 - p_t) \beta_L^2 + \chi} \pi_t^* - \frac{p_t \beta_H + (1 - p_t) \beta_L}{p_t \beta_H^2 + (1 - p_t) \beta_L^2 + \chi} z_t \quad (4.3)$$

Equation (4.3) shows that, under the passive learning policy, home inflation reacts linearly to foreign inflation and the observable home output shock. The extent to which the home central bank reacts depends on its beliefs, p_t , and the distaste-for-inflation parameter, χ . In general, the home central bank is more activist in responding if the distaste for inflation is low. An identical derivation defines the policy followed by the foreign central bank under passive learning. In equation (4.4), foreign inflation is a linear reaction to home inflation and the observable foreign output shock. In this case, the size of the reaction depends on the beliefs of the foreign central bank, p_t^* .

$$\pi_t^* = \frac{p_t^* \beta_H^2 + (1 - p_t^*) \beta_L^2}{p_t^* \beta_H^2 + (1 - p_t^*) \beta_L^2 + \chi} \pi_t - \frac{p_t^* \beta_H + (1 - p_t^*) \beta_L}{p_t^* \beta_H^2 + (1 - p_t^*) \beta_L^2 + \chi} z_t^* \quad (4.4)$$

Equations (4.3) and (4.4) simultaneously define non-cooperative Nash equilibrium in the model.

Figure 2 shows numerically the equilibrium inflation choices made by the passive learning home central bank in the calibrated economy, as a function of prior beliefs, p_t , and the observable home output shock, z_t . The upper line is the response to a one standard deviation negative shock, $z_t = -1$, and the lower line is for a one standard deviation positive shock, $z_t = +1$. The vertical

distance between the upper and lower lines measures the extent to which the home central bank reacts to the observable home output shock for a given prior belief. The figure shows that changes in beliefs have a significant effect on the extent to which the home central bank reacts to the observable home output shock. An increased belief that the Phillips curve parameter is high is associated with a stronger reaction.

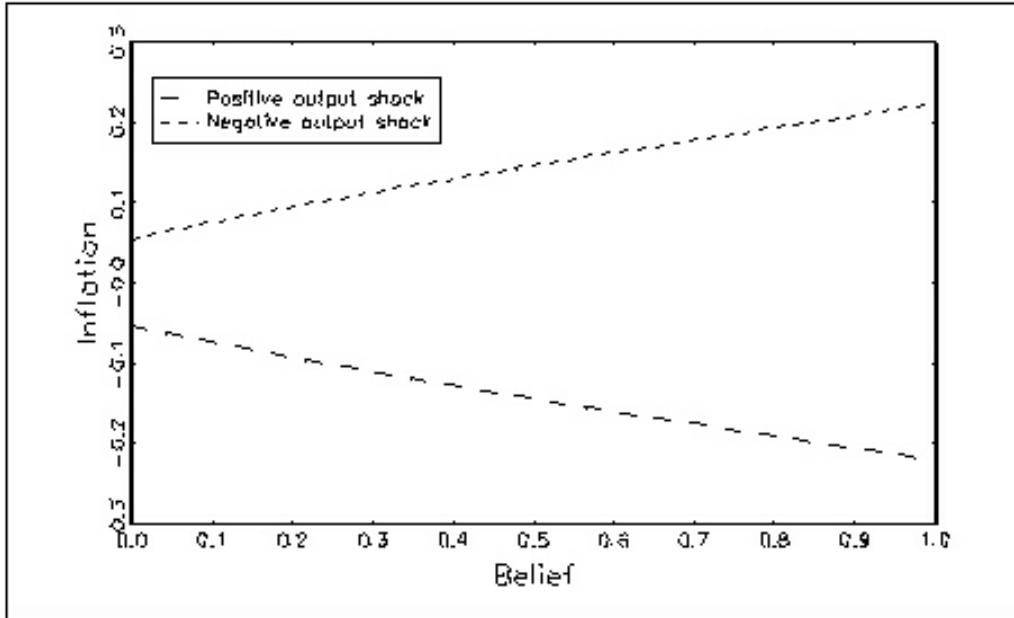


Figure 2: Inflation choices under the passive learning policy

Although learning issues are ignored under the passive learning policy, there will still be learning in the economy. As time passes, both central banks receive information which helps them to infer the current value of the Phillips curve parameter. The central banks update their beliefs using the learning procedure discussed in Section 2.4. Since equation (2.9) for the Bayesian updating of beliefs is non-linear, simulations are needed to gain insights into the dynamic behaviour of the economy. Table 2 shows some stylised facts for the calibrated economy, based on 100 simulations of 250 periods each. In the table, σ^2 denotes variance. We report the variance of quarterly inflation to maintain comparability with the output variance figures.⁷

⁷Monthly inflation figures are by definition small so the variance of monthly inflation is much less than the variance of monthly output. Quarterly inflation and output have approximately the same variance and so more comparable.

Passive learning policy		
Home inflation	σ_{π}^2	0.254
Foreign inflation	$\sigma_{\pi^*}^2$	0.253
Home output	σ_y^2	0.746
Foreign output	$\sigma_{y^*}^2$	0.747
Inflation differential	$\sigma_{\pi-\pi^*}^2$	0.154
Beliefs	σ_p^2	0.120
Welfare loss	\mathcal{L}	1.000

Table 2: Stylised facts of the passive learning policy

The dynamic simulations show the symmetric nature of non-cooperative equilibrium in the model. Inflation and output are equally volatile in both the home and foreign countries. Since there is always joint learning in the model, we only report the variance of the beliefs of the home central bank. The final row of Table 2 shows the average per-period welfare loss of the home central bank, calculated according to equation (2.3).

4.2 Active learning policy

The academic arguments of Bertocchi and Spagat (1993) and Beck and Wieland (2002) suggest that the passive learning policy is not optimal because it does not internalise the benefits of learning. They argue that central bank policy should be more activist in its response to the observed output shocks because this provides valuable information about the state of the economy. By increased activism, the central bank learns quicker about the economy and so can stabilise the economy better in the face of future observable output shocks.

To assess this argument in our open economy model, we calculate the active learning policy followed by the home central bank when it takes all the costs and benefits of learning into account, and then ask whether it is more activist or cautious than the passive learning policy. We assume throughout that the foreign central bank continues to follow a passive learning policy. The active learning problem of the home central bank solves the dynamic loss-minimisation problem (4.5), in which the home central bank minimises the net present value of expected losses, subject to four constraints. The first two constraints are the Phillips curve and the updating equation (2.9) for the beliefs of the home central bank. The third constraint recognises that all learning is joint with the foreign central bank so future beliefs of the home and foreign central banks will coincide. The fourth constraint is that the foreign central bank follows the passive learning policy (4.4) in the future. The minimisation problem is intertemporal because future beliefs depend on current actions.

$$\begin{aligned}
& \min_{\{\pi_t\}} E_t \sum_{n=0}^{\infty} \delta^n \left[\mathcal{L}_{t+n}(y_{t+n}, \pi_{t+n} | \pi_t^*, z_t, p_t) \right] \\
& \text{s.t.} \\
& y_t = \beta_t(\pi_t - \pi_t^*) + z_t + \varepsilon_t \\
& p_{t+1} = \mathcal{B}(p_t, \pi_t - \pi_t^*, z_t, z_t^*, y_t, y_t^*) \\
& p_{t+1}^* = p_{t+1} \\
& \pi_{t+1}^* = \frac{p_{t+1}^* \beta_H^2 + (1-p_{t+1}^*) \beta_L^2}{p_{t+1}^* \beta_H^2 + (1-p_{t+1}^*) \beta_L^2 + \chi} \pi_{t+1} - \frac{p_{t+1}^* \beta_H + (1-p_{t+1}^*) \beta_L}{p_{t+1}^* \beta_H^2 + (1-p_{t+1}^*) \beta_L^2 + \chi} z_{t+1}^*
\end{aligned} \tag{4.5}$$

This problem has a recursive nature so the active learning policy must satisfy the Bellman equation (4.6).

$$V(p_t, z_t, \pi_t^*) = \min_{\pi_t} E_t \left[\mathcal{L}(p_t, z_t, \pi_t^*, \pi_t) + \delta V(p_{t+1}, z_{t+1}, \pi_{t+1}^*) \right] \tag{4.6}$$

It is not possible to derive a closed-form solution to this problem because of the non-linearity in the equation for updating beliefs. However, Wieland (2000a) shows that standard dynamic programming algorithms can be applied to obtain a numerical approximation to the Bellman equation and the active learning policy.⁸

Figure 3 shows the policies followed by the home central bank under passive and active learning. As the figure demonstrates, there is a marked difference in the degree of activism between the two policies. In contrast to the passive learning policy, the active learning policy involves less reaction to the observable home output shock. In other words, the home central bank's reaction to the observable home output shock is dampened. The active learning policy is more cautious than the passive learning policy.

⁸Blackwell's sufficiency conditions are satisfied for this class of problems, see Kiefer and Nyarko (1989), so it is possible to define a contraction mapping that converges to a unique fixed point. Repeated iterations over the Bellman equation will therefore converge to the stationary optimal policy and value function. More details about this solution technique are given in Appendix A.

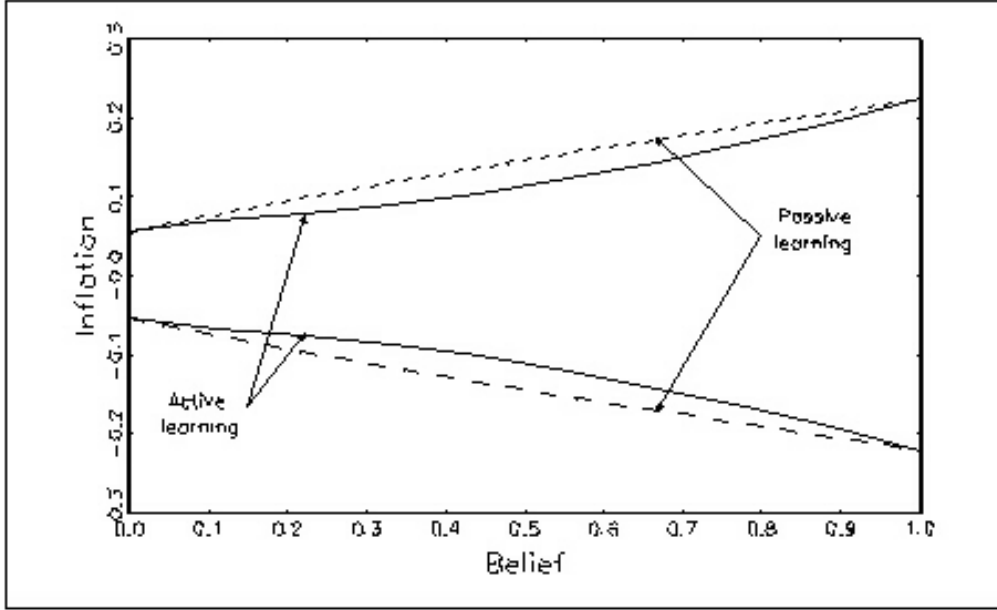


Figure 3: Inflation choices under the active learning policy

The dynamic properties of the active and passive learning policies are compared in Table 3. Under the active learning policy, the volatility of home inflation naturally falls as the home central bank becomes more cautious. As policy is more cautious, the inflation choices of the central banks are less informative and beliefs are updated more slowly. Indeed, the variance of beliefs drops by approximately 13%.

		Passive learning policy	Active learning policy
Home inflation	σ_{π}^2	0.254	0.223
Foreign inflation	$\sigma_{\pi^*}^2$	0.253	0.253
Home output	σ_y^2	0.746	0.772
Foreign output	$\sigma_{y^*}^2$	0.747	0.770
Inflation differential	$\sigma_{\pi-\pi^*}^2$	0.154	0.121
Beliefs	σ_p^2	0.120	0.104
Welfare loss	\mathcal{L}	1.000	0.995

Table 3: Stylised facts of the active learning policy

The decreased volatility of beliefs has a positive effect on the welfare of the home central bank. It therefore appears that there are no incentives for activist policy in our open economy model. On the contrary, active learning policy that internalises the costs and benefits of learning is more cautious than the passive learning policy.

4.3 Sensitivity analysis

To establish the robustness of our result, we show that the active learning policy is more cautious than the passive learning policy for a wide range of calibrated parameter values. We do this by examining whether taking learning into account creates an incentive to decrease or increase activism relative to the passive learning policy. If the incentive is to decrease activism then the active learning policy will be more cautious than the passive learning policy.

The incentive to decrease or increase activism in turn depends on the convexity or concavity of the expected central bank loss function with respect to beliefs when the home and foreign central banks follow the passive learning policies.⁹ If the expected loss function is convex with respect to beliefs then the central bank strictly prefers a cautious policy that does not reduce uncertainty. For example, if the expected loss function is convex then the central bank *ex ante* prefers uncertainty (eg $p_t = 0.5$) to certainty (ie, $p_t = 0$ or $p_t = 1$) about the parameter β_t . If the expected loss function is concave with respect to beliefs then the opposite is true and the central bank strictly prefers an activist policy which decreases uncertainty.

The expected central bank loss function can be written as a function, $\mathcal{L}^e(p_t, p_t^*)$, of the beliefs of the home and foreign central banks by substituting the passive learning policies (4.3) and (4.4) into the objective function (2.3) and taking expectations. Its convexity or concavity depends on the sign of its second derivative with respect to beliefs. Total differentiation of the expected loss function gives equation (4.7), in which the time subscripts on beliefs have been dropped for ease of notation.

$$d^2 \mathcal{L}^e(p_t, p_t^*) = \mathcal{L}_{pp} dp^2 + \mathcal{L}_{p^*p^*} dp^{*2} + 2\mathcal{L}_{pp^*} dp dp^* \quad (4.7)$$

Due to the symmetric nature of the model there is always joint learning of the home and foreign central banks, so $dp_t = dp_t^*$ for all t . We therefore evaluate the convexity or concavity of the expected loss function by looking at the sign of the second derivative along the path of joint learning. Equation (4.8) shows that this second derivative has three components. The first, \mathcal{L}_{pp} , is the second partial derivative with respect to the beliefs of the home central bank. The second, $\mathcal{L}_{p^*p^*}$, is the second partial derivative with respect to the beliefs of the foreign central bank. The third, $2\mathcal{L}_{pp^*}$, depends on the cross partial derivative with respect to the joint beliefs of both the home and foreign central banks.

$$\left. \frac{d^2 \mathcal{L}^e(p_t, p_t^*)}{dp^2} \right|_{dp=dp^*} = \mathcal{L}_{pp} + \mathcal{L}_{p^*p^*} + 2\mathcal{L}_{pp^*} \quad (4.8)$$

The components of the second derivative of the expected loss function for the baseline model, evaluated at $p_t = p_t^* = 0.5$, are shown in Table 4. The expected central bank loss function is actually concave with respect to the beliefs of both the home and foreign central banks since \mathcal{L}_{pp} and $\mathcal{L}_{p^*p^*}$ are both negative. If

⁹See Ellison and Vilmunen (2002) for more details of this approach to calculating the incentives for policy activism under learning.

the home central bank were able to learn in isolation then it would *ex ante* strictly prefer the certainty of knowing the parameter β_t to the uncertainty represented by $p_t = 0.5$. Similarly, if the foreign central bank were able to learn in isolation then the home central bank would be better off. Hence, if we consider the learning of the home and foreign central banks in isolation there will be an incentive for increased activism to reduce uncertainty. However, the home and foreign central banks do not learn in isolation in our model. There is always joint learning so we also have to consider whether the expected loss function is convex or concave with respect to the joint beliefs of the home and foreign central banks. Since \mathcal{L}_{pp^*} is positive, we see that the expected loss function turns out to be convex in the joint beliefs and therefore there is an incentive for more caution in monetary policy. This effect dominates the effects of learning in isolation so overall the active learning policy is cautious. The strategic interaction in our model is so strong that even though learning in isolation is beneficial, the costs of joint learning are sufficiently high that the active learning policy is more cautious than the passive learning policy.

β_H	β_L	\mathcal{L}_{pp}	$\mathcal{L}_{p^*p^*}$	\mathcal{L}_{pp^*}	$\left. \frac{d^2 \mathcal{L}^e}{dp^2} \right _{dp=dp^*}$	<i>Active learning policy</i>
3	0.5	-0.37	-0.15	0.60	0.68	cautious

Table 4: Activism of monetary policy in the baseline calibration

Table 5 reports whether the active learning policy is more activist or cautious than the passive learning policy under a wide range of alternative calibrations for the parameters β_H and β_L . In all cases, the value of \mathcal{L}_{pp} is positive and \mathcal{L}_{pp^*} is negative so learning of the home central bank in isolation is beneficial but joint learning by both central banks is costly. The latter effect dominates for all but one of the calibrations and so the active learning policy is typically more cautious than the passive learning policy. Only if β_H and β_L are small or of very similar magnitude is it possible for the active learning policy to be more activist than its passive learning counterpart. Since these cases are of limited practical interest, we conclude that our result is robust to alternative calibrations.

β_H	β_L	\mathcal{L}_{pp}	$\mathcal{L}_{p^*p^*}$	\mathcal{L}_{pp^*}	$\frac{d^2 \mathcal{L}^e}{dp^2} \Big _{dp=dp^*}$	<i>Active learning policy</i>
6	0	-1.16	-0.85	3.00	3.98	cautious
6	1.5	-0.93	-1.09	2.10	2.18	cautious
6	3	-0.48	-0.64	1.00	0.87	cautious
6	4.5	-0.13	-0.17	0.25	0.20	cautious
3	0	-0.55	-0.05	0.70	0.81	cautious
3	0.75	-0.30	-0.17	0.52	0.57	cautious
3	1.5	-0.13	-0.13	0.26	0.25	cautious
3	2.25	-0.03	-0.04	0.07	0.06	cautious
1.5	0	-0.30	0.02	0.11	-0.06	activist
1.5	0.375	-0.15	0.00	0.09	0.03	cautious
1.5	0.75	-0.06	-0.00	0.05	0.04	cautious
1.5	1.125	-0.01	-0.00	0.01	0.01	cautious

Table 5: Activism of monetary policy for alternative calibrations

5 Conclusions

At the beginning of this paper, we highlighted the controversy that exists between academics and central bankers about whether monetary policy should be activist or cautious. Our results support the claim of the central bankers. In an open economy with two countries playing a non-cooperative game, caution prevails. Monetary policy that internalises the costs and benefits of learning is actually even more cautious than if learning is ignored. Our result is robust to a wide range of calibrations.

Our conclusion differs from the results of Bertocchi and Spagat (1993) and Beck and Wieland (2002) due to the presence of the foreign country in the open economy. There is a strategic interaction between the two central banks so the home central bank cannot consider its learning in isolation – it has to recognise that the foreign central bank is also learning in tandem. *Joint* learning of both central banks is costly in our model because they are effectively learning to play a ‘beggar thy neighbour’ non-cooperative Nash game, in which they both lose out. The cautious nature of the active learning policy is aimed at delaying the two central banks learning how to play this game.

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A Appendix

Numerical approximation of the active learning policy

The approximation to the active learning policy is obtained by solving the Bellman equation (4.6) numerically. This requires expressions for the expected one-period loss, $E_t \mathcal{L}_t$, and the expected continuation value, $E_t V_{t+1}$, for an inflation choice π_t . The one-period loss is given by equation (2.3) and the expected continuation value is given by equation (A.1), where future beliefs p_{t+1} have been substituted out using the non-linear updating equation (2.9).

$$E_t V_{t+1} = E_t V(\mathcal{B}(p_t, z_t, z_t^*, \pi_t - \pi_t^*, y_t, y_t^*), z_{t+1}, \pi_{t+1}^*) \quad (\text{A.1})$$

The expectation in equation (A.1) is evaluated by the home central bank before the realisation of current output, y_t and y_t^* , and the observable output shocks for next period, z_{t+1} and z_{t+1}^* . The home central bank therefore evaluates the quadruple integral in equation (A.2).

$$E_t V_{t+1} = \iiint \int V(\mathcal{B}(p_t, z_t, z_t^*, \pi_t - \pi_t^*, y_t, y_t^*), z_{t+1}, \pi_{t+1}^*) f(y_t | \pi_t^*, z_t, p_t, \pi_t) f(y_t^* | \pi_t, z_t^*, p_t, \pi_t^*) f(z_{t+1}) f(z_{t+1}^*) dy_t dy_t^* dz_{t+1} dz_{t+1}^* \quad (\text{A.2})$$

$f(z_{t+1})$ and $f(z_{t+1}^*)$ are the distributions of z_{t+1} and z_{t+1}^* . $f(y_t | \pi_t^*, z_t, p_t, \pi_t)$ and $f(y_t^* | \pi_t, z_t^*, p_t, \pi_t^*)$ are the predictive distributions of y_t and y_t^* . They have independent distributions, a normal and mix of normals respectively, as described respectively by equations (A.3) and (A.4).

$$\begin{aligned} f(z_{t+1}) &= N[0; \Omega_{11}] \\ f(z_{t+1}^*) &= N[0; \Omega_{22}] \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} f \left(\begin{array}{c} y_t \\ y_t^* \end{array} \middle| \begin{array}{c} \pi_t^*, z_t, p_t, \pi_t \end{array} \right) &= p_t N \left[\begin{array}{c} \beta_H(\pi_t - \pi_t^*) + z_t \\ \beta_H(\pi_t^* - \pi_t) + z_t^* \end{array}; \Sigma \right] \\ &+ (1 - p_t) N \left[\begin{array}{c} \beta_L(\pi_t - \pi_t^*) + z_t \\ \beta_L(\pi_t^* - \pi_t) + z_t^* \end{array}; \Sigma \right] \end{aligned} \quad (\text{A.4})$$

The computational algorithm starts by defining a grid of points in the state space (p_t, z_t, z_t^*) . The grid points for beliefs, p_t , are distributed uniformly across the interval $[0,1]$, but grid points for the observable output shocks z_t and z_t^* are bunched around zero according to a cosine weighting function to increase accuracy. For each gridpoint, we assign an inflation choice for the home central bank, π_t , and an initial value for the value function, V_t , using the inflation choices of the passive learning central bank (4.3) and the expected one-period loss (2.3) as starting values.

An iteration of the Bellman equation involves passing through the grid point by point. For each gridpoint, the inflation choice is reoptimised by minimising the right hand side of the Bellman equation (4.6), using

equation (2.3) for the expected one-period loss and equations (A.2) to (A.4) for the expected continuation value. Numerical evaluation of the expected continuation value requires Gaussian Quadrature methods and linear interpolation of adjacent gridpoints to evaluate the quadruple integral in (A.2). After each new inflation choice has been calculated, new values of π_t and V_t are assigned to the gridpoint. Each iteration of the Bellman equation is complete when the inflation choice and value function have been updated for each gridpoint.

Repeated application of the iterative procedure outlined above converges to the active learning policy. Using a grid of $10 \times 10 \times 10$ points, we accept convergence when the values associated with each gridpoint change by less than 0.0001 between successive iterations. When optimising the inflation choice at each gridpoint we use a convergence tolerance of 0.00001. 32 ordinates were used in the Gaussian Quadrature approximation to equation (A.2).

To calculate the non-cooperative Nash equilibrium we use a simple iterative algorithm. In the first stage, the inflation choices of the central bank are calculated for the given inflation choices of the foreign central bank, using the procedure described above. In the next stage, the inflation choices of the foreign central bank are updated according to the passive learning policy (4.4). These foreign inflation choices are then used as the basis for calculating the new inflation choices of the home central bank. This procedure is iterated until convergence to non-cooperative equilibrium is achieved. Convergence is accepted when the change in the passive learning policy of the foreign central bank is less than 0.000001 for each gridpoint.

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