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Simulation-based stress testing of banks' regulatory capital adequacy

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The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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Abstract

Banks' holding of reasonable capital buffers in excess of minimum requirements could alleviate the procyclicality problem potentially exacerbated by the rating-sensitive capital charges of Basel II. Determining the required buffer size is an important risk management issue for banks, which the Basle Committee (2002) suggests should be approached via stress testing. We present here a simulation-based approach to stress testing of capital adequacy where rating transitions are conditioned on business-cycle phase and business-cycle dynamics are taken into account. Our approach is an extension of the standard credit portfolio analysis in that we simulate actual bank capital and minimum capital requirements simultaneously. Actual bank capital (absent mark-to-market accounting) is driven by bank income and default losses, whereas capital requirements within the Basel II framework are driven by rating transitions. The joint dynamics of these determine the necessary capital buffers, given bank management's specified confidence level for capital adequacy. We provide a tentative calibration of this confidence level to data on actual bank capital ratios, which enables a ceteris-paribus extrapolation of bank capital under the current regime to bank capital under Basel II.

Key words: Basel II, Pillar 2, bank capital, stress tests, procyclicality

JEL classification numbers: G21, G32

Pankkien pääomapuskurien analyysi uusilla vakavaraisuussäännöksillä

Suomen Pankin keskustelualoitteita 4/2003

Samu Peura – Esa Jokivuolle
Rahoitusmarkkinaosasto

Tiivistelmä

Vakavaraisuussääntelyn uudistus, jolla pyritään sitomaan pankkien minimipääomavaatimukset saamisten luottokelpoisuusluokituksiin, saattaa voimistaa talouden suhdannevaihteluita. Baselin komitea (2002) on ehdottanut, että tämän vaikutuksen hillitsemiseksi pankkien tulisi pitää ylimääräisiä pääomapuskureita. Ne tulisi mitoittaa minimipääomavaatimuksia koskevien stressitestien avulla. Tämä saattaa edellyttää pankeilta niiden käyttämien riskienhallintamallien laajentamista.

Tässä työssä esitetään simulointiin perustuva vakavaraisuuden stressitestikehikko, jossa otetaan huomioon suhdannevaihtelut ja suhdannetilan vaikutus luottokelpoisuusluokitusten muutostodennäköisyyksiin. Työssä laajennetaan tyypillistä luottoportfoliomallia simuloiden samanaikaisesti pankin kirjanpidollisen pääoman ja minimivakavaraisuusvaatimusten kehitystä. Pääoman määrään vaikuttavat marginaalituotot ja luottotappiot, kun taas vakavaraisuusvaatimusten muutokset aiheutuvat luottokelpoisuusluokitusten muutoksista. Tarvittavan ylimääräisen pääomapuskurin koko määräytyy näiden yhteisdynamiikan sekä valitun tilastollisen luottamustason perusteella. Tämä luottamustaso on pyritty kalibroimaan istuttamalla malli nykyisiin vakavaraisuusvaatimuksiin ja yhdysvaltalaisten pankkien viime vuosien keskimääräiseen pääomasuhteeseen. Kalibrointi mahdollistaa myös alustavan ceteris paribus -arvion siitä, miten pankkien pääomien määrä saattaisi kehittyä Baselin komitean ehdottamien uudistusten myötä.

Avainsanat: vakavaraisuusuudistus, pilari II, stresstitestit, myötäsyklisyys

JEL-luokittelu: G21, G32

Contents

1 Introduction

The macroeconomic consequences of rating sensitive capital requirements have been debated actively during the consultation process on the new Basel Capital Accord. The critics argue that risk sensitive capital rules may amplify the natural procyclicality in banking in that they force banks to significantly cut back lending in recessions (see the views expressed eg by Danielsson et al, 2001, and Erwin and Wilde, 2001). This concern is to some extent valid under the current Basel capital regime since credit losses consume bank capital in downturns, but the likely procyclical consequences of minimum capital constraints are supposed to be much more pronounced under risk sensitive capital regimes such as the proposed Basel II. Hence risk sensitive capital requirements are thought to trade off fairness in capital allocation across banks against macroeconomic stability.

Determining the macroeconomic consequences of a given minimum capital requirement regime is a complex task which calls for an understanding of how individual banks would react under a given regime. In particular, does it automatically follow from the fact that the minimum capital requirement is more volatile that cyclical fluctuations will be re-enforced? After all, there are different measures that banks can take to be better prepared against shocks to capital, and the macroeconomic effects of the new capital regulation are likely to depend on the extent to which individual banks find it optimal to hedge, eg through holding extra capital or through risk shifting, against shocks to capital.

There is by now a lot of theoretical research at microeconomic level which shows that financially constrained firms optimally hold buffer stocks of assets to protect against the adverse consequences of running out of liquid assets or capital (eg Bhattacharya et al, 2002, Hojgaard and Taksar, 1999, Holt, 2003, Milne and Robertson, 1996, Milne and Whalley, 2001, and Peura, 2002). This research also shows that the precautionary capital stocks are the larger the more severe are the financial constraints and the more illiquid or costly to hedge are the firms' primary assets. Although these theories are highly stylised, it is quite natural to think of them as applying to banks, and some of the previously listed contributions have indeed been directly concerned with banks or insurance companies. A quick look at banks' actual capital ratios reveals that banks do hold buffer stocks of capital (see eg Section 2 in this paper). This could be due to behaviour as suggested by the above theories, since bank portfolios are quite illiquid, and banks already under the current Basel regime face significant costs of regulatory capital violation resulting from strengthened supervisory scrutiny and adverse market reactions. Ideally one would like to test these theories with actual data on bank portfolios, to see if they can explain the observed bank capital ratios.[1] This is not an easy task, however, because most of the existing theories are so stylised that it is not at all clear how to eg represent different banking portfolios in these theories. Also, the stylised models fail to incorporate the essential structure of the current versus the proposed new minimum capital regulation.

Yet in light of this theoretical knowledge, it is no surprise that several authors have suggested that the procyclicality problem with risk sensitive capital requirements be remedied through adjustment of banks' capital buffers (eg Borio et al, 2001, and Lowe, 2002). It appears that this argumentation has also been adopted by the Basel Committee itself.[2] The idea is that under 'normal' business conditions banks should hold capital over minimum requirements, while the extra capital would be (partially) 'consumed' during severe downturns through credit losses and through increases in minimum capital requirements. If the capital buffers were sufficient to outlast a downturn, lending would not have to be severely cut down, and hence there would be no credit crunch accelerating the downturn.

As to the size of banks' capital buffers, there are at least two perspectives to the issue: i) what is the optimal level of capital buffers from the macroeconomic point of view? and ii) how much capital would individual banks optimally hold under a given minimum capital regime? These are both very challenging questions to answer using fully specified optimization models, in particular if a realistic level of institutional detail is sought for. Therefore, rather than to give up realism in the description of minimum capital rules, we in this paper approach the latter question from a single bank's risk management view point. That is, we identify how much capital a bank needs (in a risk sensitive capital regime) in order to be protected, at a desired statistical confidence level, against shocks to its actual capital and to its minimum capital requirement. Hence we use a Value-at-Risk type criterion to determine the required capital buffer, which is applied to the distribution of the bank's buffer capital over a relevant future horizon. The confidence level used in the criterion can be interpreted to be determined from the trade-off that the bank faces between the costs from violating the minimum capital constraint and the costs of holding extra capital. We also calibrate our model to actual bank capital data through the selection of this confidence level. This allows for a ceteris paribus extrapolation of banks' capital buffers under the current Basel regime into banks' capital buffers under the proposed new regimes.

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[1] Furfine (2001) performs calibrations of this type. He also presents evidence that banks reacted to the current Basel Accord by increasing their capital ratios, which suggests that banks' holdings of buffer capital are not purely 'economic capital' (in the standard sense of the term economic capital), but a genuine response to minimum capital requirements.

[2] Basel Committee (2002) states that 'to help address potential concerns about the cyclicality of the IRB approaches, the Committee agreed that meaningfully conservative credit risk stress testing by banks should be a requirement under the IRB approaches as a means of ensuring that banks hold a sufficient capital buffer under Pillar Two of the new Accord'.

Our model of bank capital is an extension of a credit risk model (of the CreditMetrics^{TM}-type (J.P. Morgan, 1997)) which is used to quantify the distribution of shocks to bank capital. Because credit risk models treat the risks in bank portfolios at obligor level, they can accurately account for the detail of the minimum capital rules. Our extended risk model jointly simulates a bank's actual book capital and its minimum capital requirement. Actual capital (in the absence of mark-to-market accounting) is driven by default losses and bank income, whereas capital requirements are driven by ratings transitions. Hence a credit portfolio model which simulates rating changes and defaults, and incorporates the minimum capital formulas, is suitable for keeping track of the evolution of both the actual and the minimum capital. Required initial capital buffers, and the resulting capital ratios, are solved from a multi-period Value-at-Risk type criterion (extending the analysis of Jokivuolle and Peura, 2001), which incorporates a confidence level chosen by bank management. It is this confidence level through which our model is calibrated to actual bank capital ratios under the current Basel regime. Using this calibrated value in turn, we can look at the capital ratios that the model generates under the rating sensitive Internal Ratings Based (IRB) approach under Basel II.[3]

Another aspect in which we extend a typical credit portfolio model is that we employ an underlying conditioning variable which represents the state of the business cycle. The business cycle variable follows a two state time homogenous Markov process. The ratings transition probabilities that we use are then conditioned on the state of the business cycle. Our simulation is a multi-period one, and time is in quarterly increments. We use the conditional transition matrices as well as the transition probabilities for the business cycle variable reported by Bangia et al (2002). Our quarterly simulation period is non-standard in credit risk contexts, but is well grounded since most banks report their capital adequacy to their regulators quarterly. We use data on average bank portfolios in the US.

Basel (2002) has suggested using stress tests to identify the size of the capital buffers that banks would need under the new Basel regime. The suggestion contained no details on how stress testing should be done. While a typical stress test would be a deterministic move in the portfolio ratings distribution (corresponding to some historical period of credit distress, as eg in Erwin and Wilde, 2001, or Catarineu-Rabell et al, 2002), we feel that stress testing could also be approached using probabilistic risk models. A credit portfolio model generates adverse scenarios subject to prespecified probability laws estimated from long horizon data. If these probability laws are not deemed pessimistic enough, a credit portfolio model may be parameterized with historical transition probabilities and asset correlations corresponding to particularly severe macroeconomic downturns,

[3] The IRB approach is briefly described in Section 3.

in order to generate conditional simulations which can be interpreted as stress test. In effect, we suggest stress testing of credit risk through varying the key parameters in a stochastic simulation, rather than doing stress testing in a deterministic sense.[4] We find it most intuitive to formulate stress tests within our simulation framework around the transition probabilities of the business cycle variable. These control for the expected duration of recessions, and hence our stress tests have intuitive interpretations in terms of average recession lengths that our multi-period simulations correspond to.

The paper is organized as follows. Section 2 presents empirical evidence on bank capital ratios under the current Basel regime. Section 3 briefly summarizes the proposed Basel II approaches for setting minimum capital requirements on credit risk. Section 4 presents our capital simulation framework, extending the work presented in Jokivuolle and Peura (2001). Section 5 presents our multi-period ratings transition model with an underlying business cycle variable driven by a two state Markov process, and discusses the parameterization of the model. Section 6 contains our main numerical results on the behaviour of bank capital buffers. Section 7 concludes.

2 Bank capital ratios under the current Basel regime

Consistent with predictions of theories on capital constrained firms, banks do hold capital in excess of their minimum requirements. We provide in this section a brief summary on bank capital ratios under the current Basel regime. We use Bankscope data, and limit ourselves to large banks in G10 countries, defined as those banks which on average have Tier-1 capital in excess of 3 billion Euros over the period 1997–2001.[5] This sample contains 128 banks (also included in this set are banks which do not have data on all the five years). Summary statistics of the sample are presented in Table 1.

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[4] Deterministic stress tests are straightforward to implement on the market risk side. A typical example of a stress test would be: 'all stocks 10% down'. In the context of illiquid credit portfolios, where the main risk is that of defaults, the corresponding test would be a deterministic transition scenario, which stipulates transition and default frequencies to be applied to each rating category.

[5] We have restricted ourselves to this group of banks on two grounds. First, our bank portfolios, taken from Gordy (2000), are averages over large US banks. Second, the Basel Committee has looked at this group of banks separately from others when reporting the results of its Quantitative Impact Studies (QIS).

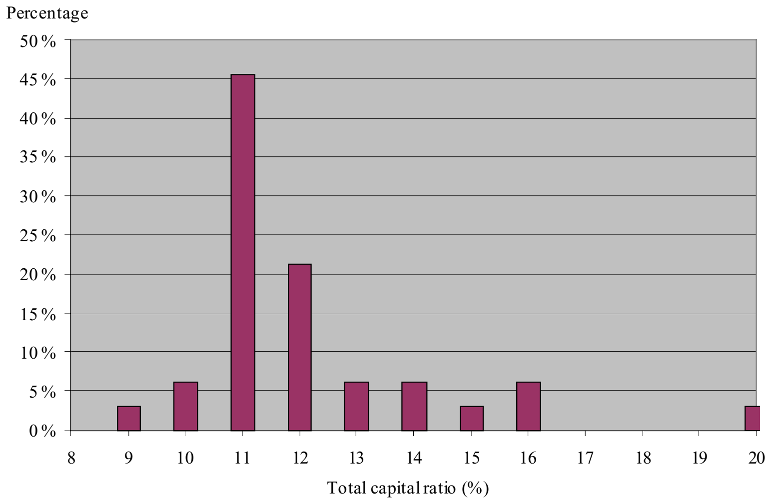
Table 1. **Data on large G10 banks' capital and portfolio 1997–2001**

		All	US	Europe	Japan
Total capital ratio (%)	Median	11.2	11.9	10.8	10.9
	StDev	2.4	2.2	2.9	1.3
Tier-1 capital ratio (%)	Median	7.3	8.6	7.4	5.9
	StDev	2.8	2.5	3.0	1.7
Loan portfolio (mrd Eur)	Median	87	33	91	117
	StDev	116	92	74	168
Loan portfolio / total assets (%)	Median	56	62	51	61
	StDev	15	19	13	10
Loan loss provision / loans (%)	Median	0.67	0.52	0.53	1.90
	StDev	0.91	0.53	0.35	1.15
Number of banks		128	33	57	33

We note that the median total capital ratio across the G10 banks is 11.2%. In the US, the median total capital ratio is 11.9%, while in Japan and Europe the median ratio is below 11%. The median Tier-1 capital ratio across the G10 banks is 7.3%. Again, median Tier-1 capital ratio in the US is higher than the G10 average, but in Japan it is considerably lower than the G10 average. The lower capital ratios of Japanese banks may be explained by the ongoing banking crises in the country, which has resulted in capital ratios falling below their optimal or target values. That Japanese banks indeed have lower quality portfolios than US or European banks is evident from the loan loss provision statistics in Table 1. These reveal that the median annual loan loss provision, as a percentage of the loan portfolio, over the 1997–2001 period has equalled 1.9% in Japan, compared to 0.52% and 0.53% in the US and Europe, respectively. Hence the difference between capital levels of US and Japanese banks could be explained by the fact that US and European banks are close to their target capitalizations, while Japanese banks are below their target capitalization levels. Moreover, since the US has been in a good economic phase over the 1997–2001 period, it appears reasonable to expect that the capital ratios of the US banks indeed are representative of the target levels chosen by the banks.

The evidence from Table 1 indicates that the median large bank holds over 11% of capital, which is over 3% in excess of the minimum requirement. There is some variation in capital ratios between banks, but no banks are close to the 8% minimum, and very few banks even have capital ratios under 10%. Figure 2 eg shows the distribution of capital ratios of the US banks in our sample. This sample contains 33 banks, and here we observe that only one bank has a total capital ratio less than 10%. Moreover, over 45% of the banks have total capital ratios between 11% and 12%. The distribution of capital ratios is somewhat asymmetric to the right, but there are only few banks with capital ratios over 13%. Hence one could argue that US banks are capitalized in a quite homogeneous manner. We think of these facts as supporting the stylized models on the behaviour of capital constrained firms. In particular, banks appear to hold considerable buffer capital in excess of their minimum requirements, and moreover, there seem to be a target level or band of capital over which most banks are located.

Figure 2. **Distribution of large US banks' average total capital ratios 1997–2001**



Source: Bankscope. The bar at 8 per cent eg indicates the percentage of banks that have average capital ratio between 8 and 9 per cent.

The parameter data and the portfolios on which our numerical analyses are based on are from the US. Therefore we base our calibrations in Section 6 on the median capital ratio of US banks shown in Table 1.

3 Basel II minimum capital requirements for credit risk

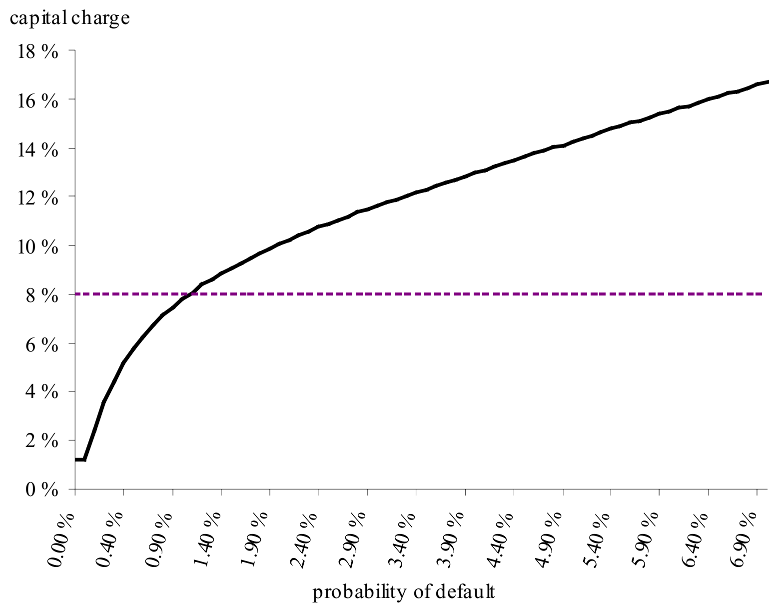
The current Basel Accord from 1988 stipulates a 4% Tier-1 capital charge and an 8% total capital charge on corporate exposures. Since it does not depend on counterparty credit quality, the capital charge in the current approach does not vary over time with counterparty ratings. A bank's capital charge only increases when new assets are purchased, and decreases as existing assets leave the portfolio either through expiry, asset sale, or default. When a write-off for credit losses is made, the credit loss is deducted from bank equity, but the written-off asset no more contributes to the capital charge. Hence a default, assuming a typical loss-given-default equal to 50%, is associated with a reduction in free bank capital equal to 42% (50%–8%), not 50%, of the face value of the defaulted asset. Also for this reason there is a negative correlation between risk-weighted assets and credit losses under the current Basel regime, given a fixed initial portfolio.

The proposed new Basel Accord (Basel Committee, 2002) offers two principal approaches for calculation of capital requirements. In the Standardized Approach the capital charge on corporate exposures depends on the issuer's external rating, should one exist. Issuers which do not have public ratings receive a 100% risk weight as in the current approach. For this reason the standardized approach practically reduces to the current Basel approach for banks whose portfolios are dominantly composed of non-publicly rated loans. As the standardized approach is a modest variation of the current approach, we in this paper concentrate on the comparison between the current approach and the Internal Ratings Based (IRB) approach.

Under the IRB approach, banks' internal ratings determine the capital charges on corporate exposures. Under an advanced version of the IRB approach, banks are also allowed to form their own loss-given-default (LGD) estimates. The treatment of default probabilities, and their effect on capital charges, is basically identical under the foundation and the advanced version of the IRB approach, so that our analysis in this paper which assumes that LGDs are calculated as in the foundation approach is also relevant concerning the advanced IRB approach.

Figure 3 plots the minimum corporate capital charge under the IRB approach, calculated according to an 8% total capital level. We observe that the capital charge over its 'credit sensitive region' is a concave function of the default probability. This implies, via Jensen's inequality, that a symmetric mean preserving volatility in the default probability reduces the expected capital charge over time. A floor of 0.03% is applied to the default probability in the calculation of the capital charge, which results in a local convex region on the capital charge function in the lower end of default probabilities. We also observe that the IRB capital charge intersects the current 8% rule from below, at a default probability of 1.07%. This probability corresponds roughly to a BB rating on a rating agency scale, so that the IRB rule assigns a lower capital charge than the current 8% rule on corporate exposures rated BBB or better, that is for all investment grade exposures. This implies that a bank whose portfolio is concentrated in investment grade assets will benefit from the IRB approach in terms of a lower minimum capital charge.

Figure 3. **Minimum capital requirements on corporate exposures**



The solid line is the IRB capital formula from October 2002 (Basel Committee, 2002). A floor of 0.03% is applied to the default probability in the calculation of the capital charge, yielding a minimum capital charge of 1.18% on (senior uncollateralized) corporate exposures under the foundation IRB approach. The LGD is assumed to be 45%, which is the default assumption in the foundation IRB approach. The dotted line is the current 8% rule.

An additional property of the IRB capital charge is that the capital charge for sufficiently low grade assets is of the same order of magnitude as the LGD estimate. This fact has important consequences for the behaviour of bank capital in the event of a default. When a default occurs, the credit loss from a defaulted asset is deducted from bank equity, while the capital charge associated with non-defaulted assets is reduced by the amount of the pre-default capital charge of the defaulted asset. Given that realized LGD is close to the 45% estimate underlying the IRB capital charge formula, there will not be much of a shock to a bank's equity buffer (defined as the bank's total equity less its minimum capital requirement) at the time of default. Typically the default probability of the defaulted asset will have been timely driven up during the months and years preceding default, which will have consumed the bank's equity buffer gradually due to the increasing IRB capital charge over the period of deterioration.

The previous discussion will help us interpret the results that will be presented in Section 6 of this paper. We also noted earlier that under the current Basel regime, there is a negative correlation between risk-weighted assets and credit losses associated with a fixed initial portfolio. In the IRB approach, the corresponding correlation is likely to be positive. The logic behind this finding is the following. On one hand, credit losses reduce risk-weighted assets in the IRB approach, even more so than under the current approach. On the other hand, the risk sensitivity of the capital charge in the IRB approach implies that in a downturn where most credit losses take place, default probabilities and therefore capital charges on all (non-defaulted) assets will have gone up. For typical bank portfolios, and for typical recessionary scenarios, the increase in the aggregate capital charge due to non-defaulted assets is more than sufficient to dominate the reduction in the capital charge due to defaults. Hence the overall correlation between credit losses and capital charges is positive, given a fixed initial portfolio. This reasoning however strongly depends on the assumption that the portfolio composition is held fixed and only changes through defaults. An individual bank may of course reduce its capital charge through loan sales or other risk transfers even in a period characterized by higher than average credit losses. The same is not likely to be true for the banking sector as a whole, since an equilibrium where all banks are liquidating their holdings will have quite dramatic consequences. This concern is at the heart of the discussion on the vices of procyclical capital charges.

4 Framework for stress testing of capital requirements

In this section we extend the framework for capital adequacy simulations presented in Jokivuolle and Peura (2001). As discussed in the introduction, our solution to determining the required buffer capital is a probabilistic Value-at-Risk type criterion which measures the complacency of actual bank capital with minimum capital requirements over a defined horizon. Our framework is an extension of a typical credit portfolio model, in that we simulate actual bank capital and minimum capital requirements simultaneously. Within Basel II these are both stochastic, and their joint dynamics determines the required initial capital, or equivalently, the required capital buffer, given a confidence level for capital adequacy chosen by bank management.

4.1 Accounting measures of capital

We develop the VaR criterion here from an accounting identity governing bank capital dynamics. Our definition of capital is the book equity which is eligible in calculations of bank capital adequacy.[6] We simulate shocks to book capital and to book capital requirements, rather than to the market value of bank equity. We find that it is imperative to perform an analysis of capital adequacy in book value terms. First, banks' regulatory capital requirements apply to an accounting measure of capital, so that this is the measure of capital which is to be simulated in a forward looking analysis of capital adequacy. Second, under the current accounting standards, banking books are not market to market, so that changes in the market valuation of bank loans do not feed into banks' income and book equity. Actual credit losses are deducted from bank's book equity, however, and it is correct to simulate credit losses to reproduce bank's book capital dynamics. Third, bank equity will continue to be virtually insulated from changes in the market valuation of illiquid loan portfolios even after the introduction of the new IAS rules. Therefore the dynamics of bank capital will continue to be driven by credit (default) losses even under the new accounting rules. Fourth, we find that our approach is mainly consistent with general banking risk management practice,

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[6] Own funds eligible as Tier-1 capital include share capital, reserve funds and premium funds. Own funds eligible as Tier-2 capital include revaluation reserves and subordinated capital. The bank capital that we simulate should be interpreted as the total of own funds (Tier-1 plus Tier-2) eligible in capital adequacy calculations. We simulate the aggregate of this capital, and not the individual components.

since Value-at-Risk analyses on illiquid bank loan portfolios are quite often performed on a nominal value basis, rather than in a mark-to-market mode.

4.2 Bank capital dynamics and required capital buffers

We imagine a bank with assets consisting of illiquid corporate loans. We let E_t be time t bank capital (ex time t dividends) and R_t be the bank's regulatory capital charge at time t. Then the regulatory capital requirement that must hold at each point in time is

$$E_t \geq R_t. \tag{4.1}$$

We show in the following how the periodic regulatory capital requirement (4.1), when applied to a bank operating subject to certain capital market imperfections, leads banks to hold capital buffers over the minimum regulatory requirement. Our analysis is based on a simplified bank capital dynamics. We let I_t be the bank's profit before credit losses during period t, L_t be the bank's credit losses during period t, D_t be the dividends paid out of the bank capital at time t, and N_t be the issues of new equity at time t. The bank's capital dynamics then satisfies (this is not the only possible decomposition of capital dynamics, but most useful for our purposes)

$$E_{t+1} = E_t + I_{t+1} - L_{t+1} - D_{t+1} + N_{t+1}. \tag{4.2}$$

The necessity to hold capital buffers is motivated by capital market imperfections. In practice, these imperfections are likely to hold in severe macroeconomic downturns, but such scenarios matter the most in our capital adequacy calculations, so that it is with little loss of generality that we assume the imperfections to hold in all scenarios. In particular, we assume that sales of the existing portfolio are ruled out, and that new issues of equity are not a viable alternative. Moreover, in severe macroeconomic downturns, dividends are likely to be withdrawn and no new assets are likely to be bought, in order to minimize the burden on capital. Therefore we assume that both the D and the N terms in (4.2) will be zero, and that the capital dynamics in (4.2) corresponds to the initial (time 0) portfolio, so that the capital dynamics that we simulate is given by

$$E_{t+1} = E_t + I_{t+1} - L_{t+1}. \tag{4.3}$$

Rolling the difference equation (4.3) forward gives us time t capital as

$$E_t = E_0 + \sum_{s=1}^{t} I_s - \sum_{s=1}^{t} L_s. \tag{4.4}$$

We now define the bank's time t capital buffer B_t as

$$B_t = E_t - R_t. \tag{4.5}$$

Then substituting (4.4) into (4.5), and applying the inequality (4.1), gives us an expression of the regulatory capital requirement at time t

$$B_t = B_0 + \sum_{s=1}^{t} I_s - \sum_{s=1}^{t} L_s - R_t + R_0 \geq 0, \tag{4.6}$$

in terms of the initial capital buffer, the inflows and outflows of capital between time 0 and time t, as well as the change in the regulatory capital charge R between time t and time 0. Equation (4.6) reflects the fact that dividends have been suspended, while new issues of equity and sales of assets are effectively ruled out. No new business is taken in order to minimize the burden on capital, so that all the components in (4.6) are functions of the bank's initial (time 0) portfolio. As asset sales have been ruled out, R_t here reflects the capital charge associated with the bank's initial portfolio, net of the capital relief due to any expirations and defaults of assets up to time t.

The constraint (4.6) states that the bank's initial capital buffer must cover for two stochastic elements, the cumulative net profit (profit before credit losses less credit losses) and the cumulative change in the minimum capital requirement. Given a fixed initial portfolio with maturity T, condition (4.6) is to be monitored at each time t between time 0 and the maturity of the initial portfolio T. Yet, given that holding of buffer capital will be somewhat costly (eg lost tax benefits), requiring (4.6) to hold in all possible states of the world is likely to be uneconomical to the bank. Therefore it is natural to require (4.6) to hold for each t, at a sufficiently high probability. This yields a Value-at-Risk type probabilistic capital requirement

$$P\left[\min_{0 \leq t \leq T} B_t \geq 0\right] \geq \alpha, \tag{4.7}$$

where B_t is given by (4.6), and where α is a confidence level, such as 99%. (4.7) is a constraint on the initial capital buffer B_0. Because B_t is increasing in B_0, we expect there to a minimum value for B_0 so that (4.7) is satisfied. Hence the required initial capital buffer $\hat{B}_0$ is a solution to

$$\hat{B}_0 = \inf\left\{B_0 : P\left[\min_{0\le t\le T} B_t \ge 0\right] \ge \alpha\right\} \tag{4.8}$$

An alternative constraint for capital adequacy is what is commonly known as 'economic capital constraint'. This states that the bank must not run completely out of capital over the period from 0 to T, at a sufficiently high confidence level β. Stated mathematically using our notation, this becomes

$$\begin{gathered} P\left[\min_{0\le t\le T} E_t \ge 0\right] \ge \beta \\ \Leftrightarrow \\ P\left[\min_{0\le t\le T}\left(B_t + R_t\right) \ge 0\right] \ge \beta. \end{gathered} \tag{4.9}$$

where the equivalence follows from applying the definition of the buffer (4.5). The minimum initial capital buffer satisfying the economic capital requirement (4.9) can be formulated analogously to (4.8) as

$$\tilde{B}_0 = \inf\left\{B_0 : P\left[\min_{0\le t\le T}\left(B_t + R_t\right) \ge 0\right] \ge \beta\right\} \tag{4.10}$$

where B_t is given by (4.6). Because $R_t \ge 0$, comparison of (4.8) and (4.10) shows that when $\alpha = \beta$, (4.10) never yields a higher initial capital buffer than (4.8). Therefore the new capital requirement (4.8) makes the standard economic capital constraint redundant when the confidence levels used in the criteria are equal. On obvious grounds, however, we would expect the confidence level applied to regulatory capital adequacy, α, to be lower than the confidence applied in an economic capital constraint, β. Therefore the capital buffer solving (4.8) may not in all cases dominate the capital buffer solving (4.10), but for typical values of α and β (such as 99% and 99.9%, respectively) as well as for typical bank portfolios this will be the case.[7]

Assuming that bank capital buffer is determined from (4.8), initial bank capital is $E_0 = R_0 + \hat{B}_0$, and the bank's capital ratio can be expressed as

$$\left(1 + \frac{\hat{B}_0}{R_0}\right) 8\%. \tag{4.11}$$

Different rating systems and capital regimes vary in terms of the distribution of the minimum capital charge, and hence in terms of the distribution of

[7] This is rather obvious since (4.8) may never yield a capital requirement less than R_0, the current regulatory minimum capital charge, while economic capital of bank portfolios of average quality is known to be typically less than the 8% regulatory capital charge. Numerical simulations on our model (not reported here) confirm this.

$\Delta R_t = R_t - R_0$. When ΔR_t has very low volatility, such as in the current Basel regime, it is the distribution of net profit (credit losses) that determines the size of the required buffers. In the IRB approach within Basel II, it may be that ΔR_t is equally or more volatile than net profit, so that the volatility in the capital requirement is the more important determinant of capital buffers. We will evaluate these volatilities in Section 6 based on representative bank portfolios.

4.3 Determination of α

In our framework α is an exogenous parameter. In a more complete optimization model of the bank, this confidence level would be determined from the trade-offs that influence the bank's choice of capital. Therefore α would be influenced by factors such as the costs and penalties associated with violation of the regulatory capital constraint, the capital market frictions that affect the recapitalization of the bank, the sensitivity of the bank's funding cost to the amount of capital held by the bank, and the availability of growth options to the bank. Also regulator's concerns regarding the viability of bank capitalization would be reflected in α through the Pillar II of the new Basel Accord.

Within our reduced form approach, the parameter α may be calibrated based on information on actual bank capital ratios. Given a bank's portfolio and its capital ratio, there is an implied value of α which makes the capital ratio solved from the model equal to the observed capital ratio. We perform this type of calibration of α in Section 6.

5 Ratings transition model with business cycle dynamics

The model of the previous section implies that once the dynamics of buffer capital can be simulated according to (4.6), the resulting minimum capital buffers can be solved from (4.8). The dynamics in (4.6) depends on the model of rating transitions used. Rating transitions determine the evolution of the minimum capital charge, and defaults (and hence credit losses) are just special cases of rating transitions. This section presents the rating transition model on which our simulations are based on, describes the parameterisation of this model, and discusses the business cycle scenarios that we use in our simulations.

5.1 Ratings dynamics

We assume that counterparty ratings form a correlated vector Markov Chain. The correlations between ratings are generated by an underlying asset value model which is a one-factor version of the CreditMetrics^{TM} framework (J.P. Morgan, 1997), and which is described eg in Gordy (2000) and in Bangia et al (2002). The standard form of the CreditMetrics^{TM} framework assumes that the Markov Chain of ratings evolution is time homogenous. We depart from this assumption by specifying that the evolution of ratings is dependent on an underlying business cycle variable, which itself follows first order Markov dynamics. The business cycle variable in our model may be in two possible states, one referred to as 'expansion' and the other referred to as 'recession'. The state of the business cycle variable determines the transition probabilities of the ratings. In particular, the transition matrix associated with the recession state is expected to display higher volatility in rating changes, and higher default probabilities, as is the transition matrix associated with the expansion state. Hence our model of rating dynamics is a vector Markov Chain model with an underlying latent variable that itself follows a two state Markov Chain. Models of ratings dynamics of this type have been suggested by Bangia et al (2002).

We assume that the underlying asset value correlations, which together with the transition probabilities determine the rating transition correlations, do not depend on the state of the business cycle. Consistent with industry standards and with the IRB capital charge formula,[8] we use 20% asset correlation across all counterparties.

We perform multiperiod simulations of rating changes in quarterly time increments. Credit portfolio models are typically implemented as one period simulations with an annual horizon, but we find the quarterly time interval justified because banks in most countries report their capital adequacy to their regulators quarterly. We parameterize the ratings model using data from Bangia et al (2002). In particular, both the rating transition probabilities and the regime transition probabilities that we use are quarterly probabilities estimated based on US data. The conditional transition matrices for the expansion and the recession states we use are from Table 4 in Bangia et al (2002), which are based on Standard and Poor's data on US corporate ratings over the period 1981–1998. We show these transition matrices in Table 4.

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[8] In the October 2002 version of the IRB rules, the asset correlation on which the IRB capital charge is based on depends on the default probability of the counterparty, varying between 12% and 24%.

Table 4. **Transition matrices conditioned on state of the business cycle**

1/4-year US Expansion matrix

Initial rating	Terminal rating							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	98.21%	1.66%	0.11%	0.02%	0.02%	0%	0%	0%
A	0.15%	98.08%	1.61%	0.12%	0.01%	0.03%	0.01%	0%
A	0.02%	0.53%	98.06%	1.21%	0.11%	0.06%	0.00%	0.00%
BBB	0.01%	0.07%	1.47%	96.94%	1.25%	0.22%	0.02%	0.02%
BB	0.01%	0.03%	0.19%	1.93%	95.31%	2.25%	0.16%	0.12%
B	0%	0.02%	0.07%	0.10%	1.70%	95.91%	1.31%	0.88%
CCC	0.05%	0%	0.19%	0.23%	0.47%	3.57%	87.32%	8.17%

Source: Bangia et al (2002), based on S&P data 1981–1998.

1/4-year US Recession matrix

Initial rating	Terminal rating							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.99%	1.76%	0.25%	0%	0%	0%	0%	0%
A	0.18%	96.89%	2.79%	0.05%	0.09%	0%	0%	0%
A	0.02%	0.88%	96.44%	2.59%	0.07%	0%	0%	0%
BBB	0.04%	0.04%	1.11%	96.31%	2.33%	0.07%	0%	0.11%
BB	0%	0.06%	0.06%	1.39%	94.98%	2.72%	0.42%	0.36%
B	0%	0.06%	0.06%	0.11%	0.72%	95.02%	2.27%	1.77%
CCC	0%	0%	0%	0%	0%	1.20%	85.60%	13.20%

Source: Bangia et al (2002), based on S&P data 1981–1998.

The regime switching matrix for the underlying business cycle variable that we use is from Table 5 in Bangia et al (2002). We represent this matrix in the upper part of Table 5. The regime switching probabilities have been estimated from quarterly data on US business cycles over 1959 to 1998, as classified by the NBER. The stationary distribution of the business cycle state implied by this transition matrix is (79%, 21%), implying that roughly every fifth quarter is classified as recession. However, following a quarter that has been classified as recession, over 40% of the times the next quarter will be a recession, resulting in an expected recession length of 1.74 quarters. In other words the business cycle state is slightly positively autocorrelated, and our framework enables us to study the consequences of this autocorrelation on bank capital adequacy in recessions. The estimate of first-order autocorrelation from the transition matrix in Table 5 is 27%.

Table 5. **Regime transition matrices used in the scenarios**

'Recession' scenario: Expected recession length 1.74 quarters

Initial state	Terminal state Expansion	Recession	Stationary distribution	Expected duration	First-order autocorrelation
Expansion	84.80%	15.20%	79.12%	6.58	27%
Recession	57.60%	42.40%	20.88%	1.74	

Source: Bangia et al (2002), based on NBER data 1959–1998.

'Long recession' scenario: Expected recession length 4 quarters

Initial state	Terminal state Expansion	Recession	Stationary distribution	Expected duration	First-order autocorrelation
Expansion	84.80%	15.20%	62.19%	6.58	60%
Recession	25.00%	75.00%	37.81%	**4.00**	

'Prolonged recession' scenario: Expected recession length 8 quarters

Initial state	Terminal state Expansion	Recession	Stationary distribution	Expected duration	First-order autocorrelation
Expansion	84.80%	15.20%	45.13%	6.58	72%
Recession	12.50%	87.50%	54.87%	**8.00**	

5.2 Business cycle scenarios

Here we introduce the business cycle scenarios on which our capital adequacy calculations are based on. In particular, our multiperiod analysis with embedded business cycle dynamics allows us to calculate bank capital requirements under various assumptions concerning the initial business cycle state as well as the duration of recessions. Since we are interested in the behaviour of bank capital buffers in recessions, we assume in all of our scenarios that the first quarter of the simulation is a recession state. Then we let the business cycle state evolve stochastically according to specified switching probabilities, which determine the expected length of the recession.

Our first scenario is based on the historical regime switching matrix in the upper part of Table 5. In this scenario the expected length of a recession is 1.74 quarters. In our second scenario we assume that the expected length of a recession is four quarters. We achieve this by changing the transition probability from the recession to the expansion state appropriately. The theory of first-order Markov Chains tells us that the expected duration of a visit to a state is given by $1/(1-p)$, where p is the probability of remaining in the state after one period. Then a targeted expected visit duration, denoted l, for the particular state is achieved by setting the probability p according to $p = 1-1/l$. The transition matrix generated in this manner is presented in the middle part of Table 5. In our third scenario, we assume that the expected length of the recession is eight quarters, and again obtain this by changing the single transition probability from the recession to the expansion state. The resulting transition matrix is shown in the lower part of Table 5. Table 5 also shows the first-order autocorrelations of the business cycle variable in the three scenarios, which are 27%, 60% and 72%, respectively.

We find it helpful to name the three business cycle scenarios. From now on, the three scenarios will be called ‘recession’, ‘long recession’, and ‘Prolonged recession’, respectively. We will also perform unconditional simulations, in which we do not fix the initial business cycle state, but select it based on the stationary distribution of the business cycle variable, (79%, 21%).

5.3 Average bank portfolios

We take representative portfolios of US banks from a Federal Reserve Board survey as reported by Gordy (2000). In Table 6 we report two different quality distributions, referred to as ‘average quality’ and ‘high quality’. The table also shows the weighted average default probabilities of the quality distributions, using annual default probabilities corresponding to the same S&P data as our rating transition matrices. We note at this point that as we calculate risk weighted assets under the Basel II regime, we use these annual default probabilities in the risk weight formulas.

Table 6. **Average bank portfolios**

S&P grade	Default probability	US average quality	US high quality
AAA	0.00%	3%	4%
AA	0.00%	5%	6%
A	0.04%	13%	29%
BBB	0.24%	29%	36%
BB	1.01%	35%	21%
B	5.45%	12%	3%
CCC	23.69%	3%	1%
Average DP		1.79%	0.71%

US portfolios are from Federal Reserve Board survey, as reported in Gordy (2000).
Default probabilities are based on S&P data 1981–1998.

For our numerical analysis, we form portfolios according to the given quality distributions that each have 500 equal sized loans. The loans are ex ante identical in all other respects except the initial obligor rating. We assume that all loans are bullet loans with a maturity of T years. T here refers to the simulation horizon, and is a parameter of the model presented in Section 4. We also note that the calibration of the parameter α in Section 6 is based on the average portfolio in Table 6, which we assume to correspond to the average portfolio of the US banks in our Bankscope data.

As discussed in Section 4, all our simulations are based on the assumption that no new assets are bought, and no existing assets are sold, over the simulation horizon. This assumption is not descriptive of actual bank portfolio dynamics. However, the size of the capital buffers is determined by portfolio dynamics over multiperiod recessions, and in such circumstances the assumption of no new business is likely to be much closer to reality.

6 Results

The plan for presenting our numerical results is the following. We first compare credit loss distributions with distributions of capital requirements, both under the current Basel regime as well as under the IRB approach under Basel II. We then present and discuss the capital buffers derived from our model. We perform a calibration of the parameter α to the data on actual bank capital presented in Section 2, which allows us to generate a ceteris paribus prophecy on how capital buffers are likely to change as banks move to the IRB regime within Basel II. We also provide comparative static analyses with respect to the key parameters in our framework. Towards the end of this section, we discuss the importance of two methodological choices that differentiate our framework from previous contributions in the literature. First, we contrast the results from our stochastic framework with the results obtained from deterministic scenarios, as applied eg by Erwin and Wilde (2001) and Catharineau-Rabell et al (2002). Second, we compare our Value-at-Risk criterion which is based on periodic monitoring of the minimum capital requirement with the Value-at-Risk criterion applied in Jokivuolle and Peura (2001) where capital adequacy is only monitored at a terminal date. Periodic monitoring yields higher capital buffers than monitoring at terminal date only, given a fixed coincidence level applied in both criteria. We show the difference in capital ratios resulting from the two related criteria.

6.1 Net losses and changes in capital requirements

According to equation (4.6), we think of the bank's capital buffer as a hedge against two stochastic elements: the net loss from bearing credit risk (L–I), and the net change in the minimum capital requirement, ΔR. In Table 7 we report the key statistics of the distributions of these two quantities, over a one year horizon.

We base this table on the standard one year horizon in order to make comparisons to other studies possible and to facilitate testing against one's own intuition. We make several observations from the table concerning the relative importance of the two stochastic variables.

Table 7. **Distributions of net losses and capital requirements over 1 year horizon**

Portfolio quality	Business cycle scenario	Net loss (L–I)			ΔR, current Basel regime			ΔR, Basel II IRB approach		
		E	SD	99%	E	SD	99%	E	SD	99%
High	Unconditional	0.00	0.32	1.16	–0.06	0.05	0.00	0.07	0.27	0.86
	Recession	0.09	0.39	1.48	–0.07	0.06	0.00	0.10	0.27	0.87
	Long recession	0.17	0.43	1.71	–0.08	0.07	0.00	0.13	0.26	0.88
	Prolonged recession	0.21	0.45	1.82	–0.09	0.08	0.00	0.14	0.26	0.89
Average	Unconditional	0.00	0.69	2.41	–0.15	0.11	0.00	–0.20	0.31	0.55
	Recession	0.22	0.80	2.97	–0.18	0.13	–0.02	–0.18	0.30	0.54
	Long recession	0.39	0.89	3.41	–0.21	0.15	–0.02	–0.15	0.28	0.54
	Prolonged recession	0.48	0.93	3.60	–0.23	0.15	–0.02	–0.14	0.28	0.54

E = expected value, SD = standard deviation, 99% = 99^{th} percentile point. Total exposure in each portfolio equals 100, so that the numbers can be interpreted as percentages of portfolio nominal value. The profit flow before credit losses (I) is assumed to equal the unconditional expected loss from the portfolio ($\theta = 1$), so that expected net loss in the 'unconditional' scenarios is zero. Because the simulation horizon is only 1 year, the results for the 'prolonged recession' scenario do not differ significantly from the results of the 'recession' scenario.

First, under the current Basel regime, the volatility of credit losses is an order of magnitude larger than the volatility in capital requirements. For a fixed initial portfolio, the volatility in the minimum capital requirement is entirely due to defaults (defaults reduce the risk-weighted assets associated with a given initial portfolio). Therefore the correlation between the minimum capital requirement and net losses is negative and close to one under the current Accord. Moreover, the expected change in the minimum capital requirement can be calculated as a function of the expected credit loss. To show this, we note that the expected credit loss of the average quality portfolio in the unconditional scenario is 0.84 (Figure 7 only shows net losses). Given an assumed loss-given-default of 45%, this implies that the expected nominal value of defaulted assets is 1.89 = 0.84/0.45. Given an 8% capital requirement on nominal assets, this implies that the change in the aggregate capital charge attributable to expected defaults is –0.15 = 8% · –1.89, which is the figure found in Table 7.

Second, under the IRB approach within Basel II, the volatility in the minimum capital charge is no more negligible compared to the volatility in credit losses. For high quality portfolios, the two volatilities are of the same order of magnitude, while for average quality portfolios, the volatility of credit losses is around twice as high as the volatility in minimum capital requirements. This comparison hence supports the view that acknowledging the volatility in minimum capital requirements, in addition to the volatility in credit losses, should be an important component of capital adequacy analysis under Basel II.

Third, as for the IRB regime, the distribution of credit losses appears more 'long-tailed' than the distribution of minimum capital requirements. For a high quality portfolio, eg, the 99th percentile point of the credit loss distribution is approximately one standard deviation further away from the mean than is the corresponding point of the minimum capital distribution. As for the average quality portfolio, the distribution of minimum capital charges appears even more concentrated relative to the credit loss distribution. Information on the lower tail of the minimum capital distribution (not shown in Table 7) in fact indicates that this distribution is rather symmetric. One could expect that the distribution of minimum capital charges is rather skewed since the default probability interpretations of ratings usually have an exponential shape. There are two reasons to the fact that the distribution of minimum capital requirements is not 'long tailed' after all. For one thing, when viewed globally, the IRB risk weight function (shown in Figure 3) is both concave in default probabilities and bounded from above. For another reason, a default of a low rated asset in the IRB regime reduces the minimum capital charge of the non-defaulted portfolio by an amount which is usually close to the realized credit loss (again we refer to the discussion in Section 3). These both are compensating forces which in a sense work to lower capital requirements of distressed portfolios, although both effects come to dominate the behaviour of the aggregate (bank) capital charge only as the portfolio has reached a sufficiently low aggregate quality.

Fourth, in the IRB regime, the drift in minimum capital requirements is positive for the high quality portfolio, but negative for the average quality portfolio. This is partly explained by the drifts present in the rating transition matrices, but also by the facts already discussed in the previous paragraph. Default probabilities attached to high and average quality ratings have an exponential shape, which (by Jensen's inequality) causes volatility in these default probabilities to translate into higher capital charges, on average. When a portfolio is of sufficiently low average quality, on the other hand, the reductions in capital charge associated with defaults contribute towards a negative drift to the capital charge.

Finally, we see from Table 7 that the scenarios that we have generated have true economic significance. The difference between the 99th percentile points of the credit loss distribution corresponding to the 'recession' scenario and the unconditional simulation is 23% and 28% for the average and the high quality portfolio, respectively. The corresponding differences between the 'long recession' scenario and the unconditional simulation are 41% and 47%. As a point of reference, Bangia et al (2002) have generated credit risk scenarios by assuming that either an extension or a recession state prevails for the entire year (we let the state vary stochastically after the initial quarter's fixing). Their 99th percentile point credit value loss is about 30% larger, given a recession scenario, than given an expansion scenario. This difference is of the same order of magnitude than ours, although the analyses are not directly comparable.[9]

We have also calculated the distribution of credit losses under longer maturities (the results are not shown here). Here we observe that as the horizon increases, the importance of conditioning on the initial business cycle state diminishes. At a four-year horizon eg the difference in the 99th percentile of the credit loss distribution is only about 5%, between the unconditional distribution and the distribution which is conditioned on the first quarter being a recession (compared to 23% at one year horizon). This suggests that typical recessions are rather temporary phenomena, which is truthfully reflected in the business cycle transition probabilities shown in Table 5. Of course, if more severe scenarios are applied instead of the basic 'recession' scenario, the difference in estimated credit risks again starts widening with respect to the unconditional case. Our 'long recession' scenario would imply a 21% higher 99th percentile point of credit losses over a four year horizon for the average portfolio, relative to the unconditional case.

6.2 Capital ratios

We present our main comparison on capital ratios under the current Basel regime and under the IRB approach subject to the following base case parameters: portfolio maturity T equal to 2.5 years, a profit flow before credit losses equal to the unconditional expected credit loss from the portfolio ($\theta = 1$), and a confidence level α of 99%. The average maturity of 2.5 years is the default assumption underlying the Basel II IRB risk weight formulas. The assumption that profit before credit losses just covers expected losses is conservative, not so much because actual margin income is always higher, but because of other (fee) income that banks typically accrue through loan sales. The choice of 99% for α will be discussed in the next subsection where we perform a tentative calibration of this parameter to empirical data. We will show sensitivities of capital ratios with respect to the model parameters later in this section. The capital buffers, calculated from (4.11) under the base case parameters, are shown in Table 8.

[9] Our portfolio is from a different source, and we only calculate the distribution of credit (default) losses, whereas they calculate a distribution of mark-to-model values.

Table 8. **Capital ratios**

		Current Basel regime				Basel II IRB approach			
Portfolio quality	Business cycle scenario	Minimum capital	Capital buffer	Total capital	Capital ratio	Minimum capital	Capital buffer	Total capital	Capital ratio
High	Unconditional	8.0	1.7	9.7	9.7%	4.0	3.2	7.2	14.3%
	Recession	8.0	1.9	9.9	9.9%	4.0	3.4	7.5	14.8%
	Long recession	8.0	2.4	10.4	10.4%	4.0	4.0	8.0	15.9%
	Prolonged recession	8.0	2.7	10.7	10.7%	4.0	4.3	8.4	16.6%
Average	Unconditional	8.0	3.1	11.1	11.1%	6.6	4.1	10.7	13.0%
	Recession	8.0	3.6	11.6	11.6%	6.6	4.6	11.2	13.6%
	Long recession	8.0	4.4	12.4	12.4%	6.6	5.5	12.1	14.7%
	Prolonged recession	8.0	4.8	12.8	12.8%	6.6	6.0	12.6	15.4%

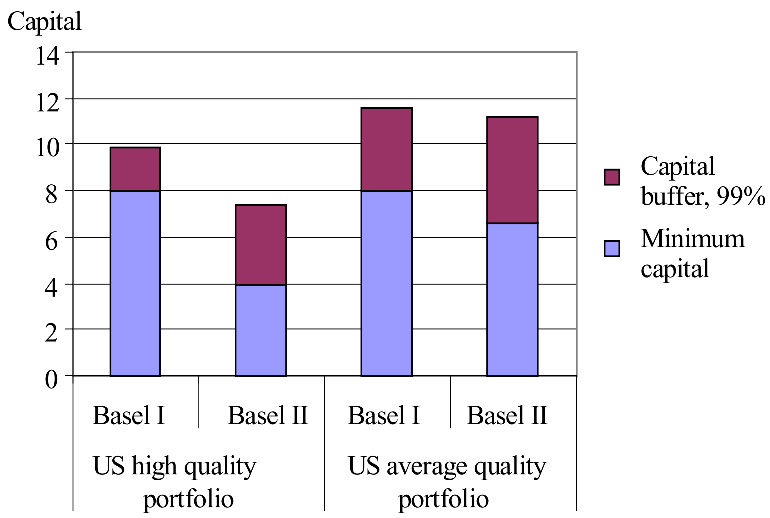
Capital buffer has been calculated from (4.8), while capital ratio has been calculated from (4.11). Other parameters: $T = 2.5$ years, $\theta = 1$, $\alpha = 99\%$.

Table 8 points to several conclusions. First, the capital ratios of our example portfolios in the Basel II IRB regime are substantially higher than in the current regime. In the unconditional scenario, eg, the capital ratio of the high quality portfolio rises from less than 10% to over 14%, moving from the current regime to the IRB regime. As for the average portfolio, the corresponding rise is roughly from 11% to 13%. Irrespective of the chosen scenario, the capital ratios of the high quality portfolio in the IRB regime are approximately 5 percentage points higher than under the current regime, while the difference is around 2 percentage points as for the average quality portfolio.

Capital ratio indicates how large the required capital buffer is relative to the minimum capital requirement. Our results indicate that in the IRB regime, the capital buffer for high quality portfolios is likely to be close to the same order of magnitude as the minimum capital requirement alone. The capital buffer for an average portfolio will be roughly two thirds of the minimum capital charge. In the current regime, on the other hand, the median US total capital ratio of 11.9% implies a median capital buffer equal to 49% of the minimum requirement.

This rise in capital ratios partially offsets the capital relieves resulting from lower minimum capital requirements. The banks with high quality portfolios who in terms of lower minimum capital requirements benefit the most from the IRB regime relative to the current regime, are also to lose more of this gain through increases in their capital ratios. The 'Minimum capital' and 'Total capital' columns in Table 8 allow us to calculate this effect. While a high quality portfolio's minimum capital requirement is reduced by 50% ((8.0–4.0)/8.0) going from the current regime into the IRB regime, its total capital is reduced only by 26% ((9.7–7.2)/9.7). Therefore roughly one half of the capital relief is consumed by the need for relatively higher capital buffers. The corresponding percentages for the average quality portfolio are 18% and 4%, respectively. Total capital holdings of a bank with an average quality portfolio, according to our calculations, therefore remain virtually unchanged through the changeover into the IRB regime, although the bank's minimum capital requirement on credit risk is reduced by roughly 18%. We have drawn the total capital and its components, both for the high and the average quality portfolio, in both Basel regimes in Figure 9 in order to illustrate this conclusion. Moreover, given the charge for operational risk that is likely to be imposed on top of the credit risk capital charges, the total capital of a bank with an average portfolio is likely to increase with the new IRB approach.[10]

Figure 9. **Decomposition of total capital into minimum capital requirement and the capital buffer**



Figures are taken from the corresponding columns in Table 8. 'Recession' scenario.

Second, we observe that the capital ratio in the IRB regime decreases as portfolio quality deteriorates, while in the current regime the reverse happens. In order to understand this, note that in the current regime, only realized credit losses generate shocks to bank capital buffers (given a fixed portfolio), whereas within Basel II, shocks are also generated by rating changes. As the volatility of credit losses increases with decreases in portfolio quality, the current regime necessitates higher buffers for lower quality portfolios. Under the IRB approach, on the other hand, shocks to capital buffers caused by rating migrations are typically more dominant than are shocks caused by actual defaults. The IRB minimum capital requirement for low rated credits is of the same order of magnitude as is the actual credit loss in the event of a default (see the discussion in Section 3), so that a default by a low rated asset results is less of a shock to bank's buffer capital in the

[10] This view is not inconsistent with the findings of most industry analyses produced on this subject.

IRB regime than in the current regime. Capital buffer volatility in the IRB regime is therefore predominantly driven by ratings transitions, which is also the driver behind the high capital ratios of high quality portfolios. As for low quality portfolios, ratings transitions increasingly generate positive shocks to buffer capital, which decreases capital ratios under the IRB regime. In fact, it turns out that for sufficiently low-quality portfolios the size of the capital buffer, both in relative and in absolute terms, is actually less in the IRB regime than under the current Basel regime (Table 8 does not contain these numbers).

We note that our previous discussion on capital ratios under Basel II assumes that the confidence level α is equal to 99% and, more importantly, that this value remains unchanged through the changeover into Basel II. Therefore the entire discussion is based on a ceteris paribus extrapolation of current bank capital holdings into bank capital holdings under Basel II. We acknowledge that many factors, such as changes in the perceived costs of supervisory intervention, could influence the value of α in connection with the Basel II reform. Should there be a systematic change in α, our previous conclusions would not necessarily hold any more.

6.3 Calibration of α

In the previous discussion, we have not said anything about the selection of the business cycle scenario. As is evident from Table 8, the choice of scenario has an economically significant impact on the resulting capital ratio. Because we have no data on banks based on which to identify the scenario, however, we suggest that the scenario be selected jointly with the value of the confidence level so that these choices together yield capital ratios consistent with empirical evidence. We perform a simple calibration along these lines in this subsection.

The calibration procedure is the following. We take the observed median capital ratio of US banks, 11.9%, from Table 1 as the ratio against which we match our model ratios. The portfolio that we use is the average portfolio of large US banks, reported in Table 6. Then we iterate on α until we find a value that yields a capital ratio, calculated from (4.11), sufficiently close to the observed value. We repeat this for each of the business cycle scenarios, which results in a set of four calibrated (scenario, α) pairs. The results are shown in Table 10. We have drawn a grey line into the table to indicate the value of α which, for any given scenario, yields a model capital ratio equal to the observed value.

Table 10. **Capital ratios as a function of α**

α	Business cycle scenario: Unconditional	Recession	Long recession	Prolonged recession
95%	9.8%	10.2%	10.9%	11.3%
96%	10.0%	10.4%	11.1%	11.5%
97%	10.2%	10.6%	11.3%	11.8%
98%	10.6%	11.0%	11.7%	12.2%
99%	11.1%	11.6%	12.4%	12.8%
99.50%	11.7%	12.1%	13.0%	13.5%
99.90%	12.9%	13.5%	14.5%	15.0%
99.95%	13.5%	14.2%	15.2%	15.5%
99.97%	13.8%	14.6%	15.6%	16.1%

Capital ratios are calculated from (4.11). The grey line indicates the value of α which yields a capital ratio equal to the empirically observed value 11.9%. Other parameters: T = 2.5 years, θ = 1.

We interpret the results in Table 10 as indicating the implied confidence level as a function of the assumed scenario. The analysis is performed on several scenarios simply because the true scenario which the average bank uses in its capital adequacy analysis is unknown. If the average US bank used a 'recession' scenario in its capital adequacy analysis, the average (or median) bank capital ratio of 11.9% were consistent with a confidence level of approximately 99.3%. If the average bank used a 'long recession' type scenario in its capital adequacy analysis, the capital ratio of 11.9% were consistent with a 98.4% confidence level, and if the average bank used a 'prolonged recession' scenario, the implied confidence level would be 97.2%. Of course, these confidence levels are sensitive not only to the scenario, but to basically all the model parameters. If eg the underlying asset correlations were increased, the value of α which would produce the empirically observed capital ratios would decline. Therefore we think of our calibration exercise as an attempt to identify a confidence level which is consistent with the other parameterisation of the model. One could also suggest that α is the calibrated parameter only because the least of it is known a priori.

The value of α on which the capital ratios in Table 8 were based was 99%. Now we can observe from Table 10 that in light of the empirical evidence, an α value of 99% is roughly consistent with a bank using something between the 'recession' and the 'long recession' scenario in its capital simulations. We can interpret this as suggesting that the relevant scenario in Table 8 for the median bank is 'recession', or perhaps 'long recession'. If we formulate our ceteris paribus extrapolation of bank capital under Basel II around these scenarios, we observe that the average US bank, which now holds 11.9% capital, would have a capital ratio of roughly 14% under the IRB regime within Basel II. For a bank with a high quality portfolio, the predicted capital ratio corresponding to the same (scenario, α) pair would be little over 15%.

We acknowledge the shortcomings of our calibration procedure. Only partial information on bank portfolios is available to the outsider. Any deviation between composition of the model portfolio and the true portfolio will be reflected in the resulting estimate for α. Also, the capital ratio within our model depends on other parameters besides α that are unknown or not directly observable. These include basically all the parameters of the transition model, and the scenario based on which banks determine their required capital ratio. If we parameterize our model differently from what banks do on average, our estimate of α is likely to be biased, even though our model were the correct description of decision making within banks. More generally, there is the model risk that our simple framework does not capture some of the essential aspects of banks' capitalization decision. Our framework supposes that these other aspects are reflected in the value of α. But many of the missing factors are likely to be bank specific, and hence there is no basis to assume that the true α would be the same across banks. Our analysis which is based on an average bank portfolio and an average bank capital ratio may not identify the average α, but some weighted combination of individual bank's α's.

Some of the previous concerns may not matter so much from the point of view of our target, which is to use the implied value of α to generate a prediction on bank capital ratios within the forthcoming Basel II regime. Suppose that we make a systematic error in parameterizing the portfolio model, which results in a biased estimate of implied α. Because we use the same biased parameter estimates as well as the same biased α, when we calculate capital ratios under Basel II, the effects of the biases are likely to be offsetting. In particular this will be the case when the portfolio does not change. As the portfolio changes, the effects of the biases are not likely to be offsetting any more.

6.4 Comparative statics

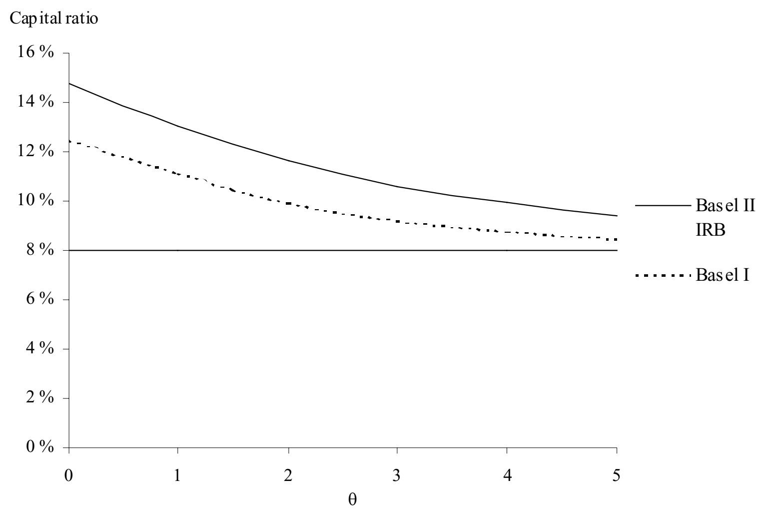
Table 10 already illustrates the sensitivity of the model capital ratios to the value of α. In this subsection we look at the sensitivities to other key parameters.

Level of profit flow

One key parameter in our analysis is the level of the bank's profit flow (before credit losses), the I_t terms in the bank capital dynamics (4.6). High positive profit flow can act as a partial hedge against credit losses and net increases in capital requirements. It is unlikely, however, that a profit flow can completely substitute for the need to hold buffer capital, since profit by definition is a flow quantity which in the short term is likely to be dominated by unexpected credit losses and, under the IRB regime, also by changes in the minimum capital requirement.

We have parameterized the profit flow to be a multiple of the unconditional expected losses associated with the bank's portfolio. Figure 11 illustrates the behaviour of the capital ratio as a function of this multiplier, θ, for the average quality portfolio. The figure shows the capital ratios both under the current Basel regime and under the IRB regime. The other parameters are set equal to their base case values. Hence the capital buffers in this figure are comparable to those in Table 8.

Figure 11. **The effect of profit flow (before credit losses) on capital ratios**



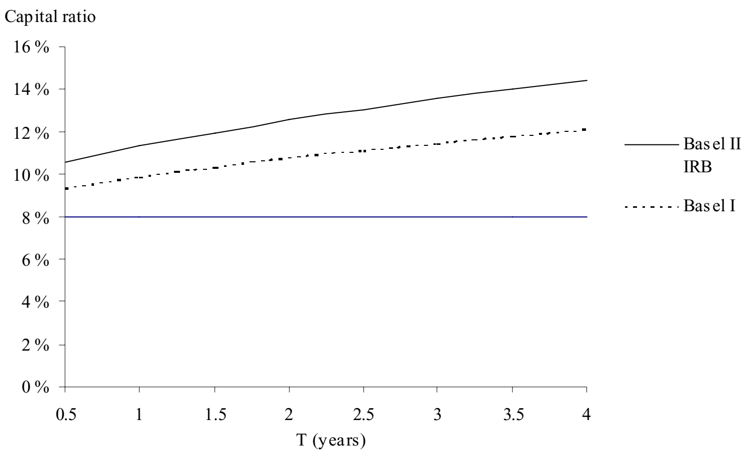
Other parameters: $\alpha = 99\%$, $T = 2.5$, 'average' portfolio, 'unconditional' simulation.

Figure 11 shows that capital ratio is a declining function of the level of the bank's profit flow, and slowly approaches the 8% minimum as the profit-to-credit risk multiple increases. The capital ratios associated with the value $\theta = 1$ correspond to the capital ratios under the unconditional scenarios in Table 8, 11.1% and 13.0%, respectively. The $\theta = 1$ case corresponds to a situation where the bank on average 'over the cycle' makes no profit, and may be deemed a conservative estimate of bank profitability. Figure 11 then shows that a highly profitable bank, eg one with $\theta = 5$, could under the current capital regime do with a capital ratio well under 9%, but would need a capital ratio close to 10% under the IRB regime.

Portfolio maturity

The simulation horizon T in our analysis is to be interpreted as (average) portfolio maturity, or alternatively the time it takes for the bank to access external capital. Figure 12 shows that the capital ratio of the average portfolio is an increasing function of T. This result holds for all portfolios in our model, which can be verified by noting that if the criterion (4.7) for a given T is satisfied by a given initial buffer B_0, then it is satisfied by the same B_0 for all T' < T. Hence the required capital buffer must be non-decreasing in T.

Figure 12. **The effect of portfolio maturity on capital ratios**



Other parameters: α = 99%, θ = 1, 'average' portfolio, 'unconditional' simulation.

6.5 Comparison to deterministic scenarios

The current paper, together with Jokivuolle and Peura (2001), is to the best of our knowledge the first study to apply a probabilistic simulation based approach to stress testing capital adequacy within Basel II. Some previous studies have presented deterministic stress test of capital adequacy, most notably Caterineu-Rabell et al (2002), and Erwin and Wilde (2001). In these papers, the basic idea is to take realized rating transition frequencies, including default frequencies, from a period corresponding to a particularly adverse ratings development, and to apply these transition frequencies to a given initial rating distribution.[11] This yields a hypothetical end-of-period ratings distribution for the initial portfolio, and the capital requirement of this stressed portfolio is then compared to the capital

[11] This amounts to multiplying the initial portfolio ratings distribution vector with the matrix of realized transition frequencies over the chosen period.

requirement of the initial portfolio. The change (increase) in the capital requirement is used as a measure of the required capital buffer.

This deterministic approach to stress testing bank capital is reminiscent of historical simulation approaches to risk measurement in market risk contexts. The appeal of this approach is in the use of adverse rating transitions from a period that actually took place in the history. The obvious shortcoming of such an analysis is the absence of any likelihoods attached to the chosen rating transition scenario itself. In other words, this approach to stress testing of capital adequacy does not yield a complete description of a bank's capital dynamics, unlike our stochastic analysis which yields a well defined stochastic process for bank's capital dynamics. Moreover, the deterministic approach may yield misleading results if the granularity of the bank's portfolio is significantly different from the granularity of the portfolio which has generated the transition matrix.

An important point demonstrated by Caterineu-Rabell et al (2002) is that a bank's chosen internal rating methodology has an impact on its regulatory capital volatility, and therefore on its needed capital buffers. In particular, Caterineu-Rabell et al compare a stock market based rating system (à lá KMV) to traditional 'over-the-cycle' agency ratings in terms of their implied capital buffers. The significantly higher rating migration volatility of the former approach results in a much higher volatility of minimum capital requirements. This in turn necessitates higher capital buffers. In this paper, we have used transition matrices of agency ratings, implying that our analysis applies to a bank whose portfolio is mainly agency rated, or correspondingly internally rated using a system analogous to agency ratings in terms of the volatility of rating changes. Analysis in our framework can also be based on a 'point-in-time' stock market based rating system as long as the rating transition probabilities are changed accordingly.

6.6 Periodic vs terminal Value-at-Risk analysis

Jokivuolle and Peura (2001) solve bank capital buffers subject to the same capital dynamics as we do here but in a single-period setting, where the Value-at-Risk criterion only applies at the terminal simulation date. In our multiperiod setting, this criterion corresponds to evaluating capital adequacy only at the terminal date T. Using our notation from Section 4, this criterion becomes

$$\hat{B}_0 = \inf\{B_0 : P[B_T \geq 0] \geq \alpha\}. \tag{6.1}$$

Banks report their capital adequacy to their regulators in most countries quarterly. Therefore our quarterly simulation period fits well into this institutional practice. In a multiquarter analysis (ie one where the terminal date extends to many quarters), monitoring of capital adequacy at the terminal date only will result in a downward bias in the estimated probability of capital adequacy violation. Therefore the capital buffer solving (6.1) is going to be a lower than the capital buffer solving (4.8), given a fixed confidence level α.

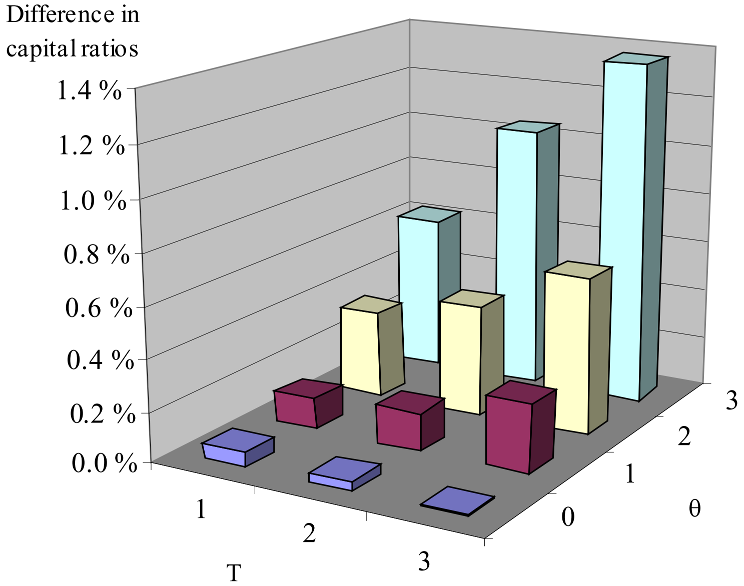
We illustrate here the magnitude of the error which results from applying the terminal Value-at-Risk criterion (6.1) instead of the periodic Value-at-Risk criterion (4.8). This comparison is contained in Table 13 and in Figure 14. Table 13 shows capital ratios calculated both under periodic monitoring (criterion (4.8)) and terminal monitoring (criterion (6.1)), as a function of portfolio maturity T and the level of the profit flow parameter θ. Also shown in Table 13 is the difference between these capital ratios. These differences are further graphed in Figure 14.

Table 13. **Capital ratios with periodic and with terminal monitoring**

		Capital ratio with periodic monitoring (A)			Capital ratio with terminal monitoring (B)			Difference (A–B) in capital ratios		
		T			T			T		
		1	2	3	1	2	3	1	2	3
θ	0	12.1%	13.9%	15.5%	12.0%	13.9%	15.5%	0.1%	0.0%	0.0%
	1	11.3%	12.5%	13.6%	11.2%	12.4%	13.3%	0.1%	0.1%	0.3%
	2	10.7%	11.4%	12.0%	10.4%	11.0%	11.3%	0.3%	0.4%	0.6%
	3	10.1%	10.5%	10.7%	9.5%	9.5%	9.3%	0.6%	1.0%	1.4%

Parameters: α = 99%, 'average' portfolio, 'unconditional' simulation, Basel II IRB regime.

Figure 14. **Difference in capital ratios with periodic/terminal monitoring**



The figures are from the right-hand side of Table 13.

The results show that the downward bias in the capital buffer resulting from terminal monitoring is the higher, the higher is the bank's profit flow. There is an intuitive explanation to this. A sufficiently high profit flow causes the bank's capital buffer to drift upwards. For this reason the terminal distribution of the capital buffer may stochastically dominate the distribution of the capital buffer at some earlier points of time. The terminal Value-at-Risk criterion that is evaluated based on the terminal distribution only will therefore yield lower buffer requirements than does the criterion based on periodic monitoring, simply because the terminal criterion does not acknowledge the possibility that the capital requirement is likely to be violated at some intermediate points in time. Moreover, when the profit flow is sufficiently high so that the drift of the capital buffer is positive, this effect increases with maturity. In a one-period (one quarter) analysis, periodic monitoring reduces to terminal monitoring, and there is no difference between the two criteria. Even over short (but multiquarter) horizons, the difference is likely to be quite low since the volatility of the capital buffer over short horizons dominates its drift, so that the terminal distribution is likely to be the one yielding most pressure on capital adequacy. The longer the horizon, however, the more significant will be the drift in the capital buffer relative to volatility, and the criterion with periodic monitoring will yield increasingly different results from terminal monitoring. Terminal monitoring in these cases goes wrong because for a bank with high profit flow bank (with a low capital ratio), the most critical events in terms of capital adequacy are credit losses and ratings changes which take place in the near future. After all, if the bank does not face credit losses in the immediate future, it will have accumulated a capital buffer which can sustain quite severe losses. In descriptive terms, periodic monitoring of capital adequacy does not allow the bank to 'borrow' from its future profit flow to fulfil its capital adequacy requirements in the immediate future. This is exactly what terminal monitoring amounts to.

7 Conclusions

We have presented a framework for capital adequacy analysis, which is based on simulating the difference between a bank's actual (book) capital and its minimum (book) capital requirement, ie the bank's capital buffer. The framework is an extension of a typical Value-at-Risk analysis applied to banking portfolios. Our framework is entirely probabilistic and parameterized with empirical data. As such we believe the framework is well suited for measuring and stress testing bank capital adequacy, a task which the Basel Committee (2002) has proposed as an additional requirement to banks under the Basel II regimes.

The aim of stress testing bank capital adequacy is to determine the sufficiency, over a business cycle, of bank's free capital buffer. Given serious illiquidity of bank lending portfolios and the capital market imperfections which are particularly severe in economic downturns, hedging through holding buffer capital is widely seen as instrumental in curbing the procyclical effects of risk sensitive minimum capital requirements. Our framework is a quantitative tool for generating an estimate of the initial buffer capital required for the sufficiency of bank capital over a downturn.

Our results indicate that the introduction of rating sensitive capital requirements will necessitate higher bank capital ratios than are currently observed, should banks (or regulators) desire to maintain, or increase, the current confidence levels associated with regulatory capital adequacy. The capital ratios of banks with high quality portfolios will rise the most, but some capital savings relative to the current regime may remain. As for banks with average quality portfolios, the minimum capital requirement on credit risk will be reduced by some 20%, but this capital relief is consumed by the need for relatively higher buffers. Taken together with the additional operational risk charge, this will lead to an increase in average bank capital levels.

Many of the assumptions underlying our analysis could be relaxed. For reasons of space and clarity, we have assumed a very simplistic form for the bank's flow profit. Moreover, we have assumed away dividend payments and the (perhaps costly) option to issue new capital. These features can be incorporated into the capital adequacy simulation subject to some technical constraints. The Monte Carlo simulation framework allows one to specify eg a stochastic profit flow subject to the constraint that it be driven by ratings changes and defaults, in addition to a deterministic component. Within this constraint, also the planned growth of the bank's portfolio could be accounted for. In general, we believe this type of bank capital simulation is a natural way of complementing currently commonplace economic capital calculations, which are based on conventional credit Value-at-Risk models, but fail to account for regulatory capital constraints.

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