Mikael Bask

Long swings and chaos in the exchange rate in a DSGE model with a Taylor rule

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This paper has benefited from presentations at a seminar at the Bank of Finland and the 2nd International Conference on Economic Sciences with Heterogeneous Interacting Agents in Fairfax, Virginia, USA, June 17–19, 2007. Comments made by Antti Ripatti are also gratefully acknowledged. The usual disclaimer applies.

http://www.bof.fi

ISBN 978-952-462-386-5
ISSN 0785-3572
(print)

ISBN 978-952-462-387-2
ISSN 1456-6184
(online)

Helsinki 2007

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Bank of Finland Research
Discussion Papers 19/2007

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Abstract

A DSGE model with a Taylor rule is augmented with an evolutionary switching between technical and fundamental analyses in currency trade, where the fractions of these trading tools are determined within the model. Then, a shock hits the economy. As a result, chaotic dynamics and long swings may occur in the exchange rate, which are appealing features of the model given existing empirical evidence on chaos and long swings in exchange rate fluctuations.

Keywords: chaotic dynamics, foreign exchange, fundamental analysis, monetary policy, technical analysis

JEL classification numbers: C65, E32, E44, E52, F31

Rahapolitiikan korkosäännön vaikutus valuuttakurssin dynaamisiin vaihteluihin avotaloudessa

Suomen Pankin keskustelualoitteita 19/2007

Mikael Bask
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkimuksessa tarkastellaan valuuttamarkkinoiden kaupankäyntistrategioiden dynaamisia vaikutuksia avotaloudessa. Avotalouden dynaamista käyttäytymistä kuvataan rahapolitiikan korkosäännöllä täydennetyllä modernilla dynaamisen stokastisen yleisen tasapainon makromallilla. Valuuttamarkkinoilla kauppaa käyvät valitsevat vaihtoehtoisesti teknisen analyysin tai rakennemallin valuuttakaupan apuvälineeksi. Tällöin teknisen kaupankäynnin levinneisyys valuuttamarkkinoilla määräytyy mallissa endogeenisesti. Tarkastelun tulosten mukaan eksogeenisen häiriön iskettyä talouteen nimellisen valuuttakurssin muutokset saattavat kestää pitkään ja dynaaminen sopeutuminen kohti tasapainoa voi olla kaoottista. Tulokset ovat sikäli rohkaisevia, että asianmukaisen empiirisen näytön mukaan valuuttakurssien dynamiikka voi olla kaaosmaista ja niiden muutokset näyttävät kestävän pitkään.

Avainsanat: kaoottinen dynamiikka, valuuttamarkkinat, rahapolitiikka, tekninen analyysi, rakenneanalyysi

JEL-luokittelu: C65, E32, E44, E52, F31

Contents

1 Introduction

Aim

How is the behavior of the economy affected when technical analysis is used in currency trade and the central bank is using a Taylor rule in policy-making? What kind of dynamics can be observed? Are there some features of the economy that are observed in data?

Model setup

A dynamic stochastic general equilibrium (DSGE) model is augmented with an evolutionary switching between technical and fundamental analyses in currency trade, where the fractions of these trading tools are determined within the model due to a mechanism suggested by Brock and Hommes (1997). Specifically, currency traders choose between costly rational expectation forecasts and free trend extrapolations, where the fractions of these tools are determined by their relative performance in currency trade.

The central bank is using a policy-rule in which the interest rate is set in response to the output gap, the inflation rate, the exchange rate change, but also in response to the output gap in the previous time period to mimic optimal policy under commitment. Thus, we include a 'commitment term' in the Taylor rule, even though we do not explicitly derive an optimal policy rule for the central bank. However, the inclusion of the previous output gap in the policy-rule might give a glimpse of an idea for future research.

Main findings

Two findings stick out. Firstly, we may have chaotic dynamics in the exchange rate when the intensity of choice of the best trading strategy in currency trade is large enough and when there is a 'commitment term' in the Taylor rule. However, if the previous output gap is not included in the policy-rule, the dynamics become non-chaotic, which is an interesting finding since it is a counter-intuitive result from an economic point of view. When it comes to chaotic dynamics in exchange rates, the empirical results are mixed (see Bask, 1996–2002, Das and Das, 2007, and references therein).

A second finding is that we may have long swings in the exchange rate, which also is interesting since Engel and Hamilton (1990) have found long swings in exchange rates for the US Dollar. The origin of their research was the strong appreciation of the US Dollar in the early 1980s and the subsequent depreciation of the same currency in the second half of the same decade. Therefore, since we are not aware of any model that is able to generate this type of behavior in the exchange rate, we believe that our model could be a fruitful starting point for future research.

Relation to the literature

The mechanism in Brock and Hommes (1997) has been adopted in several papers in which different trading tools are available in asset trade (see Brock et al, 2006, Brock and Hommes, 1998, Chiarella et al, 2006, and De Grauwe and Grimaldi, 2006a, for an exchange rate application), but there are also papers in which the fractions of trading tools are determined endogenously in a different fashion (see Bask, 2007a, and Bask and Selander, 2007a, for two exchange rate applications).

An evolutionary switching between trading strategies in currency trade has never been implemented into a DSGE model that includes a Taylor rule for the central bank, but there are papers in which the fractions of trading tools are exogenously given (see Bask, 2007b, Bask and Selander, 2007b, and Pierdzioch, 2005, for a model without an interest rate rule). Branch and McGough (2006) is a closely related paper to our paper since the mechanism in Brock and Hommes (1997) is implemented into a DSGE model but for a closed economy. They introduce heterogeneity in the forecasts of the output gap and the inflation rate, and find complicated dynamics in the economy.[1]

Organization of the paper

A prototype DSGE model is presented in Section 2, whereas the trading strategies in currency trade, including the mechanism in Brock and Hommes (1997), are in focus in Section 3. Thereafter, in Section 4, we present the Taylor rule that the central bank is using in policy-making, and, in Section 5, we analyze the behavior of the economy. Section 6 concludes the paper with a discussion.

2 A prototype DSGE model

The prototype model consists of IS and AS curves for the domestic economy

$$\begin{cases} x_t = E_t\left(x_{t+1}\right) - \alpha\left(r_t - E_t\left(\pi_{d,t+1}\right)\right) \\ \pi_{d,t} = \beta E_t\left(\pi_{d,t+1}\right) + \gamma x_t + \delta r_t + \varepsilon_t \end{cases} \tag{2.1}$$

where x is the output gap, r is the interest rate, π_d is the domestic inflation rate, and ε is a cost-push shock. Thus, even though there is an endogenous cost channel in the AS curve, we also allow for exogenous cost-push shocks. However, we will only make use of a shock in the initial time period when analyzing the model since our focus is on its behavior when there are no transients. Finally, $E_t(\cdot)$ is rational expectations of the variable in focus, conditioned on the structure of the complete model and realized values of all variables in the model up to and including time t, meaning that the dating of expectations is time t.

The model in (2.1) can be viewed as Galí and Monacelli's (2005) DSGE model for a small open economy that has been augmented with a cost channel. Barth and Ramey (2001) and Chowdhury et al (2006) provide empirical evidence for a cost channel, meaning that firms' marginal costs are directly affected by the interest rate. The intuition is that firms have to pay their production factors before they receive revenues from selling their products, and, therefore, need to borrow money from financial intermediaries. Based on

[1] See Hommes (2006) for a literature survey on heterogeneous agent models in economics and finance, and De Grauwe and Grimaldi (2006b) for an introduction to exchange rate determination in a behavioral finance framework.

this intuition, Ravenna and Walsh (2006) derive a DSGE model with a cost channel. Therefore, the model in (2.1) can also be viewed as an open economy version of their model.

Unfortunately, there are no exchange rate terms in (2.1) that is necessary when incorporating market expectations in currency trade into the model. However, it is possible to rewrite (2.1) using equations that are derived in Galí and Monacelli (2005) to have a model that includes exchange rate terms. As a secondary effect, the IS and AS curves for the domestic economy are no longer functions of the domestic inflation rate, but instead the CPI inflation rate, π

$$\begin{cases} x_t = E_t\left(x_{t+1}\right) - \alpha\left(r_t - \frac{1}{1-\zeta}\cdot\left(E_t\left(\pi_{t+1}\right) - \zeta \cdot {}_t\,\Delta e^{e,m}_{t+1}\right)\right) \\ \pi_t = \beta E_t\left(\pi_{t+1}\right) + \gamma\left(1-\zeta\right)x_t + \delta\left(1-\zeta\right)r_t + \zeta\left(\Delta e_t - \beta \cdot {}_t\,\Delta e^{e,m}_{t+1}\right) + \varepsilon_t \end{cases} \tag{2.2}$$

where e is the exchange rate that is the domestic price of the foreign currency, ${}_t\Delta e^{e,m}_{t+1}$ is the expected exchange rate change according to the market that is dated at time t, and $\zeta \in [0,1]$ is an index of openness of the economy. See the Appendix for a derivation of (2.2).

A third equation in the prototype model, which is derived in Galí and Monacelli (2005), is the condition for uncovered interest rate parity

$$r_t = {}_t\,\Delta e^{e,m}_{t+1} \tag{2.3}$$

Note that we have ignored variables in the foreign economy in (2.2)–(2.3) since these variables can be treated as constants.

We will now look into what determines the expected exchange rate change according to the market.

3 Trading strategies in currency trade

When fundamental analysis is used in currency trade, agents have rational expectations regarding the next time period's exchange rate change

$${}_t\Delta e^{e,fa}_{t+1} = E_t\left(\Delta e_{t+1}\right) \tag{3.1}$$

where the cost of using fundamental analysis is c. Further on, when technical analysis is used in currency trade, which is free to use, agents extrapolate the trend in the exchange rate

$${}_t\Delta e^{e,ta}_{t+1} = \eta \Delta e_t \tag{3.2}$$

where η is the strength in trend extrapolations.

The reason that we incorporate technical analysis into the model is that questionnaire surveys made at currency markets around the world reveal that currency trade to a large extent not only is determined by an economy's performance or expected performance. In fact, a non-negligible fraction is guided by technical analysis, meaning that past exchange rates are assumed to provide information about future exchange rate movements.[2] Also, because of Friedman's (1953) argument that 'irrational' traders cannot survive in the market since they would loose money, and, therefore, be driven out of the market, we believe that it is important that the fractions of the trading tools in currency trade are determined within the model.

Now, the expected exchange rate change according to the market is

$$_t\Delta e_{t+1}^{e,m} = \omega_t \cdot_t \Delta e_{t+1}^{e,ta} + (1-\omega_t) \cdot_t \Delta e_{t+1}^{e,fa} \tag{3.3}$$

where $\omega \in [0,1]$ is the weight attached to technical analysis, which is determined by the relative performance of technical and fundamental analyses in currency trade

$$\omega_t = \frac{\exp\left(\theta \Pi_t^{ta}\right)}{\exp\left(\theta \Pi_t^{ta}\right) + \exp\left(\theta \Pi_t^{fa}\right)} \tag{3.4}$$

where θ is the intensity in which agents switch from one trading strategy to the other. Specifically, when $\theta = 0$, agents are insensitive to the performance of the trading strategies, whereas when $\theta \to \infty$, all weight is attached to the trading strategy that has performed better. Thus, the weights attached to technical and fundamental analyses are updated by the probabilities according to the logit model since we assume that the number of currency traders that use each trading tool is large (see Manski and McFadden, 1981). Also, since we assume a finite θ, we take into account the 'status quo bias' that is revealed in the psychological literature (see Kahneman et al, 1991).

Turning to the performance of the trading strategies, it is the most recent squared errors that matter

$$\begin{cases} \Pi_t^{ta} = -\left(\Delta e_t -_{t-1} \Delta e_t^{e,ta}\right)^2 = -\left(\Delta e_t - \eta \Delta e_{t-1}\right)^2 \\ \Pi_t^{fa} = -\left(\Delta e_t -_{t-1} \Delta e_t^{e,fa}\right)^2 - c = -c \end{cases} \tag{3.5}$$

Of course, one could argue that the expected performance of the trading strategies is more suitable when agents can choose the rational expectation forecast in currency trade. However, this would cause an inconsistency problem since, to be able to evaluate the expected performance of the trading strategies, agents have to know the exchange rate change according to rational expectations in advance. To pose a rhetorical question, why should agents choose technical analysis when they know the rational expectation forecast?[3]

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[2] See Oberlechner (2004) for an in-depth discussion of two large questionnaire surveys conducted at the European and the North American markets, Gehrig and Menkhoff (2006) for a survey on trading behavior that includes references to several other surveys made at currency markets (eg, Cheung and Chinn, 2001, Lui and Mole, 1998, Menkhoff, 1997, Oberlechner, 2001, and Taylor and Allen, 1992), and Neely (1997) for a layman's guide on technical analysis.

[3] Brock et al (2006) propose a solution to this problem, in the context of an asset pricing model, by saying that there is an expert manager who sells the rational expectation forecast. However, they have to implement this expert manager into the model in such a way that agents are not able to derive this forecast for free. We do not find this setup convincing, and Brock et al (2006) do not either claim that it is a realistic description of market behavior, meaning that we instead assume that agents evaluate technical and fundamental analyses by their most recent performance in currency trade.

We will now close the model with an interest rate rule for the central bank, before we analyze the behavior of the model.

4 A Taylor rule in policy-making

In 1993, Taylor (1993) demonstrated that Federal Reserve's policy could be described by the following interest rate rule

$$r_t = 0.04 + 0.5\,(y_t - \overline{y}) + 1.5\,(\pi_t - 0.02) \tag{4.1}$$

where r is Federal Reserve's operating target for the funds rate, y is real GDP, $\overline{y}$ is potential real GDP, meaning that $y - \overline{y}$ is the output gap, and π is the inflation rate according to the GDP deflator. This rule has been the center of attention in the monetary policy literature since it was presented and is often referred to as a Taylor rule, and Taylor (1999) argues that since the rule in (4.1) describes Federal Reserve's policy during a successful period, one should adopt a rule like this in policy-making.

For this reason, the central bank is using a Taylor rule in the model when setting the interest rate, and responds to the output gap, the CPI inflation rate, the exchange rate change, and the output gap in the previous time period when making its policy-decision

$$r_t = \kappa_1 x_{t-1} + \kappa_2 x_t + \kappa_3 \pi_t + \kappa_4 \Delta e_t \tag{4.2}$$

The inclusion of the previous output gap in the rule is motivated by the fact that such a term shows up in rules when the central bank commits to a policy that is optimal over time. Therefore, even though we do not explicitly derive an optimal policy rule for the central bank, we include a 'commitment term' when analyzing the behavior of the model to see the effects.[4]

5 The behavior of the economy

We illustrate our findings using the following calibrated values of the structural parameters: $\alpha = \frac{1}{2}$ since it has been estimated to be $\frac{1}{2.04}$ and $\frac{1}{1.86}$ for the US economy (see Levin et al, 2005, and Lubik and Schorfheide, 2004); $\beta = 0.99$; $\gamma = 0.072$ since this is an estimate for the US economy under the assumption of unit intertemporal substitution elasticities in consumption and labor supply (see Chowdhury et al, 2006, for details); $\delta = 0.03$ since this is an estimate for the US economy (see Chowdhury et al, 2006); and $\zeta = 0.4$, which is the parameter setting in Galí and Monacelli (2005).

We assume that the economy is hit by a cost-push shock in the initial time period when it is in steady state, $\varepsilon_0 = 0.1$, but that there are no shocks thereafter, $\varepsilon_t = 0$, $t > 0$. Moreover, the cost of using fundamental analysis in

[4] See Clarida et al (1999) for an exposition of interest rate rules in DSGE models, and Zimmermann (2003) for a more introductory text on the same topic. Woodford's (2003) seminal work on interest rate rules in policy-making should also be part of the reading list.

currency trade is $c = 0.1$, the strength in trend extrapolations in the exchange rate is $\eta = 0.9$, and the intensity in which agents switch from one trading strategy in currency trade to the other is $\theta = 5$. However, when we search for periodic cycles in the economy, the aforementioned parameters belong to the sets $c \in [0, 1]$, $\eta \in [0, 2]$ and $\theta \in [0, 20]$.

Obviously, the economy's behavior can be described in numerous ways. Also, since we are dealing with a non-linear dynamic system, several of the findings are sensitive to the parameter setting. For example, as we will see below, cycles of different orders may be close in parameter space, meaning that small changes in one of the parameters constantly change the order of the cycle. However, when the intensity of choice of the best trading strategy in currency trade is large enough, we almost always have chaotic dynamics in the economy when there is a 'commitment term' in the Taylor rule. In fact, it is not necessary to have an endogenous cost channel in the model to have chaotic dynamics.

The length of a time series in the calculations is 1000 time periods, where the first 100 periods is excluded to avoid transients. Thus, there is a fundamental difference between our analysis and the analysis of typical DSGE models. In the latter models, the focus is on the transients' behavior, whereas we focus on the dynamics when the transients have disappeared. The reason is that the economy returns to a unique steady state in typical DSGE models, meaning that there are no dynamics to study when there are no transients. As we will see below, this is clearly not the case in our model. We use *'E&F Chaos'* when analyzing the model (see Diks et al, 2006).

Periodic cycles in the economy

We start the analysis by looking at figures that show the presence of cycles of different orders in the economy. In the first figure, we make use of the findings in Clarida et al (2000) for the US economy during the Volcker-Greenspan period and set $\kappa_2 = 0.5$ and $\kappa_3 = 1.5$ in the Taylor rule. Turning to the parameters for the 'commitment term' and the exchange rate change in the rule, they belong to the sets $\kappa_1 \in [-2, 0]$ and $\kappa_4 \in [-1, 1]$. See Figure 1 for periodic cycles of order 1 (blue), 2 (red), 4 (yellow), 6 (orange) and 8 (green).

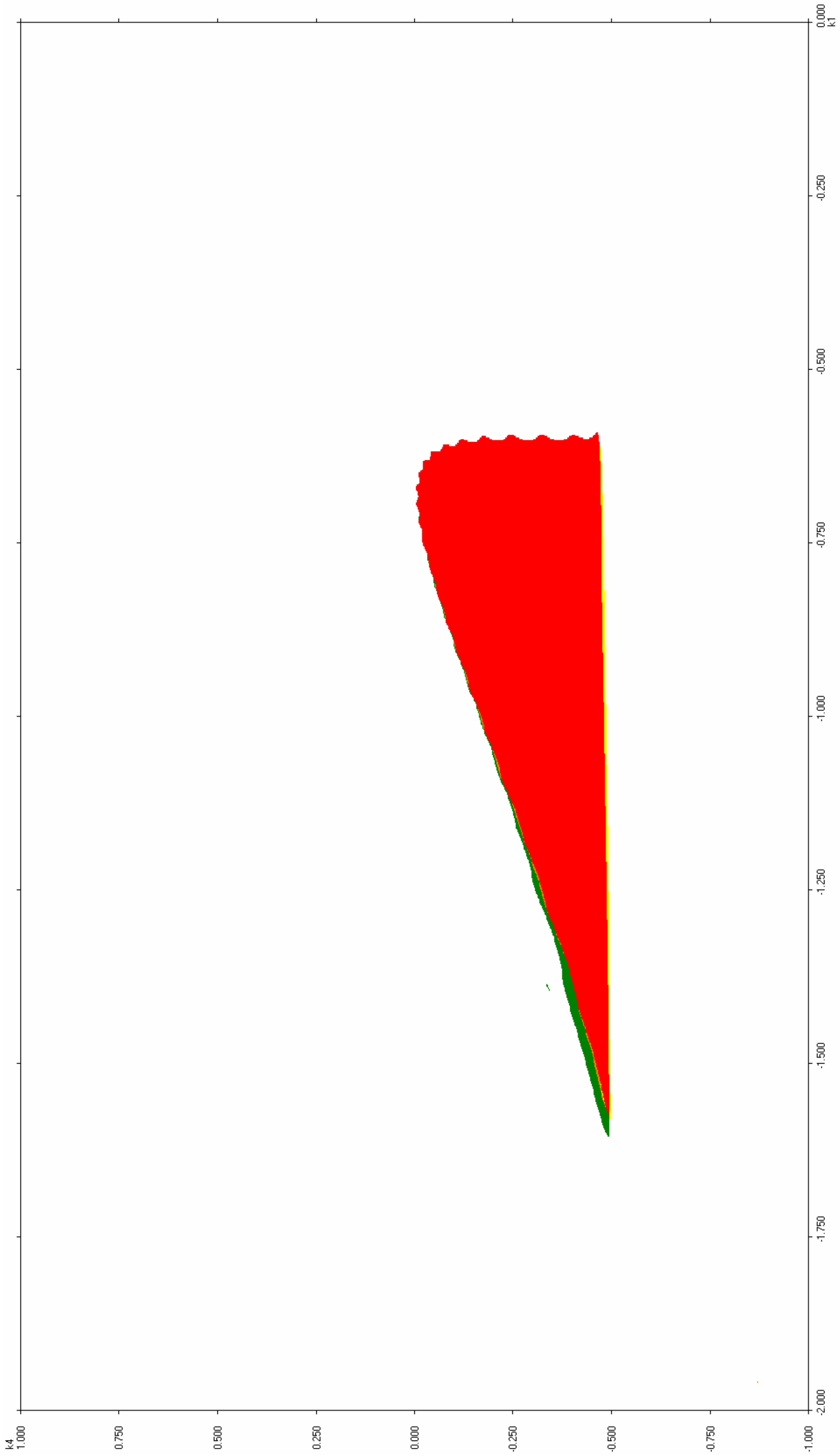

Figure 1. **Periodic cycles of order 1 (blue), 2 (red),** 4 **(yellow),** 6 **(orange) and** 8 **(green) when the rule is** $r_t = \kappa_1 x_{t-1} + 0.5 x_t + 1.5\pi_t + \kappa_4 \Delta e_t$**.** $\kappa_1 \in [-2, 0]$ **is on the horizontal axis, and** $\kappa_4 \in [-1, 1]$ **is on the vertical axis.** $c = 0.1$**,** $\eta = 0.9$ **and** $\theta = 5$**.**

The message in the figure is that the 2-cycle is a common feature of the economy for the parameter settings investigated, but also that there are cycles of orders 4, 6 and 8 in the economy. Since the structural parameters in the model are based on quarterly data, the length of these cycles range from half of a year to 2 years. Further on, because the economy is never globally unstable for the parameter settings examined, the white areas in the figure must contain cycles of higher orders[5], a-periodic cycles or even more complicated dynamics. More on this below when searching for chaotic dynamics in the economy.

In the second figure, we have set the parameters for the 'commitment term' and the exchange rate change in the Taylor rule to $\kappa_1 = -1$ and $\kappa_4 = -0.5$. This is because the first parameter in a typical DSGE model with optimal policy-making has a negative value (see Evans and Honkapohja, 2006), and that an expansionary policy in case of a depreciating exchange rate is a reasonable assumption to make. The parameters in focus are now the cost of using fundamental analysis in currency trade and the intensity in which agents switch from one trading strategy in currency trade to the other, and they belong to the sets $c \in [0, 1]$ and $\theta \in [0, 20]$. See Figure 2 for periodic cycles of the same orders as above.

Also in this figure, the message is that the 2-cycle is a common feature of the economy for the parameter settings investigated, but a difference is now that higher-order cycles are more common, especially the 6-cycle. Again, the white areas in the figure may hide more complicated dynamics since the economy is never globally unstable for the parameter settings examined.

The convexities that are visible in the figure are stable for different parameter settings. Loosely speaking, these convexities mean that if, for example, the economy is characterized by a 4-cycle, then, to still have a 4-cycle in the economy when the cost of using fundamental analysis in currency trade increases, the intensity in which agents switch from one trading strategy in currency trade to the other must decrease.

The parameters in focus in the third figure are the strength in trend extrapolations in the exchange rate and, again, the intensity of choice of the best trading strategy in currency trade, and they belong to the sets $\eta \in [0, 2]$ and $\theta \in [0, 20]$. See Figure 3 for periodic cycles of the same orders as above.

Compared with previous figures, it is now relatively common that the economy is in rest, even though the 2-cycle still dominates when examining different parameter settings. Periodic cycles of order 4 are also slightly more common than previously, but cycles of order 6 are now less common. Another feature of the model that is visible in the figure, especially when the strength in trend extrapolations in the exchange rate is large, is that cycles of different orders are close in parameter space, meaning that small changes in one of the parameters constantly change the order of the cycle.

A bifurcation diagram is found in Figure 4, where the bifurcation parameter is the intensity in which agents switch from one trading strategy in currency trade to the other. This parameter belongs to the set $\theta \in [0, 20]$, and typical bifurcation routes to more complicated dynamics are visible in the figure, where the exchange rate change is on the vertical axis.

[5] This is because we have not found periodic cycles of order 3, 5 and 7.

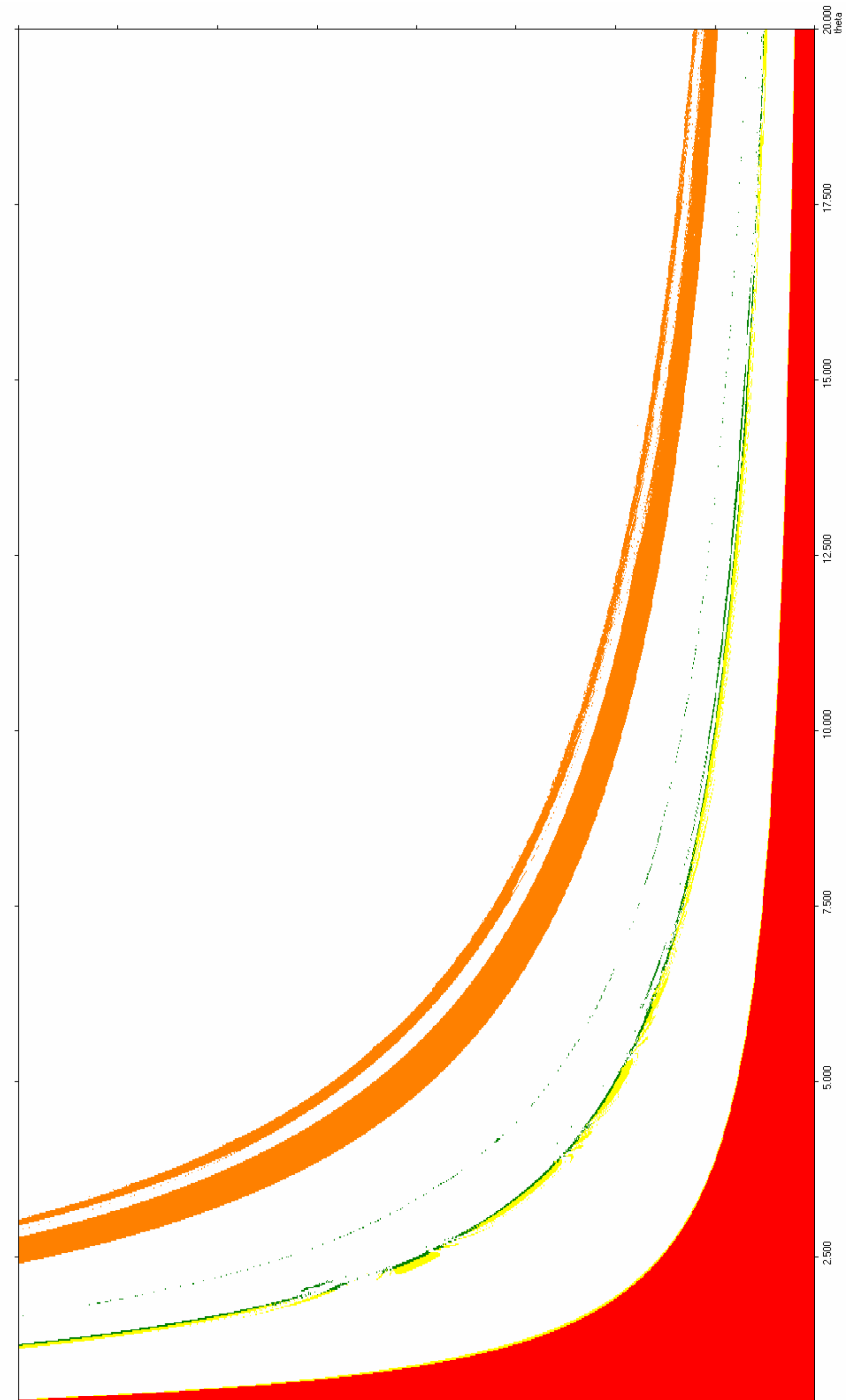

Figure 2. **Periodic cycles of order** 1 **(blue),** 2 **(red),** 4 **(yellow),** 6 **(orange) and** 8 **(green) when the rule is** $r_t = -x_{t-1} + 0.5x_t + 1.5\pi_t - 0.5\Delta e_t$**.** $\theta \in [0, 20]$ **is on the horizontal axis, and** $c \in [0, 1]$ **is on the vertical axis.** $\eta = 0.9$**.**

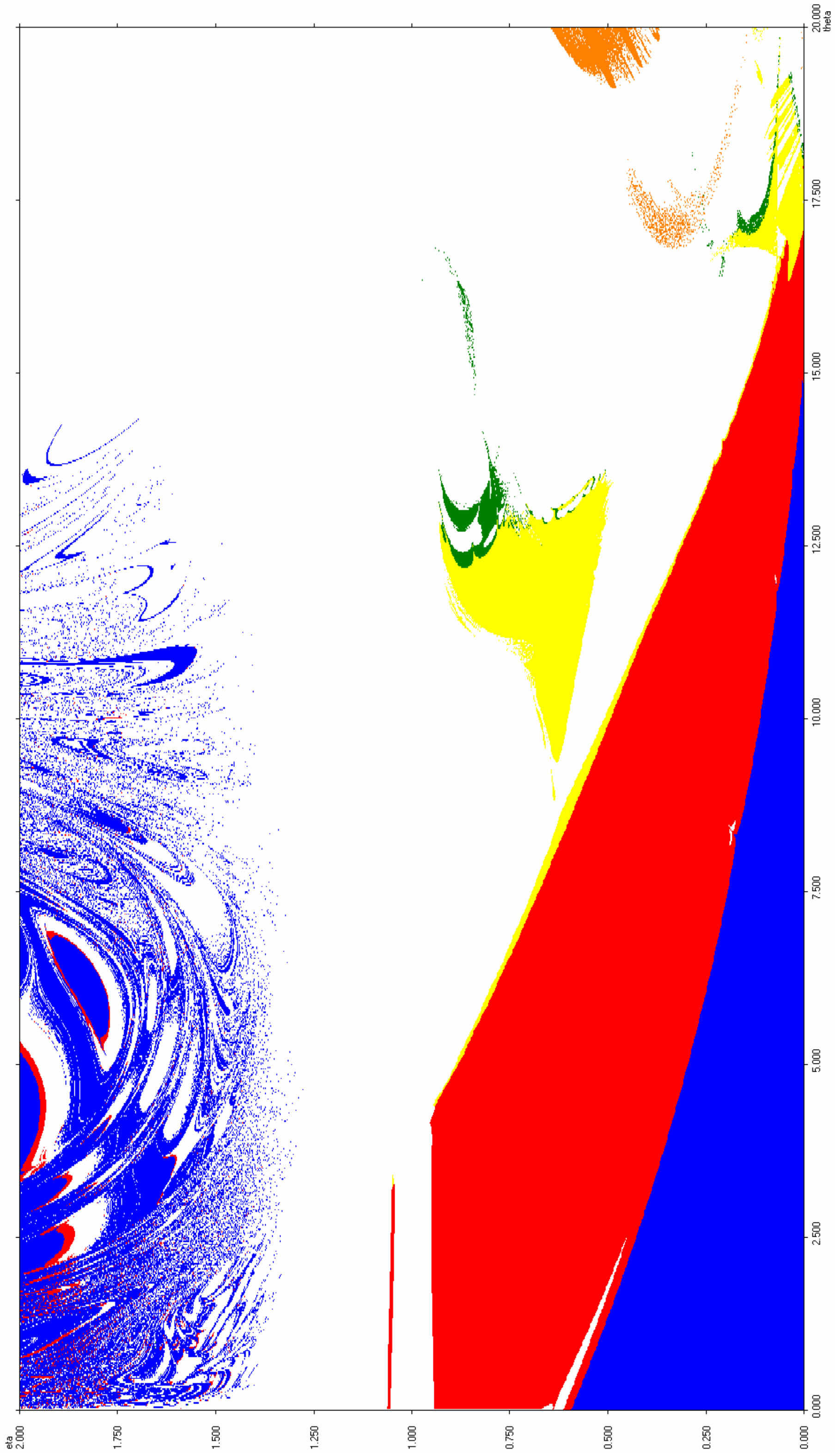

Figure 3. **Periodic cycles of order** 1 **(blue),** 2 **(red),** 4 **(yellow),** 6 **(orange) and** 8 **(green) when the rule is** $r_t = -x_{t-1} + 0.5x_t + 1.5\pi_t - 0.5\Delta e_t$**.** $\theta \in [0, 20]$ **is on the horizontal axis, and** $\eta \in [0, 2]$ **is on the vertical axis.** $c = 0.1$**.**

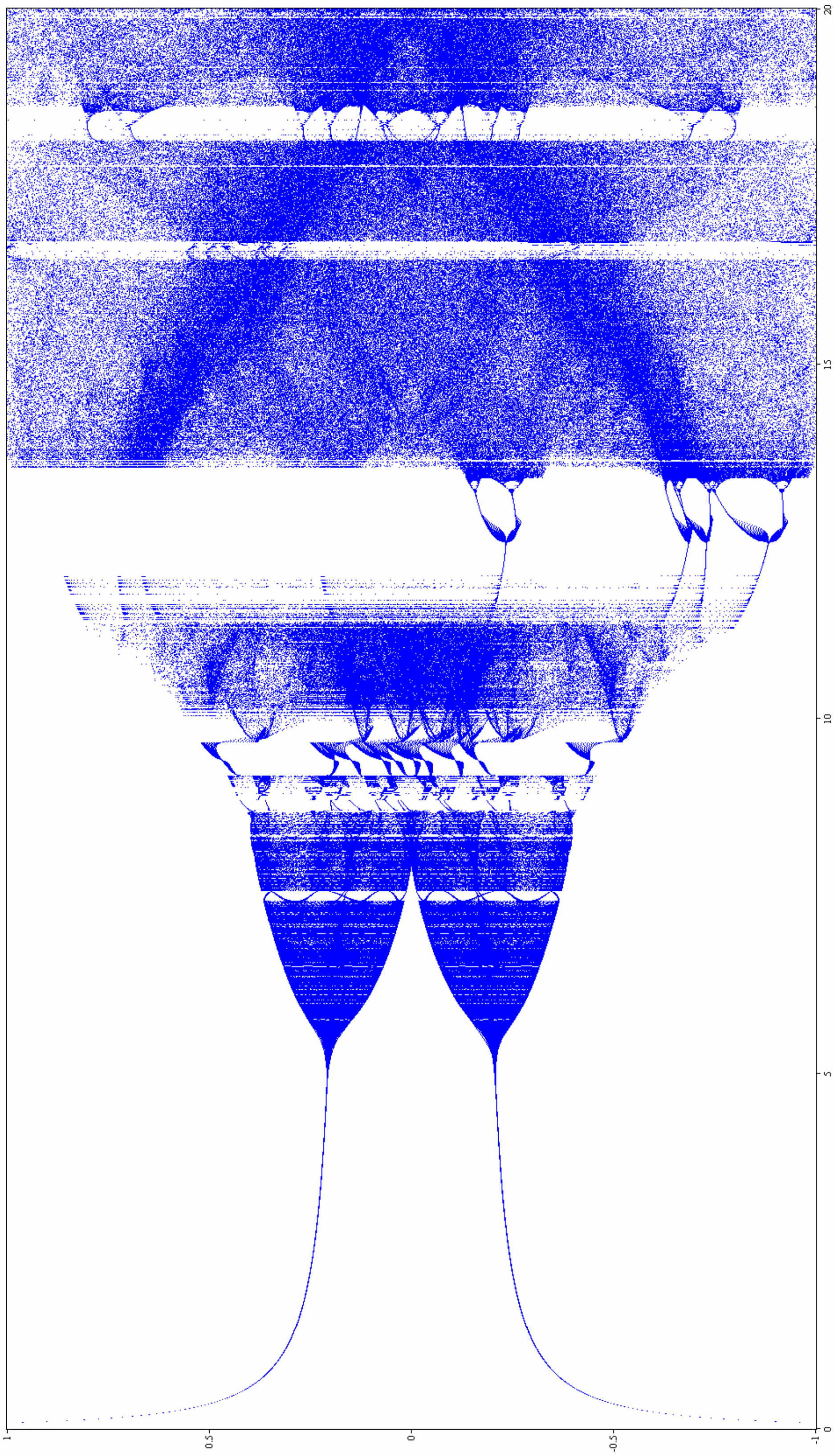

Figure 4. **Bifurcation diagram when the rule is $r_t = -x_{t-1} + 0.5x_t + 1.5\pi_t - 0.5\Delta e_t$. $\theta \in [0, 20]$ is on the horizontal axis, which is the bifurcation parameter, and $\Delta e_t \in [-1, 1]$ is on the vertical axis. $c = 0.1$ and $\eta = 0.9$.**

Let us now shift focus from periodic cycles in the economy to more complicated dynamics, such as chaotic dynamics, that may be hidden in the white areas in previous figures.

Chaotic dynamics and long swings in the economy

A typical result in the asset pricing literature that make use of the mechanism in Brock and Hommes (1997) is the presence of chaotic dynamics when the intensity of choice of the best trading strategy in asset trade is large enough (see Brock et al, 2006, Brock and Hommes, 1997–1998, and Chiarella et al, 2006). For this reason, but also encouraged by Figure 4 above, we search for chaotic dynamics in the model. The characterizing feature of this type of dynamics is the property of 'sensitive dependence on initial conditions': any two trajectories with arbitrarily close, but not equal, initial conditions will diverge from each other at exponential rates. Globally, however, the trajectories are bounded.

An operational definition of chaotic dynamics is that the largest Lyapunov exponent should be positive, and this is because this exponent measures the average exponential convergence or divergence of two trajectories with nearby, but not equal, initial conditions. Therefore, in Figure 5, the largest Lyapunov exponent as a function of the intensity in which agents switch from one trading strategy in currency trade to the other is shown.

Clearly, the economy is characterized by chaotic dynamics when the aforementioned switching between trading strategies is intense enough. Also, by comparing Figures 4 and 5, one can see that areas in the bifurcation diagram that seems to contain more complicated dynamics most often are associated with a positive largest Lyapunov exponent, meaning that we have chaotic dynamics.

In Figure 6, a typical exchange rate change series is plotted when the economy is characterized by chaotic dynamics, where the intensity of choice of the best trading strategy in currency trade is $\theta = 15$, and the length of the time series is 400 time periods.

Besides that cycles in the economy never repeats itself exactly, it is also clear in the figure that there are cycles at different frequencies, both high and low frequencies. Thus, if we start with the latter observation, it seems that the model is able to reproduce the long swings that Engel and Hamilton (1990) found in exchange rates for the US Dollar. When it comes to chaotic dynamics in exchange rates, the empirical results are mixed (see Bask, 1996–2002, Das and Das, 2007, and references therein).

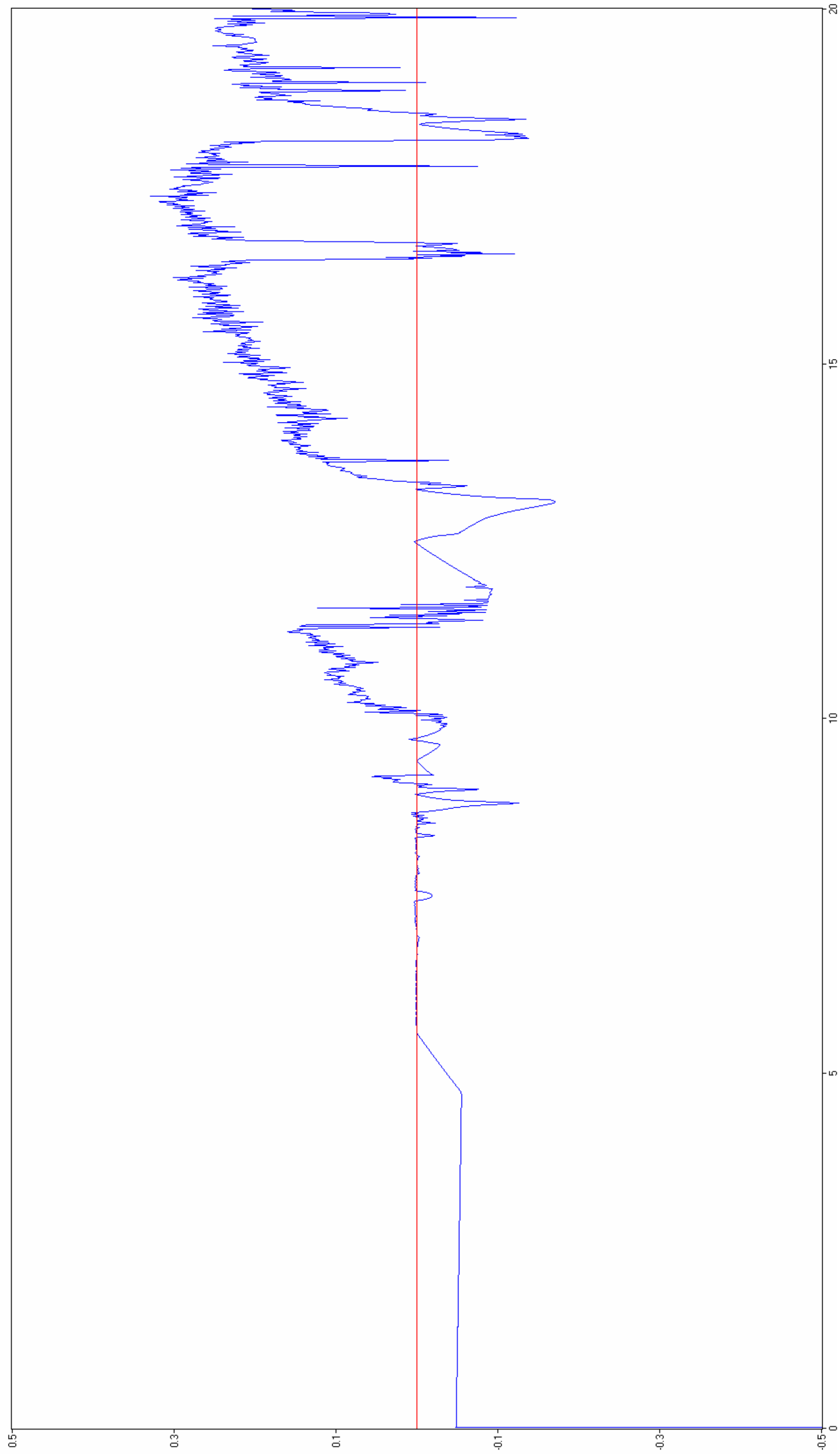

Figure 5. **The largest Lyapunov exponent,** $\lambda_1 \in [-0.5, 0.5]$**, as a function of** $\theta \in [0, 20]$ **when the rule is** $r_t = -x_{t-1} + 0.5x_t + 1.5\pi_t - 0.5\Delta e_t$**.** $c = 0.1$ **and** $\eta = 0.9$**. There is chaotic dynamics when** $\lambda_1 > 0$**.**

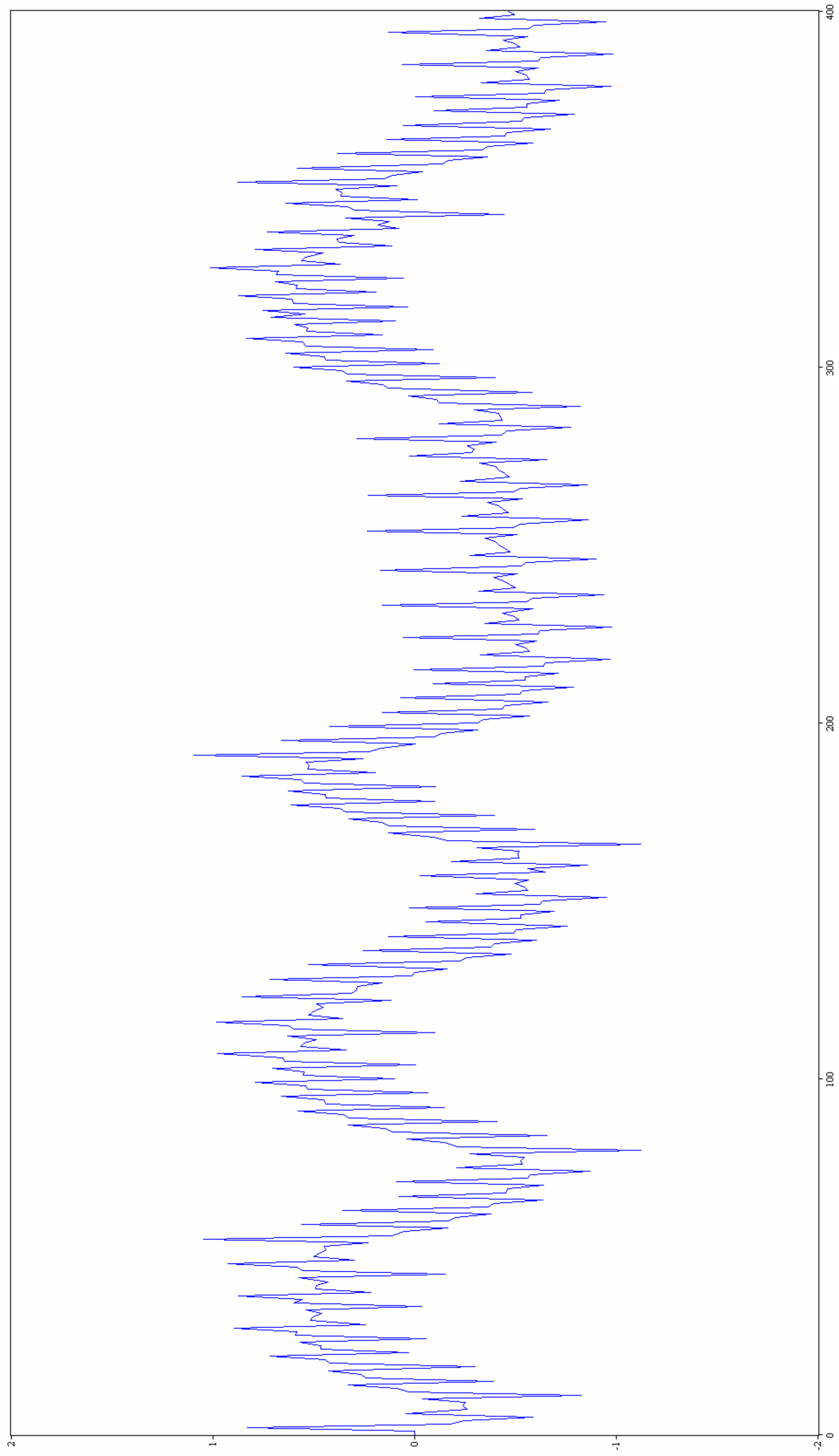

Figure 6. **A time series for Δe_t when the rule is $r_t = -x_{t-1} + 0.5x_t + 1.5\pi_t - 0.5\Delta e_t$. $c = 0.1$, $\eta = 0.9$ and $\theta = 15$.**

6 Discussion

Main findings

Our aim has been to examine how the use of technical analysis in currency trade may affect the behavior of the economy when the central bank is using a Taylor rule in policy-making. Two findings stick out: (i) chaotic dynamics in the exchange rate may occur when the intensity of choice of the best trading strategy in currency trade is large enough and when there is a 'commitment term' in the Taylor rule; and (ii) long swings in the exchange rate may also occur under the same circumstances.

If we start with the first finding, the predictability of monetary policy might get lost for agents in the economy, which is an important ingredient in the present-day era of independent central banks, openness and inflation rate targeting, at least among central banks in industrialized countries. It is interesting to note that without a 'commitment term' in the Taylor rule, the economy is no longer characterized by chaotic dynamics. More on this below when we outline an agenda for future research.

When it comes to long swings in the exchange rate, this finding is very interesting due to the fact that Engel and Hamilton (1990) found this phenomenon when examining several exchange rates for the US Dollar. The starting point for their research was the strong appreciation of the US Dollar in the early 1980s and the subsequent depreciation of the same currency in the second half of the same decade. Having in mind that the structural parameters in the model are based on quarterly data, a typical period of appreciation or depreciation of the exchange rate is around 10 years long. Even though this period is a bit too long, the interesting thing is that the model, in fact, is able to generate long swings in the exchange rate.

Finally, it is worth emphasizing that our model essentially is a deterministic model since there is only a cost-push shock in the initial time period and that we have analyzed the behavior of the model when there are no transients. In other words, it is not necessary to have a stochastic model to be able to reproduce long swings in the exchange rate and to have erratic behavior in the model's variables.

Research agenda

One can easily think of several important directions for future research. First, it would be interesting to implement the evolutionary switching mechanism in a slightly different manner than we did in this paper. For example, one could augment the model with bond markets at home and abroad, and use past profits in currency trade as the performance measure. De Grauwe and Grimaldi (2006a) is, therefore, a useful starting point since they implement such a performance measure into an asset pricing model for the exchange rate.

Another research direction is to derive interest rate rules that implement optimal policy, both under discretion and commitment in policy-making. This type of research would be interesting to accomplish since it could give rise to counter-intuitive results. Recall that we have chaotic dynamics when there is a 'commitment term' in the Taylor rule, but that the dynamics is non-chaotic when this term is excluded from the rule. In other words, a policy that is superior from a welfare perspective might introduce complicated dynamics in the economy and a less predictable monetary policy. Of course, such a finding would call for a closer examination of the microeconomic foundations of the welfare function that the central bank is optimizing.

Finally, a third research direction would be to implement the mechanism in Brock and Hommes (1997) into a DSGE model that has not been linearized around steady state that, otherwise, is the typical case in the literature. However, Eusepi (2007) is an interesting exception in which such a linearization has not been done. It goes without saying that this type of research, especially when the central bank is using an optimal policy rule in policy-making, can be quite challenging, but at the same time interesting to accomplish.

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Appendix

We make use of the following equations that are derived in Galí and Monacelli (2005)

$$\begin{cases} \pi_t = \pi_{d,t} + \zeta \Delta s_t \\ s_t = e_t + p_t^* - p_{d,t} \end{cases} \tag{A.1}$$

where s is the terms of trade, p^* is the index of foreign goods prices, p_d is the index of domestic goods prices, and the asterisk denotes a foreign quantity. Firstly, shift the first equation in (A.1) one time period forward in time

$$E_t(\pi_{d,t+1}) = E_t(\pi_{t+1}) - \zeta E_t(\Delta s_{t+1}) \tag{A.2}$$

Secondly, shift the second equation in (A.1) one time period forward in time, and take differences

$$\begin{aligned} E_t(\Delta s_{t+1}) &= {}_t\Delta e_{t+1}^{e,m} + E_t\left(\Delta p_{t+1}^*\right) - E_t(\Delta p_{d,t+1}) \\ &= {}_t\Delta e_{t+1}^{e,m} + E_t\left(\pi_{t+1}^*\right) - E_t(\pi_{d,t+1}) \end{aligned} \tag{A.3}$$

Thirdly, substitute (A.3) into (A.2)

$$E_t(\pi_{d,t+1}) = \frac{1}{1-\zeta} \cdot \left(E_t(\pi_{t+1}) - \zeta\left({}_t\Delta e_{t+1}^{e,m} + E_t\left(\pi_{t+1}^*\right)\right)\right) \tag{A.4}$$

Fourthly, shift (A.4) one time period backward in time

$$\pi_{d,t} = \frac{1}{1-\zeta} \cdot (\pi_t - \zeta(\Delta e_t + \pi_t^*)) \tag{A.5}$$

Finally, substitute (A.4) into the first equation in (2.1), substitute (A.4)–(A.5) into the second equation in (2.1), ignore variables in the foreign economy since they can be treated as constants, and (2.2) is derived.

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