Hanna-Leena Männistö

Forecasting with a forward-looking DGE model – combining long-run views of financial markets with macro forecasting

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Abstract

To develop forecasting procedures with a forward-looking dynamic general equilibrium model, we built a small New-Keynesian model and calibrated it to euro area data. It was essential in this context that we allowed for long-run growth in GDP. We brought additional asset price equations based on the expectations hypothesis and the Gordon growth model, into the standard open economy model, in order to extract information on private sector long-run expectations on fundamentals, and to combine that information into the macro economic forecast.

We propose a method of transforming the model in forecasting use in such a way, as to match, in an economically meaningful way, the short-term forecast levels, especially of the model's jump-variables, to the parameters affecting the long-run trends of the key macroeconomic variables. More specifically, in the model we have used for illustrative purposes, we pinned down the long-run inflation expectations and domestic and foreign potential growth-rates using the model's steady state solution in combination with, by assumption, forward looking information in up-to-date financial market data. Consequently, our proposed solution preserves consistency with market expectations and results, as a favourable by-product, in forecast paths with no initial, first forecast period jumps. Furthermore, no ad hoc re-calibration is called for in the proposed forecasting procedures, which clearly is an advantage from point of view of transparency in communication.

Key words: forecasting, New Keynesian model, DSGE model, rational expectations, open economy

JEL classification numbers: E17, E30, E31, F41

Ennustamisesta eteenpäin katsovalla yleisen tasapainon mallilla – rahoitusmarkkinoiden pitkän ajan odotusten yhdistäminen kokonaistaloudelliseen ennusteeseen

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Hanna-Leena Männistö
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Tiivistelmä

Tässä tutkimuksessa kehitetään eteenpäin katsovan yleisen tasapainon mallin ennustekäyttöä. Tätä varten rakennetaan pieni uuskeynesiläinen malli ja kalibroidaan se käyttäen euroalueen dataa. Ennustekäyttöä ajatellen on oleellista, että mallissa määräytyy BKT:n pitkän ajan kasvu. Avoimen talouden yleistä mallispesifikaatiota laajennetaan varallisuusvaateiden hintayhtälöillä, jotka perustuvat korkojen aikarakenteen odotushypoteesiin sekä osakkeiden hintojen osalta Gordonin kasvumalliin. Näin on mahdollista yhdistää kokonaistaloudelliseen ennusteeseen rahoitusmarkkinainformaatiota, joka kuvaa yksityisen sektorin odotuksia talouden perustekijöiden pitkän ajan trendeistä.

Työssä ehdotetaan mallin ennustekäyttöön menetelmää, jossa malli ratkaistaan käännettynä siten, että sen eteenpäin katsovien muuttujien lähtötasot saatetaan talousteorian mukaisesti yhteensopiviksi mallin makromuuttujien pitkän ajan trendien kanssa. Erityisesti esimerkkimallin tapauksessa pitkän ajan inflaatio-odotukset sekä koti- ja ulkomainen potentiaalinen BKT:n kasvu ratkaistaan yhdistämällä mallin pitkän ajan ratkaisuun ajantasainen rahoitusmarkkinadata, joka sisältää tulevaa talouskehitystä koskevaa informaatiota. Työssä ehdotettu ratkaisumenetelmä käsittelee markkinaodotuksia johdonmukaisesti, ja lisäetuna seuraa, ettei ennusteuriin synny ensimmäisen ennustejakon perusteettomia hyppyjä. Ehdotettu ennustemenettely ei myöskään sisällä mallin parametrien ad hoc uudelleenkalibrointia, mikä on etu avoimen viestinnän kannalta.

Avainsanat: ennustaminen, uuskeynesiläinen malli, dynaaminen stokastinen yleisen tasapainon malli, rationaaliset odotukset, avoin talous

JEL-luokittelu: E17, E30, E31, F41

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1 Introduction: Combining expectation extraction and DGE model based macro forecasting

We build a small New Keynesian dynamic general equilibrium model for the euro area to be used as a tool in discussing forecasting procedures. We aim at a transparent specification that allows us to focus on the specific issue of how assessment of factors effecting long-run trends effect the short-term forecast. As a benchmark assessment of long-run trends, we use market expectations.

As the use of DSGE models in forecasting is currently not yet widespread, the documentation of forecasting practices is rather scarce and discussion on best practicies is only evolving. Our motivation to take part in that discussion comes from the fact that the Bank of Finland has already built two DSGE models – one for the euro area (the EDGE model[1]), another for the domestic economy (the AINO model[2]) – and has started to use these models in forecasting and policy analysis. The small model discussed here is hoped to provide a transparent 'exercise' set-up that could support developing procedures for these main models.

With a forward-looking model, first-period jumps in variables with leads, like private consumption or the exchange rate, has been considered a problem in practical forecasting.[3] In some early applications, the information implied by the jumps – big changes in values accuring at the first forecast period compared to historical level – was simply ignored. When the Bank of Finland macroeconomic model BOF5[4], a structural model based on inter temporal optimisation, was used for forecasting and policy analysis (in 1996–2004), the rational expectations hypothesis was fully applied only in policy analysis and scenarios. In forecasting, for practical reasons, the RE was not fully applied. Rather, the equations with leads were replaced with backward-looking transformations, some analytical, some re-estimated as error-correction type of equations with the attractor taken from the original forward-looking specification. This alternative version of the model produced a somewhat different dynamics but the same long-run solution as the version with leads. The first period forecast jumps were avoided, and short-term forecast paths could be manipulated with add-factors to bring in off-model judgemental information.

With a new generation of general equilibrium models, formerly used for policy simulations, now taken also into active forecasting use, forecasters face new challenges. Questions about the forecasting performance of these type of models arise. Typically, in the short-term, the DSGE models can not compete in forecast accuracy with more traditional tools, while in the medium-term, the

[1] Kortelainen (2002) and Kortelainen – Tarkka (2001).
[2] Kilponen – Ripatti – Vilmunen (2004).
[3] Some typical examples of these jumps are illustrated in Figure A1 in Appendix 4.
[4] Willman – Kortelainen – Männistö – Tujula (2000).

results are more promising.[5] This is easy to understand as in a forward-looking model, even with a constant steady state, key variables may jump in the first forecast period. If we allow for growth in the steady state, the model becomes even more difficult to control as a short-term forecasting tool.

Within general equilibrium framework, exogenising variables or manipulating their forecast paths with add-factors is difficult and potentially mis-leading without breaking the logic of the model. One approach, taken first by the Bank of England in their new model[6], is to build a backward-looking non-core structure on and above the forward-looking DSGE core model. Generally speaking, while forecasting teams working with DSGE models – also at the Bank of Finland – respect the DSGE model structure especially in the long-run, they try to find helpful ways of bringing in short-term information or to smoothen the first period jumps.

Ideally, the DSGE modelling approach and rational expectations would imply that the first period jumps should not be considered a problem. Instead, one could use all the information provided by the model and the data, ie one could find economically meaningful ways of adjusting short-term levels so that they would not conflict with assessment of the parameters effecting long-run trends. In particular, we suggest the long-term assessment be based on market expectations. The motivation of this paper is to develop a forecasting procedure that incorporates these ideas.

Extracting market expectations from financial market instruments has become popular in the 1990's.[7] At the Bank of Finland, Junttila (2002) applied the so called Economic Tracking Portfolios methodology to forecast future values of inflation and changes in industrial production in the US and in major euro countries. Kajanoja (2004) developed a framework that builds mainly on the same economic thinking as our DGE model but also utilizes information on equity index futures. This framework is regularly used at the Bank in monitoring growth and inflation expectations extracted from financial market data on a real-time basis, both for the euro area and for the United States. However, we are not aware of any previous application of these ideas to a forward-looking DGE model in a macro economic forecasting context.

To demonstrate these thoughts, we build a small model and present simulation exercises with it. The model is based on intertemporal utility maximisation and rational expectations. As we, for simplicity, abstract from capital and wealth accumulation, and as we further do not bring any inertia in the form of myopic behaviour into the model, this sketchy model is likely to show relatively big first period jumps. More generally, in a forward-looking DGE model, the first forecast

[5] Zimmermann (2001), Del Negro – Schorfheide – Smets – Wouters (2004).
[6] Harrison – Nikolov – Quinn – Ramsay – Scott and Thomas (2005).
[7] Söderlind – Svensson (1997) give a survey.

period jumps may reflect inconsistencies in long-term assumptions. In this type of a forecasting framework, the long-term assumptions are of crucial importance.

The model we use for demonstration consists of an optimizing-based IS-equation, a New Keynesian Phillips -curve, where we rely on recent work by Paloviita (2004), and a standard Taylor rule for monetary policy, parameter values of which are taken from the literature. In contrast to most recent DGE models, even those rich enough in structure to include capital accumulation and labour markets, we build an open economy model. This reflects the view that the exchange rate plays an important role in monetary policy transmission in the euro area. An uncovered interest parity determines short-term dynamics of the exchange rate while in the steady state, real exchange rate balances trade, thus adjusting to productivity differentials at home and abroad.

In the forecasting context, we would like to use market information on the private sector's growth and inflation expectations. Therefore, we bring into the standard model a long-term interest rate equation based on the expectations hypothesis and an equity price equation based on the Gordon growth model. For the forecasting use of the model, it is essential that we allow for GDP growth in the steady state, and that short-term growth rates may deviate from long-term trends. Potential output growth is modelled as an exogenous process.

For forecasting use of the model, we suggest a model transformation or invertion method, and compare forecasts produced with the transformed model to those produced with the original model. The economic thinking does not change in this transformation. The transformation method is related to 'reverse engineering' as refered to in financial markets literature.

In the model transformation, equations for some chosen jump variables, in our example those for the asset prices, are lagged one period. As a result, the transformed model produces not jumpy but smooth forecast paths for the asset prices, which is a nice by-product. More importantly, the steady state conditions of these, in essence forward-looking equations, are instead used to govern long-term of the endogenised potential GDP growth and inflation rate. Thus, instead of using exogenous assumptions on long-term trends, like in the original model, we endogenise them in forecasting use so that the steady state values of long-term growth and inflation match with private sector ecpextations as revealed by the model and latest financial market data.

In calibrating the model, we use euro area aggregate national accounts data for 1977q1–2003q4, and for the foreign variables, corresponding US data. Some parameter values are fixed based eg on sample averages, but we also apply the GMM estimation method to discuss the key parameters of the IS-curve, the deep parameters of the utility function, that are crucial in determining economic growth. We find some support for the canonical optimizing-based IS-curve and use the forward-looking model as a benchmark. In the forecasting application, to check the transformation method works cross various specifications, we bring some inertia into the model by applying alternative hybrid specifications.

The report is organised as follows. To motivate the chosen modelling approach, models with optimising-based IS-equations are first discussed in Chapter 2. After presenting derivation of the IS-curve of the model in Chapter 3, we turn to the data and calibration of parameters in Chapter 4. Model specification is discussed in Chapter 5. Calibration is finalised based on simulations reported in Chapter 6. In Chapter 7 we report simulations to illustrate how parameters determining long-run trends affect short-term forecast levels. We suggset a model transformation method, that combines expectations extraction from financial market data into a macro economic forecasting context. Chapter 8 discusses robustness of the transformation method cross model specifications and Chapter 9 concludes by summarisizing the suggested forecasting procedures.

2 A small New Keynesian policy model, motivation for specification

Small New Keynesian models, based on an aggregate demand equation or an IS-curve, an inflation equation or a Phillips-curve and policy rule, a Taylor rule, for the short-term nominal interest rate, have since the late 1990's become extensevily used to study macroeconomic dynamics and to analyse monetary policy rules[8], and more recently, also fiscal policy rules. In these models, the IS-curve and the New Keynesian Phillips Curve are based on intertemporal optimising behaviour of private sector agents, and the interest rate feedback rule may be based on some optimisation of social welfare, if derived from an optimisation problem of the policy-maker. Essentially, the forward-looking IS-curve captures consumption smoothing, and the forward-looking Phillips-curve reflects optimal price setting in the presence of price rigidities. Potential output is modelled as an exogenous process. For theoretical foundations of these optimizing models, often labelelled as Small New Keynesian Policy Models, we refer to text-book presentation by Woodford (2003).

To motivate the structure of our small policy and forecasting model, and to give some background for the calibration issues, we'll discuss briefly some theoretical and empirical issues on these models, focusing mainly on the IS-equation, the specification of which is vital to our excersise.

While the theoretical underpinnings of the optimizing policy models imply a forward-looking 'canonical' specification, empirical applications often include

[8] See Rotemberg – Woodford (1997), Clarida – Gali – Gertler (1999) and Christiano – Eichenbaum – Evans (2005).

lagged endogenous explanatory variables in either the IS-curve or the Phillips-curve or both. In the specification of a forwad-looking Taylor-rule, as well, a lagged interest rate is often added to capture interest rate smoothing. These 'hybrid' specifications are often motivated so that otherwise the model would not be able to replicate persistence that is typically found in macroeconomic time series data.

The performance of forward-looking vs backward-looking specifications of the New Keynesian Phillips curve (NKPC) has been subject of extensive empirical research. Gali and Gertler (1999) is an example of empirical studies where the lagged inflation term is found important. As a theoretical motivation for the hybrid specification they assume some price setters are not optimizing but follow a rule-of-thumb instead.

Paloviita (2004) uses both aggregate and pooled data to examine the empirical performance of the NKPC and its hybrid specification in the euro area. Instead of imposing rational expectations, OECD forecasts are used in this study as proxies for agents' inflation expectations. It is found that OECD inflation forecasts perform relatively well as a proxy for inflation expectations, since under this approach the European inflation process can be modelled using the NKPC. However, inflation can be modelled even more accurately by the hybrid Phillips curve.

In contrast, empirical studies of the New Keynesian IS-curve are, until recently, relatively few. Rotemberg and Woodford (1997) estimate a forward-looking IS-equation as part of a small policy model. Smets (2000) estimates a hybrid specification both for output and inflation using euro area synthetic data over the period 1974–1998, and finds a sligthly larger weight for the forward-looking components, and finds a significant negative coefficient for the expected real interest rate in the IS-equation. Domenech, Ledo and Taguas (2001) find rather similar parameter values for the euro area using quarterly data over 1986 to 2000, the period over which they find stability of the empirical interest rate rule applied in the study.

The range of empirical results reflects both specification and estimation issues. Proper treatment of expectations in estimation – either succesful instrumenting of variables with leads or succesful use of direct measures of expectations – is one key issue. Kara and Nelson (2004) point out that parameters of the IS-equation are empirically more stable if a forward-looking specification is chosen. They conclude that the estimate of interest elasticity of aggregate demand in the US is about -0.2, accross several studies. McCallum (2001) argues for correcting the often reached empirical result of -0.2 to a value of -0.4 to reflect investment spending that is not explicitly modelled in the aggregative models.

The most compelling puzzle in some empirical IS curves is that the real interest rate does not seem to have a significantly negative effect on output. Goodhart and Hofmann (2003) asses the performance of the New Keynesian IS curve for the G7 countries, and conclude this puzzle is due to omission of determinants of aggregate demand. By extending the specification – albeit without discussing the underlying modified optimisation problem – to include asset prices (real residential property prices), they get rid of the puzzle.

An obvious candidate for omitted variables in the standard closed economy models are the foreign variables that would extend the model into an open economy one. Svensson (2000) extends the optimizing-based model by building a model of a small open economy and the rest-of-the-world. The exchange rate enters both the IS-equation and the Phillips-curve, and allows additional channels for the transmission of monetary policy. As an asset price, the exchange rate contributes to the role of expectations. Foreign demand directly affects aggregate demand in the IS-equation. While empirical relevance of foreign demand in the IS-equation is found in many studies, that of the exchange rate is more of an open issue.

While there is no consensus regarding the optimizing-based empirical small policy model, we summarize below, for the purposes of this study, key properties of the model we prefer to work with.

- The forward-looking specification will be our benchmark case.
- We specify the IS-equation to imply growth in the steady state.
- Accordingly, the most interesting calibration issue is that of the coefficient of the expected real interest rate in the IS-equation.
- Government consumption is included in the IS-equation to make GDP less trivially dependent on private consumption.
- We prefer the open economy version to include more realistic monetary policy transmission and the foreign demand channel.
- Of asset prices, the real exchange rate is included in the IS-equation, so it is a part of the simultaneous block of the model.
- Additional asset price equations, based on the Gordon growth model and the expectation hypothesis, are motivated by the forecasting use of the model, ie by the expectation extraction that will be combined to the macro model.
- As for the Phillips-curve, we rely on Paloviita (2004).
- For the Taylor rule, a standard specification is used.

3 Derivation of the expectational IS-equation

3.1 Basic results with growth

Let's start with a closed economy model[9]. Abstracting from capital accumulation, the resource constraint is of the form

$$Y_t = C_t + G_t \tag{3.1}$$

where Y is real income, C private consumption and G exogenous government consumption. Fiscal policy is not explicitly modelled here and hence, implicitly, government consumption is financed by non-distortionary lump-sum taxes.

A representative household, consumer-producer, is, as we abstract from the labour hours vs. leisure decision, assumed to maximise expected welfare W, ie a discounted sum of utilities of the form

$$W = \sum_{t=0}^{\infty} \beta^t (U(C_t)) \tag{3.2}$$

subject to the resource constraint (3.1), where $0 < \beta < 1$ is the subjective rate of discount and $U(C_t)$ is a standard utility function with $\frac{\partial U(C_t)}{\partial C} \equiv U'(C_t) > 0$ and $U''(C_t) < 0$.

The first order condition for inter temporal optimisation is

$$U'(C_t) = R_t \beta U'(C_{t+1}) \tag{3.3}$$

where R is the expected real rate of interest $R_t = 1 + i_t - E_t \pi_{t+1}$, i is the riskless nominal one period interest rate determined by the central bank according to the Taylor rule, and expected inflation is $E_t \pi_{t+1} = \frac{E_t P_{t+1}}{P_t}$.

We follow the notation that lower case letter denote logs, deviations around the steady state are denoted by hat and the steady state by bar, so $\hat{x}_t \equiv x_t - \bar{x}$ where $x_t = \log(X_t)$ and $\bar{x}$ refers to the steady state value of x.

To get a specification that can be estimated or calibrated, let's make the common assumption that preferences are of the constant relative risk aversion (CRRA) type, so the utility function is of the form

[9] We follow Woodford (2003) Chapter 4 loosely in the standard case of no growth in the steady state.

$$U_t(C) = \frac{C^{1-\theta}}{1-\theta} \tag{3.4}$$

where θ is the coefficient of risk aversion and $\frac{1}{\theta}$ is the intertemporal elasticity of substitution.

We want to derive the IS-equation of this economy. The standard text-book solution would be to logaritmise the inter temporal first order condition for consumption and to log-linearise the resource constraint around the steady state. Typically, neither real growth nor inflation is assumed to take place in the steady state. We manipulate the resource constraint differently in order to cover a general case with growth in the steady state. Further, we do not want to restrict ourselves to any specific assumption such as linear trend regarding the steady state growth pattern.

To write the first order Taylor expansion[10] we rewrite the resource constraint as

$$e^{y_t} = e^{c_t} + e^{g_t} \tag{3.5}$$

A useful approximation of the resource constraint around values for period t-1 is given by

$$e^{y_{t-1}}(y_t - y_{t-1}) + e^{y_{t-1}} = e^{c_{t-1}}(c_t - c_{t-1}) + e^{c_{t-1}} + e^{g_{t-1}}(g_t - g_{t-1}) + e^{g_{t-1}} \tag{3.6}$$

or

$$e^{y_{t-1}}(y_t - y_{t-1}) + Y_{t-1} = e^{c_{t-1}}(c_t - c_{t-1}) + C_{t-1} + e^{g_{t-1}}(g_t - g_{t-1}) + G_{t-1} \tag{3.7}$$

which simplies further using (3.1) into

$$Y_{t-1}(y_t - y_{t-1}) = C_{t-1}(c_t - c_{t-1}) + G_{t-1}(g_t - g_{t-1}) \tag{3.8}$$

and, devided by Y_{t-1}

$$(y_t - y_{t-1}) = \frac{C_{t-1}}{Y_{t-1}}(c_t - c_{t-1}) + \frac{G_{t-1}}{Y_{t-1}}(g_t - g_{t-1}) \tag{3.9}$$

[10] If f(x) is a differentiable function then the first order Taylor approximation of f at c is $f(x) \approx f(c) + f'(c)(x - c)$.

Assuming that the shares of consumption and government expenditure of GDP remain fairly constant over time, we can approximate (3.9) by using the steady state shares, and leading one period,

$$(y_{t+1} - y_t) = \frac{\overline{C}}{\overline{\overline{Y}}}(c_{t+1} - c_t) + \frac{\overline{G}}{\overline{\overline{Y}}}(g_{t+1} - g_t) \tag{3.10}$$

The first order condition for inter temporal utility maximisation in the CRRA case is

$$C_t^{-\theta} = \beta(1 + i_t)E_t \frac{P_t}{P_{t+1}} C_{t+1}^{-\theta} \tag{3.11}$$

and logaritmised and rearranging

$$c_{t+1} - c_t = \frac{1}{\theta}\left[\log(\beta(1 + i_t)) - (E_t p_{t+1} - p_t)\right] \tag{3.12}$$

The IS-curve can now be derived by substituting the FOC (3.12) into the chosen approximation of the resource constraint (3.10)

$$y_t = y_{t+1} - \frac{1}{\theta}\frac{\overline{C}}{\overline{\overline{Y}}}\left[\log(\beta(1 + i_t)) - (E_t p_{t+1} - p_t)\right] - \frac{\overline{G}}{\overline{\overline{Y}}}(g_{t+1} - g_t) \tag{3.13}$$

Using the approximation $\log(1 + i_t) \approx i_t$ and $\log(\beta) \equiv -\log(1 + \rho_t) \approx -\rho$ and noting that expectations for period t+1 are formed at period t, this can be presented as

$$y_t = E_t y_{t+1} - \frac{1}{\theta}\frac{\overline{C}}{\overline{\overline{Y}}}\left[i_t - E_t \pi_{t+1} - \rho\right] + \frac{\overline{G}}{\overline{\overline{Y}}}(g_t - E_t g_{t+1}) \tag{3.14}$$

This is the expectational IS-equation in the closed economy case.

In comparison, the standard expectational closed economy IS-curve derived assuming no growth in the steady state is

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\theta}\frac{\overline{C}}{\overline{\overline{Y}}}\left(\hat{i} - E_t \pi_{t+1}\right) + \frac{\overline{G}}{\overline{\overline{Y}}}\left(\hat{g}_t - E\hat{g}_{t+1}\right) \tag{3.15}$$

Clearly, we have in (3.14) a more general equation in levels, whereas in (3.15) we have deviations from a constant steady state. In our preferred formulation (3.14) we have not assumed anything regarding the steady state equilibrium interest rate level. This gives us the opportunity to relate the steady state growth to the deep parameters and the real equilibrium interest rate as follows.

Let us assume that government expenditure and GDP grow at the same rate, ie if

$$y_{t+1} - y_t = g_{t+1} - g_t \equiv \gamma_t \tag{3.16}$$

we can derive from the IS curve a presentation of this growth rate as a function of the interest rate as follows. Substituting definition (3.16) into equation (3.14),

$$-\gamma = -\frac{\overline{C}}{\overline{Y}}\frac{1}{\theta}\left[i_t - E_t\pi_{t+1} - \rho\right] - \frac{\overline{G}}{\overline{Y}}\gamma \tag{3.17}$$

Rearranging, and deviding by $-\frac{\overline{C}}{\overline{Y}}$

$$\gamma = \frac{1}{\theta}\left[i_t - E_t\pi_{t+1} - \rho\right] \tag{3.18}$$

Equation (3.18) holds also for the steady state values, ie

$$\overline{\gamma} = \frac{1}{\theta}\left[\overline{i} - \overline{\pi} - \rho\right] \tag{3.19}$$

Thus, it is easy to see that the steady state growth rate depends positively on the intertemporal elasticity of substitution and on the difference of the equilibrium real interest rate from the subjective rate of discount.

3.2 Extension of the model – the open economy case

We extend the previous analysis sketching a two-country case, but without explicitly modelling the foreign economy, as we are only interested in bringing the exchange rate channel of monetary policy and the foreign income channel to the IS-equation[11]. This goal will be achieved here using very straightforward assumptions concerning consumers preferences and producers pricing decisions.[12]

[11] A two-country general equilibrium model is derived in Obstfeld and Rogoff (1996) Ch. 10.

[12] We follow loosely Guender (2003) in derivation of the open economy IS-curve, but the resource constraint approximation is again different from the standard case, analogously with the closed economy case above.

The utility function is now of the type $U(C_t(C_t^h, C_t^f))$, where aggregate consumption consists of domestically produced goods C_t^h and imported goods C_t^f. Assuming again CRRA preferences

$$U_t(C_t^h, C_t^f) = \frac{C(C_{t,}^h C_t^f)^{1-\theta}}{1-\theta} \tag{3.20}$$

The resource constraint of the economy, including foreign trade, is

$$Y_t = C_t + G_t + X_t - M_t \tag{3.21}$$

Resource constraint (3.21), linearised using the approximation method described above for the closed economy case, gives (3.22)

$$(y_t - y_{t-1}) = \frac{C_{t-1}}{Y_{t-1}}(c_t - c_{t-1}) + \frac{G_{t-1}}{Y_{t-1}}(g_t - g_{t-1}) + \frac{X_{t-1}}{Y_{t-1}}(x_t - x_{t-1}) - \frac{M_{t-1}}{Y_{t-1}}(m_t - m_{t-1})$$

Assuming that the shares of consumption, government expenditure and foreign trade of GDP remain fairly constant over time, we can approximate (3.22) by using the steady state shares, and leading one period

$$(y_{t+1} - y_t) = \frac{\overline{C}}{\overline{\overline{Y}}}(c_{t+1} - c_t) + \frac{\overline{G}}{\overline{\overline{Y}}}(g_{t+1} - g_t) + \frac{\overline{X}}{\overline{\overline{Y}}}(x_{t+1} - x_t) - \frac{\overline{M}}{\overline{\overline{Y}}}(m_{t+1} - m_t) \tag{3.23}$$

The inter temporal FOC condition for the consumption basket is again

$$c_{t+1} - c_t = \frac{1}{\theta}\left[i_t - E_t\pi_{t+1} - \rho\right] \tag{3.24}$$

In addition, we have the intra temporal FOC conditions for domestic and imported consumption goods, both in the home country and analogously in the foreign county. From these we can derive standard trade functions.

Let's assume a Cobb-Douglas basket of home (h) and foreign (f) goods, with ψ the weight of the foreign good. The intra temporal FOC gives demand for the domestic good that is propotional to the consumption basket and inversley dependend on the relative price

$$c_t^h = -\eta(p_t^{hc} - p_t^C) + c_t \tag{3.25}$$

where p_t^C is the weighted average for the consumption basket price

$$p_t^C = (1-\psi)p_t^{hc} + \psi(p_t^{fc} - s_t). \tag{3.26}$$

We have used the symbol η for the elasticity of substitution between the domestic and foreign consumption good. Note that the spot exchange rate s_t is defined here as the foreign currency value of one unit of domestic currency. We assume producer pricing, full pass-trough of the exchange rate to import prices, and further that the law of one price holds. We define the real exchange rate taking into account the difference of domestic and foreign aggregate price developments

$$q_t = s_t + p_t - p_t^f \tag{3.27}$$

Assuming that home country and foreign country consumers' preferences are symmetric, we end up with the following log linear export and import functions

$$x_t = \kappa_1 y_t^f - \kappa_2 q_t, \quad m_t = \kappa_1 y_t + \kappa_2 q_t \tag{3.28}$$

where income elasticity of exports and imports is κ_1 and price elasticity is κ_2. We assume further[13] that

$$\frac{\overline{X}}{\overline{Y}} = \frac{\overline{M}}{\overline{Y}} \tag{3.29}$$

and substituting trade assumptions (3.28) and (3.29) into the approximation of the resource constraint gives

$$\begin{aligned}(y_{t+1} - y_t) = {} & \frac{\overline{C}}{\overline{Y}}(c_{t+1} - c_t) + \frac{\overline{G}}{\overline{Y}}(g_{t+1} - g_t) + \frac{\overline{X}}{\overline{Y}}(\kappa_1 y_{t+1}^f - \kappa_2 q_{t+1} - \kappa_1 y_t^f + \kappa_2 q_t) \\ & - \frac{\overline{X}}{\overline{Y}}(\kappa_1 y_{t+1} + \kappa_2 q_{t+1} - \kappa_1 y_t - \kappa_2 q_t)\end{aligned} \tag{3.30}$$

which, collecting terms, and rearranging

$$\begin{aligned}y_t = y_{t+1} - \frac{1}{(1 + \frac{\overline{X}}{\overline{Y}}\kappa_1)} \Bigg[& \frac{\overline{C}}{\overline{Y}}(c_{t+1} - c_t) + \frac{\overline{G}}{\overline{Y}}(g_{t+1} - g_t) \\ & + \frac{\overline{X}}{\overline{Y}}\kappa_1(y_{t+1}^f - y_t^f) - \frac{\overline{X}}{\overline{Y}}2\kappa_2(q_{t+1} - q_t)\Bigg]\end{aligned} \tag{3.31}$$

[13] This approximation is in accordance with actual statistics on euro area intra trade.

Substituting finally the intertemporal FOC for consumption, equation (3.24), into (3.31) we get the open economy IS-equation as

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \alpha \frac{\overline{C}}{\overline{Y}} \frac{1}{\theta} \left[i_t - E_t \pi_{t+1} - \rho \right] + \\ & + \alpha \frac{\overline{G}}{\overline{Y}} (g_t - E g_{t+1}) + \alpha \frac{\overline{X}}{\overline{Y}} \kappa_1 (y_t^f - E y_{t+1}^f) - \alpha \frac{\overline{X}}{\overline{Y}} 2\kappa_2 (q_t - E q_{t+1}) \end{aligned} \tag{3.32}$$

where $\alpha = \dfrac{1}{(1 + \frac{\overline{X}}{\overline{Y}} \kappa_1)} > 0$, $\dfrac{\overline{C}}{\overline{Y}}, \dfrac{\overline{G}}{\overline{Y}}, \dfrac{\overline{X}}{\overline{Y}} > 0$, parameters $\theta, \kappa_1, \kappa_2 > 0$, and $0 < \beta < 1$.

We notice that the larger the share of foreign trade, with given trade elasticities, the larger the coefficients in absolute value for the expected changes of the foreign GDP and the real exchange rate. If all the steady state shares can be assumed constant, implying that the income elasticity of exports and imports is one, (3.32) simplifies to

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \alpha_i \left[i_t - E_t \pi_{t+1} - \rho \right] + \alpha_g (g_t - E g_{t+1}) + \alpha_f (y_t^f - E y_{t+1}^f) \\ & - \alpha_q (q_t - E q_{t+1}) \end{aligned} \tag{3.33}$$

where the coefficients $\alpha_j > 0$ for all j.

Finally we note that if there is habit formation, ie time non-separability in households' preferences, we get a lagged consumption term into the Euler equation for consumption as in Dennis (2004), and therefore also a lagged output term into the IS-equation.

4 Data and preliminary calibration of the IS-equation

4.1 Data

We use synthetic euro area data for the domestic economy and US data for the foreign economy. All data are quarterly and, except for the interest rates and exchange rates, seasonally adjusted. For the euro area, we employ data aggregated by constant 1995 PPP weights. This data has been constructed for the ECB Area Wide Model (Fagan, Henry and Mestre, 2001). The original set covers period 1970q1–1999q4 and the EU11 countries. The update we employ covers EU12 and the sample has been extended to 2003q4.[14] For the US, we use data from the Bank of Finland EDGE model data bank, that builds on the BEA data.

Using synthetic data for the euro area prior to the start of the monetary union is common in empirical literature. A shift in monetary policy regime is, however, an issue with a sample covering even the period of negative real interest rates of the 1970's. We find that existence of meaningful enough proxies for market interest rates and the exchange rate actually restrict the sample to that of 1977q1–2003q4. It is also the case that OECD forecasts for the EU12, even on annual basis, are only available since 1977.

Key variables used in calibration are GDP, GDP deflator, real government expenditure and the nominal 3-month (respective 12-month) interest rate for both the euro area and the US, and the bilateral nominal exchange rate, ecu rate prior to 1999.[15] Additional data on euro area investment, investment deflator and real wages are taken from the AWM data bank, and used in instrumentation. OECD forecasts for euro area GDP-deflator inflation and German and French GDP growth are also utilised when constructing instrumental variables. Further data descriptions are given in Appendix 1.

4.2 Preliminary calibration

The open economy version of the IS-equation is written in levels, but as long as we impose the coefficient of the lead of the output term to unity, there should not be any problems regarding stationarity.[16] Our main interest is in the quarterly model, but as a robustnesss check, we will also use annual data. The chosen sample is relatively short, however, and panel methods would likely improve efficiency of estimation compared to our aggregate approach.

We apply the single equation instrumental variables approach. As we assume rational expectations, we can in principle apply actual data for the lead values, and instrument the leaded explanatory variables. In the GMM method, the instruments shoud be valid and relevant, ie uncorrelated with the error term and closely correlated with the variables they are to instrument. We assume the information set includes lagged variables and OECD forecasts, all known by economic agents at the preceeding period. As our variables are typically both correleted with each other and strongly autocorrelated, there are diminishing returns to increasing the number of instruments, and in particular, in increasing the number of lags.

[14] The data update has been published for Euro Area Business Cycle Network members on the EABCN internet pages.

[15] Additional financial market data is described in Chapter 6.3.

[16] ADF-test reveal no surprises, stationarity of neither differenced data nor of the detrended series can not be rejected.

An indicative unrestricted estimation of equation (3.33) leads to some tentative conclusions. The interest rate coefficient in the canonical specification tends to be economically meaningful (see Table 2 below), while that is not the case in the hybrid specification. The failure to get any meaningful intererest rate coefficient in the closed economy specifications – either with levels or detrended data – is taken as evidence in favour of the open economy specification.

Next we fix the structural parameters of the IS-equation, the share parameters as well as the foreign trade elasticities, and estimate the deep parameters of the utility function. We recall the open economy specification (3.32) on p. 19 and make the following assumptions.

<u>The trade parameters:</u>
Income elasticity of trade: $\kappa_1 = 1$.
Price elasticity of trade (absolute value): $\kappa_2 = 1$.
These are rather typical baseline calibration values, and the income elasticity of trade is consistent with the share of trade remaining constant in the steady state. The assumed price elasticity is on the low side, however.

<u>The share parameters:</u>
$\overline{G}/\overline{Y} = .23$, sample mean of share of real government expenditure in GDP.
$\overline{C}/\overline{Y} = 1 - \overline{G}/\overline{Y} = .77$, sample mean of share of domestic private expenditure.
$\overline{X}/\overline{Y} = .15$, based on GDP share of extra (non-euro area) exports in 2003. As trade shares are on an increasing trend, the sample mean is not applied. Instead, a break in trend is assumed.

Fixing all the parameters above and replacing the real exchange rate by the RUIP condition gives the IS-equation

$$\begin{aligned} y_t = {} & E_t y_{t+1} + 0.67\frac{1}{\theta}\rho - 0.67\frac{1}{\theta}(i_t - E_t\pi_{t+1}) + +0.20(g_t - E_t g_{t+1}) \\ & + 0.13(y_t^f - E_t y_{t+1}^f) - 0.26(i_t - E_t\pi_{t+1} - (i_t^f - E_t\pi_{t+1}^f)) \end{aligned} \tag{4.1}$$

Whether this partly postulated parametrisation confirms data coherence or not is not really tested here. Nevertheless, we estimate this equation to get some starting values for calibration of the most interesting parameters, the rate of time preference β[17] and the intertemporal elasticity of substitution $1/\theta$, reported in Table 2.

[17] $\beta = (1 + \rho)^{**}(-1)$, annual, or $\beta = (1 + \rho)^{**}(-.25)$, quarterly.

Table 2. **Main calibration results of equations (3.33) and (4.1) based on GMM estimation.**[18] **Indicative t-statistics in parenthesis.**

Eq No, frequency	θ	β	Constant	α_i	α_g	α_f	α_{RUIP}	J-statistic (p)	# Obs
(3.33), Q			.0017	–.28	.23	.45	.50	.03	107
								(.21)	
(3.33), A			.0051	–.25	.34	.39	–.049	.04	26
								(.61)	
(4.1), Q	1.3	.997	.00152	–.50				0.05	107
			(0.58)	(1.87)				(.07)	
(4.1), A	1.3	.988	.00679	–.52				0.03	25
			(0.57)	(1.63)				(.65)	

The quarterly and annual results seem to be rather consistent with each other. The estimates for the subjective time preference β are quite in line, but these estimates are not received with much precision, however. The same holds for the estimates for the inverse of the intertemporal substitution rate θ. The values we get compare as rather plausible to the parameters in the empirical studies refered to in Chapter 2, but the subjective time preference is compatible with an annual discount rate of only 1.2. Compared to the unrestricted case, the restricted estimates for the coefficient of the interest rate is bigger in absolute value.

As these estimation results suffer from weak identification, we can not exclude bias in the parameter values nor can we rely on standard distributions of test statistics. Nevertheless, we use the results of the quarterly equation (4.1) of Table 2 as a starting point in calibration. The results should be understood here as a means of supporting judgemental calibration. A final step of calibration takes place when we discuss simulation results with the whole model in Chapter 6.

5 Model specification

5.1 Modeling the exchange rate

The quarterly model consists of the open economy IS-equation, a New Keynesian Phillips curve, a Taylor rule and definitions. The Phillips curve could also be modified to include the exchange rate, but evidence is mixed so we do not apply such a modification. We will next discuss in detail modeling of the exchange rate.

[18] GMM estimation was done using a quadratic kernel, fixed Newey-West bandwith, and no prewhitening, in both cases. Number of instruments is 4 in the quarterly case, 7 in the annual case. According to the J-statistics the validity of overidentifying restrictions is not rejected. More detailed reporting of the estimation is given in Appendix 2.

Two further asset price equations, those for the long-term interest rate and for the stock price index, will then be added. The full model specification is listed on p. 26.

Recall that lower case letters y, ypot, g, y^f denote logs of GDP, potential GDP, government expenditure and foreign GDP. Inflation target is denoted by π^*, the real interest rate by r and the subjective rate of time preference by ρ. Potential output growth follows an exogenous process with growth parameter γ. In addition to domestic government expenditure, three exogenous foreign variables – GDP, inflation and interest rate – enter the model.

We keep the model a one country model so that all the foreign variables are exogenous. For the forecasting use of the model, it is essential that we allow for GDP growth in the steady state, and that short-term growth rates may deviate from long-term trends. Furthermore, both short-term developments and long-term trends may deviate from those of the foreign country. This leads us to the following solution for modelling the exchange rate.

The real exchange rate change is modelled to follow real interest rate parity without a risk premium. The exchange rate level is calculated by cumulating observed level with these changes. If the RUIP condition were the only anchor for the exchange rate, however, we would not gain any information on the equilibrium level of the exchange rate. To discuss the equilibrium level, we need to let the real exchange rate react to productivity differentials home vs. abroad in the steady state.

The steady state equation for the real exchange rate level can be motivated as follows. We introduced above, when deriving the IS-equation, the dynamic trade equations.

$$x_t = \kappa_1 y_t^f - \kappa_2 q_t + c_1$$
$$m_t = \kappa_1 y_t + \kappa_2 q_t + c_2$$

When deriving the IS-equation, we made the assumption that export and import volume are of roughly equal size. As a steady state condition, we introduce the trade balance condition in nominal terms, leading to $x_t + p_t - (m_t + p_t^f - s) = 0$, so that $q_t = m_t - x_t$, as the real exchange rate is defined as $q_t = s_t + p_t - p_t^f$.

Solving for the real exchange rate we get

$$q_t = \kappa_1 / (2\kappa_2 - 1)(y_t^f - y_t) + c$$

where $c = (c_1 - c_2)/(2\kappa_2 - 1)$.

With parameter values $\kappa_1 = \kappa_2 = 1$, the steady state equation is

$$q_t = (y_t^f - y_t) + c \tag{5.1}$$

How to reconcile this long-term relationship with the dynamic uncovered real interest rate parity condition in model simulations? When domestic and foreign growth rates differ, there is indeed potential discrepancy between the dynamic model and the steady state model. The order of this discrepancy is, however, limited with any reasonable range of growth and real interest rate assumptions. This can be seen by the following reasoning. If the RUIP and the Euler condition both hold in the steady state for both the countries, and if we for simplicity assume an equal value for the inter temporal substitution parameter in both countries, there is a steady state condition that ties the change in the real exchange rate to the growth differential

$$q_{t+1} - q_t = (-1)\left(i_t - E_t\pi_{t+1} - \left(i_t^f - E_t\pi_{t+1}^f\right)\right) = \theta(\gamma^f - \gamma)$$

This implies a long-run linkage between the exogenous foreign growth rate and the real interest rate differential of the type

$$\gamma^f = \gamma - (1/\theta)\left(\bar{i} - \bar{\pi} - \left(\bar{i}^f - \bar{\pi}^f\right)\right)$$

We prefer to choose values for the exogenous variables in baseline simulations in such a way that this condition holds for the long-run. However, there is nothing to restrict assumptions concerning foreign country productivity or real interest rate developments in simulations.

5.2 Equations for the long-term interest rate and stock prices

We complete the model by adding two jump variables, ie variables that carry information on private sector expectations on long-term growth and inflation. According to the expectations hypotheses the long-term interest rate is determined as a weighted average of expected short-term interest rates[19]

$$l_t = \sum_{i=0}^{n} \omega_i E_t i_{t+i} + \text{risk premium} \tag{5.2}$$

[19] Shiller, Campbell and Schoenholtz (1983). This modelling approach has also been used in the Bank of Finland models BOF5 and EDGE.

where i is short-term interest rate, l is long-term interest rate and weights ω_i sum to unity.

We approximate the finite-maturity bond by an infinite-maturity (consol) bond with geometric weights that make the duration of the bond equal to duration of the finite-maturity bond. The infinite-maturity approximation of finite-maturity bond with constant duration D is thus

$$l_t = \frac{1}{1+D}\sum_{i=0}^{\infty}\left(\frac{D}{1+D}\right)^i (E_t i_{t+i} + \text{risk premium})$$
$$= \frac{1}{1+D}(i_t + \text{risk premium}) + \frac{1}{1+D}\frac{D}{1+D}(E_t i_{t+1} + \text{risk premium}) + \ldots$$

Leading forward, taking expectations and multiplying by $\frac{D}{1+D}$ yields

$$\frac{D}{1+D}E_t l_{t+1} = \frac{1}{1+D}\frac{D}{1+D}(E_t i_{t+1} + \text{risk premium})$$
$$+ \frac{1}{1+D}\left(\frac{D}{1+D}\right)^2 (E_t i_{t+2} + \text{risk premium}) + \ldots$$

Subtracting this from the first equation above yields

$$l_t = (1-\alpha)(i_t + \text{risk premium}) + \alpha E_t l_{t+1}$$

where $\alpha = \frac{D}{1+D}$.

Assuming no risk premium, the above equation simplifies to

$$l_t = i_t + D(E_t l_{t+1} - l_t) \quad (5.3)$$

The steady state version of this equation is

$$\bar{l} = \bar{i} \quad (5.4)$$

Equity price equation according to the Gordon growth model[20] is

$$DIV_t / E_t = i_t - (E_{t+1} / E_t - 1) + \text{Risk premium} \quad (5.5)$$

where DIV is dividends, E is equity price index and i is short-term interest rate.

[20] See, eg Elton, Gruber, Brown and Goetzman (2003) p. 447.

We simplify again assuming no risk premium. Dividends are assumed to grow in line with nominal GDP

$$DIV_t = \nu Y_t P_t$$

The steady state version of (5.5) is

$$DIV / E = \bar{i} - (\bar{\pi} + \bar{\gamma}) \qquad (5.6)$$

where $\bar{i}$ is the steady state short-term interest rate, $\bar{\pi}$ is the steady state inflation rate and γ is the steady state growth rate for real GDP.

These asset price equations, the long-term rate and the equity prices, do not affect the long-term properties of the rest of the model. Instead, they are helpful in the forecasting context as the steady state values are related to expectations about long-term growth and inflation.

While the model can be numerically solved without explicitly solving each equation for one of the endogenous variables, we hope the presentation of the steady state below sheds light on the long-run properties of the model.[21] We solve the IS equation for the equilibrium nominal interest rate, the Phillips curve for the steady state output, and substituting these into the Taylor rule, get the expression for the steady state inflation. In the steady state, the economy is growing at the assumed potential rate of growth γ, inflation equals the inflation target and the equilibrium real interest rate fulfills the condition $r = \bar{i} - \pi^*$.

The model specification

The dynamic equations:

IS-equation

$$y_t = y_{t+1} - c_y \frac{1}{\theta}\left[i_t - \pi_{t+1} - \rho\right] + g_y(g_t - g_{t+1}) + f_y(y_t^f - y_{t+1}^f) - q_y(-\lambda_t)$$

NKPC	$\pi_t - \pi_t^* = \beta(\pi_{t+1} - \pi_t^*) + \kappa(y_t - ypot_t)$
Taylor rule	$i_t = r^* + \pi_t + \phi\left(\pi_t - \pi_t^*\right) + \eta(y_t - ypot_t)$
RUIP	$\lambda_t = i_t^f - i_t - \pi_{t+1}^f + \pi_{t+1}$
Exchange rate level	$q_{t+1} = q_t + \lambda_t$

[21] If output elasticity w.r.t. the real exchange rate and foreign output q_y, f_y are zero and if $1-g_y = c_y$, we get the closed economy case, and further, if in the closed economy case there is no growth ($\gamma = 0$), $r = \rho$.

Long term interest rate $l_t = \delta l_{t+1} + (1-\delta) i_t$

Equity price $e_t = \varphi e_{t+1} - \varphi i_t + (1-\varphi) y_t + \Psi$

Potential growth $ypot_t = ypot_{t-1} + \gamma_t$

Foreign growth $y_t^f = y_{t-1}^f + \gamma_t^f$

Definitions:

Inflation $\pi_t = p_t - p_{t-1}$

Real interest rate $r_t = i_t - E_t \pi_{t+1}$

Nominal exchange rate $s_t = q_t - p_t + p_t^f$

Exogenous variables $g_t, y_t^f, \pi_t^f, i_t^f$

Exogenous policy target π^*

The steady state equations:

$$\bar{i} = \bar{\pi} + \rho + ((1-g_y)/c_y)\theta\gamma - (f_y/c_y)\theta\gamma^f + (q_y/c_y)\theta\lambda$$

$$\bar{y} = \overline{ypot} + ((1-\beta)/\kappa))(\bar{\pi} - \pi^*)$$

$$\bar{\pi} = \pi^* + (1/\Omega)\left[\rho - r + (\theta/c_y)((1-g_y)\gamma - f_y\gamma^f + q_y\lambda)\right]$$

where

$$\Omega \equiv \phi + \eta(1-\beta)/\kappa$$

$$\lambda_t = i_t^f - i_t - \pi_{t+1}^f + \pi_{t+1}$$

$$q_t = (\bar{y}^f - \overline{ypot}) + const$$

$$\bar{l} = \bar{i}$$

$$e = y + (\varphi/(1-\varphi))\left(\gamma + \bar{\pi} - \bar{i}\right) + (1/1-\varphi))\Psi$$

$$ypot = ypot_{-1} + \gamma$$

$$y_t^f = y_{t-1}^f + \gamma_t^f$$

6 Model calibration by simulation

6.1 Initial parameter values

We start with using parameters of the quarterly IS-equation (3.34) of Table 2. For the NKPC we apply parameters received from GMM panel estimation by Paloviita (2004). In her study, conflicting our results, the subjective rate of time preference is assumed to be 3% pa, but Paloviita and Mayes (2004) report that the estimation results in Paloviita (2004) are not very sensitive to variation in this assumption. We set parameters of the Taylor rule such that they are consistent with the Taylor principle.

For an in-sample simulation 1977q1–2003q4 we could apply sample mean for GDP growth (2.1%) as potential output growth and sample mean of inflation (5.0%) as inflation target, but we apply a 1.5 per cent growth assumption and an inflation target of 2.0 per cent, more in line with the latter part of the sample. In both in-sample and out-of-sample simulations the model is stable, ie fulfils the Blanchard-Kahn condition. However, the resulting steady state real interest rate is high, between 3 and 4 per cent pa, depending on assumed rate for domestic and foreign growth and the foreign real interest rate. We recall that the estimates for the deep parameters of the utility function were not received with much precision (Table 2).

6.2 Final parameter values of the utility function

For the forecasting use of the model, we need to check calibration of the deep parameters of the utility function such that

1) The model solves with a reasonable range of values for the long-term fundamentals

2) The jump variables react in a plausible way to changes in the long-term fundamentals.

With long-term fundamentals we mean the steady state rate of domestic and foreign growth, steady state inflation, and exogenous steady state foreign real interest rate. One obvious stability condition is that the equilibrium real interest rate be higher than the rate of growth of real GDP.

We want the model to be able to cope with as low as 0.5 per cent pa growth rate and as high as 3.0 per cent pa growth rate. This should cover most interesting euro area long-term growth rate scenarios. It results from this that θ, the inverse of the inter temporal elasticity of substitution, should be no smaller than 0.7. To be sure the model is stable we choose ρ, the rate of subjective time preference, no smaller than 1.2 per cent pa. This reasoning follows directly from the steady state relation

$$\bar{\gamma} = \frac{1}{\theta}\left[\bar{i} - \bar{\pi} - \rho\right] \tag{6.1}$$

when steady sate inflation expectations are assumed 2% pa. With parameter values $\theta = 0.7$, $\rho = 1.2$ and $\pi^* = 2.0$, we get the following combinations of real GDP growth and real interest rate.

Table 3. **Steady state growth and real interest rate combinations**

GDP Growth γ	0.5	1.0	1.5	2.0	2.5	3.0
Real interest rate $\bar{i} - \pi^*$	1.55	1.9	2.25	2.6	2.95	3.3

The range or real growth is as wide as we prefer, but with this parametrisation, the exceptionally low real interest rate level prevailing in summer 2005 (below 1.5 per cent pa), remains a puzzle within this framework.

To check the second condition, we simulate permanent shocks in the long-term parameters and calculate first quarter elasticities of the forward-looking variables with respect to these shocks.

We present in Table 4 below elasticities based on fairly large shocks, both positive and negative. A positive permanent one percentage point domestic potential output shock makes the stock prices jump to a new higher level, almost double as high as the previous level. The stock price elasticity wrt the domestic and foreign productivity shock is clearly stronger than that of the real exchange rate. Higher volatility of stock prices compared to the exchange rate matches with actual data. If expectations of the inflation target change, inflation and long-term interest rate jump to match the new expected level. Finally, an increase in long-term foreign interest rate causes an opposite effect compared with that of a positive shock in the foreign growth rate.

To interpret results in Table 4, one should keep in mind that they are based on the benchmark model with no inertia in the equations. We conclude that with this model, keeping θ unchanged and increasing ρ would lessen the stock price elasticity to growth. With ρ unchanged, increasing θ would dampen stock price reactions to growth. Given the trade-offs, we decide to work with this current parametrisation of the utility function.

Table 4. **Elasticities of the forward-looking variables GDP, Inflation, and asset prices, wrt long-term exogenous trends with parameter values θ = 0.7, ρ = 1.2, π^* = 2.0. Permanent shocks. Shock simulation vs. control simulation.**

Shock in exogenous parameter or variable	Shock, %-pts	GDP, %	Inflation, %-pts	Real exchange rate, %	Stock prices, %	Long-term interest rate, %-pts
Domestic pot. growth	+1	0.25	0.00	–19.2	82.0	0.65
	–1	–0.25	0.00	24.0	–31.4	–0.65
Foreign pot. growth	+1	0.0	0.00	73.7	16.3	–0.10
	–1	0.0	0.00	–42.7	–12.4	0.10
Inflation target	+1	0.0	1.00	0.0	0.25	1.00
	–1	0.0	–1.00	0.0	–0.25	–1.00
Foreign real interest rate	+1	0.0	0.00	–38.8	–22.2	0.21
	–1	0.0	0.00	63.5	39.8	–0.21

6.3 Final parameter values of the asset price equations

To conclude calibration, we check if the parameters of the asset price equations need any fine-tuning based on full-model simulations.

For the interest rates, we apply the 3-month market rate as the short-term rate, and the 10-year government bond yield as the long-term rate, so the modified duration is in this case about 6.5. In the equation for the long-term interest rate, the parameter $\delta = D/(1+D)$ takes the value 0.87.

In modelling the bilateral real exchange rate, we need to calibrate the constant c in the steady state condition. We could solve the equation for the parameter c in a period when the bilateral trade was in balance. In practice, we note that trade balance of the euro area viz a viz the rest of the world has been rather close to zero in recent years. We set the value of c based on euro area and US data for the average of the year 1999, implicitly assuming bilateral trade balance was zero in that period. The parameter value c = –.60 implies a value of 1.07 USD for one euro for the long-term equilibrium level of the bilateral real exchange rate.

Finally, as stock price data we apply the DJ Euro Stoxx 50 index. Quarterly averages of daily data, available from 1990q1 onwards, are used, and values for 1990q1–2003q4 are first used to calibrate a value for the parameter ν. Above we have the log-linearized equation for the stock prices. For the original form of the equation, simulations with the model suggest half of the preliminary parameter value, and we settle with final calibration of $\nu = 0.00000252$.

We can now summarise the chosen model parametrisation in Table 5 below.

Table 5. **The parameters of the model and growth of exogenous variables in the control simulation (annualised values)**

Equation	Parameter	Symbol	Value
IS-equation	Subjective rate of time preference	ρ	1.2
	Inverse of elasticity of inter temporal substitution	θ	0.7 (Initial 1.3)
	Open economy share parameters for – consumption – government consumption – trade (foreign demand) – trade (real exchange rate)	 y_c y_g y_f y_q	 0.67 0.20 0.13 0.26
NKPC	Parameter for the driving variable	κ	0.228
Taylor rule		ϕ	0.5
		η	0.5
	Inflation target	π^*	2.0
Exchange rate	SS-condition	C	–0.60
Long-term rate	Duration	δ	0.867
Stock prices	Dividend share	ν (φ)	$2.52*10^{-6}$
Potential GDP	Growth rate	γ	1.5
Foreign GDP medium-term	Growth rate	γ^f	2.0
Foreign GDP steady-state	Growth rate	$\bar{\gamma}^f$	1.5
Foreign real interest rate		$\bar{i}^f - \bar{\pi}^f$	2.25

7 How do parameters determining long-run trends effect short-term forecast levels?

7.1 Model transformation method for forecasting

Elasticities of the forward-looking variables wrt permanent shocks in parameters effecting long-term trends (as in Table 4) are as such a useful guide to iterating the forecast. More information on the elasticities would be needed, however, with a non-linear model with richer dynamics. In practice, by iterating one might fail to reach a satisfactory outcome that utilizes all available data and the model structure to the full.

Instead of repeating iterations, searching for a mapping between the long-term trends and the initial jumps, can we do any better? Can we solve the model in such a way that we endogenise (but still keep constant) the parameters determining long-run trends, and get smooth paths for the financial market jump-variables, given fixed starting values for these variables? In the following we show, that we actually can do this, if we only rewrite the model slightly without changing the economic logic of the model. The forecasting version of the model is listed below.

We recall that in normal simulation use (the model is listed on p. 26), the growth rate parameters and the inflation target (γ, γ^f, π^*) are exogenous parameters. Now, we lag the equations for the financial market variables to get rid of the leads in these equations. This implies that the forecast paths of the real exchange rate, long-term interest rate and stock prices (q, l, e) now start from the given starting values without a jump.

With financial market equations lagged one period, is there any role for the implied steady state conditions in the transformed model? Yes there is, once we transfer the growth rate parameters and the inflation target into forward-looking equations, and take note of the relationships of these parameters and the original jump-variables in the original steady state equations. We endogenise variables γ, γ^f, π^*, the domestic and foreign growth rate and the inflation target, in such a way that the steady state conditions of the original jump variables, q, l, e, still hold. Let's look at them one by one.

The transformed version of the model for forecasting use:

IS-equation

$$y_t = y_{t+1} - c_y \frac{1}{\theta}[i_t - \pi_{t+1} - \rho] + g_y(g_t - g_{t+1}) + f_y(y_t^f - y_{t+1}^f) - q_y(-\lambda_t)$$

NKPC $\pi_t - \pi_t^* = \beta(\pi_{t+1} - \pi_t^*) + \kappa(y_t - ypot_t)$

Taylor rule $i_t = r^* + \pi_t + \phi(\pi_t - \pi_t^*) + \eta(y_t - ypot_t)$

RUIP $\lambda_t = i_t^f - i_t - \pi_{t+1}^f + \pi_{t+1}$

Asset price equations of the original model lagged one period:

Exchange rate level $q_t = q_{t-1} + \lambda_{t-1}$

Long term interest rate $l_t = (1/\delta)\, l_{t-1} - (1-\delta)/\delta\, i_{t-1}$

Equity price $e_{t-1} = \varphi\, e_t - \varphi\, i_{t-1} + (1-\varphi) y_{t-1} + \Psi$

Potential growth $ypot_t = ypot_{t-1} + \gamma_t$

Foreign growth $y_t^f = y_{t-1}^f + \gamma_t^f$

The steady state (constant growth rate) form of the model is

$$\bar{i} = \bar{\pi} + \rho + ((1-g_y)/c_y)\theta\gamma - (f_y/c_y)\theta\gamma^f + (q_y/c_y)\theta\lambda$$
$$\bar{y} = \overline{ypot} + ((1-\beta)/\kappa))(\bar{\pi} - \pi^*)$$
$$\bar{\pi} = \pi^* + (1/\Omega)\left[\rho - r + (\theta/c_y)((1-g_y)\gamma - f_y\gamma^f + q_y\lambda)\right]$$

where

$$\Omega \equiv \phi + \eta(1-\beta)/\kappa$$

$$\lambda_t = i_t^f - i_t - \pi_{t+1}^f + \pi_{t+1}$$
$$ypot = ypot_{-1} + \gamma$$
$$y_t^f = y_{t-1}^f + \gamma_t^f$$
$$r = r^* = \bar{i} - \pi^*$$

Steady state conditions of the original forward-looking asset price equations

$$q_t = (\bar{y}^f - \overline{ypot}) + c$$
$$l = i$$
$$e = y + (\varphi/(1-\varphi))\left(\gamma + \bar{\pi} - \bar{i}\right) + (1/1-\varphi))\Psi$$

now govern steady state of the endogenised variables γ, γ^f, π^*, for which a forward-looking equation (a transformation equation) needs to be postulated.

Steady state conditions of the financial market variables:

Exchange rate $\quad q = y^f - y + c$

This can be converted to a <u>terminal condition for foreign output</u>. To use this condition, we need to transform the foreign growth rate parameter to a time series obeying some forward-looking equation, the simplest alternative being a constant growth rate $\gamma_t^f = \gamma_{t+1}^f$.

Long term interest rate $\quad l = i$

This can be converted to a <u>terminal condition for the inflation target</u>. To use this condition, we also need to transform the inflation target to a time series obeying some forward-looking equation, the simplest alternative being a constant target $\pi_t^* = \pi_{t+1}^*$.

Equity price $$e = y + \frac{\varphi}{1-\varphi}(\gamma + \pi - i) + \frac{1}{1-\varphi}\Psi$$

This can be converted to a terminal condition for domestic output. To use this condition, we also need to transform the growth rate parameter into a time series obeying some forward-looking equation, the simplest alternative being a constant growth rate $\gamma_t = \gamma_{t+1}$.

7.2 Elasticities of the long-term trends wrt changes in starting values of the financial-market variables

Typically, when forecasting, there are more observations available for the financial market variables than for the GDP or other national accounts data. This 'ragged edge' case is as such easy to deal with, as one can fix these additional data points variable by variable in a forecast simulation. In the following we demonstrate the ideas developed in Chapter 7.1. We assume for clarity that there is one additional observation for the jump variables, ie the real exchange rate, the stock prices and the long-term interest rate, and also for the domestic and foreign short-term interest rate. The transformation method as such does not depend on whether we have these additional observations or not.

With the transformed model, we do not let the financial variables jump but we impose the observed values on them. Instead, we let the long-term growth trends shift to a new level, consistent with the observed levels of the financial market variables, interpreted through the model. Whereas the original model can be solved stepwise, first solving the steady state and then the dynamic path, the transformed model needs to be solved in one step.[22]

To get more intuition for the transformed model and the choices we made above on the steady state conditions, let's look at how the key long-term parameters react to variation in starting values of the financial market variables. This is reported in Table 6 below. When interpreting these results, we need to keep in mind that we are still working with the model with no inertia in behavioral equations. We also simplify at this stage and make no difference between medium-term growth rate and steady-state growth rate. As we let domestic growth rate adjust, we also let domestic government expenditure follow, thus keeping the GDP share of government expenditure unchanged in these simulations.

In this model, as shown in Table 6, the long-term interest rate carries information mainly on the long-term inflation expectations. Stock prices reflect mostly domestic growth expectations, while the real exchange rate reflects expectations on foreign growth potential. The results show further that the model

[22] We use the stacked time algorithm by Laffargue – Boucekkine – Juillard in TROLL software in both cases. TROLL programme for the forecasting procedure was written by Mika Kortelainen.

is quite symmetric, as response to a negative shock is typically the opposite of a response to a positive shock.

Table 6. **Elasticities of the long-term trends wrt changes in starting values for the financial market variables. Parameter values and exogenous variables as in Table 5. Shock simulation vs. control simulation.**

		Domestic pot. growth rate, %-pts	Foreign pot. growth rate, %-pts	Inflation target, %-pts
Shock in starting value of	Shock			
Real exchange rate	+10%	–0.10	0.31	0.10
	–10%	0.11	–0.34	–0.11
Long-term interest rate	+0.5%-pts	–0.01	0.00	0.58
	–0.5%-pts	0.01	0.00	–0.58
Stock prices	+10%	0.16	0.06	–0.10
	–10%	–0.20	–0.08	0.12

7.3 The transformation method in practice

The model transformation method can be summarised to consist of three steps,

1) Lag the dynamic equations of those original jump-variables, that carry most information on long-term fundamentals, ie the financial market variables.

2) Write forward-looking equations for the long-term trend parameters.

3) Convert original steady state conditions of the chosen jump-variables, the financial market variables, to terminal conditions of the long-term parameters.

We illustrate how this transformation method works in practice with the presented forward-looking model specification. Let's make a forecast both with the original model and with the transformed model. For the original model, we assume two alternative sets of exogenous variables. Exogenous input not listed in Table 7 below is equal cross the three forecasts and equals that of Table 5. Until 2003Q4 we have data, and the forecast starts from 2004Q1. For the financial market variables, observations for 2004Q1 are also applied with the transformed model. Simulation horizon is 31 years, ie longer than the period shown in the graphs.

Table 7. **Two jumpy forecasts and a smooth one, long-term solution with the transformed model, assumptions used with the original model**

	Original model (Forecast S2)	Original model (Forecast S1)	Transformed model (Forecast CO)
Domestic potential growth, % pa	0.5	1.5	1.6
Foreign potential growth, % pa	2.5	2.0	1.9
Inflation target, % pa	2.0	2.0	2.4

What if we make an exogenous assumption about a very low long-term growth rate, 0.5% pa, in the euro area (Forecast S2)? The transformed model utilizes market expectations and gives an endogenous result of 1.6% pa (Forecast CO). This long-term potential growth rate is between recent estimates by the OECD and the ECB, but the definitions for the relevant time horizon vary somewhat in these studies. For the US long-term growth rate we get, as expected, a slightly higher estimate, 1.9%, which is on the low side compared to recent estimates by eg the OECD. For the euro area inflation target, we suggest an exogenous target of 2.0% pa, and find out that private sector expectations, as revealed by the transformed model, give a somewhat higher rate, 2.4%.

As illustrated in Figure 1 (See Appendix 4), the forecast paths for the financial market variables are clearly different in these two cases, as the original model produces jumpy outcomes while the transformed model gives smooth ones, starting from the known observations for 2004Q1. Even if we give more reasonable exogenous growth and inflation target assumptions with the original model (Forecast S1), the first-period jumps of the financial market variables are again marked, contrasted to the smooth paths with the transformed model.

7.4 Short-term macro forecasts with the transformed model

The model transformation is successful in producing smooth forecast paths for the asset prices. How the paths of GDP, inflation and interest rates change, when solved with the transformed model as compared to the original model, is dependent both on the dynamic specification of the original model, and on the dynamic specification of the transformation equations in the transformed model. Forecast paths for the exogenous variables also play a role here. In particular, we assume foreign interest rates are roughly based on the observed yield curve (in spring 2004) in the reported forecast simulations. We will clarify and illustrate these points by bringing in more dynamics into the model specification.

Let's look at the short-term forecast paths first, for the typical three-year forecasting horizon. In Figure 2 (in Appendix 4) we present two forecast paths, one with the original model and another with the transformed model. Forecast with the original model is based on the same assumptions as forecast S1 we already discussed in Table 7 and Figure 1. However, instead of using a purely forward-looking model, we now add some inertia into the behavioural part of the model. For the Forecast with the transformed model, we additionally put some inertia into the transformation equations. Hybrid specifications are discussed at length in Chapter 8 below. Here we briefly mention that for the illustrative forecast simulations in Figure 2, the following parameter values were applied. For lagged GDP in the IS-equation, $\mu = 0.45$, and for lagged inflation in the Phillips curve, $\omega = 0.4$, and, finally, for the lagged term in the transformation equations for the long-term parameters, $\psi = 0.4$.

In Figure 2 we report not only the financial market variables but all the variables of the model, including foreign output and interest rate. It is shown that in the transformed case, the long-term parameters, potential growth rate and inflation target, converge gradually to the new steady state level. With the transformed model, the short-term output and inflation developments are smooth. GDP growth rate converges to the potential within three years, and likewise inflation converges to the target rate within this period.

The short-term paths for GDP and inflation, produced with the transformed model, are not very different from those of the original model in Figure 2. The comparison is here to the case where the exogenous assumptions concerning potential growth rate and inflation target are not far from the private sector's expectations as revealed by the transformed model. A comparison with a forecast using the original model with clearly different assumptions – like those of Forecast S2 discussed above – would highlight the difference of resulting forecasts, as the transformed model would give smoother paths not only for the financial variables but also for GDP.

8 Robustness of the model transformation method

8.1 The long-run solution of a transformed model

To summarise findings with the purely forward-looking model specification, the transformation moves the steady state of the solution and smoothens short-term dynamics of the asset prices. This is our benchmark case, as we want to tie down the long-term growth-rates and the long-term inflation expectations based on the values extracted from up-to-date financial market data.

If the current rates of GDP growth or inflation were to deviate a lot from the expected long-term values, one could desire a smooth convergence to this value rather than a quick jump. This can easily be achieved, without altering the steady state solution of *the transformed model*, by adding dynamics into the transformation equations as follows

$$\pi_t^* = (1-\psi)\pi_{t+1}^* + \psi \cdot \pi_{t-1}^*$$
$$\gamma_t = (1-\psi)\gamma_{t+1} + \psi \cdot \gamma_{t-1}$$
$$\gamma_t^f = (1-\psi)\gamma_{t+1}^f + \psi \cdot \gamma_{t-1}^f$$

The parameter ψ may in principle vary between (0,1], but to guarantee the Blanchard-Kahn stability condition is not violated, the available range for a particular model specification should be checked. The larger the weight on the lagged term, the slower the convergence of the long-term parameters π^*, γ, γ^f to their steady state levels, and hence the smoother the dynamic path of inflation, output, and foreign output. For example, if parameter ψ takes the value 0.4, the shift of the parameters π^*, γ, γ^f to the new level will practically take place within 12 quarters, as is the case in Figure 2 in Appendix 4.

What if we have richer dynamics in the behaviourial part of the model? Let's look at hybrid specifications one by one. First, we replace the New Keynesian Phillips Curve (NKPC) by a Hybrid Phillips Curve (HPC) to bring in more inflation persistance and hence smoothness into the short-term path of inflation. We get a Fuhrer- Moore (1995) type of hybrid specification simply by adding a lagged inflation term into the NKPC as follows.

NKPC $\quad \pi_t - \pi_t^* = \beta(\pi_{t+1} - \pi_t^*) + \kappa(y_t - ypot_t)$

HPC $\quad \pi_t - \pi_t^* = (1-\omega)\beta(\pi_{t+1} - \pi_t^*) + \omega(\pi_{t-1} - \pi_t^*) + \kappa(y_t - ypot_t)$

The larger the weight ω on the lagged term, the smoother the response of inflation in response to shocks in the exogenous variables. For example, the value $\omega = .63$

is taken from Paloviita's (2004) estimations using aggregated euro area data for the period 1977–2003. One could vary this parameter value in the range (0,1] as long as stability is not violated, without moving the steady state solution of *the transformed model*.

Analogously, one could replace the static Taylor rule with a rule with interest rate smoothing as follows

Taylor rule $\quad i_t = r^* + \pi_t + \phi(\pi_t - \pi_t^*) + \eta(y_t - ypot_t)$

Taylor, smoothing $\quad i_t = \tau \cdot i_{t-1} + (1-\tau)\left(r^* + \pi_t + \phi(\pi_t - \pi_t^*) + \eta(y_t - ypot_t)\right)$

Again, the larger the weight on the lagged interest rate – an often cited value is $\tau = 0.8$ – the smoother the path of the interest rate. However, smoothing the interest rate might, with some parameter values, result in more volatility in the short-term GDP and inflation responses compared to the static rule. In the illustrative forecast simulations in this report we apply the static Taylor rule.

Even if smoothing does not alter the steady state of the original model, the steady state solution of *the transformed model* does not remain totally unaltered in simulations compared to the purely canonical case, but changes slightly, the more the bigger the smoothing parameter in the range (0,1). Quantitatively, this result is not a cause for concern.

The most challenging dynamics in this model is obviously that of the IS-equation. Habit formation in consumption or liquidity constrained consumers could be motivation for a following hybrid specification.

IS-equation, canonical

$$y_t = y_{t+1} - c_y \frac{1}{\theta}\left[i_t - \pi_{t+1} - \rho\right] + g_y(g_t - g_{t+1}) + f_y(y_t^f - y_{t+1}^f) - q_y(-\lambda_t)$$

IS-equation, hybrid

$$\begin{aligned} y_t &= \mu \cdot y_{t-1} + (1-\mu) \cdot y_{t+1} \\ &+ (1-2\mu) \cdot \left\{ -c_y \frac{1}{\theta}\left[i_t - \pi_{t+1} - \rho\right] + g_y(g_t - g_{t+1}) + f_y(y_t^f - y_{t+1}^f) - q_y(-\lambda_t) \right\} \end{aligned}$$

If we derived the IS-equation from an optimizing framework, we would not expect the steady state solution to change compared to the canonical case, at least in the case of habit formation. If we, however, as a shortcut postulate the hybrid specification, as above, we need to multiply the fundamentals by a term $(1 - 2\mu)$

in order to make sure that the parameter μ does not affect the steady state solution of the model (see Appendix 3 for the algebra).

If the core macro part of the model is purely forward-looking, ie if it builds on the same assumption on rational expectations as the asset price equations do, combining the model with expectation extraction from financial market data is straight-forward. Further, we have shown that we can add richer dynamics into the model, either in the transformation equations or in the behaviourial part, without breaking this result. As shown above for the IS-equation, one should be careful with ad hoc hybrid specifications, however. If the core macro model were a complex mix of forward-looking and backward-looking expectations, the steady state of a transformed model could also be a function of weights of the leads and lags. This would make combination of such a model and market expectations less transparent, and would first call for respecification such that the steady state no longer reacts to the dynamic parameters.

The lesson we draw from this is, that when optimisation based DGE models are used for forecasting, we can get rid of unrealistic jumpy forecast paths by imposing realistic steady state values. As criteria for manipulating the steady state we suggest imposing market expectations on fundamentals as reflected in financial data. If we, alternatively, were to manipulate short-term dynamics to get rid of the first-period jumps, we could risk making ad hoc changes in the adjustment cost parameters and alike. We would not suggest recalibrating deep parameters of the model (like those of the utility function) in the forecasting context. With a complex model, we suggest the forecaster checks the model specification, before applying the model transformation method, to be sure of an economically meaningful interpretation.

8.2 Elasticities of the long-term trends wrt changes in starting values of the jump variables; hybrid model

To further illustrate properties of the transformed model, we repeat, now with the hybrid model, the excersise of checking how sensitive the long-term trends are wrt variations in starting values of the financial market variables. This was reported for the canonical model in Table 6 above. We now impose inertia in the IS-equation ($\mu = 0.45$), in the Phillips curve ($\omega = 0.4$) and in the transformation equations ($\psi = 0.4$). We report below not only the steady state response of the potential growth rates and the inflation target, but also the short-term adjustement of domestic and foreign potential GDP-growth and inflation.

Table 8. **Elasticities of the long-term trends wrt changes in starting values for the financial market variables. Parameter values and exogenous variables as in Table 5, control simulation the transformed forecast of Figure 2. First year average and long-term (SS) elasticity.**

		Domestic potential growth rate, %-pts		Foreign potential growth rate, %-pts		Inflation target, %-pts	
Shock in starting value of	Shock	Year 1	SS	Year 1	SS	Year 1	SS
Real exchange rate	+10%	–0.03	–0.10	0.10	0.32	0.06	0.12
	–10%	0.04	0.11	–0.12	–0.35	–0.07	–0.14
Long-term interest rate	+0.5%-pts	0.01	–0.01	0.00	0.00	0.44	0.61
	–0.5%-pts	–0.01	0.01	0.00	0.00	–0.44	–0.61
Stock prices	+10%	0.06	0.16	0.02	0.06	–0.06	–0.14
	–10%	–0.07	–0.20	–0.02	–0.07	0.07	0.18

First, the long-term results are almost identical to those of the canonical case, as expected. The only difference seems to be the slightly larger elasticities for the inflation target in the hybrid case.[23] Second, the dynamics is smooth, without any peaks or kinks, with this parametrization of the model. The long-term potential growth rates and inflation target converge smoothly to the new level within about three years time, as dictated by the chosen parameter of the transformation equations.

To gain more intuition to what these elasticities imply to practical forecasting, in terms of how much the assessment of the long-term parameters could move from forecast round to the next, let's look at actual data. To simplify we only look partially at one variable at a time. Within the sample period, real exchange rate has jumped from previous quarter by more than 10% (the shock shown in Table 9) only once. Typically, the rate floats around the equilibrium level, hence implying no big shifts in the long term-trends. Cumulatively, over several quarters, it may appreciate or depreciate more considerably, thereby shifting the assessment of the foreign long-term potential growth rate, and to a lesser degree, that of domestic potential growth and inflation.

Likewise, for the long-term interest rate, 0.5%-pts (the shock applied in Table 9), is rather big compared to developments in subsequent quarterly data. Such a jump would shift assessment of long-term inflation but, practically, would not affect assessment of growth. Finally, the stock prices have the potential to drift up or down, so that 10% is not an exceptional change between two subsequent

[23] This is also evident in Figure 2, where the steady state level of the inflation target is a bit higher than that of the canonical case, reported in Table 7.

quarters. Would such a change take place, assessment of all long-term parameters would be affected, mostly that of the domestic potential growth rate, which would change, given the model and the calibration, by 0.2 %-pts.

9 Conclusions

We add asset price equations into a standard New Keynesian open economy general equilibrium model and show that a simple transformation of the model is all we need in order to utilize information on the long-term expectations, ie on the domestic and foreign long-term growth rate and on the inflation target, carried by observed levels of financial market variables.

To summarise the suggested model transformation method, one needs to identify long-term trend variables that are controlled by the steady state conditions of a set of informative jump-variables of the model. Lagging the chosen original jump-variable equations and postulating instead forward-looking equations for the corresponding long-term parameters, makes it possible to fix the starting points of the original jump-variables and also to force their steady state conditions govern the adjustment of the long-term trend variables.

Based on work with a sketchy standard model we conclude that macro forecasters using DGE models should not ignore information in the jump-variables of their model. A fairly straight-forward transformation of the model makes that information explicit in the forecast and forces a careful discussion on the implied long-term trends. As a benchmark assessment of long-run trends, we suggest market expectations. The proposed model transformation method combines expectation extraction and model-based macro forecasting in a transparent way, and is technically fairly easy to implement. Based on robustness analysis, applicability of the method seems good cross model specifications that are derived from optimisation behaviour.

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Appendix 1

Summary of model data

Mean and standard deviation for 1977q1–2003q4, for the stock price, 1990q1–2003q4. Data sources are discussed in the text.

Variable	Transf./ Description	Symbol	Source	Unit	Mean	Standard deviation
Euro area						
GDP	Log	y	AWM	95-EUR mill.	14.01	0.173
Potential GDP	Log	ypot	AWM	95-EUR mill.	14.01	0.172
Government expenditure	Log	g	AWM	95-EUR mill.	12.54	0.169
Inflation, GDP deflator	Annualised one quarter change	π	AWM	%, 1995=1	4.95	3.18
3-month interest rate	Interbank	i	AWM	%	8.22	3.48
10-year govt bond yield		l	AWM	%	8.91	2.96
Real bilat exchange rate	Real USD price of euro	Q		EUR/USD	1.011	0.185
Real bilat exchange rate	Log	q	Page 12, (3.27)		–.0074	0.194
Nominal exchange rate	USD price of one euro	S	ECB	EUR/USD	1.116	0.173
Stock price index	DJ Euro Stoxx 50	E	Bloomberg	31.12.99 =1000	2315.7	1295.1
Stock price index	Log	e	Bloomberg		7.60	0.555
US						
GDP	Log	y^f	BEA	95-USD mill.	14.58	0.241
Inflation, GDP deflator	Annualised one quarter change	π^f	BEA	%, 1995=1	3.53	2.38
3-month interest rate	Interbank	i^f	Reuters	%	7.36	3.73

Appendix 2

Documentation for estimation of the IS-equation (3.33), reported in Table 2.

In the quarterly specification the set of instruments is inflation, relative investment prices, US GDP growth, all lagged by one period, and a constant. In the annual case, the set includes a real time HP-filtered output gap for France and inflation (GDP deflator) forecast for the euro area, both based on OECD forecasts released in December of the previous year. The other instruments are, lagged one period, relative investment prices, euro area and US government expenditure growth, euro area long rate vs short rate differential, and a constant. Relative investment prices are outside of the model framework, but contain information on the business cycle.

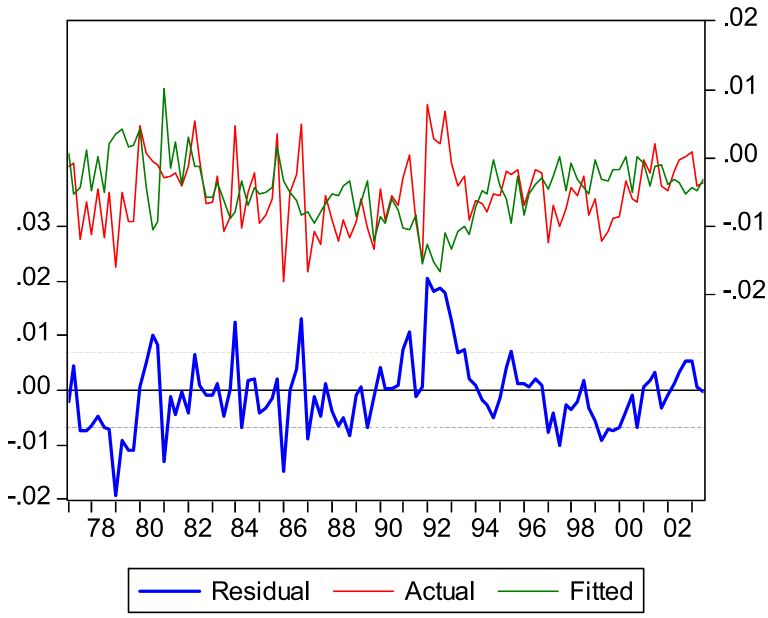
Calculation of the French real time output gap, which is based on OECD GDP forecasts and historical data vintages of the time, is explained in detail in Paloviita and Mayes (2004). French data is used here as a proxy for euro area data. If, instead, lagged euro area GDP growth is used as an instrument, the parameter estimates for θ and β remain fairly unchanged, but the related t-statistics diminish slightly. If OECD inflation forecast is replaced by lagged GDP deflator inflation, precision of the estimation suffers somewhat more and parameter values are also affected. When both the instruments are replaced by lagged endogenous variables, the parameter values become less plausible. We conclude that using published forecasts as instruments improves robustness of these results.

The annual specification seems to be more robust w.r.t. variations in the set of instruments than the quarterly specification. It is also less sensitive to corrections for heteroscedasticity and serial correlation. When the RUIP is substituted for the real exchange rate change, identification of the constant of the IS-equation is potentially disturbed by a risk premium term, not modelled. The quarterly estimates are from a specification with the RUIP condition, but the annual ones are received using real exchange rate change in deviation from trend. Further, lagging the interest rate by one period does not effect the estimates much.

The values we get compare as rather plausible to the parameters in the empirical studies refered to in Chapter 2. Both Rotemberg and Woodford (1997) and McCallum and Nelson (1999) fix $\beta = .99$, the value suggested already by the real business cycle litterature of the 1980's, and estimate θ around 5. Bayoumi, Laxton and Pesenti (2004) calibrate a quarterly model for the euro area with subjective time preference $\beta = 1.03^{**}(-.25)$ and elasticity of substitution 5. Smets and Wouters (2004) set $\beta = .99$ and estimate $\theta = 1.1$ for the euro area. In the Bank of Finland quarterly EDGE model, Kortelainen (2002) calibrates parameter values $\theta = 1.0$ and $\beta = 1.04^{**}(-.25)$.

How good is the fit of these calibrated IS-equations? Not very good, as in the quarterly specification, the standard error of regression is about the same size as that of the dependend variable, and in the annual specification, it is larger. Residuals of only the annual equation pass Jarque-Bera normality test. The time pattern, visible in both frequencies, is shown in the Chart below. Before 1987, the residuals are relatively large, which could be a reflection of the then prevailing negative real interest rate and exchange rate fluctuations not captured by the model. The IS-equation seems to work reasonably well only in the period with more stable monetary policy. German unification is another phenomen challenging the equation in the early 1990's.

Chart **Residuals of the quarterly IS-equation (Table 2)**



Appendix 3

Steady state equations in the hybrid IS-equation case

We assume the IS-equation is a hybrid of the type

$$y_t = \mu \cdot y_{t-1} + (1-\mu) \cdot y_{t+1} + (1-2\mu) \cdot \{X_t\}$$

where $\{X_t\}$ is the open economy IS-curve fundamentals as derived in the text. Substituting $y_{t+1} - y_t = \gamma$, and assuming $y_t - y_{t-1} = y_{t+1} - y_t$ in the steady state, results in $y_t = \mu \cdot (y_t - \gamma) + (1-\mu) \cdot (y_t + \gamma) + (1-2\mu) \cdot \{X_t\}$, so that $\gamma \cdot (1-2\mu)/(1-2\mu) = -\{X_t\}$, and finally $\gamma = -\{X_t\}$.

This shows that the steady state of this hybrid specification equals that of the canonical case (ie the case $\mu = 0$).

Appendix 4

Figure 1. **The real exchange rate, long-term interest rate and stock prices, two jumpy forecasts and a smooth one**

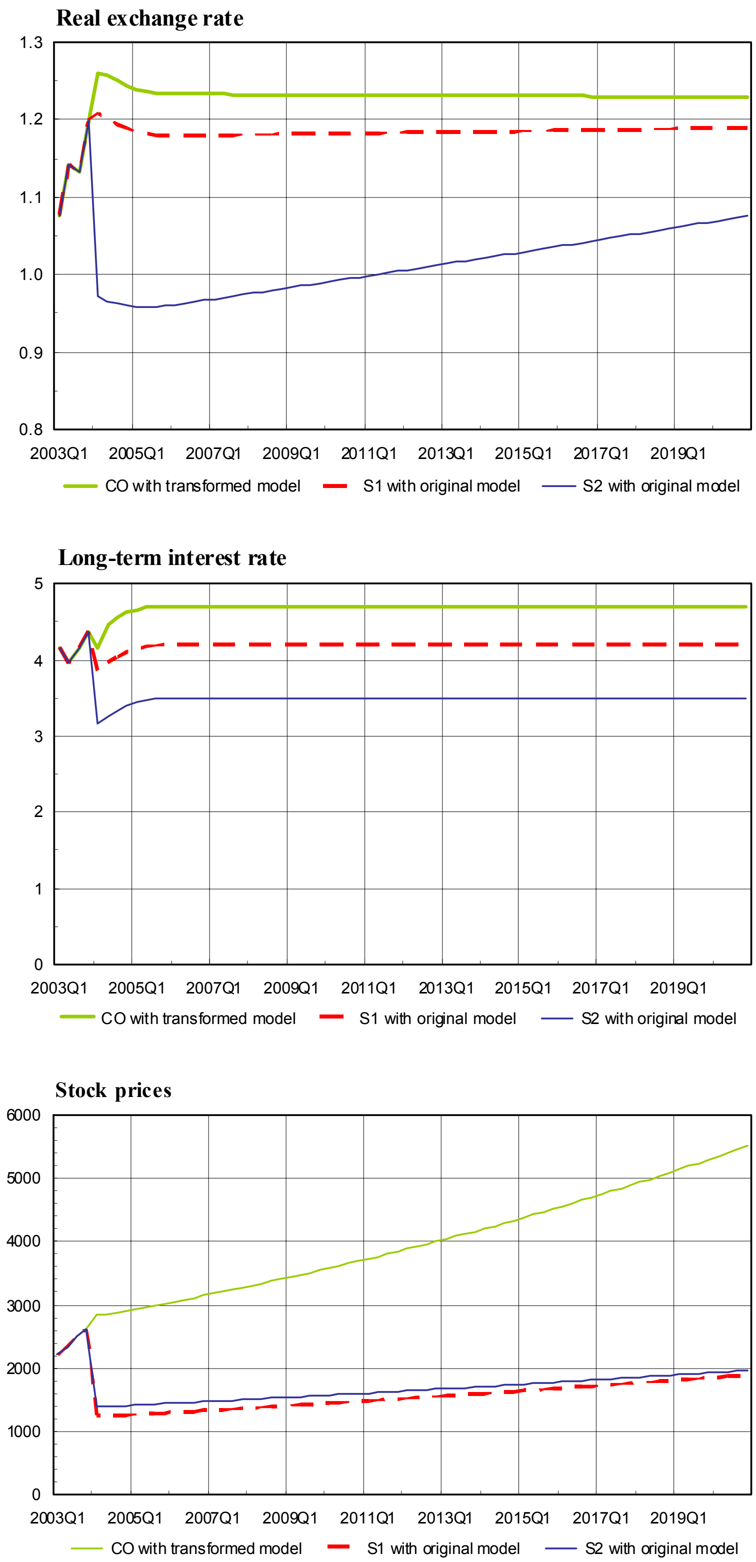
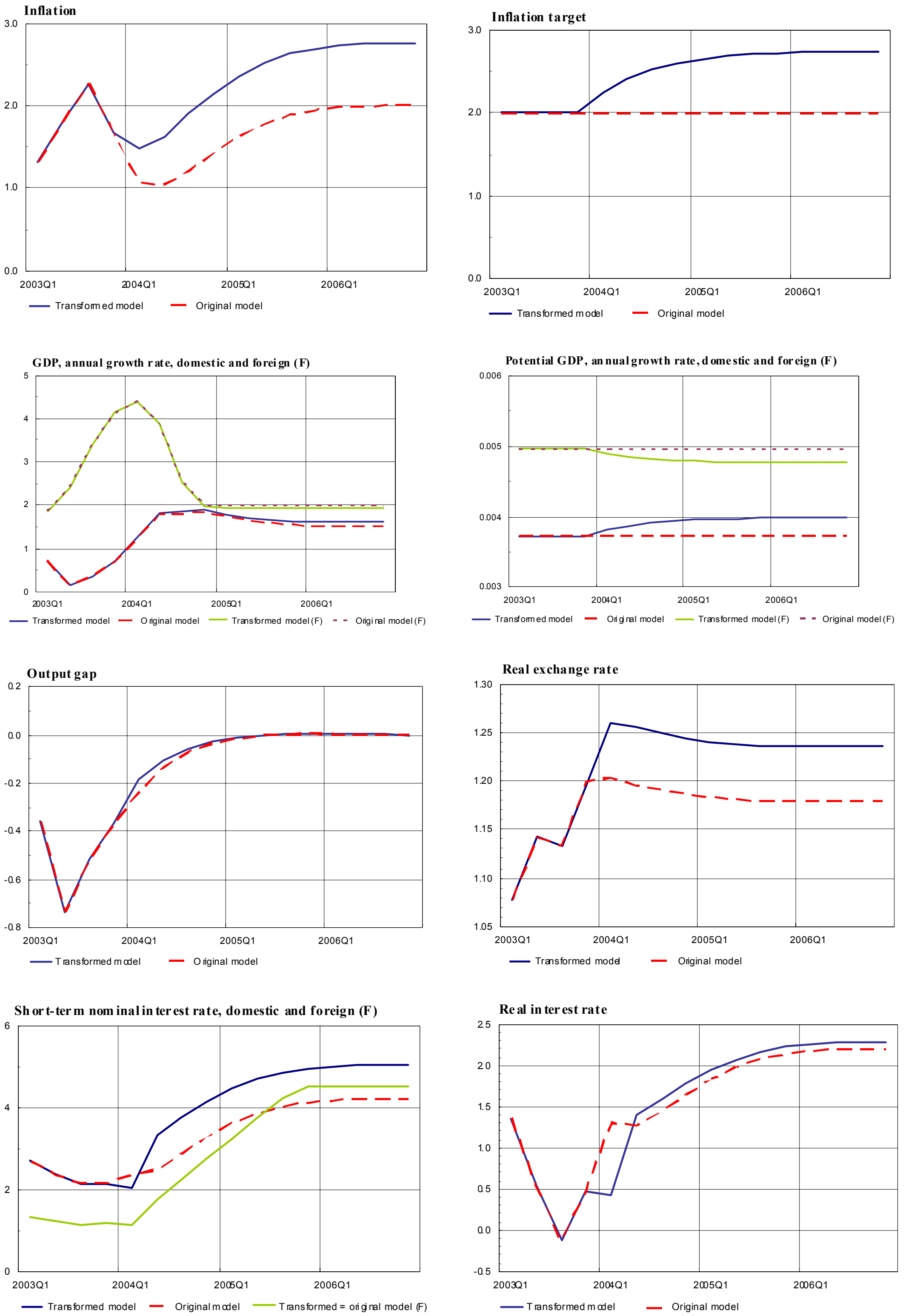
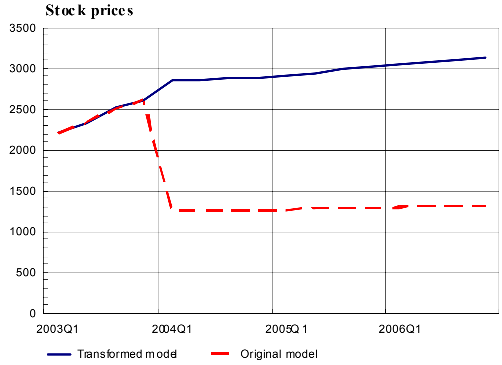


Figure 2. **Short-term forecast paths with the transformed and the original model**




Long-term interest rate
6
5
4
3
2
1
0
2003Q1
2004Q1
2005Q1
2006Q1
Transformed model
Original model


Stock prices
3500
3000
2500
2000
1500
1000
500
0
2003Q1
2004Q1
2005Q1
2006Q1
Transformed model
Original model

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