Essi Eerola – Niku Määttänen

The optimal tax treatment of housing capital in the neoclassical growth model

Bank of Finland Research
Discussion Papers
10 • 2005

Suomen Pankki
Bank of Finland
P.O.Box 160
FIN-00101 HELSINKI
Finland
☎ + 358 10 8311

http://www.bof.fi

Bank of Finland Research
Discussion Papers
10 • 2005

Essi Eerola* – Niku Määttänen**

The optimal tax treatment of housing capital in the neoclassical growth model

* RUESG, University of Helsinki. E-mail: essi.eerola@helsinki.fi
** The Research Institute of the Finnish Economy and CEBR. E-mail: niku.maattanen@etla.fi

We thank Jouko Vilmunen for helpful discussions. We also thank Markus Haavio, Seppo Honkapohja, Karlo Kauko, Erkki Koskela, Henry Ohlsson, and Matti Virén for useful comments and suggestions. This research project was conducted during Eerola's visit at the Bank of Finland Research Department. She wishes to thank the Department for its hospitality during her visit. Both authors gratefully acknowledge financial support from the Yrjö Jahnsson Foundation.

http://www.bof.fi

ISBN 952-462-210-6
ISSN 0785-3572
(print)

ISBN 952-462-211-4
ISSN 1456-6184
(online)

Multiprint Oy
Helsinki 2005

The optimal tax treatment of housing capital in the neoclassical growth model

Bank of Finland Research
Discussion Papers 10/2005

Essi Eerola – Niku Määttänen
Monetary Policy and Research Department

Abstract

In a dynamic setting, housing is both an asset and a consumption good. But should it be taxed like other forms of consumption or like other forms of saving? We consider the optimal taxation of the imputed rent from owner housing within a version of the neoclassical growth model. We find that the optimal tax rate on the imputed rent is quite sensitive to the constraints imposed on the other available tax rates. In general, it is not optimal to tax the imputed rent at the same rate as the business capital income.

Key words: housing, capital taxation, optimal taxation

JEL classification numbers: H21, E21

Omistusasumisen optimaalinen verokohtelu neoklassisessa kasvumallissa

Suomen Pankin tutkimus
Keskustelualoitteita 10/2005

Essi Eerola – Niku Määttänen
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Omistusasunto on sekä varallisuuserä että kulutushyödyke. Mutta pitäisikö asumista verottaa kuten muuta kulutusta vai kuten muuta säästämistä? Täsä työssä tarkastellaan omistusasumisen laskennallisen tuoton optimaalista verokohtelua dynaamisessa yleisen tasapainon mallissa, jossa julkinen valta rahoittaa kulutusmenojaan verottamalla työtuloa, kulutusta, asumista ja pääomavoittoja. Tutkimuksen keskeinen tulos on, että asumisen optimaalinen verokohtelu riippuu voimakkaasti siitä, mitä muita veroinstrumentteja julkinen valta käyttää. Yleisesti ottaen ei ole optimaalista yhtenäistää omistusasumisen ja pääomatulon verokohtelua.

Avainsanat: asuminen, pääomaverotus, optimaalinen verotus

JEL-luokittelu: H21, E21

Contents

1 Introduction

The tax treatment of housing is an important fiscal question because housing wealth constitutes a large share of all household wealth. A common view in the public finance literature is that housing enjoys a tax favored status in most western economies, mainly because the return to owner housing, the imputed rent, usually goes untaxed while the return to business capital is taxed at a relatively high effective tax rate.[1]

Several studies have assessed the welfare consequences of a tax reform that removes this tax favored status of owner housing by setting an equal tax rate on the imputed rent and business capital income. Using quantitative dynamic general equilibrium models, Gahvari (1985), Skinner (1996), and Gervais (2002), among others, have shown that such a reform would lead to substantial efficiency gains.[2]

While these previous studies show that the current tax status of housing is highly distortionary, they do not aim to determine what the optimal tax treatment of housing is and how it depends on the overall tax system. Of course, it need not be optimal to tax the imputed rent at the same rate as business capital income.[3] Since the return to owner housing is a utility flow, it is not clear whether this return should be treated like other forms of saving rather than other forms of consumption.

The previous studies also typically consider only steady state effects of tax reforms.[4] As is often the case with dynamic optimal taxation, neglecting the transition may give very misleading welfare results. Moreover, the optimal tax rates are time varying and it is of interest to see how the tax rate on the imputed rent should evolve over time. In this regard, housing and business capital are different in an interesting way. It is well known that tax rates on business capital income tend to be very high during the first periods of an optimal tax reform. The tax rate on business capital income, however, cannot exceed 100%, unless it takes the form of an unexpected capital levy. This is because firms cannot be forced to operate capital for a certain loss. In contrast, there should always be some demand for housing even with an extremely high tax burden on it. This is true as long as the marginal utility from housing services goes to infinity as they go to zero. Consequently, there is no natural upper bound on the tax rate on the imputed rent.

In this paper, we analyze the optimal tax treatment of housing in a dynamic general equilibrium setting. We employ a version of the neoclassical growth model with a representative household. Housing is introduced following Greenwood and Hercowitz (1991). Although the model is relatively simple,

[1] See Hendershott and White (2000) for an international comparison of housing's tax status.

[2] Other studies that also consider the efficiency and welfare effects of the tax favored status of housing include Gahvari (1984), Slemrod (1982), Berkovec and Fullerton (1992), Hendershott and Won (1992), Poterba (1992), and Bye and Åvitsland (2003). Turnovsky and Okuyama (1994) focus solely on capital accumulation.

[3] Gahvari (1984, 1985) touches upon this issue and shows that the tax rate on the imputed rent need not equal the tax rate on business capital income. However, he does not consider consumption taxation nor labor income taxation.

[4] Exceptions are Skinner (1996) and Määttänen (2004).

it captures the intertemporal savings-consumption decision and the general equilibrium effects of capital taxation that we are interested in.[5]

Formally, we analyze a Ramsey problem for a government that finances government expenditure by taxing consumption, labor income, business capital income, and the imputed rent from owner housing. The government is assumed to be able to commit to future tax policies. The solution to the Ramsey problem is a tax reform which is optimal given the initial state of the economy, individual optimization, and the available tax instruments. This approach takes transitionary dynamics properly into account.

We formulate the optimal taxation problem following the line of research represented by Judd (1985), Chamley (1986), Jones et al (1997), and Atkeson et al (1999), among others. Our analysis is particularly close to Coleman (2000) in that in our benchmark case we allow for taxing consumption and impose the constraint that the tax rate on labor income must be non-negative.[6] Of course, in our set-up the government disposes an additional tax instrument, namely a tax on the imputed rent from owner housing. We also follow Coleman (2000) in assuming that part of the tax revenues are distributed back to the household.[7]

We first present analytical results about the optimal tax structure. In the benchmark set-up, in the long run, the optimal tax rate on capital income is strictly negative and the optimal tax rate on consumption is strictly positive. While we cannot determine the sign of the optimal long run tax rate on the imputed rent, we can show that when the utility function is logarithmic, it is strictly larger than the tax rate on business capital income. We also show that the optimal tax treatment of housing capital relative to business capital depends on both the availability of a consumption tax and the amount of government transfers to the household. Assuming again a logarithmic utility function, we can show that in the special case without consumption taxation and government transfers, the optimal tax rate on the imputed rent equals the optimal tax rate on business capital income, not just in steady state but also during the transition.[8]

Our numerical results shed more light on the optimal tax structure. In particular, we find that in the benchmark case both housing and consumption should be taxed at relatively high rates even in the long run. The optimal tax rate on business capital income is slightly below zero. In this sense, our

[5] As far as we know, we are the first to consider housing taxation as a part of an optimal taxation problem in a dynamic general equilibrium set-up. Cremer and Gahvari (1998) study the optimal taxation of housing in a very different context. In their static model with incomplete information, the government may use differentiated housing taxes so as to separate between different consumer types. See also Englund (2003) for general discussion on housing taxation.

[6] As shown by Coleman (2000), in the absence of such a constraint the government can obtain the first best allocation. This issue will also come up in our analysis.

[7] As discussed in Coleman (2000), in the absence of this assumption, the optimal tax rates would not be uniquely determined.

[8] Strictly speaking, this analytical result requires assuming that the non-negativity constraint on labor income is now never binding. This was the case in our numerical experiments.

results are certainly consistent with the notion that the current tax treatment of housing is far too generous.

We also evaluate the efficiency cost of not taxing housing. Given our optimal taxation approach, we find it natural to compute it by comparing the welfare gains of optimal tax reforms with and without the possibility to tax housing. In our benchmark specification (with consumption taxes and government transfers), the ability to tax housing increases welfare by about two percentage points in terms of an equivalent consumption compensation. This is about one third of the overall welfare gain of an optimal tax reform when housing can be taxed.

We proceed as follows. In the next section we describe the economy. We then analyze in section 3 the case where the government can use the full set of linear taxes with the ability to tax leisure. In this case, it is easy to characterize the optimal tax system essentially because the first best solution can be obtained. In section 4 we characterize the optimal tax structure under various constraints to the set of available tax instruments employing the primal approach to the Ramsey problem. We also consider briefly a small open economy version of the model. We present and discuss our numerical results in section 5. We conclude in section 6.

2 The model

We consider a deterministic model with an infinitely lived representative household that derives utility from the consumption of a consumption good, housing services, and leisure. The production side consists of firms that employ business capital and labor to produce output goods which can be turned into investment and consumption goods. A government imposes flat-rate taxes on labor income, business capital income, imputed rent, and consumption.

2.1 Firms

Every period t, a representative firm employs business capital, k_t, and labor, n_t, to produce output goods, y_t. The production function is

$$y_t = f(k_t, n_t). \tag{2.1}$$

We assume that the production function features constant returns to scale and that $f_{kn} > 0$ for all $k > 0$ and $n > 0$.[9] The firm's first-order conditions for profit maximization imply that the before-tax returns to business capital and labor are given by their marginal productivities, that is,

$$r_t = f_{k_t} - \delta_k \tag{2.2}$$

and

$$w_t = f_{n_t}, \tag{2.3}$$

[9] We denote $\frac{\partial}{\partial k_t} f(k_t, n_t) = f_{k_t}$ and similarly for other derivatives throughout the paper.

where δ_k is the depreciation rate of business capital. The output good may be costlessly converted into consumption good, business capital, and housing capital.

2.2 The household's problem

A representative household is endowed with one unit of time every period. It derives utility from consumption, c, leisure, $1 - n$, and the stock of housing capital, h. The periodic utility function is $u(c, h, n)$. The utility function is strictly increasing in consumption and housing and strictly decreasing in labor, is strictly concave, and satisfies the Inada conditions.

This utility function can be interpreted as a reduced form of the preference structure in Greenwood and Hercowitz (1991), where households derive utility from the consumption of goods and services produced in the market and goods and services produced at home, or 'home production'. Home production is created by combining housing capital and time not allocated to market production.[10]

Some of our analytical results concern the logarithmic and separable special case where

$$u(c, h, n) = \alpha^c \log(c) + \alpha^h \log(h) + (1 - \alpha^c - \alpha^h) \log(1 - n). \tag{2.4}$$

This is also the utility function employed in deriving our benchmark quantitative results.

The household can use both housing and business capital as a savings vehicle. The maximization problem of the household at time $t = 1$ is

$$\max \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, h_t, n_t) \tag{2.5}$$

subject to the intertemporal budget constraint

$$\sum_{t=1}^{\infty} p_t [(1 + \tau_t^c)\, c_t + k_{t+1} + h_{t+1} - R_t k_t - (1 - \tau_t^n) n_t w_t - g_2 - R_t^h h_t] \leq 0 \tag{2.6}$$

where

$$R_t = 1 + (1 - \tau_t^k) r_t$$

and

$$R_t^h = 1 - \delta_h - \tau_t^h r_t.$$

The tax rates on consumption, business capital income, imputed rent, and labor income are denoted by τ^c, τ^k, τ^h, and τ^n, respectively. We set the price

[10] A more general formulation would allow for allocating time to 'leisure', 'home production' and 'market production'. For studies using this approach, see eg Gomme et al (2001), Baxter and Jerman (1999) and McGrattan et al (1997). The two approaches result in the same allocations under a logarithmic specification. For more discussion on this issue, see Greenwood et al (1995).

of one unit of the consumption good in period 1 equal to one. The price of period t consumption in terms of period 1 consumption is denoted by p_t. Lump-sum transfers from the government are denoted by g_2. The parameter δ_h is the depreciation rate of housing capital.

Housing taxation is based on the imputed rent, which is defined as the rental price of housing services. If rental markets existed, the return to rental housing should equal the return to business capital. Thus, the rental price of housing would be $r_t + \delta_h$, assuming that landlords pay for the depreciation and that the tax rate on rental income equals the tax rate on business capital income. The tax base for an amount h_t of housing capital is the imputed rent net of depreciation, ie $r_t h_t$.[11]

The first-order conditions of the household's problem are

$$c_t \; : \; \beta^{t-1} u_{c_t} - \lambda p_t \left(1 + \tau_t^c\right) = 0 \tag{2.7}$$

$$n_t \; : \; \beta^{t-1} u_{n_t} + \lambda p_t (1 - \tau_t^n) w_t = 0 \tag{2.8}$$

$$h_{t+1} \; : \; \beta^t u_{h_{t+1}} - \lambda \left(p_t - p_{t+1} R_{t+1}^h\right) = 0 \tag{2.9}$$

$$k_{t+1} \; : \; p_t - p_{t+1} R_{t+1} = 0 \tag{2.10}$$

where λ is the Lagrange multiplier for the household's budget constraint.

2.3 Government

Each period, total government expenditure is $g = g_1 + g_2$ where g_1 is public consumption that does not affect households at the margin and g_2 denotes transfers to the households. The budget need not be balanced on a period by period basis. The government faces the following intertemporal budget constraint:

$$\sum p_t [\tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^n n_t w_t + \tau_t^h r_t h_t - g] \geq 0 \tag{2.11}$$

2.4 Equilibrium

For a given sequence of tax rates, a competitive equilibrium consists of individual policies and prices such that the individual policies solve the household's problem in (2.5) and (2.6), factor returns are given by equations in (2.2) and (2.3), the government budget constraint in (2.11) is satisfied, and the aggregate resource constraint

$$c_t + k_{t+1} + h_{t+1} + g_1 = f(k_t, n_t) + (1 - \delta_k) k_t + (1 - \delta_h) h_t \tag{2.12}$$

is satisfied for all t.

[11] The tax rate on the imputed rent should be understood as the overall effective tax rate on housing. Alternatively, and equivalently, housing taxation could take the form of a property tax.

2.5 Optimal taxation

The objective of the government is to maximize household welfare by announcing at period $t = 1$ a sequence of tax rates $\{\tau_t^n, \tau_t^c, \tau_{t+1}^k, \tau_{t+1}^h\}_{t=1}^{\infty}$. We assume that the government takes as given the first period tax rates on business capital income and the imputed rent, τ_1^k and τ_1^h. As is always the case with capital taxation, we must impose *some* restriction on the government's ability to tax past investments. Otherwise, the government could trivially reach the first best allocation by confiscating part of the existing capital stocks.

3 First best

It is instructive to first consider the optimal tax structure when the government can use a full set of linear taxes. The first best solution is obtained by assuming that the government can directly choose the allocation (or dictate households' consumption, savings, and labor-leisure decisions). The first best allocations are determined by the following first-order conditions (together with transversality conditions):

$$\begin{aligned} u_{n_t} + u_{c_t} f_{n_t} &= 0 \\ u_{c_t} - \beta u_{c_{t+1}} (1 + r_{t+1}) &= 0 \\ \beta u_{h_{t+1}} - u_{c_t} + \beta u_{c_{t+1}} (1 - \delta_h) &= 0. \end{aligned}$$

The first-order conditions characterizing individually optimal behavior in (2.7)–(2.10) can be written as

$$\begin{aligned} u_{c_t} \eta_t (1 - \tau_t^n) w_t + u_{n_t} &= 0 && (3.1) \\ u_{c_t} \eta_t - \beta u_{c_{t+1}} \eta_{t+1} R_{t+1} &= 0 && (3.2) \\ \beta u_{h_{t+1}} - u_{c_t} \eta_t + \beta u_{c_{t+1}} \eta_{t+1} R_{t+1}^h &= 0. && (3.3) \end{aligned}$$

where $\eta_t = \frac{1}{1+\tau_t^c}$.

Consider then the following tax rates for all $t \geq 1$:

$$\begin{aligned} \tau_t^c &= -\tau_t^n = \tau \geq 0 && (3.4) \\ \tau_{t+1}^k &= 0 && (3.5) \\ \tau_{t+1}^h &= \tau \left(1 + \frac{\delta_h}{r_{t+1}}\right) && (3.6) \end{aligned}$$

When the above tax rates are inserted into the household's first-order conditions in (3.1)–(3.3), the conditions become identical to the first-order conditions characterizing the first best allocation. Hence, if the tax policy in (3.4)–(3.6) is feasible, the resulting competitive equilibrium will correspond to the first best allocation.

To check under what conditions the proposed tax policy is feasible, we first note that with the tax rates in (3.4)–(3.6), the government budget constraint in (2.11) can be written as

$$\tau \sum_{t=1}^{\infty} \left[p_t c_t - p_t n_t w_t + p_{t+1} (r_{t+1} + \delta_h) h_{t+1} \right] = g \sum_{t=1}^{\infty} p_t - T_1,$$

where $T_1 = \left(\tau_1^h h_1 + \tau_1^k k_1\right) r_1$. Similarly, the budget constraint of the household (2.6) can be written as

$$(1+\tau)\sum_{t=1}^{\infty}\left[p_t c_t - p_t n_t w_t + p_{t+1}\left(r_{t+1}+\delta_h\right)h_{t+1}\right] = g_2\sum_{t=1}^{\infty}p_t + A_1.$$

where $A_1 = R_1 k_1 + R_1^h h_1$. Combining these two constraints yields

$$\tau\left[A_1 - \left(g_1\sum_{t=1}^{\infty}p_t - T_1\right)\right] = g\sum_{t=1}^{\infty}p_t - T_1. \tag{3.7}$$

Clearly, $\tau \leq -1$ is not possible in equilibrium. Hence, the above equation may be used to solve for τ provided that

$$\frac{g\sum_{t=1}^{\infty}p_t - T_1}{A_1 - \left(g_1\sum_{t=1}^{\infty}p_t - T_1\right)} > -1. \tag{3.8}$$

Therefore, we obtain

Result 1 *Assuming that condition (3.8) holds, the first best allocation can be achieved with a constant strictly positive tax on consumption, a constant subsidy on labor income, zero tax on business capital income, and a strictly positive tax on the imputed rent.*

The result closely parallels Result 1 in Coleman (2000). Essentially, the first best solution is a form of a lump-sum tax. As a negative tax on labor income is a positive tax on leisure, the government can tax all goods and it need not distort household behavior. We note that the tax treatment of housing is similar to that of consumption in that they are both taxed at a positive rate whereas the tax rate on business capital income is zero.[12]

4 Ramsey problems

In this section, we will consider different restrictions on the available tax instruments. An obvious problem with the first best tax scheme is that it requires subsidizing labor. Such a subsidy would give households an incentive to misrepresent hours of work. Therefore, we begin the section by analyzing the case where labor may not be subsidized and maintain this assumption throughout the section. We also impose the common constraint that the tax

[12] In order to be consistent with the previous literature on housing taxation, we have assumed that depreciation of housing is tax deductible. Disallowing the deduction would simply scale the tax rate on housing downwards. In that case, the first best tax rates would be $\tau^c = \tau^h = -\tau^n = \tau > 0$ and $\tau^k = 0$.

rate on business capital income may not exceed unity.[13] We consider this as our benchmark case.

It is also interesting to see how the optimal tax treatment of housing depends on the availability of a consumption tax. Therefore, we solve the Ramsey problem also for the case without a consumption tax. In addition, we consider restrictions on the tax rate on the imputed rent. The extreme case where housing taxation is ruled out is useful in evaluating the efficiency cost of not taxing housing.

This section consists of four subsections. The first subsection presents the general methodology for solving the government problem in our benchmark case. The two following subsections discuss three different constraints on the tax instruments: the tax rate on the imputed rent may not exceed unity, taxing housing is ruled out, and taxing consumption is ruled out. The emphasis in the presentation is on how the optimal tax reform in the benchmark case changes with the introduction of additional constraints. The last subsection presents the implications of the housing tax reform in a small open economy where the government may not tax the return to international lending.

4.1 No tax on leisure

The problem of the government is to choose the tax policy so as to maximize the utility of the representative household subject to the aggregate resource constraint, household's optimizing behavior, and the constraints imposed on the tax rates. Following the approach taken in Chamley (1986), Judd (1985), and others, we start by eliminating prices and tax rates from the government's problem. Rewriting the budget constraint of the household in (2.6) using the first-order conditions in (2.7)–(2.10) gives

$$\sum \beta^{t-1} \left(u_{c_t} c_t + u_{n_t} n_t + \beta u_{h_{t+1}} h_{t+1} - u_{c_t} \eta_t g_2 \right) = u_{c_1} \eta_1 A_1. \tag{4.1}$$

This is the so-called implementability constraint for the government. Essentially, it states that the allocation must be compatible with individual optimization.

As in Coleman (2000), we can now formulate the problem of the government so that it chooses the consumption tax rates $\{\tau_t^c\}_{t=1}^{\infty}$ and the allocations $\{c_t, n_t, k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$ subject to the aggregate resource constraint, the implementability constraint, and the constraints on the tax rates. Given the consumption tax rates and the optimal allocations, the other tax rates are determined from the household's first-order conditions.

As mentioned above, we require all tax reforms to be such that $\tau_{t+1}^k \leq 1$. This is equivalent to $R_{t+1} \geq 1$. Together with the household's first-order condition in (3.2), this constraint can be written as

$$u_{c_t} \eta_t \geq \beta u_{c_{t+1}} \eta_{t+1}. \tag{4.2}$$

[13] Also slightly different resctrictions have been imposed in the literature. See eg Atkeson et al (1999) and Jones et al (1993) for a discussion on the effects of changing the bounds.

In addition, the government is constrained to set a non-negative tax rate on labor income. Combining constraint $\tau_t^n \geq 0$ with the household's first-order condition in (3.1) gives the following constraint:

$$u_{n_t} + u_{c_t}\eta_t f_{n_t} \geq 0. \tag{4.3}$$

The Lagrangian for the government may now be written as:

$$\begin{aligned}
\mathcal{L} \;=\; & \sum \beta^{t-1} u(c_t, h_t, n_t) \\
& +\lambda \left[\sum \beta^{t-1} \left(u_{c_t} c_t + u_{n_t} n_t + \beta u_{h_{t+1}} h_{t+1} - u_{c_t}\eta_t g_2 \right) - u_{c_1}\eta_1 A_1 \right] \\
& +\sum \beta^{t-1} \mu_t (f(k_t, n_t) + (1-\delta_k)k_t + (1-\delta_h)h_t \\
& -c_t - k_{t+1} - h_{t+1} - g_1) \\
& +\sum \beta^{t-1} \theta_t \left(u_{c_t}\eta_t - \beta u_{c_{t+1}}\eta_{t+1} \right) \\
& +\sum \beta^{t-1} \omega_t \left(u_{n_t} + u_{c_t}\eta_t f_{n_t} \right).
\end{aligned} \tag{4.4}$$

The first constraint is the implementability constraint. The second set of constraints contains an aggregate resource constraint for each period. The third and fourth sets of constraints are the restrictions on the tax rates.

For periods $t > 1$, the first-order conditions for the government are:[14,15]

$$\begin{aligned}
n_t &: W_{n_t} + B_t u_{cn_t} + \mu_t f_{n_t} + \omega_t \left(u_{nn_t} + u_{c_t}\eta_t f_{nn_t} \right) = 0 && (4.5)\\
c_t &: W_{c_t} + B_t u_{cc_t} - \mu_t + \omega_t u_{nc_t} = 0 && (4.6)\\
k_{t+1} &: -\mu_t + \beta\mu_{t+1}(1 + r_{t+1}) + \beta\omega_{t+1} u_{c_{t+1}}\eta_{t+1} f_{nk_{t+1}} = 0 && (4.7)\\
h_{t+1} &: \beta W_{h_{t+1}} + \beta B_{t+1} u_{ch_{t+1}} - \mu_t + \beta\mu_{t+1}(1-\delta_h) && (4.8)\\
&: +\beta\omega_{t+1} u_{nh_{t+1}} = 0 \\
\eta_t &: \omega_t f_{n_t} - \lambda g_2 = 0. && (4.9)
\end{aligned}$$

where $B_t = \omega_t \eta_t f_{n_t} - \lambda g_2 \eta_t$ and

$$\begin{aligned}
W_{n_t} &= u_{n_t} + \lambda(u_{hn_t} h_t + u_{cn_t} c_t + u_{nn_t} n_t + u_{n_t}) \\
W_{c_t} &= u_{c_t} + \lambda(u_{hc_t} h_t + u_{cc_t} c_t + u_{c_t} + u_{nc_t} n_t) \\
W_{h_t} &= u_{h_t} + \lambda(u_{hh_t} h_t + u_{h_t} + u_{ch_t} c_t + u_{nh_t} n_t)
\end{aligned}$$

These first-order conditions determine, together with the aggregate resource constraints and the implementability constraint, the allocations $\{c_t, n_t, k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$, the consumption tax rates $\{\tau_t^c\}_{t=1}^{\infty}$, and λ, the multiplier of the implementability constraint. After an allocation has been found, prices $\{r_{t,} w_t\}$ are determined from equations in (2.2) and (2.3). Finally, the tax rates on labor income, business capital income, and the imputed rent are determined from equations in (3.1), (3.2), and (3.3), respectively.

[14] For presentational purposes, we drop here the constraint in (4.2). One can show that it can only be binding during the first periods of the transition.

[15] The first-order conditions for period 1 will be different as period 1 variables enter the government problem independently of the other period variables through the implementability constraint.

Several interesting results may be obtained by inspecting the first-order conditions of the government Ramsey problem. Note first that if the constraint in (4.2) is not binding, the first-order condition in (4.9) directly implies that

$$\lambda g_2 = \omega_t f_{n_t}. \tag{4.10}$$

Since the implementability constraint must be binding, $\lambda > 0$. Therefore, this condition requires that if $g_2 > 0$ then $\omega_t > 0$ for $t > 1$. Therefore, whenever the constraint in (4.2) is not binding, the optimal tax system must involve $\tau^n = 0$.[16]

Consider then the optimal tax rate on business capital income. Denote the steady state value of μ_t by μ (and similarly for other variables). Then, the steady state version of condition (4.7) can be written as

$$1 = \beta (1 + r) + \frac{\beta \omega u_c \eta f_{nk}}{\mu}.$$

In the same manner, in a steady state, the household first-order condition that determines the tax rate on business capital income in (3.2) becomes

$$1 = \beta \left(1 + \left(1 - \tau^k\right) r\right).$$

By combining these two conditions we obtain

$$\tau^k = -\frac{\omega u_c \eta f_{nk}}{r\mu}. \tag{4.11}$$

Since $\omega > 0$, it follows that the long run tax rate on the return to business capital is negative.

These two results are the same as in Coleman (2000). The tax rate on labor income should be zero because the consumption tax rate is more efficient. In addition, given that labor is not taxed it is beneficial to subsidize business capital accumulation. This is because increased business capital accumulation increases the marginal productivity of labor. This in turn lowers the cost of imposing the constraint that labor may not be subsidized.

The government first-order conditions with respect to consumption and housing are somewhat complicated and it turns out that it is not possible to draw general conclusions. This is because, in general, the signs of W_c, W_h, and W_n depend on the magnitude of λ, the multiplier of the implementability constraint.

However, we can characterize these tax rates assuming logarithmic utility function, as in (2.4).[17] Consider first the tax rate on consumption. Rewriting (4.6) using (4.5) and (4.8) gives

$$\frac{u_n}{u_c} + f_n = -\frac{\lambda (u_{nn} n + u_n)}{u_c} - \frac{\omega (u_{nn} + u_c \eta f_{nn})}{u_c}$$

[16] If $g_2 = 0$, the condition in (4.10) implies that $\omega_t = 0$. But then the government first-order condition in (4.9) is satisfied for all allocations and consumption tax rates. In the absence of transfers, there need not be a unique optimal path of tax rates. This point is also discussed in Coleman (2000).

[17] This implies that $W_{n_t} = u_{n_t} + \lambda (u_{nn_t} n_t + u_{n_t})$, $W_{c_t} = u_{c_t}$, and $W_{h_t} = u_{h_t}$.

where the right hand side is strictly positive. The household first-order condition in (3.1) in turn implies that

$$\frac{u_n}{u_c} + \frac{f_n}{(1+\tau^c)} = 0.$$

Comparison of these two constraints shows that $\tau^c > 0$.

Let us then analyze the long run tax rate on the imputed rent. Using equations in (4.6), (4.7) and (4.9), we can write the steady state version of the first-order condition in (4.8) as

$$\frac{u_h}{u_c} - (\delta_h + r) = \omega\eta f_{nk}. \tag{4.12}$$

Combining the first-order conditions of the household in (3.2) and (3.3) in turn leads to

$$\frac{u_h}{u_c} = \eta\left(R - R^h\right). \tag{4.13}$$

Combining (4.12) and (4.13) results in[18]

$$\tau^h - \tau^k - \tau^c\left(1 + \frac{\delta_h}{r}\right) = \frac{\omega f_{nk}}{r}.$$

By collecting these results, we obtain

Result 2 *If the government cannot subsidize labor and gives transfers to the households, in a steady state*

i) the tax rate on labor income is zero.

ii) the tax rate on business capital income is negative.

iii) if the utility function is logarithmic, the tax rate on consumption is strictly positive and the relationship between the tax rates on the imputed rent, business capital income and consumption is

$$\tau^h - \tau^k - \tau^c\left(1 + \frac{\delta_h}{r}\right) = \frac{\omega f_{nk}}{r}.$$

These results leave open whether the long run tax rate on the imputed rent should be positive or negative. They do show, however, that the tax rate on the imputed rent should be higher than the tax rate on business capital income.

[18]As in the first best, the relationship between the optimal tax rates would be slightly different if we didn't allow tax deduction of housing depreciation. In that case, we would obtain $\tau^h - \tau^c - \tau^k \frac{r}{r+\delta_h} = \frac{\omega f_{nk}}{r+\delta_h}$.

4.2 Restrictions on the tax rate on the imputed rent

The optimal tax system considered in the previous subsection may involve very high tax rates on the imputed rent during the transition. In our quantitative analysis, we therefore consider how the results change if we impose an upper bound on the tax rate on the imputed rent. In particular, we require that $\tau^h_{t+1} \leq 1$. In addition, in order to assess the importance of taxing the imputed rent, we consider a tax reform that requires setting $\tau^h_{t+1} = 0$ in all periods.

The first constraint implies that $R^h_{t+1} \geq 1 - \delta_h - r_{t+1}$. This inequality, together with the household's first-order condition in (3.3), implies that the government is constrained to choose allocations that satisfy

$$u_{c_t}\eta_t \geq \beta u_{c_{t+1}}\eta_{t+1}\left(1 - \delta_h - r_{t+1}\right) + \beta u_{h_{t+1}}. \tag{4.14}$$

As long as this constraint is not binding in the steady state, the characterization of the steady state tax rates remains the same as in Result 2.

The second constraint, $\tau^h_{t+1} = 0$, implies that $R^h_{t+1} = 1 - \delta_h$ in all periods. Together with the household's first-order condition it translates into the following constraint to the government's problem:

$$u_{c_t}\eta_t = \beta u_{c_{t+1}}\eta_{t+1}\left(1 - \delta_h\right) + \beta u_{h_{t+1}}. \tag{4.15}$$

This constraint will be binding also in the long run.[19] Now (4.9) becomes

$$-\lambda g_2 + \gamma_{t+1} - \gamma_t\left(1 - \delta_h\right) + \omega_{t+1} f_{n_{t+1}} = 0$$

where γ is the Lagrange multiplier on constraint (4.15). Since γ is always strictly positive, it may be that $\omega = 0$. If this is the case, the condition in (4.11) implies that $\tau^k = 0$ in the long run.

4.3 No consumption taxes

In the previous literature on housing taxation, consumption taxation is typically ignored altogether. In order to examine the importance of allowing for consumption taxes, we now impose an additional restriction requiring the tax rate on consumption to be zero.

The problem of the government now looks very similar to the one above except that now we do not have the first-order conditions related to the consumption tax rate in (4.9). As a result, we are no longer able to determine analytically whether the non-negativity constraint on the labor income tax rate is binding or not.

The case where $\omega > 0$ is quite cumbersome and not very insightful. Also, our numerical analysis shows that for a low τ^c, $\omega = 0$, i.e. labor income should be taxed. We will therefore discuss here only the case where $\omega = 0$.

[19] It can be shown that simply setting $\tau^h = 0$ and ignoring this constraint leads to an allocation that cannot be decentralized using the remaining instruments. Consequently, this constraint must be imposed separately and is always binding.

If $\omega = 0$, the tax rate on labor income is strictly positive.[20] In addition, from the equation in (4.11) it follows that $\tau^k = 0$.

Again determining the tax rate on housing is more complicated. However, under logarithmic utility, using the first-order conditions in (4.6) and (4.7), the first-order condition in (4.8) can be written as

$$\frac{u_{h_{t+1}}}{u_{c_{t+1}}} - (\delta_h + r_{t+1}) = \frac{-\lambda g_2 u_{cc_{t+1}} (\delta_h + r_{t+1})}{u_{c_{t+1}}}. \tag{4.16}$$

Combining this with the first-order condition of the household

$$\frac{u_{h_{t+1}}}{u_{c_{t+1}}} = R_{t+1} - R^h_{t+1} \tag{4.17}$$

gives

$$\tau^h_{t+1} - \tau^k_{t+1} = \frac{\lambda g_2}{c_{t+1}} \left(\frac{\delta_h}{r_{t+1}} + 1 \right). \tag{4.18}$$

By collecting these results, we obtain

Result 3 *In the absence of consumption taxation, if the non-negativity constraint on labor income tax rate is not binding.*

i) in a steady state, tax rate on business capital income is zero.

ii) in a steady state, if the utility function is logarithmic, the tax rate on the imputed rent is given by

$$\tau^h = \frac{\lambda g_2}{c} \left(1 + \frac{\delta_h}{r} \right).$$

iii) if the utility function is logarithmic, in the absence of transfers to the households, $\tau^h_t = \tau^k_t$ in all periods and both are zero in the steady state.

Thus, when consumption taxes are ruled out, the tax rate on the imputed rent depends on the amount of transfers from the government to the households. Only in the special case where there are no transfers, the optimal tax rate on the imputed rent equals the optimal tax rate on business capital income.

These results clearly indicate that the effective tax rate on housing should be compared not just to the effective tax rate on business capital income (as is typically done) but also to the effective tax rate on consumption.

[20] Ignoring the case where the corner solution is optimal.

4.4 Small open economy

Our motivation to consider a small open economy case are related to tax competition. Accordingly, we consider a somewhat extreme case where the government cannot tax capital income from abroad. We focus on our benchmark case where the government cannot subsidize labor but is allowed to tax consumption.

The first-order conditions of the household are the same as in section 2.2 except that we now have an additional condition stating that

$$\left(1-\tau_t^k\right) r_t = r^*, \tag{4.19}$$

where r^* denotes the international interest rate, which is assumed to be constant over time. The constraint implies that now the international interest rate and the domestic tax rate on business capital income directly pin down the marginal productivity of capital.

Apart from the constraint in (4.19), the only difference to the government problem in (4.4) is that now the aggregate resource constraint for the economy is given by

$$c_t + k_{t+1} + b_{t+1} + h_{t+1} + g_1 = (1-\delta_k)k_t + (1-\delta_h)h_t + f(k_t, n_t) + (1+r_t^*)\, b_t$$

where b_t denotes international bonds.

It is straightforward to show that

Result 4 *In a small open economy, if the government cannot subsidize labor and gives transfers to the households*

i) labor income tax rate is zero in all periods.

ii) tax rate on business capital income is negative for all $t > 1$.

iii) if utility is logarithmic, relationship of the tax rate on the imputed rent and the consumption tax rate is given by

$$\tau_{t+1}^h = \tau_{t+1}^c \left(\frac{\delta_h}{r_{t+1}} + \frac{r^*}{r_{t+1}}\right) \text{ in all periods.}$$

These analytical results suggest that, except for the first periods, the optimal tax reform in a small open economy is similar to the one in a closed economy.

5 Quantitative analysis

In this section, we present our numerical results. We begin by explaining the calibration of the model. We then present the optimal tax reforms and the corresponding welfare effects in the first best and under the different constraints discussed in the previous section. In the last subsection, we experiment with different parameter values to test the robustness of the results.

5.1 Benchmark calibration

In our benchmark calibration, we consider the logarithmic utility function in (2.4). This utility function is consistent with the fact that US households have spent a roughly constant fraction of their expenditures in housing over time even though the relative price of housing has declined. (See eg Kydland, 1995).

The production function is Cobb-Douglas:

$$f(k, n) = k^{\alpha} n^{1-\alpha},$$

where α is the capital share.

We calibrate the model to the US economy. Following previous studies with a similar set-up, we set the technology parameters at the following values: $\alpha = 0.33$, $\delta_k = 0.08$, $\delta_h = 0.05$. We assume that in the initial tax system $\tau^n = 0.27$, $\tau^c = 0$, $\tau^h = 0$, and $\tau^k = 0.5$. These tax rates are within the range of empirical estimates in the literature.[21] Given the other parameter values chosen, these tax rates imply a government revenue-to-total output ratio of 0.21.

Parameters β, α^c, α^h, g_1, and g_2 are chosen so as to match the following aggregate targets. 1) Business capital-to-housing ratio $k/h = 1$. 2) Total capital-to-total output ratio $(k + h)/y = 3.0$, where $y = k^{\alpha} n^{1-\alpha} + (r + \delta_h)\, h$. 3) Transfers-to-total government spending ratio $g_2/\,(g_1 + g_2) = 0.43$. 4) Labor supply $n = 0.333$. 5) The government budget is balanced and there is no government debt.

The first two of these targets are based on the National Income and Product Accounts (NIPA). The Fixed Asset Table in NIPA contains private residential and non-residential assets. We interpret all business capital in the model as private non-residential assets and all housing capital in the model as private residential assets. The third target is from the 2004 Economic Report of the President.[22] The fourth target implies that households spend one third of their time working. The last constraint pins down the sum of g_1 and g_2. All parameter values of this benchmark calibration are collected in table 1.

---

[21] For the labor income tax rate, see e.g. Mendoza et al (1994). The tax rate on the imputed rent is in the range of what was estimated by Fullerton (1987) to be the effective tax rate on housing in the US. In a similar model, Greenwood et al (1995) set the tax rate on business capital equal to 0.70, arguing that it includes not just taxes but also various regulatory costs. The tax rate on consumption is typically estimated to be around 0.05 (See eg Carey and Rabesona, 2004). In order to make it easier to compare the cases with and without consumption taxes we set it to zero.

[22] We use Table B-83 to first calculate the sum of government consumption expenditures and transfer payments. We then calculate the average share of transfer payments of this sum for years 1999–2003.

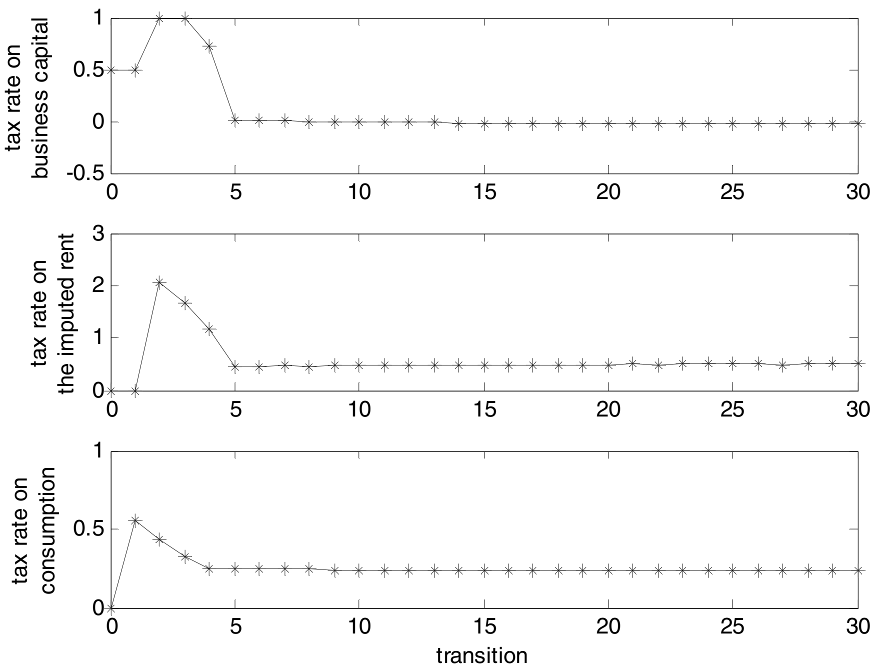
Table 1: Benchmark calibration

Preferences	β	0.9556
	α^c	0.3395
	α^h	0.1045
Technology	α	0.33
	δ_k	0.08
	δ_h	0.05
Tax system	τ^n	0.27
	τ^h	0
	τ^c	0
	τ^k	0.50
Government expenditures	g_1	0.0703
	g_2	0.0531

5.2 Results

We first present the transitional dynamics of the economy in the benchmark case. Figure 1 shows the paths of the optimal tax rates. The tax rates in the initial steady state are depicted in the figure as period 0 tax rates. The tax rate on labor income, being zero from period one onwards, is not shown in the figure.

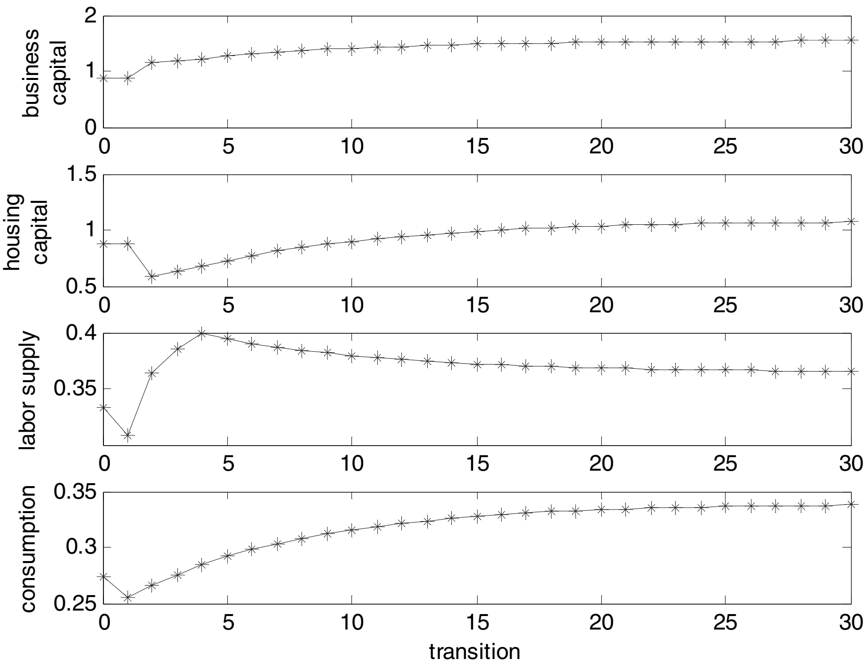
Figure 1: Optimal tax rates when $\tau^n \geq 0$ and $\tau^k \leq 1$.



Recall that we require the first period tax rates on business capital income and the imputed rent to remain fixed. In addition, the tax rate on business capital income is required not to exceed one during the transition. In periods 2 and 3, this constraint is binding. The tax rate on the imputed rent is remarkably high during the first periods of the transition. The tax rate on consumption increases immediately. All tax rates converge close to their new steady state levels in just about five periods.

Figure 2 below shows the corresponding transitionary dynamics of business and housing capital stocks, labor supply, and consumption. Again period 0 values refer to the initial steady state.

Figure 2: Transition paths when $\tau^n \geq 0$ and $\tau^k \leq 1$.



Business capital stock increases steadily during the transition towards its post-reform steady state level which is substantially higher than in the initial steady state. Housing capital stock first decreases after which it increases towards the new steady state level which is also somewhat higher than the initial steady state level.[23] The labor supply increases first since the tax rate on labor income drops drastically with the announcement of the reform. By the same token, consumption first diminishes following the introduction of the consumption tax.

Table 2 below shows the optimal tax rates in the different cases discussed above in both the short run and the long run. The first four rows present the tax system which supports the first best allocation as explained in section 3. The second case is the benchmark case where we restrict the labor income tax rate to be non-negative and the business capital income tax rate to be at most one. In the last three cases, we impose additional constraints on the tax instruments.

[23] Of course, the large changes in the two capital stocks are related to the fact there are no adjustment costs or irreversibility constraints in the model.

Table 2: Optimal tax rates.

Constraints on tax policy		Tax rate paths year 1	year 2	year 3	year 4	year 5	year 10	year ∞
Case 1	τ^k	0.5	0	0	0	0	0	0
First best	τ^h	0	6.03	6.23	6.42	6.60	7.30	8.20
	τ^c	3.95	3.95	3.95	3.95	3.95	3.95	3.95
	τ^n	-3.95	-3.95	-3.95	-3.95	-3.95	-3.95	-3.95
Case 2	τ^k	0.5	1.00	1.00	0.73	0.00	-0.01	-0.03
$\tau^n \geq 0$	τ^h	0	2.06	1.66	1.17	0.45	0.47	0.50
and $\tau^k \leq 1$	τ^c	0.56	0.43	0.32	0.25	0.25	0.24	0.23
	τ^n	0	0	0	0	0	0	0
Case 3	τ^k	0.5	0.66	0.52	0.50	0.30	-0.01	-0.03
$\tau^n \geq 0$, $\tau^k \leq 1$	τ^h	0	1.00	1.00	1.00	0.78	0.51	0.54
and $\tau^h \leq 1$	τ^c	0.49	0.40	0.34	0.29	0.27	0.26	0.25
	τ^n	0	0	0	0	0	0	0
Case 4	τ^k	0.5	1.0	1.0	0.99	0.88	0.01	0
$\tau^c = 0$, $\tau^n \geq 0$	τ^h	0	1.0	1.0	1.0	1.0	0.11	0.11
$\tau^k \leq 1$	τ^c	0	0	0	0	0	0	0
and $\tau^h \leq 1$	τ^n	0	0.03	0.10	0.17	0.22	0.28	0.28
Case 5	τ^k	0.5	0.18	0.17	0.15	0.14	0.09	0.00
$\tau^h = 0$, $\tau^n \geq 0$	τ^h	0	0	0	0	0	0	0
$\tau^k \leq 1$	τ^c	0.42	0.40	0.38	0.37	0.36	0.31	0.24
	τ^n	0	0.03	0.04	0.05	0.05	0.08	0.12

In the first best case, the required subsidy on labor income is very high. Consequently, all other tax rates are very high as well.

Considering the optimal tax structure in the constrained cases reveals several important insights about the optimal tax treatment of the imputed rent. Consider first the benchmark case (case 2 in table 2). In the long run, both the optimal tax rate on the imputed rent and the tax rate on consumption are relatively high (0.50 and 0.23, respectively), whereas the tax rate on business capital income is slightly below zero (-0.03). In this example, the optimal tax treatment of housing appears to be closer to the tax treatment of consumption than that of business capital.

Inspection of case 4 in table 2 shows that the optimal tax treatment of housing is very sensitive to whether or not we can freely tax consumption. The optimal long run tax rate on the imputed rent falls from 0.50 in the benchmark case to 0.11 when we rule out consumption taxation.

The constraints imposed on the tax rates on business capital income and the imputed rent influence always both of these tax rates. In the benchmark case, the constraint $\tau^k \leq 1$ is binding during the first periods of the transition. However, when there is an upper bound on the tax rate on the imputed rent (case 3 in table 2), also the tax rate on business capital income is set at a lower level and the constraint $\tau^k \leq 1$ is never binding.

Interestingly, ruling out housing taxation altogether (case 5 in table 2) changes the dynamics of the optimal business capital taxation completely. The tax rate on business capital income starts diminishing from the very first period after the reform is announced. Furthermore, it now converges to zero very slowly.[24] In other words, if housing cannot be taxed, the optimal tax rate on business capital income does not feature the usual dynamics with very high tax rates in the first periods and a rapid convergence to the new steady state tax rate. This is due to the fact that households have two savings vehicles, housing and business capital, and the government can tax only one of them. Ruling out housing taxation also changes the evolution of the labor income tax rate. Contrary to the case where both forms of saving may be taxed, it is now optimal to tax labor income.

Our welfare measure is the 'equivalent consumption variation'. It tells how much consumption should be increased in the initial steady state so as to make the household indifferent between the status quo and the tax reform when leisure and housing choices are kept fixed.[25] The overall welfare effect in table 3 takes household welfare during the transition periods into account. The steady state gain compares welfare in the initial steady state to the welfare in the new steady state.

Table 3: Welfare effects of the tax reforms.

Tax reform	Overall welfare gain	Steady state gain
first best	8.8%	28.9%
$\tau^n \geq 0$ and $\tau^k \leq 1$	6.7%	22.2%
$\tau^n \geq 0$, $\tau^k \leq 1$ and $\tau^h \leq 1$	6.6%	21.5%
$\tau^c = 0$, $\tau^n \geq 0$, $\tau^k \leq 1$ and $\tau^h \leq 1$	4.7%	15.3%
$\tau^h = 0$, $\tau^n \geq 0$, and $\tau^k \leq 1$	4.7%	19.8%

These welfare results suggest two important conclusions. First, the ability to tax the return to housing at a very high rate (exceeding 100%) is not important. Imposing an upper bound of unity to the tax rate on the imputed rent decreases the overall welfare gain only by one tenth of a percentage point. This is despite the fact that the optimal tax rate on the imputed rent is indeed quite high during the first periods after the reform. Apparently, very high initial tax rates on the imputed rent distort households' resource allocation so much that the gain from the increased tax revenue remains small. Second, the ability to tax consumption is important. The overall welfare gain of the tax reform fall substantially if consumption taxes are ruled out.

The last row presents the welfare gain when housing is not taxed at all. The overall welfare gain is two percentage points lower than in the benchmark case where only labor subsidies and confiscatorily high capital income taxes are excluded. This is our preferred measure for the welfare cost of not taxing housing.

The steady state effects are always much larger than the overall welfare effect. The post-reform steady state is associated with higher total stock of capital which is built up by higher saving levels during the transition. Comparing the steady state gains in the last two cases reveals that in the long run the welfare cost of not taxing housing is far larger than the welfare cost of not taxing consumption.

[24] For instance, we find that the tax rate on business capital income is still about 0.04 after 20 periods.

[25] Of course, the required compensation would be smaller if households were allowed to reoptimize after receiving the compensation.

5.3 Sensitivity analysis

In this section, we experiment with different assumptions about the elasticity of substitution between consumption, housing, and leisure. In doing so, we focus on the benchmark case where labor may not be subsidized and the tax rate on business capital income may not exceed unity.

Following Greenwood and Hercowitz (1991), we define the following 'home production function':

$$c^*(h,n) = (\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h})^{1/\gamma^h},$$

where $0 < \theta^h < 1$ is the weight of housing services in the home production function. The elasticity of substitution between housing services and leisure is given by $\varepsilon^h = \frac{1}{1-\gamma^h}$ with γ^h satisfying $\gamma^h < 1$ and $\gamma^h \neq 0$. The utility function is then given by

$$u(c,h,n) = \frac{[(\theta^c c^{\gamma^c} + (1-\theta^c)c^{*\gamma^c})^{1/\gamma^c}]^{1-\sigma}}{1-\sigma}, \text{ for } \sigma > 0,\ \sigma \neq 1$$
$$u(c,h,n) = \log(\theta^c c^{\gamma^c} + (1-\theta^c)c^{*\gamma^c})^{1/\gamma^c}), \text{ for } \sigma = 1$$

where $0 < \theta^c < 1$ is the utility weight of consumption, σ is the inverse of the intertemporal elasticity of substitution. The elasticity of substitution between 'home production' and consumption is given by $\varepsilon^c = \frac{1}{1-\gamma^c}$ with γ^c satisfying $\gamma^c < 1$ and $\gamma^c \neq 0$.

We will fix $\sigma = 1$ and consider different elasticities of substitution ε^c and ε^h.[26] For both elasticities, we consider values 2 and 1/2. In all cases, we calibrate the other preference parameters so as to match the same targets as in section 5.1 with the same initial tax system and market technology parameters.

The logarithmic utility function employed in section 5.1 is a special case of the utility function considered here with $\varepsilon^c = 1$ and $\varepsilon^h = 1$. So as to facilitate comparison of the different cases, we also report here the steady state tax rates under the logarithmic utility. The results on the optimal steady state tax rates in different cases are summarized in table 4.

Table 4. Optimal steady state tax rates under different ε^c and ε^h.

	Steady state tax rates					
	$\varepsilon^c = 1$			$\varepsilon^h = 1$		
	$\varepsilon^h = 2$	$\varepsilon^h = 1$	$\varepsilon^h = 1/2$	$\varepsilon^c = 2$	$\varepsilon^c = 1$	$\varepsilon^c = 1/2$
τ^k	-0.03	-0.03	-0.02	-0.02	-0.03	-0.02
τ^h	0.40	0.50	0.64	0.56	0.50	0.39
τ^c	0.24	0.23	0.22	0.20	0.23	0.26
τ^n	0	0	0	0	0	0

The variation in the degree of substitutability between leisure and housing services or between home production and consumption has virtually no effect on the optimal tax rate on business capital income. Also the labor income tax rate remains unaffected as the non-negativity constraint continues to bind.

[26] We found that different reasonable values for σ did not change the results concerning the optimal long run tax rates substantially.

In contrast, the long run tax rate on the imputed rent is quite sensitive not only to the substitutability between leisure and housing services but also the substitutability between consumption and home production. As the elasticity of substitution between housing services and leisure falls from 2 to 1/2, the long run tax rate on the imputed rent increases from 0.40 to 0.64. Similarly, when ε^c drops from 2 to 1/2, the optimal tax rate on the imputed rent decreases from 0.56 to 0.39. Variation in the substitutability has a much more modest impact on the optimal tax rate on consumption. Changes in ε^h leave the optimal consumption tax rate almost unaffected while a reduction in ε^c from 2 to 1/2, increases the optimal consumption tax rate from 0.20 to 0.26.

Table 5 shows the overall welfare effects and steady state welfare effects under the different model specifications considered above. The upper part reports the results related to changes in the elasticity of substitution between housing and leisure when $\varepsilon^c = 1$ while the bottom of the table shows the same results for different elasticities between home production and consumption keeping the substitutability between housing services and leisure fixed at $\varepsilon^h = 1$.

Table 5: Welfare effects in different model specifications.

Elasticity of substitution		Overall welfare gain	Steady state gain
	$\varepsilon^h = 2$	8.2%	32.4%
$\varepsilon^c = 1$	$\varepsilon^h = 1$	6.7%	22.2%
	$\varepsilon^h = 1/2$	6.3%	18.4%
	$\varepsilon^c = 2$	11.0%	28.5%
$\varepsilon^h = 1$	$\varepsilon^c = 1$	6.7%	22.2%
	$\varepsilon^c = 1/2$	5.1%	21.2%

The table shows a clear pattern in the magnitude of welfare effects: As the elasticity of substitution increases, the welfare effects also increase. The rationale is straightforward: When the elasticity of substitution is high, there exist a lot of scope for substituting leisure for housing services and vice versa. This means, other things equal, that the distortions created by the initial tax system grow larger as the substitutability increases. The larger the distortions in the initial steady state, the bigger will be the welfare gains associated from moving to the optimal tax structure.

6 Conclusions

We have considered the optimal tax status of housing within a dynamic general equilibrium model. Our analytical results demonstrate that in general the optimal tax rate on the imputed rent should not equal the tax rate on the business capital income. Quantitatively, we found that in the arguably realistic case where consumption can be taxed but labor cannot be subsidized, both housing and other consumption should be taxed at relatively high tax rates, whereas the tax rate on business capital income should be close to zero. In this sense, our results certainly support the view that the current tax treatment of housing is far from optimal in most western economies.

Our experiments with different preference parameters reveal that the long run tax rate on the imputed rent depends on the elasticity of substitution between leisure (or, alternatively, time spent on homework) and housing: the lower the elasticity, the higher should be the tax on housing. However, the general structure of the tax system is not affected by changes in the elasticity.

The model we employed to derive these results is, intentionally, very simple in many respects. As a next step, it would be interesting to study the optimal tax treatment of housing with models that involve some more real world complications. One possibly important extension would be to take land into account by assuming that housing services are provided by the combination of land and structures.

References

Atkeson, A – Chari, V V – Kehoe, P J (1999) **Taxing Capital Income: A Bad Idea.** Federal Reserve Bank of Minneapolis Review 23(3), 3–17.

Baxter, M – Jerman, U J (1999) **Household Production and the Excess Sensitivity of Consumption to Current Income.** American Economic Review 89(4), 902–920.

Berkovec, J – Fullerton, D (1992) **A General Equilibrium Model of Housing, Taxes, and Portfolio Choice.** Journal of Political Economy 100(2), 390–429.

Bye, B – Åvitsland, T (2003) **The welfare effects of housing taxation in a distorted economy: a general equilibrium analysis.** Economic Modelling 20, 895–921.

Carey, D – Rabesona, J (2004) **Tax Ratios on Labor and Capital Income and on Consumption.** In Sorensen, P B (ed.) Measuring the Tax Burden on Capital and Labor, The MIT Press, Cambridge, Massachusetts.

Chamley, C (1986) **Optimal taxation capital income in general equilibrium with infinite lives.** Econometrica 54, 607–622.

Coleman II, W J (2000) **Welfare and optimum dynamic taxation of consumption and income.** Journal of Public Economics 76, 1–39.

Cremer, H – Gahvari, F (1998) **On Optimal Taxation of Housing.** Journal of Urban Economics 43, 315–335.

Englund, P (2003) **Taxing Residential Housing Capital.** Urban Studies, 40(5–6), 937–952.

Fullerton, D (1987) **The Indexation of Interest, Depreciation, and Capital Gains and Tax Reform in the United States.** Journal of Public Economics 32(1), 25–52.

Gahvari, F (1984) **Incidence and efficiency aspects of differential taxation of residential and industrial capital in a growing economy.** Journal of Public Economics 25, 211–233.

Gahvari, F (1985) **Taxation of housing, capital accumulation, and welfare: a study in dynamic tax reform.** Public Finance Quarterly 13(2), 132–160.

Gervais, M (2002) **Housing Taxation and Capital Accumulation.** Journal of Monetary Economics 49(7), 1461–1489.

Gomme, P – Kydland, F E – Rupert, P (2001) **Home Production Meets Time to Build.** Journal of Political Economy 109(5), 1115–1131.

Greenwood, J – Hercowitz, Z (1991) **The Allocation of Capital and Time over the Business Cycle.** Journal of Political Economy 99(6), 1188–1214.

Greenwood, J – Rogerson, R – Wright, R (1995) **Household Production in Real Business Cycle Theory.** In Cooley, T (ed.) Frontiers of Business Cycle Research, Princeton University Press.

Hendershott, P H. – White, M (2000) **Taxing and Subsidizing Housing Investment: The Rise and Fall of Housing's Favored Status.** Journal of Housing Research 11(2), 257–275.

Hendershott, P H – Won, Y (1992) **Introducing Risky Housing and Endogenous Tenure Choice into a Portfolio-Based General Equilibrium Model.** Journal of Public Economics 48, 293–316.

Jones, L E – Manuelli, R E – Rossi, P E (1993) **Optimal Taxation in Models of Endogenous Growth.** Journal of Political Economy 101(3), 485–517.

Jones, L E – Manuelli, R E – Rossi, P E (1997) **On the Optimal Taxation of Capital Income.** Journal of Economic Theory 73, 93–117.

Judd, K L (1985) **Redistributive taxation in a simple perfect foresight model.** Journal of Public Economics 28, 59–83.

Kydland, F (1995) **Business cycles and aggregate labor market fluctuations.** In Cooley, T (ed.) Frontiers of Business Cycle Research, Princeton University Press.

McGrattan, E R – Rogerson, R – Wright, R (1997) **An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy.** International Economic Review 38(2), 267–290.

Mendoza, E G – Razin, A – Thesar, L L (1994) **Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption.** Journal of Monetary Economics 34, 297–323.

Määttänen, N (2004) **On the distributional effects of taxing housing.** Mimeo, Universitat Pompeu Fabra.

Poterba, J (1992) **Taxation and Housing: Old Questions, New Answers.** American Economic Review 82(2), 237–242.

Skinner, J (1996) **The Dynamic Efficiency Cost of Not Taxing Housing.** Journal of Public Economics 59(3), 397–417.

Slemrod, J (1982) **Down-payment constraints: tax policy effects in a growing economy with rental and owner-occupied housing.** Public Finance Quarterly 10(2), 193–217.

Turnovsky, S J – Okuyama, T (1994) **Taxes, housing and capital accumulation in a two-sector growing economy.** Journal of Public Economics 53, 245–267.

BANK OF FINLAND RESEARCH
DISCUSSION PAPERS

ISSN 0785-3572, print; ISSN 1456-6184, online

1/2005 Patrick M. Crowley **An intuitive guide to wavelets for economists.** 2005. 68 p. ISBN 952-462-188-6, print; ISBN 952-462-189-4, online.

2/2005 Tuomas Saarenheimo **Ageing, interest rates, and financial flows.** 2005. 30 p. ISBN 952-462-190-8, print; ISBN 952-462-191-6, online.

3/2005 Timo Vesala **Relationship lending and competition: Higher switching cost does not necessarily imply greater relationship benefits.** 2005. 27 p. ISBN 952-462-192-4, print; ISBN 952-462-193-2, online.

4/2005 Johanna Sinkkonen **Labour productivity growth and industry structure. The impact of industry structure on productivity growth, export prices and labour compensation.** 2005. 88 p. ISBN 952-462-196-7, print; ISBN 952-462-197-5, online.

5/2005 Matti Virén **Why do capital intensive companies pay higher wages?** 2005. 23 p. ISBN 952-462-198-3, print; ISBN 952-462-199-1, online.

6/2005 Maritta Paloviita – Matti Virén **The role of expectations in the inflation process in the euro area.** 2005. 33 p. ISBN 952-462-200-9, print; ISBN 952-462-201-7, online.

7/2005 Marketta Henriksson **Productivity differentials and external balance in ERM II.** 2005. 32 p. ISBN 952-462-202-5, print; ISBN 952-462-203-3, online.

8/2005 Matti Virén **Government size and output volatility: is there a relationship?** 2005. 24 p. ISBN 952-462-204-1, print; ISBN 952-462-205-X, online.

9/2005 Karlo Kauko **Bank interest rates in a small European economy: Some exploratory macro level analyses using Finnish data.** 2005. 44 p. ISBN 952-462-206-8, print; ISBN 952-462-207-6, online.

10/2005 Essi Eerola – Niku Määttänen **The optimal tax treatment of housing capital in the neoclassical growth model.** 2005. 30 p. ISBN 952-462-210-6, print; ISBN 952-462-211-4, online.