Mikael Bask

A case for interest rate smoothing

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Abstract

The aim of this paper is to determine whether it would be desirable from the perspective of macroeconomic balance for central banks to take account of nominal exchange rate movements when framing monetary policy. The theoretical framework is a small, open DSGE economy that is closed by a Taylor rule for the monetary authority, and a determinate REE that is least-squares learnable is defined as a desirable outcome in the economy. When the policy rule contains contemporaneous data on the output gap and the CPI inflation rate, the monetary authority does not have to consider the exchange rate as long as there is sufficient inertia in policy-making. In fact, due to a parity condition on the international asset market, interest-rate smoothing and a response to changes in the nominal exchange rate are perfectly intersubstitutable in monetary policy. In other words, we give a rationale for the monetary authority to focus on the *change* in the nominal interest rate rather than its *level* in policy-making. Thus, we have a case for interest-rate smoothing.

Keywords: determinacy, E-stability, foreign exchange, inertia, Taylor rule

JEL classification numbers: E52, F31

Korkosäännöt ja rahapolitiikan jatkuvuus

Suomen Pankin keskustelualoitteita 25/2007

Mikael Bask
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkimuksessa tarkastellaan, olisiko kokonaistaloudellisen tasapainon kannalta toivottavaa, että keskuspankki ottaisi rahapoliittisia päätöksiä tehdessään huomioon nimellisten valuuttakurssien vaihteluihin sisältyvän informaation. Työn teoreettisissa tarkasteluissa käytetään modernissa rahapolitiikan analyysissa yleistä pienen avotalouden dynaamista makromallia, jossa keskuspankki toteuttaa rahapolitiikkaa ohjailemalla korkoja. Korko-ohjailun tehokkuutta ja onnistumista arvioidaan sen mukaan, onko talouden kehitys sopusoinnussa hyvin määritellyn rationaalisten odotusten tasapainon kanssa. Tämän tasapainon ominaisuudet taloudenpitäjät oppivat hyödyntämällä estimoituja tilastollisia malleja ennusteiden laatimiseksi. Jos korkopäätökset perustuvat tuotantokuilua ja inflaatiota koskeviin julkaistuihin tilastotietoihin, keskuspankin ei analyysin tulosten mukaan tarvitse erikseen ottaa huomioon valuuttakurssin muutoksia korkopäätöksiä tehdessään, mikäli keskuspankilla on riittävän voimakas tarve tasata korkojen vaihteluita eli mikäli halu turvata harjoitetun rahapolitiikan jatkuvuus on riittävän voimakas. Täysin vapaiden pääomaliikkeiden oloissa kotimaiset ja ulkomaiset sijoituskohteet ovat mallissa itse asiassa täydellisiä substituutteja, joten korkovaihteluiden tasaaminen ja reagointi nimellisen valuuttakurssin muutoksiin korvaavat rahapolitiikan kannalta toisensa täydellisesti. Tämän tuloksen avulla voidaan näin ollen perustella keskuspankin halua tasoittaa korkojen vaihteluita.

Avainsanat: määrittyneisyys, E-stabiilius, ulkomaanvaluutta, rahapolitiikan jatkuvuus, korkosääntö

JEL-luokittelu: E52, F31

Contents

1 Introduction

It is argued in Taylor (2001) that it is not necessary to react to movements in exchange rates in monetary policy to have a desirable outcome in the economy. Taylor's (2001) argument is that the indirect effects that exchange rates have on monetary policy, via its effects on output and the inflation rate, are to prefer since it results in fewer and less erratic changes in the nominal interest rate.

In this paper, we re-examine the question whether one should respond to nominal exchange rate movements in policy-making by embedding two specifications of an interest rate rule into a small open economy, where we include an exchange rate term in the policy-rules. In the first rule, contemporaneous data on the output gap, the CPI inflation rate and the nominal exchange rate change are included, whereas, in the second rule, forward expectations of the same variables are included in the rule.

We also augment the policy-rules with the interest rate in the previous time period to allow for inertia in monetary policy. This means that we are able to answer the following question: should the monetary authority care about the interest rate *level* or the *change* in the interest rate to have a desirable outcome in the economy? Recall that inertia in policy-making has sometimes been the source of criticism in the financial press of central banking behavior since the adjustments of the interest rate have been claimed to be too slow. Thus, the posed question is relevant to answer.

What is then a desirable outcome in the economy? We focus on two things. First, we search for the regions in an interest rate rule's parameter space that give rise to a unique and stable rational expectations equilibrium (REE). Second, such a REE should also be learnable in least squares sense, and this is because rational expectation is a rather strong assumption since it assumes that agents often have an outstanding capacity when it comes to deriving equilibrium outcomes of the variables in a model. We make use of the E-stability concept when doing the learnability analysis (see Evans and Honkapohja, 2001, for an introduction to this literature).

There are two papers that are closely related to this paper. The first is Llosa and Tuesta (2007) who examine the inclusion of an exchange rate term in the interest rate rule but neglects from inertia in monetary policy, and the second is Bullard and Mitra (2007) who allow for inertia in policy-making but for a closed economy.[1] It goes without saying that we are able to replicate their findings. But, more importantly, it turns out that our paper is able to fill the gap between Bullard and Mitra (2007) and Llosa and Tuesta (2007) with some interesting findings.

[1] The inclusion of an exchange rate term in the interest rate rule in actual policy-making is examined in Lubik and Schorfheide (2007), and they find that the Bank of Canada and the Bank of England react to movements in nominal exchange rates, whereas this is not the case for the Reserve Bank of Australia and the Reserve Bank of New Zealand. Turning to interest rate inertia in monetary policy, this is a well-documented feature of central banking behavior in developed countries (see Bullard and Mitra, 2007). For example, Rudebusch (1995) finds that the Federal Reserve's policy can be characterized by interest rate smoothing.

Three findings should be emphasized. First, it is more likely to have a desirable outcome in the economy when the central bank cares about the *change* in the interest rate rather than its *level*. Second, the specification of the interest rate rule that contains contemporaneous data of the variables is more likely to deliver a desirable outcome in the economy than the specification with forward expectations of the same variables.

Third, it is not necessary to respond to the exchange rate in monetary policy to achieve a determinate REE that is least squares learnable. The reason is that interest rate inertia is a perfect substitute to the nominal exchange rate change response (when contemporaneous data are used in the policy-rule). Thus, it is not because it would result in fewer and less erratic changes in the interest rate nor that a reaction to the output gap or the CPI inflation rate would be a perfect substitute in policy-making.

Instead, the intuition to this result is that a parity condition at the international asset market (namely, uncovered interest rate parity), ties the current interest rate and the expected exchange rate change together. Further, since this parity condition is assumed to hold in every time period, it also holds in the previous time period, meaning that the parity condition also ties the lagged interest rate and the current exchange rate change together.[2]

Thus, we have the following link between Bullard and Mitra (2007), Llosa and Tuesta (2007) and our paper: Llosa and Tuesta (2007) show that policy-rules that include an exchange rate term can help alleviate the indeterminacy problem, but at the cost of greater volatility in the economy (see also Taylor, 2001). We therefore show that the monetary authority can shift focus from an exchange rate response in monetary policy to interest rate smoothing.

Consequently, interest rate smoothing should help alleviate the indeterminacy problem. This is also the main finding in Bullard and Mitra (2007) but for a closed economy, meaning that we broaden their finding. Thus, we give a rationale for the monetary authority to focus on the *change* in the interest rate rather than its *level* to have a desirable outcome in the economy. This is interesting from the point of view of practical policy.

The rest of this paper is organized as follows: The model economy is outlined in Section 2, whereas determinacy and E-stability results are presented in Section 3 for both interest rate rules in policy-making. Section 4 concludes the paper with a discussion of our main finding.

[2] Note that the foreign interest rate in the parity condition is exogenous since we are dealing with a small economy, and that shocks to expectations also are exogenous, meaning that the conditions for a desirable outcome in the economy are not affected by changes in these constants. Of course, if one would like to estimate the model economy from data and then compute impulse-response functions for key variables, one must estimate these shocks to expectations.

2 A small open economy

The baseline model is outlined in Section 2.1, which is a model for a small open economy. Thereafter, in Section 2.2, we present two specifications of an interest rate rule for monetary policy that are embedded into the baseline model in Section 3.

2.1 Baseline model

A dynamic stochastic general equilibrium (DSGE) model with imperfect competition and nominal rigidities is presented in Galí and Monacelli (2005) for a small open economy.[3] After extensive derivations, their model can be reduced to a dynamic IS-type equation and a new Keynesian Phillips curve

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \pi^e_{H,t+1} - \overline{rr}_t\right) \\ \pi_{H,t} = \beta\pi^e_{H,t+1} + \gamma x_t \end{cases} \tag{2.1}$$

where x is the output gap, r is the nominal interest rate, π_H is the domestic inflation rate, $\overline{rr}$ is the natural rate of interest, and the superscript e denotes rational expectation of the variable in focus, where the dating of expectations is time period t. All variables are in natural logarithms, except the interest rates.

Unfortunately, (2.1) is not in an appropriate form since there is no exchange rate term in the equations. It is, however, possible to use the following equations that are derived in Galí and Monacelli (2005) to rewrite (2.1) into a suitable form

$$\begin{cases} \pi_t = \pi_{H,t} + \delta\Delta s_t \\ s_t = e_t + p^*_t - p_{H,t} \end{cases} \tag{2.2}$$

where π is the CPI inflation rate, s is the terms of trade, e is the nominal exchange rate that is the domestic price of the foreign currency, p^* is the index of foreign goods prices, p_H is the index of domestic goods prices, and the asterisk denotes a foreign quantity. For interpretations of the structural parameters, we refer to Galí and Monacelli (2005).

If we rewrite the equations in (2.1) with help of those in (2.2), we get the first two equations in the model examined (see the Appendix for a derivation of (2.3))

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \frac{1}{1-\delta}\cdot\left(\pi^e_{t+1} - \delta\left(\Delta e^e_{t+1} + \pi^{e,*}_{t+1}\right)\right) - \overline{rr}_t\right) \\ \pi_t = \beta\pi^e_{t+1} + \gamma\left(1-\delta\right)x_t + \delta\left(\Delta e_t - \beta\Delta e^e_{t+1} + \pi^*_t - \beta\pi^{e,*}_{t+1}\right) + \varepsilon_t \end{cases} \tag{2.3}$$

The third equation in the model, which also is derived in Galí and Monacelli (2005), is the condition for uncovered interest rate parity (UIP)

$$r_t - r^*_t = \Delta e^e_{t+1} \tag{2.4}$$

To complete the model in (2.3)–(2.4), we will augment it with an interest rate rule for the monetary authority.

[3] Llosa and Tuesta (2007) also use the Galí and Monacelli (2005) model, and Bullard and Mitra (2007) use a closed economy version of the Galí and Monacelli (2005) model.

2.2 Interest rate rules examined

The monetary authority is using a Taylor (instrument) rule when setting the nominal interest rate, where two specifications of the rule are examined: (i) contemporaneous data in the rule

$$r_t = \zeta_r r_{t-1} + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t \tag{2.5}$$

and (ii) forward expectations in the rule

$$r_t = \zeta_r r_{t-1} + \zeta_x x^e_{t+1} + \zeta_\pi \pi^e_{t+1} + \zeta_e \Delta e^e_{t+1} \tag{2.6}$$

We have also included the interest rate in the previous time period in the rules to allow for inertia in policy-making. An interesting special case of inertia that we will focus on is $\zeta_r = 1$. This is because this allows us to answer a relevant question in practical policy: should the monetary authority care about the interest rate *level* or the *change* in the interest rate to have a desirable outcome in the economy?

One could also think of a lagged data specification of the Taylor rule. However, since we assume time-t dating of expectations, we neglect from such a specification in the analysis. To pose a rhetorical question: is it reasonable to believe that the monetary authority is reacting to old information when setting the interest rate?

Finally, the vigilant reader might object that Taylor (2001) is referring to the real exchange rate in his discussion, whereas we have included the nominal exchange rate in the interest rate rules. It is, however, an easy exercise to transform the rules in (2.5)–(2.6) to rules that are functions of the real exchange rate, q, via the following identity: $\Delta q_t \equiv \Delta e_t + \pi^*_t - \pi_t$.

3 Properties of the model economy

In Section 3.1, the specification of the policy-rule that contains contemporaneous data on the output gap, the CPI inflation rate and the nominal exchange rate change is embedded into the baseline model and then analyzed. Specifically, conditions for a determinate REE that is least squares learnable are derived. In Section 3.2, the specification of the rule that contains forward expectations of the same variables is embedded into the baseline model and then analyzed.

3.1 Contemporaneous data in the interest rate rule

After substituting the Taylor rule in (2.5) into the baseline model in (2.3)–(2.4), the complete model in matrix form is

$$\mathbf{\Gamma} \cdot \mathbf{y}_t = \mathbf{\Theta} \cdot \mathbf{y}^e_{t+1} + \mathbf{\Lambda} \cdot \mathbf{y}_{t-1} \tag{3.1}$$

where the state of the economy is

$$\mathbf{y}_t = [x_t, \pi_t, \Delta e_t, r_t]' \tag{3.2}$$

and the coefficient matrices are

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ -\gamma(1-\delta) & 1 & -\delta & 0 \\ 0 & 0 & 0 & 1 \\ -\zeta_x & -\zeta_\pi & -\zeta_e & 1 \end{bmatrix} \tag{3.3}$$

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & -\frac{\alpha\delta}{1-\delta} & 0 \\ 0 & \beta & -\beta\delta & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

and

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta_r \end{bmatrix} \tag{3.5}$$

We have neglected from a constant in (3.1) since it does not affect our findings when it comes to conditions for a desirable outcome in the economy.

To be able to determine whether the complete model has a determinate REE, a first step is to rewrite the model into first-order form, and then to compare the number of predetermined variables with the number of eigenvalues of a certain matrix that are outside the unit circle (see Blanchard and Kahn, 1980). Specifically, we make use of the following variable vector when rewriting the model in (3.1)–(3.5)

$$\mathbf{y}_{d,t} = \left[\mathbf{y}_t, r_t^L \equiv r_{t-1}\right]' \tag{3.6}$$

meaning that the coefficient matrices are

$$\mathbf{\Gamma}_d = \begin{bmatrix} \mathbf{\Gamma} & & -\mathbf{\Lambda}_4 \\ \mathbf{0}_{(1\times3)} & 1 & 0 \end{bmatrix} \tag{3.7}$$

where the vector $\mathbf{\Lambda}_4$ is the fourth column in matrix $\mathbf{\Lambda}$, and

$$\mathbf{\Theta}_d = \begin{bmatrix} \mathbf{\Theta} & \mathbf{0}_{(4\times1)} \\ \mathbf{0}_{(1\times4)} & 1 \end{bmatrix} \tag{3.8}$$

because the complete model in matrix form is now

$$\mathbf{\Gamma}_d \cdot \mathbf{y}_{d,t} = \mathbf{\Theta}_d \cdot \mathbf{y}_{d,t+1}^e \tag{3.9}$$

Unfortunately, the matrix $\mathbf{\Gamma}_d$ (and $\mathbf{\Theta}_d$) is singular, meaning that the matrix $\mathbf{\Gamma}_d^{-1} \cdot \mathbf{\Theta}_d$ (and $\mathbf{\Theta}_d^{-1} \cdot \mathbf{\Gamma}_d$), which is the relevant matrix when determining whether the complete model has a determinate REE, does not exist. One way to solve this problem is by substituting out the current and expected exchange rate changes from the equations. That is, to use the UIP condition in (2.4) to substitute out Δe_{t+1}^e, and to use the same equation lagged one time period to substitute out Δe_t. In the latter case, the UIP condition is

$$r_{t-1} - r_{t-1}^* = \Delta e_t^e = \Delta e_t + \epsilon_t \tag{3.10}$$

where the dating of expectations is time period $t-1$, and ϵ is the realized error in the expectation formation, which is a constant that does not affect our findings when it comes to conditions for a desirable outcome in the economy. Putting it differently, ϵ is an exogenous shock to expectations.

After doing these substitutions, the variable vector in (3.9) is

$$\mathbf{y}_{d,t} = \left[x_t, \pi_t, r_t, r_t^L \equiv r_{t-1}\right]' \tag{3.11}$$

and the coefficient matrices in (3.9) are now

$$\mathbf{\Gamma}_d = \begin{bmatrix} 1 & 0 & \frac{\alpha}{1-\delta} & 0 \\ -\gamma(1-\delta) & 1 & \beta\delta & -\delta \\ -\zeta_x & -\zeta_\pi & 1 & -(\zeta_r+\zeta_e) \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.12}$$

and

$$\mathbf{\Theta}_d = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.13}$$

One thing that we immediately observe is that interest rate inertia and a reaction to the nominal exchange rate change are perfect substitutes in monetary policy, and this is because the parameters ζ_r and ζ_e in the Taylor rule in (2.5) appear at the same place in the matrix $\mathbf{\Gamma}_d$ and also with the same coefficient (-1). The intuition to this finding, which is a strong result, is that the UIP condition in (3.10) ties the lagged interest rate and the exchange rate change together.

Thus, our first finding is this:

Finding 1 *When contemporaneous data are used in the policy-rule, interest rate inertia and a reaction to the exchange rate change are perfect substitutes in policy-making.*

When deriving conditions for a determinate REE, it is not always self-evident which variables in a model that are predetermined. But by looking at the entries in the relevant matrix

$$\mathbf{\Gamma}_d^{-1} \cdot \mathbf{\Theta}_d = \begin{bmatrix} - & - & 0 & - \\ - & - & 0 & - \\ 0 & 0 & 0 & 1 \\ - & - & 0 & - \end{bmatrix} \tag{3.14}$$

we can conclude that r in (3.11) is predetermined. For this reason, exactly one eigenvalue of the matrix $\mathbf{\Gamma}_d^{-1} \cdot \mathbf{\Theta}_d$ must be outside the unit circle to have a determinate REE. In other words, if more than one eigenvalue are outside the unit circle, we have an indeterminate REE, and if no eigenvalue is outside the unit circle, there is no stable REE in the economy.

Turning to the learnability analysis, to have a REE that is E-stable and therefore least squares learnable, the parameter values in the agents' perceived law of motion (PLM) of the economy have to converge to the economy's actual law of motion (ALM). Further, it is shown in McCallum (2007) that for a broad class of linear rational expectations models, which includes the model in (3.1), a determinate REE is E-stable when the dating of expectations is time period t. Consequently, all determinacy regions found below are also regions for a least squares learnable REE.

This means that we have our second finding:

Finding 2 *When contemporaneous data are used in the policy-rule, and a main objective in monetary policy is to achieve a unique REE that is learnable, interest rate inertia and a reaction to the exchange rate change are perfect substitutes in policy-making.*

Thus, we have a case for interest rate smoothing since it is not necessary to respond to the exchange rate to achieve a determinate REE that is least squares learnable. The intuition to this result is that a parity condition at the international asset market ties the lagged interest rate and the exchange rate change together.

How strong must interest rate smoothing be to have a desirable outcome in the economy? Are there any restrictions on the other parameters in the Taylor rule in (2.5)?

Deriving analytical conditions for determinacy and E-stability is not meaningful since these expressions would be too large and cumbersome to interpret. Instead, we adopt the same strategy as other papers within this research area and illustrate our findings graphically using calibrated values of the structural parameters.[4] To be more precise, the following parameter values are used in the analysis:[5]

$$\begin{cases} \alpha = \frac{1}{0.157} & \beta = 0.99 & \gamma = 0.024 \\ \delta = 0.4 & \zeta_r = 0,\ 1 & \zeta_x = 0.5 \end{cases} \tag{3.15}$$

See Woodford (1999) for the values of α, β and γ. Moreover, $\delta \in [0, 1]$ is the economy's openness index since it is the share of consumption allocated to imported goods. When $\delta = 0.4$, this index approximates the import/GDP ratio in an open economy such as Canada.[6]

Turning to the parameters in the interest rate rule, we set the output gap reaction to $\zeta_x = 0.5$ since this value is close to what typically is found in data (see Clarida et al, 2000). When it comes to the importance of the lagged interest rate in policy-making, we examine two cases: (i) the monetary authority cares about the interest rate *level* (no inertia); and (ii) the monetary authority cares about the *change* in the interest rate (inertia).

[4] MATLAB routines for this purpose are available on request from the author.

[5] Bullard and Mitra (2002)–(2007) use the same calibration of the structural parameters as we do.

[6] This value of δ is also used in Galí and Monacelli (2005) and Llosa and Tuesta (2007).

The central bank cares about the interest rate level[7]

In Figure 1, there is no inertia in policy-making, meaning that the monetary authority is focusing on the interest rate level in monetary policy.

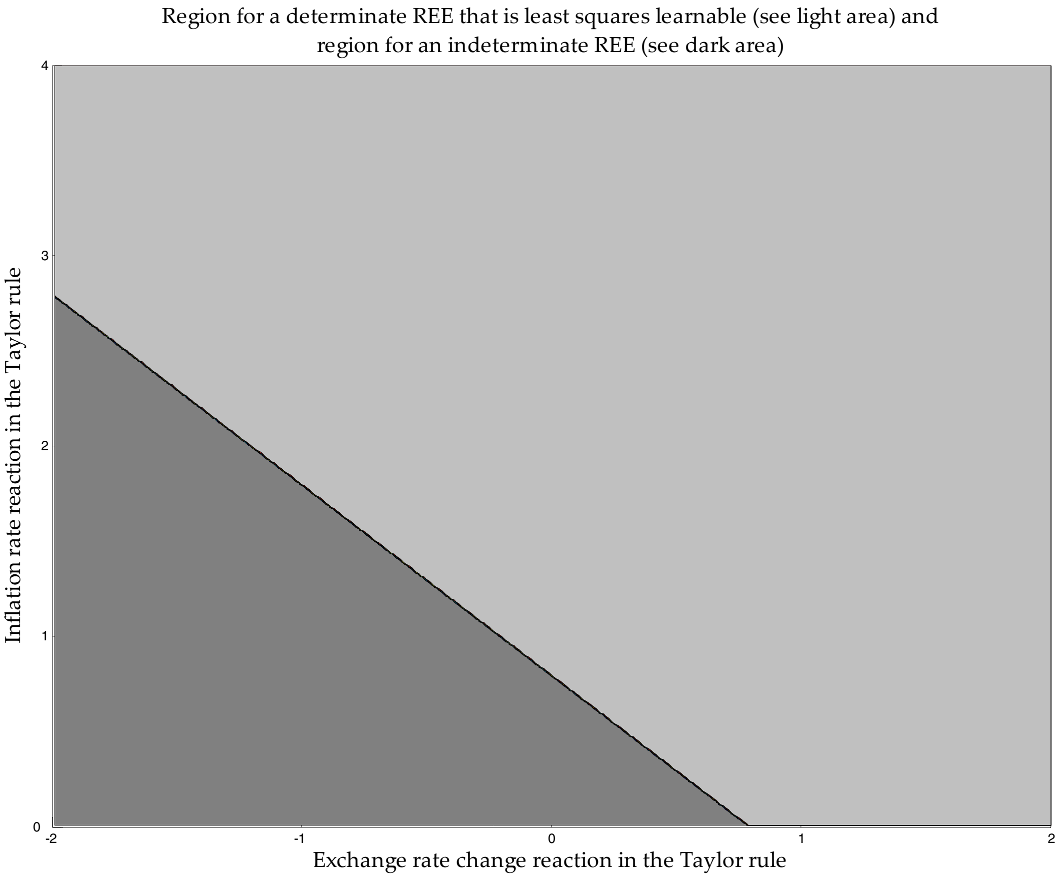

Figure 1. Contemporaneous data are used in the interest rate rule and it is the interest rate level that matters in policy-making.

Two things in the figure can be noted. First, if the inflation rate response in the Taylor rule is large enough, the monetary authority does not have to think about the exchange rate to achieve a determinate REE that is least squares learnable. Second, if the inflation rate response in the rule is at the lower bound to achieve the same outcome, there is a one-to-one trade-off between reactions to the inflation rate and the exchange rate change.

We also have the following findings (that are not shown graphically[8]): (i) the determinacy-learnability region is not visibly affected by the openness index, which is a somewhat surprising result; and (ii) a larger (smaller) output gap reaction in the Taylor rule shifts the lower bound of the determinacy-learnability region slightly downwards (upwards).

[7] See Bullard and Mitra (2002), especially Figure 1, since our results limit their results when $\delta \to 0$. See also Llosa and Tuesta (2007), especially Figure 2, but be aware that they use a somewhat different calibration of the structural parameters than we do.

[8] These and other results discussed in the paper, but not shown graphically, are available on request from the author in the form of graphs.

We summarize the results as follows:

Finding 3 *When the interest rate* ***level*** *matters in policy-making, contemporaneous data are used in the policy-rule, and a main objective in monetary policy is to achieve a unique REE that is learnable, there is no need to consider the exchange rate change in policy-making, if the inflation rate reaction is large enough.*

To make the needed inflation rate reaction in the interest rate rule more precise, if the monetary authority does not care about the exchange rate in policy-making, it is sufficient that the Taylor principle is satisfied to achieve a determinate REE that is least squares learnable. The Taylor principle says that the monetary authority can stabilize the economy by increasing (decreasing) the nominal interest rate more than one-for-one in response to a higher (lower) inflation rate, meaning that the real interest rate also is increasing (decreasing).

The central bank cares about the change in the interest rate[9]

In Figure 2, there is inertia in policy-making.

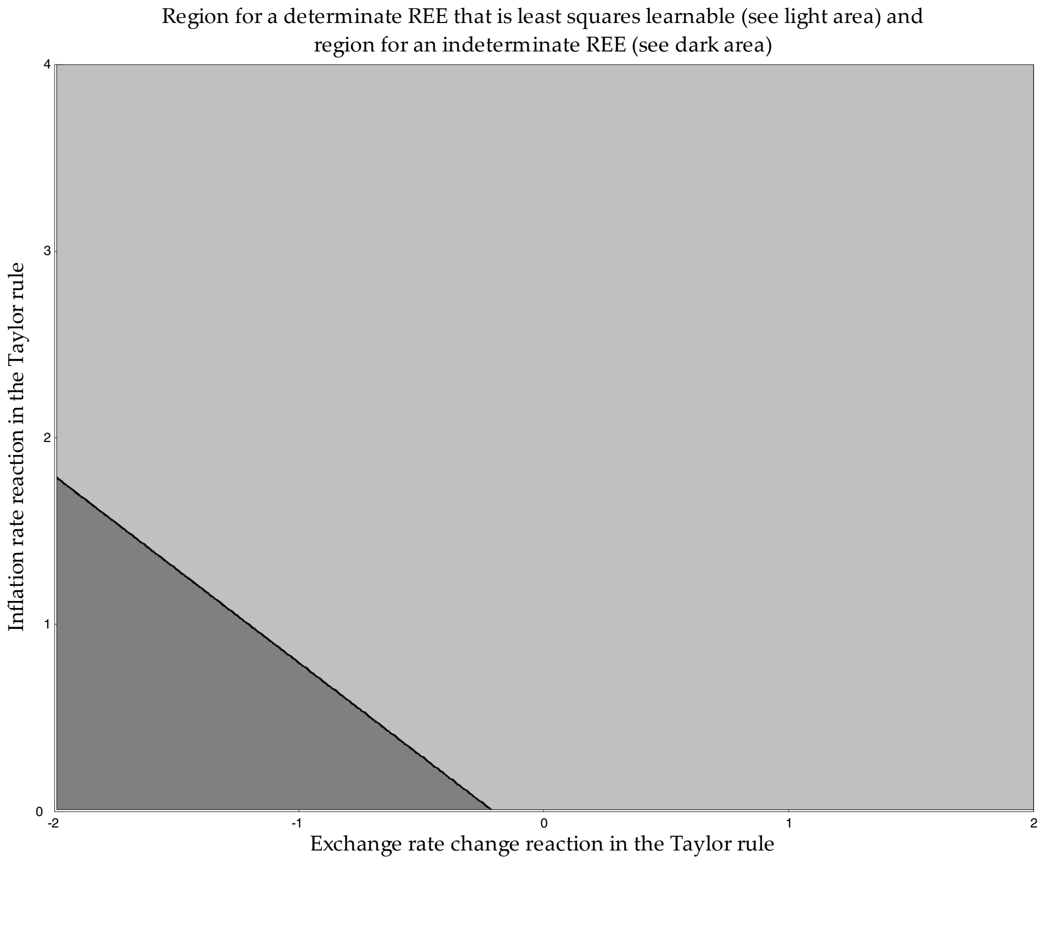

Figure 2. Contemporaneous data are used in the interest rate rule and it is the change in the interest rate that matters in policy-making.

[9] Even though Bullard and Mitra (2007) allow for inertia in policy-making, they do not consider a contemporaneous data specification of the interest rate rule.

In fact, the inertia is so strong ($\zeta_r = 1$) that the monetary authority shifts focus from the interest rate *level* to the interest rate *change* in the Taylor rule

$$\Delta r_t \equiv r_t - r_{t-1} = \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t \tag{3.16}$$

It turns out that the results mentioned above still hold, but there is one important exception. Due to inertia in monetary policy, the lower bound of the determinacy-learnability region shifts downwards. In fact, when inertia is $\zeta_r = 1$, the shift is so large that the monetary authority does not have to think about the inflation rate nor the exchange rate to achieve a determinate REE that is least squares learnable. This is a strong result.

We can also formulate this finding as follows:

Finding 4 *When the* ***change*** *in the interest rate matters in policy-making, contemporaneous data are used in the policy-rule, and a main objective in monetary policy is to achieve a unique REE that is learnable, it is enough with a modest output gap reaction in policy-making.*

In fact, the output gap reaction in policy-making can be so modest that it is almost negligible.

Let us now examine the properties of the model economy when the monetary authority is using the Taylor rule in (2.6) in monetary policy. That is, forward expectations of the output gap, the CPI inflation rate and the exchange rate change replace contemporaneous data of the same variables.

3.2 Forward expectations in the interest rate rule

Due to the fact that the analysis of the model economy in this section is similar to the analysis in Section 3.1, it is not necessary to repeat every step. We are therefore more concise here.

First, the complete model in matrix form is again (3.1)–(3.5), but with the exception that the elements in the last row in each matrix are replaced with the appropriate elements as described by the Taylor rule in (2.6). After rewriting the model into first-order form, it is also easily verified that we again have problems with singular matrices when trying to derive conditions for a desirable outcome in the economy.

For this reason, we substitute out the current and expected exchange rate changes from the equations with help of the UIP conditions in (2.4) and (3.10). This means that when the Taylor rule in (2.6) is used in policy-making, the complete model in matrix form is (3.9), the variable vector is (3.11), and the coefficient matrices are

$$\mathbf{\Gamma}_d = \begin{bmatrix} 1 & 0 & \frac{\alpha}{1-\delta} & 0 \\ -\gamma(1-\delta) & 1 & \beta\delta & -\delta \\ 0 & 0 & 1-\zeta_e & -\zeta_r \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.17}$$

and

$$\mathbf{\Theta}_d = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & 0 & 0 \\ 0 & \beta & 0 & 0 \\ \zeta_x & \zeta_\pi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.18}$$

Second, the relevant matrix for the determinacy and E-stability results, $\mathbf{\Gamma}_d^{-1} \cdot \mathbf{\Theta}_d$, has the same form as in (3.14), meaning that r in (3.11) is predetermined even in this case. This means that we have the same eigenvalue-conditions as above to have a determinate REE that is least squares learnable, an indeterminate REE and no stable REE in the economy.

Turning to the findings (and having Findings 1 and 2 above in mind), interest rate inertia and a reaction to the expected exchange rate change are not perfect substitutes in monetary policy, and the reason is that the UIP conditions in (2.4) and (3.10) are not able to tie the lagged interest rate and the expected exchange rate change together. This result is also visible in the matrix $\mathbf{\Gamma}_d$ since the parameters ζ_r and ζ_e in the Taylor rule in (2.6) do not appear at the same place in the matrix.

This means that we have our fifth finding:

Finding 5 *When forward expectations are used in the policy-rule, and a main objective in monetary policy is to achieve a unique REE that is learnable, interest rate inertia and a reaction to the expected exchange rate change are not perfect substitutes in policy-making.*

We examine two cases when it comes to the importance of the lagged interest rate in policy-making: (i) the monetary authority cares about the interest rate *level* (no inertia); and (ii) the monetary authority cares about the *change* in the interest rate (inertia). As we also did in Section 3.1, we illustrate our findings graphically using the parameter values in (3.15).

The central bank cares about the interest rate level[10]

In Figure 3, there is no inertia in policy-making.

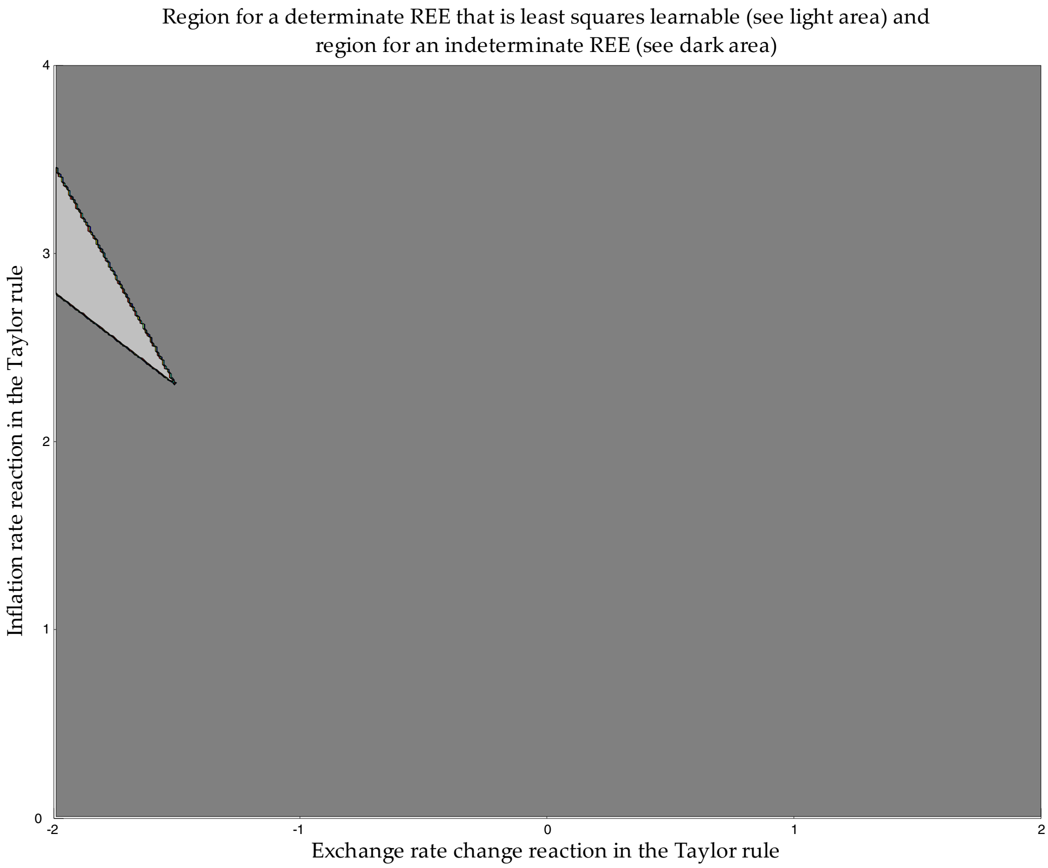

Figure 3. Forward expectations are used in the interest rate rule and it is the interest rate level that matters in policy-making.

Two things in the figure can be noted. First, since we assume that $\zeta_x = 0.5$ in the policy-rule, the monetary authority must respond to the expected exchange rate to achieve a determinate REE that is least squares learnable, and that this is irrespective of the strength in the expected inflation rate response. To be more precise, the central bank should decrease (increase) the interest rate when the exchange rate is expected to depreciate (appreciate) since, otherwise, we would have a multiplicity of REE in the economy. However, if there would not be any expected output response in the rule ($\zeta_x = 0$), an expected inflation rate response that is not too weak or too strong would secure a unique and learnable REE (that is not shown graphically).

Second, if the expected inflation rate response in the policy-rule is at the lower bound to achieve a desirable outcome in the economy, there is a one-to-one trade-off between reactions to the expected inflation rate and

[10] See Bullard and Mitra (2002), especially Figure 3, since our results limit their results when $\delta \to 0$. See also Llosa and Tuesta (2007), especially Figure 6, but be aware that they use a somewhat different calibration of the structural parameters than we do.

the expected exchange rate change. Recall that a similar result holds when the monetary authority responds to contemporaneous data of the variables as described by the Taylor rule in (2.5).

Also recall that when the policy-rule in (2.5) is used in monetary policy, the determinacy-learnability region is not visibly affected by the openness index. This is no longer true when the monetary authority is using the rule in (2.6). In the latter case, a less (more) open economy increases (decreases) the determinacy-learnability region (that is not shown graphically).

Let us summarize the aforementioned results as follows:

Finding 6 *When the interest rate* ***level*** *matters in policy-making, forward expectations are used in the policy-rule, and a main objective in monetary policy is to achieve a unique REE that is learnable, one must consider the expected exchange rate change in policy-making, if there is an expected output reaction.*

The central bank cares about the change in the interest rate[11]

In Figure 4, there is inertia in policy-making.

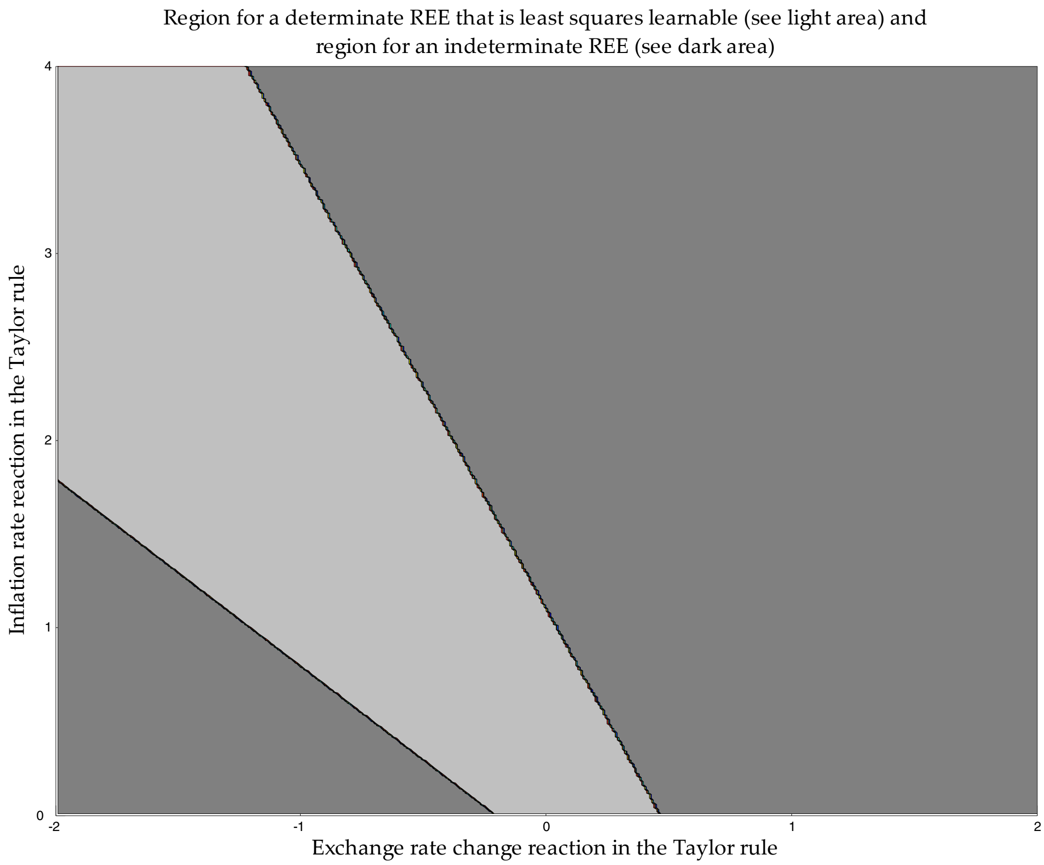

Figure 4. Forward expectations are used in the interest rate rule and it is the change in the interest rate that matters in policy-making.

[11] See Bullard and Mitra (2007), especially Figure 2.

A shift in focus from the interest rate *level* to the interest rate *change* has the effect that the determinacy-learnability region increases in size. In fact, this increase is so large that it is no longer necessary for the monetary authority to respond to the expected exchange rate to achieve a determinate REE that is least squares learnable. At the same time, the expected inflation rate response cannot be too strong to have a desirable outcome in the economy.

We can formulate this finding as follows:

Finding 7 *When the* ***change*** *in the interest rate matters in policy-making, forward expectations are used in the policy-rule, and a main objective in monetary policy is to achieve a unique REE that is learnable, there is no need to consider the expected exchange rate change in policy-making, if the expected inflation rate reaction is small enough.*

4 Discussion

For a small open economy, it is natural to ask whether one should augment the Taylor rule with an exchange rate term to achieve a desirable outcome in the economy such as a determinate REE that is least squares learnable. Taylor's (2001) answer to this question is *no*, and his argument is that the indirect effects that exchange rates have on monetary policy, via its effects on output and the inflation rate, are to prefer since it results in fewer and less erratic changes in the nominal interest rate.

We agree with Taylor (2001) that there is no need to include an exchange rate term in the interest rate rule, even though our argument is somewhat different. If the policy-rule contains contemporaneous data on the output gap and the CPI inflation rate, the monetary authority does not have to care about the exchange rate as long as there is enough with inertia in policy-making. To be more precise, due to a parity condition at the international asset market, interest rate smoothing and a response to the nominal exchange rate change are perfect substitutes in monetary policy.

Bullard and Mitra (2007) show the merits of interest rate smoothing in policy-making but for a closed economy, and Llosa and Tuesta (2007) examine the inclusion of an exchange rate term in the interest rate rule but neglects from inertia. However, even though the inclusion of such a term helps alleviate the indeterminacy problem, the cost is greater volatility in the economy (see also Taylor, 2001). We therefore show that the central bank can shift focus from an exchange rate response in monetary policy to interest rate smoothing.

Thus, we are able to fill the gap between Bullard and Mitra (2007) and Llosa and Tuesta (2007) with an important finding since we give a rationale for the monetary authority to focus on the *change* in the interest rate rather than its *level* to have a desirable outcome in the economy. This is interesting from the point of view of practical policy, and be aware that this finding does not rely on any specific calibration of the structural parameters in the model economy.

References

Blanchard, O J – Kahn, C M (1980) **The Solution of Linear Difference Models under Rational Expectations.** Econometrica, 48, 1305–1311.

Bullard, J – Mitra, K (2002) **Learning About Monetary Policy Rules.** Journal of Monetary Economics, 49, 1105–1129.

Bullard, J – Mitra, K (2007) **Determinacy, Learnability, and Monetary Policy Inertia.** Journal of Money, Credit and Banking, 39, 1177–1212.

Clarida, R – Galí, J – Gertler, M (2000) **Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.** Quarterly Journal of Economics, 115, 147–180.

Evans, G W – Honkapohja, S (2001) **Learning and Expectations in Macroeconomics.** Princeton, New Jersey: Princeton University Press.

Galí, J – Monacelli, T (2005) **Monetary Policy and Exchange Rate Volatility in a Small Open Economy.** Review of Economic Studies, 72, 707–734.

Llosa, L-G – Tuesta, V (2007) **Determinacy and Learnability of Monetary Policy Rules in Small Open Economies.** Forthcoming in Journal of Money, Credit and Banking.

Lubik, T A – Schorfheide, F (2007) **Do Central Banks Respond to Exchange Rate Movements? A Structural Investigation.** Journal of Monetary Economics, 54, 1069–1087.

McCallum, B T (2007) **E-Stability vis-a-vis Determinacy Results for a Broad Class of Linear Rational Expectations Models.** Journal of Economic Dynamics and Control, 31, 1376–1391.

Rudebusch, G D (1995) **Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure.** Journal of Monetary Economics, 35, 245–274.

Taylor, J B (2001) **The Role of the Exchange Rate in Monetary-Policy Rules.** American Economic Review (Papers and Proceedings), 91, 263–267.

Woodford, M (1999) **Optimal Monetary Policy Inertia.** NBER Working Paper No. 7261.

Appendix

First, shift the first equation in (2.2) one time period forward in time

$$\pi^e_{H,t+1} = \pi^e_{t+1} - \delta \Delta s^e_{t+1} \tag{A1.1}$$

Second, shift the second equation in (2.2) one time period forward in time, and take differences

$$\Delta s^e_{t+1} = \Delta e^{e,m}_{t+1} + \Delta p^{e,*}_{t+1} - \Delta p^e_{H,t+1} = \Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1} - \pi^e_{H,t+1} \tag{A1.2}$$

Third, substitute (A1.2) into (A1.1)

$$\pi^e_{H,t+1} = \frac{1}{1-\delta} \cdot \left(\pi^e_{t+1} - \delta \left(\Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1}\right)\right) \tag{A1.3}$$

Fourth, shift (A1.3) one time period backward in time

$$\pi^e_{H,t} = \frac{1}{1-\delta} \cdot \left(\pi^e_t - \delta \left(\Delta e^{e,m}_t + \pi^{e,*}_t\right)\right) \tag{A1.4}$$

where the dating of expectations is time period $t-1$. Fifth, by construction

$$[\pi_{H,t} + \varepsilon_{1,t}] = \frac{1}{1-\delta} \cdot \left([\pi_t + \varepsilon_{2,t}] - \delta \left([\Delta e_t + \varepsilon_{3,t}] + [\pi^*_t + \varepsilon_{4,t}]\right)\right) \tag{A1.5}$$

or

$$\pi_{H,t} = \frac{1}{1-\delta} \cdot \left(\pi_t - \delta \left(\Delta e_t + \pi^*_t\right)\right) + \varepsilon_{5,t} \tag{A1.6}$$

where the ε:s are the realized errors in the expectations formations. Finally, substitute (A1.3) into the first equation in (2.1), substitute (A1.3) and (A1.6) into the second equation in (2.1), and (2.3) is derived.

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