BOFIT Discussion Papers

1999 • No. 7

Vadims Sarajevs

Macroeconomic Model of Transition Economy: A Stochastic Calculus Approach

Bank of Finland
Institute for Economies in Transition
BOFIT

Bank of Finland Institute for Economies in Transition BOFIT

Economists

Mr Pekka Sutela, head
Russian economy and economic policy
Russia's international economic relations
Baltic economies

Mr Tuomas Komulainen, economist
Russian financial system
Currency crises

Mr Iikka Korhonen, economist
Baltic economies
Russian financial system

Ms Seija Lainela, economist
Baltic economies
EU policy and Eastern relations
Russian economy and economic policy

Mr Juhani Laurila, senior adviser
Russian economy and economic policy
Baltic countries' external relations

Mr Tom Nordman, senior adviser
Chinese economy
Foreign exchange markets

Mr Jukka Pirttilä, research supervisor
Public economics
Transition economics

Mr Jouko Rautava, economist
Russian economy and economic policy

Mr Jian-Guang Shen, economist
Chinese and East Asian economies
Currency crises

Ms Laura Solanko, economist
Fiscal federalism
Russian regional issues

Information Services

Ms Joanna Ahlberg, department secretary
Department coordinator, publications traffic

Mr Timo Harell, senior clerk
Press monitoring

Ms Päivi Määttä, information specialist
Institute's library, information services

Ms Tiina Saajasto, research secretary
Statistical analysis, statistical data bases

Ms Liisa Sipola, research secretary
Information retrieval, Institute's library and publications

Publications

Russian and Baltic Economies – The Week in Review
A two-page weekly review of the previous week's focal events in Russia and the Baltic states.

Russian Economy – The Month in Review
A four-page review with analysis of developments in Russia's economy, fiscal policy and financial markets.

Baltic Economies – The Quarter in Review
A four-page review with analysis of recent economic developments in Estonia, Latvia and Lithuania.

BOFIT Discussion Papers
A series devoted to transition issues. Contains academic studies on transition economics by BOFIT economists and guest researchers.

BOFIT Online
An electronic publishing series devoted to transition issues. The focus is on descriptive work on economic policy and economic developments in transition economies. Papers are either in English or Finnish.

Contact us

Bank of Finland Institute for Economies in Transition (BOFIT)
P.O. Box 160 FIN- 00101 Helsinki
Phone: +358 9 183 2268 Fax: +358 9 183 2294 Email: bofit@bof.fi (firstname.surname@bof.fi)
Web: www.bof.fi/bofit

BOFIT
Discussion Papers

1999 • No. 7

Vadims Sarajevs

Macroeconomic Model of Transition Economy: A Stochastic Calculus Approach

Bank of Finland
Institute for Economies in Transition
BOFIT

ISBN 951-686-912-2 (print)
ISSN 1456-4564 (print)

ISBN 951-686-913-0 (online)
ISSN 1456-5889 (online)

Suomen Pankin monistuskeskus
Helsinki 1999

Contents

Vadims Sarajevs*

Macroeconomic Model of Transition Economy: A Stochastic Calculus Approach

Abstract

An integrated stochastic macroeconomic model of transition economy at the early stage of reforms with optimising representative risk averse agents is constructed. The equilibrium growth rate of the economy, real asset returns, domestic money demand, and expected inflation rate are determined as functions of the exogenous risks in the economy. The main issue addressed are: domestic money demand, currency substitution ratio, expected rate of inflation, real asset returns, the equilibrium growth rate of the economy as well as government ability to control these variables. Analysis of the model finds that the equilibrium growth rate of the economy is not independent on the monetary and fiscal policies but can be affected by the government through its ability to fix the real cost of capital for the firm, expenditure and monetary policy parameters.

JEL classification numbers: D80, D90, E41, E44, E52, F41, O11, O23.

This research was undertaken with support from the European Commission's PHARE - ACE Programme 1996 (P96-7114-S)

---

* Correspondence address: Economics Department, Queen Mary & Westfield College (University of London). Mile End Road, London E1 4NS, United Kingdom, E-mail: v.saraev@qmw.ac.uk

1 Introduction

The process of transition from the centrally planned to an open free market economy has been on its way for about nine years already for the countries in Eastern Europe and slightly less for the countries of the former Soviet Union, but still the level of theoretical understanding and the number of theoretical models of transition, addressing the questions of how do specific features of transition economies (menu of assets available to economic agents, absence of well developed financial markets such as - stock and bond markets, and the system of financial intermediaries) affect the level of macroeconomic performance and alter conventional results, remain fragmented, limited in scope and quantity.

Apart from setting up a specific model and solving it, the paper rises and addresses the following three issues: the issue of macroeconomic modelling, the issue of uncertainty, and the issue of integrating financial markets into macroeconomic framework. All of them are considered in relation to the problems of transition and development economies.

Most of macroeconomic models of transition use a deterministic framework, for example Bennett, J. and Dixon, H. D. (1996), Agenor, P-R. and Montiel, P.J. (1996), with a very few articles trying to approach the problem of uncertainty which is intrinsic to the process of transition. When modelling uncertainty articles predominantly rely on the rational expectations methodology or use *ad hoc* mechanisms to deal with some specific problems. This approach, especially models which are not derived from the optimising framework, brings forth a danger of dynamic inconsistency, misspecification, and, as the result, erroneous conclusions about the question under research. In part this is a problem caused by the usual conflict between partial and general equilibrium approaches. I would argue that the process of transition with the state of the economy far from macroeconomic equilibrium creates a highly interdependent system. Therefore, to account for the integrity of macroeconomic system with all its interactions between the markets and economic agents properly, the general equilibrium optimising approach should be preferred. A short clarification of my position may be necessary here. In no way I am arguing against so called *ad hoc* modelling in principle. Often it is the only way to start the research of a new and complicated phenomenon, an attempt to capture its most salient features and stylised facts. Hence, it is a necessary stage of a research process. I only wish to emphasise that it is the beginning and not the end of the research process. Also taking into account the above mentioned deficit of all kind of research on transition phenomenon, I believe, all of us who work on this topic would greatly benefit from more, much more research of both approaches.

Secondly, the issue of uncertainty is central to modelling transition. Uncertainty of all kinds increase dramatically during transition, because a whole new legislation system is introduced to replace and reform the old one, in particular, laws governing the behaviour of the government with respect to its fiscal stance and budget, the conduct of the monetary policy by newly established central bank or currency board, new system of reporting macroeconomic indicators, bankruptcy law, pension and social security reforms, privatisation to mention all but few major changes. All this has a direct impact on the behaviour and decision making process of economic agents. Therefore, an every possible effort should be made to account for the uncertainty in transition. To assess for the presence of uncertainty is important because it may alter significantly the outcomes of different policies pursued by the government. Also to the extent that the government is able to control the stochastic environment it may have a wider range of policy tools at its disposal than predicted by conventional models. Especially, it will be shown below that macroeconomic equilibrium depends on the both means and variances of government policies. Moreover, changes in uncertainty, as measured by the variances, affect the equilibrium behaviour of the economy.

Finally, the issue of integrating financial markets into macroeconomic model is of particular interest and importance. During transition financial markets undergo radical transformations. As soon as the role of central bank is clarified and the system of commercial banks is established, government starts to issue government debt (to cover its budget deficit) in the form of bonds, thereby, creating the *bond* market. Later in the transition, when large-scale privatisation takes place, the *stock* market is created. These alter the asset menu available to economic agents, who, therefore, will face a new consumption-portfolio investment allocation problem. Hence, the need for a model, which can account for that kind of changes in financial structure of the economy.

It should be noted that the above issues are relevant for many development countries, which despite having rather different initial conditions, during economic liberalisation programmes, i.e. lifting of capital controls on foreign exchange markets and privatisation, undergo through somewhat similar changes as transition economies.

To address the questions of interest, such as: domestic money demand, the currency substitution ratio, the real rates of return on assets, and the equilibrium growth rate of the economy as well as government ability to influence these variables, we develop a theoretical model of transition economy at the early stage of transition based on intertemporal optimisation of risk averse representative agents, and employ still relatively new and unfamiliar to many macroeconomists stochastic calculus methodology which allow one to deal coherently with the problem of uncertainty in macroeconomic framework, in continuous time.

Despite the fact that many stochastic calculus problems are analytically tractable only under a set of restrictive assumptions the solutions, when obtained, are lucid and deliver a great deal of understanding of the characteristics of the equilibrium. In deriving the equilibrium, means and variances of relevant endogenous variables are jointly and consistently determined. Such a mean-variance equilibrium is analogous to that of finance theory, the elements of which, therefore, may be more satisfactory incorporated into a complete macroeconomic framework. The general strategy accepted by the stochastic calculus approach to determine the macroeconomic equilibrium is to impose specific forms for the stochastic processes facing the agents in the economy and then to determine the restrictions on these processes which make them consistent with optimising behaviour and market clearance conditions. Basically, this is an application of the method of undetermined coefficients, which is a standard procedure for the solution of linear rational expectations models. All stochastic processes in the economy are presented as Brownian motions. There are two advantages of using this assumption. First, the rates of return on assets can be characterised in terms of only two variables, their instantaneous means and variances. This lead to the type of asset valuation principle that has been extensively studied in the finance literature for both discrete and continuous time cases. Second, the stochastic calculus applications to Brownian motions have been well developed and available to economists for some time, and is known to deliver a workable framework. For basic technical exposition of stochastic calculus methodology and applications one can refer to Chow, G. (1979), Malliaris, A.G. and W.A. Brock (1982), with the reference to finance see Ingersoll, J. E. (1987), Merton, R.C. (1990), and Duffie, D. (1992). For more applications in economics see Turnovsky, S.J. (1995), Benavie, A., Grinols, E. and Turnovsky, S. J. (1996), Grinols, E. and S.J. Turnovsky (1993), Grinols, E. and S.J. Turnovsky (1994), Turnovsky, S.J. (1993).

Comparing this approach with the Rational Expectations methodology, one can notice that in the Rational Expectations methodology the objective has been the development of an internally consistent, stochastic system to examine the economy, under the assumption that the underlying relationships depend only upon the means (first moments) of the relevant variables. That approach delivers a certainty equivalence macroeconomic framework, where the mean values (levels) do not depend on variances. Here, in the stochastic calculus problems, the objective is to develop a model that is internally consistent in both the means and variances, hence, able to address important trade-offs that, in general, exist between the level of macroeconomic performance and the associated risk.

Many governments during transition rely heavily on monetary policy to achieve macroeconomic stabilisation but the effectiveness of monetary policy has been seriously hindered by a high level of currency substitution and dollarization of national economies. It is still remains very high in the former Soviet Union and I address this problem in the article by investigating the factors determining the level of currency substitution. The key results obtained are as follows. It was found that the government has an ability to influence the equilibrium growth rate of the economy because it can fix the real cost of capital for the firm. A reduction of the real cost of capital was found to be growth improving. Also the equilibrium growth rate was found to depend on the choice of the government policy parameters, the mean share of government expenditures (g) and the mean rate of monetary growth (μ). In addition to the extent that the government can affect the stochastic environment, for example by making monetary and fiscal policy rules more simple and transparent, and by releasing the precise information on monetary affairs more frequently, it has an extra policy instruments to influence economic variables. Also it is derived that the currency substitution ratio falls with the rise in the foreign currency real balances - capital ratio (s), the variance of the productivity shocks, the real cost of capital, the variance of the monetary policy shocks. An increase in the mean rate of monetary growth, the mean share of government expenditures and its variance causes the currency substitution ratio to rise as well.

The rest of the paper proceeds as follows. Section 2 describes the structure of the economy and some specific features of the early stage of transition. In Section 3 the stochastic macroeconomic equilibrium is derived and all solutions are obtained and discussed. In Section 4 the conclusions and remarks on future research are presented.

2 Stochastic Framework of the Economy

2.1 Main Features

In our description of the national economy at the early stage of transition we shall refer, especially, to the case of Latvia, one of the three Baltic states. But to a large extent with minor variations it is applicable to the most of transition economies in Eastern Europe and the former Soviet Union. What follow is a justification of modelling Latvian economy at this stage of transition as the closed economy, despite the fact that consumer asset menu includes foreign currency.

First, consider the external economic environment for Latvia. Even now, in 1997, Russia is the main economic partner for Latvia, see *Economic Development of Latvia*, Report (1996), and at the beginning of the reforms in the early 1990s its role was even more important. The situation can be described as one of free asset flows between Russia and Latvia due to loose or zero regulation on capital movements and a high openness of Latvian banking system; also, despite the existence of some tariffs and quotas in Russia on a number of strategic commodities, oil for example, the situation was close to a free trade one (partly because of the small size of Latvian economy compared with Russia). However, because trade and asset flows with the rest of the world were initially highly insignificant we can consider the two-country system, Latvia-Russia, as closed with respect to the rest of the world. Therefore, in our model the rest of the world plays the role of an infinitely elastic supplier of foreign currency, in our case the US dollars, which are used in business operations between Latvia and Russia as an intermediate currency. Historically, the dollar was brought to the existence on the domestic markets of Latvia and Russia by a large number of individual retail traders. This initial inflow of dollars was accumulated by the commercial banks through the currency exchange cash points. Since the above process was quite slow and small in magnitude relative to international capital flows it has not altered anyhow the dollar exchange rate on the world markets.

The supply of the US dollars is determined by the demand from the Latvian households. An argument supporting such an assumption is the Deposit-Loan Matching phenomenon which exists in Latvian banking system. The Deposit-Loan Matching means that Latvian commercial banks were matching the size of the accepted deposits to the size of the loans distributed to firms. It works as follows. A large share of Latvian foreign trade with Russia has been represented by reexport and transit businesses, as well as by large purchases by Latvian firms of fuel and raw materials in Russia for productive domestic purposes. These activities were all conducted with the use of US dollars as an intermediate currency. Therefore, Latvian firms were taking US dollars trade credits in domestic commercial banks and in return paid interest, which was determined by the profit opportunities of their trade operations with Russia. In other words, the return on the foreign currency deposits was determined by existing production technology. The very high real interest rates paid by Latvian commercial banks to the households on their US dollars time-deposits were determined by such returns and not by interest rate parity condition of any kind with the rest of the world. The commercial banks were matching the amount of accepted deposits to the size of loans they were able to place with the firms, and would not accept a large deposit if there were not matching loan opportunities. Therefore, the interest paid on the US dollars time deposits was determined by their usefulness in commercial activity in the East.

Thus the use of foreign currency holdings were widely used by Latvian firms in their operations with the East and purchases of an intermediate inputs for domestic production purposes (e.g. oil, gas, energy, base metals), provides justification for the use of the real foreign currency balances as an input in production function in our model[1]. It is worth to

1. Lehvari, D. and D. Patinkin (1968) is one of the first examples of money in the production function specification. They argue that higher real money balances let the firm to have larger working capital and to purchase more inputs and hence produce more output. Another, more recent justification is the presence of high and variable inflation, so characteristic for the transition economies at the early stages, which is very disrupting for the production process (see M. Baxter's comment on Rebelo, S. and C.A. Vegh (1995, pp.: 177-178).

remind here that by reputation reasons it was impossible for the newly established commercial banks/firms to receive credits in world market.

Turning to the structure of the economy. There are three players in the economy: the government, a representative risk averse consumer (the household), and a representative firm. The situation of the early phase of transition (the very beginning of the reforms) is characterised by the absence of the stock and bond markets. Even now the stock market in Latvia is very thin and underdeveloped because of the slow process of privatisation. Hence, the government is assumed to be the owner of all existing capital in the economy. It rents capital to the firm at a real price r_K, which is in general non-zero.

There are two more assets in the economy, which are held by the household. These are domestic and foreign real money balances. The government prints money, collect taxes from the household, and rents capital to the firm. The representative consumer maximises his expected lifetime utility by choosing consumption and holdings of real money balances subject to the real wealth constraint. He lends foreign currency to the firm. The firm uses available capital and real foreign currency balances to produce output. Produced output which is not consumed or purchased by the government becomes part of the domestic capital stock. Government expenditures, domestic money creation and production are all described as continuous-time stochastic processes. Finally, equilibrium in the goods market and market for assets then determines the equilibrium stochastic processes for the domestic price level, the real rates of return, consumption and capital formation.

2.2 Consumer Optimisation

The representative consumer is assumed to optimise his expected lifetime utility. His asset holdings are subject to the wealth constraint

$$W = \frac{M}{P} + \frac{EM^s}{P} \qquad (1)$$

where M is domestic currency, M^S is foreign currency (in our case it is US dollars, \$), P is domestic price level, E is an exchange rate in terms of the number of units of domestic currency paid per one unit of foreign currency (in our case number of lats per one dollar, *(LVL/$)*).

Let $n_M = (M/P)/W$ and $n_S = (EM^S/P)/W$ be the portfolio shares of domestic and foreign currency in consumer's total portfolio, $n_M + n_S = 1$. The consumer objective is to maximise the expected value of his lifetime utility as function of consumption and money services,

$$\max_{C,M/P} E_0 \int_0^{+\infty} U(C(t), M(t)/P(t)) \cdot e^{-\beta t} dt \tag{2}$$

subject to the stochastic wealth accumulation constraint

$$dW/W = [n_M\, dR_M + n_S\, dR_S] - C\, dt - dT\ , \tag{3}$$

and taxes paid by the consumer to the government are given by

$$dT = \tau W\, dt + W\, dv. \tag{4}$$

The question of including money in the utility function is the one with a long history. Traditionally, it is argued that the real money balances provide utility to a consumer by reducing cost and facilitating his transactions. Also one can refer to Feenstra, R.C. (1986) who showed the equivalence between cash-in-advance approach and money in the utility function approach. More on this questions can be found in Blanchard, O.J. and Fischer, S. (1993).

Assume, as in Grinols, E. and S.J. Turnovsky (1993), logarithmic utility for analytical convenience

$$U = \theta \ln C(t) + (1-\theta) \ln (M(t)/P(t)) \tag{5}$$

A closed form solution can also be obtained in the case of the more general constant elasticity utility function as, for example, in Turnovsky, S.J. (1995), Merton, R.C. (1990). Assume that domestic money has a zero nominal rate of return. The household perceives that the price level evolves stochastically as given by

$$\frac{dP}{P} = \pi dt + dp \tag{6}$$

where π is the expected instantaneous rate of change, inflation, and *dp* is a temporally independent, normally distributed random variable with zero mean and variances $\sigma_p^2 dt$; it is generated by Wiener process. *P* is nondifferentiable function because dp is proportional to $\sqrt{dt}$ and the variance $\sigma_p^2 dt$, being a term of the first order, affects the mean behaviour of the system.

The real rates of returns are given by:

$$dR_M = r_M\, dt - dp,\; r_M = -\pi + \sigma_p^2$$
$$dR_S = r_S\, dt + ds \qquad (7)$$

where the return on domestic money is defined as (Ito's lemma is used for derivations)

$$dR_M \equiv \frac{d\left(\frac{1}{P}\right)}{\frac{1}{P}} \cong -\left(\frac{dP}{P}\right) + \left(\frac{dP}{P}\right)^2 = [-\pi + \sigma_p^2]dt - dp.$$

The return on money now depends on the stochastic properties of the inflation rate, it is rising with inflation variance[2].

The real mean rate of return on real foreign currency balances, r_S, will be specified below after the production sector is described.

The consumer's stochastic optimisation problem is to choose consumption and portfolio shares to maximise his expected lifetime utility

$$\max_{C, n_S, n_M} E_0 \int_0^{+\infty} [\theta \ln C + (1-\theta)\ln(n_M W)] e^{-\beta t} dt \qquad (8)$$

subject to the stochastic wealth accumulation constraint

2. This is generally due to the fact that the real rates of return are convex functions of the price level, i.e. $R_M = f(1/P)$, and, in general, $E(1/X) \neq 1/E(X)$, see Fischer, S. (1975, p. 513) for more explanations.

$$dW = W[\psi\, dt + dw] \tag{9}$$

where $\psi = n_M r_M + n_S r_S - C/W - \tau$,

$dw = - n_M\, dp + n_S\, ds - dv$,

$\sigma_w^2 \equiv lim_{t\to 0}\, E(dw)^2/dt = n_M^2\sigma_p^2 + n_S^2\sigma_S^2 + \sigma_v^2 - 2n_M n_S\sigma_{pS} -$

$2n_S\sigma_{vS} + 2n_M\sigma_{pv}$,

and $\sigma_{ij}\, dt = cov(di,dj),\ i,j = p, S, v.$

While performing optimisation the representative consumer takes all rates of return (r_S and r_M), τ and all variances and covariances as given. Although, all of them will be determined in the stochastic equilibrium derived later. Performing the optimisation (see Appendix A) yields the following first order conditions

$$C/W = \beta\theta ,$$

$$(1-\theta)\beta/n_M + r_M = \lambda\beta + \sigma_{w,-p} ,$$

$$r_S = \lambda\beta + \sigma_{wS} , \tag{10}$$

$$n_M + n_S = 1 .$$

The second and the third equations in (10) are asset pricing relationships, similar to the ones from the finance theory. In the spirit of the finance literature on asset-pricing models one can assume that the term $\lambda\beta$ represents the real rate of return on the asset whose return is uncorrelated with *dw*. Basically, $\lambda\beta$ is the equilibrium rate of return on consumption. In the absence of risk (zero covariances) first order conditions imply that all real rates of return must be equal, in particular, the real rate of return on money, including its utility return, $(1-\theta)\beta/n_M$, would be equal to the real rate of return on foreign currency holdings, r_S, provided both are held.

From the second and third equations we can derive the money demand equation, subtracting second equation from the third and rearranging yields

$$n_M = \frac{(1-\theta)\beta}{(r_S - r_M) - \sigma_{w,S+p}} . \tag{11}$$

The portfolio share of real money balances depends on the difference between the real rates of return on domestic and foreign money holdings, and on the stochastic characteristics of the economy defined by the covariance $\sigma_{w,S+p}$ which will be derived later in the equilibrium. Given n_M the portfolio share of the foreign currency is determined residually $n_S = 1 - n_M$.

For the future applications we can define the ratio

$$\xi = n_S / n_M \tag{12}$$

which is the simplest measure of currency substitution and dollarization of the economy. Its behaviour will be investigated after the solution for the stochastic macroeconomic equilibrium will be derived. But one can see already now that it will move in the opposite direction with the portfolio share of domestic money n_M, which is hardly surprising, so that factors increasing n_M will lower the currency substitution ratio and would improve the efficiency of monetary policy. Under the efficiency of monetary policy we mean the ability and the relative efforts which the monetary authorities need to apply to control/target some monetary aggregates, the dynamic of the exchange rate, and the revenue from inflation tax. Obviously, in the presence of currency substitution households can easily switch between domestic and foreign currencies. This can make money supply process endogenous and increase the instability of the money demand function, which impairs the ability of the monetary authorities to conduct its policies. For the full account of these effects see van der Ploeg, F. (1996, pp. 394-404).

2.3 Production Sector

The representative firm produces output by means of the stochastic production technology

$$dY(t) = F(K, EM^S/P)\,dt + F(K, EM^S/P)\,dy\,, \tag{13}$$

where the first term on the right hand side is deterministic and $F(K, EM^S/P)$ represents the mean rate of output per unit of time. The second, stochastic term dy represents productivity shocks and is assumed to be a temporally independent, normally distributed random variable with zero mean and variance $\sigma_y^2\,dt$. K is the physical stock of capital rented by the firm from the government (the owner of all capital in the economy) and (EM^S/P) is real foreign currency balances borrowed by the firm from the representative household. In return for capital the firm pays real interest on it, r_K, which is non-stochastic and given (set by the government). In return for the real foreign currency balances the firm pays real interest $dR_S = r_S\,dt + ds$, which will be determined below. Here we specify the contract in real terms for the purpose of convenience only. The rules of transformations from real to nominal variables and back are easily derived (see Appendix B).

In our model the firm's objective cannot be the maximisation of its stock market value as in Grinols, E. and S.J. Turnovsky, (1993) because of non-existence of the latter. An alternative and reasonable objective is profit maximisation. Profit is given by

$$d\Pi(t) = dY(t) - r_K \cdot K\,dt - (EM^S/P) \cdot dR_S \tag{14}$$

which is output less input costs. If we assume a competitive market, then there is no time inconsistency problem, because the competitive means also perfect, full market that is there are no hidden information and knowledge, everything (all laws of motion) is known by everybody. Therefore the firm can maximise its expected profit at each instant of time by choosing K and (EM^S/P). Firm's expected profit is given by

$$\Pi^e(t) = F(K, EM^S/P) - r_K \cdot K - r_S \cdot (EM^S/P). \qquad (15)$$

The stochastic and deterministic components of the real rate of return on foreign currency balances dR_S can be found from the following simple consideration. One can consider what return does the consumer get over the time period from *t* to *(t + dt)* from lending 1$ to the firm, and what return does the firm get. Then one can equate both returns in equilibrium. For the consumer the return is an interest paid on 1$, over the period *dt*, he rented initially at the time *t* plus gain/loss due to the change of exchange rate over the period *dt*, which is

$$1\$ * E * dR_S + dE.$$

For the firm's decision ex-ante considerations are important, and the return on 1$ borrowed by the firm from the consumer is determined by its production technology and is equal to the marginal product of foreign currency balances, which is

$$1\$ * E * [\, d\,F(K, EM^S/P)/d(EM^S/P)\,dt + d\,F(K, EM^S/P)/d(EM^S/P)\,dy].$$

Since both returns must be equal in the equilibrium, this yields

$$dR_S = d\,F(K, EM^S/P)/d(EM^S/P)\,dt +$$

$$d\,F(K, EM^S/P)/d(EM^S/P)\,dy - dE/E \qquad (16)$$

where $d\,F(K, EM^S/P)/d(EM^S/P) \equiv \boldsymbol{F'}$ is the marginal product of the real foreign currency balances. We assume that the exchange rate evolves in accordance with the Brownian motion process

$$dE/E = e\,dt + de, \qquad (17)$$

where e is the expected rate of depreciation of the exchange rate and de is a temporally independent, normally distributed random variable with zero mean and variance $\sigma_e^2\, dt$. Our assumption of random walk process for the exchange rate is not unusual, see van der Ploeg (1994, pp. 304, 350-369) for the discussion of the unit-root process in the log of the nominal exchange rate and their martingale properties. Also Hallwood, C.P. and R. MacDonald (1994, pp. 176-180) and the famous article by Meese, R. and K. Rogoff (1983), in which they showed that a simple random walk model of the exchange rate performs as well as any other one. The recent development still suggests that the random walk behaviour of the exchange rate remains a plausible assumption for the short run time horizon, which is employed in this paper modelling the early phase of the reforms. It follows then that

$$r_S = d\, F(K, EM^S/P)/d(EM^S/P) - e, \qquad (18)$$

$$ds = d\, F(K, EM^S/P)/d(EM^S/P)\, dy - de.$$

While performing the optimisation the firm can consider r_S as being under its control. To see it recall from the first order conditions for consumer (10) that

$$r_S = \lambda\beta + \sigma_{wS}\text{, where now from (18)}$$

$$\sigma_{wS} = d\, F(K, EM^S/P)/d(EM^S/P) \cdot \sigma_{wy} + \sigma_e^2 \qquad \text{so that}$$

$$r_S = \lambda\beta + (d\, F(K, EM^S/P)/d(EM^S/P) \cdot \sigma_{wy} + \sigma_e^2). \qquad (19)$$

It is reasonable to assume that taking $\lambda\beta$, consumer's marginal utility of wealth, and all variances and covariances as given the firm, nevertheless, can affect r_S through the marginal product of the real foreign currency balances by its choice of productive inputs. The simplest way to take this into account is to substitute the above equation into the expression for the firm's expected profit

$$\Pi^e(t) = F(K, EM^S/P) - r_K \cdot K - \qquad (20)$$

$$(\lambda\beta + d\,F(K, EM^S/P)/d(EM^S/P) \cdot \sigma_{wy} + \sigma_e^2)\,(EM^S/P).$$

Optimisation yields the following first order conditions for the firm's problem

$$\frac{\partial F}{\partial K} - r_K - \frac{\partial^2 F}{\partial (EM^S/P)\partial K} \cdot \sigma_{wy} \cdot (EM^S/P) = 0$$

$$\frac{\partial F}{\partial (EM^S/P)} - (\lambda\beta + \frac{\partial F}{\partial (EM^S/P)} \cdot \sigma_{wy} + \sigma_e^2) - \qquad (21)$$

$$(EM^S/P) \cdot \frac{\partial^2 F}{\partial (EM^S/P)^2} \cdot \sigma_{wy} = 0$$

Both equations may be seen as equating 'risk-adjusted' marginal products of capital and real foreign currency holdings to their real costs, r_K and $\lambda\beta$, the real cost of capital and the consumer rate of return correspondingly.

2.4 Government

In our model government policies are: expenditure policy, monetary policy (the printing of money), and tax policy. All of them are chosen subject to the budget constraint, which in real terms is given by

$$d(M/P) + r_K \cdot K\,dt = (dG - dT) + (M/P)\,dR_M\,, \qquad (22)$$

where on the right hand side we have the government deficit, which is equal to the value of government expenditures on goods and services plus the interest payments on the outstanding debt less tax revenue. On the left hand side are the means of financing the government deficit by printing more money and by lending physical capital. Note that the term $(M/P)\,dR_M$ on the right hand side represents the interest payments on the outstanding stock of money. Normally, this term would be negative (from eq.(7) the appropriate condition is $r_M = (-\pi + \sigma_p^2) < 0)$, i.e. it is real revenue for the government from inflation - seigniorage. Accumulation of capital, investments, *dK* is determined residually from the product market equilibrium condition

$$dK = dY - dC - dG \, . \qquad (23)$$

Tax policy is given by

$$dT = \tau W\, dt + W\, dv \qquad (24)$$

so that the representative household pay tax on the total real wealth, where τ is a fixed share of wealth and *dv* is a temporally independent, normally distributed random variable with zero mean and variance $\sigma_v^2\, dt$. Both variables are set so as to maintain a balanced budget.

Expenditure policy is given by

$$dG = g\, F(K, EM^S/P)\, dt + F(K, EM^S/P)\, dz \qquad (25)$$

where *dz* is a temporally independent, normally distributed random variable with zero mean and variance $\sigma_z^2\, dt$. Hence, the instantaneous mean level of public expenditures is a fraction of the mean level of output, and the stochastic error is proportional to the mean level of output.

Monetary policy is given by

$$dM/M = \mu\, dt + dx \, , \qquad (26)$$

where μ is the mean rate of nominal monetary growth subject to a stochastic disturbance *dx*, which is a temporally independent, normally distributed random variable with zero mean and variance $\sigma_x^2\, dt$. We assume that the mean rate of nominal monetary growth is the one decided upon and controlled directly by the monetary authority, while the stochastic component reflects exogenous failures to meet this target.

Finally, using the first order conditions for consumer and the policies specified above we can write down the capital accumulation *dK* as

$$dK = [(1\text{-}g)F - \beta\theta W]\, dt + F(dy - dz) \qquad (27)$$

it responds positively to the productivity shocks (*dy*) and negatively to the increase in government expenditures (*g*).

3 Macroeconomic Equilibrium and Model Solution

The Notion of Equilibrium. Joining together the elements developed in the previous sections we can derive the overall stochastic macroeconomic equilibrium. The exogenous factors are: the mean rate of government expenditures *g*, the mean rate of monetary growth μ, the real cost of capital r_K, as well as the preferences parameters θ, β and any parameters describing the production technology. The exogenous stochastic processes include: *dz* (government expenditures), *dx* (monetary growth), *dy* (productivity shocks), and *de* (exchange rate shocks), all of which are assumed to be mutually uncorrelated. The remaining of stochastic disturbances: *dv* (taxes), *dp* (inflation shocks), *dw* (wealth) and *ds* (return on foreign currency) are endogenous and reflect stochastic adjustments, and can be expressed as functions of the exogenous shocks.

The optimality conditions for consumer problem (10) suggest that it is reasonable to assume that if assets have the same stochastic characteristics through time (the means and variance-covariance matrix of asset returns are stationary), they will yield the same allocation of portfolio shares. This kind of repeated equilibrium has the property that the portfolio shares are non-stochastic functions of underlying parameters and are constant through time. Therefore, we will look for an equilibrium where portfolio shares have this property. Now, to close the model we need to describe the functional relationships between state variables (M, M^S, K) and asset returns, including expected inflation π.

Equilibrium Tax Adjustments. First, consider the government budget constraint (22). Substituting (23 - 27) into it one can get

$$d(M/P) + r_K \cdot K\,dt = (gF - \tau W)\,dt + F dz - W\,dv + (M/P)\,dR_M\,, \quad (28)$$

divide it by the total wealth *W*,

$$n_M \frac{d\left(M/P\right)}{\left(M/P\right)} = \left[\frac{gF - r_K K}{W} - \tau\right]dt + \frac{F}{W}dz - dv + n_M dR_M \quad (29)$$

By Ito’s lemma for stochastic calculus

$$d(M/P)/(M/P) = dM/M - dP/P - (dM/M)\cdot(dP/P) + (dP/P)^2 ,$$

and from (6) and (26) it follows that

$$d(M/P)/(M/P) = (\mu - \pi + \sigma_p^2 - \sigma_{px})\ dt + (dx - dp).$$

Substitute this and (7) into (29) to yield (by equating the deterministic and stochastic parts of the resulted expression) the following two relationships

$$dv = (F/W)\ dz - n_M\ dx = n_S\ (F/(EM^S/P))\ dz - n_M\ dx, \qquad (30)$$

$$\tau = n_S \frac{gF - r_K K}{\left(EM^S/P\right)} - n_M(\mu - \sigma_{px})\ .$$

These equations show the endogenous adjustments in taxes necessary to maintain the balanced government budget for the given policy specifications. The mean tax rate τ should be set only once because after the government made its choice of the policies all covariances are known and stationary in the equilibrium as well as all other variables. However, the adjustment in the stochastic component *dv* is continuous in response to fluctuations in government expenditures (*dz*) and monetary growth (*dx*). From the above equations one can see that a higher monetary growth rate *(μ)* will raise seigniorage revenue and allow for a lower mean tax rate. Since the covariance between the monetary growth rate and the price level is positive it reduces the growth rate of the real stock of money, therefore, requires a higher mean tax rate. Of course higher government expenditures *(g)* will increase the mean tax rate. Also an increase in the price of capital will lower the mean tax rate because the government revenue will rise from lending the capital.

Equilibrium Rate of Inflation. From the assumption of the constant portfolio shares and the relationships $(M/P) = n_M W$ and $(EM^S/P) = (M^S/P^*) = n_S W$ the domestic price level can be expressed as $P = constant * M/(M^S/P^*)$. Using (26) and the fact that the supply of foreign currency balances is determined by the demand from the representative household $(EM^S/P) = n_S W$, hence,

$$\frac{d\left(EM^S/P\right)}{\left(EM^S/P\right)} = \frac{dW}{W} = \psi dt + dw \tag{31}$$

and we can derive

$$\frac{dP}{P} = \frac{d\left(\frac{M}{EM^S/P}\right)}{\left(\frac{M}{EM^S/P}\right)} = (\mu - \psi - \sigma_{wx} + \sigma_w^2)dt + (dx - dw). \tag{32}$$

Therefore, we can write for the expected rate of change and for the stochastic component of prices the following equations

$$\pi = \mu - \psi - \sigma_{wx} + \sigma_w^2,$$

$$dp = dx - dw. \tag{33}$$

Consider, first, the stochastic component, $dp = dx - [- n_M\, dp + n_S\, ds - dv]$. Substitute for the *ds* and *dv* from (18) and (30) to yield

$$dp = dx + [F/(EM^S/P)]\, dz - \mathbf{F'}\, dy + de. \qquad (34)$$

The stochastic component of prices responds positively to the shocks of money supply *(dx)*, government expenditures *(dz)* and the expected rate of depreciation of the exchange rate *(de)*, while positive productivity shocks *(dy)* drives the stochastic component of prices down.

Now we can collect together all stochastic adjustments.

Summary of Stochastic Adjustments (35)

$$dv = n_S\, (F/(EM^S/P))\, dz - n_M\, dx,$$

$$dp = dx + [F/(EM^S/P)]\, dz - \mathbf{F'}\, dy + de,$$

$$ds = \mathbf{F'}\, dy - de,$$

$$dw = \mathbf{F'}\, dy - [F/(EM^S/P)]\, dz - de.$$

These equations allow one to calculate all the variances and covariances which are met in the optimality conditions and solutions.

Summary of Variances and Covariances (36)

$$\sigma_w^{\,2} = [F/(EM^S/P)]^2\,\sigma_z^{\,2} + (\mathbf{F'})^2\,\sigma_y^{\,2} + \sigma_e^{\,2},$$

$$\sigma_{wy} = (\mathbf{F'})^2\,\sigma_y^{\,2},$$

$$\sigma_{w,-p} = \sigma_w^{\,2},$$

$$\sigma_{w,S} = (\mathbf{F'})^2\,\sigma_y^{\,2} + \sigma_e^{\,2},$$

$$\sigma_{w,\,S+p} = -\,[F/(EM^S/P)]^2\,\sigma_z^{\,2},$$

$$\sigma_p^{\,2} = [F/(EM^S/P)]^2\,\sigma_z^{\,2} + (\mathbf{F'})^2\,\sigma_y^{\,2} + \sigma_e^{\,2} + \sigma_x^{\,2} = \sigma_w^{\,2} + \sigma_x^{\,2},$$

$$\sigma_{px} = \sigma_x^{\,2},$$

$$\sigma_{wx} = 0.$$

To proceed further with the solution we assume a standard Cobb-Douglas production function with the constant return to scale

$$F(K,S) = a\,K^{\alpha}\,S^{1-\alpha} \qquad (37)$$

where for the sake of notation convenience we denote the real foreign currency balances as *S*, $S \equiv (EM^S/P)$. Then from the firm's first order conditions, the first equation in (21), with the use of (37) and σ_{wy} from (36) we can derive

$$\alpha \frac{F}{S}\frac{S}{K}\left[1-\frac{F}{S}(1-\alpha)\alpha\sigma_y^2\right]=r_K \ . \qquad (38)$$

From the homogeneity of the production function we can write $\frac{F(K,S)}{S}=\frac{f(s)}{s}$ where $s \equiv S/K$ is the real foreign currency balances-to-capital ratio. Now we can rewrite (38) as

$$\alpha f(s)\left[1-\frac{f(s)}{s}(1-\alpha)\alpha\sigma_y^2\right]=r_K \ . \qquad (39)$$

Equation (39) (as equation 38 before) describes the optimality condition for capital, i.e. it equates the 'risk-adjusted' marginal product of capital to the given real cost (r_K). For this equation to have economic sense we require

$$0 < \frac{f(s)}{s}(1-\alpha)\alpha\sigma_y^2 < 1.$$

Equation (39) is an implicit function in *s* which can be resolved in the form

$$s = s(\sigma_y^2, r_K) \qquad (40)$$

and with the help of the implicit function differentiation rule it can be shown that

$$ds/d\sigma_y^2 > 0, \text{ and } ds/dr_K > 0,$$

so that an increase in the variance of productivity shocks lowers the ‘risk-adjusted’ marginal product of capital, hence, raises the real foreign currency balances-to-capital ratio; and a rise in the real cost of capital causes an increase in the real foreign currency balances-to-capital ratio.

We can solve now for the real rate of return on foreign currency balances, from (18) and (40) we obtain

$$r_S = f'(s(\sigma_y^2, r_K)) - e \ . \tag{41}$$

It follows that the derivatives of r_S with respect to e, r_K, σ_y^2, and s are all negative, i.e. rise in the rate of the exchange rate depreciation causes r_S to fall. In Latvia, actually, the rate of the exchange rate appreciation is slowing down, hence, r_S is rising. Increases in the variance of productivity shocks and in the real cost of capital, raises the real foreign currency balances-to-capital ratio (s), hence, cause the real rate of return on the foreign currency balances to fall. Finally, an increase in the real foreign currency balances-to-capital ratio drives the real rate of return on the foreign currency balances down, because of the diminishing marginal return property of the production function.

Now using the first order conditions for consumer problem (10, third and second equations), (41) and (7) one can derive the following expression for inflation

$$\pi = \frac{(1-\theta)\beta}{n_M} - (f' - e) + (f')^2 \sigma_y^2 + \sigma_x^2 + \sigma_e^2 \tag{42}$$

Also using equations (33, first equation), (7), and (41) one can derive another expression for inflation

$$\pi = \mu - (f' - e) + \frac{gF - r_K}{s} + \sigma_w^2 + \frac{\beta\theta}{1 - n_M} . \tag{43}$$

Substructing (43) from (42) we can resolve it with respect to n_M, portfolio share of the domestic real money balances, in the form of a quadratic equation

$$A\, n_M^2 - (A + \beta)\, n_M + (1 - \theta)\, \beta = 0\,, \qquad (44)$$

$$A = \mu + \frac{gf(s) - r_K}{s} + \left(\frac{f(s)}{s}\right)^2 \sigma_z^2 - \sigma_x^2\,.$$

The solution of (44) is given by

$$(n_M)_{1,2} = \frac{(A+\beta) \pm \sqrt{(A+\beta)^2 - 4A\beta(1-\theta)}}{2A} \qquad (45)$$

It is naturally to assume $A > 0$. Also we will assume that domestic money dominates foreign currency and, hence, we should choose the plus sign in front of the square root expression. From this equation the portfolio share of the real foreign currency balances is determined residually, $n_S = 1 - n_M$, and the currency substitution ratio (12) can be easily derived.

From (45) one can see that n_M (basically, money demand) is the function of seven parameters: μ the rate of monetary growth, g the share of government expenditures, s the real foreign currency balances - capital ratio, σ_z^2 the variance of government expenditures, σ_x^2 the variance of the monetary growth, σ_y^2 the variance of productivity shocks. It is easier to investigate the behaviour of n_M using (44) as an implicit function equation, $\Phi\,(n_M, X) = 0$, where X is the vector of parameters, and applying to it the implicit function differentiation rule, $dn_M/dX = - \Phi_X/\Phi_n$, where Φ_X denotes partial derivative of Φ with respect to one of the parameters of vector X, and Φ_n denotes partial derivative of Φ with respect to n_M , rather than to use expression (45). In our case $\Phi_n = A(2n_M - 1) - \beta$. To proceed further one needs to sign this expression. We can do it from the obvious case of an increase in the rate of monetary growth (μ). When μ rises, inflation rises, money own interest falls (see eq.(7)) and, hence, the demand for money should fall, therefore one should have $dn_M/d\mu < 0$, but $\Phi_\mu = n_M^2 - n_M < 0$ (since $n_M < 1$). This means that for negative derivative $dn_M/d\mu$ we need to assume $\Phi_n < 0$ which require $A < \beta$. With this assumption we obtain the following rule for signing the derivatives of n_M with respect to the above mentioned seven exogeneous variables

$$\text{sign}\ (dn_M/dX) = \text{sign}\ (\Phi_X) = -\ \text{sign}\ (A_X) \qquad (46)$$

where A_X means partial derivative of A from (44) with respect to X. The results of this exercise are summarised in the **Summary Table** at the end of this section. From the table one can see that the demand for money responds negatively to the rise in the mean rate of monetary growth (μ), the share of government expenditures (g) and to an increase in the variance of government expenditures (σ_z^2), because an increase in the government expenditures will require higher seigniorage revenue and inflation. The demand for money is increasing when the real foreign currency balances - capital ratio (s) is increasing, because by (41) and the law of diminishing return it leads to a fall in the real rate of return on foreign currency balances (r_S) making domestic money holdings more attractive. Also it responds positively to an increase in the real cost of capital (r_K) and in the variance of the monetary uncertainty (σ_x^2), because the rise in the variance of the monetary uncertainty (σ_x^2) drive the real rate of return on the domestic money balances up[3], and this causes for the portfolio share of the real domestic money balances in the representative consumer's total real wealth to rise as well. Finally, the demand for money rises with an increase in the variance of productivity shocks (σ_y^2), this effect (as partly in case with r_K) works through the real foreign currency balances - capital ratio (s).

Turning to the currency substitution ratio (12) one can notice that an increase in any of the factors which cause the demand for money to fall causes the currency substitution ratio (ξ) to rise, negatively affecting the effectiveness of monetary policy and vice a versa. Therefore, the government has five instruments (μ, g, σ_x^2, σ_z^2, r_K) at its disposal to influence the rate of currency substitution. This is an important result which shows that except a usual way of influencing the economy by choosing levels of variables of government policies the authorities have additional policy instruments at their disposal, namely (σ_x^2 and σ_z^2) the variance of the monetary uncertainty and the variance of government expenditures which are some of the stochastic characteristics of the

3. see equation (36) to notice that the variance of inflation include the variance of monetary growth with positive sign and, hence, by (7) leads to an increase in the real rate of return on the real domestic money balances.

economy. They can influence these variables by, for example, choosing the frequency and quality of announcements/reviews of monetary and budget reports. Also as reforms proceed we can expect that new legislation will be introduced which regulates the behaviour of the central bank and the fiscal authorities by setting transparent rules for the monetary authorities and by defining the limits of the budget deficit. These measures when implemented will reduce the level of uncertainty in the economy and respectively the variances will decrease. Then a simple static analysis exercise can reveal how do the decreasing variances affect the currency substitution ratio.

To complete the solution one can find the real rate of return on domestic money balances (r_M) and the mean rate of inflation (π). Expression for inflation rate is given by (42). From first order conditions for consumer problem (10) we can derive

$$r_M = -\frac{(1-\theta)\beta}{n_M} + (f' - e) + \left(\frac{f(s)}{s}\right)^2 \sigma_z^2 \qquad (47)$$

The behaviour of these variables is summarised in the **Summary Table** at the end of this section.

Equilibrium Growth Rate. All assets in the equilibrium grow at the same rate which is equal to the mean rate of total real wealth accumulation, $E[dW/W] = \psi$. From the definitions in (9) and the obtained solutions we can derive

$$\psi = f'(s(\sigma_y^2, r_K)) - e - \beta + \frac{gf(s) - r_K}{s} + n_M A \ . \qquad (48)$$

where A is from (44).

The derivatives of the equilibrium growth rate with respect to an increase in real foreign currency balances - capital ratio (s), the real cost of capital (r_K), the variance of productivity shocks (σ_y^2), and the rate of the exchange rate depreciation ($e\uparrow$) are negative. In particular, the rise in the real cost of capital (r_K) causes the equilibrium growth rate to fall. Therefore, if privatisation is delayed and there is no stock market which determines the price of capital, the reduction by the government of the real cost of capital

(r_K) to the firm to the lowest possible level (keeping, of course, an appropriate level of monitoring) will be growth promoting. The same is true for the effect of the exchange rate stabilisation ($e\downarrow$).

Finally, one can notice that monetary policy does affect the equilibrium growth rate (ψ), i.e. an increase in the rate of monetary growth causes ψ to rise (first-order effect), but rise in the rate of monetary growth is correlated with an increase in monetary uncertainty measured by σ_x^2, which push the equilibrium growth rate (ψ) down (second-order effect). As μ is rising the second effect can become dominant, hence, only moderate (how moderate depends on particular conditions) inflation rate can be growth improving. Also ‘fiscal’ policy, as far as it concerns government ability to set the real cost of capital (r_K) and the mean proportion of government expenditures (g), can affect the equilibrium growth rate of the economy.

To conclude one can investigate the variance of the equilibrium growth rate of the economy, from (36)

$$\sigma_w^2 = \sigma_e^2 + (f'(s))^2 \sigma_y^2 + \left(\frac{f(s)}{s}\right)^2 \sigma_z^2 . \tag{49}$$

The derivative of the variance of the equilibrium growth rate with respect to an increase in the variance of the exchange rate (σ_e^2) is positive, i.e. increased exogenous risk causes a destabilisation effect. The derivative of the variance of the equilibrium growth rate with respect to an increase in the real foreign currency balances - capital ratio (s) is negative; and with respect to an increase in the variance of productivity shocks (σ_y^2) is uncertain, that is, paradoxically, the increased exogenous risk may be stabilising for growth. An increase in the real cost of capital (r_K) also is stabilising.

Summary Table

Effects on the endogenous variables of an increase in model parameters

Increase in parameter	**Endogeneous variables**							
	s	$\mathbf{r_S}$	$\mathbf{n_M}^a$	ξ	$\mathbf{r_M}^a$	π^a	ψ^a	σ_w^2
μ	n.a.	n.a.	↓	↑	↓	↑	↑	n.a.
G	n.a.	n.a.	↓	↑	↓	↑	↑	n.a.
$\mathbf{r_K}$	↑	↓	↑	↓	?	?	↓	↓
σ_y^2	↑	↓	↑	↓	?	?	↓	?
σ_z^2	n.a.	n.a.	↓	↑	↑	↑	↑	↑
σ_x^2	n.a.	n.a.	↑	↓	↑	↑[b]	↓	n.a.
σ_e^2	n.a.	n.a.	n.a.	n.a.	n.a.	↑[b]	n.a.	↑
e	n.a.	↓	n.a.	n.a.	↓	↑	↓	n.a.

where the following notations are used:
n.a. means that this endogenous variable does not depend on the given parameter;
? means that the expression (derivative of the variable with respect to the parameter) for this variable cannot be definitely signed because there are two or more competing effects of the opposite signs;
a: to sign this expression some simplifications and additional assumptions were made, see Appendix C;
b: the opposite sign is possible if the second-order effects become dominants, e.g. the variance of the monetary growth (σ_x^2) impacts on inflation rate directly by (42) and indirectly through the demand for money (n_M), these two effects are of opposite signs.

4 Conclusions and Remarks on future research

This paper has presented a macroeconomic model of a transition economy at the early stage of transition from centrally planned economy to a free market economy based on an optimising representative risk averse agent set in a stochastic framework. The macroeconomy consists of consumers, producers and the government. Behavioural relationships in the private sector (consumers and producers) were obtained from the underlying optimisation of representative economic agents. The analytical solution to this stochastic macroeconomic model was derived.

This allows for a coherent treatment of an economy with different sources of risk: risk from production sector, risks from monetary and fiscal policy shocks, and risk from exchange rate shocks. The stochastic real rates of return on assets and the endogenous stochastic process for inflation are derived from these underlying risks.

The questions of domestic money demand, the currency substitution ratio, and the equilibrium growth rate of the economy as well as government ability to influence these variables were addressed. The effects of changes in the underlying model's parameters and risks on these variables were discussed.

It was found that in the present model the government has an ability to influence the equilibrium growth rate of the economy because it can fix the real cost of capital for the firm, as well as by setting the government policies parameters, share of government expenditures (g) and the rate of monetary growth (μ). A reduction of the real cost of capital was found to be growth improving. In addition to the extent the government can affect the stochastic environment, for example by making monetary and fiscal policy rules more simple and transparent, and by releasing precise information on monetary and fiscal affairs more frequently, it has an extra policy instrument to influence economic variables. Notice, that the real rate of return on domestic money as well as the portfolio share of domestic money in the representative consumer's total wealth depend on the variances of monetary and fiscal policies in equilibrium and, hence, may be affected by the changes in these variances. The model can be used as a benchmark to study the effects of introduction of the fully fledge financial markets in the economy, *bond market* through creation of government securities and commercial banks, and *stock market* through privatisation of state owned enterprises.

The present paper has left a number of interesting questions to be explored in the future research. First of all, as reforms proceeds stock and bond markets can be expected to be created. It would be interesting to consider how do the introduction of the government bond market and the stock market (after privatisation) alter the results of our model. Secondly, it may be worthwhile to consider this model in an international set up, in the context of a small open economy, to address the questions of foreign trade and balance of payments crisis; it would be possible also to endogenize the process for the exchange rate and to consider different exchange rate regimes. During the process of reforms government expenditures on economic infrastructure and on the creation of a social safety net are very important, therefore, one can also relax the assumption of no impact from government expenditures on private utility. Additionally, the questions of welfare analysis can be easily approached in the context of a representative agent model, just by using his utility as a measure of welfare.

Appendix A

Define the differential operator of the value function *V(W,t)* by

$$L_W[V(W,t)] \equiv \left\{ \frac{\partial}{\partial t} + \psi W \frac{\partial}{\partial W} + \frac{1}{2}\sigma_w^2 W^2 \frac{\partial^2}{\partial W^2} \right\} \cdot V(W,t)\ .$$

Assume that the value function is of the time-separable form

$$V(W,t) \equiv X(W) \exp(-\beta t)$$

Then

$$L_W[V(W,t)] = e^{-\beta t}[-\beta X + \psi W \frac{dX}{dW} + \frac{1}{2}\sigma_w^2 W^2 \frac{d^2 X}{dW^2}]\ .$$

The Lagrangean of the problem is

$L = \exp(-\beta t)\ [\theta \ln C + (1-\theta)\ln(n_M W)] + L_W[V(W,t)] + \lambda \exp(-\beta t)$
$[1 - n_M - n_S]$

which is maximised with respect of consumption (*C*), the portfolio shares (n_M and n_S), and the Lagrange multiplier (λ).

The first order conditions are

$$\theta/C - dX/dW = 0\ ,$$

$$(1-\theta)/n_M + r_M W \cdot dX/dW + W^2 \cdot d^2X/dW^2\ [\sigma_p^2 n_M - n_S\sigma_{pS} + \sigma_{pv}] = \lambda\ ,$$

$$r_S W \cdot dX/dW + W^2 \cdot d^2X/dW^2\ [\sigma_S^2 n_S - n_M\sigma_{pS} + \sigma_{Sv}] = \lambda\ , \quad \text{(A.1)}$$

$$n_M + n_S = 1 \ .$$

These equations determine the optimal values for consumption and portfolio shares as functions of the derivatives *dX/dW* and *d²X/dW²* of the value function *X(W)*.

In addition, the value function must satisfy the stochastic Bellman equation

$$\max_{C, n_M, n_S} [\theta \ln C + (1-\theta)\ln(n_M W)]e^{-\beta t} + L_W[e^{-\beta t} X(W)] = 0,$$
$$\theta \ln \hat{C} + (1-\theta)\ln(\hat{n}_M W) - \beta X + \hat{\psi} W \frac{dX}{dW} + \frac{1}{2}\hat{\sigma}_w^2 W^2 \frac{d^2 X}{dW^2} = 0, \qquad \text{(A.2)}$$

where hat (^) denotes optimised values obtained from the first order conditions. To solve this differential equation for *X(W)* we postulate the solution of the form

$$X(W) = b_0 + b_1 \ln W, \qquad \text{(A.3)}$$

where b_0 and b_1 are to be determined. From the form of the value function it follows that

$$dX/dW = b_1/W \text{ and } d^2X/dW^2 = -b_1/W^2 \ . \quad \text{(A.4)}$$

Substituting this into the first order condition yields

$$\hat{C} = \frac{\theta}{b_1} W,$$
$$\ln \hat{C} = \ln\theta - \ln b_1 + \ln W \qquad \text{(A.5)}$$

Now we substitute this into Bellman equation to obtain

$$\theta[\ln\theta - \ln b_1 + \ln W] + (1-\theta)[\ln \hat{n}_M + \ln W] - \beta[b_0 + b_1 \ln W] + \hat{\psi} b_1 - \frac{1}{2}\hat{\sigma}_w^2 b_1 = 0$$

This consists of constants and terms involving *lnW*. For the function

$X(W) = b_0 + b_1 \ln W$ to be a viable solution b_0 and b_1 must be chosen to satisfy

$$b_1 = 1/\beta ,$$

$$\beta b_0 = \frac{1}{\beta}[\hat{\psi} - \frac{1}{2}\hat{\sigma}_w^2] + \theta\ln\theta + \theta\ln\beta + (1-\theta)\ln\hat{n}_M . \quad \text{(A.6)}$$

Therefor, the value function is

$$X(W) = b_0 + \ln W/\beta ,$$

and $$dX/dW = 1/(\beta W),\ d^2X/dW^2 = -1/(\beta W^2). \quad \text{(A.7)}$$

Substituting this into the first order conditions yields

$$C/W = \beta\theta ,$$

$$(1-\theta)\beta/n_M + r_M = \lambda\beta + \sigma_{w,-p} ,$$

$$r_S = \lambda\beta + \sigma_{wS} , \quad \text{(A.8)}$$

$$n_M + n_S = 1 .$$

which are the first order conditions (10) in the text.

One can further establish that the transversality condition is satisfied

$$\lim_{t \to +\infty} E[e^{-\beta t} X(W)] = 0 \ .$$

Appendix B

Let the evolution of price level is given by eq.(6)

$$\frac{dP}{P} = \pi dt + dp$$

Then, if the real rate of return on capital r_K is certain then the return on capital is given by

$$\frac{dK}{K} = r_K dt + \frac{dP}{P} = (r_K + \pi)dt + dp$$

so that the nominal rate of return is given by the sum of the real one and the mean rate of inflation.

An opposite example is when the certain nominal return is known, on, let say, government bonds

$$dB/B = R_B \, dt$$

Then the real return can be calculated, and the application of the Ito's lemma yields

$$\frac{d\left(B/P\right)}{\left(B/P\right)} = (R_B - \pi + \sigma_p^2)dt - dp = r_B dt - dp \ ,$$

where r_B is the real rate of return on bonds.

For more derivations one can refer to Fischer, S. (1975).

Appendix C

To sign the derivatives of n_M with respect to s, σ_y^2 and r_K we need to consider first the derivative of A from (44) with respect to s. It can be written as

$$A_s = \frac{MPK}{s^2}\left[\varepsilon - (g + 2\left(\frac{f(s)}{s}\right)\sigma_z^2)\right],$$

where $MPK = \alpha f(s) > 0$ is the marginal product of capital, $\varepsilon = 1 - \frac{f(s)}{s}(1-\alpha)\alpha\sigma_y^2$ is the 'risk-adjustment' factor from (39). Obviously that the sign of A_s depends on the expression in square brackets; in the range of reasonably small s, $(f(s)/s \sim 1/s^{\alpha}) >> 1$, and, hence, A_s will be negative. Recalling that $s \equiv S/K$ is the real foreign currency balances - capital ratio ($S \equiv (EM^S/P)$) we would argue that the above assumption holds for the wide range of non-pathological economies.

Further, to simplify and sign the expression $n_M A$ in (48) we made two assumptions. First one is that A is significantly smaller than β, the time discounting factor, so that we can neglect $(A/\beta)^2$ and use an approximation $\sqrt{1 + x} \cong 1 + 0.5\,x$. The theoretical justification for this is that at the beginning of transition the level of uncertainty about future is very high and, therefore, the time discounting factor can be quite large as well. Second assumption is that $\theta > 0.5$ where from utility function (5) θ is the share of consumption relative to the utility of real money balances. Again it seems reasonable to assume that despite liquidity services delivered by real money balances holdings consumer will put a higher weight to consumption, for which he uses his real money balances, hence, θ would be higher than a half.

Bibliography

Agenor, P-R. and Montiel, P.J. (1996) *Development Macroeconomics*, Princeton University Press: Princeton, New Jersey.

Benavie, A., Grinols, E. and Turnovsky, S. J. (1996) “Adjustment costs and investment in a stochastic endogenous growth model”, Journal of Monetary Economics, 38: 77-100.

Bennett, J. and Dixon, H. D. (1996) “A Macrotheoretic Model of the Chinese Economy”, Journal of Comparative Economics, 22: 277-294.

Bhandari, J. S., Nadeen Ul Haque, and Turnovsky, S. J. (1990) “Growth, External Debt, and Sovereign Risk in a Small Open Economy”, IMF Staff Papers, 37 (June): 388-417.

Blanchard, O.J. and Fischer, S. (1993) *Lectures on Macroeconomics*, Ch.4. MIT Press.

Chow, G. (1979) “Optimum control of stochastic differential equation systems”, Journal of Economic Dynamic and Control, 1: 132-175.

Duffie, D. (1992) *Dynamic asset pricing theory* (Princeton, New Jersey: Princeton University Press,), pp. 75-186.

Economic Development of Latvia, Report, Ministry of Economy, RIGA, December 1996.

Feenstra, R.C. (1986) “Functional equivalence between liquidity costs and the utility of money”, Journal of Monetary Economics 17: 271-291.

Fischer, S. (1975) “The demand for indexed bonds”, Journal of Political Economy 83: 509-534.

Grinols, E. and S.J. Turnovsky (1993) “Risk, the Financial Market, and Macroeconomic Equilibrium”, Journal of Economic Dynamic and Control, 17: 1-36.

Grinols, E. and S.J. Turnovsky (1994) "Exchange Rate Determination and Asset Prices in a Stochastic Small Open Economy", Journal of International Economics, 36: 75-97.

Hallwood, C.P. and R. MacDonald (1994) *International Money and Finance*, Ch.9. Blackwell.

Ingersoll, J. E. (1987) Theory of financial decision making (Maryland, US: Rowman & Littlefield), pp. 298-447.

Lehvari, D. and D. Patinkin (1968) "The Role of Money in a Simple Growth Model", American Economic Review 58: 713-754.

Malliaris, A.G. and W.A. Brock (1982) *Stochastic Methods in Economics and Finance*, (Amsterdam: North-Holland).

Meese, R. and K. Rogoff (1983) "Empirical exchange rate models of the seventies: do they fit out of the sample?", Journal of International Economics 14: 3-24.

Merton, R. C. (1990) *Continuous-Time Finance*. (Oxford: Blackwell), Ch.3, 4, 5.

Rebelo, S. and C.A. Vegh (1995) "Real Effects of Exchange-Rate-Based Stabilisation: An Analysis of Competing Theories", NBER Annual 1995: 125-188.

Turnovsky, S. J. (1995) *Methods of Macroeconomic Dynamics*, (Cambridge, Massachusetts: MIT Press), Ch.14.

Turnovsky, S.J. (1993) "Macroeconomic Policies, Growth, and Welfare in a Stochastic Economy", International Economic Review, 35: 953-981.

Van Der Ploeg, F. (1994) *The Handbook of International Macroeconomics*, (Oxford: Basil Blackwell), Ch.11, 12.

BOFIT Discussion Papers

No 1 Tuomas Komulainen: Currency Crisis Theories – Some Explanations for the Russian Case

No 2 Jukka Pirttilä: Tax Evasion and Economies in Transition: Lessons from Tax Theory

No 3 Andrei Yakovlev: Black cash tax evasion in Russia: Its forms, incentives and consequences at firm level

No 4 Barbara Lehmbruch: Managing uncertainty: Hierarchies, Markets and "Networks" in the Russian Timber Industry. 1991–1998

No 5 Rupinder Singh – Juhani Laurila: Azerbaijan: Recent Economic Developments and Policy Issues in Sustainability of Growth

No 6 Iikka Korhonen: Currency Boards in the Baltic Countries: What have we learned?

No 7 Vadims Sarajevs: Macroeconomic Model of Transition Economy: A Stochastic Calculus Approach

BOFIT
Discussion Papers
ISBN 951-686-912-2(print)
ISSN 1456-4564 (print)

ISBN 951-686-913-0(online)
ISSN 1456-5889 (online)

Editor-in-Chief **Jukka Pirttilä**

Bank of Finland Institute for Economies inTransition
(BOFIT)
P.O. Box 160
FIN-00101 Helsinki
Phone: +358 9 183 2268
Fax: +358 9 183 2294
bofit@bof.fi
www.bof.fi/bofit