THE IMPACT OF A COST MINIMISATION USER COST MODEL ON PUBLIC TRANSPORT SUBSIDY

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ABSTRACT :

We show that optimal public transport subsidy is sensitive to the use of alternative user cost models, and that a model based on cost minimisation principles may lead to an improvement in subsidy estimates. For the case of homogeneous consumers and non-peaked demand, the cost minimisation user cost model yields optimal subsidy estimates which differ by up to 55% from those generated by existing models. Multiple optimal subsidy equilibria may also exist arising from a kink in the user cost schedule at a critical frequency and a resulting discontinuity in the marginal benefit of frequency enhancement schedule.

GLOSSARY OF NOTATION

Variables

C_p	total producer costs incurred by the bus operator
d^P	frequency delay
D	frequency delay cost
ES	economic surplus
f	unit frequency delay cost
F	frequency of buses
g	generalised cost of travel
H	mean, or scheduled, headway between buses
I	information cost
L	bus network route-kilometres
MB	marginal benefit of frequency enhancement
MB_p	marginal benefit per potential passenger-trip
MC^P	marginal producer cost of frequency enhancement, $\frac{\partial C_p}{\partial F}$
n	vehicle-kilometres of service provided
OH_H	number of hours per annum over which bus service is provided
P	fare charged
q	quantity of trips undertaken in the bus market
S	subsidy
t_A	departure time of bus immediately after t_p
t_B	departure time of bus immediately before t_p
t_p	the time at which a consumer would prefer a bus departure to occur
U	utility
UC_H	headway related user cost
UC_o	non-headway related user cost
UC_p	user cost under planned behaviour
UC_r	user cost under random behaviour
UC_n	minimum expected headway-related user cost, or effective user cost, for the cost minimisation model (i.e. the smaller of $E(UC_r)$ and $E(UC_p)$)
v_{IV}	marginal value of in-vehicle travel time savings
v_w	unit waiting time cost
w	average waiting time
W	waiting time cost
α	level of potential passenger demand
σ	standard deviation of headway, indicator of bus unreliability
λ	marginal utility of money income

Bus User Cost Models

A	Cost minimisation user cost model
AL	Model A : Low parameter values
AH	Model A : High parameter values
B	Simple random waiting time cost model
C	Dodgson polynomial waiting time cost model

1. INTRODUCTION

Since Mohring (1972) first identified the importance of user costs in the determination of bus subsidy, the user economies of scale argument in favour of public transport subsidy has received increased attention (e.g. Turvey and Mohring, 1975; Jansson, 1979; Evans, 1987; Forsyth and Hocking, 1978; Findlay, 1983). It could be argued that, in the case of urban public transport, this basis for subsidy has become at least as important as the original road congestion argument (see, for example, Glaister and Lewis, 1978 for an exposition of the latter). Firstly, Mohring's argument provides a more direct case for subsidy, relying solely on considerations of the public transport market rather than the related road market. Secondly, given that cross-elasticities of demand between travel by road and public transport are generally low (Dodgson, 1985), considerable road travel demand, and thus congestion, is required before the original subsidy argument becomes significant. In contrast, the Mohring argument turns out to be stronger when travel demand is lower (Jansson, 1980).

Given the significance of the user economies of scale argument, and the key role played in it by frequency related user costs, the way these costs have been modelled in the subsidy literature is of some concern. Tisato (1990) has identified a number of deficiencies in existing models (including the lack of a sound theoretical economic framework and non-recognition of bus unreliability) which raise doubts about their usefulness for the purpose of determining optimal levels of subsidy, and has proposed an alternative cost minimisation model which eliminates these deficiencies. The aim of this paper is to consider the impact on public transport subsidy determination of use of this cost minimisation user cost model.

Although for convenience our discussion will generally be expressed in terms of bus transport, the arguments are relevant to all forms of public transport. In fact, the phenomenon of user economies of scale, and the corresponding case for subsidy, have already been generalised to the even broader category of scheduled transport services (Jansson, 1979), of which bus transport is a subset. Section 2 briefly reviews a number of existing frequency related user cost models and describes the alternative cost minimisation model. Section 3 derives mathematical expressions for optimal subsidy from a welfare maximisation formulation which are then used in sections 4 and 5 to assess the impact of use of the cost minimisation user cost model. Finally, in section 6 some concluding comments and suggestions for further research are presented.

2. HEADWAY RELATED[1] USER COST MODELS[2]

Two types of headway related user cost model can generally be found in the public transport subsidy literature : waiting time models and the frequency delay model. In most studies of public transport subsidy, user costs have been perceived as consisting exclusively of the former, i.e. the cost of time spent by passengers waiting at bus stops for the next bus (Mohring, 1972; Dodgson, 1985; Glaister, 1987; Travers Morgan, 1988).

More recently, frequency delay cost has been used as an alternative model of headway related user cost (Evans, 1987). In this case user costs take the form of activity rescheduling costs incurred by passengers. These costs occur because bus departure times do not generally coincide with the time, t_p, at which the consumer would prefer a bus to depart from the bus stop, thus resulting in an "inconvenience" cost[3] to him. Passengers are inconvenienced because they must select a

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[1] Throughout the paper, it will be more convenient to refer to user costs as "headway" related rather than "frequency" related. We should note however that the two terms are implicitly equivalent and thus can be used interchangeably. Frequency (F) is the number of buses departing from a bus stop over a given interval of time (e.g. one hour). Headway, on the other hand, is the interval of time between successive buses. We will denote the mean, or scheduled, headway as H. If headway is expressed in minutes and frequency in buses per hour, the relationship between them is :

$$F = 60/H \qquad (1)$$

[2] The material in this section is drawn from a more comprehensive discussion of headway related user cost models found in Tisato (1990).

[3] The terms rescheduling cost, inconvenience cost and frequency delay cost can be used interchangeably. From here on we will mainly use the latter to minimise confusion.

rescheduled non-optimal pattern of daily activities as a result of the unavailability of buses at t_p.

Frequency delay cost has also been widely recognised for many years as a key determinant of subsidy in the airline market (Douglas and Miller, 1974; Forsyth and Hocking, 1978; Findlay, 1983; Panzar, 1979). Waters (1982) and Vickrey (1980) have recognised the importance of frequency delay in the case of urban buses. However, both assign only passing reference to it, and do not incorporate it in their models which they base instead on random arrival waiting costs. Overall, therefore, the concept of frequency delay cost has not been widely used in the economic analysis of buses, in contrast to its widespread use in the economic analysis of air transport.

The cost minimisation model developed in Tisato (1990) is a combination of waiting time and frequency delay models. The impact of this alternative model on subsidy determination can best be assessed by comparing the subsidy results it generates with those produced by some of the existing user cost models. Two existing models will be used for this comparison : the random and polynomial waiting time models. A brief summary of the three models for which this comparison will be undertaken now follows.

2.1 Existing Models

The Simple Random Waiting Time Model

The most widely used model in the subsidy literature has been the simple random waiting time model which assumes that passengers arrive at bus stops in a random fashion (for example Mohring, 1972; Turvey and Mohring, 1975; Vickrey, 1980). This model also implicitly assumes that buses run perfectly according to schedule. Average[4] waiting time is then equal to half the headway, and factoring by the unit waiting time cost, v_w, yields the waiting time cost, W :

$$W = v_w.H/2 \qquad (2)$$

The Polynomial Waiting Time Model

More recently, a polynomial function has been used as an indicator of waiting time (Dodgson 1985, 1987)[5], in response to the empirical observation that waiting time ceases to correspond with a random passenger behaviour assumption as service headway increases. The resulting expression for waiting time cost used by Dodgson was :

$$W = (11.39 + 0.49H - 0.00009H^2)/60 \;.\; v_w \qquad (3)$$

where the term in brackets is waiting time, and it and H are in seconds. The model predicts waiting times approximately consistent with random arrivals up to a headway of about 10 minutes, and with non-random arrivals thereafter.

2.2 A Cost Minimisation User Cost Model

The key features of this model are (Tisato, 1990) that it :

(a) considers both frequency delay cost and waiting time cost

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[4] The "average" wait time is the relevant unit because buses are jointly consumed by the passengers they carry. With this public good characteristic, the marginal private user cost is the same for all users and equal to the average user cost.

[5] Non-linear waiting time functions have also been reported for Stockholm by Jansson (1979, 1984).

simultaneously;

(b) explicitly recognises service unreliability as an important determinant of the level of user cost;

(c) is firmly grounded in economic principles, namely, user cost (or disutility) minimisation; and

(d) provides a possible explanation for the empiricly observed switching of passengers between random and non-random behaviour as headway increases.

None of these features can be found in any of the existing models. The lack of some or all of these features constitute the deficiencies of existing models.

2.2.1 Random vs Planned Behaviour : The Choice Facing Consumers

In the cost minimisation model, the consumer is faced with choosing between two modes of behaviour, namely, acting in either a random or a planned manner. The driving force in his decision process is assumed to be a desire to minimise headway related user costs.

An important parameter in the decision process is the level of information held by the consumer. Individuals with imperfect knowledge are assumed to be aware of headway and the degree of bus unreliability, but are not aware of exact bus departure times. Perfect knowledge is attained once the latter is also known (either through experience or by acquiring and using a timetable).

Individuals with imperfect knowledge have no choice, in the immediate term, other than to arrive at the bus stop in a random fashion. Beyond the immediate term, the consumer may switch to planned behaviour by acquiring the necessary information on departure times (and thus have full information). Since this task is generally not costless, there will be an associated information cost (I) of moving from imperfect to perfect knowledge and thus from random to planned behaviour.

2.2.2 User Costs Under Random Behaviour

The average waiting time under random behaviour is given by (Bowman and Turnquist,1981) :

$$w = \frac{H}{2} \left[1 + (\sigma/H)^2 \right] \qquad (4)$$

where H = mean (scheduled) headway

σ = standard deviation of headway.

When bus departures are highly reliable (i.e. $\sigma \rightarrow 0$), w tends to H/2, and (4), factored by v_w, therefore reverts to the simple random waiting time cost model of expression (2).

The consumer perceives that on average he will be subject to this amount of waiting at the bus stop, i.e. the expected wait time. The randomly arriving consumer cannot influence this outcome. Consequently, in order to cost minimise, he must turn his attention to frequency delay costs. The cost minimising consumer will choose to arrive exactly at t_p and thus incur no frequency delay costs.

The total expected user cost under random behaviour ($E(UC_r)$) therefore consists solely of waiting time costs and is given by :

$$E(UC_r) = \frac{H}{2} (1 + (\sigma/H)^2) v_w \qquad (5)$$

2.2.3 User Costs Under Planned Behaviour

Waiting time cost

Based on the work of Bowman and Turnquist (1981), Tisato (1990) shows that planned waiting time cost can be described by the following expression :

$$W = (1.105 H^{0.557} \sigma^{0.324}) v_w \qquad (6)$$

where the expression in brackets is waiting time.

Therefore, waiting time cost will increase with both headway and bus unreliability. Assuming H is constant[6], the expected waiting time will be the same for each bus. The cost minimising consumer will therefore concentrate on frequency delay in order to determine which bus to catch.

Frequency delay cost

The frequency delay (d) associated with any particular scheduled bus is simply the time difference between t_p and the scheduled departure time. If we assume (following Evans, 1987) that unit frequency delay cost, f, is constant and the same for both forward and backward rescheduling, then frequency delay cost (D) is a linear function of d :

$$D = fd \qquad (7)$$

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[6] Given that for most of the time scheduled headway is constant for consecutive pairs of buses, this is not an unreasonable assumption and characterises by far the most frequent situation facing consumers. The obvious exception is of course when we move between peak and off-peak periods where headway does change.

The consumer is faced with a choice between catching one of two buses : the one arriving immediately before t_p, i.e. at t_B, and the one arriving immediately after t_p, i.e. at t_A[7]. He will select the bus which minimises frequency delay cost. If t_p coincides with t_A or t_B, then d and D will each be zero. On the other hand, if t_p is equidistant between t_A and t_B, the consumer will thus incur the maximum frequency delay of H/2 and maximum frequency delay cost fH/2[8]. Assuming that t_p has an equal probability of lying anywhere in the range between t_B and t_A, the consumer's expected frequency delay cost will be :

$$E(D) = fH/4 \tag{8}$$

Total planned user cost

The total planned user cost (UC_p) will consist of the sum of waiting time and frequency delay costs plus any information costs incurred. Therefore, from (6) and (8), and adding I, the expected value will be :

$$E(UC_p) = 0.25Hf + 1.105\ H^{0.557}\sigma^{0.324}v_w + I \tag{9}$$

2.2.4 Cost Minimising Behaviour and Choice

We are now in a position to bring our random and planned user cost sub-models into a unified cost minimisation framework. The consumer will choose between random and planned behaviour so as to minimise his expected total headway related

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[7] All other earlier and later buses must have a higher frequency delay cost and can therefore be ignored given our cost minimisation objective.

[8] These maxima will only lie exactly at t_B+H/2 for the special case assumed here of equal f for both forward and backward rescheduling.

user costs, UC_n, i.e.

$$\min \quad UC_n$$

$$\text{where} \quad UC_n = \text{the smaller of :} \qquad (10)$$

$$E(UC_r) \text{ (see equation (5))}$$

$$\text{and} \quad E(UC_p) \text{ (see equation (9))}$$

Therefore, a cost minimising consumer will either :

(a) act in a random fashion, if $E(UC_r) < E(UC_p)$; or

(b) act in a planned fashion, if $E(UC_r) > E(UC_p)$.

He will be indifferent between the two modes of behaviour when $E(UC_r) = E(UC_p)$.

Consider the situation illustrated in figure 1 which is drawn, for illustrative purposes, for some given non-zero values of σ, f and I. At the lower headways, the $E(UC_r)$ schedule lies below the $E(UC_p)$ schedule, thus random behaviour yields lower user costs and the cost minimising consumer would therefore act randomly. As headway (H) increases, a critical value H_c is reached beyond which $E(UC_r)$ lies above $E(UC_p)$. Therefore, for $H > H_c$, random behaviour costs exceed planned behaviour costs (including information costs), and the cost minimising consumer will switch to planned behaviour. Clearly, he is indifferent between random and planned behaviour when $H = H_c$.

The effective minimum user cost curve (UC_n) is thus ABC, consisting of that part of the $E(UC_r)$ schedule up to the critical headway (H_c), and the $E(UC_p)$ schedule thereafter, with a kink at H_c. The value of H_c will vary between consumers (as the values of f, v_w and I differ from one person to the

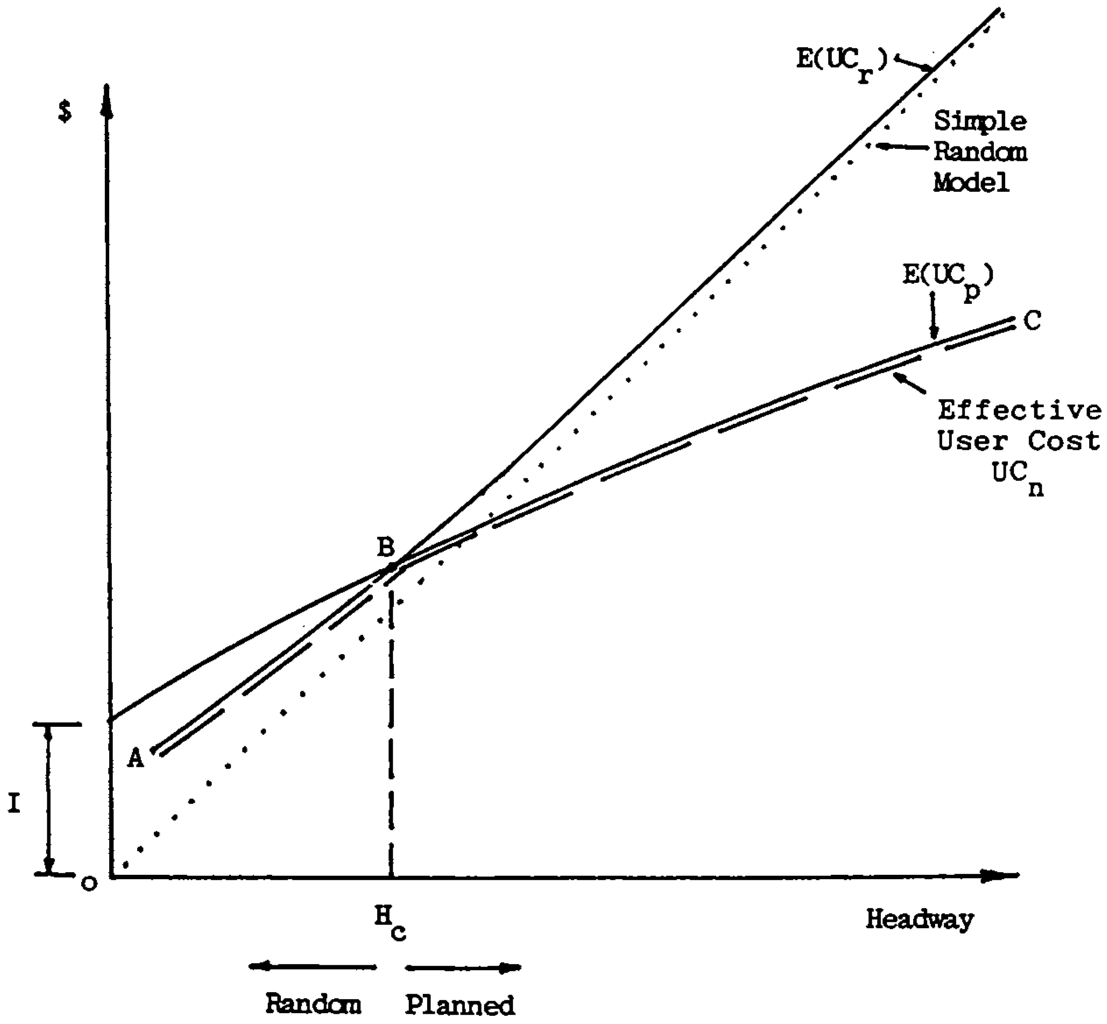

FIGURE 1:

Choice Between Random and Planned Behaviour in the Cost Minimisation Model next) and between situations for the same consumer (as s varies, or as f varies, for example, within the day).

3. THE USER ECONOMIES OF SCALE ARGUMENT FOR SUBSIDY

Most analyses of public transport subsidy have used vehicle-kilometres, n, as an indicator of service frequency and as one of the policy variables to be optimised. Our analysis here differs slightly in that we use frequency (F) directly as a policy variable. The optimisation results are identical for the two approaches if we also assume that the service network is of a given size and density. We also assume that vehicle size is fixed and determined independently and therefore does not form part of the optimisation[9].

For our purposes here, we only require an exposition of the bus subsidy argument which will enable us to evaluate the impact of alternative user cost models, in particular the cost minimisation model. Consequently we have abstracted away from many of the extensions that have appeared in the literature

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[9] This may or may not be considered to be a reasonable assumption depending on how one views the debate over the importance of vehicle size in the determination of optimal subsidy (Jansson, 1979; Walters, 1982). The critical factor in this debate appears to be the nature of the trade-off between economies of vehicle size and economies of vehicle numbers (i.e. of frequency). The appropriate level of subsidy will vary depending on the nature of the trade-off, the existence of any institutional constraints restricting the ability to vary vehicle size and the market regime which operates. Whilst the consideration of vehicle size is important, we consider the fixed vehicle size assumption to be reasonable given that we will be mainly concerned here with a comparative analysis of the impact on subsidy of alternative user cost models rather than the determination of the correct absolute level of subsidy.

and have chosen to present only the key aspects of the user economies of scale analysis.

We undertake our analysis for a first best world where optimal conditions prevail in the rest of the economy with respect to the existence of the Pareto conditions, and public funds can be raised in a costless manner through non-distorting lump sum taxation (and thus there is no opportunity cost of public funds). In addition we assume no peaks in demand for bus travel. Therefore, our analysis, and the subsequent results in sections 4 and 5, are representative of average conditions only. Clearly, the analysis could usefully be extended by relaxing these assumptions.

Let the demand for bus services be given by :

$$q = q(g) \tag{11}$$

where q = the quantity of travel demanded (passenger-trips)

g = the generalised cost of bus travel, ie. the sum of monetary and user costs.

$$= P + UC_H + UC_0 \tag{12}$$

P = the fare paid

UC_H = headway related user costs

UC_0 = other user costs which do not vary with headway, e.g. in-vehicle time costs.

We use this particular formulation for g because it clearly distinguishes between costs that do and do not vary with frequency, thus simplifying the analysis of the benefits of frequency enhancement. In addition, we also assume that the demand curve is downward sloping, i.e. $dq/dg < 0$.

Following Glaister (1987) and Evans (1987), we use an exponential functional form for the demand function :

$$q = \alpha.\exp(-\beta g) \qquad (13)$$

where α = the level of potential demand (passenger-trip units), i.e. the value taken by q when g=0

β = a constant

We also abstract away from passenger congestion effects. Therefore, user costs will not be related to the quantity of travel (q), and thus UC_H will be a function of headway (and thus frequency) only.

The optimal level of subsidy can be determined from the unconstrained welfare maximisation problem. Our analysis is similar to previous expositions (see for example Evans, 1987; Else, 1985). We wish to maximise economic surplus (ES), the sum of consumer surplus (CS)[10] and producer surplus (PS), i.e.

$$\max \qquad ES = CS + PS \qquad (14)$$

$$= \int_g^\infty q.dg + (Pq - C_p) \qquad (15)$$

where C_p = costs to the producer, and are assumed to vary with both the quantity of travel (q) and frequency (F), i.e.

$$C_p = C_p(q,F) \qquad (16)$$

[10] We make the usual assumption that the area to the left of the Marshallian demand curve is an adequate measure of consumer surplus.

We also assume that there are constant returns to scale in producer costs. Therefore, C_p is homogeneous of degree 1 and by Euler's theorem :

$$C_p = \frac{\partial C_p}{\partial F}.F + \frac{\partial C_p}{\partial q}.q \qquad (17)$$

and $\frac{\partial C_p}{\partial F}$ and $\frac{\partial C_p}{\partial q}$ are constants.

We now derive the first order conditions for maximising ES with respect to P and F, the two policy variables. Setting $\frac{\partial ES}{\partial P}$ equal to zero :

$$\frac{\partial ES}{\partial P} = \frac{\partial}{\partial g}.\int_g^{\infty} q.dg.\frac{\partial g}{\partial P} + q + P.\frac{\partial q}{\partial P} - \frac{\partial C_p}{\partial P} = 0 \qquad (18)$$

We note that $\frac{\partial g}{\partial P} = 1$, thus :

$$-q.1 + q + P.\frac{\partial q}{\partial P} - \frac{\partial C_p}{\partial q}.\frac{\partial q}{\partial P} = 0 \qquad (19)$$

Therefore :

$$P = \frac{\partial C_p}{\partial q} \qquad (20)$$

This is the general expression for the optimal fare, i.e. it must be set equal to the marginal cost to the producer of an additional bus trip.

Setting $\frac{\partial ES}{\partial F}$ equal to zero :

$$\frac{\partial ES}{\partial F} = \frac{\partial}{\partial g}.\int_g^{\infty} q.dg.\frac{\partial g}{\partial F} + P.\frac{\partial q}{\partial F} - \frac{\partial C_p}{\partial F} - \frac{\partial C_p}{\partial q}.\frac{\partial q}{\partial F} = 0 \qquad (21)$$

Substituting for P from (20), and noting that $\frac{\partial g}{\partial F} = \frac{\partial UC_H}{\partial F}$:

$$-q.\frac{\partial UC_H}{\partial F} + \frac{\partial C_p}{\partial q}.\frac{\partial q}{\partial F} - \frac{\partial C_p}{\partial F} - \frac{\partial C_p}{\partial q}.\frac{\partial q}{\partial F} \qquad (22)$$

thus :

$$-q.\frac{\partial UC_H}{\partial F} = \frac{\partial C_p}{\partial F} \qquad (23)$$

Expression (23) indicates that we require equality between the marginal benefit of frequency enhancement (the LHS of (23)) and the marginal cost to the producer of its provision (the RHS of (23)).

Equation (23) can be expressed in a more useful form by stating $\partial C_p/\partial F$ in terms of $\partial C_p/\partial n$ (where n is the vehicle-kilometres of service). In a public transport network of fixed routes and unduplicated length L route-kilometres, the number of vehicle-kilometres of service provided over OH_H hours is given by (Dodgson,1985) :

$$n = F.2L.OH_H \tag{24}$$

Thus :

$$\frac{\partial n}{\partial F} = 2L.OH_H$$

and since

$$\frac{\partial C_p}{\partial F} = \frac{\partial C_p}{\partial n}.\frac{\partial n}{\partial F}$$

then

$$\frac{\partial C_p}{\partial F} = \frac{\partial C_p}{\partial n}.2L.OH_H \tag{25}$$

Substituting (25) into (23) yields :

$$-q.\frac{\partial UC_H}{\partial F} = \frac{\partial C_p}{\partial n}.2L.OH_H \tag{26}$$

It is important to note that in this expression, demand (q) is also defined over the period OH_H hours.

Subsidy (S) is given by :

$$S = k.(C_p - Pq) \tag{27}$$

where $k = 1$ when $Pq < C_p$

0 when $Pq > C_p$

Substituting (20) and (17) into (27) :

$$S = k.\frac{\partial C_p}{\partial F}.F \tag{28}$$

or alternatively, substituting (25) into (28) :

$$S = k(\frac{\partial C_p}{\partial n}.2L.OH_H.F) \tag{29}$$

Therefore, the optimal subsidy will equal the fixed cost component of producer costs.

In the next two sections we use the above expressions to quantify the impact of alternative user cost models on subsidy.

4. SUBSIDY DETERMINATION : COST MINIMISATION MODEL

4.1 Marginal Benefit per Potential Passenger

In order to fully understand some of the outcomes in the analysis of subsidy, it is necessary to first consider how the marginal benefit of frequency enhancement behaves. The left hand side of equation (26) is the marginal benefit (MB) of frequency enhancement. Since the marginal cost of frequency enhancement (the right hand side of (26)) is constant at any given point in time, determination of optimal frequency and subsidy levels therefore hinges critically on MB.

In considering how MB behaves, our task is simplified by removing the influence of the underlying level of potential demand, α[11] (see expression (13)). We thus consider the marginal benefit per potential passenger-trip (MB_p) where :

$$MB_p = \frac{MB}{\alpha} = \frac{q}{\alpha} \cdot -\frac{\partial UC_H}{\partial F} \qquad (30)$$

where q/α = the propensity to travel.

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[11] We use potential demand here rather than actual demand because the latter is directly influenced by the policy choice variables (P and F) whereas the former is not and is thus more effective at normalising MB in an unbiased manner.

Figure 2 plots MB_p vs F for the cost minimisation model for three combinations of values of f and σ (i.e. (0,1), (1.5,2) and (3,3))[12]. There are two key underlying features. Firstly, in each case, a discontinuity exists at the critical frequency, F_c[13]. The discontinuity, which appears as a sharp jump in the value of MB_p, is a direct result of the change in slope at the kink in the user cost (UC_n) vs H schedule at H_c[14] (see figure 1) as behaviour switches between random and planned. For each combination of f and σ, F_c takes a different value. As the values of f and σ increase, there is a corresponding increase in H_c (Tisato, 1990) and therefore a decrease in F_c.

The second feature is that (outside the discontinuity) MB_p declines as F increases, which implies, not unexpectedly, that there are diminishing returns to frequency enhancement.

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12 These values were chosen to represent gradually increasing parameter values subject to the requirement that they all lay within the range of feasible values suggested in appendix A.

13 From (1), we can note that $F_c = 60/H_c$ where H_c is the critical headway (see section 2.2.4). Therefore, at F_c behaviour switches from planned to random as F increases.

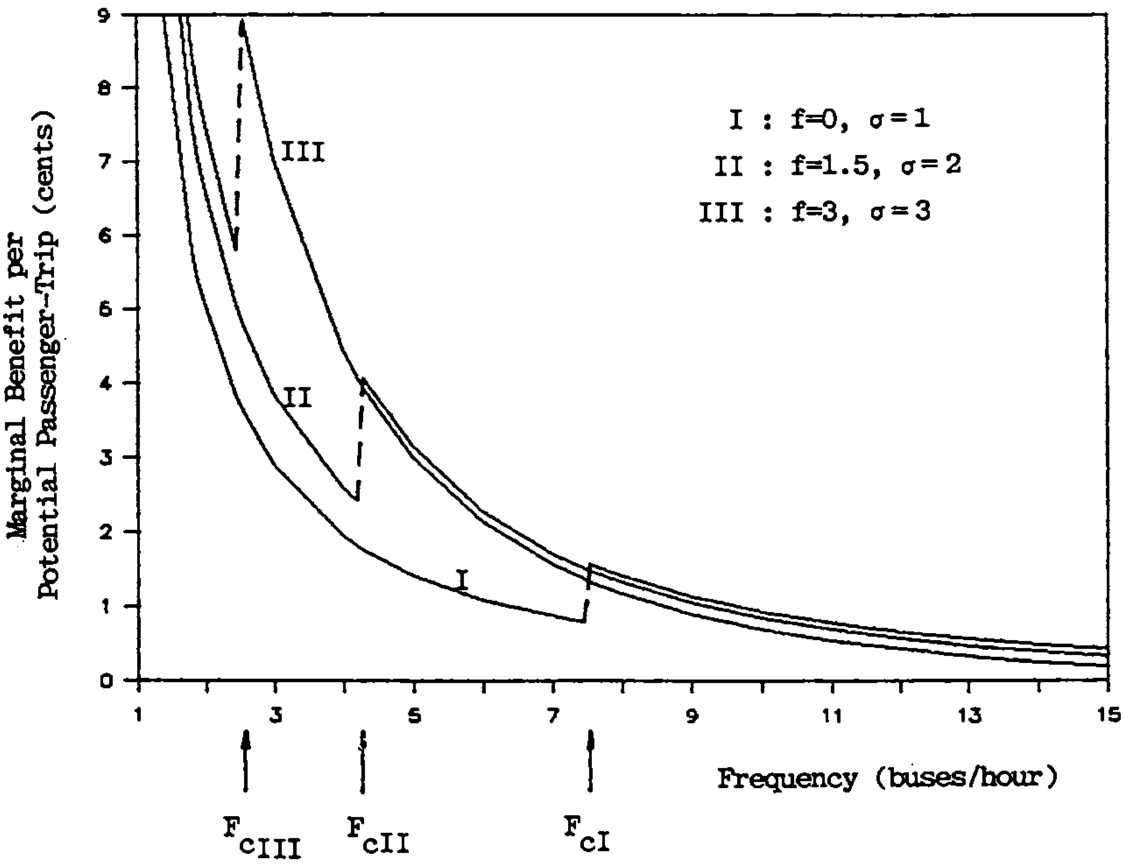
14 Expression (30) can also be rewritten as :

$$MB_p = \frac{q}{\alpha} \cdot \frac{-\partial UC_H}{\partial H} \cdot \frac{\partial H}{\partial F}$$

From (1), $\frac{\partial H}{\partial F} = \frac{-60}{F^2}$,

therefore, $$MB_p = \frac{q}{\alpha} \cdot \frac{60}{F^2} \cdot \frac{\partial UC_H}{\partial H} \qquad (31)$$

and thus for any given value of F, MB_p is proportional to $\frac{\partial UC_H}{\partial H}$.



NOTE : f = unit frequency delay cost (cents/min)
σ = standard deviation of headway (mins)

FIGURE 2 :

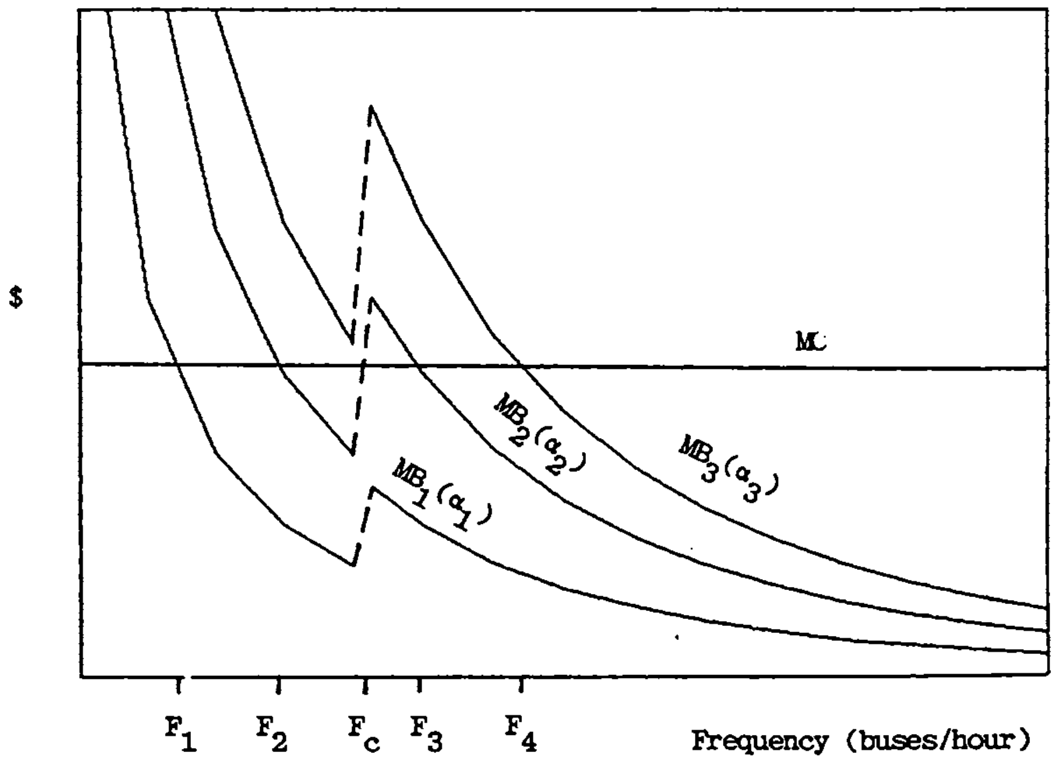
Marginal Benefit per Potential Passenger-Trip
for the Cost Minimisation Model

There is another interesting feature in figure 2. At the lowest frequencies, the highest MB_p values are predicted for the highest parameter value case. However, at the highest frequencies, this trend is reversed, with the highest parameter value case now predicting the lowest MB_p values. This can be explained with reference to the behaviour at low and high frequencies of $\partial UC_H/\partial H$, the slope of the UC_n schedule (see figure 1) and a key determinant of MB_p (see footnote 14). At high frequencies (i.e. low headways), higher values of σ result in a flatter UC_n schedule, and thus lower MB_p. At low frequencies (i.e. high headways), higher values of f and σ yield a steeper UC_n schedule and thus higher MB_p. This feature is relevant to explaining the behaviour of optimal subsidy under the cost minimisation model in section 4.3 .

4.2 Equilibrium

The discontinuity in the MB schedule at F_c produces some interesting multiple optimisation equilibria results. As a result of the discontinuity, multiple maxima in economic surplus may arise at different levels of frequency.

We limit our considerations here to the case of homogeneous consumers. Each person will have identical values of f, v_w and I and thus F_c will take the same value for all consumers. Figure 3 shows the marginal cost of frequency enhancement (MC), and MB schedules (for given values of f and σ) as potential demand level (α) is allowed to vary. As α increases, the MB curve moves upwards.



NOTE : α = Potential Demand (passenger-trips/min)

$\alpha_1 < \alpha_2 < \alpha_3$

<u>FIGURE 3</u> :

Marginal Benefit vs Marginal Cost : Cost Minimisation Model

An equilibrium will occur wherever MB = MC, at which point economic surplus will reach a maximum (local and possibly global). Cases with a single equilibrium only will exist whenever MC does not pass through the discontinuity in the MB schedule, and thus MB and MC only equate once. For example, when $\alpha = \alpha_1$ or α_3 in figure 3, an equilibrium exists at F_1 and F_4 respectively. Equilibria at values of $F < F_c$ (e.g. F_1) coincide with planned behaviour whilst those at values of $F > F_c$ (e.g. F_4) coincide with random behaviour.

Now consider the existence of multiple equilibria. For any given value of MC, a range of values of α exist for which MC passes through the discontinuity in MB resulting in MB and MC equating twice at different levels of frequency (and thus two local maxima in economic surplus). This occurs for the following reason (- consider for example the case of $\alpha = \alpha_2$ (i.e. MB_2) in figure 3). Starting at low frequencies, we see that as frequency increases, MB declines progressively (i.e. we move along the MB schedule). MB eventually drops below MC for $F > F_2$. When we reach F_c, as a result of the discontinuity in the MB schedule, MB rises above MC once again. As frequency increases further, MB once again declines progressively and eventually drops below MC a second time for $F > F_3$. As a result of this process we have two local equilibria, at F_2 and F_3[15]. For this example ($\alpha = \alpha_2$), the equilibrium at F_2 coincides with planned behaviour (since $F_2 < F_c$), whilst that

[15] If the analysis were extended to the heterogeneous consumers case (and thus more than one value of F_c existed in the market) the number of local equilibria will exceed two. In the simplest case, where there are two F_c values, three local equilibria will result (Tisato, 1990).

at F_3 displays random behaviour (since $F_3 > F_C$). Twin equlibria will occur every time that MC passes through the discontinuity in MB.

Although the difference between economic surplus at the two maxima may in some cases be small, the existence of multiple equilibria is an interesting and important feature. It suggests that an analyst or policy maker trying to identify the frequency which maximises economic surplus must be aware that there may be multiple equilibria. In selecting a welfare maximising outcome, a comparison of both equilibria must be undertaken. At the very least this will identify whether behaviour at the global optimum is random or planned. It is also worth noting that the two local equilibria could yield very similar economic surplus values yet one occurs at considerably higher frequency and, from (29), requires considerably higher subsidy. Therefore it may be the case that to increase economic surplus by a small amount may require, in some cases, a considerable increase in frequency and subsidy.

4.3 Optimal Subsidy

Once the issue of multiple equilibria has been resolved, a single global optimum frequency (F_O) will be known for each level of potential demand (α). Then, restating expression (29) for k = 1 we have :

$$S = \frac{\partial C_p}{\partial n}.2L.OH_H.F \qquad (32)$$

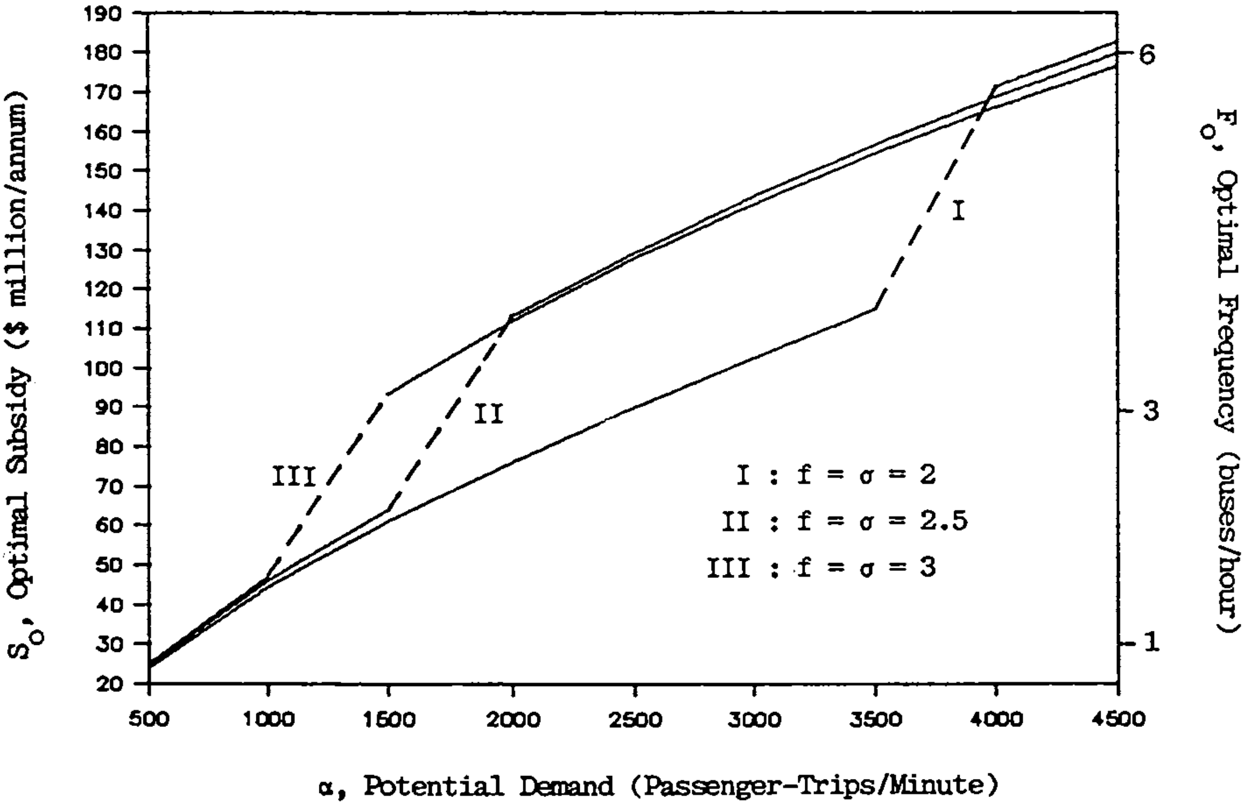
Given that $\partial C_p/\partial n$, L and OH_H are constant at any point in time, optimal subsidy, S_O, will therefore change proportionally with optimal frequency, F_O. The plots of F_O and

S_o against α (see figure 4) therefore have an identical pattern, differing only by the factor $\partial C_p/\partial F$ (see (28)). Figure 4 plots these curves for 3 cases of increasing parameter values : $f = \sigma = 2$, 2.5 and 3 respectively.

The key feature of figure 4 is the sudden increase in S_o once α reaches a particular level. At the lowest values of α, the MB schedule will be fairly low and therefore equilibrium will occur at low F values on the planned behaviour segment of the schedule (i.e. below F_c). As α increases, the MB schedule is lifted upwards (as we saw in figure 3) and F_o increases steadily. As α continues to increase, the global equilibrium will eventually switch to the random part of the MB schedule (i.e. above F_c). As this happens, the resulting sudden jump in F_o will also result in a sudden jump in S_o. Therefore, that part of the S_o schedule below the jump corresponds to planned behaviour whilst that above the jump corresponds with random behaviour.

Figure 4 is plotted for increments of 500 in α. As a result, the jump in S_o only appears to occur between consecutive increments of 500. In reality the jump would be much sharper, occuring at a particular value of α.

The location of the jump will vary depending on the parameter values used. The higher the values of f and σ, the lower will be the values of α at which the jump occurs. This is due to the impact of changes in f and σ on the location of the MB discontinuity. The higher the values of f and σ, the higher up on the MB schedule will one find the discontinuity,



NOTE : f = unit frequency delay cost (cents/min)
σ = standard deviation of headway (mins)

<u>FIGURE 4</u> :

Optimal Frequency and Optimal Subsidy :
Cost Minimisation Model

and therefore the closer one would be to the MC line. Consequently, the smaller will be the increase in α required to get us to a global equilibrium with random behaviour.

Another feature is that at low α values the highest parameter value case predicts the highest S values, whilst the reverse is true at high α values where the highest parameter value case predicts the lowest S values. This follows directly from the similar feature found for MB in figure 2 and discussed in section 4.1 .

A final feature of figure 4 is that (outside the vertical jumps) as α increases there is a corresponding increase in F_0 and S_0 but at a diminishing rate. So subsidy per passenger-trip will decline as α increases.

5. SUBSIDY DETERMINATION : MODEL COMPARISON

The impact of using the cost minimisation user cost model in the determination of optimal subsidy was assessed by comparing the results it generated with those of the two existing models summarised in section 2.1 . A summary of the parameter values used in this analysis is presented in Table 1 below. A discussion on the derivation of these parameter values is contained in appendix A.

We will label the user cost models as follows :

A : The Cost Minimisation model (UC_n, expression (10))

B : The Simple Random Waiting Time Cost model (expression (2))

C : The Polynomial Waiting Time Cost model (expression (3))[16].

TABLE 1 : SUMMARY OF PARAMETER VALUES

Value of in-vehicle travel time savings, v_{IV} (cents/minute)		4.6
Unit waiting time cost, v_w (cents/minute)		9.2
Information cost, I (cents/trip)		5.0
"Other" user costs unrelated to frequency, UC_0 (cents/trip)		147
Constant in demand function, β		0.006
Network length, L (kilometres)		1000
Annual hours of operation, OH_H		5000
Level of potential demand, α (passenger-trips/minute)	1000 →	4500
$\partial C_p/\partial n$ (cents/vehicle-kilometre)		300
$\partial C_p/\partial q$ (cents/passenger-trip)		0
	Low	High
Unit frequency delay cost, f (cents/minute)	0	4.6
Standard deviation of headway, σ (mins)	0.5	4.0

We have in turn split model A into two sub-models (A Low (AL) based on low values for both f and σ, and A High (AH) with high values for f and σ) reflecting the variation in the values of f and σ outlined in Table 1.

Figure 5 plots the relationship between S_o and F_o and α for the three models. The relative position of the curve for each model is a direct reflection of the relativities in the MB values between models within the optimal frequency range, which figure 5 shows is up to approximately 6 buses/hour for α up to 4500[17]. The plot is drawn for the range α = 1000 to 4500 for simplicity so that AH corresponds solely with random

[16] The polynomial model had to be slightly modified since user cost peaked at H = 45 mins and declined thereafter. We assumed that user cost would remain constant for H > 45 .

[17] These relativities do change as F increases beyond this range. In particular, subsidy for model AH drops below that of model C and eventually below AL as F increases further due to the effect of high bus unreliability (Tisato, 1990).

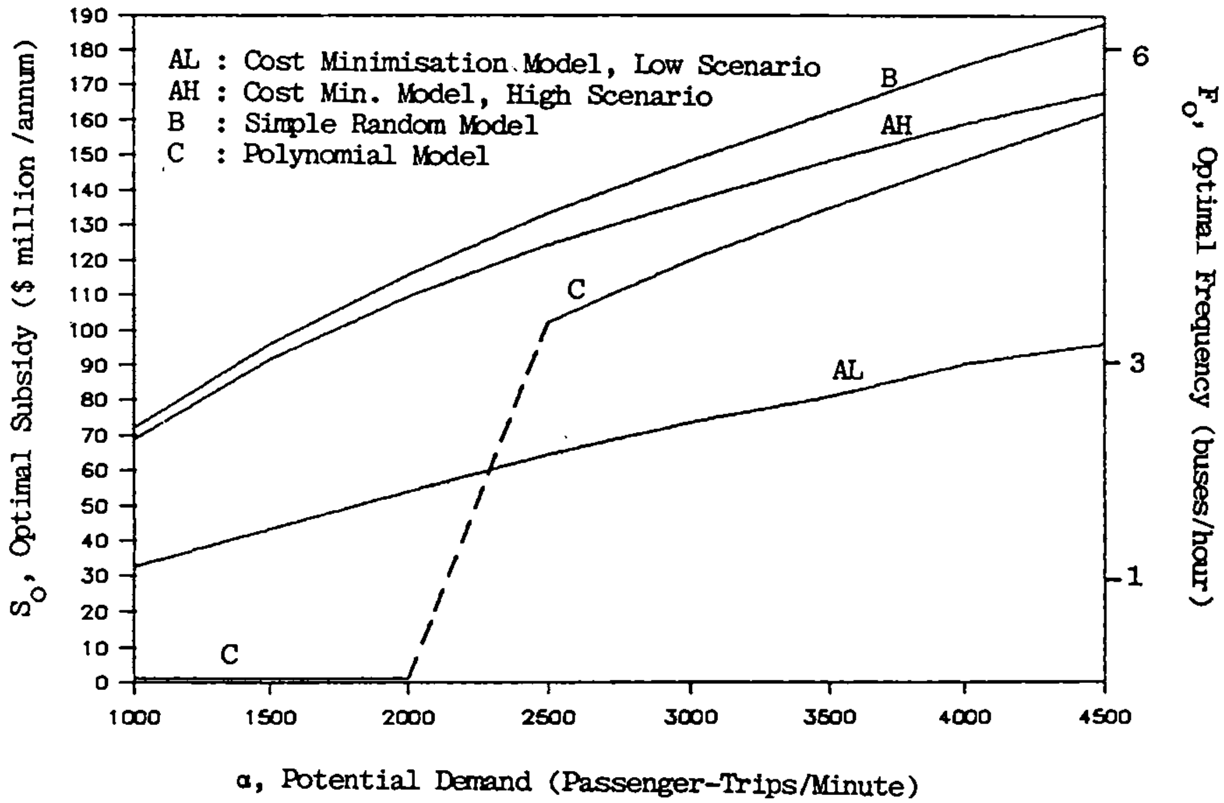


FIGURE 5 :

Optimal Frequency and Optimal Subsidy :
All User Cost Models

behaviour and AL solely with planned behaviour. This enables us to demonstrate the scope for variation in subsidy under the two types of consumer behaviour.

The simple random model (B) generates the highest subsidy (S) values. Model AH produces slightly lower S values than model B, whilst AL produces the lowest S schedule. Model C generates a zero value for S at low values of α due to the fact that at low frequencies it yields zero MB (as a result of footnote 16). When it does produce positive S values, model C lies between models AH and AL. Clearly, the two sets of parameter values used here for model A (AL and AH) produce considerable variation in optimal subsidy relative to the other models. Table 2 summarises the percentage differences between model A and the existing models, with differences of up to 55% being observed.

TABLE 2 : PERCENTAGE DIFFERENCES IN SUBSIDY BETWEEN MODELS

	relative to model B		relative to model C	
α	AL	AH	AL	AH
1000	-54	-4	*	*
1500	-55	-5	*	*
2000	-53	-5	*	*
2500	-52	-7	-37	22
3000	-51	-8	-39	14
3500	-50	-8	-40	10
4000	-49	-9	-39	7
4500	-49	-10	-41	4

Notes : 1. A -ve(+ve) value implies model A's subsidy estimate is below(above) that of the model with which the comparison is being made.

2. * - cannot be calculated since model C predicts zero subsidy at this level of α.

It appears clear that the selection of alternative user cost models can have a significant effect on the determination of optimal subsidy. The random model (B) generates results at the upper limit of the possible subsidy result spectrum, whilst the polynomial model (C) is more conservative. The cost minimisation user cost model (A), on the other hand, predicts a range of subsidy outcomes depending on the value of the key parameters unit frequency delay cost (f) and bus unreliability (σ).

6. CONCLUDING COMMENTS

Our main task throughout this paper has been to determine the impact of a cost minimisation user cost model on the determination of optimal public transport subsidy.

Our analysis has shown that the use of alternative user cost models has a considerable impact on the determination of optimal levels of frequency and subsidy. The subsidy results generated by the cost minimisation model differed from those predicted by other models by between 4% and 55% depending on the parameter values selected and the level of potential demand. It follows that the selection of an accurate user cost model is a critical element in subsidy determination. Given that the cost minimisation model has a stronger theoretical basis than existing models, one can conclude that its use may lead to an important improvement in the estimates of optimal public transport subsidy.

The analysis presented here has been adequate to demonstrate the potential of the cost minimisation user cost model. However, further work is required before truely policy relevant subsidy figures could be generated. This would include rigorous empirical estimation of the model (Tisato, 1990) and relaxation of some of the assumptions which have been made including introducing peak loads in demand, congestion effects amongst passengers, recognising that raising public funds for subsidy is not costless but involves an opportunity cost and that subsidy may have a negative impact on production efficiency.

APPENDIX A : PARAMETER VALUE DETERMINATION[a1]

A.1 Marginal Value of In-Vehicle Travel Time Savings, v_{IV} - This is a key parameter because v_w and f will be expressed in terms of it. All in-vehicle time spent on public transport was assumed to be valued as non-working time (Dodgson, 1985). We have used a value of time savings of 4.6 cents/minute (1988 prices) from survey work undertaken in Perth (Director-General of Transport, Western Australia, 1976). Starrs (1984) based her analysis of Adelaide public transport on similar values.

A.2 Unit Waiting Time Cost, v_w - Following common practice (BTE, 1982; Truong and Hensher, 1985), we have set v_w to double the value of v_{IV} (i.e. 9.2 cents/minute).

A.3 Unit Frequency Delay Cost, f - Estimates of f were derived by considering the relationship between it and the value of in-vehicle time savings (for which empirical estimates are widely available) using the theory of time allocation (Tisato, 1990). It is possible to show, using the theory of time allocation, that :

$$f = \frac{(1 - x_0)}{(1 - y_0)} \cdot v_{IV} \qquad \text{(A1)}$$

where x_o = the ratio of the marginal value of time spent in rescheduled activities to the value of leisure time

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[a1] It was beyond the scope of this research to undertake detailed and rigorous testing of parameter values. The values selected were judged to be of the correct order of magnitude and were thus adequate for the task at hand (i.e. undertaking a comparative analysis of subsidy for alternative user cost models). The discussion in this section is a summary of the material in Tisato (1990).

and y_o = the ratio of the value of in-vehicle travel time to the value of leisure time.

We would normally expect x_0 to be positive and y_0 to be negative. We assumed that y_o might reasonably fall within the range 0 to -0.5 and x_o in the range 0 to 1. These ranges generate the following low and high values for f :

	low	high
x_0	1	0
y_0	-0.5	0
$(1-x_0)$	0	1
$1/(1-y_0)$	0.67	1.0
f	0	1.0 v_{IV}

With v_{IV} = 4.6 cents/min, we have chosen final low and high values for f of 0 and 4.6 cents/min.

A.4 Unreliability of Buses, σ - The values used for σ were those generated by variation in road congestion, which, by its nature, is outside the control of the bus operator. We assume that additional increments in σ which are caused by the operator's inefficiencies are reduced by appropriate supply side measures. The effect of road congestion on bus unreliability was assessed using the following road travel time function (Davidson,1978)

$$t_t = t_f \cdot \frac{(1 - mx_1)}{(1 - x_1)} \qquad \text{(A2)}$$

where t_t = travel time per unit distance

t_f = travel time under zero traffic flow conditions

x_1 = the ratio of traffic volume to road capacity and is an indicator of congestion

m = a factor which varies with road type

The variation in road travel time, δt_t, will be:

$$\delta t_t = \frac{\partial t_t}{\partial x_1}.\delta x_1 \qquad \text{(A3)}$$

where δx_1 = the variability in volume/capacity ratio, and thus road congestion.

From (A2) and (A3) we obtain :

$$\delta t_t = \frac{t_f(1 - m)}{(1 - x_1^2)}.\ \delta x_1 \qquad \text{(A4)}$$

The parameter values which we used to determine δt_t were :

	low	high
m	0.82	0.88
$s_f (=1/t_f)$	40	50
δx_1	0.1	0.2

The units of s_f (i.e. speed under zero traffic flow conditions) are kilometres/hour, whilst m and δx_1 are unitless. The values for m, and s_f were based on the work of Dodgson(1985) and Starrs(1984). Data was not readily available on δx_1. The values chosen seem intuitively reasonable for the purpose of comparative subsidy analysis. Using these values, the resulting low and high values for σ were approximately 0.5 and 4 mins respectively.

A.5 Information Cost, I - No direct data was available on information cost, I. A value was selected on the basis of ensuring consistency with the empirical findings on the headway at which behaviour makes the transition from random to planned, i.e. critical headway H_c (see section 2.2.4). Except for the higher values of f and σ, a value of I = 5 cents/trip produces outcomes which are fairly consistent with the limited empirical evidence (see Seddon and Day, 1974; and Bowman and

Turnquist, 1981). A figure of I = 5 cents/trip was therefore adopted.

A.6 "Other" Non-Headway Related User Costs, UC_0 - This comprises 2 costs: walk time, and in-vehicle time. As is the case for waiting time, the value of walking time savings is conventionally valued at twice in-vehicle time savings. Therefore :

$$UC_0 = wk.2.v_{IV} + t_{IV}.v_{IV} \qquad (A5)$$

where wk = walk time (mins)

t_{IV} = in-vehicle time (mins)

Assuming a 25 km/hr average bus speed (State Transport Authority, 1989), a 9 km average trip length (for Australian cities (Australian Government, Commonwealth Grants Commission (1988)) and a 5 minute walk time, then UC_0 = 147 cents (1988 dollars).

A.7 Constant in Demand Function, β - For the exponential demand function (13)

$$\varepsilon_p = -\beta P \qquad (A6)$$

where ε_p = own price elasticity of demand

P = the fare charged

A widely used value for ε_p is -0.3 (Transport and Road Research Laboratory, 1980). Using a value for average fare level of 50 cents/trip (approximate average for the 5 largest Australian cities in 1987/88) the resulting value of β is 0.006.

A.8 Network Length, L - A value of 1000 km was used since it simplified calculations and it was approximately the median value for the five largest Australian cities.

A.9 Annual Hours of Operation, OH_H - Following Dodgson (1985) we adopted a figure of 5000 hours.

A.10 Potential Passenger Demand Level, α - α values up to 4500 passenger-trips/minute were considered. The choice of this upper value is somewhat arbitrary. However, to put this figure in perspective, the largest Australian city (Sydney) had a value of 3100 in 1987/88. So our results at least cover Australian cities.

A.11 $\partial C_p/\partial n$ - It was assumed that $\partial C_p/\partial n$ was reasonably approximated by the average cost per vehicle-km (C_p/n). A representative value for the five largest Australian cities was around $3.00 per veh-km in 1987/88.

A.12 $\partial C_p/\partial q$ - We assume that $\partial C_p/\partial q$ is zero. In reality a small positive value could be expected, but it was felt that its exclusion would not affect results significantly.

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