Job Mix, Performance Pay, and Matching Outcomes: Contracting with Multiple Heterogeneous Agents

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Abstract

We examine the problem of designing performance contracts with multiple agents when principals must compete for quality teams from a heterogeneous pool of agents. The trade-off principals face between good recruiting and good team performance provides micro foundations for agents to form stable matches, and for initially identical principals to adopt different organizational schemes. The equilibrium pattern of team formation exhibits two distinct, and inversely related, forms of assortative matching. We find that a greater share of principals offering diverse performance incentives across teammates (extensive margin), leads to a lesser degree of heterogeneity in abilities within teams on average (intensive margin). We apply the model to firm behavior to examine the mix of jobs offered and the degree of performance pay in a general equilibrium environment. At the aggregate level, increases in the supply of high-skilled workers leads to a polarization of jobs offered, i.e. relatively greater use of high- and low-skill occupations, consistent with changing labor demands in recent history. Moreover, skill accumulation among the labor force induces more firms to offer a steep set of performance contracts.

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Introduction

Many principal-agent relationships begin at recruitment so that principals are concerned with attracting high-quality teams of agents, as well as eliciting optimal performance from the team during production. Firms are obvious examples since they must compete with one another for a high-ability workforce, and then write incentive contracts that allow them to be competitive in product markets. In general equilibrium, firms need to manage a trade-off between contracts that lead to good recruiting and contracts that lead to good team performance. We examine the contracting problem with multiple agents, where agents are heterogeneous and form teams (matches) voluntarily. Principals have organizational choices regarding how to manage the incomplete information environment by directing the efforts of agents toward success. Put differently, principals can offer different job mixes to balance the incentives of agents to form stable matches and exert effort. We apply this principal-agent framework to the contracting problems faced by firms and explore how changing labor and product market conditions alter the provision of performance pay, the mix of occupations offered and matching outcomes among the labor force.

A central thesis of this analysis is that identical principals may endogenously select different organizational forms when balancing recruiting and performance incentives. The availability of micro-data has revealed flexibility in both the composition of labor demands and the provision of performance payments within individual firms. Cuñat and Guadalupe (2009) link firm provision of incentive contracts among US manufacturing firms to recent waves of globalization, while Guadalupe and Wulf (2010) and Bresnahan *et al.* (2002) demonstrate that labor demands within firms have shifted across skills and occupations. Understanding the economic forces which drive these patterns, particularly at the micro-level, requires a framework with multiple-agent contracting, agent heterogeneity and organizational choices available to principals.

While we are interested primarily in the organization of individual firms, we consider the contracting problem in a competitive environment for two reasons. First, principals balance the incentives for quality performance, within the firm, against the incentives of the labor force, outside the firm, to form production teams. A competitive matching market for agents best describes the outside forces that principals must consider. Second, the organizational flexibility of firms has manifested into aggregate labor market trends in recent decades. Many countries have witnessed job polarization - jobs which require the highest, and lowest, skills have become larger shares of total employment, while demand for moderate skill levels has waned[1] -

[1] This polarization of the work force is observed across several developed countries: see Goos and Manning (2007), Autor, Katz and Kerney (2006), Goos, Manning and Salomons (2009), and Autor and Dorn (2009). At the aggregate level, some have argued for a hybrid of forces as determinants of polarizing labor demands such as globalization plus non-neutral technological change. Goos, Manning, and Salomons (2009) find weak evidence that routine moderate-skilled tasks are being offshored by European economies while technologies have shifted towards use of workers performing non-routine tasks. See also Autor, Levy and Murnane (2003). Feenstra and Hanson (2003) and Trefler and Zhu (2005) argue that offshoring and trade with developing countries has lead to changes in the specific factor services demanded and rising inequality.

and an increasing share of the labor force receiving some version of performance-pay[2]. In order to connect these aggregate trends to firm-level adjustments, the principal's contracting and organizational problems must be considered in a competitive environment[3].

The framework developed here embeds a familiar principal-agent relationship in to a general equilibrium setting. Principals are concerned with writing contracts that allow them to be competitive in both input and output markets. Our approach uses the insights from Lazear (2000) that the provision of performance-pay brings productivity gains largely through better recruiting of workers. Principals recruit a team of agents from a pool with a continuum of abilities without costs or frictions. Yet, the agents recruited must agree to be matched and assigned to their respective jobs. Information problems inhibit agents from conveying efforts to their supervising principal, though agents who exert more effort send better signals of performance.

Principals must decide how to reward (potentially) heterogeneous agents in an attempt to elicit optimal effort. Each principal also has options regarding how to interact with agents during production. A principal can offer a relatively diverse mix of jobs and interact with a foreman more closely, or interact with each member of the team similarly. Workers elected as foremen of their team are able to send more reliable signals of their performance, while constraints on the ability of principals to exchange information dictate that the efforts of the foreman's counterpart are generally less productive. Interaction with a worker leads to proportional increases in signal quality, favoring high skilled workers more heavily. Thus the two organizational choices of job mix are equivalent to offering either a pair of intermediate-skill occupations, or one high-skill and one low-skill job in each firm.

The choice of job mix is aimed at optimizing recruiting outcomes. To understand how principals can improve recruiting by altering the composition of jobs they offer, consider team formation by agents. Agents form matches voluntarily and must compete for the best partners and best jobs. When motivating a team of heterogeneous agents, principals optimally adopt a wage scheme that rewards team successes more intensively[4]. In response of each agent seeks out the highest skilled partner available to raise the possibility of earning team rewards. As a result the equilibrium pattern of team formation is characterized by positive

[2] See Lemieux, MacLeod, and Parent (2009)

[3] The distinction between firm-level and aggregate labor market outcomes is important, as Dunne *et al.* (2004) find that wage inequality rose across US manufacturing firms between 1970 and 1992, while wage dispersion fell or remained constant within firms over the same period. To be specific, within-firm wage dispersion of all workers, and of production workers, exhibited no trend over time, while wage dispersion among non-production workers fell.

[4] In equilibrium all matches will be between agents of different ability. We present the simplest version of the principal-agent relationship where agents work on independent tasks simultaneously. This environment contains the weakest incentive for assortative matching or heterogeneous organizations to arise in equilibrium because there are no strategic interaction built into the production process. In essences we are stacking the deck against ourselves. One may prefer to model production at the firm such that a foreman takes a leadership role, and the remaining agent chooses her effort level in response. Though a hierarchy is a more intuitive representation of the firm production process, the qualitative results are reenforced by the coordination of agent efforts. For simplicity, and to avoid the results driven only by exogenously given production processes, we focus on simultaneous actions over independent tasks.

assortative matching: relatively high ability agents paired together and relatively low ability agents matched together.

Principals with the misfortune of hiring teams of two low-quality agents are not likely to be satisfied. Some may decide to alter their strategy, and offer one job as a foreman with relatively closer monitoring and one job with little supervision. The possibility of obtaining a job as a foreman is welcomed; high ability agents will seek out the opportunity to send more reliable signals to their supervisor. Thus a principal can avoid weak recruiting outcomes by offering one job as a foreman, and a job with little supervision - as opposed to two jobs with an similar division of labor.

We characterize an equilibrium such that no agent has an incentive to rematch with a different partner, and no principal has an incentive to change her payment scheme or job composition. That is, we focus on a stable allocation in the core. Several striking features emerge. First, though principals are identical initially, *ex post* they are endogenously heterogenous with respect to the composition of jobs offered and agents recruited. Differences in skill compositions across teams arise without bias. All principals have the same capability to supervise agents, so that competition results in similar average recruiting outcomes. Principals that offer similar occupations hire two agents with intermediate ability levels. Principals that offer a diverse job mix hire the most able workers in the economy as foremen and the least skilled agents to receive little supervision. Though average recruiting success is similar across principals, the composition of workers varies depending on the chosen organizational form.

Second, team formation by agents exhibits two distinct types of positive assortative matching. Agents employed in a job mix with similar occupations form matches with positive assortment from a conjoined ability set. On the other hand, the highest skilled agents obtain jobs as foremen within a diverse job mix, match with lower ability agents from a disjoint ability set. There is positive assortative matching across occupations, between the most and least able workers. As a result the equilibrium mix of jobs available is crucial in describing the overall degree of heterogeneity within teams.

A third feature of an equilibrium is that differences in organizational choices lead to variation in the returns to skill across agents. The appointment of high skilled agents to be foremen allows them to send more reliable signals of their performance, while moderate and low skill workers are paid less on average (effort constant) because of their weaker signals. This is consistent with the results in Lemieux (2006) that returns to ability are heterogeneous across skills/education, and Lemieux, MacLeod, and Parent's (2009) finding high ability workers receive payments more closely tied to their performance.

In a static environment the trade-off firms principal's between good performance and good recruiting generates an interesting, and empirically relevant, allocation of jobs, incentive contracts, and agents. We also consider a key comparative static to explain changes in the employment shares of occupations that have been observed over the last three decades. In particular we show that an increase in the relative supply of high skilled workers leads to a greater prevalence of firms that offer high-skill occupations and low-skill occupations. Polarization of the workforce, or hollowing-out of the middle class, is an endogenous response to relative increases in the supply of high skilled workers[5]. Likewise, more firms offer strong performance pay in response to skill accumulation. The reason being that as more relatively high-skilled agents become available, organizational forms with the benefit of recruiting a single high-performing agent begin to dominate organizational forms that divide production risks across moderately-skilled agents.

We feel the contribution of this article is three-fold. First, we demonstrate how firm heterogeneity in skill compositions, rather than skill biases, arises endogenously in a competitive environment. There are several analyses of why firms select technologies that favor a particular type of skilled worker: for example see Acemoglu (1999 & 2001), Acemoglu and Shimer (2000), Albrecht and Vroman (2002), and Yeaple (2005) for examples in different labor and product market environments. Little attention has been paid to the causes of differences in skill composition at the micro, or firm, level. Regardless of looking at aggregate or firm-specific labor demands, the evidence does not support skill-biased technological change as a single force affecting labor markets in recent history[6].

Second, by allowing differences in the organization of firms to arise naturally, we are able to describe a new feature of equilibrium team formation. We find that the diversity of occupations offered in equilibrium is inversely related to the degree of heterogeneity in matches. In other words, as more firms offer a diverse mix of jobs (extensive margin), the degree of worker heterogeneity within firms shrinks on average (intensive margin). A great deal of the matching literature emphasizes the conditions sufficient for assortative matching to arise. (See Shimmer and Smith 2000 for search environments, Legros and Newman 2002 for frictionless markets and Legros and Newman 2007 & 2009 for settings with non-transferable utilities.) Focusing on job mix, we are able to describe both the extent and intensity of assortative matching in competitive environments.

[5] Acemoglu (1997, 1998, 2001) first demonstrated that an increase in the supply of high-skilled labor could induce more firms to offer high-skill jobs in models of one-to-one firm-worker matching. Note the difference here that increases in aggregate skill levels generate increases in both high-skill and low-skill occupations, consistent with stylized facts discussed above.

[6] Two notable exceptions that highlight the endogenous structure of firms given labor market conditions are Acemoglu and Newman (2002) and Garicano and Rossi-Hansberg (2006). In the former, the authors relate labor market conditions across countries to differences in employment structure. Whereas here, we are interested in the variation in the mix of jobs offered by firms competing in the same labor market, and facing a heterogenous supply of workers. Garicano and Rossi-Hansberg describe the organization of firms that face the problem of effectively organizing the knowledge of heterogeneous workers. As we have done here, they show that skill accumulation can lead to changes in the overall distribution of income, as well as wage dispersion within occupations, that fit recent trends. The primary distinction between our analyses is our focus on the composition of occupations and the prevalence of performance pay. Here ex ante identical firms adopt different organizational forms. The endogenous behavior of firms provides a unified explanation of polarizing labor demands as the supply of high-skilled workers rises, the greater incidence of performance payments, and changes in wage dispersion within and across firms.

Finally, our analysis is also related to a long literature on agency theory. In particular, our model draws on analyses of contracting with multiple agents; see Green and Stokey (1983), Mookherjee (1984), Itoh (1991) and Che and Yoo (2001). While much of this literature is aimed at issues of providing optimal incentives to agents, our framework is distinct in that the participants of each contract are determined endogenously via matching and assignment behavior. The analyses most closely related to ours are Serfes (2005) and Dam and Perez-Castrillo (2006). They each explore the matching outcomes between a principal and a single agent, rather than the contracting problem with multiple agents. Using their parlance, we highlight the *one-sided* matching problem between heterogeneous agents, whereas they focus solely on the *two-sided* matching problem between principals and agents[7].

The paper proceeds as follows. The next section builds a model of production, and describes the actors in the economy. Section 2 derives a unique equilibrium, defined as a stable allocation in the core. Section 3 discusses the properties of the equilibrium. Section 4 examines how skill accumulation among the labor force shifts the aggregate mix of occupations available in the labor force, and the prevalence of performance pay. In section 5 we show how the model and results can be generalized to an environment with many job types available at each firm. The final section concludes.

1 Model

The model described in this section provides a framework to examine the mix of occupations, as measured by skill-content, and the provision of incentive contracts that are available at individual firms. Thus we require three elements. First, each firm (principal) can offer multiple types of employment opportunities. This allows us to describe the composition of jobs within firms, as well as across firms. Second, worker (agent) heterogeneity is introduced to correspond to the measuring of occupations by skill. Third, inability to observe worker effort necessitates some form of performance payments. These three elements are detailed below.

1.1 Production and Information

Production at each firm occurs in teams of two agents supervised by a single principal. Within the team, each agent works on a specific task that is physically independent of her partner's duties. The production

[7]The literature on micro-finance has also considered team formation under different risk strategies. For example, lenders make choices about group- or individual- liability agreements with borrower heterogeneity. See Genicot and Ray (2003), Ahlin and Townsend (2006), and Maderia and Townsend (2008). Our current framework differs from previous analyses in that principals have organizational choices regarding how to mitigate the risk from individuals in addition to the choice of payment scheme. More importantly, in the general equilibrium environment here, the outside option of each agent is endogenously determined by the competitive environment.

process is replete with information problems. Each principal supervises production, but even though the agents' abilities are known, principals cannot perfectly observe the efforts of the agents working under her. There are two important features of the information problem inherent during production. First, each agent can put forth effort to improve the signal that he sends to the principal. An agent sends a signal of success with probability $p(\cdot)$, which is continuously increasing in her effort level, e.

The second feature of the information problem is that principals have the capability to advise agents in their efforts. That is, principals can provide direction that benefits agent effort, though efforts are still unobserved. Interaction with the principal improves the quality of agent performance by a total factor $\Gamma > 0$. For example the principal observes successful completion by both agents with probability $\Gamma p\left(e_1\right) p\left(e_2\right)$.

The specific characteristics of the efforts put forth by agents and the supervision by principals are described below. For now we impose a few regularity conditions on the production environment. Effort improves the probability of good performance with decreasing returns. Principals can never fully direct the activities of those working under her. To be specific, the information exchanged through effort and supervision satisfies

$$0 \leq \Gamma p(e) < 1,\ p'\left(e\right) > 0,\ \text{and}\ \ p''\left(e\right) \leq 0.$$

In order to ensure an equilibrium where strategic interactions between agents in a team exists, and that principals are concerned with their relative incentives to provide effort, we also assume that

$$p''\left(e_1\right) p''\left(e_2\right) p\left(e_1\right) p\left(e_2\right) - \left[p'\left(e_1\right) p'\left(e_2\right)\right]^2 > 0$$

With these informational constraints the principal can realize four potential states during production: receive positive signal from both, receive negative signal from both, and receive positive signal from either agent accompanied by a negative signal from the other agent. Success by either agent generates a single unit of the numeraire good[8].

In what follows we will describe the behavior of principals and agents within this production and information environment. Yet it is worthwhile to first provide a picture of how the contest evolves. All decisions, agent matching & efforts, as well as job mix & performance contracts from principals, are made simultaneously. Within the single period agents are free to reconsider their matches, given the behavior of their

[8] Assuming that agents work independently and simultaneously erodes any complementarities that would arise from strategic interaction. However, production environments that seem more intuitive representations of the firm, say hierarchies with sequential moves or effort coordination, only reinforce the results derived below. Kim (2010) studies the endogenous formation of agent hierarchies and sequential games. He shows that equilibrium payment schemes will be identical to what follows. See also Che and Yoo (2001)

employing principal. Likewise principals are free to alter the wage scheme and occupations offered, given the team they recruited. Put differently, employment at firms and matching between agents are at-will agreements. Both agents and principals understand that renegotiation or reconsideration of behavior requires them to enter a new phase of competition for good jobs and good teams. A stable equilibrium occurs when no agent or principal has an incentive to seek new opportunities.

To fix ideas, consider the behavior of both principals and agents just prior to production. Each principal announces the wage scheme she will use to reward agents, and the manner in which she will advise each member of the team. Considering the menu of opportunities available from principals, agents compete for quality matches, quality occupations and subsequently decide on effort levels. Principals then recruit teams competitively. Given the allocation of agents to principals, production could occur.

Yet, some principals may not be satisfied with the team that they recruit and therefore may have an incentive to alter their original contract. As some principals offer different contracts, agents reconsider their initial matches in light of a new menu of options. This process of reconsideration continues until all principals are satisfied with their recruits and all agents are satisfied with their match, given the expected competition. Upon reaching stable matching outcomes, job mixes, and incentive contracts, production occurs immediately. As more details about the model emerge the reader should bear in mind that the equilibrium concept we employ is that of the core. We are seeking a stable allocation with the composition of jobs offered and matches determined endogenously.

1.2 Principals

There is an unbounded mass of identical principals who can choose to engage in production freely. Immediately before production each principal must compete for a team of agents in a competitive market. The objective of each principal is to maximize profits net of payments to the two agents working under her. The behavior of each principal is characterized a by mix of jobs offered (intended to attract high quality teams) and a choice of payment scheme (used to elicit effort from individual team members).

1.2.1 Job Mix

Informational constraints inhibit the ability of principals to observe their agents' efforts perfectly. All principals can improve the reliability of the performance signals they receive from their team members with the same proficiency $\Gamma > 0$. However each principal has two options regarding the composition of jobs to offer agents. Each organizational form allows the principal to appoint one team member to be a foreman who can be advised more closely. For example, two different fund managers may be responsible for separate accounts, but the investment brokerage may choose to direct the activities of one fund manger more closely toward the clients' goals. Another possibility is that two product designers can be assigned to develop a new component for use in production, and management can choose to observe the progress made by one developer more than the other. Nearly all firms are multi-product firms. Each agent may be assigned to a single product line, while the firm makes different investments in advertising each product to consumers. So the foreman can be interpreted as the agent in charge of the firm's core product line, and her counterpart in charge of an ancillary product line.

While both organizational forms appoint one worker to be a foreman, they differ in how the monitoring capability of the principal, Γ, is divided across agents. The first composition of jobs (k=1) allows the principal to give similar direction regarding how efforts can be steered toward success. That is, she can offer jobs with a relatively similar division of labor. The second (k=2) provides a more diverse composition of jobs such that the foreman receives much direction from the principal, while the efforts of the remaining agent go relatively undirected.

To be specific let $\overline{\gamma_k}$ be the improvement to the signal of the foreman under organizational form k, while $\underline{\gamma_k}$ is the direction given to her counterpart. The differences in the composition of jobs across organizational forms are given by

$$\underline{\gamma_2} < \underline{\gamma_1} < \overline{\gamma_1} < \overline{\gamma_2} \qquad \text{(JM.1)}$$

where the principal limited span of control requires

$$\overline{\gamma_1}\underline{\gamma_1} = \Gamma = \overline{\gamma_2}\underline{\gamma_2} \qquad \text{(JM.1)}$$

Devoting more time to one agent improves the chance that her efforts lead to success. The probability that agent i, assigned to occupation γ_i, sends a positive signal of performance, $\gamma_i p(e_i)$, is enhanced for any e_i. Yet agents are free to vary their behavior by choosing different effort levels under varying degrees of supervision. In this sense, abilities translate across occupations[9].

Principals have choices about how to manage information across agents, but they face the same information problem over the team as a whole, regardless of the job mix or incentive contracts. Restricting principals' spans of control of information, Γ, to be identical simplifies the setting to that of one-sided heterogeneity[10]. More importantly it highlights the tradeoff that principals face – between recruiting quality

[9]Bandiera *et al.* (2007) find that management makes specific choices about which workers to direct more or less intensively. Interestingly, this choice reflects the incentive contracts that individual managers face.

[10]See Dam and Perez-Castrillo (2006) for two-side matching between principals and heterogeneous agents. Note the key

teams, and eliciting good agent performance – because principals with the same ability must compete for high-quality teams.

1.2.2 Performance Payments

In addition to choosing which occupations to offer the principal also chooses incentive contracts to motivate workers where effort cannot be observed. Incentive contracts vary payments based on the individual performance of agents. Every principal sets the rewards for each type of signal received: she chooses the payments V^s when both agents succeed, V^w to the agent with a good signal and V^l to the agent with a bad signal when they are different, and V^o to each agent when neither succeeds[11] .

Though these payments can vary continuously, and be made commensurate with the ability of agents, we can classify them in the typical way. A payment scheme is said to be Joint Performance Evaluation (JPE) if $\left(V^s, V^l\right) > (V^w, V^o)$ and Relative Performance Evaluation (RPE) if $\left(V^s, V^l\right) < (V^w, V^o)$. Note that under JPE there are strong team incentives since an agent in made better off by good performance of her partner. On the other hand, RPE provides strong competitive incentives since agents are best off when they outperform their partners.

1.2.3 Principal's Objective

Principals maximize expected profits by choosing (1) the composition of jobs to offer, and (2) a payment scheme in order to attract the highest potential quality team and to incentivize worker effort. We impose a limited liability constraint such that agents cannot be forced to make payments to principals regardless of their performance.

$$V^s, V^w, V^l, V^o \geq 0$$

With a limited liability constraint on performance payments note that the agents' participation constraints are non-binding. Then any $V^l, V^o > 0$ decreases agent effort, inducing principals to optimally set $V^l = V^o = 0$. Setting the price of output to be the numeraire, each principal solves the following problem subject to JM.1 and JM.2:

difference between our analyses in that we examine the multi-agent contracting problem where heterogeneous agents form matches voluntarily.

[11]Note that firms offer the same performance contract for each member of the team, rather than writing individual contracts for each agent. This does not preclude the principal from offering individual incentive schemes such that $v^s = v^w$. It also corresponds to the findings of wage compression in Baker *et al.* (1994) due to strong cohort effects and in Fang and Moscarini (2005) due to the potential hazards of poor morale.

$$\max_{\substack{\gamma_i,\gamma_j \\ V^s,V^w,V^l,V^o}} \quad E\{\gamma_i p\left(e_i\right)\gamma_j p\left(e_j\right)\left[1-2V^s\right]+\left[1-\gamma_i p\left(e_i\right)\gamma_j p\left(e_j\right)-\left(1-\gamma_i p\left(e_i\right)\right)\left(1-\gamma_j p\left(e_j\right)\right)\right]\left[1-V^w\right]$$

The next section describes the behavior of agents which provides endogenous incentive compatibility constraints on the principal's optimization problem above.

1.3 Agents

The mass of agents is normalized to unity[12]. Individual agents are differentiated with respect to ability. Higher ability agents can exert effort, e, at a lower marginal cost to their overall utility. The ability of an agent is given by a, and her marginal cost of effort is $\frac{1}{a}$. The distribution of ability is assumed to be atomless and given by $G(a)$ on $[a_{min}, a_{max}]$[13].

Agents are risk adverse and make two choices to maximize their utility. First, agents must decide with whom they are willing to form a team knowing that they will have to compete for quality matches, and perhaps jobs that offer relatively close direction. Second, given their match, job type and wage scheme imposed, agents must decide what level of effort to put forth. Agent behavior is best illustrated by working backwards from a stable matching allocation. Once matched and employed, each agent maximizes her indirect utility function, $\mathcal{V}$, for the benefits of income realized from performance payments, $u(\cdot)$, by choosing her effort level, given her own ability, the ability of her partner, and job type. That is, agent i solves the following problem

$$\mathcal{V}_i = \max_{e_i} \quad \gamma_i p\left(e_i\right)\gamma_j p\left(e_j\right)u\left(V^s\right)+\gamma_i p\left(e_i\right)\left[1-\gamma_j p\left(e_j\right)\right]u\left(V^w\right)-\frac{1}{\alpha_i}e_i$$

where γ_i satisfies equations (JM.1) and (JM.2) and the utility function satisfies $u'(\cdot) > u(0) = 0 > u''(\cdot)$. Having fully described the actors in the economy as well as the production environment the next step is to derive an equilibrium.

[12]Note that with two job types at each firm, this limits the equilibrium mass of principals to 1/2. We maintain the assumption of an unbounded mass of potential entrants as principals to accommodate the generalization of the model to multiple occupations in section 5.

[13]By assuming that abilities have an atomless distribution we are implicitly assuming that there is an even mass of agents with each type of ability. Hence if one agent with ability a is assigned to a particular occupation, then they all are. This simplifies the analysis without changing any of the qualitative results.

2 Equilibrium

In this economy, a full equilibrium is comprised of five elements: (1) the composition of jobs offered by all firms - we define Σ as the share of jobs with similar direction according to $k = 1$; (2) an assignment of workers to jobs with different degrees of supervision- let Ω_γ be the sets of abilities assigned to a job with given levels of direction d where $\gamma = \underline{\gamma_1}, \underline{\gamma_2}, \overline{\gamma_1}, \overline{\gamma_2},$ as in JM.1 and JM.2; (3) an allocation of workers into teams- define $M_1 : \Omega_{\underline{\gamma_1}} \rightarrow \Omega_{\overline{\gamma_1}}$ and $M_2 : \Omega_{\underline{\gamma_2}} \rightarrow \Omega_{\overline{\gamma_2}}$ to be the matching functions that describe teams within firms; (4) performance payments to agents; and (5) degrees of effort put forth by agents during production. We are specifically interested in a core equilibrium where matching behavior and the composition of jobs are stable given the competitive environment. The derivation of an equilibrium will proceed backwards from a stable allocation, beginning with agent effort.

2.1 Agent Effort

Agent i's behaves according to the following first order condition, choosing an effort level e_i which satisfies the following first order condition

$$\frac{1}{\alpha_i} \frac{1}{p'(e_i)} \equiv \gamma_i \left[p(e_j) \gamma_j (u(V^s) - u(V^w)) + u(V^w) \right] \tag{1}$$

Clearly, every agent's effort is increasing in her own ability, given any wage scheme or occupation. For simplicity we will describe agent preferences for matches and principal preferences for teams in terms of the abilities of agents. Yet, the fact that greater ability leads to greater effort indicates that ability is indeed the relevant feature during the matching process. An agent may put forth different levels of effort when matched with various partners. But since agents cannot alter the incentives of their partners, i.e. equation (1) must hold, variation in effort levels across different patterns of team formation has only second order effects. One might like to change the effort level of her partner, but must resort to new matches in order to experience real differences. As indicated by equation (1), agent efforts do respond to the payments offered by principals, which are derived next.

2.2 Payments to Agents

The dual problem to the principal is to minimize payments to agents, subject the agents' incentive compatibility conditions in equation (1). As mentioned above, payments for poor performance (V^l or V^0) are disincentives for agents to exert effort. Thus principals optimally set rewards for poor performance to zero.

The following proposition describes the optimal incentive scheme chosen by principals given the willingness of agents to provide effort.

Proposition 1 *For either job mix (k=1,2), all principals will implement a Joint Performance Evaluation wage scheme such that $v_s > v_w$, when teams are comprised of heterogeneous agents.*

Proof. See Appendix ∎

According to proposition 1, principals use relatively strong group incentives to motivate teammates with different abilities. Even though agents work simultaneously on independent tasks, relative reward schemes optimally balance the incentives of heterogeneous agents to provide effort[14]. A JPE wage scheme avoids discouraging a lesser ability team member, where the marginal return to effort is greatest, given that she is likely to be outperformed by her partner.

Though JPE is always the optimal type of incentive contract, the actual value of each payment varies across teams. Principals recognize that agents exert effort with differing marginal cost to their overall utility, and so make performance payments commensurate with the ability of the team recruited. The next section turns to equilibrium team formation as workers match based on the anticipated payment scheme from principals.

2.3 Agent Matching & Team Formation

Under a Joint Performance Evaluation payment scheme agents view the efforts of their partner as strategic complements for their own; $\frac{de_i}{de_j} > 0$. As a result, every agent would prefer to obtain a high ability partner, all else equal. However, agents must compete for their preferred partners, and so it is relative preferences for partners' abilities that determine matching outcomes. The relative benefits of own and partner's abilities to agent i's expected payoff,$\mathcal{V}_i$, are given by

$$\frac{\partial^2 \mathcal{V}_i}{\partial a_j \partial a_i} = \Gamma p'(e_i) p'(e_j) \frac{de_i}{da_i} \frac{de_j}{da_j} \left[u(V^s) - u(V^w) \right] > 0 \qquad (2)$$

The relative benefits of payments (term in brackets), and the relative effort levels across abilities ($\frac{de_i}{da_i}$) are both positive when principals offer a JPE wage scheme. So equation (2) guarantees that individual payoffs are supermodular in agent abilities. We cite the well known results from Legros and Newman (2007) that supermodularity of individual payoffs is sufficient for positive assortative matching to arise for any

[14]Homogeneous agents face the same chances of success for their optimal effort level and hence individual rewards are sufficient motivation. As shown in the appendix, the optimal incentive contract offered to homogeneous agents in Individual Performance Evaluation with $v_s = v_w$. Proposition 1 is the relevant result in the current context since the organizational choices regarding job mix will lead to heterogeneous matches in equilibrium for non-degenerate distributions of ability.

distribution of ability. Thus M_1 and M_2 are non-decreasing and unique matching functions between workers in corresponding occupations.

Matches are consummated between foremen and those with little direct monitoring according to positive assortment within each job mix. Of course the assignments of workers to jobs occur endogenously, as derived immediately below.

2.4 Agent Task Assignment

All workers benefit from greater supervision by their principal; hence all agents prefer jobs as foremen. Yet, agents must compete for their preferred types of employment, and so it is relative preferences for job types that determine assignments. Closer monitoring results in a proportional increase in signal quality that agents send. Since higher ability agents can provide more effort (and hence perform better) at a lower cost to their overall utility, greater abilities benefit proportionally more from employment as foreman. Given the discrete differences in supervision one receives in each type of employment, and the fact information constraints dictate $\frac{\Delta\gamma_j}{\Delta\gamma_i} < 0$, the relative benefits of supervision across agents are described formally by

$$\frac{\Delta}{\Delta\gamma_i}\left(\frac{\partial\mathcal{V}^i}{\partial a_i}\right) = p'\left(e_i\right)\frac{de_i}{da_i}u(V^w) > 0 \tag{3}$$

According to equation (3) agent payoff functions are supermodular in direction from principals and own ability, which is sufficient for positive assignment of agents to occupations. Agents with the greatest ability will be hired as foremen at firms offering diverse occupations; those with moderate ability will receive jobs with moderate levels of supervision at firms with a similar division of labor; and the lowest ability agents will obtain jobs with little supervision.

Figure 1 illustrates equilibrium matching outcomes and occupation assignments consistent with positive assortment. Job assignments are characterized by the most able workers employed as foreman, and matched positively with the remaining agents employed in the respective job mix. As seen in Figure 1, the highest ability foremen are matched with the highest ability agent to not be employed as a foreman in the same job mix. (Similarly the lowest ability foremen are matched with the lowest ability agents not employed as a foreman in the same job mix, illustrated by the dashed arrows.) Intermediately skilled workers within A_1 and A_2 are employed in similar jobs that each offer moderate performance incentives. Those agents with abilities greater than A_2 and below A_1 are employed by firms that offer a relatively diverse occupational mix. Because an equal number of agents must be employed in each occupation, agents with abilities between A_1 and A_2 form matches positively from either side of the median ability level.

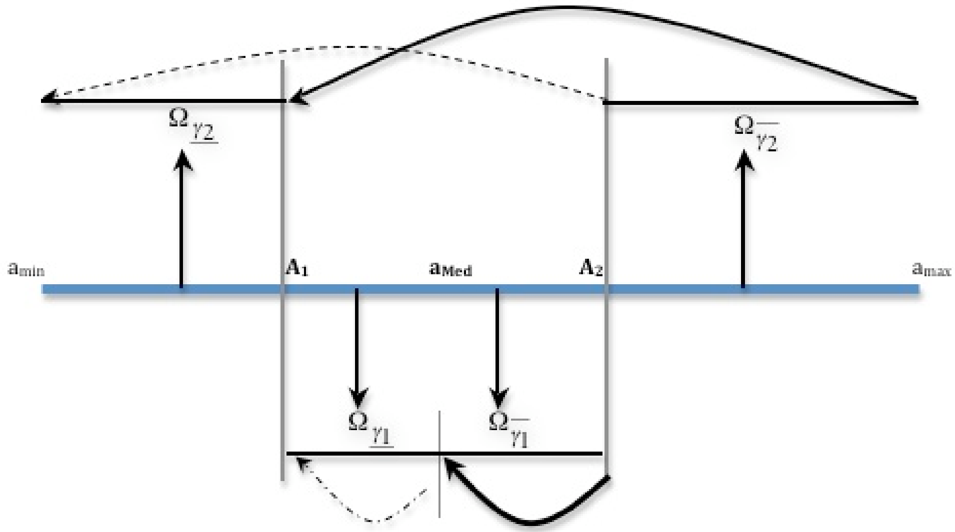

Figure 1: Equilibrium Matching and Job Assignments

Consistent with positive assignment of workers to tasks, in equilibrium the sets Ω_γ are given by[15]

$$\Omega_{\underline{\gamma_2}} = [a_{min}, A_1), \quad \Omega_{\underline{\gamma_1}} = [A_1, a_{Med}), \quad \Omega_{\overline{\gamma_1}} = [a_{Med}, A_2), \quad \text{and} \quad \Omega_{\overline{\gamma_2}} = [A_2, a_{max}] \tag{4}$$

Equations (2) and (3) are sufficient conditions for positive matching between agents and positive assignment of workers to occupations. At this point it is worthwhile to discuss which features of the model drive these results. The jobs that agents perform within each firm are physically independent, so principals care only about the incentives of agents to provide effort during production. The production process has no complementarities between the occupations. Positive assortative matching is a result of the strategic behaviors about efforts which are endogenous to the model[16]. On the other hand, positive assignment of workers is due to the modeled proportional increase in signal quality that comes from better jobs. If better employment guaranteed only a fixed (additive) increase in monitoring, positive assignment may not arise. As we are interested in describing an economy with various occupation types defined by their skill content, we feel the proportional modeling choice is appropriate.

With behavior fully described across and within occupations, the two cutoffs A_1 and A_2 are sufficient to describe the assignment of workers to jobs across firms and matching between agents within firms. Hence the last component of a full equilibrium is the mass of each type of job available, as determined by these

[15]Without loss of generality agents with abilities equal to the cutoff values are assigned occupations with relatively higher performance incentives.

[16]Any production environment such that principals offer JPE wage schemes would still lead to positive matching. Under JPE incentive contracts $u(V^s) - u(V^w) > 0$ and hence equation (2) is still positive. Potential alternatives include agent hierarchies with sequential moves and the potential for collusion on effort levels.

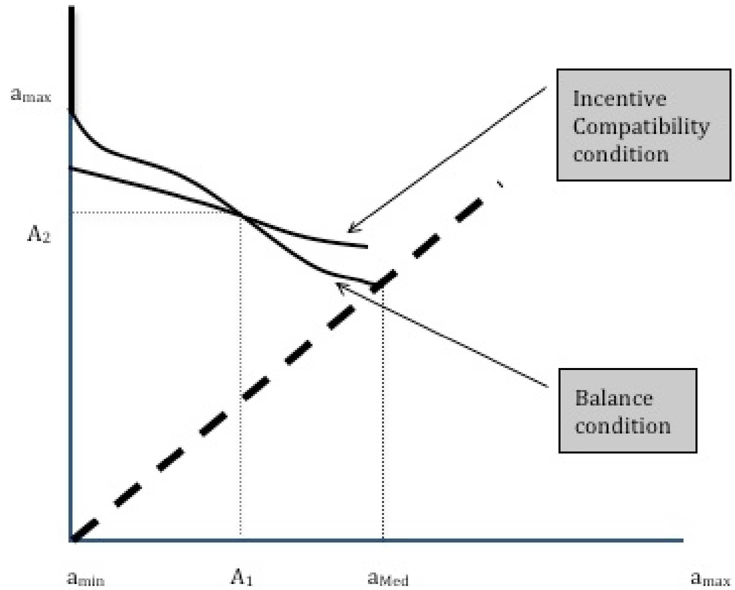

Figure 2: Determining the Equilibrium Composition of Jobs

cutoffs.

2.5 Equilibrium Job Composition

Let Σ be equilibrium share of intermediate-skilled workers employed in similar occupations (k=1). Given positive assignment across occupations, Σ is calculated by

$$\Sigma = \int_{A_1}^{A_2} dG(a) \tag{5}$$

The two equilibrium conditions that determine the two abilities cutoffs reflect the incentives and possibilities of firms to offer each type of employment contract.

Principals decide which types of occupations they wish to offer, given that they must compete for quality recruits. Thus our first equilibrium condition is the principal's Recruiting Indifference Condition: the expected profit from the team a principal anticipates to recruit under job mix k=1 (i.e. the average intermediate team) must yield the same expected profit yielded by the anticipated team from offering occupations according to job mix k=2 (i.e. the average diverse team).

Equilibrium assignments to occupations determine the quality of teams that each principal can expect to recruit. Since principals are identical, they draw teams randomly according to their chosen job mix. For a general distribution of ability, principals expect to draw agents with the average rank of ability; that is each principal expects to draw a team comprised of the median ability employed in the respective occupations[17]. Define a^{γ_k} the ability that any principal expects to recruit when offering a particular occupation γ_k, such that

$$a^{\underline{\gamma_1}} \equiv med(\Omega_{\underline{\gamma_1}})\ , \qquad a^{\overline{\gamma_1}} \equiv med(\Omega_{\overline{\gamma_1}})\ , \qquad a^{\underline{\gamma_2}} \equiv med(\Omega_{\underline{\gamma_2}}), \qquad \text{and} \qquad a^{\overline{\gamma_2}} \equiv med(\Omega_{\overline{\gamma_2}}).$$

Let $\Pi(\cdot)$ denote expected profits earned per the principal's objective function for expected recruiting outcomes. Then the principal's Recruiting Incentive Compatibility condition is

$$\Pi(a^{\underline{\gamma_1}}, a^{\overline{\gamma_1}}, \underline{\gamma_1}, \overline{\gamma_1}) \equiv \Pi(a^{\underline{\gamma_2}}, a^{\overline{\gamma_2}}, \underline{\gamma_2}, \overline{\gamma_2}) \tag{6}$$

and is an implicit function of the equilibrium values A_1 and A_2. (See equation (4).)

The Recruiting Indifference Condition condition for each principal describes the relative incentives to offer each type of job. The second equilibrium condition reflects the constraints on the occupations that principals can actually make available. Every team is formed by two employment positions with a fixed monitoring capability. Employment on either side of A_1 and A_2 must be balanced in that

$$\int_{a_{min}}^{A_1} dG(a) \equiv \int_{A_2}^{a_{max}} dG(a) \tag{7}$$

The intersection of the Recruiting Indifference Condition and Balance Condition on employment jointly determine the equilibrium job composition in that their intersection pins down equilibrium values of A_1 and A_2.

A full equilibrium consists of the following: the mix of jobs offered and task assignments of agents, $\Omega_{\underline{\gamma_2}}$, $\Omega_{\underline{\gamma_1}}$, $\Omega_{\overline{\gamma_1}}$, $\Omega_{\overline{\gamma_2}}$, which are determined by positive assignment according to equation (3), plus the Recruiting Indifference Condition and Balance Conditions in (6) and (7); matching functions M_1 and M_2 consistent with positive assortative matching a prescribed by equation (2); JPE incentive contracts subject to proposition 1; and agent effort levels that satisfy the first order condition in (1).

Proposition 2 *An full equilibrium exists and is unique.*

Proof. See appendix ∎

The determination of the equilibrium job composition for a core allocation of the labor force is illustrated in Figure 2. By definition of the cutoff values A_1 and A_2 from (4), each condition must lie above the 45^o

[17] Principals cannot expect to recruit agents with abilities equal to $\sup \Omega_{\overline{\gamma}_k}$ and $\inf \Omega_{\underline{\gamma}_k}$ in the same team since this violates positive assortative matching for any non-trivial assignment of agents to occupations. Moreover, principals cannot form expectations with regard to the average skill assigned to each occupation. For general distributions of ability, the expected values may not correspond to the same ranking across occupations, and hence cannot be considered as potential equilibrium recruits.

diagonal and to the left of the median ability level. Implicit differentiation demonstrates that the slope of the Balance Condition is strictly negative and more steep than the Recruiting Indifference Condition on the interior. (See proof of proposition 2 in the Appendix.).

3 Discussion

In this economy principals choose an organizational form to balance a trade-off between quality recruiting and quality team performance. For any interior solution of A_1 & A_2, there are some principals who offer each type of job mix. Differences in organizational form can arise endogenously, even though principals are identical in their capabilities to direct agents. A second source of heterogeneity across firms is the skill composition of the workforce. These differences that arise in any equilibrium reflect the endogenous assignments of workers to tasks in matching markets.

A last source of heterogeneity across firms is the actual recruiting outcomes. As principals all have the same capability to monitor agents, variation in team quality occurs randomly among the shares of firms that choose each job mix. It is worthwhile to note that (weighted) average recruiting outcomes are similar across firms. The two workforce compositions are either high ability foremen are matched with low ability agents, or firms that offer a similar division of labor over two moderately skilled agents. The choice of occupations offered by principals is a best-response to the competition for a quality workforce. Yet, neither mix of jobs represents a dominant strategy.

The potential for an equilibrium with heterogenous organizational choices by principals depends in the underlying distribution of ability. However the following proposition states that only one organizational choice can support an equilibrium with symmetric behavior by principals.

Proposition 3 *For any distribution of agent ability, if all principals select the same job mix in equilibrium they necessarily offer a similar division of labor according to k=1.*

Proof. See Appendix. ∎

All principals prefer to mitigate the risk of poor agent performance by interacting with both agents during production. The only reason to appoint a single foreman with close interaction is if the reward is better recruiting outcomes. When all principals offer the same job mix expected recruiting outcomes are identical, and hence must reflect the benefits of hedging agent performances.

The equilibrium composition of jobs influences the outcomes for agents as well. Each agent's maximum utility is strictly greater when employed in occupations that offer relatively stronger performance payments (i.e. foreman or moderate tasks). Moreover, there is second-order effect of better advising to agent effort.

Equation (1) ensures that agents with jobs that provide better direction choose to exert more effort during production. While the envelope condition ensures that this has negligible effects on the overall welfare of agents, it does ensure that higher skilled agents are expected to earn greater observed wages via better performance. This result is consistent with the findings of Lemieux (2006) of heterogeneous returns to skill.

Matching markets have two distinct problems to solve: (1) match intermediate ability agents and (2) match high ability foremen to low skill agents. The strategic behavior of agents chasing the best rewards for their efforts causes both of these matching problems to be resolved in a pattern of positive assortment. Intermediately skilled agents form matches from the conjoined ability set between A_1 and A_2. On the other hand, the highest skilled foreman, ability above A_2, will be matched with agents from the disjoint set of abilities below A_1. These two types of positive assortative matching – heterogeneous matching within a conjoined set of ability, and heterogenous matching across disjoint sets – are inversely related as the forces that lead more principals to offer similar job types, necessarily reduces the share of principals that spread production across diverse occupations. The implications of this inverse relationship for matching outcomes are stated in the following proposition.

Proposition 4 *In equilibrium, the greater the share of teams formed across a diverse job mix according to k=2 (extensive margin), the smaller the differences in ability within teams (intensive margin) will be on average.*

Proof. Consider the average matching outcome between foremen and low skill workers with little supervision. That is consider the team $\left(a^{\underline{\gamma_2}}, a^{\overline{\gamma_2}}\right)$. The Balance condition is downward sloping so that across potential equilibria, A_1 and A_2 are inversely related. A larger share of teams are formed across a diverse job mix (k=2) is equivalent to an increase in A_1 and a decrease in A_2. As $a^{\underline{\gamma_2}}$ and $a^{\overline{\gamma_2}}$ are both increasing in the respective cutoff, and $a^{\underline{\gamma_1}} < a^{\overline{\gamma_2}}$, it must be that an increase in A_1 and a decrease in A_2 causes $a^{\underline{\gamma_1}}$ and $a^{\overline{\gamma_2}}$ to converge. A parallel argument can be made for matching outcomes between A_1 and A_2. ▮

4 Skill Accumulation

The fundamental intuition supporting this model is that firms consider the abilities of workers they expect to recruit when they decide what mix of jobs, and what performance incentives to offer. How then do changes in the shape of labor endowments influence the equilibrium composition of occupations and the provision of performance payments? In this section we consider changes in the degree of competition in factor markets by examining the impact of a shift in the overall distribution of agent ability toward high skills. The comparative static exercise is meant to capture the effects of skill accumulation among the labor force.

Definition 1 *$G'(\cdot)$ is said to be more a more skill abundant distribution of ability than $G(\cdot)$ if $G'(\cdot)$ first-order stochastically dominates $G(\cdot)$.*

This definition implies that an abundance of skills corresponds to greater masses of higher ability workers at or below each skill level. Since firms take the overall distribution of recruits into account, definition 1 is needed to relate the behavior of principals across distributions of abilities. The impact of skill accumulation on the aggregate composition of jobs is stated in the next proposition.

Proposition 5 *An accumulation of skills among the labor force leads to polarization of the work force. Specifically, the employment shares of high- and low- skill occupations (k=2) rise, while intermediate-skill occupations account for a smaller share of employment.*

Proof. Let the share of intermediate-skill employment in an abundant distribution of ability be given by Σ', and let Σ be the respective employment share in a less abundant distribution. Then job polarization is equivalent to $\Sigma' < \Sigma$, or by calculation using equation (8), $G'(A_2) - G'(A_1) < G(A_2) - G(A_1)$. The equilibrium Balance condition implies that $1 - F(A_2) = F(A_1)$ for any distribution of ability $F(\cdot)$. Direct substitution of the Balance condition establishes the result immediately. This result can be seen in Figure 2 noting that moving to a stochastically dominant distribution of ability rotates the Balance condition downward about the vertical intercept. ∎

Polarized labor demands across occupations are consistent with the experience of several countries, particularly the US, realizing smaller shares of middle-skill employment at the same time that high-skill workers became more abundant. Goos and Manning (2007) find evidence of polarization in the UK. Autor, Katz and Kerney (2006) provide evidence from the US, particularly in the 1990s. Goos, Manning and Salomons (2009) find broad evidence of polarization in sixteen European nations. Demand shifts away from the middle of the skill distribution appear large in magnitude; in the US for example, Autor and Dorn (2009) find that the lowest and highest deciles of occupations each account for 10%-25% more of employment in 2005 than observed twenty-five years earlier. The effects of these shifts in labor demand correspond to recent changes in the aggregate wage distribution.

In addition, proposition 5 speaks to directly wage dispersion within firms as the supply of high ability labor rises. Polarization is equivalent to an increase in A_1 and a decrease in A_2. Note that the average skill of foremen in diverse teams, $a^{\overline{\gamma_2}}$, falls as A_2 declines, while the average skill of lightly supervised workers, $a^{\underline{\gamma_2}}$, will rise with A_1. Skill accumulation causes the the average abilities workers across these occupations to converge, and the differences in their wages to fall on average. These effects are consistent with the evidence provided by Dunne *et al.* (2004) for US manufacturing firms that rising wage dispersion was concentrated across firms, rather than within.

An important corollary to proposition 5 relates skill accumulation to incidence of performance pay and incentive contracts.

Corollary 1 *An accumulation of skills among the labor force leads to a greater prevalence of strong performance payments, most concentrated among the highest ability workers.*

As high-skill workers become more abundant a larger share of firms find it optimal to try to recruit these high-performing agents by offering them relatively stronger performance incentives. Corollary 1 matches the empirical findings of Lemieux *et al.* (2009). The greater use of performance payments reflects firm recruiting behavior, while the concentration of these contracts among high ability workers is a consequence of worker assignment in competitive matching markets.

Proposition 5 relates aggregate labor demands to shape of the aggregate labor supply through the behavior of individual workers and firms. As the supply of high ability workers increases, more firms have an incentive to offer employment opportunities that better accommodate them. This is similar to the intuition put forth by Acemoglu (1997, 1998). The key distinction here is that shifts in labor demands are not wholly skill-biased. Informational constraints surrounding production inhibit firms from directing the efforts of all their workers perfectly, providing micro foundations for job polarization as opposed skill-biased-technological change.

We should point out that these are not competing insights, nor do they speak necessarily to the same economic phenomenon. The framework here emphasizes information constraints that exist for workers of any ability. Job composition is a tool used to alter recruiting quality, rather than mitigate price effects of scarce factors, avoid search costs, or capture larger markets for skill-specific technologies. In any case, recent trends do not suggest that shifts in labor demand have been solely-skill biased.

5 Generalization

Our last goal is to demonstrate how the results above can be generalized to an economy with principals that offer multiple occupations, as opposed to just two. Like before, each principal a fixed span of control over the information environment, but can divide that capability over n_k employment positions, for positive and finite values of n_k. Again suppose firms can choose between two organizational forms; the first (GJM.1) divides the production process into jobs with relatively similar degrees of monitoring, while the second (GJM.2) creates some occupations with very strong performance payments (e.g. upper-management) and subsequently some jobs with weak incentive contracts. Furthermore, the number of different occupations, described by n_k, can vary across each job mix. Then the two generalized job mixes available to firms are

$$\gamma_1^1 < \gamma_2^1 < ... < \gamma_{n_1}^1 \qquad \text{with} \qquad \prod_{i=1}^{n_1} \gamma_i = \Gamma \tag{GJM.1}$$

and

$$\gamma_1^2 < \gamma_2^2 < ... < \gamma_{n_2}^2 \qquad \text{with} \qquad \prod_{i=1}^{n_2} \gamma_i = \Gamma \tag{GJM.2}$$

The first organizational form is assumed to provide a more equal division of labor in that occupations do not differ greatly in terms of monitoring. Specifically, let the relative supervision of each task across strategies satisfy the monotone likelihood ratio property about a particular occupation $M > 1$, so that

$$\frac{\gamma_{M-j-1}^2}{\gamma_{M-j-1}^1} < \frac{\gamma_{M-j}^2}{\gamma_{M-j}^1} \qquad \text{and} \qquad \frac{\gamma_{M+j}^2}{\gamma_{M+j}^1} < \frac{\gamma_{M+j+1}^2}{\gamma_{M+j+1}^1} \tag{MLRP}$$

for all $j \geq 1$. We also assume $n_1 \leq n_2$ and that $\gamma_h^1 \neq \gamma_i^2 \ \forall \ h, i$.[18] Under these generalizations the following result is obtained:

Proposition 6 *Suppose that the compositions of jobs that principals can offer are given by GJM.1 and GJM.2, and that MLRP holds. Then a full equilibrium exists and is unique.*

Proof. See Appendix ∎

The qualitative features of the basic model are maintained by the generalized composition of jobs: principals offer JPE wage schemes; agents are positively assigned to occupations and positively matched to other agents in the same job mix; and firms differ in the skill-content of their work force without bias. Proposition 2 is maintained. As more firms choose to offer a highly diverse array of occupations, à la GIM.2, they will necessarily recruit more agents from the top and bottom of the skill distribution, based on the positive assignment of workers to occupations. The subsequent matching of workers causes the averages abilities of those employed by the same firm to converge. Likewise, skill accumulation leads to job polarization, as fewer firms will offer jobs according to GJM.1 when high ability workers are abundant[19].

[18]Restricting $n_1 \leq n_2$ ensures that the relative diversity of job mix GJM.2 is satisfied for all occupations. Lastly, using the maximum likelihood ratio property to relate the different job mixes is a sufficient, but not necessary, condition to maintain the insights of the basic model in the generalized case. MLRP is chosen because of it familiarity.

[19]Following the derivation of a unique equilibrium under the generalized model in the appendix, it is a simple comparative statics exercise to extend propositions 2 and 3 to an environment with multiple occupations.

6 Conclusion

We examined the multiple-agent contracting problem in a general equilibrium setting where principals recruit agents from a heterogenous pool. The trade-off that principals face between quality recruiting outcomes and quality performance leads to endogenous differences in organizational structure, the provision of performance incentives, matching patterns across teams. Though different organizational choices lead to different compositions of agents recruited, average recruiting outcomes are similar for all principals.

When principals must coordinate performance incentives in a general equilibrium environment, we have shown that team formation between agents exhibits two distinct, and inversely related, forms of positive assortative matching. As a result, the degree of heterogeneity within teams (intensive margin) is inversely related to the share of firms that offer a diverse mix of jobs. A key feature of these matching patterns is that they are the result of strategical complementarities between agent efforts, rather than exogenous physical complementarities in the production process. Environments with sequential effort decisions or other coordination of effort levels would generate stronger incentives for assortative matching, and thus organizational differences to arise in equilibrium. Therefore we feel these results apply generally to firms with many common production processes.

We applied the principal-agent framework developed to the relationship between firms and workers to examine labor market outcomes with regard to job mix and performance payments. An abundance of high-ability workers was shown to generate job polarization, or hollowing-out of the middle class. Also, a greater share of firms offer a steep mix of incentive contracts when the distribution of labor productivity is skewed upward. As high-ability workers obtain payments more closely tied to their performance, overall wage inequality rises. Yet, because of worker matching behavior across occupations, skill accumulation and the ensuing polarization soften wage dispersion within firms.

Though the main exposition of the model is highly stylized, we have shown how it could be extended for applications to other environments. Any choices regarding the compositions of jobs that respect the monotone likelihood ratio property would produce similar insights to the simple model in the text. As many familiar distributions (e.g. exponential, or normal) satisfy the property, and have density functions with simple parameterizations, our approach can be used to model firm behavior over several occupations in more sophisticated contexts.

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Appendix

Derivation of Principal's Choice of JPE Wage Scheme: Proof of Proposition 1

Given that there is an optimal level of effort that each principal wishes to induce, the dual problem is to minimize reward payments, subject the agents' incentive compatibility conditions. The optimal wage scheme aligns the principal's unit cost function

$$2\Gamma p(e_i)p(e_j)[V^s - V^w] + \gamma_i p(e_i)V^w + \gamma_j p(e_j)V^w$$

with agents' willingness to supply effort in equation (1). Implicit differentiation of the unit cost function condition gives

$$\frac{dV^s}{dV^w} = \frac{-[\gamma_i p(e_i)(1-\gamma_j p(e_j)) - \gamma_j p(e_j)(1-\gamma_i p(e))]}{2\Gamma p(e_i)p(e_j)} \tag{A.1}$$

Calculating the slope of the first order condition for effort provided by agent i gives

$$\frac{dV^s}{dV^w} = \frac{-(1-\gamma_j p(e_j))U'(V^w)}{\gamma p(e_j)U'(V^s)} \tag{A.2}$$

Equating (A.1) and (A.2) yields

$$\frac{U'(V^w)}{U'(V^s)} = \frac{\gamma_i p(e_i)(1-\gamma_j p(e_j)) + \gamma_j p(e_j)(1-\gamma_i p(e_i))}{2\gamma_i p(e_i)(1-\gamma_j p(e_j))} > 1$$

Agents are risk averse, and so $U'(V^w) > U'(V^s)$ implies that $V^s > V^w$.

When principals recruit heterogeneous agents, only one of the agent's effort decision will bind the performance contract offered[20]. With decreasing returns the marginal return to effort is greatest for agents who put forth less effort, all else equal. Thus equation (A.2) must correspond to the lowest ability agent recruited by the principal. When agent i puts forth less effort that agent j, the choice by each principal to offer JPE incentive contracts after equating (A.1) and (A.2) is readily apparent.

Existence and Uniqueness of Full Equilibrium: Proof of Propositions 2 & 3

Proposition 1 and the properties of equations (1) through (3) ensure that efforts, performance payments to agents, matching and job assignments are uniquely determined given the compositions of occupations available. Thus the existence of an unique equilibrium hinges on unique values of the ability cutoffs between occupations: A_1 & A_2. The composition of jobs available in equilibrium is determined by the incentives of firms to offer each job mix, as constrained by the fact that employment must be balanced.[21]

Consider the Balance condition. Total differentiation reveals that it has slope equal to $\frac{dA_2}{dA_1} = -\frac{g(A_1)}{g(A_2)}$. An unique equilibrium exists if the Recruiting Indifference Condition (equation 6) insects the Balance condition (equation 7) exactly once. Thus it is sufficient to show that the Balance Condition is everywhere more steeply sloped, and lies on or above the Recruiting Indifference Condition condition for any potential value of A_1.

Implicit differentiation of the Recruiting Indifference Condition conditions yields

$$\frac{dA_2}{dA_1} = -\frac{g(A_1)}{g(A_2)}\frac{A_1}{A_2}\left(\frac{\Pi_{a\underline{\gamma_2}} - \Pi_{a\underline{\gamma_1}}}{\Pi_{a\overline{\gamma_2}} - \Pi_{a\overline{\gamma_1}}}\right) \tag{A.3}$$

Consider the right hand side of equation (A.3). First, $A_1 \leq A_2$ and secondly, profits increase faster in the skills of agents receiving greater direction from the principal, i.e. $\Pi_{a\overline{\gamma_2}} - \Pi_{a\underline{\gamma_2}} + \Pi_{a\overline{\gamma_1}} - \Pi_{a\underline{\gamma_1}} > 0$. Therefore the right hand side is strictly greater than $-\frac{g(A_1)}{g(A_2)}$. The Balance condition is everywhere more steeply sloped,

[20] From equation (3) there positive assortment across occupations such that all teams are heterogeneous in a non-degenerate distribution of ability.

[21] Existence and uniqueness of an equilibrium when the intersection of the Balance and Recruiting Indifference conditions has a corner solution is trivial. As described in Proposition 3, a corner solution can only exist where principals all select the same organization form with a job mix according to k=1.

intersecting the Recruiting Indifference Condition at most once. This establishes the criterion for uniqueness.

The Balance condition places a lower bound of a_{Med} on any equilibrium value of A_2. Existence then requires that the Recruiting Indifference Condition intersects the Balance condition for A_2 no less than the median ability. Given the relative slopes of the two equilibrium conditions, it is sufficient to show that the Balance condition lies on or below the Recruiting Indifference Condition at a_{Med}.

For each cutoff value A_2, the expected ability of a recruited foreman in job mix k=2 will be at least the cutoff value, since higher skilled agents are potential recruits. So as $A_2 \to a_{Med}$ it must be that $a^{\overline{\gamma_2}} > a_{Med}$. On the other hand, as $A_2 \to a_{Med}$ it must be that $a^{\overline{\gamma_1}} \to a_{Med}$. Note that profits increase faster in the ability of an agent with closer supervision (i.e. the foreman in job mix k=2), and the expected ability of agents recruited to be foreman exceeds the median level of ability. Therefore values of A_2 which satisfy the IC condition are no less than the median level: that is, for $A_2 = a_{Med}$ we have $\Pi(a^{\underline{\gamma_1}}, a^{\overline{\gamma_1}}, \underline{\gamma_1}, \overline{\gamma_1}) < \Pi(a^{\underline{\gamma_2}}, a^{\overline{\gamma_2}}, \underline{\gamma_2}, \overline{\gamma_2})$. Since the value $A_2 = a_{Med}$ satisfies the Balance condition, the criterion for existence is met.

The fact that for $A_2 = a_{Med}$ we have $\Pi(a^{\underline{\gamma_1}}, a^{\overline{\gamma_1}}, \underline{\gamma_1}, \overline{\gamma_1}) < \Pi(a^{\underline{\gamma_2}}, a^{\overline{\gamma_2}}, \underline{\gamma_2}, \overline{\gamma_2})$ also establishes the result in Proposition 3.

Existence and Uniqueness of Full Equilibrium-General Case: Proof of Proposition 6

Under the generalized definitions of job compositions in GJM.1 and GJM.2, many of the previous equilibrium conditions are unchanged: agent efforts still satisfy equation (1); principals offer a JPE wage scheme to manage the incentives of heterogeneous teammates as in equation (2); for any mix of jobs, agents' abilities are supermodular in payoffs, as in equation (3), so that matches between workers are positively assorted; agent abilities and the advising associated with occupations in GJM.1 and GJM.2 are supermodular in the agents' payoffs, so that agents are positively assigned to jobs as in (4).

It remains the case that principals must be indifferent between their choice of job mix and that firms must hire workers for each occupation so that employment is balanced in equilibrium. But the Recruiting Indifference Condition and Balance conditions must be modified to accommodate the greater array of jobs available.

With several occupations there is a series of Balance conditions that must be satisfied. Regardless of the mix of occupations that firms choose, they will hire one worker per job type. Using γ_1^2 as the reference occupation for each job mix without loss of generality, the Generalized Balance conditions requires that for all occupations $j > 1$, in job mix $k = 2$ the following holds

$$\int_{\Omega_{\gamma_1^2}} dG(a) = \int_{\Omega_{\gamma_j^2}} dG(a) \tag{GBC.1}$$

Employment in the alternate job mix must also be balanced and evenly divide the remaining mass of workers. The series of Balance conditions for job mix $k = 1$ across all occupations j is

$$1 - n_2 \int_{\Omega_{\gamma_1^2}} dG(a) = n_1 \int_{\Omega_{\gamma_j^1}} dG(a) \tag{GBC.2}$$

Given positive assignment over the number $n_1 + n_2$ of different occupations available there are $n_1 + n_2 - 1$ cutoff values that need to be determined to describe a full equilibrium (Walras' Law). The Generalized Balance conditions provide $n_1 + n_2 - 2$ implicit functions of the ability cutoff levels; the missing conditions arise because the mass of agents in one occupation cannot be related to itself, and the upper bound of $\Omega_{\gamma^2_{n_2}}$ is fixed exogenously at a_{max} (or symmetrically the lower bound of $\Omega_{\gamma^2_{n_2}}$ is fixed exogenously at a_{min}). The final equilibrium condition is Recruiting Indifference Condition regarding job mix offered by principals.

Define $a^{\gamma_j^k}$ as the expected ability of a worker employed in the j^{th} occupation (equal to the median ability level of agents assigned to γ_j^k) at firm choosing the composition of jobs given by k, according to GJM.1 and

GJM.2. Define the n_k vector of supervision allocated to each occupation when offering job mix k as

$$\vec{\gamma}^k = [\gamma_1^k, \gamma_2^k, ..., \gamma_{n_k}^j]$$

and the corresponding expected recruits as

$$\vec{a}^{\gamma_k} = [a^{\gamma_1^k}, a^{\gamma_2^k}, ..., a^{\gamma_{n_j}^k}] \ .$$

Then the Recruiting Indifference Condition is

$$\Pi\left(\vec{a}^{\gamma_1}, \vec{\gamma}^1\right) \equiv \Pi\left(\vec{a}^{\gamma_2}, \vec{\gamma}^2\right) \quad \text{(GIC)}$$

The Recruiting Indifference Condition provides an implicit function of the equilibrium cutoff values for each occupation via the definitions of $a^{\gamma_j^k}$.

Together the Recruiting Indifference Condition and Balance conditions provide $n_1 + n_2 - 1$ equations in $n_1 + n_2 - 1$ unknown ability cutoffs for different occupations. Therefore it only remains to show that the Recruiting Indifference Condition and Balance conditions determine unique ability cutoffs between occupations.

To verify existence of an equilibrium we find a correspondence, $\mathbf{F}(\cdot)$, that maps the $[a_{min}, a_{max}]^{n_1+n_2-1}$ space onto itself, such that any fixed point is an equilibrium. Then we demonstrate that the correspondence indeed contains a fixed point.

As the correspondence must be defined such that any fixed point is an equilibrium we write $\mathbf{F}(\cdot) = [F_{IC}, F_{BC_1^2}, F_{BC_2^2}, ..., F_{BC_{n_1}^1}]$ to accommodate each equilibrium requirement. We need to define three types of correspondences: one for indifference, one for balance within job mix k=2, and one for balance in job mix k=1. Let

$$\begin{aligned} F_{IC}\left(\vec{a}^{\gamma_1}, \vec{a}^{\gamma_2}\right) = \Big\{ & \left(\vec{y}^{\gamma_1}, \vec{y}^{\gamma_2}\right) \in [a_{min}, a_{max}]^{n_1+n_2-1} : \\ & \left|\Pi\left(\vec{y}^{\gamma_1}, \vec{\gamma_1}\right) - \Pi\left(\vec{y}^{\gamma_2}, \vec{\gamma_2}\right)\right| \le \left|\Pi\left(\vec{\dot{y}}^{\gamma_1}, \vec{\gamma_1}\right) - \Pi\left(\vec{\dot{y}}^{\gamma_2}, \vec{\gamma_2}\right)\right|, \\ & \forall (\vec{\dot{y}}^{\gamma_1}, \vec{\dot{y}}^{\gamma_2}) \in [a_{min}, a_{max}]^{n_1+n_2-1} \Big\} \end{aligned} \quad \text{(A.4)}$$

so that the correspondence maps vectors of abilities into the least non-negative value of the Recruiting Indifference Condition. Also, for occupations j define

$$\begin{aligned} F_{BC_j^2}\left(\vec{a}^{\gamma_1}, \vec{a}^{\gamma_2}\right) = \Big\{ & \left(\vec{b}^{\gamma_1}, \vec{b}^{\gamma_2}\right) \in [a_{min}, a_{max}]^{n_1+n_2-1} : \\ & \left| \int\limits_{\Omega\left(\vec{b}^{\gamma_1}, \vec{b}^{\gamma_2}\right)_{\gamma_1^2}} dG(a) - \int\limits_{\Omega\left(\vec{b}^{\gamma_1}, \vec{b}^{\gamma_2}\right)_{\gamma_j^2}} dG(a) \right| \le \left| \int\limits_{\Omega\left(\vec{\dot{b}}^{\gamma_1}, \vec{\dot{b}}^{\gamma_2}\right)_{\gamma_1^2}} dG(a) - \int\limits_{\Omega\left(\vec{\dot{b}}^{\gamma_1}, \vec{\dot{b}}^{\gamma_2}\right)_{\gamma_j^2}} dG(a) \right|, \\ & \forall (\vec{\dot{b}}^{\gamma_1}, \vec{\dot{b}}^{\gamma_2}) \in [a_{min}, a_{max}]^{n_1+n_2-1} \Big\} \end{aligned} \quad \text{(A.5)}$$

and

$$F_{BC_j^1}\left(\vec{a}^{\gamma_1}, \vec{a}^{\gamma_2}\right) = \Big\{ \left(\vec{b}^{\gamma_1}, \vec{b}^{\gamma_2}\right) \in [a_{min}, a_{max}]^{n_1+n_2-1} :$$

$$\left| \left(\frac{1-(2n_2-1)}{2n_1-1} \right) \int_{\Omega\left(\vec{b}^{\gamma_1}, \vec{b}^{\gamma_2}\right)_{\gamma_1^2}} dG(a) - \int_{\Omega\left(\vec{b}^{\gamma_1}, \vec{b}^{\gamma_2}\right)_{\gamma_j^2}} dG(a) \right| \leq$$

$$\left| \left(\frac{1-(2n_2-1)}{2n_1-1} \right) \int_{\Omega(\vec{\tilde{b}}^{\gamma_1}, \vec{\tilde{b}}^{\gamma_2})_{\gamma_1^2}} dG(a) - \int_{\Omega(\vec{\tilde{b}}^{\gamma_1}, \vec{\tilde{b}}^{\gamma_2})_{\gamma_j^2}} dG(a) \right|,$$

$$\forall (\vec{\tilde{b}}^{\gamma_1}, \vec{\tilde{b}}^{\gamma_2}) \in [a_{min}, a_{max}]^{n_1+n_2-1} \Big\} \quad \text{(A.6)}$$

so that these correspondences map vectors of abilities into the least non-negative values of the Balance conditions.

The are several things to note about these correspondences. First, they are everywhere hemicontinuous. Second, they are each defined over the convex, compact set of abilities $[a_{min}, a_{max}]$. Lastly, all together, $\mathbf{F}(\cdot)$ maps $[a_{min}, a_{max}]^{n_1+n_2-1}$ onto $[a_{min}, a_{max}]^{n_1+n_2-1}$. So by Kakutani's theorem, these correspondences have a fixed point. Each correspondence is defined to map vectors of ability into the least non-negative values of the equilibrium conditions. If each correspondence maps to a fixed point, then it must return cutoff vectors that satisfy equal expected profits across job mixes and equal mass of employment across occupations. Because such a fixed point must exist, so must an equilibrium. The last step is to establish uniqueness.

A sufficient condition for uniqueness is that the Jacobian matrix of the equilibrium conditions is sign-definite. (Notice the similarity with the proof of uniqueness for the simple case above. Sign-definiteness corresponds to the intersecting equilibrium conditions everywhere having slopes greater or less than one another, and hence crossing only once.) With positive assignment, the range of abilities employed in each occupation is convex so that we can describe the equilibrium using the upper-ability cutoffs for each occupation[22]; we define the upper-ability cutoff for occupation γ_j^k as A_j^k. Without loss of generality we write the column-elements of the Jacobian matrix as the derivative of each equilibrium condition, with respect to A_j^k for job mix $k = 2$ and then $k = 1$, in ascending order of j. Then we write the row-elements with the Recruiting Indifference Condition (GIC) stacked over the Balance conditions (GBC.2 then GBC.1).

Then the Jacobian matrix of the system of equilibrium conditions is

$$\begin{bmatrix}
\frac{d\Pi^2}{dA_1^2} & \frac{d\Pi^2}{dA_2^2} & \frac{d\Pi^2}{dA_3^2} & \cdots & \frac{d\Pi^1}{dA_1^1} & \frac{d\Pi^1}{dA_2^1} & \cdots & \frac{d\Pi^2}{dA_{n_1}^1} \\
G(A_1^2) & -G(A_2^2) & 0 & 0 & 0 & 0 & \cdots & 0 \\
G(A_1^2) & 0 & -G(A_3^2) & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & 0 & 0 & \cdots & 0 \\
G(A_1^2) & 0 & 0 & 0 & -\frac{n_1}{n_2}G(A_1^1) & 0 & \cdots & 0 \\
G(A_1^2) & 0 & 0 & 0 & 0 & -\frac{n_1}{n_2}G(A_2^2) & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\
G(A_1^2) & 0 & 0 & 0 & 0 & 0 & \cdots & -\frac{n_1}{n_2}G(A_{n_1}^2)
\end{bmatrix}$$

As we are only interested in the sign-definiteness of the matrix above, it is helpful is write it simply in

[22] Recall that the definition in (MLRP) does not make pairwise comparisons for all occupations so that each A_j^k may demark a cutoff between occupations in the same, or alternate job mixes. Such a definition is not be necessary to establish a unique equilibrium. We allow arbitrary orderings to maintain generality.

terms of the signs of each element.

$$\begin{bmatrix} + & + & + & + & + & + & \cdots & + \\ + & - & 0 & 0 & 0 & 0 & \cdots & 0 \\ + & 0 & - & 0 & 0 & 0 & \cdots & 0 \\ + & 0 & 0 & - & 0 & 0 & \cdots & 0 \\ + & 0 & 0 & 0 & - & 0 & \cdots & 0 \\ + & 0 & 0 & 0 & 0 & - & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ + & 0 & 0 & 0 & 0 & 0 & \cdots & - \end{bmatrix}$$

It is straightforward to verify that the determinants of the principal minors of this matrix alternate in sign, so that the Jacobian must be a negative-definite matrix. This verifies uniqueness of the generalized equilibrium.